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A GAME MODEL FOR PRICE AND OUTPUT CHANGING AT THE SAME TIME:

AN EXAMPLE FROM FIXED COMMUNICATION MARKETING IN CHINA[1]

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Abstract: This paper present 4 game models which can be used to analysis several important stages of fixed network of communication industry in China. Analysis method in this paper can be reference to complete information dynamic game when price and output changes in the same time.
Key words: Game theory, Communication market, Game model

1. INTRODUCTION

Development of communication industry in China was several adjusted by government severally which have great impact on the relation of competition and cooperation for several oligopolies in Chinese communication marketing. This paper select adjusting policies as dividing line to analysis marketing structure and game results of oligopolies in Chinese communication marketing. Analysis method in this paper can be reference to complete information dynamic game when price and output changes in the same time.

China Telecom, as first professional service supplier in fixed network of communication, appearance in communication marketing. Then communication service of China Mobile and China Unicom which appear in marketing one after the other have substitute effect for fixed network of China Telecom. China Unicom, though as all- operation supplier, has been supplied little communication service on fixed network, and has little impact on fixed network communication marketing.

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2. LITERATURE REVIEW

Cournot competition model (Cournot, 1838) and Stackelberg competition model (Von Stackelberg, 1934) can be a good description for oligopoly market competition. One of the focuses of research on oligopolistic competition is to discuss the circumstances under which oligarchs to obtain the advantages of action. Sherali, etc. (1983) structured and researched Stackelberg-Nash-Cournot equilibrium of one leading oligarch and N after mover oligopolies and testify the existence and uniqueness of solution. Later, Sherali (1984) expanded Stackelberg duopoly model, constructed and studied competitive model of multi first move oligarchs multi follow oligarchs, pointed out that classic Stackelberg model and Cournot model are special cases and that profits of first move oligopoly is higher than follow oligopolies in order to yield competitive output, draw the conclusion that new entering oligopolies will inevitably reduce the profits of the existing oligarchies. Gal-Or (1985) pointed out the difference of first mover and second mover advantages under various reaction function. Dowrick (1986) proposes by duopoly model that oligarchs will be the first mover for the corresponding advantages or for second mover advantage as long as the oligarchs' response to competition in the market by different functions. Daughety (1990) analyzed that Stackelberg equilibrium game solution of m first mover oligarchies and n-m follow oligopolies under normal circumstances. Haan, Marks (1996) researched duopoly of Stackelberg model and the Cournot model from the welfare point of view and pointed out that Stackelberg competition does not necessarily improve the welfare when the market entry barriers exist. Wolf, Smeers (1997) constructed stochastic Stackelberg-Nash-Cournot model, discussed on equilibrium solution and corresponding characterizes and verified by European gas market. But Vander Werf etc. (1997) investigated experimental methodology of first move advantage by comparative analysis and think that different experimental methods have significant impact on the results. Muceller (1997) investigated path dependency of first move advantage for leading oligarchs in special industrial period. Oligopoly production and social welfare aspects is on attention of oligopolistic competition of game. Matsumura (1999) research role of stock in a multi-period game and conditions of become a Stackelberg leader by analysis on limited stage Cournot duopoly game model. Okuguchi (1999) pointed out that different reaction function cause not only different game equilibrium but also led to various Stackelberg model and the Cournot model output and profits through analysis of the duopoly model. Rassenti, etc. (2000) studied on the convergence of equilibrium solution for repeated Cournot competition game. Huck, etc. (2001) study learning model in Cournot competition game, researched duopoly Stackelberg model and the Cournot model by the way of experimental economics and compared the total output and efficiency of different models. Then, Huck, etc. (2002) investigated on influence of External conditions on game results from the perspective of experimental economics in duopoly Stackelberg model. Game model mentioned above is mainly focus on duopoly classic competition.

From the current literature, as restrictions of demand function of the, linear function of demand cannot be used to construction game model when prices and production changes at the same time. However, in the communications market, the case is that the price and output (here is number of users) changes at the same time, thus demand cannot be used to direct analysis China's oligarchs telecommunications market. And more, there are no literature on multi oligarchies Cournot model and multi oligarchies Stackelberg model to analysis move advantage, output and profit for follow oligarchies. This paper present further study on multi oligarchies game and price and output changing at the same time game which can be used to explain China's oligarchs telecommunications market.

