CSCanada
Progress in Applied Mathematics
Vol. 2, No. 2, 2011, pp. 12-15
DOI: 10.3968/j.pam.1925252820110302.1668

ISSN 1923-8444 [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
www.cscanada.org

The Application of Probability Method on Mathematical Analysis

YU Xing[1,*]; SUN Hongguo; LIAO Xiaolian[1]

[1] Department of Mathematics & Applied Mathematics Humanities & Science and Technology Institute of Hunan Loudi, 417000, P.R. China
*Corresponding author.
Address: Department of Mathematics & Applied Mathematics Humanities & Science and Technology Institute of Hunan Loudi, 417000, P.R. China
Email: hnyuxing@163.com

Supported by This work was supported in part by Teaching Research Fund of Hunan Institute of Humanities, Science and Technology of China (No. RKJGZ1012), Hunan province teaching reform subjects 2010(32).

Received 10 September, 2011; accepted 26 October, 2011

Abstract
This paper summarizes the applications of probability method on mathematical analysis, including a class of generalized integra1 and acquiring the sum of infinite series £ so that contact up the mathematical analysis and probability and statistics knowledge ,explain the inner contacts among different branches of mathematics in some discussions.

Key words
Probability method; Mathematical analysis; Generalized integral; The sum of infinite series

XingYu*, Xiaolian Liao (2011). The application of probability method on mathematical analysis. *Progress in Applied Mathematics, 2*(2), 12-15. Available from: URL: http://www.cscanada.net/index.php/pam/article/view/j.pam.1925252820110302.1668
DOI: http://dx.doi.org/10.3968/j.pam.1925252820110302.1668

1. INTRODUCTION

The normal distribution and the exponential distribution are the most commonly used distributions of probability and statistic, whose expectation, variance and moments function are expressed in form of infinite interval generalized integral. How to solve some mathematical analysis problems using them? This paper will give some examples to explain, including a class of generalized integra1 and acquiring the sum of infinite series and the limit of multi-integral. This paper is organized as follows: some related properties of normal distribution and exponential distribution will be given in section 2.we will explain the application of the two distribution's moment to solve some generalized integral, and give six important integral formula and its applications in examples. In part 3, acquiring the sum of infinite series using probability method, some examples will also be showed in this section.

2. THE PROPERTIES OF NORMAL AND EXPONENTIAL DISTRIBUTION

2.1 Definitions

Definition 1[1]. It is said that a random variable X has a normal distribution with mean $\mu$and variance σ^2 if

X has a continuous distribution for which the $p.d.f$ $f(x)$ is as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \text{ for } -\infty < x < \infty$$

Definition 2[1]It is said that a random Xhas an exponential distribution with parameter $\lambda\,(\lambda > 0)$if Xhas a continuous distribution for which the $p.d.f$ $f(x)$is as follows:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$$

2.2 The Properties

(1) The $k-$order moment of normal distribution is:

$$E(X) = \int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \mu$$

$$\begin{aligned} E\left(X^2\right) &= D(X) + (E(X))^2 \\ &= \int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx \\ &= \sigma^2 + \mu^2 \end{aligned}$$

We can get some important integral formulas are:

$$\int_{-\infty}^{+\infty} x \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \sqrt{2\pi}\mu\sigma \tag{1}$$

$$\int_{-\infty}^{+\infty} x^2 \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \sqrt{2\pi}\sigma\left(\sigma^2 + \mu^2\right) \tag{2}$$

We also can get an integral formula from the properties of$p.d.f$, that is:

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = 1$$

So

$$\int_{-\infty}^{+\infty} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx = \sqrt{2\pi}\sigma \tag{3}$$

(2) The $k-$order moment of exponential distribution is:
The same as exponential distribution, some useful formulas are given:

$$\int_0^{+\infty} e^{-\lambda x} dx = \frac{1}{\lambda} \tag{4}$$

$$\int_0^{+\infty} x e^{-\lambda x} dx = \frac{1}{\lambda^2} \tag{5}$$

$$\int_0^{+\infty} x^2 e^{-\lambda x} dx = \frac{2}{\lambda^2} \tag{6}$$

2.3 Examples

Example 1. To calculate generalized integra1 $\int_{-\infty}^{+\infty}(x-2)e^{-\frac{(x-6)^2}{6}}$

Solution:

$$\int_{-\infty}^{+\infty}(x-2)e^{-\frac{(x-6)^2}{6}}dx = \int_{-\infty}^{+\infty}xe^{-\frac{(x-6)^2}{6}}dx - \int_{-\infty}^{+\infty}2e^{-\frac{(x-6)^2}{6}}dx$$

$$\underset{\substack{\mu=6\\ \sigma=\sqrt{3}}}{\overset{(1)(3)}{=}} \sqrt{2\pi}6\sqrt{3} - 2\sqrt{2\pi}\sqrt{3}$$

$$= 4\sqrt{6\pi}$$

3. ACQUIRING THE SUM OF INFINITE SERIES USING PROBABILITY METHOD

By the idea of constructing a stochastic model to obtain the sum of infinite series, such as:

