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Statistical Analysis of MOBVE Distribution with TFR Model Under Step-Stress Accelerated Life Test

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Abstract: We obtain the maximum likelihood estimates of parameters of MOBVE distribution with tampered failure rate model under step-stress accelerated life test. Thereafter we show the feasibility of this method by using the Monte-Carlo simulation.

Key words: MOBVE; Step-stress accelerated life test; Tampered failure rate model; Maximum likelihood estimate

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1. INTRODUCTION

Marshall and Olkin proposed a bivariate exponential distribution in 1967, which has the joint survival function

$$\bar{F}(x, y) = \exp\{-\lambda_1 x - \lambda_2 y - \lambda_{12} \max(x, y)\}, \quad x > 0, \quad y > 0$$

And the corresponding joint density function is

$$f(x, y) = \begin{cases} \lambda_1(\lambda_2 + \lambda_{12}) \exp\{-\lambda_1 x - (\lambda_2 + \lambda_{12})y\}, & y > x > 0 \\ \lambda_2(\lambda_1 + \lambda_{12}) \exp\{-(\lambda_1 + \lambda_{12})x - \lambda_2 y\}, & x > y > 0 \\ \lambda_{12} \exp\{-(\lambda_1 + \lambda_2 + \lambda_{12})x\}, & x = y > 0 \end{cases}$$

where the parameters are $\lambda_1, \lambda_2, \lambda_{12} > 0$. This bivariate exponential distribution is usually called MOBVE distribution, which is denoted by $(X, Y) \sim MOBVE(\lambda_1, \lambda_2, \lambda_{12})$. Assumed that the system consists of two components and is impacted by three independent impact sources controlled by Poisson process with the strength $\lambda_1, \lambda_2, \lambda_{12}$, the first two impact sources cause a component to fail and the third impact source causes two components to fail at the same time. So MOBVE distribution can be derived by this fatal shock model.

Previous researches mainly focused on the properties of MOBVE distribution [2–4] and parameter estimates under the constant stress situation [5]. However, there are few researches about the parameter estimates of MOBVE distribution in the changeable stress situation. In this paper, we discuss the step-stress accelerated life test of MOBVE distribution based on TFR model and obtain the maximum likelihood estimates of parameters. Thereafter, the example is illustrated to show the feasibility of this method by Monte-Carlo simulations.

2. PARAMETER ESTIMATES OF MOBVE DISTRIBUTION UNDER TFR MODEL

n systems are randomly selected from a large number of systems with two components and put on the simple step-stress accelerated life test. The life of two components is $(X, Y) \sim MOBVE(\lambda_1, \lambda_2, \lambda_{12})$. Supposed that the stress is S_1 at the beginning of test and the stress is increasing from S_1 to S_2 at the time t_1 , the failure rate function of TFR model [7] is

$$\lambda^*(t) = \begin{cases} \lambda(t) & t \leq t_1 \\ \alpha\lambda(t) & t > t_1 \end{cases}$$

Supposed that the failure rate is increasing from $\lambda_1, \lambda_2, \lambda_{12}$ to $\alpha_1\lambda_1, \alpha_2\lambda_2, \alpha_{12}\lambda_{12}$, $\alpha_1, \alpha_2, \alpha_{12} > 1$ at the time t_1 , where $\alpha_1, \alpha_2, \alpha_{12} > 1$ depend on S_1, S_2 and maybe depend on the time t_1 , which is usually called tamper factor, we have

$$\bar{F}^*(t) = \begin{cases} \bar{F}(t) & t \leq t_1 \\ \bar{F}(t_1) \left[\frac{\bar{F}(t)}{\bar{F}(t_1)} \right]^\alpha & t > t_1 \end{cases}$$

2.1. The Lives X, Y of Two Components Are Both Observed

Assumed that component X has r_1 failure, component Y has r_2 failure and system $Z = \min(X, Y)$ has r failure at the time $t \leq t_1$, the test is continuous under stress

