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E-mail: mse@cscanada.org; caooc@hotmail.com

# Application of Rough Classification of Multi-objective Extension Group Decision-making under Uncertainty

ZHU Jia-jun<sup>1</sup> ZHENG Jian-guo<sup>2</sup> QIN Chao-yong<sup>3</sup>

**Abstract:** On account of the problem of incomplete information system in classification of extension group decision-making, this paper studies attribution reduction with decision-making function based on the group interaction and individual preferences assembly for achieving the goal of rough classification of multi-objective extension group decision-making under uncertainty. Then, this paper describes the idea and operating processes of multi-objective extension classification model in order to provide decision-makers with more practical, easy to operate and objective classification. Finally, an example concerning practical problem is given to demonstrate the classification process. Combining by extension association and rough reduction, this method not only takes the advantages of dynamic classification in extension decision-making, but also achieves the elimination of redundant attributes, conducive to the promotion on the accuracy and the reliability of the classification results in multi-objective extension group decision-making.

**Key words**: extension group decision-making; matter-element analysis; extension association; rough set; attribution reduction

#### 1. INTRODUCTION

Extenics is a new science, which studies the extension possibility and extension laws of things, and explores means for extension and innovation. The cognition in basic concept and theoretical frame is deepening step by step. As an important component of extenics, extension decision-making is a new

<sup>&</sup>lt;sup>1</sup> School of Business & Management, Donghua University. Department of Accountancy & Finance, Wuxi Institute of Commerce, Shanghai, P.R.China.

Email: zhujiajun@mail.dhu.edu.cn

<sup>&</sup>lt;sup>2</sup> School of Business & Management, Donghua University, Shanghai, P.R.China.

<sup>&</sup>lt;sup>3</sup> School of Business & Management, Donghua University, School of Mathematics & Information Science, Guangxi University, Nanning, P.R.China.

Email: qcy@mail.dhu.edu.cn

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sub-discipline which integrates scientific thinking, systems science and mathematics through the correlation function and the extension transformation to seek satisfaction in the decision-making space. Extension decision-making analyses the various sub-system compatibility with correlation function based on a mathematical tool of extension set, and through matter-element transformation to change contradictory issue into a compatibility issue in order to extend relevant decision-making strategy. Matter-element theory has a good adaptability and feasibility in description and analysis of natural language to achieve dynamic and systematic decision-making based on extension transformation, it makes artificial intelligence based on matter-element extension decision has a broader use of space.

Developing rapidly, extenics acquires quite a great progress in basic theory and application research. Based on the concept of n-dimensional matter element extension set (CAO, YANG. 2006), gives the concepts of multilayer multidimensional matter element system extension set and its positive field, negative field, zero boundary and its extension field as well as its stable field in order to study contradictory problems of multilayer multidimensional complex systems. By using knowledge presentation and reasoning technique in extension theory (CAO, PENG, 2006), established intelligent decision support system based on extension expert system (ZHANG, WANG, 2000). develops fuzzy gray matter-element space and fuzzy extension economic space which is combined with newly emerging fields such as fuzzy sets and fuzzy systems, extension sets, gray system and set pair analysis, and then some fuzzy extension mathematical models are suggested, several sets of fuzzy decision support systems based on the extension theory are presented applied to the large scale systems. Based on extension matter-element theory (SHENG, ZHAO, 2006), presents an automatic on-line measuring method of distributed production plan track using the multi-sensor and a new extension measurement method which can realize the right time to finish the production plan and to supply data guarantee for the production plan and control in core enterprise under supply chain. According to limitation of FGES-DSS (YANG, ZHANG, 2007), puts forward a new approach for decision-making that is called Set Pair Extension Space Decision Support System based on set pair analysis and extension theory, the model can characterize both the favoring evidence and the opposing evidence for every scheme. Based on extension theory and extension engineering methods (LIU, LIU, 2007), brings forward a new kind of machine-learning method that is called extension machine method which can pile up experience in the continually use and obtain the exact knowledge about decision, corrects its parameter and ameliorate the arithmetic of itself, thus improving its capability of self-learning (Wang, Tseng, 2009). presents a novel classified method that is called Extension Genetic Algorithm (EGA) which combines extension theory and genetic algorithm (GA), is extremely innovative, in order to eliminate try and error adjustment of modeling parameters and increase accuracy of the classification..

In addition, the extension method also applies to the land development and consolidation project management (ZHANG, WEI, 2007), decision-making of risk investment (BAI, 2008), comprehensive evaluation (XIE, LI, 2008; ZHAO, ZHU, 2008), intelligent control (CHAO, LEE, YEN, 2008; ZHANG, CHENG, 2007), data mining (CHEN, 2003), fault diagnosis (JIN, CHEN, 2006; YE, 2009), pattern recognition (HUNG, FENG, 2008), etc.

