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Research on EPQ Model Based on Random Defective Rate

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Abstract
In the real economic life, it is inevitable that a lot of phenomena will happen, such as damage in transportation and machine failure, which may generate a certain percentage of defective products in the process of logistics and production. Especially in the production process, the stoppage on the production line often brings about defective products. To provide mathematical models that more closely conform to actual inventories and respond to the factors that contribute to inventory costs, based on the classical EPQ model, this paper develops an EPQ model for defective items with a certain price relative to the defective level. And this paper also considers the issue that defective items are sold at a lower price which depends on the degree of product defects. A mathematical model is developed and numerical examples are provided to illustrate the solution procedure. The research will enrich researches and it has important practical significance.

Key words: EPQ/EOQ; Imperfect quality; Defectives

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INTRODUCTION

The problems concerning ordering and inventory have always been the emphasis of research for many scholars. In recent years, with further deepening of the research on ordering and inventory problem, the practical research is getting much closer to the actual situation. Moreover, more scholars have a tendency to expand the research from different perspectives.

Ever since the EOQ/EPQ (economic order/production quantity) inventory control model was introduced in the early decades of this century, it appears that it has been widely accepted by many industries and has achieved success in the present inventory management, EPQ model can be regarded as a development based on EOQ model.

The research on EOQ and EPQ is both based on the assumption that the arrival of goods is at a stable and limited rate, namely, the arrival rate of goods is a limited constant. However, both the traditional EOQ model and EPQ model are also based on the assumption that there is no defect existing in the products, namely, all products are perfect, but it is not commensurate with the real ordering and production situation. It is likely that owing to imperfect production system, improper packing, or transportation damage, and so on. There is a certain percentage of defective products, also called imperfect products, in the practical production and ordering process. The imperfect products will exert some impacts on the order decisions from customers. Therefore the research on the defects of the production orders will bring practical significance to enterprises and academia.

For the sake of the unreasonable assumptions of traditional EOQ/EPQ model, many scholars have done some research on the EOQ/EPQ model concerning quality defects in order inventory model with the desire that they will make some improvements. The research on this aspect is as follows:

Rosenblatt（1986） is the first one who put forward the production inventory model concerning the impact of manufacturing defects. In this paper, the production system is divided into two kinds of conditions, namely, the condition under control (producing perfect products) and the condition out of control (producing defective products). In the initial stage, the production system is controllable, and the time of transforming to uncontrollable state accords with the exponential distribution. Kim（1999）further improved the above model based on the assumption that the time of transforming production system into an uncontrollable state is commensurate with the random distribution. Cheng（1999） put forward an expanded EOQ model concerning the defective production process, assumed that the unit cost of production is related with demand, and established the function to build relationships among the unit cost of production, the demand rate and the reliability of the production process. The inventory problem was regarded as a geometric one which could get the optimal solution on the basis of the theory of GP. Zhang（1990）expanded the EOQ model by considering the joint batch and the inspection program on the assumption that the defect rate accords with a certain distribution. In this model, the defective products can be replaced by qualified products. Schwaller (1998) assumed that the ordered products contain a certain percentage of defective products, and the inspection will push up the cost of fixed and variable parts, which can also be regarded as a development of EOQ model. Porteus (1986) considered the impact of defective products based on improving the product process. The above researches has not taken the handling of defective products into consideration.

The researches on defective products contain several different ways of handlings, such as reprocessing, or selling at a lower price, or simply discarded. Based on the EOQ model that product defects are commensurate with random distribution, Salameh（2000）assumed that there was no error existent in the process of hypothesis test and all of the goods should be inspected after the arrival. After the inspection process, all the defective products should be sold at a lower price for one time. The model is referred to as S-J model in subsequent researches, which is considered very important. Based on the S-J model, Goyal（2003） put forward a simplified algorithm of the optimal solution, and improved the model by taking the joint inventory of the suppliers and the retailers into account. On the basis of Salameh and other one's research achievement, Papachristos（2006） discussed the impacts of the processing time of defective products in the inventory. Maddah（2010） put forward a solution to solve the problems of shortages existent in the inspection process, and gave a more accurate expression to calculate the optimal expected profits and the optimal order quantity. The above researches regard the defect rate as a random variable, make some improvements on the basis of the traditional EOQ/EPQ model, and determine the economic order quantity aiming at the maximum profits.

