



Progress in Applied Mathematics

Vol. 3, No. 1, 2012, pp. 16-21

DOI: 10.3968/j.pam.1925252820120301.1700

ISSN 1923-8444 [Print]

ISSN 1925-2528 [Online]

www.cscanada.net

www.cscanada.org

The Study of a Class of Multidimension Stochastic System

XIA Xuewen^{1,2,*}

¹Hunan Institute of Engineering, Xiangtan, 411104, China

²Hunan Normal University, Changsha, 410082, China

*Corresponding author.

Address: Hunan Institute of Engineering, Xiangtan, 411104, China

Received September 10, 2011; accepted January 11, 2012

Abstract

In this paper, we use wavelet methods to analyse a class of multidimension linear stochastic system, we obtain its average power and density degree, wavelet expansion and relation of expansion coefficient.

Key words

Stochastic system; Density degree; Wavelet expansion; Average power; Relation

XIA Xuewen (2012). The Study of a Class of Multidimension Stochastic System. *Progress in Applied Mathematics*, 3(1), 16-21.

Available from: URL: http://www.cscanada.net/index.php/pam/article/view/j.pam.192525282012_0301.1700

DOI: <http://dx.doi.org/10.3968/j.pam.1925252820120301.1700>

1. INTRODUCTION

The stochastic system is very important in many aspects.

Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering. The basic idea is to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks come in different sizes, and are suitable for describing features with a resolution commensurate with their sizes.

There are two important aspects to wavelets, which we shall call “mathematical” and “algorithmic”. Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates. As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain, wavelet based algorithms exhibit a number of new and important properties. Recently some persons have studied wavelet problems of stochastic process or stochastic system (see [1]-[17]). In this paper, we study the system (see [7]) as follows to use wavelet methods.

We will take wavelet and use them in a series expansion of signal or function. Wavelet has its energy concentrated in time to give a tool for the analysis of transient, nonstationary, or time-varying phenomena. It still has the oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. We take wavelet and use them in a series expansion of signals or functions much the same way a Fourier series the wave or sinusoid to represent a signal or function. In order to use the idea of multiresolution, we will start by defining the scaling function and then define the wavelet in terms of it.

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In fields ranging from Extragalactic Astronomy to Molecu-

lar Spectroscopy to Medical Imaging to computer vision, One must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data. Wavelets have contributed to this already intensely developed and rapidly advancing field.

2. BASIC MODEL

We consider system

$$X(t) = X_0 + \int_{t_0}^t F(\tau)X(\tau)d\tau + \int_{t_0}^t B(\tau)u(\tau)d\tau + \int_{t_0}^t G(\tau)d\beta(\tau) \quad (1)$$

Where $X(t)$ is n -state variable, $u(t)$ is a deterministic input, $F(t)$ is $n \times n$ matrix, $B(t)$ is $n \times n$ input matrix, $G(t)$ is $n \times n$ matrix, $\beta(t)$ is n -Brownian motion.

Suppose (1) satisfy:

1) $E\{d\beta(t) d\beta^T(t)\} = Q(t)dt;$

2) X_0 is Gauss random variable, the average is \bar{X}_0 , X_0 and $\beta(t)$ is independent each other

3) \int_{R^2} .

We have^[8]

$$X(t) = e^{F(t-t_0)}X(t_0) + \int_{t_0}^t e^{F(t-\tau)}B(\tau)u(\tau)d\tau + \int_{t_0}^t e^{F(t-\tau)}G(\tau)d\beta(\tau) \quad (2)$$

Where $F(t) = F$ is constant matrix.

We can let

$$X(t) = e^{F(t-t_0)}X_0 + \int_{t_0}^t e^{F(t-\tau)}G(\tau)d\beta(\tau) \quad (3)$$

Then have

$$m_x(t) = E\{X(t)\} = (u),$$

$$P_x(t) = E\{[X(t) - m_x(t)][X(t) - m_x(t)]^T\} = e^{F(t-t_0)}P_0e^{F^T(t-t_0)} + \int_{t_0}^t e^{F(t-\tau)}GQG^Te^{F^T(t-\tau)}d\tau,$$

Where $\bar{X}_0 = E\{\bar{X}_0\}$, $P_0 = E[(X_0 - \bar{X}_0)(X_0 - \bar{X}_0)^T]$.

Let $H = \{X(t), t \in T\}$ is continuous n -random processes, $E\{[X(t)][X(t)]^T\} < \infty$.