Based on matter-element extension theory and rough set theory, this paper makes a study of multi-objective classification optimization of extension group decision-making. Through studying the extension transformation under uncertainty, this paper analyzes advantages and disadvantages of extension classification, thus, the attribute reduction methods of rough set is introduced to improve the effect of extension classification under uncertainty. This improved extension classification model can help decision-makes to observe the effect of classification from the dynamic point of view, and to identify the main factors which impact program's classification changes under different decision-making preferences. As a result, systematic classification problems of multi-objective extension group decision-making under uncertainty can be solved.

## 2. EXTENSION CLASSIFICATION AND TRANSFORMATION OF EXTENSION GROUP DECISION-MAKING

Let  $R_i = \{R_1, R_2, \cdots, R_n\}$  means n schemes,  $c_j = \{c_1, c_2, \cdots, c_m\}$  means m decision-makers of  $R_i$ , the value of  $R_i$  is  $c_j(R_i) = (c_1(R_i), c_2(R_i), \cdots, c_m(R_i))$   $= (u_{i1}, u_{i2}, \cdots u_{im})$ ,  $(i = 1, 2, \cdots n, j = 1, 2, \cdots m)$ . Then the composite matter-element of multi-dimensional group decision-making is  $R_i = (N, c_j, u_{ij})$ .

Definition 1: let R =  $\{R\} = \{R \mid R_i = (N, c_j, u_{ij}) \in_{\mathbb{R}}, u_{ij} \in V \}$  is the composite element set of group decision-making,  $\widetilde{A} = \{(u_{ij}, y) \mid u_{ij} \in V, y = k(u_{ij})\}$  is the extension set, then a matter-element extension set of group decision making in R is as follows:

$$\widetilde{A}(R) = \{ (R, y) \mid R_i = (N, c_j, u_{ij}) \in \mathbb{R}, y = K(R), y' = T_k K(T_u(R)) \}$$
(1)

Among them,  $R_p = (M, c_j, [e_{pj}, b_{pj}])$  is the joint field of the matter-element extension set, M is the joint field  $v_{pj} = [e_{pj}, b_{pj}]$  which is composed of standard things and things which can be transformed into standard things, in other words, it is the range of evaluation value of joint field about decision-makers  $c_j$ ,  $R_l = (M_l, c_j, [e_{lj}, b_{lj}])$  is the classical field of the matter-element extension set,  $M_l$  is the standard object  $v_{lj} = [e_{lj}, b_{lj}]$  which means the range of evaluation value of standard object  $M_l$  about decision-makers  $c_j$ ,  $v_{lj} \in v_{pj}$ ,  $(l = 1, 2, \cdots g)$ .

The association degree between value and interval of assessment as follows (YANG, ZHANG, CAI, 2002):

$$k_{j}(u_{ij}) = \begin{cases} \frac{-\rho(u_{ij}, v_{lj})}{|v_{lj}|} & (u_{ij} \in v_{lj}) \\ \frac{\rho(u_{ij}, v_{lj})}{\rho(u_{ij}, v_{pj}) - \rho(u_{ij}, v_{lj})} & (u_{ij} \notin v_{lj}) \end{cases}$$
(2)

 $\rho(u_{ij},v_{lj})$  means the distance between  $u_{ij}$  and limited interval  $v_{lj}$  of classical field and  $\rho(u_{ij},v_{pj})$  means the distance between  $u_{ij}$  and limited interval  $v_{pj}$  of joint field. The formula of the distance between point u and limited interval v(e,b) is:

$$\rho(u,v) = |u + (e+b)/2| - (b-e)/2 \tag{3}$$

Thus, the integrated association degree based on weights  $\beta_j$  of decision-maker  $c_j$  is:

$$K_{i}(R_{i}) = \frac{1}{\bigvee_{1 \le j \le m} \left| k_{j}(u_{ij}) \right|} \sum_{j=1}^{m} \beta_{j} k_{j}(u_{ij})$$
(4)

Based on the extended association degree, the evaluation value of scheme  $R_i$  about decision-makers  $c_j$  can be judge whether it is belong to l-type.

$$L_{g}(c_{j}(R_{i})) = \begin{cases} L_{g} = g & y_{g-1} \leq 0 \pm y_{g} \geq 0 \\ L_{g} \neq g & y_{g-1} < 0 \pm y_{g} < 0 \end{cases}$$
(5)

Through to summary the judgment results of decision-makers  $c_j$  about scheme  $R_i$  , then:

$$L(c_{j}(R_{i})) = \begin{cases} L_{g}(c_{j}(R_{i})) & y_{g} \geq 0 \\ \emptyset & y_{i} < 0 \end{cases}$$
(6)

If  $y_g^{'} \geq 0$ ,  $L(c_j(R_i))$  means that the evaluation results of the decision-makers  $c_j^{'}$  on the scheme  $R_i^{'}$  is belong to l-type. Otherwise, if  $y_l^{'} < 0$ ,  $L(c_j(R_i))$  means that the evaluation results of  $R_i^{'}$  is not belong to l-type.

