CSCanada

Progress in Applied Mathematics
Vol. 4, No. 2, 2012, pp. [87–91]
DOI: 10.3968/j.pam.1925252820120402.ZT937

ISSN 1925-251X [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
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A General Model on Intertemporal Measure Correlation

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Supported by National Natural Science Foundation of China (NSFC) Grants 60972162.

Received: August 10, 2012/ Accepted: September 24, 2012/ Published: October 31, 2012

Abstract: In this paper, we proposed the error growth curve model for the integration of intertemporal measure errors correlation which usually exist in quality control process. This model can work well in the usual error distributions, such as normal, uniform, Rayleigh and some other error distributions. Simulation results show that the proposed estimators of the model parameters perform well especially in small sample situations.

Key words: Error growth curve model; Intertemporal errors correlation; COPLS; Error distribution; AAB

LIU, F. (2012). A General Model on Intertemporal Measure Correlation. *Progress in Applied Mathematics, 4*(2), 87–91. Available from http://www.cscanada.net/index.php/pam/article/view/j.pam.1925252820120402.ZT937 DOI: 10.3968/j.pam.1925252820120402.ZT937

1. INTRODUCTION

Observations that occur in natural science and social science are usually measured over multiple time points on a particular characteristic to investigate temporal pattern of change on the characteristic. Especially, every product in production line usually will be measured in all different steps. In general, quality management and technical statistics [1] seems to pay more attention to the error controlling in every step of producing. In fact, those errors in different steps of the whole production line maybe exist some certain correlations each other which have not been attracted enough interest up to now. Meanwhile, these error terms maybe some more general error distributions instead of normal distributions [1].

In this paper, we propose the framework of the investigation on these intertemporal correlation in more general error distributions. The paper are organized as follows:

Our main interest is the investigation on the multiple measurement error correlation structure, so the regression structure may be trivial. For convenience, we consider the following error growth curve model.

$$Y_{n\times p} = X_{n\times m}\Theta_{m\times p} + \varepsilon_{n\times p},\ E(\varepsilon_{n\times p}) = 0,\ \text{and } Cov(\varepsilon_{n\times p}) = I_n \otimes \Sigma_{p\times p} \tag{1}$$

Where $Y_{n\times p}$ is the observation matrix of the response consisting of p measurements taken on n products or individuals, $X_{n\times m}$ is the treatment design matrix with order $n \times m$, $\Theta_{m\times p}$ is the unknown regression coefficient matrix with order $m\times p$. Assume that observations on individuals are independent, so that the rows of the random error matrix $\varepsilon_{n\times p}$ are independent and identically distributed (i.i.d.) by a general continuous type distribution Γ with zero mean and a common covariance matrix $Sigma$ of order p. And we assume that the rank of the treatment design matrix $X_{n\times m}$ is less than the number of observations, i.e., $r(X_{n\times m}) < n$. For convenience, these notations will be omitted its subscripts in the following part. In the recent years, many growth curve problems are attracted in the researchers' interest, and some kinds of growth curve models are deeply studied. An interested reader can refer to Kollo and von Rosen [2], Pan and Fang [3], Hu, Liu and Ahmed [4] or Hu, Liu and You [5], Reinsel [8], Ohlson and von Rosen [9].

2. ESTIMATORS AND ITS PROPERTIES

Without the assumption of normality, the ML estimators [6,7] can't be suitable with this situation. Obviously, looking for the general ML estimators in general error distributions is impossible. Here we directly propose the corresponding estimator of covariance by COPLS approach for the model (1) without assumption of normality.

$$\widehat{\Sigma} = \frac{1}{n-r} Y' M_X Y \tag{2}$$

and two-stage GLS estimator for the regression coefficient matrix

$$\widehat{\Theta} = (X'X)^{-1} X'Y \tag{3}$$

where $M_X = I_n - P_X = I_n - X(X'X)^{-1}X'$ denotes the orthogonal projection matrix onto the orthogonal complement $\mathcal{C}(X)^{\perp}$ of the column space $\mathcal{C}(X)$ of a matrix X. And the COPLS method and the corresponding two-stage GLS estimation have been discussed carefully in [4,5], so here we omit some unnecessary theoretical details.

