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Dynamic Stochastic Multi-Criteria Decision Making Method Based on Prospect Theory and Conjoint Analysis

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Abstract
A method based on prospect theory and conjoint analysis is proposed for dynamic stochastic multi-criteria decision making problems, in which the information about criteria weight is unknown and criteria values follow some kinds of distributions. Decision-maker's attitude towards risk is introduced into this paper. First, data is collected by investigation and criteria weights are derived by conjoint analysis. The prospect values of each alternative in different periods are calculated according to distribution function. Then, index distribution decides time sequence weight, and overall prospect values of each alternative are obtained and ranked by aggregating prospect values in different periods. Finally, an example of choosing the best product illustrates the feasibility and effectiveness of this method.

Key words: Dynamic stochastic; Multi-criteria decision making; Prospect theory; Conjoint analysis

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INTRODUCTION

Stochastic multi-criteria decision making (SMCDM) is a problem under uncertainty in which the natural states faced by decision makers (DMs) are randomly appeared. The main feature of SMCDM is that criteria values are random variables. In the field of solving SMCDM, expect utility theory (EUT) is widely employed. But in actuality, it is infeasible for decision makers to provide utility function with complete preference information (Miyamoto & Multiattribute, 1996). Accordingly, stochastic dominance (SD) theory become another effective method. For example, Zaras (1999) proposed a ranking method, where the criteria values are random variables, combining rough set theory and stochastic dominance. Lahdelma and Hokkanen (1998) presented stochastic multi-objective acceptability analysis (SMAA) method, and subsequently SMAA-2 (Lahdelma & Salminen, 2001), SMAA-3 (Lahdelma & Salminen, 2002) were proposed to deal with SMCDM.

However, most of the existing methods are based on expect utility theory (EUT) and assume that decision-makers (DMs) are totally rational. But in fact, due to the ambiguity of the problems, individual cognitive limitation and lacking of knowledge, people are not fully rational. Prospect theory (PT) (Kahneman & Tversky, 1979) and cumulative prospect theory (CPT) (Tversky & Kahneman, 1992) proposed by Kahneman and Tverskycan well reflect the DMs' subjective risk preference. In recent years, PT has been applied to multi-criteria decision making problems gradually (Hu, Chen, & Liu, 2009; Hu & Xu, 2011; Wang & Zhou, 2010; Lahdelma & Salminen, 2009), for instance, Lahdelma and Salminen (2009) proposed SMAA-P, which combined PT with SMAA-2. However, these methods have been only devoted to investigating single period MCDM. But in real decision making problems, data information may come from different periods or stages. Fan et al. (1993) call this decision making problems as Dynamic Multiple Criteria Decision Making (DMCDM). Currently, methods about Dynamic Stochastic Multiple Criteria Decision Making (DSMCDM) are still less. Lin proposed dynamic multi-attribute decision making model with grey number evaluations (Lin, Lee, & Ting, 2008), Wang and Yang (2010) put forward an approach to solve DSMCDM, where the information about criteria weights is incompletely certain and the criteria values are normal distribution stochastic variables.

Conjoint analysis (CA) was proposed by Luce and Tukey in 1964 and applied to marketing by Green and Rao in 1971. CA is an evaluation and decision method, which the evaluation objects can be optimized and ranked based on a overall utility model. At present, CA has been widely employed in many fields, for example, Zhu (2000) applied the principle of CA to new products' development to determine how important each attribute was to the product.

On the basis of existing studies, this paper applies CA to dynamic stochastic multiple criteria decision making, considering DMs' subjective risk preference, and proposes dynamic stochastic multiple criteria decision making method based on prospect theory and conjoint analysis.