Furthermore, based on the integrated association degree, the evaluation value of scheme  $R_i$  can be judge whether it is belong to l-type.

$$d_{g}(R_{i}) = \begin{cases} d_{g} = g & y_{g-1}(K_{i}(R_{i})) \leq 0 \coprod y_{g}^{'}(K_{i}(R_{i})) > 0 \\ d_{g} \neq g & y_{g-1}(K_{i}(R_{i})) \leq 0 \coprod y_{g}^{'}(K_{i}(R_{i})) \leq 0 \end{cases}$$
(7)

Through to summary the judgment results about scheme  $R_i$ , then:

$$d(R_i) = \begin{cases} d_g(R_i) & y_g(K_i(R_i)) \ge 0 \\ \emptyset & y_i < 0 \end{cases}$$
(8)

If  $y_g'(K_i(R_i)) \ge 0$ ,  $d(R_i)$  means that the evaluation results of the scheme  $R_i$  is belong to l-type. But, if  $y_l' < 0$ ,  $d(R_i)$  means that the evaluation results of the scheme  $R_i$  is not belongs to l-type.

#### 3. MULTI-OBJECTIVE CONVERSION AND STANDARDIZATION OF EXTENSION GROUP DECISION-MAKING BASED ON DECISION-MAKING PREFERENCES

Definition 2 (Cai, 1999): let matter-element  $R_1=(N_1,c_1,v_1)$  and  $R_2=(N_2,c_2,v_2)$ . "And" refers to both get  $R_1$  and  $R_2$ , call  $R=R_1\wedge R_2$ . "Or" means taking either  $R_1$  or  $R_2$ , call  $R=R_1\vee R_2$ . All

appearance:

$$R_1 \wedge R_2 = R_2 \wedge R_1 \tag{9}$$

$$R_1 \vee R_2 = R_2 \vee R_1 \tag{10}$$

Definition 3: let matter-element  $R=(N,c,V_0)$  . If  $R_1=(N,c,u)$  ,  $u\not\in V_0$  , call  $R_1$  is a non-matter-element of R ,  $\overline{R}=R_1$  ; if  $V_0=\{v_0\}$  , that  $\overline{R}=(N,c,u)$  ,  $u\not=v_0$  ,  $\neg R$  means "Not" operation which change matter-element R to  $\overline{R}$  .

Inference 1: the rules of logic operation under the matter-element with same matter:

$$R_1 \vee R_2 = (N, c, v_1) \vee (N, c, v_2) = (N, c, v_1 \vee v_2)$$
 (11)

$$R_1 \wedge R_2 = (N, c, v_1) \wedge (N, c, v_2) = (N, c, v_1 \wedge v_2)$$
 (12)

Inference 2: the rules of logic operation under the matter-element with same features:

$$R_1 \vee R_2 = (N_1, c, v_1) \vee (N_2, c, v_2) = (N_1 \vee N_2, c, v)$$
 (13)

$$R_1 \wedge R_2 = (N_1, c, v_1) \wedge (N_2, c, v_2) = (N_1 \wedge N_2, c, v)$$
 (14)

Matter-element combines the thing, its characteristics and feature values into one set. For a multiple dimension matter-element can describe multiple aspects of a thing, it is possible to build a modal which can describe systematic decision-making problems of multi-objective conversion and multi-index evaluation in group decision-making by matter-element.

Let  $O=O_1\times O_2\times \cdots O_s$ ,  $R_t\in O_t$ ,  $(t=1,2,\cdots s)$  and  $R_i^t=(R_1,R_2,\cdots ,R_n)$ ,  $(i=1,2,\cdots n)$ ,  $c_j^t=(c_1,c_2,\cdots ,c_m)$ ,  $(j=1,2,\cdots m)$  means m decision-makers of  $R_i^t$ ,  $R_i^t$  of field t is  $c_j^t(R_i^t)=(c_1^t(R_i),c_2^t(R_i),\cdots ,c_m^t(R_i))=(v_{i1}^t,v_{i2}^t,\cdots v_{im}^t)$ , then the composite matter-element of multi-objective and multi-dimensional group decision-making is  $R_i=(O_t(N,c_j,v_{li}))$ .

