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Relation Equations in the Set of Finite Natural Numbers and Its Maximal Solution

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Abstract: Definition of relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ in the set of finite natural numbers is given. Rapid method of solving the maximal solution of relation equations $Q_{uv}\circ X_{vw}=S_{uw}$ in the set of finite natural numbers is provided.

Key words: Set of finite natural numbers; Relation equations; Maximal solution

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1. INTRODUCTION

$N=\{0, 1, 2,\ldots\}$ is a set of natural numbers[1]. $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ is a set of finite natural numbers, among them, n^* is an arbitrary natural number larger than zero. Starting from Jieshengjishi, human ancestors began to use a set of finite natural numbers. Since the discovery of the set of natural numbers, the set of rational numbers, the set of real numbers, the set of finite natural numbers seem to fade out of sight. The emergence of computer, and the line of sight of people back to the set of finite natural numbers. In the computer, the set $\{0, 1\}$ is the most simple set of finite natural numbers[2]. In the image processing, the set $\{0, 1, 2,\ldots, 255\}$ is the set of finite natural numbers[3].The relation equation on the set of natural numbers has been discussed very mature, but the relation equation on the set of finite natural numbers is not mentioned. One of the reasons is that conventional operators on the set of natural numbers does not satisfy closed on the set of finite natural numbers. But that information processing dependent the set of finite natural numbers show importance of relation equations on the set of finite natural numbers.

2. OPERATORS AND RELATION EQUATIONON IN THE SET OF FINITE NATURAL NUMBERS

Addition and multiplication on elements in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows:

Definition 1 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, multiplication is defined as follows:

$$m\times n=\min\{m, n\}= m\wedge n\ . \tag{1}$$

Definition 2 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, addition is defined as follows:

$$m+n=\max\{m, n\}= m\vee n\ . \tag{2}$$

Definition 3 With $F(N_{n^*})$ representative power set in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers, for given set $U=\{u_1, u_2,\ldots, u_m\}\in F(N_{n^*})$ and $V=\{ v_1, v_2,\ldots, v_n\}\in F(N_{n^*})$, the product set of U and V is defined as follows

$$P=U\times V=\{p(u,v)|u\in U,v\in V\}$$

among them, $0\le u_1\le u_2\le\ldots\le u_m\le n^*$ and $0\le v_1\le v_2\le\ldots\le v_n\le n^*$.

Definition 4 With $\boldsymbol{F}(U\times V)$ representative all product set on the U and V, for given product set $Q=\{q(u,v)|u\in U,\ v\in V\}\in \boldsymbol{F}(U\times V)$, $X=\{x(u,v)|u\in U,\ v\in V\}\in \boldsymbol{F}(V\times W)$ and $S=\{s(u,v)|u\in U,\ v\in V\}\in \boldsymbol{F}(U\times W)$, relationship equations for opertor $(\wedge,\vee)$in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows:

$$Q_{uv}\circ X_{vw}=S_{uw}\ , \tag{3}$$

among them $S_{uw}=[s_{uw}]=[\underset{v}{\vee}(q_{uv}\underset{u,w}{\wedge} r_{vw})]$.

Definition 5 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, operator α is defined as follows:

$$m\alpha n=\begin{cases} n & m>n \\ n^* & m\le n \end{cases}. \tag{4}$$

Definition 6 For given element $m\in N_{n^*}$ and element $n\in N_{n^*}$, operator β is defined as follows:

$$m\beta n=\begin{cases} n & m\ge n \\ 0 & m<n \end{cases}. \tag{5}$$

Definition 7 For given product set $Q=\{q(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(U\times V)$,$R=\{r(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(V\times W)$, opertoration for opertor $(\alpha,\wedge)$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows:

$$S=Q@R\in \boldsymbol{F}(U\times W)\ , \tag{6}$$

among them $S_{uw}=[s_{uw}]=[\underset{v}{\wedge}(q_{uv}\underset{u,w}{\alpha} r_{vw})]$.

Definition 8 For given product set $Q=\{q(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(U\times V)$, $R=\{r(u,v)|u\in U,v\in V\}\in \boldsymbol{F}(V\times W)$, opertoration for opertor $(\beta,\vee)$ in the set $N_{n^*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers is defined as follows:

$$S=Q\&R\in \boldsymbol{F}(U\times W)\ , \tag{7}$$

among them $S_{uw}=[s_{uw}]=[\underset{v}{\vee}(q_{uv}\underset{u,w}{\beta} r_{vw})]$.