Example 2[2]: to calculate the sum of infinite series

$$\frac{a^2}{k^2} + \left(1-\frac{a^2}{k^2}\right)\frac{a^2}{(k+1)^2} + \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\frac{a^2}{(k+2)^2} + \cdots + \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\cdots\left(1-\frac{a^2}{(k+n-2)^2}\right)\frac{a^2}{(k+n-1)^2} + \cdots$$

Solution: Consider the following model: suppose that there are k balls in the bag with the same size, shape, but different color, where a white balls ($0 < a < k, a, k \in N$), $k-a$ black balls. There are twice back to touch the ball, if the white ball is removed, then that trial is successful; otherwise considered a failure, in this case, then put into a black ball and take out two balls again, so keep on going, as well as infinite number, find the probability of a successful test.

In fact, suppose A denotes the event is successful, and Adenotes the first test successfully at $n-th$ times , with the probability formula of independent events,

$$P(A_1) = \frac{a^2}{k^2}, P(A_2) = \left(1-\frac{a^2}{k^2}\right)\frac{a^2}{(k+1)^2}, \cdots$$
$$P(A_n) = \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\cdots\left(1-\frac{a^2}{(k+n-2)^2}\right)\frac{a^2}{(k+n-1)^2}, \cdots$$

And $A = A_1 \cup A_2 \cup \cdots \cup A_n \cup \cdots$ where $\{A_n\}$are exclusive, so

$$\begin{aligned} P(A) &= P(A_1) + P(A_2) + \cdots + P(A_n) + \cdots \\ &= \frac{a^2}{k^2} + \left(1-\frac{a^2}{k^2}\right)\frac{a^2}{(k+1)^2} + \cdots + \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\cdots\left(1-\frac{a^2}{(k+n-2)^2}\right) + \cdots \end{aligned}$$

On the other hand, with the probability of opposite event,

$$\begin{aligned} P\left(\bar{A}\right) &= P\left(\bar{A}_1\right)P\left(\bar{A}_2\right)\cdots P\left(\bar{A}_n\right)\cdots \\ &= \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\cdots\left(1-\frac{a^2}{(k+n-1)^2}\right)\cdots \\ &= \lim_{n\to\infty}\left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\cdots\left(1-\frac{a^2}{(k+n-1)^2}\right)\cdots \end{aligned}$$

$$
\begin{aligned}
&= \lim_{n\to\infty} \frac{k-a}{k}\frac{k+a}{k}\frac{k-a+1}{k+1}\frac{k+1+a}{k+1}\cdots\frac{k}{k+a}\frac{k+a+a}{k+a}\frac{k+1}{k+a+1} \\
&\quad \frac{k+1+a+a}{k+a+1}\cdots\frac{k-a+n-1}{k+n-1}\frac{k+n-1-a}{k+n-1} \\
&= \frac{k-a}{k}\cdots\frac{k-a+1}{k+1}\frac{k-a+2}{k+2}\frac{k-a+3}{k+3}\cdots\frac{k-1}{k+a-1} \\
&P(A) = 1 - P\left(\bar{A}\right) = 1 - \frac{k-a}{k}\cdots\frac{k-a+1}{k+1}\frac{k-a+2}{k+2}\frac{k-a+3}{k+3}\cdots\frac{k-1}{k+a-1}
\end{aligned}
$$

So

$$
\begin{aligned}
&\frac{a^2}{k^2} + \left(1-\frac{a^2}{k^2}\right)\frac{a^2}{(k+1)^2} + \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\frac{a^2}{(k+2)^2} + \\
&\quad \cdots + \left(1-\frac{a^2}{k^2}\right)\left(1-\frac{a^2}{(k+1)^2}\right)\cdots\left(1-\frac{a^2}{(k+n-2)^2}\right)\frac{a^2}{(k+n-1)^2} + \cdots \\
&= 1 - \frac{k-a}{k}\cdots\frac{k-a+1}{k+1}\frac{k-a+2}{k+2}\frac{k-a+3}{k+3}\cdots\frac{k-1}{k+a-1}
\end{aligned}
$$

REFERENCES

[1] DeGroot, Morris H., & Schervish, Mark J. (2005). *Probability and Statistics* (3rd. edition). Beijing: Higher Education Press.

[2] XU Xianghong (2002). Probability Method of Acquiring the Sum of Infinite Series and the Limit of Multi-Integral. *Journal of Mathematics for Technology*, *18*(1), 105-107.