S_2 until all units are failed. Under stress S_1 , r_1 failure time of component X are $X_1 \leq X_2 \leq \dots \leq X_{r_1}$; r_2 failure time of component Y are $Y_1 \leq Y_2 \leq \dots \leq Y_{r_2}$; r failure time of system are $Z_1 \leq Z_2 \leq \dots \leq Z_r$. Under stress S_2 , $n - r_1$ failure time of component X are $X_{r_1+1} \leq X_{r_1+2} \leq \dots \leq X_n$; $n - r_2$ failure time of component Y are $Y_{r_2+1} \leq Y_{r_2+2} \leq \dots \leq Y_n$; $n - r$ failure time of system are $Z_{r+1} \leq Z_{r+2} \leq \dots \leq Z_n$.

Under stress S_1 , we have $\overline{F}_X(x) = e^{-(\lambda_1 + \lambda_{12})x}$, $\overline{F}_Y(y) = e^{-(\lambda_2 + \lambda_{12})y}$, $\overline{F}_Z(z) = e^{-(\lambda_1 + \lambda_2 + \lambda_{12})z}$.

Under stress S_2 , we have $\overline{F}_X(x) = e^{-(\alpha_1 \lambda_1 + \alpha_{12} \lambda_{12})x}$, $\overline{F}_Y(y) = e^{-(\alpha_2 \lambda_2 + \alpha_{12} \lambda_{12})y}$, $\overline{F}_Z(z) = e^{-(\alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_{12} \lambda_{12})z}$.

The log-likelihood function of life of component X is

$$\begin{aligned} \ln L_X &= r_1 \ln(\lambda_1 + \lambda_{12}) + (n - r_1) \ln(\alpha_1 \lambda_1 + \alpha_{12} \lambda_{12}) \\ &\quad - (\lambda_1 + \lambda_{12}) \sum_{j=1}^{r_1} x_j - (\alpha_1 \lambda_1 + \alpha_{12} \lambda_{12}) \sum_{j=r_1+1}^n x_j \\ &\quad - (\lambda_1 + \lambda_{12} - \alpha_1 \lambda_1 - \alpha_{12} \lambda_{12})(n - r_1) t_1 \end{aligned}$$

Setting $\lambda_1 + \lambda_{12} = a_X$, $\alpha_1 \lambda_1 + \alpha_{12} \lambda_{12} = b_X$, $\frac{\partial \ln L_X}{\partial a_X} = 0$, $\frac{\partial \ln L_X}{\partial b_X} = 0$, the likelihood equation set is

$$\begin{cases} \frac{r_1}{a_X} - \sum_{j=1}^{r_1} x_j - (n - r_1) t_1 = 0 \\ \frac{n - r_1}{b_X} - \sum_{j=r_1+1}^n x_j + (n - r_1) t_1 = 0 \end{cases}$$

It can be solved that the maximum likelihood estimates of a_X , b_X are

$$\begin{cases} \widehat{a}_X = \frac{r_1}{\sum_{j=1}^{r_1} x_j + (n - r_1) t_1} \\ \widehat{b}_X = \frac{n - r_1}{\sum_{j=r_1+1}^n x_j - (n - r_1) t_1} \end{cases}$$

The log-likelihood function of life of component Y is

$$\begin{aligned} \ln L_Y &= r_2 \ln(\lambda_2 + \lambda_{12}) + (n - r_2) \ln(\alpha_2 \lambda_2 + \alpha_{12} \lambda_{12}) \\ &\quad - (\lambda_2 + \lambda_{12}) \sum_{j=1}^{r_2} y_j - (\alpha_2 \lambda_2 + \alpha_{12} \lambda_{12}) \sum_{j=r_2+1}^n x_j \\ &\quad - (\lambda_2 + \lambda_{12} - \alpha_2 \lambda_2 - \alpha_{12} \lambda_{12})(n - r_2) t_1 \end{aligned}$$