Due to differences goals would affect the outcome of the decision-making, through the composite matter-element should to be standardization in order to meet the needs of data processing under the multi-objective matter-element with same matter or same features. According to Definition 3, let multi-objective group decision-making matter-element  $R = (O_t(N,c,V_0))$ . The smaller the better for the composite matter-element is  $R_i = (O_1(N,c,u))$ ,  $u \in V_0$ ,  $\neg R_i$  means that  $R_i$  is able to change to the bigger the better for the composite matter-element under target  $O_1, u_0 \notin u$ ,  $u_0 \in V_0$ .

$$\neg R_i = (O_1(N, c, v_0 - u)) = (O_1(N, c, u_0))$$
 (15)

The same principle, as well as the object that is changed from the bigger the better to the smaller the better for the composite matter-element.

According to Definition 2 and Inference 1, based on target conformity under decision-making preference  $\alpha$ , a correlation matrix  $k^{\alpha}(c_j^{\alpha}(R))$  of the target  $O_i$  is established with  $c_j^{\alpha}(R_i)$  of  $R_i$  about  $c_j$ .

$$c_j^{\alpha}(R_i) = u_{ij} = (\alpha \times \bigvee_{i=1}^{n} c_j(R_i) + (1 - \alpha) \times \bigwedge_{i=1}^{n} c_j(R_i))$$
 (16)

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If  $\alpha=0$ , then  $u_{ij}=\bigwedge_{i=1}^n c_j(R_i)$ , which is pessimistic decision-making method; if  $\alpha=1$ , then  $u_{ij} = \bigvee_{i=1}^{n} c_{j}(R_{i})$  , which is optimistic decision-making method;  $\alpha = 0.5$  $u_{ij} = 0.5 \times (\bigvee_{i=1}^{n} c_{j}(R_{i}) + \bigwedge_{i=1}^{n} c_{j}(R_{i}))$ , which is compromise decision-making method. Then

$$k^{\alpha}(c_{j}^{\alpha}(R)) = (k_{j}^{\alpha}(u_{ij}))$$

$$= \begin{bmatrix} k_{1}^{\alpha}(u_{11}) & k_{2}^{\alpha}(u_{12}) & \cdots & k_{m}^{\alpha}(u_{1m}) \\ k_{1}^{\alpha}(u_{21}) & k_{2}^{\alpha}(u_{22}) & \cdots & k_{m}^{\alpha}(u_{2m}) \\ \vdots & \vdots & & \vdots \\ k_{1}^{\alpha}(u_{n1}) & k_{2}^{\alpha}(u_{n2}) & \cdots & k_{m}^{\alpha}(u_{nm}) \end{bmatrix}$$

$$(17)$$

The comprehensive association degree is:

$$K_{i}(R_{i}) = \frac{1}{\bigvee_{1 \le j \le m} \left| k_{j}^{\alpha}(u_{ij}) \right|} \sum_{j=1}^{m} \beta_{j} k_{j}^{\alpha}(u_{ij})$$
 (18)

which of  $R_i$  about  $c_i$ ,  $\beta_i$  is the weight factor of  $c_i$ .

Thus, the comprehensive association degree of R about decision makers  $c_i$  and weight factors  $\gamma_i$ is:

$$K(R) = \sum_{i=1}^{n} \gamma_i K_i(R_i)$$
(19)

To make matter-element extension set  $\widetilde{X}$  and to give transform  $T_i = (T_{w_i}, T_{K_i}, T_{R_i})$  under field  $V(c_{ii})$ , call:

$$\begin{split} \widetilde{A}(R_{i})(T_{i}) &= \left\{ \left. (R_{i}, Y_{i}, Y_{i}^{'}) \middle| R_{i} \in T_{w_{i}} W_{R_{i}}, \right. \\ Y_{i} &= K(R) = \sum_{i=1}^{n} \gamma_{i} K_{i}(R_{i}) \in (-\infty, +\infty) \\ Y_{i}^{'} &= T_{K} K(T_{R_{i}} R) = \sum_{i=1}^{n} \gamma_{i} T_{K_{i}} K_{i}(T_{R_{i}} R_{i}) \in (-\infty, +\infty) \right. \right\} (20) \end{split}$$

Based on changing classical field and preferences of extension group decision-making, we are able to observe the changes of optimal scheme from the dynamic point of view and compare optimal scheme with other schemes under different conditions in order to obtain a optimal classification under no preference. However, this classification remains in a simple classification can not analyze the decision-makers on the impact of the decision-making options and can not reflect the correlation between schemes. In addition, sometimes the judgment result of some policy makers or schemes could be belong to more categories based on extension transformation, thus the formation of incomplete decision-making situations, it also adds uncertainty to the scheme classification of decision-making.