Firstly, we list some regular conditions of the model (1).

Assumption 1. $E(\varepsilon_1) = 0$, $E(\varepsilon_1\varepsilon_1') = \Sigma > 0$, $E(\varepsilon_1 \otimes \varepsilon_1\varepsilon_1') = 0_{p^2\times p}$ and $E\|\varepsilon_1\|^4 < \infty$, where ε_1' is the first row vector of the error matrix ε.

Assumption 2. Assume that $\lim_{n\to\infty} \frac{1}{n} X'X = R > 0$

Theorem 1 The covariance estimator given by (2) is strong consistent to covariance Σ. Meanwhile, under assumption 1, the statistic $\sqrt{n}\left(\widehat{\Sigma} - \Sigma\right)$ converges in distribution to the multivariate normal distribution $N_{p^2}\left(0, Cov(\epsilon_1 \otimes \varepsilon_1)\right)$.

Theorem 2 Under assumption 2, the estimator $\widehat{\Theta}$ is consistent to regression coefficients Θ.

Theorem 3 Under assumptions 1 and 2, then (a) $\sqrt{n}\left(\widehat{\Theta} - \Theta\right)$ converges in distribution to the multivariate normal distribution $N_{mp}(0, R^{-1}\otimes\Sigma)$ and (b) $\sqrt{n}\left(\widehat{\Sigma} - \Sigma\right)$ and $\sqrt{n}\left(\widehat{\Theta} - \Theta\right)$ are asymptotically independent.

The corresponding proofs of these theorems above will be given in the appendix.

3. SIMULATION STUDIES

In this section, we conduct some simulation studies to show the finite sample performance of the proposed procedure in previous section. The data are generated from the following model

$$Y_{n\times p} = X_{n\times m}\Theta_{m\times p} + \varepsilon_{n\times p},\ Cov(\varepsilon_{n\times p} = I_n \otimes \Sigma_{p\times p}), \text{and } E(\varepsilon_{n\times p}) = 0$$

where $p = 4$, the sample size $n = 30,\ 50,\ 80$ with the design matrices $X = diag(\frac{1n}{2}\ \ \frac{1n}{2})$, the true $\Theta = \begin{pmatrix} -1 & 1 & 2 & 1 \\ 1 & 3 & 5 & 2 \end{pmatrix}$. We assume the covariance Σ is the autoregressive structure, namely $\Sigma = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{pmatrix}$ with $\rho = 0.0,\ 0.1,\ 0.3,\ 0.5$, and the marginal distribution of ε_1 is assumed the uniform distribution $U(-2, 2)$.

For evaluating the performance of these matrix form estimators, we define the accumulation of absolute value of estimators' bias (abbr. AAB) as follows, $AAB(\widehat{\Theta}) = \sum_{i,j}\left|bias(\hat{\theta}_{i,j})\right|$, $AAB(\widehat{\Sigma}) = \sum_{i,j}|bias(\hat{\sigma}_{i,j})|$. The number of simulated realizations is $1,000$, some results are summarized as follows.

$n = 30,\ \rho$	0	0.1	0.3	0.5
$AAB(\widehat{\Theta})$	0.0486	0.0789	0.0730	0.0985
$AAB(\widehat{\Sigma})$	0.0758	0.06561	0.0571	0.1581
$n = 50,\ \rho$	0	0.1	0.3	0.5
$AAB(\widehat{\Theta})$	0.0321	0.0494	0.0543	0.0638
$AAB(\widehat{\Sigma})$	0.0324	0.0367	0.0630	0.1029
$n = 80,\ \rho$	0	0.1	0.3	0.5
$AAB(\widehat{\Theta})$	0.0656	0.0705	0.0843	0.0851
$AAB(\widehat{\Sigma})$	0.0302	0.0378	0.0477	0.0490

The simulation results show that the performance of the proposed estimators perform certain robust statistical properties in the more general error distributions, especially in the small sample situations. Some other error distributions are also investigated in these models, and some similar performance can be obtained, so the results are omitted here.