Setting $\lambda_2 + \lambda_{12} = a_Y$, $\alpha_2 \lambda_2 + \alpha_{12} \lambda_{12} = b_Y$, $\frac{\partial \ln L_Y}{\partial a_Y} = 0$, $\frac{\partial \ln L_Y}{\partial b_Y} = 0$, the likelihood equation set is

$$\begin{cases} \frac{r_2}{a_Y} - \sum_{j=1}^{r_2} y_j - (n - r_2) t_1 = 0 \\ \frac{n - r_2}{b_Y} - \sum_{j=r_2+1}^n y_j + (n - r_2) t_1 = 0 \end{cases}$$

It can be solved that the maximum likelihood estimates of a_Y , b_Y are

$$\left\{ \begin{array}{l} \widehat{a}_Y = \frac{r_2}{\sum_{j=1}^{r_2} y_j + (n - r_2) t_1} \\ \widehat{b}_Y = \frac{n - r_2}{\sum_{j=r_2+1}^n y_j - (n - r_2) t_1} \end{array} \right.$$

The log-likelihood function of system life Z is

$$\begin{aligned} \ln L_Z &= r \ln(\lambda_1 + \lambda_2 + \lambda_{12}) + (n - r) \ln(\alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_{12} \lambda_{12}) \\ &- (\lambda_1 + \lambda_2 + \lambda_{12}) \sum_{j=1}^r z_j - (\alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_{12} \lambda_{12}) \sum_{j=r+1}^n z_j \\ &- (\lambda_1 + \lambda_2 + \lambda_{12} - \alpha_1 \lambda_1 - \alpha_2 \lambda_2 - \alpha_{12} \lambda_{12}) (n - r) t_1 \end{aligned}$$

Setting $\lambda_1 + \lambda_2 + \lambda_{12} = a_Z$, $\alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \alpha_{12} \lambda_{12} = b_Z$, $\frac{\partial \ln L_Z}{\partial a_Z} = 0$, $\frac{\partial \ln L_Z}{\partial b_Z} = 0$, the likelihood equation set is

$$\left\{ \begin{array}{l} \frac{r}{a_Z} - \sum_{j=1}^r z_j - (n - r) t_1 = 0 \\ \frac{n - r}{b_Z} - \sum_{j=r+1}^n z_j + (n - r) t_1 = 0 \end{array} \right.$$

It can be solved that the maximum likelihood estimates of a_Z , b_Z are

$$\left\{ \begin{array}{l} \widehat{a}_Z = \frac{r}{\sum_{j=1}^r z_j + (n - r) t_1} \\ \widehat{b}_Z = \frac{n - r}{\sum_{j=r+1}^n z_j - (n - r) t_1} \end{array} \right.$$

Therefore, the maximum likelihood estimates of parameters λ_1 , λ_2 , λ_{12} , α_1 , α_2 , α_{12} are

$$\begin{aligned} \widehat{\lambda}_1 &= \widehat{a}_Z - \widehat{a}_Y = \frac{r}{\sum_{j=1}^r z_j + (n - r) t_1} - \frac{r_2}{\sum_{j=1}^{r_2} y_j + (n - r_2) t_1} \\ \widehat{\lambda}_2 &= \widehat{a}_Z - \widehat{a}_X = \frac{r}{\sum_{j=1}^r z_j + (n - r) t_1} - \frac{r_1}{\sum_{j=1}^{r_1} x_j + (n - r_1) t_1} \\ \widehat{\lambda}_{12} &= \widehat{a}_X + \widehat{a}_Y - \widehat{a}_Z = \frac{r_1}{\sum_{j=1}^{r_1} x_j + (n - r_1) t_1} + \frac{r_2}{\sum_{j=1}^{r_2} y_j + (n - r_2) t_1} - \frac{r}{\sum_{j=1}^r z_j + (n - r) t_1} \\ \widehat{\alpha}_1 &= \frac{\widehat{b}_Z - \widehat{b}_Y}{\widehat{\lambda}_1} = \frac{\frac{n - r}{\sum_{j=r+1}^n z_j - (n - r) t_1} - \frac{n - r_2}{\sum_{j=r_2+1}^{n - r_2} y_j - (n - r_2) t_1}}{\frac{r}{\sum_{j=1}^r z_j + (n - r) t_1} - \frac{r_2}{\sum_{j=1}^{r_2} y_j + (n - r_2) t_1}} \end{aligned}$$