The researches above have solved the problem of shortage existent in the inspection process, and deal with inspection cost independently, and bring us some positive reference significances. But the methods of handling of defective products need to be improved further more.

In the economic life, some of the defective products are not disposed in the way mentioned above, such as reprocessing and selling at a lower price, and so on. In most cases, defective products are sold at different prices according to the levels of product defects. For example, the famous electronic retailers--Amazon, has already begun to sell some defective goods or second-hand products, which have a few quality problems. This mechanism can not only bring the recycling of the goods which just have minor defective problems, but also provide consumers with a safe channel of buying goods at a discount, thus is beneficial to them. In real life, the defective products may be set at different prices because of the extent of defects. It will meet different market demands because of the price differences. The quantity of goods brings a certain amount of defective ones, while the defective degree and the price will affect the quantity of products.

In conclusion, this paper established the model of economic order quantity with the condition of random defective rate and no shortage of supply. According to actual investigation, if a same batch of order comes from the same production line, the defective is often due to a failure of the production line. Therefore, it can be concluded that the defects are of the same level if the items are produced in the same production line. Under this assumption, owing to the infinite range of inventory time, we can regard each ordering cycle as a renewal process so as to get the expression of the expected profit and optimize the ordering strategy, and draw the conclusion of the impact of product defect rate, degree of product defects, and other related factors related to the ordering strategy. Finally we examine the model by virtue of numerical examples.

1. PROBLEM DESRIPTION AND MATHEMATICAL MODEL

1.1 Problem Description

The case is as follows: The commodities of size y which needs to be inspected are delivered instantaneously. As the inspection costs are considered as a part of the unit order cost, we can simplify the model further. Setting the amount unit purchasing and inspection price as a whole, we named each unit is c and each ordering is k. It is assumed that each received commodities contains a certain percentage defectives. The percentage is p, which obeys a known probability density function, $I(p)$. Each unit of the good-quality items' selling price is s. The good-quality items cannot be affected by defects, thus the demand of good-quality items only relates to the price. And the demand function of good-quality items can be expressed as $f(s)$. Unlike previous research, in this paper, it is assumed that defective items can be sold as at a discounted price v, and the price depends on the degree of defects e, thus the demand function can be expressed as $g(v,e)$. All items of different quality are kept in stock, and different items are sold at different prices, and have different quantity demand.

So the behavior of the inventory level is shown in Figure 1 and Figure 2.