1. CUMULATIVE PROSPECT THEORY AND CONJOINT ANALYSIS

1.1 Cumulative Prospect Theory

In CPT, prospect value $V(f)$ depends not only on value function V but also on decision weight function π, which can be expressed as:

$$V(f)=V(f^-)+V(f^+)=\sum_{i=1}^{h}\pi_i^- v(x_i)+\sum_{i=h+1}^{n}\pi_i^+ v(x_i). \qquad (1)$$

Kahneman and Tversky (1992) proposed a form of value function, which can satisfy the DMs' preference characteristics that they tend to risk aversion for gains and risk seeking for loss. The concrete expression of the value function is defined as:

$$v(x)=\begin{cases} x^{\alpha} & x\geq 0 \\ -\lambda(-x)^{\beta} & x<0 \end{cases}, \qquad (2)$$

where x is the difference between criteria value and reference point, gains are positive and losses are negative. α and β are risk attitude coefficients, $0<\alpha, \beta<1$. The larger the parameters, the more the DMs are willing to take risk. DMs can be seen as a risk neutral where $\alpha=\beta=1$. λ is loss aversion coefficient. $\lambda>1$ indicates DMs are more sensitive to loss. Kahneman and Tversky (1992) considered that $\lambda=2.25$, $\alpha=\beta=0.88$; Wu and Gonzalez (1996) considered that $\lambda=2.25$, $\alpha=\beta=0.52$; Zeng (2007) considered that $\lambda=2.25$, $\alpha=121$, $\beta=1.02$.

The cumulative calculation formulas of decision weight function defined by Kahneman and Tversky (1992) are given as follows:

$$\begin{cases} \pi_i^+ = w^+(\sum_{j=i}^{n} p_j) - w^+(\sum_{j=i+1}^{n} p_j) \\ \pi_i^- = w^-(\sum_{j=1}^{i} p_j) - w^-(\sum_{j=1}^{i-1} p_j) \end{cases}, \qquad (3)$$

$$\begin{cases} w^+(p)=\dfrac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}} \\ w^-(p)=\dfrac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}} \end{cases}. \qquad (4)$$

Where w^+ and w^- are the weighting function of the gains and the losses, respectively. γ is the risk gain attitude coefficient and δ is the risk loss attitude coefficient. p is the probability of events.

For the situation where there exists more than two prospect values, Prelec (1998) defined another function form which is expressed as:

$$\begin{cases} w^+(\sum_{j=h}^{n} p_j) = \exp(-\gamma^+(-\ln(\sum_{j=h}^{n} p_j))^{\phi}) \\ w^-(\sum_{j=1}^{h} p_j) = \exp(-\gamma^-(-\ln(\sum_{j=1}^{h} p_j))^{\phi}) \end{cases}, \qquad (5)$$

where γ^+, $\gamma^->0$, $\varphi>0$.

Scholars at home and abroad have done some research on the coefficient of weighting function: Kahneman and Tversky (1992) suggested $\gamma=0.74$, $\delta=0.72$; Wu and Gonzale (1996) suggested $\gamma=0.74$, $\delta=0.74$; Zeng (2007) suggested $\gamma=0.5$, $\delta=0.74$ in Chinese background. Goda and Hong (2008) suggested $\gamma^+=0.8$, $\gamma^-=0.8$, $\varphi>1.0$ in Equation (5).

1.2 Conjoint Analysis

Conjoint analysis (CA) is a kind of multivariate statistical analysis method. The basic process of CA can be concluded as follows: Firstly, products' attributes and their levels should be determined, so that we can simulate consumers' preference. Then, after accumulating the data about consumers' preference, we can construct a utility function through mathematical statistics method. Eventually, not only the relative importance of product' attributes, but also the utility of each level with respect to the product's attributes can be obtained from the utility function.

The common CA model is a regression model, which is shown as follows:

$$U(X)=a+\sum_{i=1}^{m}\sum_{j=1}^{k_i} a_{ij}\cdot x_{ij}+\varepsilon. \qquad (6)$$

Where $U(X)$ denotes the total utility of a particular product portfolio$X$. a denotes the intercept. ε denotes the error. k_i denotes the number of all the possible values of attribute i and we look each value as a level of an attribute. m denotes the number of attributes. a_{ij} denotes the jth level's utility with respect to attribute i, which, in fact, is the fitting parameter of regression equation. When a_{ij} occurs, $x_{ij}=2$, otherwise $x_{ij}=0$.

