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Stock Investment Value Analysis Model Based on AHP and Gray Relational Degree*

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Abstract: The article presents a method for stock selection from the view of investors who contemplate stocks of a new investment. The AHP (Analytic Hierarchy Process) and Grey Relational Analysis are used as two integral parts of the method. By distilling information from the Judgment matrix, the AHP-GRA method provides a framework to assist investors in analyzing various investment factors, evaluating stock investment alternatives, and making final investment selections. The primary principle of the method is to match decision-makers' preferences with stock characteristics. The model requires that a number of potential stocks have been proposed. Alternatives are then evaluated and compared under various factors. It allows investor to incorporate personal preference and judgement in the solution process. An example of evaluating eight listed companies in the steel industry of China is showed to illustrate the solution process, the results of which are promising.

Key words: Stock Investment Decision; Judgment Matrix; AHP; Grey Relational Analysis

1. INTRODUCTION

The study on stock investment value starts early overseas, and several forecasting models such as DDM, CAPM Models have been developed quite well. All of these models are based on a common idea---investment value theory, which considers any investment tool, including stocks, has a stable investment foundation, which is also known as the intrinsic value. Although in recent years, this idea has been generally accepted by the domestic academic circle, these models are still impractical for stock investment applications. Instead, it is the synthetic evaluation model with a variety of financial indicators that is widely used in China now. The synthetic evaluation model is close to perfect; however, it neglects a basic question: the assessment of the value of stocks is an essential element to choose good stocks, but it can not be equated to a good stock investment decision. Selecting a stock as an investment is a decision-making process for the

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investors, that is to say, different investors with different expectations will have different trade-offs. Therefore, a full consideration of the investor preferences and stock evaluation is strongly needed when evaluating a stock.

Combined AHP with Grey Relational Analysis, this paper presents a feasible and practical model for selecting a stock. The AHP (Analytical Hierarchy Process), initially proposed by Professor T.L. Saaty, is a multi-objective decision methodology. The AHP has three major steps: First, identify the factors from the problem and divide them into different hierarchies corresponding to different evaluation levels; Then evaluate relative importance of each factor in a lower hierarchy relative to the corresponding factor in an upper hierarchy; And finally, by forming judgement matrix, the relative importance of every factor can be calculated. AHP can help resolve such multi-criteria evaluation problem as stock assessment issue. The gray relational analysis is a statistical analysis method. By using the grey relational grade to describe the relevance and the order of different factors, it can cope well with any vagueness problems which can not adequately be treated by probability distributions. The integrated applications of these two theories in the stock investment analysis can satisfy investors' expectations while appraising a stock comprehensively. By sorting the factors, investors can select a relatively good stock under the premise of satisfaction. In the end of this paper, the model is illustrated by evaluating eight listed companies in the steel industry of China, and the result shows that the method is feasible and reliable.

2. STEPS OF STOCK INVESTMENT VALUE ANALYSIS

The model involves the following major steps:

2.1 Develop a Hierarchy of Factors Explaining the Stock

This step is actually a process of analyzing matter. AHP model divides the factor system into three levels, namely the objective level (top level), the criteria level (middle level) and the alternative level (bottom level), in order to make evaluation clearer and easier. This step is of critical importance to synthetic evaluation model. The factor hierarchy [5] is obtained after processing all the listed companies' various financial indicator data in China by using the grey clustering method, which is of comparatively high reliability. Covering most information of a stock, the hierarchy provides an empirical basis for the application of artificial intelligence methods in the stock investment assessment.