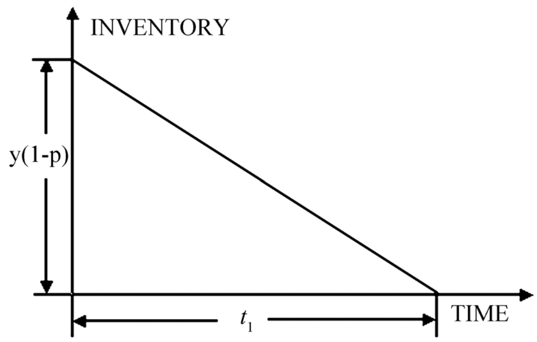

Figure 1
Behavior of the Perfect Products Inventory Level

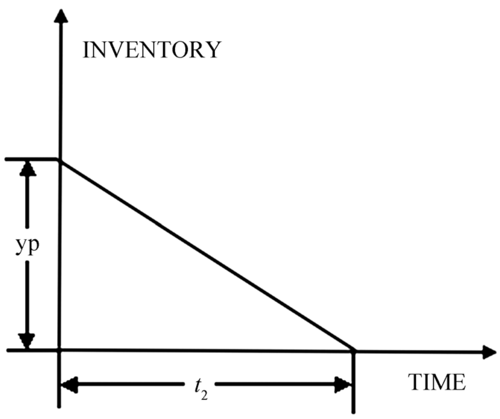

Figure 2
Behavior of the Defectives Inventory Level

In Figure 1, t_1 is the cycle length, (1-p) is the number of perfect products with drawn from inventory and (1-p) is the total selling time of perfect units ordered per cycle. In Figure 2, t_2 is the cycle length, is the number of defectives with drawn from inventory and is the total selling time of defectives units per cycle. Because the demand rate of defectives is bigger than the rate of perfect products, and the number of perfect items is bigger than defectives, we can come to the conclusion that $t_1 > t_2$. However, there will still be a certain percentage of perfect items in stock after the defectives sold out, thus it will cost to sell all items. The ensemble inventory tendency is illustrated in Figure. 3.

In Figure 3, T is the cycle length, and T=t_1, y is the number of products with drawn from inventory and t_2 is the selling time of perfect units and defectives per cycle.

The optimum operating inventory doctrine is obtained by trading off total revenues per unit time, procurement cost per unit time, the inventory carrying cost per unit time so that their sum will be a maximum.

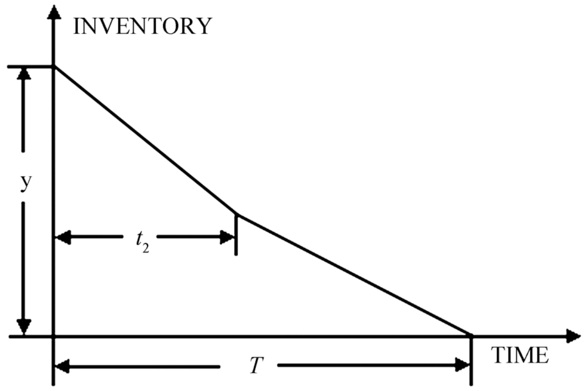

Figure 3
Behavior of the Inventory Level Over Time

1.2 Assumptions and Model Parameters

1.2.1 Assumptions

In this paper, the model is built under the condition of the following assumptions:

1) The demand rate is stable in a cycle;

2) The arrival of the goods immediately thus there is no need to consider the impact of the shortage;

3) Each order will contain a certain imperfect products, and the imperfect products account for the proportion of all products for p;

4) Different items are sold at different prices, and the price of defectives is lower;

5) In each cycle, the arrival of the products is from the same batch production, thus we can assume that the degree of defect is the same, which is expressed as e, and in this paper, e will be a mathematical expectation value;

6) Different items have different quantity demand. For the perfect products, which have a very stable and mature marketing environment, then the demand only relate to price; for the imperfect products, thus defectives, the demand relate to price and the degree of defects both, the degree of defects is higher, the demand will be higher.

1.2.2 Model Parameters

y: order size

c: unit variable cost, c has already included the cost of products screening

K: fixed cost of placing an order

p: percentage of defective items in y

$I(\rho)$: probability density function of p

s: unit selling price of items of good quality

v: unit selling price of defective items

e: the degree of defects, and in this paper, e will be an expectation

$f(s)$: the demand function of items of good quality

g(v, e): the demand function of defective items

1.3 Mathematical Model and Analysis

The optimum operating inventory doctrine is obtained by trading of total revenues per unit time, procurement cost per unit time, the inventory carrying cost per unit time so that their sum will be a maximum.

Define $M(y, p)$ as the number of good items in each order, lot size less defective items, and it is represented as $M(y, p)=y(1-p)$; and $N(y, p)$ as the number of defective items in each order, and it is represented as $N(y, p)=py$.

To avoid shortages, it is assumed that number of good items, $N(y, p)$, is at least equal to the demand during selling time T, that is $M(y, p)\geqslant f(s)T$.