Let $\langle X(t), X(s) \rangle = E\{X(t) \cdot X(s)\}$, $t, s \in T \subset R$, let

$$\|X\| = \langle X(t), X(t) \rangle^{1/2} = (E\{[X(t)][X(t)]^T\})^{1/2}$$

To $X(t) \in H$, its wavelet transform is^[4]

$$WX(s, x) = \frac{1}{s} \int_R X(t)\Psi\left(\frac{x-t}{s}\right)dt, \quad s \in R_+, t \in R \quad (4)$$

Where ψ is mother wavelet^[4]. We have

Theorem 1

$$\int_0^\infty \int_R WX(s, x) \cdot Wg(s, x)dx s^{-2}ds = C_\Psi \langle x, g \rangle \quad (5)$$

Where $C_\Psi = \int_0^\infty |\hat{\Psi}(t)|^2 t^{-1}dt < \infty$.

Proof: to each s , $WX(s, x) \in H$, then

$$\begin{aligned} \int_0^\infty \int_R WX(s, x) \cdot Wg(s, x)dx s^{-2}ds &= 2\pi \int_0^\infty \int_R \hat{X}(\xi)\hat{g}(\xi) |\hat{\Psi}(s, \xi)|^2 d\xi s^{-1}ds \\ &= 2\pi \int_R \int_0^\infty |\hat{\Psi}(s, \xi)|^2 s^{-1}ds \hat{x}(\xi)\hat{g}(\xi)d\xi = C_\Psi \langle x, g \rangle. \end{aligned}$$

3. SOME PROPERTIES OF WAVELET TRANSFORM

To (3), we have

$$\begin{aligned} \text{WX}(s, x) &= \frac{1}{s} \int_R \left[e^{F(t-t_0)} \mathbf{X}_0 + \int_{t_0}^t e^{F(t-\tau)} \mathbf{G}(\tau) d\beta(\tau) \right] \Psi \left(\frac{x-t}{s} \right) dt \\ &= \frac{1}{s} \int_R \left[e^{F(t-t_0)} \mathbf{X}_0 \Psi \left(\frac{x-t}{s} \right) dt + \int_R \int_{t_0}^t e^{F(t-\tau)} \mathbf{G}(\tau) d\beta(\tau) \Psi \left(\frac{x-t}{s} \right) dt \right], \\ [\text{WX}(s, x)]^2 &= \frac{1}{s^2} \left\{ \left[\int_R e^{F(t-t_0)} \mathbf{X}_0 \Psi \left(\frac{x-t}{s} \right) dt \right]^2 \right. \\ &\quad + 2 \int_R e^{F(t-t_0)} \mathbf{X}_0 \Psi \left(\frac{x-t}{s} \right) dt \int_R \int_{t_0}^t e^{F(t-\tau)} \mathbf{G}(\tau) d\beta(\tau) \Psi \left(\frac{x-t}{s} \right) dt \\ &\quad \left. + \left[\int_R \int_{t_0}^t e^{F(t-\tau)} \mathbf{G}(\tau) d\beta(\tau) \Psi \left(\frac{x-t}{s} \right) dt \right]^2 \right\} \end{aligned}$$

Then

$$\begin{aligned} E[\text{WX}(s, x)]^2 &= \frac{1}{s^2} \left\{ E \left[\int_R e^{F(t-t_0)} \mathbf{X}_0 \Psi \left(\frac{x-t}{s} \right) dt \right]^2 \right. \\ &\quad \left. + E \left[\int_R \int_{t_0}^t e^{F(t-\tau)} \mathbf{G}(\tau) \Psi \left(\frac{x-t}{s} \right) d\beta(\tau) dt \right]^2 \right\}. \end{aligned}$$

We have:

Theorem 2 The average power of $\text{WX}(s, x)$ is

$$E[\text{WX}(s, x)]^2 = \frac{1}{s^2} \left\{ E \left[\int_R e^{F(t-t_0)} \mathbf{X}_0 \Psi \left(\frac{x-t}{s} \right) dt \right]^2 + E \left[\int_R \int_{t_0}^t e^{F(t-\tau)} \mathbf{G}(\tau) \Psi \left(\frac{x-t}{s} \right) d\beta(\tau) dt \right]^2 \right\}.$$

4. DENSITY

Consider

$$\begin{aligned} R(\tau) &\triangleq E[\text{WX}(s, x + \tau) \text{WX}(s, x)] \\ &= \frac{1}{s^2} \iint_{R^2} E[X(u)X(v)] \Psi \left(\frac{u - (x + \tau)}{s} \right) \Psi \left(\frac{v - x}{s} \right) dudv \end{aligned}$$