Table 1

Objective level	Criteria level	Indicator level
stock investment value	Profitability (A_1)	Profit margin on sales A_{11} Net profit margin on total assets A_{12} Net profit margin on current assets A_{13}
	Development capability (A_2)	Total assets growth rate A_{21} Profit growth rate A_{22} Sales growth rate A_{23}
	profitability to the shareholders(A_3)	Earnings per share A_{31} Net assets per share A_{32} Price to earnings ratio A_{33}
	Solvency (A_4)	Asset-liability ratio A_{41} Current assets ratio A_{42} Current ratio A_{43}
	Asset management and operating ability (A_5)	Inventory turnover A_{51} Total assets turnover A_{52} Cash per share A_{53}

2.2 Use AHP Method to Determine Index Weight

Evaluate relative importance of each factor in a lower hierarchy relative to the corresponding factor in an upper hierarchy and form judgement matrix, A=（a_{ij}）$_{n\times n.}$ To construct the judgement matrix, there are a number of scales and 1—9 scales are the commonly used. The comparison scales of judgement and their meanings are shown in Table 2.

Table 2

Scale	Verbal judgement of preferences
a_{ij}=1，a_{ji}=1	Equally preferred
a_{ij}=3，a_{ji}=1/3	Moderately preferred
a_{ij}=5，a_{ji}=1/5	Strongly preferred
a_{ij}=7，a_{ji}=1/7	Very strongly preferred
a_{ij}=9，a_{ji}=1/9	Extremely preferred
a_{ij}=2, 4, 6, 8; Mean-value of two near situations above	

2.3 Calculate Weights and Test the Consistency of the Judgement Matrix

1)After constructing the judgement matrix, weights of factors can be shown in the eigenvector of the matrix (Eq.1), W=（w_1，w_2，w_3，w_4，w_5）, which is related to the largest eigenvalue λ_{max} .

$$Aw = \lambda_{max} w \tag{1}$$

2) Test the Consistency of the Judgement Matrix

To assess whether the weights distribution of the factor in the same hierarchy is reasonable, we have to test the consistency o f the judgement matrix. The random consistency ratio C.R. can be calculated by the following equation:

$$C.R. = \frac{\lambda_{\max} - n}{(n-1)R.I.} \tag{2}$$

where R .I. is the average random consistency index.

2.4 Grey Relational Grade Analysis

The gray relational analysis uses grey relational grade to describe the relevance and the order of different factors. In the following, we present the steps of grey relational analysis:

1)Each objective stock is a compared sequence. Select the optimal value of each indicator from the overall objective stocks to construct the reference sequence, V_0= ($V_0(1)$,$V_0(2)\ldots V_0(n)$). Of all the indicators involved in this paper, except for the appropriate indicators, the optimal values refer to the largest values.

2) In the grey relational analysis, data preprocessing is first performed in order to normalize the raw data for analysis. The equation to normalize the value of a given appropriate indicators is:

$$v'_i(k) = -\left|v_i(k) - a\right| \tag{3}$$

where ‘a’ is the approximation of this indicator.

Nondimensionalize all the values by using the Eq. (4)

$$x_i(k) = \frac{v_i(k)}{\overline{v(k)}} \tag{4}$$

$\overline{v(k)}$ is the mean of all values of corresponding indicator.(Indicator k)

3) Calculate the grey relational coefficient. In traditional grey relational analysis, the grey relational coefficient between the reference sequence and compared sequence is

$$\gamma_{0i}(k)=\gamma(x_0(k),x_i(k))=\frac{\min_i\min_k\left|x_0(k)-x_i(k)\right|+\xi\max_i\max_k\left|x_0(k)-x_i(k)\right|}{\left|x_0(k)-x_i(k)\right|+\xi\max_i\max_k\left|x_0(k)-x_i(k)\right|} \tag{5}$$

where ξ is identification coefficient and is commonly 0.5. $\gamma_{0i}(k)$ is the grey relational coefficient between $x_i(k)$ and $y_j(k)$.

