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Teaching Fractions Procedurally and Conceptually to Pre-Service Elementary Education Teachers

Deborah Edwards
University of Central Florida

TEACHING FRACTIONS PROCEDURALLY AND CONCEPTUALLY TO PRE-SERVICE ELEMENTARY EDUCATION TEACHERS

by

DEBORAH M. EDWARDS
A.A. Daytona Beach Community College, 1992
B.S. Stetson University, 1995
M.S. Nova Southeastern University, 2001

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Major Professor: Carolyn Hopp

ABSTRACT

The purpose of this Dissertation in Practice was to inform pre-service elementary education teachers of conceptual and procedural methods for teaching fractions. The problem of practice began when the researcher noticed a deficiency in fraction addition knowledge for a remedial mathematics program at a local private university. Further exposure of fraction knowledge for the 2014 third-grade Florida Comprehensive Assessment Test scores at a local elementary charter school ascertained slightly above 50% of those students making a 70% percentile or higher. Now that Florida State Standards are aligned with the Common Core Standards, pre-service elementary teachers need to know how to teach fractions procedurally and conceptually. This research-based model was used to determine the level of fraction knowledge, math anxiety level, and present NCTM videos aligned with Common Core Standards. A key element of the model was the performance assessment of the participants teaching randomly selected fraction problems they had already encountered confirming the need for more professional development in this essential mathematics domain.

I dedicate this dissertation to my mother. She was a guiding light, strong woman, and my best cheerleader who always told me to be the best at whatever I chose to do and to reach for the stars. I wish she could be here to see my accomplishments so I could continue to make her proud of her daughter.

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LIST OF ACRONYMS/ABBREVIATIONS

CSA	Charter School A
CCSS	Common Core State Standards
EOC	End of Course Assessments
FCAT	Florida Comprehensive Assessment Test
FDOE	Florida Department of Education
HSCT	High School Competency Test
IEA	International Association for the Evaluation of Educational Achievement
IRB	Institutional Research Board
MAFS	Mathematics Florida Standards
MARS	Mathematics Anxiety Rating Scale
NCTM	National Council of Teachers of Mathematics
PARCC	Partnership for the Assessment of Readiness for College and Careers
PTA	Parent/Teacher Association
SSS	Sunshine State Standards
STEM	Science, Technology, Engineering, and Mathematics
TIMSS	Trends in International Mathematics and Science Study
VCSB	Volusia County School Board

CHAPTER 1: PROBLEM OF PRACTICE

Statement of the Problem

Over 50 percent of students entering two year colleges are placed in remedial classes and almost 20 percent are taking remedial classes at four year universities (Complete College America, 2012). Because of the rising level of freshmen enrolled in remedial math classes at colleges and universities, the problem of practice this dissertation will address is the conceptual and procedural teaching methods of a basic mathematical concept used in remedial mathematics specifically known as the operations of fractions. In the State of Florida, colleges and universities offer remedial mathematics programs that are growing in enrollment. Students transitioning from high school to college are not mathematically prepared, as they should have mastered specific skill sets directly from high school math classes (Manly & Ginsburg, 2010). Some of the most common issues for beginning level math college students are the knowledge of number sense, word problems, problem solving, and "a lack of proficiency with fraction concepts" (Brown & Quinn, 2006).

One would think that college students do not demonstrate the learned behavior of their teachers, but if the foundation of understanding number sense or fractions was not clear nor exemplified in the youngest years of learning, then that behavior begins to have a snowball effect and a thorough conceptual awareness of mathematics is not accomplished in the initial foundations. Not knowing where the difficulty in mathematics in prior education years began for the college students, this dissertation will begin a discussion of the onset of fraction concepts teaching. The model presented in this dissertation in practice will focus on teaching fractions conceptually and procedurally to pre-service elementary education teachers.