Use (3) we have

$$\begin{aligned} R(\tau) &= \frac{1}{s^2} \left\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^T(v-t_0)} \Psi \left(\frac{u - (x + \tau)}{s} \right) \Psi \left(\frac{v - x}{s} \right) dudv \right. \\ &\quad \left. + \iint_{R^2} \left[\int_{t_0}^{\min(u, v)} e^{F(u-\tau)} \mathbf{G}(\tau) \mathbf{Q}(\tau) \mathbf{G}^T(\tau) e^{F^T(v-\tau)} d\tau \right] \cdot \Psi \left(\frac{u - (x + \tau)}{s} \right) \Psi \left(\frac{v - x}{s} \right) dudv \right\}, \end{aligned}$$

then

$$R^{(1)}(\tau) = \frac{1}{s^2} \left\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^T(v-t_0)} \Psi \left(\frac{u - (x + \tau)}{s} \right) \Psi \left(\frac{v - x}{s} \right) dudv \right.$$

$$\begin{aligned}
 & + \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-\tau)} G(\tau) Q(\tau) G^T(\tau) e^{F^T(v-\tau)} d\tau \right] \cdot \Psi' \left(\frac{u-(x+\tau)}{s} \right) \Psi \left(\frac{v-x}{s} \right) dudv \Big\}, \\
 R^{(2)}(\tau) & = \frac{1}{s^2} \left\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^T(v-t_0)} \Psi'' \left(\frac{u-(x+\tau)}{s} \right) \Psi \left(\frac{v-x}{s} \right) dudv \right. \\
 & \left. + \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-\tau)} G(\tau) Q(\tau) G^T(\tau) e^{F^T(v-\tau)} d\tau \cdot \Psi'' \left(\frac{u-(x+\tau)}{s} \right) \Psi \left(\frac{v-x}{s} \right) \right] dudv \right\},
 \end{aligned}$$

We have

$$\begin{aligned}
 R(0) & = \frac{1}{s^2} \left\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^T(v-t_0)} \Psi \left(\frac{u-x}{s} \right) \Psi \left(\frac{v-x}{s} \right) dudv \right. \\
 & \left. + \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-t)} G(t) Q(t) G^T(t) e^{F^T(v-t)} dt \right] \cdot \Psi \left(\frac{u-x}{s} \right) \Psi \left(\frac{v-x}{s} \right) dudv \right\}, \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 R^{(2)}(0) & = \frac{1}{s^2} \left\{ \iint_{R^2} e^{F(u-t_0)} \bar{P}_0 e^{F^T(v-t_0)} \Psi'' \left(\frac{u-x}{s} \right) \Psi \left(\frac{v-x}{s} \right) dudv \right. \\
 & \left. + \iint_{R^2} \left[\int_{t_0}^{\min(u,v)} e^{F(u-t)} G(t) Q(t) G^T(t) e^{F^T(v-t)} dt \right] \cdot \Psi'' \left(\frac{u-x}{s} \right) \Psi \left(\frac{v-x}{s} \right) dudv \right\}, \quad (7)
 \end{aligned}$$

Where $\bar{P}_0 = E(X_0 X_0^T)$.

Theorem 3 the zero density of $X(t)$ is $ds = |R^{(2)}(0)|/\pi^2 R(0)^{1/2}$ can obtain use (6) and (7).

5. WAVELET REPRESENTATION

We know^[5] have $X_m(t) \in H$, then $E [X(t) - X_m(t)]^2 \rightarrow 0, m \rightarrow \infty, t \in R$,

$$\text{And } X_m(t) = \sum_{k=-\infty}^{m-1} \sum_{n=-\infty}^{\infty} b_{kn} \Psi_{kn}(t),$$

Where $b_{kn} = \int_R X(t) \Psi_{kn}(t) dt$. have

$$\begin{aligned}
 E [b_{mn} \cdot b_{kj}] & = \iint_{R^2} E [X(t) \cdot X(t)] \Psi(2^m t - n) \Psi(2^m t - n) \Psi(2^k s - j) 2^{m/2} 2^{k/2} dt ds \\
 & = \iint_{R^2} e^{F(t-t_0)} \bar{P}_0 e^{F^T(s-t_0)} \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds \\
 & \quad + \iint_{R^2} \left[\int_{t_0}^{\min(s,t)} e^{F(t-\tau)} G(\tau) Q(\tau) G^T(\tau) e^{F^T(s-\tau)} d\tau \right] \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds.
 \end{aligned}$$