Construct the grey relational coefficient matrix $\gamma=[\gamma_{0i}(k)]_{m\times n}$.

m=the number of the indicators

n=the number of the objective stocks

4) Based on the grey relational coefficient matrix and the weight vector, the calculation of the grey relational grade is usually expressed as

$$r_i=\sum_{k=1}^{m}W_k r_{0i}(k) \tag{6}$$

5) Assume that a row of a relational grade matrix is marked as $r=\{r_1,r_2,\ldots,r_n\}$ and rearrange the sequence in descending order, $r_s>r_h>r_p>\ldots$. For r_s, because the possibility of the similarity between the compared sequence and the reference sequences is the biggest, it is believed that this compared sequence belongs to the reference sequences in terms of upper hierarchy. Repeat the calculation by the steps above in order to obtain the final grades Ri , which explains the degree of correlation between a compared sequence and the reference sequence. Thus the best investment choice is achieved.

3. EMPIRICAL ANALYSIS

To illustrate the model, this paper evaluates eight listed companies in China's steel industry (Table.4). All data came from Guotai Junan database.

First, evaluate relative importance of each factor and construct judgement matrix under the guidance of experts. Calculate weight vector and test the consistency of the judgement matrix (Eq. (1).(2)). The following table shows the results of the judgement matrix at criteria level relative to the object.

Table 3: Judgement Matrix at Criteria Level Relative to the Object

U	A_1	A_2	A_3	A_4	A_5	W
A_1	1	2	1/3	2	5	0.2305
A_2	1/2	1	1/3	2	3	0.1577
A_3	3	3	1	4	5	0.4456
A_4	1/2	1/2	1/4	1	3	0.1128
A_5	1/5	1/3	1/5	1/3	1	0.0534

NOTE: $\lambda_{\max}=5.17, C.I.=0.0422, R.I.=1.12, C.R.=0.0377$

Similarly, we can derive weight vectors W1=（0.5396，0.2969，0.1634）; W2 =（0.1571，0.5936，0.2493）;W3 =（0.3325，0.1396，0.5278）;W4 =（0.6144，0.2684，0.1172）;W5=（0.3089，0.5816，0.1095）;

Table 4

Stc.	Ref.	600001	600005	600010	600022	600102	600282	600569	600581
nd.	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
A_{11}	0.0708	0.0161	0.0708	0.0209	0.0182	0.0067	0.0043	0.0034	0.0052
A_{12}	0.0751	0.0229	0.0751	0.0237	0.0303	0.0154	0.0112	0.0050	0.0092
A_{13}	0.3710	0.0431	0.3710	0.0530	0.0752	0.0420	0.0176	0.0120	0.0204
A_{21}	0.3058	0.0442	0.1288	0.3058	0.1551	0.0044	0.0256	0.1652	0.1763
A_{22}	-1.8338	-1.8338	-1.8765	-3.2424	-2.7240	-3.7589	-5.4266	-5.8163	-5.0087
A_{23}	0.0755	-0.3723	-0.2698	0.0755	-0.2497	-0.4744	-0.3742	-0.4357	-0.4896
A_{31}	43.1169	13.2291	9.3568	6.8693	24.9096	43.1169	16.8295	15.5049	26.1407
A_{32}	6.4372	4.3367	3.5292	2.2345	4.1929	6.4372	2.5898	4.3122	3.6815
A_{33}	15.0000	15.2582	7.2205	18.1429	8.0595	20.3846	39.8085	60.4000	35.5714
A_{41}	0.5000	0.5435	0.6220	0.6731	0.7370	0.6587	0.6098	0.6156	0.7676
A_{42}	0.6405	0.5199	0.1820	0.4937	0.4026	0.3453	0.6405	0.4067	0.4296
A_{43}	2.0000	1.1518	0.4586	0.9107	0.6439	1.1468	1.0977	0.9046	0.7096
A_{51}	11.8130	5.4156	7.0421	4.0697	8.8671	11.8130	6.0746	5.7140	9.4888
A_{52}	2.5676	1.4227	1.0609	1.1384	1.6643	2.2806	2.5676	1.4704	1.7835
A_{53}	0.9242	-0.1771	-0.3069	0.1873	0.9242	-0.5505	-0.2178	-0.1861	0.0163