According to the Florida Department of Education (2014), the full implementation of the revised Common Core State Standards (CCSS), now called Florida Standards, will begin in the 2014-2015 school year. Florida schools will need educators from elementary to high school levels who have the knowledge and skills to teach a more rigorous and deeper conceptual curriculum than ever before. This integrated standard system involves all levels of the K-12 educational structure, but the elementary school teachers who begin teaching number sense in depth are the first level of professionals that students will encounter. The foundation of mathematical learning begins in elementary school, even in kindergarten.

In teacher education programs, elementary education majors have experienced four years of learning how to teach children ranging from ages five- to twelve in grades kindergarten to sixth grade. Their certification is required by the state of Florida to enable these new educators to enter into the classroom. Although they may pass the new requirements of the CCSS certification exams, this is not always an indicator that they have a comprehensive understanding of mathematics and how to teach concepts and operations with fractions (Soto-Johnson et al., 2008). Elementary education teachers sometimes do not learn methods of teaching mathematics since they are not required to take any methods courses. For example, Tooke & Lindstrom (1998) states that Texas legislation banned methodology courses for education majors back in the late ‘90s.

At first, a new elementary education teacher may be excited to display pretty posters and feel ready to begin the new school year, but what if their confidence level in mathematics is extremely low? How does this attitude or behavior transfer to students in an elementary class setting? Not including the *Counting and Cardinality* found only in kindergarten, elementary education majors certified in K-6 grade levels are required to teach the four common domains*:* *Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data*, and *Geometry* in the Common Core Standards for grades K-6 (FDOE, 2014). When the sixth domain *Number and Operations in Fractions* is introduced in third grade, the lack of teacher knowledge and skills to teach fractions becomes more evident (Tooke & Lindstrom, 1998).

Pre-service teachers need to be aware of and overcome their own weaknesses in mathematics, especially in the understanding of operations among the realm of numbers such as fractions. More importantly, they must know the most effective instructional strategies to use to teach fundamental mathematical concepts.

Examples of the Problem

At a local charter school, Charter School A, there were 31 third graders who took the FCAT last year (CSA, 2014). The "fractions" domain had 10 possible points to be earned (see Table 1). Table 1 displays the scores from 1 to 10 and how many students earned each score.

Using the standard grading system of a 10-point scale, there were 25.8% (n = 8) of the students who earned an "A", 12.9% (n = 4) of the students who earned a "B", and 12.9% (n=4) earned a "C". Therefore, 48.4% (n=15) did not earn an "A", "B", or "C" grade in the fractions portion while slightly over half of the students performed at a "C" level or higher on this portion of the FCAT (CSA, 2014).

Table 1: 2014 Third-Grade Students FCAT Scores for Fraction Domain

Scores	Amount of Students
10	4
9	4
8	4
7	4
5	3
5	3
4	4
3	2
2	1
1	2

Note: Adapted from Charter School A FCAT results for 2013-2014 school year. Copyright 2014 by CSA. Reprinted with permission.

During the fall semester of 2014 at a university in the Daytona Beach area, items involving operations with fractions on the remedial math placement exam were evaluated. Question #4 is an addition of fractions problem $\frac{2}{3}+\frac{3}{4}$. Out of 283 remedial math students who took the remedial math placement exam that semester, 33.6% (n=95) of the students could not answer this question correctly while two-thirds of those remedial math students answered accurately (Edwards, 2014). Comparing the results of the elementary norm referenced test to the specific placement test question of a local university, the percentage of students understanding fractions at a "C" level or higher does not increase by much (51.6% to 66. 4 %).

When the students in elementary schools do not grasp a complete understanding of fractions, the misconceptions or misunderstandings of this crucial mathematical concept could transfer to their next level of education into middle school. Bailey et al. (2014) conducted a longitudinal study showing that the early mathematical understanding of fractions "is a predictive of much later overall mathematics achievement" (p.776). At an early age, the students may feel incompetent in fraction operations and may avoid completing problems that involve this concept. When those students enter high school, the snowball effect of not understanding fractions could continue unless there is an intervention that assists those students with their deficiency. Students in high school demonstrate an inability to be proficient in fraction concepts when asked to complete algebraic problems involving fractions (Brown & Quinn, 2006). In college, students are expected to know how to complete operations with fractions in all math classes.