Now define $TR(y)$ and $TC(y)$ as the total revenue and the total cost per cycle, respectively. $TR(y)$ is the sum of total sales volume of good quality and imperfect quality items.

$$TR(y) = sy(1-p) + vpy \quad (1)$$

The total cost contains two parts, thus the inventory cost and the ordering cost, which are defined as:

$$\frac{h}{2}\left[\frac{y^2(1-p)^2}{f(s)}+\frac{y^2p^2}{g(v,e)}\right];\ K+cy \quad (2)$$

Then $TC(y)$ can be defined as:

$$TC(y)=\frac{h}{2}\left[\frac{y^2(1-p)^2}{f(s)}+\frac{y^2p^2}{g(v,e)}\right]+K+cy \quad (3)$$

And because there exists no item in stock by the end of every cycle, and the imperfect quality items will be sold out in advance, so the cycle refresh at the time that good quality items be sold out, then the cycle length T will be defined as:

$$T=\frac{y(1-p)}{f(s)} \quad (4)$$

The total profit per cycle is the total revenue per cycle less the total cost per cycle, thus $TP(y)=TR(y)-TC(y)$, so it is given as:

$$TP(y)=sy(1-p)+vpy-\left\{K+cy+\frac{h}{2}\left[\frac{y^2(1-p)^2}{f(s)}+\frac{y^2p^2}{g(v,e)}\right]\right\} \quad (5)$$

The total profit per unit time is given by dividing the total profit per cycle by the cycle length, $TPU(y)=TP(y)/T$, then it can be written as:

$$TPU(y)=\frac{sy(1-p)+vpy-\left\{K+cy+\frac{h}{2}\left[\frac{y^2(1-p)^2}{f(s)}+\frac{y^2p^2}{g(v,e)}\right]\right\}}{y(1-p)/f(s)} \quad (6)$$

Simplified as:

$$TPU(y)=-\frac{hf(s)}{2(1-p)}\left[\frac{(1-p)^2}{f(s)}+\frac{p^2}{g(v,e)}\right]y-\frac{f(s)K}{1-p}.\frac{1}{y}+sf(s)+vf(s).\frac{p}{1-p}-\frac{cf(s)}{1-p} \quad (7)$$

Thus:

$$TPU(y)=-\frac{hf(s)}{2}\left[(1-p)\frac{1}{f(s)}+\frac{p^2}{(1-p)}.\frac{1}{g(v,e)}\right]y-\frac{f(s)K}{1-p}.\frac{1}{y}+sf(s)+vf(s).\frac{p}{1-p}-\frac{cf(s)}{1-p} \quad (8)$$

Since p is a random variable with a known probability density by definition, $f(p)$, then the expected value of $ETPU(y)$, is given as:

$$ETPU(y)=-\frac{hf(s)}{2}\left[E(1-p).\frac{1}{f(s)}+E\left(\frac{p^2}{1-p}\right).\frac{1}{g(v,e)}\right]y-\frac{f(s)K}{E(1-p)}.\frac{1}{y}+sf(s)+vf(s).E\left(\frac{p}{1-p}\right)-\frac{cf(s)}{E(1-p)} \quad (9)$$

Then the expression of $ETPU(y)$will be discussed from the following aspects:

Theorem 1: the expression of $ETPU(y)$ is a strictly concave function, and there exists a unique value y^* that maximizes $ETPU(y)$, which can be defined as:

$$y^*=\sqrt{\frac{2K}{E(1-p).h}}.\sqrt{\frac{1}{\frac{E(1-p)}{f(s)}+\frac{E\left(\frac{p^2}{1-p}\right)}{g(v,e)}}} \quad (10)$$

Proof: According to the expression of ,

$$\frac{\partial(ETPU)}{\partial y}=-\frac{hf(s)}{2}\left[E(1-p).\frac{1}{f(s)}+E\left(\frac{p^2}{1-p}\right).\frac{1}{g(v,e)}\right]+\frac{f(s)K}{E(1-p)}.\frac{1}{y^2} \quad (11)$$

$$\frac{\partial^2(ETPU)}{\partial y^2}=-\frac{f(s)K}{E(1-p)}.\frac{1}{y^3} \quad (12)$$

By the result, we can draw the conclusions that $ETPU'$, is positive in a certain value area, and is negative in another value area. And $ETPU'$ is negative for all values. Thus $\partial^2 (ETPU)/\partial y^2=(-f(s)K)/(E(1-p)y^3<0))$. So it implies that there exists a unique value that maximizes . Let $\partial^2 (ETPU)/\partial y^2=0$, then we can get Eqution (10) as theorem 1 showed

Especially, if $p=0$, it indicates that each order is perfect product, in this case, $y^*=\sqrt{2Kf(s)/h}$, it is a EPQ model without defective product, thus the traditional EOQ formulae.