Let $t_0 \rightarrow -\infty$, then have

$$\begin{aligned}
 \lim_{t_0 \rightarrow -\infty} E [b_{mn} \cdot b_{kj}] & = \iint_{R^2} \left[\int_0^{\infty} e^{Ft} G(\tau) Q(\tau) G^T(\tau) e^{F^T s} d\tau \right] \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds \\
 & = \iint_{R^2} e^{Ft} \left[\int_0^{\infty} G(\tau) Q(\tau) G^T(\tau) d\tau \right] e^{F^T s} \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds \\
 & = C \iint_{R^2} e^{Ft} e^{F^T s} \cdot \Psi(2^m t - n) \Psi(2^k s - j) 2^{(m+k)/2} dt ds,
 \end{aligned}$$

Where $C = \int_0^{\infty} G(\tau) Q(\tau) G^T(\tau) d\tau$.

Let $t \rightarrow \infty$, then

$$\lim_{t_0 \rightarrow \infty} E [b_{mm} \cdot b_{kj}] = 0$$

We know have $\varphi(t) = 2^{1/2} \sum_{k \in \mathbb{Z}} h_k \varphi(2x - k) h_k \in L^2$, $x \in \mathbb{R}$, $k \in \mathbb{Z}$.

$$\text{Let } \Psi(t) = 2^{1/2} \sum_{k \in \mathbb{Z}} (-1)^k h_{1-k} \varphi(2x - k),$$

Then we have

$$X(t) = 2^{J/2} \sum_{n \in \mathbb{Z}} C_n^J \varphi(2^{-J}t - n) + \sum_{j \leq J} 2^{-j/2} \sum_{n \in \mathbb{Z}} d_n^j \Psi(2^{-j}t - n)$$

Where $C_n^j = 2^{-j/2} \int_{\mathbb{R}} X(t) \varphi(2^{-j}t - n) dt$, $d_n^j = 2^{-j/2} \int_{\mathbb{R}} X(t) \Psi(2^{-j}t - n) dt$.

We have

$$\begin{aligned} E [d_n^j \cdot d_m^k] &= 2^{-(j+k)/2} \int_{\mathbb{R}^2} E [X(t) \cdot X(t)] \Psi(2^{-j}t - n) \Psi(2^{-k}s - m) dt ds \\ &= 2^{-(j+k)/2} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} e^{F(t-t_0)} \bar{P}_0 e^{F^T(s-t_0)} \Psi(2^{-j}t - n) \Psi(2^{-k}s - m) dt ds \\ &\quad + \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \left[\int_{t_0}^{\min(s,t)} e^{F(t-\tau)} G(\tau) G^T(\tau) e^{F^T(s-\tau)} d\tau \right] \cdot \Psi(2^{-j}t - n) \Psi(2^{-k}s - m) dt ds \end{aligned}$$

Let $\psi(t)$ have support set $[-K_1, K_2]$, $K_1, K_2 \geq 0$, and have M ,

$$\int_{\mathbb{R}} t^m \Psi(t) dt = 0, \quad 0 \leq m \leq M - 1$$

Then φ have support set $[-K_3, K_4]$, and $K_1 + K_2 = K_3 + K_4$, $K_3, K_4 \geq 0$.

Let $b(j, k) = \langle X(t), \psi_{jk} \rangle$, $a(j, k) = \langle x(t), \psi_{jk} \rangle$, then

$$\{2^{J/2} \varphi(2^J x - k), K \in \mathbb{Z}\} \cup \{2^{j/2} \psi(2^j t - k), K \in \mathbb{Z}\}_{j \geq J}$$

Is a base of $L^2(\mathbb{R})$. and have

$$X(t) = 2^{J/2} \sum_{K \in \mathbb{Z}} a(J, K) \varphi(2^J t - k) + \sum_{j \geq J} \sum_{K \in \mathbb{Z}} 2^{j/2} b(j, k) \Psi(2^j t - k).$$

We have

$$\begin{aligned} R_b(j, k; m, n) &= E[b(j, m) b(k; n)] \\ &= 2^{-(j+k)/2} \int_{\mathbb{R}^2} E [X(t) \cdot X(t)] \Psi(2^{-j}t - n) \Psi(2^{-k}s - n) dt ds \\ &= 2^{-(j+k)/2} \int_{D^2} E [X(t) \cdot X(t)] \Psi(2^{-j}t - n) \Psi(2^{-k}s - n) dt ds, \end{aligned}$$