Mathematics is a progression of learning concepts that build upon each other. Difficulties can arise when students try to apply knowledge learned in one context that is applied to another context that may be connected to a previous concept (Geiger & Galbraith, 1998). The four basic operations addition, subtraction, multiplication, and division of fractions are key elements in understanding algebraic concepts in middle school math classes (Bailey et al., 2014). New teachers will be expected to demonstrate mastery of fractions when teaching the Florida Standards directly associated with the third through fifth-grade domain, *Number and Operations in Fractions* (FDOE, 2014). Not understanding how to teach fractions can adversely affect teaching and learning. Van Steenbrugge et at. (2014) examined first-year pre-service teachers and last year pre-service teachers' ability to teach fractions conceptually and procedurally only to find that there is no difference when it comes to having limitations in knowledge of fractions.

An elementary teacher needs to be well versed in the ability to teach all grades from kindergarten to sixth grade, which means they need to understand fractions when assigned to teach third grade and above. Understanding the standards and being able to collaborate on new approaches with colleagues is crucial to their comprehension of these mathematical concepts (Wise & Darling-Hammond, 1984).

Pre-service teachers' learning and understanding effective strategies for teaching number sense and fractions to elementary level students is essential. Additionally, their own knowledge of higher level mathematics is impacted. Pre-service teachers will need to have a positive attitude toward teaching fractions rather than an anxiety level that could inhibit a deeper learning since anxiety could surface when teaching the subject (Tooke & Lindstrom, 1998). Math anxiety in an elementary education setting can lead to less time spent on the subject and negative feelings toward mathematics as a whole (Rayner et al., 2009). Less time on this mathematical concept could lead to less understanding of fractions and a weaker ability to complete harder tasks in mathematics.

Organizational Context

In general, the organization of interest is elementary schools. The specific institution of interest used to provide the framework for the discussion of teaching both procedural and conceptual knowledge is Charter School A. This charter school began with less than 300 students and reopened the doors of a school that the county had closed due to budget cuts. Their mission statement document found on the school's Internet website states (CSA, 2013):

> ***CSA's "mission is to cultivate learners and leaders who are inspired, able, and prepared to make a positive difference in the world"*** **(p.2).**

Charter School A, CSA, believes that STEM concepts are the root of meaningful and enhanced learning that will allow children to implement what they have learned in their community and life. CSA also believes in community involvement and support through partnerships. The school enjoys visitors from different businesses and stakeholders in education. Their philosophy is learning through doing. CSA offers a “project based, active learning environment that links to real life” and “fosters critical thinking, independent problem solving” (CSA website, 2013, p. 1). This school has a Board of Directors with a Management Company, EdFutures, Inc., that assists and increases productivity in public schools such as charter schools. There is a principal, assistant principal, dean of students for middle school, one exceptional student education specialist, and instructors who are all degreed and highly qualified as defined by the state of Florida in their subject areas (CSA, 2014).

History and Conceptualization (Local, National, and International)

Local

In 1872, the first public school in Volusia County was established in New Smyrna Beach rather than other areas because the first Volusia County Superintendent lived in that town (Langlotz, 2000). Each school day lasted about six hours and the school terms could vary between three to six months. The basic curriculum of the three R’s, writing, reading, and arithmetic, along with spelling, history, and geography were offered. A few times a week the students were taught farming skills and needlework. Thirteen years later, another school was developed and again, sixteen years went by for the third school to be established. This third school housed the first elementary grade classrooms with a teacher for each of the primary and middle school grades. The high school subjects were taught by three teachers. Elementary schools began to appear across Volusia County in cities such as Daytona Beach, DeLand, and Ormond (Langlotz, 2000). The age-graded schools began to pop up everywhere and little one room schoolhouses were found in almost every community.

In 1996, the first charter school law was approved by Florida in turn allowing Miami to open the doors of the first charter school in Florida, Liberty City Charter School (O'Connor, 2014). Reading Edge Academy was the first charter school to open in Volusia County (Martin, 2011). Since then many charter schools have opened and some have closed. Under Charter School law, any private group(s) can create charter schools as long as the requirements and laws are followed.