The relative importance of attribute i is defined as follows: $w_j = \dfrac{\max(a_{ij}) - \min(a_{ij})}{\sum_{i=1}^{m} \max(a_{ij}) - \min(a_{ij})}, i = 1,2,...$

Analysis results have to be tested to evaluate the validity of CA model. For example, we can calculate the correlation coefficient between factual values from consumers and predictive values and then test if it is statistically significant.

2. DYNAMIC STOCHASTIC MULTIPLE CRITERIA DECISION MAKING METHOD

Let $A = \{a_1, a_2, \cdots, a_m\}$ be a finite set of alternatives, $c = \{c_1, c_2, ..., c_n\}$ be the set of criteria which are independent. $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weight vector of criteria which is completely unknown, where, $\omega_j \in [0,1](j = 1,2,...,n), \sum_{j=1}^{n} \omega_j = 1$ Let $t_b (b = 1,2,..., p)$ be p different periods and $\omega(t) = (\omega(t_1), \omega(t_2), ..., \omega(t_p))^T$ be time series weight vector where $\omega(t_b) \in [0,1](b = 1,2,..., p), \sum_{b=1}^{p} \omega(t_b) = 1$. $X_{ij} (i = 1,2,..., m; j = 1,2,..., n)$ is the evaluation of the alternative with respect to $c_j (j = 1,2,..., n)$, obeying various distributions, such as uniform distribution, normal distribution or discrete type distribution. $h_j (j = 1,2, \cdots, n)$ is the reference point with respect to each criterion $c_j (j = 1,2,..., n)$. Then the definitions are as below:

Definition 1 Assuming that $X_{ij}(t_b)$ is discrete random variable, permute the outcome x_{ijq}^b in ascending like this: $x_{ij1}^b \le x_{ij2}^b \le ... \le h_j \le ... \le x_{ijr}^b$ where $x_{ijq}^b < h_j$ for loss, $x_{ijq}^b = h_j$ for zero, $x_{ijq}^b > h_j$ for gain. The prospect value of alternative a_j with respect to c_j at period t_b is:

$$V_{ij}^b(f) = V_{ij}^b(f^-) + V_{ij}^b(f^+) = \sum_{q=1}^{h} (\pi_q^b)^- \cdot v\left(x_{ijq}^b - h_j\right) + \sum_{q=h+1}^{r} (\pi_q^b)^+ \cdot v\left(x_{ijq}^b - h_j\right), \tag{7}$$

$$v(x_{ijq}^b - h_j) = \begin{cases} (x_{ijq}^b - h_j)^\alpha & x_{ijq}^b \ge h_j \\ -\lambda(-(x_{ijq}^b - h_j))^\beta & x_{ijq}^b < h_j \end{cases}, \tag{8}$$

$$(\pi_q^b)^+ = w^+(\sum_{s=q}^{r} p_{ijs}^b) - w^+(\sum_{s=q+1}^{r} p_{ijs}^b), \tag{9}$$

$$(\pi_q^b)^- = w^-(\sum_{s=1}^{q} p_{ijs}^b) - w^-(\sum_{s=1}^{q-1} p_{ijs}^b), \tag{10}$$

$$w^+(\sum_{s=q}^{r} p_{ijs}^b) = \exp(-\gamma^+(-\ln(\sum_{s=q}^{r} p_{ijs}^b))^\phi), \tag{11}$$

$$w^-(\sum_{s=1}^{q} p_{ijs}^b) = \exp(-\gamma^-(-\ln(\sum_{s=1}^{q} p_{ijs}^b)^{\varphi}). \tag{12}$$