Theorem 2: When K increases, thus the fixed cost of placing per order increases, then y^* will increase. This suggests that vender will increase the quantity each order so as to reduce cost.

Proof: Based on the expression of Eqution (13), we can see $(\partial y^*)/(\partial K>0)$, so it can be proofed easily.

Theorem 3: When the proportion of imperfect products in each order remains the same, thus p remains the same, if the price of perfect product or imperfect increases, the order quantity will decrease and the order batch will increase. And if the ideal level of imperfect products increases, then the order quantity will increase.

Proof: As in the assumption, the demand of perfect products only relates to price, and the price is higher, the demand is bigger, thus $\partial y^*/\partial s>0$; for the imperfect products, thus defectives, the demand relate to price and the degree of defects both, the degree of defects is higher, the demand will be higher, thus $\partial g(v, e)/\partial v<0$, $\partial g(v, e)/$

$\partial e<0$. When p, K, h remain the same, only relates to s, v, e. According to the expression of y^*, $1/y^*$, will increase with the increase of $E(1-p)/f(s)+E(p^2/1-p)/g(v,e)$, and because y^* is positive all the time, so we can draw the conclusion that y^* is correlated with$1/y^*$, and is correlated with $E(1-p)/f(s)+E(p^2/(1-p))/g(v,e)$. Then by analyzing the expression of $E(1-p)/f(s)+E(p^2/1-p)/g(v,e)$, thus we can gain the variation of y^*. Because the plus or minus $\partial(1/y^*)/\partial s$ and $-E(1-p)f(s)/f^2(s)$ is the same, because $-E(1-p)f(s)'/f^2(s)>0$. Then we can draw the following conclusion: $\partial(1/y^{*2})/\partial s>0$, $\partial(1/y^{*2})/\partial v>0$, $\partial(1/y^{*2})/\partial e<0$ so with the increase of s and v, $1/y^{*2}$ increases; with the increase of e, $1/y^{*2}$ decreases.

So we can indicate that: Either perfect product or imperfect increases, when its price increases, the demand will decrease, thus the inventory cost will increase, while the each item sale profit increases, but the sum profit increase cannot make up for the inventory additional cost because of the low demand, so the venders tend to order less items per time, thus the frequency of order will increase.

And if the production technology or transportation improves, then the imperfect items will decrease, or the imperfect products are more close to perfect products while the imperfect products have a lower price, so its demand will increase. Because the amount of imperfect products every time is not controllable, then by increasing the quantity of goods , there will be more sales profit, relative to the inventory cost, ordering cost can bring more benefit, and the benefit will make up with the additional inventory cost ,and so on, so the order quantity will increase.

Theorem 4: When p obeys a certain uniform distribution in the range from a to b, then if the expectation of p remains the same while its variance increases, thus p becomes more unstable, y^* will increase. It indicates that if the estimation of the amount of imperfect products becomes more difficult, then the venders tend to increase the order quantity so as to reduce the possibility of additional order cost result in too many imperfect products containing.

Proof: Based on the expression of Eqution (13), then we can know that $1/y^{*2}$ is positively correlated with $E(1-p)/f(s)+E(p^2/(1-p))/g(v,e)$, if $f(s)$, $g(v,e)$ remain the same, thus s, v, e remain the same, to simplify the calculation, we can regard $f(s)$, $g(v,e)$ as two constant which can be noted as m, n. Then the expression can be simplified as $[E(1-p)]^2/m+E(p^2/1-p)E(1-p)/n$, then we can think $1/y^{*2}$ is positive correlated with $[E(1-p)]^2/m+E(p^2/1-p)E(1-p)/n$.

According to the hypothesis, the expectation of p remains the same while its variance increases, $1/y^{*2}$ only relates to $E(p^2/1-p)$.