Charter School Law focuses on curriculum, baseline standards for instructional evaluation of students, methods used for determining students' success via assessments, financial and administrational stability, balanced admission of students to a charter school, qualifications of the teachers, governance structure, and a timeline of goals to be met (FCPCS, 2014).

Establishing a Charter School

Anyone can start a charter school as long as the constraints of Florida Department of Education Charter School Law are followed. These constraints consist of: a) no charge for attendance, b) financial and academic governance structure that is held accountable with audits conducted periodically, c) compliance with civil rights for children, and d) participation in the Florida's education accountability program (FDOE, 2012).

There are also several types of charter schools. Most schools that begin as charter schools are "new start-ups," but some are conversion charters such as CSA. A conversion charter school is a school that used to be a public school prior to being a charter school. Charter schools in the workplace service the children of the employees while charter schools in municipalities are started by local school districts in cooperation with the municipality to service special racial/ethnic groups of the community. Additionally, there are charter schools in community colleges to offer associate degrees to students and now current trending virtual charter schools are being formed. The interested parties have to create a vision and build a team for the charter school. There is research and development of a business plan to be completed before the application is submitted. Once approved by the district's school board, the founders have to prepare the grounds for opening. Also a governing board consisting of stakeholders, teachers, community persons, and those of interest must be assembled and continue to meet since they are the ones legally responsible for the oversight of the school (FDOE, 2012). The charter schools are evaluated every three to five years, depending on the contract created with the district for compliance of the educational laws.

National

Luo et al. (2011) state that "to provide better teacher preparation in mathematics, the United States need to re-examine the content and instruction of mathematics courses required for these pre-service teachers" (p.175). Degree requirements for elementary education majors vary from college to college and state to state. "The teacher education programs need to provide opportunities for their pre-service elementary teachers to develop fluency with fractions on a number line" (p.175). Consistency of curriculum for elementary education majors in the United States could actually assist the Common Core Standards to be more successful in its goals. Elementary education teachers have to be certified and "highly qualified" according to the Florida Department of Education (FDOE, 2014). Teachers may appear to be highly qualified according to a checklist of credentials met, but their actual knowledge of teaching fractions procedurally and conceptually to third graders according to CCSS design may be weak.

Since 1995, Trends in International Mathematics and Science Study, TIMSS, have been conducting international comparisons of mathematics and science achievement among countries around the world (Kastberg et al., 2013). The United States has participated in the studies since 1995 but no educational system has been consistent in the assessment for all five years (1995, 1999, 2003, 2007, 2011). There are 18 educational systems from the United States that have participated in the TIMSS assessments (see Table 2).

Table 2: United States Participation in the TIMSS Assessment by Year and Grade Level

Educational System	1995	1999	2003	2007	2011
Alabama	-	-	-	-	8
California	-	-	-	-	8
Colorado	4	-	-	-	8
Connecticut	-	8	-	-	8
Florida	-	-	-	-	4//8
Idaho	-	8	-	-	-
Illinois	8	8	-	-	-
Indiana	-	8	4//8	-	8
Maryland	-	8	-	-	-
Massachusetts	-	8	-	4//8	8
Michigan	-	8	-	-	-
Minnesota	4//8	-	-	4//8	8
Missouri	8	8	-	-	-
North Carolina	-	8	-	-	4//8
Oregon	8	8	-	-	-
Pennsylvania	-	8	-	-	-
South Carolina	-	8	-	-	-
Texas	-	8	-	-	-

Note: Chart revised from TIMSS Table 1 from Kastberg et al. (2013). The dash represents no participation that year for that particular educational system.