Definition 2 If $X_{ij}^b \sim N(u_{ij}, \sigma_{ij}^2)$, the continuous random variable through the method is discretized as follows:

Divide the interval $[u_{ij} - 3\sigma_{ij}, u_{ij} + 3\sigma_{ij}]$ into N parts equally, then each part is $\Delta_{ij}^b = ((u_{ij} + 3\sigma_{ij}) - (u_{ij} - 3\sigma_{ij})) / N = 6\sigma_{ij} / N$ $z_{ijk}^b = (u_{ij} - 3\sigma_{ij}) + k \cdot \Delta_{ij}^b (k = 0,1,2, \cdots, N)$ which is similar to the role of x_{ijq}^b in definition 1. The probability of $z_{ijk}^b (k = 0,1,2, \cdots, N)$ is

$$p_{ijk}^b = f(z_{ijk}^b) = \frac{1}{\sqrt{2\pi}\sigma_{ij}} \exp\left[-\frac{(z_{ijk}^b - u_{ij})^2}{2\sigma_{ij}^2}\right]. \tag{13}$$

The prospect value of alternative a_i with respect to c_j at period t_b is:

$$V_{ij}^b = \sum_{k=1}^{N} v(z_{ijk}^b) \cdot \pi_{ijk} = \sum_{k=1}^{h} v(z_{ijk}^b) \cdot \pi_{ijk}^- + \sum_{k=h+1}^{N} v(z_{ijk}^b) \cdot \pi_{ijk}^+, \tag{14}$$

$$v(z_{ijk}^b) = \begin{cases} (z_{ijk}^b - h_j)^\alpha & z_{ijk}^b \ge h_j \\ -\lambda(h_j - z_{ijk}^b)^\beta & z_{ijk}^b < h_j \end{cases}, \tag{15}$$

$$\pi_{ijk}^+ = w^+(\sum_{l=k}^{N} p_{ijl}) - w^+(\sum_{l=k+1}^{N} p_{ijl}), \tag{16}$$

$$\pi_{ijk}^- = w^-(\sum_{l=1}^{k} p_{ijl}) - w^-(\sum_{l=1}^{k-1} p_{ijl}), \tag{17}$$

$$w^+(\sum_{l=k}^{N} p_{ijl}) = \exp(-\gamma^+(-\ln(\sum_{l=k}^{N} p_{ijl}))^\phi), \tag{18}$$

$$w^-(\sum_{l=1}^{k} p_{ijl}) = \exp(-\gamma^-(-\ln(\sum_{l=1}^{k} p_{ijl}))^\phi), \tag{19}$$

where $\pi_n^+ = w^+(p_n)$, $\pi_1^- = w^-(p_1)$.

Definition 3 If $X_{ij}^b \sim U(a_{ij}, b_{ij})$, we can calculate the prospect value v_{ij}^b through the process as below:

Firstly, divide the interval $[a_{ij}, b_{ij}]$ into N parts equally, then each part is $\Delta x_{ij} = \dfrac{b_{ij} - a_{ij}}{N}$.

Secondly, calculate the value of random variable x_{ij}^b, $x_{ijk}^b = a_{ij} + k \cdot \Delta x_{ij} (k = 1,2,..., N)$ and the corresponding probability density function is $p(x_{ijk}^b) = \dfrac{1}{b_{ij} - a_{ij}}$.

Thirdly, calculate the prospect value of alternative a_j with respect to c_j at period t_b according to formulas (14)-(19).