## 4. EXTENSION GROUP DECISION-MAKING ATTRIBUTE REDUCTION AND CLASSIFICATION UNDER UNCERTAINTY

Rough set theory is a mathematical tool to deal with ambiguous and uncertainties data (PAWLAK, 1982), which has been used in various fields such as machine learning, pattern recognition, knowledge discovery, etc. Attribute reduction is a core part of rough set theory which is used to eliminate redundant attributes in the decision-making table.

Definition 4 (JELONEK, 1995): let (U, A, F) is a decision-making information systems,  $B \subseteq A$ , if  $\mathfrak{R}_B = \mathfrak{R}_A$ , call B is partition consistent set, if any real subset of B is not partition consistent set, then B is partition reduction set.

If  $\mathfrak{R}_B = \mathfrak{R}_A$ , then  $U/\mathfrak{R}_B = U/\mathfrak{R}_A$ , the results of U which are classified by attribute A and B are identical. Thus, the object set described by A also can be described by partition consistent set B and partition consistent set B.

Definition 5: let (U, A, F) is a decision-making information systems,

$$U/\mathfrak{R}_A = \left\{ \left[ x_i \right]_A \middle| x_i \in U \right\} \,,$$

$$D([x_i]_A, [x_j]_A) = \left\{ a_l \in A \middle| f_l(x_i) \neq f_l(x_j) \right\}$$
 (21)

call  $D([x_i]_A, [x_j]_A)$  is partition discrimination set of  $[x_i]_A$  and  $[x_j]_A$ , then D is the partition discrimination matrix of decision-making information systems.

$$D = (D([x_i]_A, [x_j]_A)[x_i]_A, [x_j]_A \in U/\Re_A)$$
 (22)

Theorem 1, let (U, A, F) is a decision-making information systems, for any  $x_i, x_j, x_k \in U$ , partition discrimination set has the following properties:

- (1)  $D([x_i]_A, [x_i]_A) = \emptyset$ ;
- (2)  $D([x_i]_A, [x_j]_A) = D([x_j]_A, [x_x]_A)$
- (3)  $D([x_i]_A, [x_i]_A) \subseteq D([x_i]_A, [x_k]_A) \cup$

$$D([x_k]_A,[x_j]_A)$$

Based on the attribute reduction method of rough set, incomplete decision-making system which is produced after extension transformation can be further classified, it is to added extension and improvement of the classification.

Definition 6, if the extension of group decision-making  $R_i = (N, c_j, u_{ij})$  changes to  $\tau = (U, A, F, d)$  after extension transformation, any  $L(c_j(R_i))$  and  $d(R_i)$  are the only established, then known as the perfect extension group decision-making information system, otherwise known as the incomplete information system.

Let  $\tau = (U, A, F, d)$  is a incomplete extension group decision-making information system,  $B \subseteq A$ , d is decision attribute, then recorded as:

$$U/\mathfrak{R}_{\{d\}} = \{D_1, D_2, \cdots, D_r\}$$

$$m_B(x) = \max \{ D(D_i / S_B(x)) | j \le r \} (x \in U)$$
 (23)

which  $D(E/F) = \frac{\left|E \cap F\right|}{\left|F\right|}$  is the inclusion degree on P(U) .

$$S_B(x) = \left\{ y \in U | (x, y) \in SIM(B) \right\} \tag{24}$$

which is means similar type of x,  $(a_1 \in B)$ 

$$SIM(B) = \{(x, y) \in U \times U | a_1(x) \cap a_1(y) \neq \emptyset \}$$
 (25)

which expressed the similar relationship on B,

$$\gamma_B(x) = \left\{ D_{ji} \middle| D(D_{ji} \middle/ S_B(x)) = m_B(x) \right\}$$
(26)

is the decision-making function  $(x \in U)$ . if any  $x \in U$  there is  $\gamma_B(x) = \gamma_A(x)$  set up, then B is the largest distribution consistent set of  $\tau = (U, A, F, d)$ . If B is the largest distribution consistent set, and any really subset of B is not the largest distribution consistent set of  $\tau = (U, A, F, d)$ , then B is the largest distribution reduction set of  $\tau = (U, A, F, d)$ .

Let  $S_{\delta}$  is one of all options in a incomplete extension group decision-making information system  $\tau = (U,A,F,d)$ ,  $B_{\delta}$  is the largest distribution reduction set of  $S_{\delta}$ ,  $B_h = \bigcap B_{\delta} \neq \emptyset$  is the smallest set of the largest distribution reduction set of all options,  $\delta = (1,2,\cdots,q), h = (1,2,\cdots,t)$ . If  $C = \bigcap B_h \neq \emptyset$ , then C is the partition set of core decision-makers  $c_j$ ,  $J = \bigcup B_h - C$  is the partition set of relative necessary decision-makers  $c_j$ ;  $Q = A - \bigcup B_h$  is the partition set of unnecessary decision-makers  $c_j$ .