Then p obeys a certain uniform distribution in the range from a to b, we can get that $E(p)=(a+b)/2$ and $D(p)=(b-a)^2/12$, then we can get:

$$E\left(\frac{p^2}{1-p}\right)=\int_a^b\frac{p^2}{1-p}.\frac{1}{b-a}dp=\frac{1}{b-a}In\frac{1-a}{1-b}-\frac{1}{2}(a+b)-1 \quad (13)$$

Because the expectation of p remains the same, the remains the same, only relates to $\text{In}[(1-a)/(1-b)]/(b-a)$.

$$\frac{1}{b-a}In\frac{1-a}{1-b}=\frac{1}{b-a}In\frac{1-\frac{(b-a)-(b+a)}{2}}{1-\frac{(b+a)+(b-a)}{2}}=\frac{1}{b-a}In\frac{(2+b+a)-(b-a)}{(2-b-a)-(b-a)} \quad (14)$$

Then we can get that

$$\frac{\partial\left[\frac{1}{b-a}In\frac{(2+b+a)-(b-a)}{(2-b-a)-(b-a)}\right]}{\partial(b-a)}<0 \quad (15)$$

It indicates that Eqution (14) will decrease with the increase of $(b-a)$, when $(b-a)$ increases, thus the variance of p increases, then $E(p^2/1-p)$ and $1/y^{*2}$ decreases, and y^* increases.

The results can be summarized as follows: In Makah's portfolio theory, the essence of proposing an investment decision is to make choices between uncertain risks and large benefits. The benefits and risks of these two key factors can be described by expectation and variance. The so-called expectations are the expected return portfolios, which are the expectations of individual investment returns. Variance is the income of investment decision, which reflects the risk of the portfolio. Portfolio theory is possible that the variance is one of the determinants of portfolio risk. And investment returns and risks are often positive correlation. The low risk investment cannot often bring higher returns, while higher earnings are often accompanied by high investment risks. This means that if we pursuit high yield, then we need variance of portfolio investment--increase the investment risks and the pursuit of a higher investment return. In this case, the capital of enhancing venture reduces the order costs.

Therefore, if p becomes more unstable, the variance of p will become bigger. It can be possible to conclude that the order risk will rise. Driven by profits, buyers will tend to increase the quantity of order and obtain a higher return on investment by large orders of several times.

2. NUMERICAL EXAMPLE

To illustrate the application of the model and practical significance intuitively, this paper provides a numerical analysis. In a smooth market with an uncertainty *"K"*, giving the assuming that the maximum demand of perfect products is 20,000, and the maximum demand rate of imperfect products is 30,000. And the purchase cost $c=5$/unit, the inventory *cost* $h=10$/unit, the imperfect degree $e=0.6$, and the perfect products price $s=100$/unit, the imperfect product price $v=60$/unit. Given the assumption that $f(s)=20000-100s$, $g(v,e)=e(30000-200s)$, we can gain the change rule roughly. The percentage defective random variable p obeys a certain uniform distribution in the range from 0 to 0.2, $E(p)$=0.1, $E(1-p)$=0.9.

Based on the assumptions, we observe the influence of value change in *"K"* on the quantity of goods and other dependent variables. From Table 1, we can see that, y^* will increase with the increase of K, inventory costs and total price will increase at the same time, the profit of per unit time is reduced gradually.

Assume that the percentage defective random variable p stays the same, then we observe the influence of product price (the imperfect product price is only related to degree of perfect price and the perfect price, when the degree of perfect e remains the same, the trend of the perfect price is similar to the imperfect's. Seen from Table 2, with the increase of the perfect price, the optimal order quantity is reduced, to increase the order batch. As shown in Table 3, if the ideal level of imperfect products increases, then the quantity of goods will increase.

Assuming that the expectation of p remains the same while its variance increases, we can see from Table 4 that when p becomes more unstable, will increase. It indicates that the buyer will increase the order quantity if the product quality becomes more uncontrolled.