4. CONCLUSION

In this paper, we proposed the error growth curve model for the investigation on the intermporal measure errors correlation which usually exist in quality control process. This model can work well in the usual error distributions, such as normal, uniform, Rayleigh and some other error distributions. Simulation results show that the proposed estimators of the model parameters perform well especially in small sample situations.

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APPENDIX

Proof of Theorem 1 The estimator's consistent is obvious, the discussion for the strong consistent is similar with Hu, Liu and Ahemed [4], here, we omit the details and only discuss the asymptotic distribution of the covariance estimator.

$\sqrt{n}\left(\widehat{\Sigma}-\Sigma\right)$ can be decomposed into $\sqrt{n}\left(\frac{1}{n}\varepsilon'\varepsilon-\Sigma\right)+Q\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{pmatrix}Q'$ with $A_{kl}=\sqrt{n}\left(\frac{1}{n-r}\sum_{i=1}^{n-r}w_{ik}{w_{ik}}'-\frac{1}{n}\sum_{i=1}^{n}w_{ik}{w_{ik}}'\right)$ for $k,l=1,2$ except $k=l=2$. Since

$$A_{kl}=\frac{r\sqrt{n}}{n-r}\frac{1}{n}\sum_{i=1}^{n}w_{ik}{w_{ik}}'-\frac{\sqrt{n}}{n-r}\sum_{i=1}^{n}w_{ik}{w_{ik}}'$$

A_{kl} converges to 0 in probability. By assumption, the first item converges to $N_{p^2}(0,\Psi)$ in distribution, where $\Psi=Cov(\varepsilon_1\otimes\varepsilon_1)$. Hence, it follows from Slutsky's Theorem, see Lehmann and Romano [10], that the $\sqrt{n}(\widehat{\Sigma}-\Sigma)$ converges in distribution to $N_{p^2}(0,Cov(\varepsilon_1\otimes\varepsilon_1))$, completing the proof.

Proof of Theorem 2

Note that the fact

$$P\left(\left\|\frac{1}{n}X'\varepsilon\right\|\geq\delta\right)\leq\frac{1}{n^2\delta^2}E\left(Tr(X'\varepsilon\varepsilon'X)\right)=\frac{1}{n\delta^2}Tr\left(\frac{1}{n}X'X\right)Tr\left(\Sigma\right)$$

for any $\delta>0$, then the convergence in probability is easily obtained.

Proof of Theorem 3

(a) $\sqrt{n}(\widehat{\Theta}-\Theta)$ can be transformed into $\sqrt{n}\left((X'X)^{-1}X'\varepsilon\right)$, let $L_n=(X'X)^{-1}X'\varepsilon$. By Theorem 4.2 of Hu and Yan [11], $\sqrt{n}L_n$ converges in distribution to the normal distribution $N_{mp}(0,R^{-1}\otimes\Sigma)$.

(b) It suffices to prove the asymptotically independence between $\frac{1}{\sqrt{n}}vec(X'\varepsilon)$ and $\sqrt{n}vec(\widehat{\Sigma}-\Sigma)$. Let $Q_n=X'\varepsilon=(x_1,...,x_n)(\epsilon_1,...,\epsilon_n)'$. Then $Cov(Q_n(\widehat{\Sigma}-\Sigma))=Cov\left((\sum_{i=1}^{n}x_i{\epsilon_i}')(\frac{1}{n}\sum_{i=1}^{n}\epsilon_i\epsilon_i'-\Sigma)\right)+o_p(1)=E\left((\sum_{i=1}^{n}x_i\otimes{\epsilon_i}')(\sum_{i=1}^{n}\epsilon_i\otimes\epsilon_i'-\Sigma)\right)+o_p(1)$

According to assumption 2, $Cov\left((\frac{1}{\sqrt{n}}X'\varepsilon)\sqrt{n}(\widehat{\Sigma}-\Sigma)\right)$ converges to 0 in probability, implying that $\frac{1}{\sqrt{n}}vec(X'\varepsilon)$ and $\sqrt{n}vec(\widehat{\Sigma}-\Sigma)$ are asymptotically independent. Therefore, the proof is complete.