According to the smallest principle which expresses the sum of deviation absolute value between evaluation values  $L(c_j(R_i))$  of schemes  $R_i$  and comprehensive evaluation value  $d(R_i)$  of the schemes, to determine the the root attribute of classification.

$$f(c_j) = \bigwedge_{j=1}^{m} \left( \sum_{i=1}^{n} \left| L(c_j(R_i)) - d(R_i) \right| \right) (c_j \in Q) \quad (27)$$

If  $B_h > 1$ , through the smallest principle which expresses the sum of deviation absolute value between  $B_h$  and comprehensive evaluation value  $d(R_i)$  of the schemes, to determine the sub-attributes of classification.

$$f(B_{w}) = \wedge (f(c_{i}) + f(c_{i+1}))$$
 (28)

Among them,  $B_{\psi} \in B_h$ ,  $c_j \cup c_{j+1} = B_{\psi}$ . First of all, using the root attribute  $c_j$  to create the beginning nodes of classification and to create branches based on each value of the root attribute.

Secondly, let the the minimum set  $B_{\psi}$  of the largest distribution reduction set of all options as a sub-set attributes to leads branches, in order to achieve the division of the sample.

#### 5. THE FRAMEWORK AND THE STEPS OF MULTI-OBJECTIVE EXTENSION ROUGH CLASSIFICATION OF GROUP DECISION-MAKING

#### 5.1 The framework and ideas of model

Based on combining extension group decision-making with classification of rough set method, attribute reduction is introduced to improve the extension classification, so as to enhance the classification results of extension group decision-making categories under uncertainty.

The core of the model is that through the rough reduction to solve the uncertainties of extension classification, and to realize multi-objective extension classification under decision-making preferences, so as to enhance the applicability and reliability of the extension classification. There are two major parts of extension rough classification model of group decision-making: Firstly, through the correlation function to achieve extension transformation, in order to achieve dynamic classification of the decision-making schemes; Secondly, through the attribute reduction and decision-making function to improve extension classification under uncertainty, and to analyze the impact of decision-making preference, decision-making relevance upon multi-objective classification results.

#### 5.2 The steps and content of model

Step1: To establish a multi-objective extension group decision-making information system which includes expert set, scheme set and target set in order to obtain the multi-objective extension matter-element set  $R_i = (O_t(N, c_i, v_{ii}))$ ;

Step2: To set the weights  $\beta_i$  of decision-makers  $c_i$  and decision-making preference  $\alpha$ ;

Step3: To input data, when the target is a negative index in decision-making, data of this target must be transformed with (15);

Step4: To achieve the goal of multi-objective conversion and standardization in order to gain a comprehensive matrix of multi-objective extension matter-element set under decision-making preference  $\alpha$ ;

Step5: To determine joint field  $v_{pj} = [e_{pj}, b_{pj}]$  and classical field  $v_{lj} = [e_{lj}, b_{lj}]$ , and to set grade-level l of extension classification;

Step6: Based on correlation function (18), to achieve extension conversion in order to carry out the initial classification of schemes, and to calculate the evaluation value  $L(c_j(R_i))$  and comprehensive evaluation value  $d(R_i)$  of schemes based on (2) to (8) for constituting a extension group decision-making system.;

Step7: Based on extension classification, to use of attribute reduction for re-classification under decision-making preference  $\alpha$ ;

Step8: To compare initial classification result with re-classification of classification result, if the classification results can meet the needs of classification goals, go to the last step; otherwise go to the

next step.

Step9: If the model need to update data, then go to step 3 to continue classification after re-enter the data, otherwise go to step 5 to continue classification after re-set the classical domain;

Step10: To output classification results, the classification ends here.

#### 6. A CASE STUDY

The upcoming 2010 Shanghai World Expo and the 2010 Guangzhou Asian Games have brought tremendous business opportunities to many domestic enterprises. A large toy and gift manufacturers in Wuxi hope to upgrade the production plans so as to expand the production capacity and product scale. On the basis of sales forecasts, pre-market research and verification of the expert group, this company studied out a specific combination production plan. A multi-objective matter-element extension group decision-making information model  $R_i = (O_t(N, c_j, v_{lj}))$  is established so that classify and evaluate the new plans of creative projects.(Table 1)

Among them, experts set is  $c = \{c_1, c_2, c_3, c_4, c_5\}$ , schemes set is  $R = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8\}$ , targets set is  $O = \{O_1, O_2, O_3\}$ ,  $O_1$  said that "income" (positive index),  $O_2$  said that "cost"(negative index) and  $O_3$  said that "production efficiency"(positive index). Set the weights of the experts  $\beta_j = (0.2, 0.2, 0.2, 0.2, 0.2, 0.2)$ , and set decision-making preference  $\alpha = 1$ ,  $\alpha = 0$  and  $\alpha = 0.5$ .