$$\widehat{\alpha}_2 = \frac{\widehat{b}_Z - \widehat{b}_X}{\widehat{\lambda}_2} = \frac{\frac{n-r}{\sum_{j=r+1}^n z_j - (n-r)t_1} - \frac{n-r_1}{\sum_{j=r_1+1}^n x_j - (n-r_1)t_1}}{\frac{r}{\sum_{j=1}^r z_j + (n-r)t_1} - \frac{r_1}{\sum_{j=1}^{r_1} x_j + (n-r_1)t_1}}$$

$$\widehat{\alpha}_{12} = \frac{\widehat{b}_X + \widehat{b}_Y - \widehat{b}_Z}{\widehat{\lambda}_{12}} = \frac{\frac{n-r_1}{\sum_{j=r_1+1}^n x_j - (n-r_1)t_1} + \frac{n-r_2}{\sum_{j=r_2+1}^n y_j - (n-r_2)t_1} - \frac{n-r}{\sum_{j=r+1}^n z_j - (n-r)t_1}}{\frac{r_1}{\sum_{j=1}^{r_1} x_j + (n-r_1)t_1} + \frac{r_2}{\sum_{j=1}^{r_2} y_j + (n-r_2)t_1} - \frac{r}{\sum_{j=1}^r z_j + (n-r)t_1}}$$

2.2. $Z = \min(X, Y)$ and Which Component Is Failed Are Observed

Assumed that system $Z = \min(X, Y)$ has r failure at the time $t \leq t_1$, the test is continuous under stress S_2 until all systems are failed. Under stress S_1 , r failure time of system are $Z_1 \leq Z_2 \leq \dots \leq Z_r$; Under stress S_2 , $n - r$ failure time of system are $Z_{r+1} \leq Z_{r+2} \leq \dots \leq Z_n$. The lives of two components are (X_j, Y_j) corresponding to Z_j , and it is denoted that

$$I_j = \begin{cases} 1, & Z_j = X_j < Y_j \\ 2, & Z_j = Y_j < X_j \\ 3, & Z_j = X_j = Y_j \end{cases}, \quad j = 1, 2, \dots, n.$$

$$P(I = i) = p_i, \quad i = 1, 2, 3,$$

$p_i f_i(z)$ is the joint density of (Z, I) .

It is also denoted that

$$W_1 = \begin{cases} 1, & X < Y \\ 0, & \text{others} \end{cases}, \quad W_2 = \begin{cases} 1, & X > Y \\ 0, & \text{others} \end{cases}, \quad W_3 = \begin{cases} 1, & X = Y \\ 0, & \text{others} \end{cases},$$

$$W_{1j} = \begin{cases} 1, & X_j < Y_j \\ 0, & \text{others} \end{cases}, \quad W_{2j} = \begin{cases} 1, & X_j > Y_j \\ 0, & \text{others} \end{cases}, \quad W_{3j} = \begin{cases} 1, & X_j = Y_j \\ 0, & \text{others} \end{cases},$$

$j = 1, 2, \dots, n.$

Under stress S_1 , the joint density of (Z, I) [5] is

$$p_i f_i(z) = \begin{cases} \lambda_1 e^{-\lambda z}, & i = 1, z > 0 \\ \lambda_2 e^{-\lambda z}, & i = 2, z > 0 \\ \lambda_{12} e^{-\lambda z}, & i = 3, z > 0 \end{cases},$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_{12}$.

Under stress S_2 , the joint density of (Z, I) is

$$p_i f_i(z) = \begin{cases} \gamma_1 e^{-\gamma z}, & i = 1, z > 0 \\ \gamma_2 e^{-\gamma z}, & i = 2, z > 0 \\ \gamma_{12} e^{-\gamma z}, & i = 3, z > 0 \end{cases},$$

where $\gamma_1 = \alpha_1 \lambda_1$, $\gamma_2 = \alpha_2 \lambda_2$, $\gamma_{12} = \alpha_{12} \lambda_{12}$, $\gamma = \gamma_1 + \gamma_2 + \gamma_{12}$.