3. IMPORTANT HYPOTHESES

3.1 Suppose products and service supplied by China Telecom and China Netcom has no difference.

Profit of oligopolies is decides directly by price and output, other factors have no effect on profit of communication service suppliers. Output of communication marketing is composed by communication time and number of consumer.

3.2 For analysis convenience, we assume communication time of consumer is constant. Because output of service suppliers is equal to talk span multiple with number of consumers, then output of communication is decided only by number of consumers.

3.3 Given scale of China Netcom is equal to China Telecom when separated from China Telecom in 2004, north and south marketing is absolute symmetry. This hypothesis makes analysis easy to complete.

4. DEMAND FUNCTION

Analysis is based on demand and supply function. For studying communication marketing of fixed network clearly, we suppose that fix cost of each oligopoly is no difference. And more, number of consumers developed by each service supplier is proportional to service price of each oligopoly. The relation can be express as follows:

$$p = a - bq \tag{1}$$

The cost of oligopolies mainly composed by management cost of consumers which is proportional to number of consumers. So cost of each oligopoly can be expressed as follows:

$$C(q) = qc \tag{2}$$

5. MODEL

Cournot model and Stackelberg model is on output game based on hypothesis of the same quality and price of service and goods, Bertrand model is on price game [1], Hottelling model explain price game of goods and service which have some substitute effect [2]. Now because restricted by properties of demand function, linear demand and supply function cannot be used to analysis game model on price and output change at the same time. But the instance of price and output (here is number of consumers) changes at the same time appears always in many marketing.

Service price of each oligopoly is mainly1 decided by government and has been adjusted to adapt to development demand of communication industry. Although prices always changes, they keep value sequence in marketing by meeting requires of government.

Service price of oligopolies can be expressed as:

$$p_i = t_i p_T \tag{3}$$

p_T is the price of China Telecom, i is a variable representing China Telecom, China Mobile, China Unicom and China Netcom. For example, p_M is service price of China Mobile, p_U is service price of China Unicom and p_N is service price of China Netcom.

It is important to point, t_i is a function which changes by time. Time is one of variables of this function which comprise other variables. Here for wider application we use abstract expression. In each game model, price is different by real situation of marketing.

5.1 The first game stage ---China Telecom setup, Communication was separated from post

China Telecom shares total communication marketing when China Telecom was separated from China Post. Then price function of China Telecom is:

$$\pi = pq - c$$
$$= (a - bq)q - qc \quad (4)$$

Here, $C(q)$ present cost function,

$$C(q) = qc \quad (5)$$

Price is the function of output

$$p = p(q) \quad (6)$$

q^* is equilibrium number of consumer.

Equilibrium number of consumer means

$$q^* \in \arg\max \pi(q^*)$$
$$= qP(q^*) - C(q^*) \quad (7)$$

The way to find out equilibrium is to derivative price function of China Telecom and makes it equal to zero

$$\frac{\delta\pi}{\delta q} = P(q) + qP'(q) - C'(q) = 0 \quad (8)$$

Reaction function is

$$q = R(q) \quad (9)$$

Reaction function means the best stratagem for China Telecom. For concrete results, optimal conditions can be work out when we use expression

$$C(q) = qc \quad (10)$$

$$p = p(q) = a - bq \quad (11)$$

Then optimal condition is:

$$\frac{\delta\pi}{\delta q} = a - c - 2bq = 0 \quad (12)$$

Reaction function is

$$q^* = R(q) = \frac{1}{2b}(a - c) \quad (13)$$

At this time, equilibrium number of consumers can be worked out:

$$p^* = \frac{a + c}{2} \quad (14)$$

Equilibrium profit is:

$$\pi(q^*) = (a - c)^2 / 4b \quad (15)$$

5.2 The second game stage---China Mobile setup, mobile was separated from telecom

China Mobile, as mobile communication supplier, has some substitute impact on fixed network monopolized by China Telecom. Influence of China Mobile on fixed network is a complete-information dynamic game in that China Mobile setup after China Telecom appearing. Each oligopoly, as game player, aimed at maximal profit and arrives at equilibrium status finally.

By hypothesis mentioned above, demand function of fixed network can be expressed:

$$p = a - bq \tag{16}$$

Service price of China Mobile is decided by government and has been adjusted by marketing situation, so it can be expressed by price of China Telecom:

$$p_M = t_m p_T \tag{17}$$

Time is one of variables of this function which comprise other variables.