3. THE MAXIMUM SOLUTION OF THE RELATION EQUATIONON IN THE SET OF FINITE NATURAL NUMBERS

Theorem 1. If the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\dots, n^*\}$ of finite natural numbers has solutions, then

$$[Q^T@S_{uw}]=[\underset{u}{\wedge}(q_{vu}\underset{v,w}{\alpha} s_{uw})] \tag{8}$$

is the maximum solution of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\dots, n^*\}$ of finite natural numbers.

Proof. If the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\dots, n^*\}$ of finite natural numbers has solutions, let $q^T_{uv}=q_{vu}$, then

$S_{uw}=Q_{uv}\circ R_{vw}\subseteq Q_{uv}\circ[Q^T@[Q_{uv}\circ R_{vw}]]= Q_{uv}\circ[Q^T@S_{uw}]$.

Because

$$\begin{aligned} Q_{uv}\circ[Q^T@S_{uw}] &= \underset{v}{\vee}[q_{uv}\underset{u,w}{\wedge}(\underset{t\in U}{\wedge}[q^T_{tv}\underset{v,w}{\alpha} s_{tw}])] \\ &\overset{(*)}{\le} \underset{v}{\vee}[q_{uv}\underset{u,w}{\wedge}[q^T_{u^*v}\underset{v,w}{\alpha} s_{u^*w}]] \\ &= \underset{v}{\vee}[q_{uv}\underset{u,w}{\wedge}[q_{vu^*}\underset{v,w}{\alpha} s_{u^*w}]] \\ &= \underset{v}{\vee}[q_{uv}\underset{u,w}{\wedge} s_{u^*w}] \\ &\le \underset{v}{\vee}[s_{uw}]=[s_{uw}]=S_{uw}\ . \end{aligned}$$

That is

$$S_{uw}=Q_{uv}\circ R_{vw}\subseteq Q_{uv}\circ[Q^T@S_{uw}]\subseteq S_{uw}\ \ . \tag{9}$$

The above Formula (9) shows that $[Q^T@S_{uw}]$ is the solution of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\dots, n^*\}$ of finite natural numbers.

Let R_{vw} is another solution of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\dots, n^*\}$ of finite natural numbers, then

$$\begin{aligned} [Q^T@S_{uw}] &= [Q^T@(Q\circ R_{vw})] \\ &= \underset{u}{\wedge}[q^T_{uv}\underset{v,w}{\alpha}(\underset{v}{\vee}[q_{uv}\underset{u,w}{\wedge} r_{vw}]) \\ &\overset{(*)}{\ge} \underset{u}{\wedge}[q_{vu}\underset{v,w}{\alpha}[q_{uv}\underset{u,w}{\wedge} r_{vw}] \\ &\overset{(*)}{\ge} \underset{u}{\wedge}[r_{vw}]=[r_{vw}]\ \ . \end{aligned}$$

That is

$$[Q^T@S_{uw}]\supseteq R_{vw}\ . \tag{10}$$

The above Formula (10) shows that another solution R_{vw} of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n^*}=\{0, 1, 2,\dots, n^*\}$ of finite natural numbers less than the solution

$[Q^T@S_{uw}]$ of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers.

So that the solution $[Q^T@S_{uw}]$ is the Maximum solution of the relation equation $Q_{uv}\circ X_{vw}=S_{uw}$ in the set $N_{n*}=\{0, 1, 2,\ldots, n^*\}$ of finite natural numbers.

4. CONCLUSION

Because that information processing dependent the set of finite natural numbers show importance of relation equations in the set of finite natural numbers. This paper gives definition of relation equations in the set of finite natural numbers. At the same time the papre provids a method of solving the maximal solution of relation equations in the set of finite natural numbers. In the future, we will discuss the solving method of solutions of relation equations in the set of finite natural numbers and will discuss other properties of the relation equations in the set of finite natural numbers.

REFERENCES

[1] Hua, L. G. (1963). *Advanced mathematics*. Beijing, China: Science Press.

[2] Gibas, C., & Jambeck, P. (2001). Developing bioinformatics computer skills. Cambridge, England: O Reilly＆Associates, 1nc.

[3] Liu, J. (2003). *Digital image processing and advanced applications*. Beijing, China: Science Press.

[4] Hua, L. G. (1957). *Number theoretic guidance*. Beijing, China: Science Press.

[5 Cheng, Y. Y. (1984). *Fuzzy mathematics* (pp.30-36). Wuhan, China: Huazhong Institute of Technology Press.

[6] Sanchez, E. (1976). Resolution of composite fuzzy relation equations. *Information and Control, 30,* 38-48.