Furthermore, grade-level of extension classification and the initial level variable l should be determined.  $l = \{l_1, l_2, l_3, l_4\}$ ,  $l_1$  said that "Eligible",  $l_2$  said that "Middling",  $l_3$  said that "Good" and  $l_4$  said that "Excellent". And to determine joint field  $v_{pj} = [R_i \quad [6,10]]$  and the initial classical field  $v_{lj} = [R_i \quad l_1 = [6,7], l_2 = [7,8], l_3 = [8,9], l_4 = [9,10]]$ .

Because  $O_2$  is a negative index, therefore, it should be translated into positive index with (15). Then, according to steps of model, we can establish a multi-objective composite matter-element matrix under different preference in order to achieve rough classification of schemes.

If  $\alpha = 1$  which means optimistic decision-making method, we can obtain the following Table 2.

According to Definition 6,  $R_i = (O_t(N, c_j, v_{lj}))$  is an incomplete information system, so the smallest set of the largest distribution reduction set of all options is  $B_h = \{c_2c_5\}$ , and we can obtain the root attribute of classification is  $c_1$  based on formula (27) and (28). If  $L(c_4(R_3)) = 3$ , then we can obtain rough classification as follow Figure 2; if  $L(c_4(R_3)) = 4$ , then we can gain rough classification as follow Figure 3.

The same way, if  $\alpha = 0$ , we obtain the table of extension evaluation value of group decision-making as Table 3.

Then, we can gain rough classification as follow Figure 4.

If  $\alpha = 0.5$ , we obtain the table of extension evaluation value of group decision-making as table 4:

If  $L(c_2(R_4)) = 3$ , we can obtain rough classification as follow Figure 5; if  $L(c_2(R_4)) = 4$ , then we can gain Figure 6.

From different preferences point of view we can see: if  $\alpha=1$ , the uncertainty of decision-making data does not affect the classification results of schemes,  $R_1,R_2,R_3,R_4$  belong to "Excellent"; if  $\alpha=0$ , the uncertainty of decision-making data has affected the classification results of  $R_1$  which is belongs to not only "Middling" but also "Good"; if  $\alpha=0.5$ , only  $R_8$  is belongs to "Middling". Therefore, based on multi-objective rough extension classification, the best scheme is  $R_4$ , the worst scheme is  $R_8$ , the smallest affected by the preferences is  $R_5$ .

#### 7. CONCLUSION

Through changing the classical field 'extension group decision-making achieves extension transformation, thereby extension group decision-making information systems is established in order to achieves the goal of dynamic classification and analysis for data and programs. However, incomplete information decision-making systems often generates after extension change, which has brought uncertainty to classification. Therefore, this paper combines extension group decision-making with rough set classification method to achieve dynamic classification through extension transformation, and on this basis uses the method of attribute reduction to achieve re-classification, thus improving the rationality and the practicability of classification.

Multi-objective rough extension classification is an important aspect of data analysis, knowledge extraction of extension group decision-making, which can achieve the goal of multi-project classification, multi-objective assessment based on the group interaction and individual preferences assembly. It can be applied to investment planning, project management, risk control etc. under uncertainty.

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#### **TABLES AND FIGURES**

Table 1: Composite matter-element matrix of extension classification

Scheme	$O_1(R)$				$O_{\gamma}(R)$				$O_3(R)$						
	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_1$	$c_2$	$c_3$	$c_4$	$C_5$	$c_1$	$c_2$	$c_3$	$c_4$	$C_5$
$R_1$	8.4	8.2	7.9	8.1	9.2	1.7	1.6	1.5	2.3	1.9	9.6	8.5	9.1	8.2	8.7
$R_2$	9.3	9.8	8.5	9.1	8.7	2.5	2.1	1.8	2.1	3.4	8.3	7.7	8.8	9.2	8.9
$R_3$	9.1	7.1	9.4	8.4	9.1	3.3	2.2	1.3	1.0	2.2	9.1	8.6	9.4	7.6	8.8
$R_{4}$	9.2	8.9	8.7	8.8	9.4	1.3	0.9	1.2	1.5	0.9	8.8	9.1	8.9	8.7	9.2
$R_5$	8.3	8.7	7.9	8.6	8.5	1.9	1.8	0.9	1.1	1.6	9.1	8.3	8.4	8.3	8.2
$R_6$	7.5	8.7	9.1	8.3	7.8	2.2	2.1	1.2	2.2	1.7	9.2	7.9	8.2	8.3	7.8
$R_7$	8.1	7.5	8.3	9.3	8.5	1.6	2.4	1.8	1.3	1.9	8.9	8.9	7.8	8.1	8.5
$R_{_{8}}$	8.3	8.4	8.2	7.2	8.1	2.9	2.6	1.9	1.8	3.1	8.0	8.3	8.2	8.3	8.0