For the data in Table 4, select a reference sequence and normalize the data. Table 5 shows the results. (Eq. (3),(4))

Table 5

Stc.	Ref.	600001	600005	600010	600022	600102	600282	600569	600581
Ind.	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
A_{11}	2.9446	0.6692	2.9446	0.8678	0.7564	0.2803	0.1808	0.1405	0.2157
A_{12}	2.5235	0.7690	2.5235	0.7981	1.0170	0.5164	0.3750	0.1669	0.3108
A_{13}	3.3215	0.3854	3.3215	0.4744	0.6734	0.3760	0.1578	0.1071	0.1829
A_{21}	2.0990	0.3033	0.8842	2.0990	1.0647	0.0302	0.1759	1.1339	1.2097
A_{22}	0.5236	0.5236	0.5358	0.9258	0.7778	1.0733	1.5494	1.6607	1.4301
A_{23}	-0.2703	1.3324	0.9655	-0.2703	0.8938	1.6979	1.3393	1.5594	1.7523
A_{31}	1.9493	0.5981	0.4230	0.3106	1.1261	1.9493	0.7609	0.7010	1.1818
A_{32}	1.5347	1.0339	0.8414	0.5327	0.9996	1.5347	0.6174	1.0280	0.8777
A_{33}	-1.3597	0.0234	0.7052	0.2849	0.6291	0.4881	2.2488	4.1154	1.8648
A_{41}	-6.1871	0.5383	1.5093	2.1414	2.9333	1.9633	1.3590	1.4306	3.3119
A_{42}	1.4196	1.1522	0.4033	1.0943	0.8923	0.7652	1.4196	0.9014	0.9522
A_{43}	-2.5802	1.0942	1.9885	1.4053	1.7495	1.1007	1.1641	1.4132	1.6647
A_{51}	1.5124	0.6933	0.9016	0.5210	1.1352	1.5124	0.7777	0.7315	1.2148
A_{52}	1.4483	0.8025	0.5984	0.6421	0.9387	1.2864	1.4483	0.8294	1.0060
A_{53}	13.5544	-2.5972	-4.5015	2.7473	13.5544	-8.0730	-3.193	-2.7300	0.2393

Calculate gray relational coefficient matrix under each criterion according to the Eq. (5). To explain, the following shows the gray relational coefficient matrix under criterion 'profitability'.

Use Eq. (6) to get the grey relational grade R1=（0.5745，0.5153，0.53803，0.5133，0.52813，0.5712，0.54163 ，0.51103）. It is not difficult to conclude that A1 is the best in terms of profitability.

In the same way, it is easy to get:

Development capability:

R_2 =（0.7488，0.7725，0.8338，0.6724，0.5259，0.4505，0.4541，0.4852）;

Profitability to shareholders:

R_3=（0.6913，0.6258，0.6400，0.6782，0.7873，0.5642，0.5222，0.6147）;

Solvency:

R_4=（0.5745，0.5153，0.5380，0.5133，0.5281，0.5712，0.5416，0.5110）;

Assets management and operating ability:

R_5=（0.8799，0.8726，0.8790，0.9634，0.9185， 0.9138，0.8819，0.9085）

Then, following the Eq. (6), we can get the final grade R=（0.6355，0.7359，0.6252，0.6198，0.6329，0.5234，0.4981，0.5442）, which reflects that the stock 2 (600005) is the best of all.

In fact, the stock2 represents Wuhan Iron and Steel stock, which is of promising prospects in the eyes of experts and the majority of investors. That means, the model can offer a comparatively reliable suggestion.

4. CONCLUSION

Stock Investment Value analysis is a problem that has been laid great emphasis on. This paper takes both the subject and the object in an investment activity and applies a comparatively reliable indicator hierarchy. Investors use AHP to combine personal preferences and experts opinions and gray correlation degree as judging criteria so as to achieve purpose of effective and comprehensive stock assessment. The result of the empirical analysis shows that the method offers a reliable and feasible advice and it will benefit investors.

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