CSCanada

Progress in Applied Mathematics
Vol. 5, No. 2, 2013, pp. [71–75]
DOI: 10.3968/j.pam.1925252820130502.123

ISSN 1925-251X [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
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Teaching Approaches for Innovation Ability Cultivation in the Undergraduate Linear Algebra Course

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Received: February 25, 2013/ Accepted: April 20, 2013/ Published: April 30, 2013

Abstract: Linear Algebra is one of the most important mathematical courses for undergraduate students. An overemphasization on memorization of definitions and equations on text books always weakens students' ability of innovation. To address this problem, in this paper we propose some approaches for teaching Linear Algebra. Our approaches include the following three aspects: combining the lectures with practical problems; depicting one concept or showing problem solving illustrations from different viewpoints; providing practices for students to find solutions by using scientific computation software. We believe that these methods could stimulate students' interest in learning, and encourage them to think divergently, which therefore can promote their ability for innovation.

Key words: Linear Algebra; Innovation ability; Divergent thinking; MATLAB

Li, H. (2013). Teaching Approaches for Innovation Ability Cultivation in the Undergraduate Linear Algebra Course. *Progress in Applied Mathematics, 5*(2), 71–75. Available from http://www.cscanada.net/index.php/pam/article/view/j.pam.19252528 20130502.123 DOI: 10.3968/j.pam.1925252820130502.123

1. INTRODUCTION

Linear Algebra is one of the most important and fundamental mathematical courses for undergraduate students. Many of its concepts (e.g., the determinant, matrix, linear transformation) and methods have been widely used in a lot of fields such as engineering, management, economics, computer science, other branches of mathematics (e.g., numerical computation, optimization, control theory) and so on. Besides, due to the abstract concepts and strong logic underlying in proofs, the Linear Algebra course has its advantages in promoting students' ability for abstract and logic thinking [1,5]. Therefore, Linear Algebra not only plays a very important role in the development of mathematics itself, but also could significantly improve the comprehensive quality of undergraduate students.

Despite its so many benefits, learning Linear Algebra is difficult due to the abstract concepts and proofs requiring solid logical inference ability. Moreover, the computation in Linear Algebra is always huge, tedious and complicated, which makes Linear Algebra less appealing, or even a nightmare to many students [1]. It is therefore the teachers' responsibility to prepare more interesting classes. However, few colleges have taken adequate measures to foster the innovation ability of undergraduate students in the Linear Algebra course until now. Fortunately, more and more attention has been paid on this issue recently, and many solutions have been proposed [1]. In this paper, based on our abundant practical experience in teaching Linear Algebra, we will present three feasible teaching approaches, aiming to promote the ability for analyzing and solving problems, and innovation of undergraduate students.

2. APPROACH ONE: MAKING LINEAR ALGEBRA MORE APPEALING BY USING PRACTICAL PROBLEMS

Though not been proved evidently, most students learn Linear Algebra just to pass the examination, and obtain credits. What is worse, few examples and exercises in many text books have real practical background, which makes students feel tedious. Instead, if the teachers could insert some practical examples in the lectures, the students could not only be more interested, but also gain experience in solving practical problems by using Linear Algebra [1]. As this kind of experience increases, the students will become skilled in combining the concepts and methods in Linear Algebra with problems in real world. Then when new problem comes, the students would feel more comfortable to connect the knowledge learnt with the concrete application scenes.

For instance, students always feel it difficult to understand the definition and computation of matrix multiplication. We could construct the following scene to help students master this piece of knowledge [4].

Suppose that a primary school purchases the following goods as desk, chair, and boxes of chalk in September and October, of which the prices and quantities are listed in Table 1 and Table 2, respectively.

Table 1
Prices of Purchased Goods

Goods	Desk	Chair	Chalk
Price	100	30	4

Here comes the question: what are the expenditures in September and October? Students can quickly get the results by using basic arithmetic as shown in Table 3.

Table 2
Quantities of Purchased Goods

Goods	September	October
Desk	10	15
Chair	20	30
Chalk	40	60

Table 3
Expenditure Calculated by Basic Arithmetic

	September	October
Expenditure	$10 \times 100 + 20 \times 30 + 40 \times 4$ $= 1760$	$15 \times 100 + 30 \times 30 + 60 \times 4$ $= 2640$

We can represent Table 1, 2, 3 by using matrix $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, respectively.

$$\boldsymbol{A} = \begin{pmatrix} 100 & 30 & 4 \end{pmatrix}, \ \boldsymbol{B} = \begin{pmatrix} 10 & 15 \\ 20 & 30 \\ 40 & 60 \end{pmatrix}, \ \boldsymbol{C} = \begin{pmatrix} 1760 & 2640 \end{pmatrix}.$$

The expenditure is the sum of scalar product of the price and quantity. Intuitively, we can write down the conjecture: the matrix $\boldsymbol{C}$ is the matrix product of the price matrix $\boldsymbol{A}$ and the quantity matrix $\boldsymbol{B}$. We can easily verify that the conjecture is true. Through this example, the students can get to know that the matrix and the matrix product have real backgrounds, and may become more interested.

3. APPROACH TWO: INSPIRING STUDENTS TO THINK FROM DIFFERENT VIEWPOINTS

There are a lot of abstract concepts in Linear Algebra such as the determinant, matrix, vector, linear space, linear dependent, linear independent and so on, together with tremendous lemmas and theorems. To effectively illustrate these concepts, lemmas and theorems, we should introduce some teaching skills.