The log-likelihood function is

$$\begin{aligned} \ln L &= \sum_{j=1}^r W_{1j} \bullet \ln \lambda_1 + \sum_{j=1}^r W_{2j} \bullet \ln \lambda_2 + \sum_{j=1}^r W_{3j} \bullet \ln \lambda_{12} \\ &- \lambda \sum_{j=1}^r z_j - (n-r) \lambda t_1 + \sum_{j=r+1}^n W_{1j} \bullet \ln \gamma_1 + \sum_{j=r+1}^n W_{2j} \bullet \ln \gamma_2 \\ &+ \sum_{j=r+1}^n W_{3j} \bullet \ln \gamma_{12} - \gamma \sum_{j=r+1}^n z_j + (n-r) \gamma t_1 \end{aligned}$$

Setting $\frac{\partial \ln L}{\partial \lambda_1} = 0$, $\frac{\partial \ln L}{\partial \lambda_2} = 0$, $\frac{\partial \ln L}{\partial \lambda_{12}} = 0$, $\frac{\partial \ln L}{\partial \gamma_1} = 0$, $\frac{\partial \ln L}{\partial \gamma_2} = 0$, $\frac{\partial \ln L}{\partial \gamma_{12}} = 0$, the likelihood equation set is

$$\left\{ \begin{array}{l} \frac{\sum_{j=1}^r W_{1j}}{\lambda_1} - \sum_{j=1}^r z_j - (n-r)t_1 = 0; \quad \frac{\sum_{j=1}^r W_{2j}}{\lambda_2} - \sum_{j=1}^r z_j - (n-r)t_1 = 0; \\ \frac{\sum_{j=1}^r W_{3j}}{\lambda_{12}} - \sum_{j=1}^r z_j - (n-r)t_1 = 0; \quad \frac{\sum_{j=r+1}^n W_{1j}}{\gamma_1} - \sum_{j=r+1}^n z_j + (n-r)t_1 = 0; \\ \frac{\sum_{j=r+1}^n W_{2j}}{\gamma_2} - \sum_{j=r+1}^n z_j + (n-r)t_1 = 0; \quad \frac{\sum_{j=r+1}^n W_{3j}}{\gamma_{12}} - \sum_{j=r+1}^n z_j + (n-r)t_1 = 0. \end{array} \right.$$

It can be solved that the maximum likelihood estimates of λ_1 , λ_2 , λ_{12} , α_1 , α_2 , α_{12} are

$$\widehat{\lambda}_1 = \frac{\sum_{j=1}^r W_{1j}}{\sum_{j=1}^r z_j + (n-r)t_1}; \quad \widehat{\gamma}_1 = \frac{\sum_{j=r+1}^n W_{1j}}{\sum_{j=r+1}^n z_j - (n-r)t_1};$$

$$\widehat{\alpha}_1 = \frac{\sum_{j=r+1}^n W_{1j} \bullet \sum_{j=1}^r z_j + (n-r)t_1}{\sum_{j=1}^r W_{1j} \bullet \sum_{j=r+1}^n z_j - (n-r)t_1};$$

$$\widehat{\lambda}_2 = \frac{\sum_{j=1}^r W_{2j}}{\sum_{j=1}^r z_j + (n-r)t_1}; \quad \widehat{\gamma}_2 = \frac{\sum_{j=r+1}^n W_{2j}}{\sum_{j=r+1}^n z_j - (n-r)t_1};$$

$$\widehat{\alpha}_2 = \frac{\sum_{j=r+1}^n W_{2j} \bullet \sum_{j=1}^r z_j + (n-r)t_1}{\sum_{j=1}^r W_{2j} \bullet \sum_{j=r+1}^n z_j - (n-r)t_1};$$

$$\widehat{\lambda}_{12} = \frac{\sum_{j=1}^r W_{3j}}{\sum_{j=1}^r z_j + (n-r)t_1}; \quad \widehat{\gamma}_{12} = \frac{\sum_{j=r+1}^n W_{3j}}{\sum_{j=r+1}^n z_j - (n-r)t_1};$$

$$\widehat{\alpha}_{12} = \frac{\sum_{j=r+1}^n W_{3j} \bullet \sum_{j=1}^r z_j + (n-r)t_1}{\sum_{j=1}^r W_{3j} \bullet \sum_{j=r+1}^n z_j - (n-r)t_1}$$