Where $D = [-K_1, K_2]$.

consider

$$X(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{m,n} \Psi_{m,n}(t),$$

Where $\Psi_{m,n}(t) = 2^{m/2} \Psi(2^m t - n)$, $a_{m,n}(t) = \int_{\mathbb{R}} X(u) \Psi_m(u - n/2^m) du$,

There $\Psi_m(t) = 2^{m/2} \Psi(2^m t)$, Ψ is mother wavelet, have

$$a_{m,n} = 2^{m/2} \int_{\mathbb{R}} X(u) 2^m \Psi(u - n/2^m) du = 2^{m/2} \int_{\mathbb{R}} X(u) \Psi(2^m u - n) du,$$

then $E[a_{m,n}] = 2^{m/2} \int_R e^{F(u-t_0)} \bar{X}_0 \Psi(2^m u - n) du$.

If $X(t)$ is a stationary processes, use (3), we know:

$m_x(t) = 0$,

$P_x(t) = \text{content} = P$.

Then $E[a_{m,n}] = 0$,

$$\begin{aligned} E[a_{m,n} a_{j,k}] &= 2^{(m+j)/2} \iint_{R^2} P \Psi(2^m u - n) \Psi(2^j v - k) dudv \\ &= P \cdot 2^{(m+j)/2} \iint_{R^2} \Psi(2^m u - n) \Psi(2^j v - k) dudv \end{aligned}$$

REFERENCES

- [1] Cambanis S. (1994). Wavelet Approximation of Deterministic and Random Signals. *IEEE Tran on Information Theory*, 40(4), 1013-1029
- [2] Flandrin P. (1992). Wavelet Analysis and Aynthesis of Fractional Brownian Motion. *IEEE Tran on Information Theory*, 38(2), 910-916.
- [3] Krim H. (1995). Multiresolution Analysis of a Class of Nonstationary Processes. *IEEE Tran on Information Theory*, 41(4), 1010-1019.
- [4] Priestley M B (1996). Wavelets and Time C Dependent Spectral Analysis. *J of Time Series Analysis*, 17(1), 85-103.
- [5] Zhang J & Walter G G (1994). A Wavelet Based K-L-Like Expansion for Wide-Sense Stationary Processes. *IEEE Trans Sig, Proc*, 38(2), 814-823.
- [6] Haobo R (2002). Wavelet Estimation for Jumps in a Heteroscedastic Regression Model. *Acta Mathematica Scientia, SerB*, 22(2), 269-276.
- [7] Arnold L (1974). *Stochastic Differential Eputation*. New York: John Wiley & Sons Inc.
- [8] Wong E & Hajek B (1985). *Stochastic Processes in Engineering Systeme*. New York: Springs-Verlay.
- [9] A. Hendi (2009). New Exact Travelling Wave Solutions for Some Nonlinear Evolution Equations. *International Journal of Nonlinear Science*, 7(3), 259-267.
- [10] Xuewen Xia (2005). Wavelet Analysis of the Stochastic System with Coular Stationary Noise. *Engineering Science*, 3, 43-46.
- [11] Xuewen Xia (2008). Wavelet Density Degree of Continuous Parameter AR Model. *International Journal Nonlinear Science*, 7(4), 237-242.
- [12] Xuewen Xia (2007). Wavelet Analysis of Browain Motion. *World Journal of Modelling and Simulation*, (3), 106-112.
- [13] Xuewen Xia & Ting Dai (2009). Wavelet Density Degree of a Class of Wiener Processes. *International Journal of Nonlinear Science*, 7(3), 327-332.
- [14] Xuewen Xia (2010). Haar Wavelet Density of the Linear Regress Processes with Rondon Coefficient. *Proceedings of the Third International Conference on Modeling and Simulation*, 2010(5).
- [15] Xuewen Xia (2010). The Haar Wavelet Density of the Two Order Polynomial Stochastic Processes. *Proceedings of the Third International Conference on Modeling and Simulation*, 2010(3).
- [16] Xuewen Xia (2011). The Study of Wiener Processes With N(0,1)-Random Trend Peocesses Based on Wavelet. *Journal Information and Computing Science*, (2).
- [17] Xuewen Xia (2011). The Study of a Class of the Fractional Brownian Motion Based on Wavelet. *Inter. J. of Nonlinear Science*, (3).