Definition 4 Let $V(t_b)=(V_{ij}^b)_{m\times n}$ be the prospect value matrix of period $t_b(b=1,2,\cdots,p)$ and $w(t)=(w(t_1),w(t_2),\cdots,w(t_p))^T$ be weight vector of different period, then

$$DWGA_{w(t)}(V(t_1),V(t_2),\cdots,V(t_p)) = \prod_{b=1}^{p}\left(V_{ij}^b\right)^{\omega(t_b)} \tag{20}$$

which is called Dynamic Weighted Geometry Averaging Operator (DWGA) where

$$\omega(t_b)\in[0,1](b=1,2,...,p),\sum_{b=1}^{p}\omega(t_b)=1.$$

The procedures of dynamic stochastic multiple criteria decision making method based on CPT and CA are as follows:

Step 1 Determine research objects' necessary attributes and their each level, then design questionnaire, sending them to the right subjects. Subsequently, the relative importance of attributes and the utility of each level can be obtained through the analysis of SPSS 17.0 with the data from valid questionnaires.

Step 2 If x_{ij}^b obey the discrete type distribution, calculate prospect value according to definition 1; if $X_{ij}^b\sim N(u_{ij},\sigma_{ij}^2)$, calculate prospect value according to definition 2; if $X_{ij}^b\sim U(a_{ij},b_{ij})$, calculate prospect value according to definition 3. Then normalize the prospect value matrix $V^b=(v_{ij}^b)_{m\times n}$ according to formulas (21) and the normalized prospect value matrix is $R^b=(r_{ij}^b)_{m\times n}$.

$$\begin{cases} r_{ij}^b=\dfrac{v_{ij}^b}{\max\limits_i(v_{ij}^b)}, i\in M, j\in N_1 \\ r_{ij}^b=\dfrac{\min\limits_i(v_{ij}^b)}{v_{ij}^b}, i\in M, j\in N_2 \end{cases}, \tag{21}$$

where N_1 denotes benefit criteria, N_2 denotes cost criteria.

Step 3 We acquire the time series weight $w(t)=(w(t_1),w(t_2),\cdots,w(t_p))$ based on exponential distribution where

$$w(t_b)=w^{'}(t_b)/\sum w^{'}(t_b)\left(b=1,2,...,p\right),$$

$$w(t_b)=(1/u_p)\times e^{bu}\quad\left(b=1,2,...,p\right),$$

$$u_p=\lambda(1+p). \tag{22}$$

Step 4 Calculate the prospect value at period $t_b(b=1,2,\cdots,p)$ based on DWGA operator which is shown as below:

$$v^b(a_i)=\prod_{j=1}^{n}r(x_j)^{\omega(j)}. \tag{23}$$

Step 5 Calculate the overall prospect value of alternative $a_i(i=1,2,\cdots,m)$ by aggregating the prospect values of different periods, which is expressed as follows:

$$v(a_i)=\prod_{b=1}^{p}(v^b(a_i)^{\,w(t_b)}. \tag{24}$$

Then rank the alternatives according to $v(a_i)$ and select the best. The large the $v(a_i)$, the better the alternative a_i.

3. ILLUSTRATIVE EXAMPLE

One family wants to buy a refrigerator and there are 5 different brands $a_i(i=1,2,\cdots,5)$ to be considered. The main evaluation criteria considered include: the reliability-c_1 which obeys normal distribution; cooling-c_2 which obeys normal distribution; economic-c_3 which obeys uniform distribution and artistic-c_4 which obeys discrete type distribution respectively, where the criterion c_3 is cost criterion and the other three are benefit criteria. We use 1-10 scale to evaluate the criterion c_4 whose information is demonstrated by 7 experts. All the criteria weights are thoroughly unknown. The decision matrices are listed in Tables 1-3. To select the best alternative, the proposed method is employed and the procedure is summarized as follows.