Table 2: The table of extension evaluation value of group decision-making under  $\alpha=1$ 

Scheme	$L(c_i)$	<i>d(D)</i>					
Scheme	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$d(R_i)$	
$R_{\scriptscriptstyle 1}$	4	3	4	3	4	4	
$R_2$	4	4	3	4	3	4	
$R_3$	4	3	4	3,4	4	4	
$R_{\scriptscriptstyle 4}$	4	4	3	3	4	4	
$R_{5}$	4	3	4	3	3	3	
$R_6$	4	3	4	3	3	3	
$R_7$	3	3	3	4	3	3	
$R_{_{8}}$	2	3	3	3	3	3	

Table 3: The table of extension evaluation value of group decision-making under  $\alpha=0$ 

C -1	$L(c_i)$	J(D)					
Scheme	$c_1$	$c_2$	$c_3$	$c_4$	$C_5$	$d(R_i)$	
$R_1$	3	3	2	2	3	2,3	
$R_2$	2	2	3	2	1	2	
$R_3$	1	2	3	2	2	2	
$R_{\scriptscriptstyle 4}$	3	3	3	3	4	3	
$R_{5}$	3	3	2	3	3	3	
$R_6$	2	2	3	2	2	2	
$R_7$	3	2	2	3	3	2	
$R_8$	2	2	3	2	1	2	

Table 4: The table of extension evaluation value of group decision-making under  $\alpha = 0.5$ 

Scheme	$L(c_i)$	$d(\mathbf{p})$				
Scheme	$c_1$	$c_2$	$C_3$	$c_{\scriptscriptstyle 4}$	$c_5$	$d(R_i)$
$R_1$	3	3	3	2	3	3
$R_{2}$	3	3	3	3	2	3
$R_3$	2	2	4	3	3	3
$R_{\scriptscriptstyle 4}$	3	3,4	3	3	4	3
$R_{5}$	3	3	3	3	3	3
$R_6$	3	3	3	3	3	3
$R_7$	3	3	3	3	2	3
$R_{\rm s}$	2	2	3	2	2	2

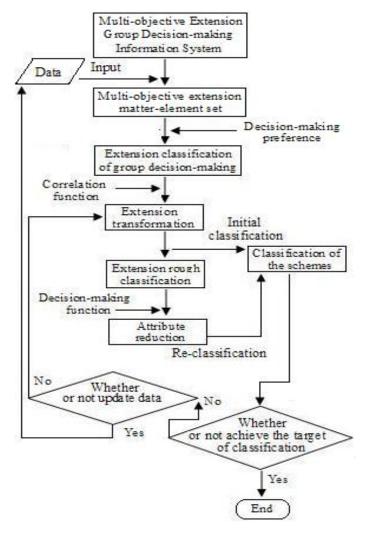


Figure 1: Operation process of the rough classification model of multi-objective extension group decision-making

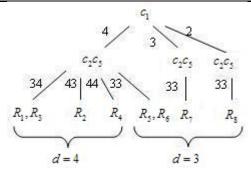


Figure 2: Extension rough classification under  $\alpha = 1$  ( I )

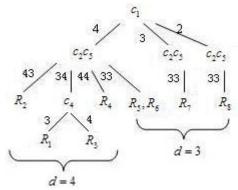


Figure 3: Extension rough classification under  $\alpha = 1$  ( II )

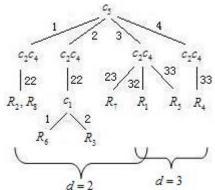


Figure 4: Extension rough classification under  $\alpha = 0$ 

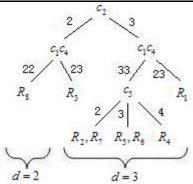


Figure 5: Extension rough classification under  $\alpha = 0.5$  ( I )

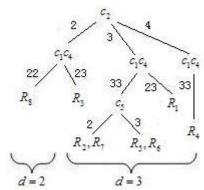


Figure 6: Extension rough classification under  $\alpha = 0.5$  (II)