Table 1
Sensitivity Analysis of Ordering Cost

K	*y**	*Inventory*	*Ordering*	*TR(y)*	*T*	*ETPU(y)*
100.00	494.37	100.11	2571.84	47459.28	0.04	995924.29
200.00	699.14	200.23	3695.71	67117.55	0.06	994880.41
300.00	856.27	300.34	4581.35	82201.88	0.08	994079.41
400.00	988.73	400.45	5343.67	94918.55	0.09	993404.13
500.00	1105.44	500.57	6027.20	106122.17	0.10	992809.21
600.00	1210.95	600.68	6654.74	116251.01	0.11	992271.35
700.00	1307.97	700.79	7239.87	125565.44	0.12	991776.74
800.00	1398.28	800.91	7791.41	134235.10	0.13	991316.37
900.00	1483.10	901.02	8315.51	142377.83	0.13	990883.98
1000.00	1563.33	1001.13	8816.64	150079.41	0.14	990475.01

Table 2
Sensitivity Analysis of Price

S	*V*	*f(s)*	*g(v, e)*	y^*	*TR(y)*	*ETPU(y)*
100.00	60.00	10000.00	10800.00	1563.33	150079.41	990475.01
105.00	63.00	9500.00	10440.00	1523.88	153606.76	990821.47
110.00	66.00	9000.00	10080.00	1483.37	156644.10	985903.34
115.00	69.00	8500.00	9720.00	1441.72	159166.36	975721.08
120.00	72.00	8000.00	9360.00	1398.83	161145.25	960275.20
125.00	75.00	7500.00	9000.00	1354.57	162548.51	939566.33
130.00	78.00	7000.00	8640.00	1308.81	163339.07	913595.15
135.00	81.00	6500.00	8280.00	1261.37	163473.86	882362.51
140.00	84.00	6000.00	7920.00	1212.07	162902.29	845869.39
145.00	87.00	5500.00	7560.00	1160.66	161564.08	804117.01

Table 3
Sensitivity Analysis of Ordering Perfect Degree

e	*g(v, e)*	y^*	*TR(y)*	*ETPU(y)*
0.30	7200.00	1559.36	145020.70	963419.02
0.35	8050.00	1560.62	145917.50	967936.73
0.40	8800.00	1561.52	146783.00	972449.53
0.45	9450.00	1562.19	147627.08	976958.99
0.50	10000.00	1562.69	148455.62	981466.04
0.55	10450.00	1563.06	149272.29	985971.26
0.60	10800.00	1563.33	150079.41	990475.01
0.65	11050.00	1563.51	150878.45	994977.55
0.70	11200.00	1563.61	151670.32	999479.02
0.75	11250.00	1563.65	152455.45	1003979.50

Table 4
Sensitivity Analysis of the Percentage Defective Random Variable

p	$E(p^2/1-p)$	y^*	*TR(y)*	*ETPU(y)*
[0.40,0.40]	—	—	—	—
[0.35,0.45]	8.11	641.16	61551.31	943356.79
[0.30,0.50]	3.87	893.04	85732.23	950171.88
[0.25,0.55]	2.45	1078.58	103543.48	953001.58
[0.20,0.60]	1.73	1229.66	118046.97	954590.17
[0.15,0.65]	1.30	1359.02	130465.91	955614.98
[0.10,0.70]	1.01	1473.43	141449.23	956332.53
[0.05,0.75]	0.80	1577.06	151398.14	956863.25
[0.00,0.80]	0.64	1672.79	160587.66	957271.64

CONCLUSION

In the actual inventory management, it is hard to avoid the defective products. When dealing with the defective products, the traditional EOQ model cannot manage to solve it. Therefore we need to establish a new model to describe this kind of reality. In this paper, we focus on the new EOQ model which has a random defect rate and the commodities are enough. Under the unlimited stock time, I use the update method to get the expression of proper order. Besides, the correlation analysis will be done to get the relations between the defect rate, defect product price and related factors and the optimal order quantity. To broad the existing research, conclusion and the model will be checked by the specific example. In order to make the model further studied, we will focus on the handing strategy of the defective product in the future investigation. For example, the relationship between the price and the optimal order quantity will be discussed when the defective products compete with the perfect products or how to formulate the price strategy, etc..

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