Table 1
Decision Matrix $D(t_1)$

Alternative	c_1	c_2	c_3	c_4									
				1	2	3	4	5	6	7	8	9	10
a_1	N(075.,0.022)	N(087.,0.052)	U[3200,3600]	0	0	0	1/7	2/7	2/7	1/7	1/7	0	0
a_2	N(073.,0.012)	N(083.,0.012)	U[3100,3450]	0	0	0	1/7	1/7	2/7	1/7	1/7	0	0
a_3	N(076.,0.042)	N(085.,0.022)	U[3150,3500]	0	0	0	2/7	1/7	1/7	1/7	1/7	1/7	0
a_4	N(074.,0.032)	N(084.,0.032)	U[3200,3500]	0	0	0	0	3/7	2/7	1/7	1/7	0	0

Table 2
Decision Matrix $D(t_2)$

Alternative	c_1	c_2	c_3	c_4									
				1	2	3	4	5	6	7	8	9	10
a_1	N(0.85,0.012)	N(0.91,0.022)	U[2800,3150]	0	0	0	0	1/7	1/7	1/7	2/7	1/7	1/7
a_2	N(0.87,0.052)	N(0.89,0.022)	U[2800,3100]	0	0	0	0	1/7	1/7	2/7	2/7	1/7	0
a_3	N(0.85,0.032)	N(0.90,0.012)	U[2800,3100]	0	0	0	0	0	3/7	2/7	2/7	0	0
a_4	N(0.88,0.0152)	N(0.92,0.032)	U[2900,3200]	0	0	0	0	0	2/7	4/7	1/7	0	0

Table 3
Decision Matrix $D(t_3)$

Alternative	c_1	c_2	c_3	c_4									
				1	2	3	4	5	6	7	8	9	10
a_1	N(0.89,0.012)	N(0.93,0.012)	U[2350,2650]	0	0	0	0	0	0	1/7	3/7	2/7	1/7
a_2	N(0.90,0.012)	N(0.94,0.012)	U[2400,2750]	0	0	0	0	0	0	2/7	2/7	3/7	0
a_3	N(0.88,0.012)	N(0.96,0.022)	U[2500,2800]	0	0	0	0	0	1/7	1/7	2/7	2/7	1/7
a_4	N(0.91,0.022)	N(0.95,0.012)	U[2500,2850]	0	0	0	0	0	0	1/7	4/7	1/7	1/7

a) Determine security, cooling, structure, reliability, economic and artistic as the main criteria of refrigerator through accumulating secondary data and visiting retailers and consumers. Then make a pre-test about the importance of these criteria by selecting consumers randomly and finally obtain 4 criteria which are more important than others. The 4 criteria and their levels are shown in Table 4.

We can acquire 9 kinds of product portfolio which are the combination of criteria and their levels through orthogonal design. The outcomes are listed in Table 5 and the analysis results from conjoint module are shown in Table 6.

The fitting degree of the model in questionnaire is 1, indicating that the model is fitting well, so the outcome based on the model has high reliability.

b) If the criteria valuex_{ij}^b obey the discrete type distribution, calculate prospect value according to definition 1; if $X_{ij}^b \sim N(u_{ij}, \sigma_{ij}^2)$, calculate prospect value according to definition 2; if $X_{ij}^b \sim U(a_{ij}, b_{ij})$, calculate prospect value according to definition 3. Assuming that the reference points with respect to c_j (j=1, 2, 3, 4) at period t_1 are 0.70, 0.80, 3200, 5 respectively and they are 0.80, 0.85, 2800, 6 respectively at period t_2 while they are 0.85, 0.90, 2400, 7 respectively at period t_3 , which are based on the technology level of different periods. The parameters α, β and λ suggested by Zen (2007) are 1.21, 1.02 and 2.25 respectively. Prelec (1998) suggested γ^+=0.8,γ^-=0.8, φ=1.0, N=5000. The prospect values of alternative a_i(i=1, 2, 3, 4)with respect to c_j(j=1, 2, 3, 4) at period t_b(b=1, 2, 3) are shown in Table 7.