Usually, there are multiple solutions to a certain problem in Linear Algebra. A new solution may potentially indicate a breakthrough in theory. Besides, multiple solutions always contain multiple points of knowledge, and could provide different thinking directions [4]. Theorem, we can use this character of the one problem multiple solution teaching scheme to help students to develop their abilities for divergent thinking, and help them master the corresponding knowledge more firmly.

Example 1: Let $\boldsymbol{A}, \boldsymbol{B}$ be two nonzero matrix with order n, and $\boldsymbol{AB} = 0$. Then the ranks of $\boldsymbol{A}$ and $\boldsymbol{B}$ satisfy ( )

(A). rank($\boldsymbol{A}$)=0, or rank($\boldsymbol{B}$)=0;

(B). both are less than n;

(C). one is less than n and the other equals to n;

(D). Both equal to n.

The answer is (B).

Solution 1. Since $\boldsymbol{AB} = 0$, $r(\boldsymbol{A}) + r(\boldsymbol{B}) \leq n$. From $\boldsymbol{A} \neq 0, \boldsymbol{B} \neq 0$, we can obtain that $r(\boldsymbol{A}) > 0, r(\boldsymbol{B}) > 0$. Therefore, $r(\boldsymbol{A}) < n, r(\boldsymbol{B}) < n$.

Solution 2. Since $\boldsymbol{A}x = 0$ has nonzero solution, $r(\boldsymbol{A}) < n$. Similarly, we can obtain that $r(\boldsymbol{B}) < n$.

Solution 3. It is easy to deduce that (A) and (C) are not correct. Form the fact that $\boldsymbol{AB} = 0$, $|\boldsymbol{A}| \cdot |\boldsymbol{B}| = 0$, i.e., $|\boldsymbol{A}| = 0$ or $|\boldsymbol{B}| = 0$. Thus, (D) is not correct, and (B) is the answer.

From Example 1, we can see that different viewpoints result in different solutions of a single problem. To obtain multiple solutions, concrete basic mathematics knowledge, acute insight and vision are required, which only can be constructed by repeated training and the long time accumulation. The aforementioned one problem multiple solutions teaching scheme provides adequate ways for fostering students' ability of divergent thinking of students.

4. APPROACH THREE: PRACTICE BY USING MATLAB

In mathematical education, there are mainly two contents: the first is the delivery of mathematical knowledge, the second is the cultivation of mathematical skills such as the computation skill, the abstract thinking skill, the logical inference skill, and application ability [2]. The mathematical software can bring a lot of convenience and intuitive understanding. Thus, it is a helpful attempt to combine the teaching and learning of Linear Algebra with scientific computation software.

Among all the scientific computation software, Matlab is an indispensable tool, widely used by undergraduate, master, or Ph.D. students, which has a lot applications in teaching Linear Algebra [2,3].

Experiment is always a popular teaching method, which many teachers are not cable of. Matlab is then a useful tool to teaching Linear Algebra in an experimental way.

For example, we have the following problems.

Example 2:

$$\begin{cases} \sin 2x - y = 0; \\ 2\cos 2x - y = 0. \end{cases}$$

It is difficult to solve the problem. However, by using Matlab, we can quickly obtain the solutions. The result can be seen intuitively from Figure 1.

Anther complex problem is as the following.

Example 3: For a given matrix $\boldsymbol{A} = \begin{pmatrix} 7 & -3 & -1 & 1 \\ -3 & 7 & 1 & -1 \\ -1 & 1 & 7 & -3 \\ 1 & -1 & -3 & 7 \end{pmatrix}$. Calculate a orthogonal matrix $\boldsymbol{T}$ such that $\boldsymbol{T}'\boldsymbol{AT}$ is a diagonal matrix.

For a matrix with order 4, calculating by hand is very difficult and error-prone. But by using Matlab, we can easily obtain the following solution

$$\boldsymbol{T} = \begin{pmatrix} -0.0000 & 0.7071 & 0.5000 & -0.5000 \\ -0.0000 & 0.7071 & -0.5000 & 0.5000 \\ 0.7071 & -0.0000 & 0.5000 & 0.5000 \\ 0.7071 & 0.0000 & -0.5000 & -0.5000 \end{pmatrix}.$$

In the aforementioned sections, we propose three approaches for improving teaching Linear Algebra. In one word, in the mathematical education, we should pay

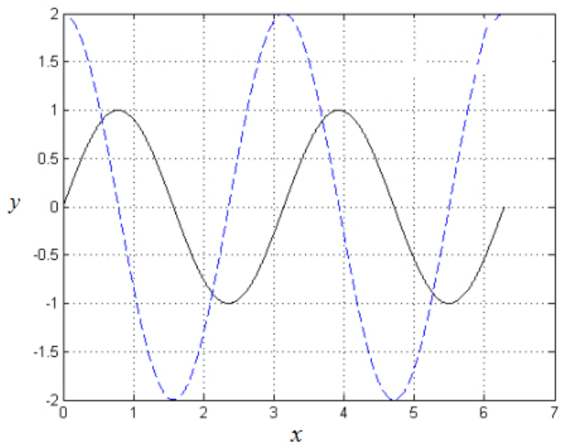

Figure 1
Solutions to Example 2 by Using MATLAB

sufficient attention on fostering the innovation ability, divergent thinking of undergraduate students.

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