Here t_m is a function which changes by time, other condition is on the same with above game model.

Profit of China Mobile connects with number of consumers of China Telecom existing in marketing, so profit function is:

$$\begin{aligned}\pi_M &= p_M q_M - c_M(q) \\ &= t_m(a - bq_M - bq_T)q_M - cq_M \\ &= t_m a q_M - t_m b q^2{}_M - t_m b q_T q_M - cq_M\end{aligned} \tag{18}$$

The way to find out equilibrium is working out derivative of profit function of China Mobile and makes it equal to zero

$$\frac{\delta\pi}{\delta q_M} = t_m a - 2t_m b q_M - t_m b q_T - c = 0 \tag{19}$$

At this time,

$$\begin{aligned}2t_m b q_M &= t_m a - t_m b q_T - c \\ &= \frac{t_m a + (t_m - 2)c}{2}\end{aligned} \tag{20}$$

Equilibrium number of consumers is:

$$q^*{}_M = \frac{t_m a + (t_m - 2)c}{4t_m b} \tag{21}$$

Equilibrium price is:

$$\pi^*{}_M = \frac{[t_m a + (t_m - 2)c]^2}{16t_m b} \tag{22}$$

5.3 The third game stage---China Unicom setup, direct competition appears

For break the monopoly status of China Telecom and China Mobile in fixed network domain and mobile communication area, the Chinese government established China Unicom. Then three players, China Telecom and China Mobile, China Unicom, appear in fixed network communication marketing. China Unicom, as all-operation supplier, has been supple little fixed network communication service, so we ignore impact on fixed network produced by it. China Unicom and China Mobile have some substitute impact on fixed network of communication marketing. Influence of China Unicom on fixed network is a complete-information dynamic game in that China Unicom setup after China Telecom and China Mobile appearing. Each oligopoly, as game player, aimed at maximal profit and arrives at equilibrium status

finally.

By hypothesis mentioned above, demand function of fixed network is

$$p = a - bq \tag{23}$$

Service price of China Unicom is decided by government and has been adjusted by marketing situation, so it can be expressed by price of China Telecom:

$$p_U = t_u p_T \tag{24}$$

Here t_u is a function which changes by time, other condition is on the same with above game model.

Profit of China Unicom connects with number of consumers of China Telecom and China Mobile, so profit function is:

$$\pi_U = p_U q_U - c_U(q)$$

$$= t_u(a - bq_U - bq_M - bq_T)q_U - cq_U = t_U aq_U - t_U bq^2{}_U - t_U bq_U q_M - t_U bq_U q_T - cq_U \tag{25}$$

The way to find out equilibrium is working out derivative of price function for China Unicom and make derivative equal to zero

$$\frac{\delta\pi}{\delta q_U} = t_U a - 2t_U bq_U - t_U bq_M - t_U bq_T - c = 0 \tag{26}$$

Then

$$2t_U bq_U = t_U a - t_U bq_T - t_U bq_M - c \tag{27}$$

Now equilibrium number of consumers of China Telecom and China Mobile getting up to maximum profits can be work out

$$q^*{}_T = \frac{1}{2b}(a-c)\ q^*{}_M = \frac{t_m a + (t_m - 2)c}{4t_m b} \tag{28}$$

Number of consumers of China Unicom turning up to equilibrium is

$$q^*{}_u = t_m t_u(a+c) + 2c(t_u - 2t_m) \Big/ 8bt_m t_u \tag{29}$$

Equilibrium profit is

$$\pi^*{}_U = [t_M t_u(a+c) + 2c(t_u - 2t_m)]^2 \Big/ 64bt^2{}_m t_u \tag{30}$$

5.4 The fourth game stage---original China Telecom was divided to China Netcom and new China Telecom

Then the operation of China Telecom dominated by Chinese government was divided into south operation and north operation. China Netcom incorporated original northern China Telecom and enters fixed network of communication marketing. New China Telecom has only southern operation of original China Telecom. New China Telecom and China Netcom conduct business all over Chinese Mainland for maximal profit and compete in north and south China.