3. SIMULATION CALCULATION

Under simple step-stress accelerated life test, the stress is increasing from S_1 to S_2 at the time $t_1 = 5$. Let tampered factors be $\alpha_1 = 10$, $\alpha_2 = 15$, $\alpha_{12} = 5$ and sample sizes be $n = 15, 20, 25, 30, 40, 50$. The mean and mean square error (MSE) of maximum likelihood estimates are calculated under two situations by 10000 Monte-Carlo simulations. The results are list on Table 1 and Table 2. From the calculation results, we find that the mean is close to the true value of parameter and the mean square error is under the acceptable range, which shows that the given estimate method is meaningful.

Table 1
The Lives of Two Components Are Both Observed

n	λ_1	λ_2	λ_{12}	$\widehat{\lambda}_1$		$\widehat{\lambda}_2$		$\widehat{\lambda}_{12}$	
				mean	MSE	mean	MSE	mean	MSE
15	0.06	0.08	0.1	0.0642	0.0023	0.0852	0.003	0.1038	0.0026
	0.05	0.07	0.09	0.0531	0.0017	0.0739	0.0023	0.0933	0.0022
	0.04	0.06	0.08	0.0427	0.0013	0.0623	0.0018	0.0829	0.0017
	0.03	0.05	0.07	0.0311	0.0008	0.0525	0.0013	0.072	0.0014
	0.06	0.08	0.1	0.0621	0.0016	0.0827	0.0021	0.1031	0.0018
20	0.05	0.07	0.09	0.0524	0.0012	0.0728	0.0017	0.0922	0.0015
	0.04	0.06	0.08	0.0412	0.0009	0.0621	0.0013	0.0817	0.0012
	0.03	0.05	0.07	0.0314	0.0006	0.0524	0.001	0.0721	0.001
	0.06	0.08	0.1	0.0622	0.0013	0.083	0.0016	0.102	0.0014
	0.05	0.07	0.09	0.0515	0.0009	0.0725	0.0013	0.0919	0.0012
25	0.04	0.06	0.08	0.0417	0.0007	0.0616	0.001	0.0814	0.001
	0.03	0.05	0.07	0.0314	0.0005	0.0515	0.0008	0.0713	0.0008
	0.06	0.08	0.1	0.0616	0.001	0.0821	0.0013	0.1015	0.0012
	0.05	0.07	0.09	0.0511	0.0008	0.0722	0.001	0.0919	0.0009
	0.04	0.06	0.08	0.0408	0.0006	0.0611	0.0008	0.0816	0.0008
30	0.03	0.05	0.07	0.0307	0.0004	0.0511	0.0006	0.0713	0.0006
	0.06	0.08	0.1	0.0617	0.0007	0.0821	0.001	0.1005	0.0009
	0.05	0.07	0.09	0.0512	0.0006	0.0714	0.0008	0.0914	0.0007
	0.04	0.06	0.08	0.041	0.0004	0.0609	0.0006	0.0814	0.0006
	0.03	0.05	0.07	0.0305	0.0003	0.0511	0.0005	0.0712	0.0005
40	0.06	0.08	0.1	0.0612	0.0006	0.0809	0.0008	0.1009	0.0007
	0.05	0.07	0.09	0.0509	0.0005	0.071	0.0006	0.0908	0.0006
	0.04	0.06	0.08	0.0404	0.0003	0.0609	0.0005	0.0808	0.0005
	0.03	0.05	0.07	0.0304	0.0002	0.051	0.0004	0.0705	0.0004