Table 4
Criteria and Levels

Criteria	Levels		
	1	2	3
Reliability	0.85	0.90	0.95
Cooling	0.85	0.90	0.95
Economic	2400	2600	2800
Artistic	Traditional	Common	Fashionable

Table 5
Orthogonal Design Table

Reliability	Cooling	Economic	Artistic	STATUS	CARD
3.00	2.00	3.00	1.00	0	1
3.00	3.00	1.00	2.00	0	2
2.00	1.00	3.00	2.00	0	3
2.00	3.00	2.00	1.00	0	4
2.00	2.00	1.00	3.00	0	5
1.00	3.00	3.00	3.00	0	6
1.00	1.00	1.00	1.00	0	7
3.00	1.00	2.00	3.00	0	8
1.00	2.00	2.00	2.00	0	9

Table 6
Outcome of Orthogonal Design

Alternative	Relative importance	Levels	Utility
Reliability	22.774%	0.85	-1.049
		0.90	0.395
		0.95	0.654
Cooling	32.607%	0.85	-1.383
		0.90	0.136
		0.95	1.247
Economic	29.412%	2400	1.025
		2600	-0.049
		2800	-0.975
Artistic	15.207%	Traditional	-0.012
		Common	-0.235
		Fashionable	0.247
Constant	5.086		
Correlation			
Correlation coefficient	Value	Sig.	
R (Pearson)	1.000	.000	
tau (Kendall)	1.000	.000	

To be continued

Continued

Table 7
Prospect Values of Alternatives With Respect to Criteria at Different Periods

Alternative	t_1				t_2				t_3			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
a_1	0.0297	0.0434	2125.8	0.9557	0.0280	0.0229	1808.6	1.9985	0.0217	0.0157	1132.5	1.8597
a_2	0.0157	0.0157	1057.7	0.6977	0.0434	0.0503	1500.9	1.2653	0.0280	0.0217	1808.6	2.7473
a_3	0.0365	0.0297	1427.8	1.3495	0.0297	0.0280	1500.9	1.1212	0.0157	0.0364	2280.8	1.2653
a_4	0.0213	0.0213	1500.9	1.4301	0.0494	0.0449	2280.8	1.0409	0.0364	0.0280	2609.4	1.6750

The normalized prospect matrix $Rb(b=1, 2, 3)$are obtained according to Equation (21) which are shown as follows:

$$R^1 = \begin{bmatrix} 0.8137 & 1 & 0.4976 & 0.6683 \\ 0.4301 & 0.3618 & 1 & 0.4879 \\ 1 & 0.6843 & 0.7408 & 0.9436 \\ 0.5836 & 0.4908 & 0.7047 & 1 \end{bmatrix}$$

$$R^2 = \begin{bmatrix} 0.5668 & 0.4553 & 0.8299 & 1 \\ 0.8785 & 1 & 1 & 0.6331 \\ 0.6012 & 0.5567 & 1 & 0.5610 \\ 1 & 0.8926 & 0.6581 & 0.5208 \end{bmatrix}$$

$$R^3 = \begin{bmatrix} 0.5962 & 0.4313 & 1 & 0.6769 \\ 0.7692 & 0.5962 & 0.6262 & 1 \\ 0.4313 & 1 & 0.4965 & 0.4606 \\ 1 & 0.7692 & 0.4340 & 0.6097 \end{bmatrix}$$

c) The time series weight $w(t)$=(0.5065, 0.3072, 0.1863) is acquired according to Equation (22).

d) Calculate the prospect values of alternative $a_i(i=1, 2, 3, 4)$ at period $t_b(b=1, 2, 3)$, which are shown in Table 8.

e) Aggregate the prospect value of different periods according to Equation (24) and obtain the overall prospect value of alternatives $a_i(i=1, 2, 3, 4)$ as follows: $v(a_1)$=0.6851, $v(a_2)$=0.6576,$v(a_3)$=0.7196. Thus we can get the ranking of alternatives $a_3 \succ a_1 \succ a_4 \succ a_2$, indicating a_3 is the best alternative.