For analysis convenience, we suppose that there exists a Pseudo-China Netcom. As a pseudo-player,

Pseudo-China Netcom enters fixed network of communication marketing and conduct four player game. Now, 3 existent and 1 new player compete for maximal profit and arrive at equilibrium status finally. This game is a complete information dynamic game.

Service price of China Netcom is decided by government and has been adjusted by marketing situation, so Pseudo-China Netcom can be expressed by price of original China Telecom:

$$p_N = t_n p_T \tag{31}$$

Here t_n is a function which changes by time, other condition is on the same with above game models. t_n is different from t_m of China Mobile and t_u of China Unicom mentioned above which express substitute effect on fixed network communication marketing, t_n express new price of Pseudo-China Netcom which origin from northern original China Telecom and is decided by competition between new China Telecom and China Netcom.

$$\begin{aligned}\pi_N &= p_N q_N - c_N(q) \\ &= t_n(a - bq_U - bq_M - bq_T - bq_n)q_n - cq_n \\ &= t_n(a - bq_U - bq_M - bq_T)q_n - t_n bq^2{}_n - cq_n\end{aligned} \tag{32}$$

The way to find out equilibrium is working out derivative of profit function of Pseudo-China Netcom and makes it equal to zero

$$\frac{\delta\pi_n}{\delta q_n} = t_n a - t_n bq_T - t_n bq_M - t_n bq_u - 2t_U bq_U - c = 0 \tag{33}$$

At this time,

$$2t_n bq_n = t_n a - t_n bq_T - t_n bq_M - t_n bq_u - c \tag{34}$$

Now equilibrium number of consumers of China Telecom, China Mobile and China Unicom getting up to maximum profits can be work out

$$\begin{aligned}q^*{}_T &= \frac{1}{2b}(a-c) \\ q^*{}_M &= \frac{t_m a + (t_m - 2)c}{4t_m b} \\ q^*{}_u &= t_m t_u(a+c) + 2c(t_u - 2t_m) \Big/ 8bt_m t_u\end{aligned} \tag{35}$$

Equilibrium number of consumer of Pseudo-China Netcom can be work out

$$q^*{}_n = \frac{t_m t_u t_n(a+c) + 2ct_u t_n + 4ct_m t_n - 8ct_u t_m}{16bt_m t_u t_n} \tag{36}$$

Equilibrium profit is

$$\pi^*{}_n = \frac{[t_m t_u t_n(a+c) + 2c(t_u t_n + 2t_m t_n - 4t_u t_m]^2}{256bt_n t^2{}_m t^2{}_u} \tag{37}$$

The game process of Pseudo-China Netcom entering fixed network of communication marketing is a

complete information dynamic game.

In fact we need new China Telecom and China Netcom which are produced by original China Telecom. So number of consumers of new China Telecom arriving at equilibrium status is:

$$q^{\bullet}_{n} = \frac{q_t}{2} + \frac{q_n}{2} = \frac{1}{4b}(a-c) + \frac{t_m t_u t_n (a+c) + 2ct_u t_n + 4ct_m t_n - 8ct_u t_m}{32bt_m t_u t_n} \quad (38)$$

Profit of New China Telecom arriving at equilibrium status is:

$$\pi^{\bullet}_{n} = \frac{\pi_t}{2} + \frac{\pi_n}{2} = \frac{(a-c)^2}{8b} + \frac{[t_m t_u t_n (a+c) + 2c(t_u t_n + 2t_m t_n - 4t_u t_m]^2}{512bt_n t^2{}_m t^2{}_u} \quad (39)$$

6. CONCLUSION

6.1 Profits of any oligopoly is not only dominated by similar service providers to compete directly, but also affected by the pressure of alternative competitors. These oligopolies can use these relations to analyze their response in different stages of the game. Government can adjust relations of oligopolies for regulations of communication industry. From our game analysis can be seen relationships of customers and market price of oligopolies which can be serving as reference to predict their potential markets for major oligopolies and as directions to make decision of long-term market development strategies to avoid over-investment and growth based on actual market Stage.

6.2 The earlier appearing in communication marketing, the higher profit obtained by oligopolies, the greater initiative are they. Otherwise, new player who want enter communication marketing must consider service price of reigning oligopolies and profit of new player is influenced by existence, their relations is shown as models.

6.3 At a specific market, these models can help each oligopoly calculate numbers of consumers and profit of main game players.

Analyses of Game process which has some substitute relations can reference to the method used in these models.

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