Example: The stress is increasing from S_1 to S_2 at the time $t_1 = 5$. Let the true value of failure rate be $\lambda_1 = 0.01$, $\lambda_2 = 0.03$, $\lambda_{12} = 0.05$, and the true value of tampered factors be $\alpha_1 = 10$, $\alpha_2 = 15$, $\alpha_{12} = 5$. Under the sample size $n = 20$, the random data are generated and calculated by Monte-Carlo simulations. When the lives of two components are both observed, the maximum likelihood estimates of failure rate λ_1 , λ_2 , λ_{12} are $\widehat{\lambda}_1 = 0.0125$, $\widehat{\lambda}_2 = 0.0268$, $\widehat{\lambda}_{12} = 0.0434$. When the

life of the first failed component is observed, the maximum likelihood estimates of failure rate λ_1 , λ_2 , λ_{12} are $\widehat{\lambda}_1 = 0.0127$, $\widehat{\lambda}_2 = 0.0270$, $\widehat{\lambda}_{12} = 0.0429$.

Table 2
The Life of First Failed Component Is Observed

n	λ_1	λ_2	λ_{12}	$\widehat{\lambda}_1$		$\widehat{\lambda}_2$		$\widehat{\lambda}_{12}$	
				mean	MSE	mean	MSE	mean	MSE
15	0.06	0.08	0.1	0.063	0.0016	0.0844	0.0021	0.1053	0.0026
	0.05	0.07	0.09	0.052	0.0012	0.0722	0.0017	0.0955	0.0023
	0.04	0.06	0.08	0.0425	9.39e-04	0.0625	0.0014	0.0848	0.002
	0.03	0.05	0.07	0.0309	6.25e-04	0.0521	0.001	0.0732	0.0016
	0.06	0.08	0.1	0.0618	0.0011	0.0828	0.0015	0.1031	0.0019
20	0.05	0.07	0.09	0.0515	8.96e-04	0.073	0.0013	0.094	0.0016
	0.04	0.06	0.08	0.0414	6.69e-04	0.0618	9.99e-04	0.0833	0.0014
	0.03	0.05	0.07	0.0308	4.59e-04	0.0518	7.85e-04	0.0729	0.0011
	0.06	0.08	0.1	0.062	8.99e-04	0.0827	0.0012	0.1028	0.0015
	0.05	0.07	0.09	0.0514	7.10e-04	0.0719	9.75e-04	0.0924	0.0012
25	0.04	0.06	0.08	0.0411	5.24e-04	0.0612	7.70e-04	0.0823	0.001
	0.03	0.05	0.07	0.0307	3.55e-04	0.0512	6.02e-04	0.0721	8.45e-04
	0.06	0.08	0.1	0.0608	7.15e-04	0.0815	9.80e-04	0.1021	0.0012
	0.05	0.07	0.09	0.0511	5.78e-04	0.0718	8.02e-04	0.0927	0.001
	0.04	0.06	0.08	0.0409	4.32e-04	0.0614	6.31e-04	0.0826	8.69e-04
30	0.03	0.05	0.07	0.0308	3.00e-04	0.0514	5.11e-04	0.0719	7.22e-04
	0.06	0.08	0.1	0.0609	5.23e-04	0.0815	7.22e-04	0.1017	9.07e-04
	0.05	0.07	0.09	0.0507	4.37e-04	0.0706	5.92e-04	0.0914	7.76e-04
	0.04	0.06	0.08	0.0406	3.18e-04	0.0612	4.73e-04	0.0819	6.59e-04
	0.03	0.05	0.07	0.0304	2.18e-04	0.0507	3.56e-04	0.0714	5.15e-04
40	0.06	0.08	0.1	0.0608	4.24e-04	0.0811	5.70e-04	0.101	7.25e-04
	0.05	0.07	0.09	0.0508	3.36e-04	0.0711	4.69e-04	0.091	6.00e-04
	0.04	0.06	0.08	0.0403	2.43e-04	0.0606	3.79e-04	0.0809	5.03e-04
	0.03	0.05	0.07	0.0303	1.72e-04	0.0505	2.93e-04	0.0708	4.16e-04

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