Table 8
Prospect Values of Alternatives at Different Periods

Alternative	Prospect value		
	t_1	t_2	t_3
a_1	0.7309	0.6436	0.6368
a_2	0.5311	0.9057	0.6935
a_3	0.8019	0.6738	0.5973
a_4	0.6328	0.7716	0.6661

CONCLUSION

This paper proposes a method based on CPT and CA with information from different periods to solve SMCDM problems, taking the risk attitude of DMs into consideration. The key of the proposed method is that it integrates DMs' risk preference with decision making process organically and determines the criteria weight with CA, qualifying the uncertainty and thus guiding

DMs scientifically. In order to make decision more reasonable, we can adjust the model parameters according to investors' risk preference characteristics in practical decision-making process, thus reducing decision risk and improving decision quality.

REFERENCES

Goda, K., & Hong, H. P. (2008). Application of cumulative prospect theory: Implied seismic design preference. *Structural Safety, 30*(6), 506-516.

Hu, J. H., Chen, X. H., & Liu, Y. M. (2009). Multi-criteria decision making method based on linguistic evaluation and prospect theory. *Control and Decision, 24*(10), 1477-1482.

Hu, J. H., & Xu, Q. (2011). Multi-criteria decision making method with interval number based on prospect theory. *Statistics & Information Forum*, *1*(9), 23-27.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society, 47*(2), 263-291.

Lahdelma, R., & Hokkanen, J. (1998). SMAA-stochastic multi-objective acceptability analysis. *European Journal of Operational Research*, *106*(1), 137-143.

Lahdelma, R., & Salminen, P. (2001). SMAA-2: Stochastic multi-criteria acceptability analysis for group decision making. *Operations Research*, *49*(3), 444-454.

Lahdelma, R., & Salminen, P. (2002). Pseudo-criteria versus linear utility function in stochastic multi-criteria acceptability analysis. *European Journal of Operational Research, 141*(2), 454-469.

Lahdelma, R., & Salminen, P. (2009). Prospect theory and stochastic multi-criteria acceptability analysis. *Omega, 37*, 961-971.

Lin, Y. H., Lee, P. C., & Ting, H. (2008). Dynamic multi-attribute decision making model with grey number evaluations. *Expert System with Application, 35*, 1638-1644.

Luce, R. D., & Tukey, J. W. (1964). Simultaneous conjoint measurement: A new type of fundamental measurement. *Journal of Mathematical Psychology, 1*(1), 1-27.

Miyamoto, J. M., & Multiattribute, W. P. (1996). Utility theory without expected utility foundations. *Operation Research*, *44*, 313-326.

Prelec, B. D. (1998). The probability weighting function. *Econometrica, 66*(3), 497-527.

Shen, H., & Ke, H. X. (1998). Principle and application of conjoint analysis. *Application of Statistics and Management, 1*(4), 39-45.

Tesfamariam, A. R. S. (2007). Probability density functions based weights for ordered weighted averaging (OWA) operators: An example of water quality indices. *European Journal of Operational Research, 182*(3), 1350-1368.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*(4), 297-323.

Wang, J. Q., & Zhou, L. (2010). Grey-stochastic multi-criteria decision making approach based on prospect theory. *Systems Engineering—Theory & Practice, 30*(9), 1658-1664.

Wang, X. F., & Yang, X. J. (2010). Dynamic stochastic multiple attribute decision making method with incomplete certain information. *Systems Engineering—Theory & Practice, 30*(2), 332-338.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science*, *42*(12), 1676-1690.

Zaras, K. (1999). Rough approximation of pairwise comparisons described by multi-attribute stochastic dominance. *Journal of Multi-Criteria Decision Analysis*, *8*, 291-297.

Zeng, J. M. (2007). An experimental test on cumulative prospect theory. *Journal of Jinan University (Nature Science Edition), 28*(1), 44-47.

Zhu, Z. P. (2000). Research on the method and the case for conjoint analysis. *Science Research Management, 21*(5), 95-102.