In the 2011 TIMSS study, Florida and North Carolina public schools are the only U.S. educational programs that participated in the recent study. Thirteen percent of the 4th graders were at or above the benchmark (*score of 625*) for “advanced” scores in comparison to the international median of 4 percent (Kastberg et al., 2013). The three content domain areas that are assessed by TIMSS are student knowledge of number, geometric shapes and measures, and data display (Kastberg et al., 2013). In the number domain, which would involve fraction knowledge, the United States performed at a score of 543, specifically 564 for North Carolina and 548 for Florida (Kastberg et al., 2013, p.17). The mean score for TIMSS is 500 with a standard deviation of 100. From the 2011 TIMSS report, the United States performed higher than the average benchmark of 500 (Kastberg, 2013).

International

The preparation of mathematics teachers in primary grades is weak and could be considered an obstacle to overcome for understanding mathematical concepts in a more thorough design (Schmidt, 2012). In 1996, the Third International Mathematics and Science Study, TIMSS, compared performance and curriculum design in mathematics and science of 40 different countries. Japan and Spain were found to teach fewer mathematical concepts while Norway, France, and United States covered a larger range of topics. For some international countries, the curriculum reform was a motto of "smaller is better." For example, in a fourth-grade math class, more time was given to fractions to develop a deeper understanding (NAS, 1996).

In Finland, primary teachers hold a master's degree in education (Tucker, 2011). These teachers also conduct class as a community of learning with common goals holding the students accountable for each other (Andersen, 2010). The 2011 TIMSS report for fourth-grade mathematics show Asian countries (Singapore, Korea, Hong Kong, Japan, and Chinese Taipei have the highest achievement of all other countries such as Belgium, Northern Ireland, the Russian Federation, England, and Finland that were in the top-ten for high achieving countries. In China and Japan, students are considered a community of learners and express their ideas verbally in class. Feedback, albeit positive or negative, is given by other students and instructor with the outcome to be considered for growth and not personal attacks especially if feedback is negative (Tucker, 2011). On the contrary, in the United States, teachers often ask students for answers to be shared with the class and only the instructor responds with feedback.

The eight educational systems that have higher TIMSS scores than the United States are as follows: Singapore, Hong Kong, Chinese Tapei, Japan, Northern Ireland, North Carolina (USA), and Belgium. Compared to the first TIMMS 1996 math scores (518), the United States has improved its mathematics average throughout the years, 2007 (529) and 2011 (541). Some countries have not improved through the years. For example, the Netherlands and Alberta, Canada educational systems have actually scored lower scores in 2011 since 1995 in the fourth-grade student assessments (see Table 3). Looking at the fourth-grade scores of the 2011 TIMSS Number domain, several Asian countries have the highest scores (see Table 4).

Table 3: Change in Average Mathematics Scores in the Education System for Fourth-Grade Students

Educational System	1995	2003	2007	2011
Singapore	590	594	599	606
Rep. of Korea	581	-	-	605
Hong Kong	557	575	607	602
Chinese Taipei	-	564	576	591
Japan	567	565	568	585
Ireland	-	-	-	527
Denmark	-	-	523	537
England	484	531	541	542
Russian Federation	-	532	544	542
Netherlands	549	540	535	540
United States	518	518	529	541
Canada	-	-	-	-
Quebec	550	506	519	533
…………..Alberta	523	-	505	507
…………..Ontario	489	511	512	518

Note: Chart revised from TIMSS Figure 1 from Kastberg et al. (2013). The dash represents no record of data for that year.

Table 4: Average Mathematics Content Domain Scores in the 2011 TIMSS Assessment for Fourth-Grade Students by Educational System

Educational System	Score
Singapore	619
Rep. of Korea	606
Hong Kong	604
Chinese Taipei	599
Japan	584
Northern Ireland	566
Belgium	552
Finland	545
Russian Federation	545
Netherlands	543
United States	543
Canada	
Quebec	531
Alberta	505
Ontario	504

Note: Chart revised from TIMSS Table 5 from Kastberg et al. (2013).

Factors that Impact the Problem

Teaching Standards

Not happy with the erratic standards of American education after Sputnik, a group of professionals in sociology, psychology, and education came together in 1958 to discuss student evaluations and the different kinds of problems in schools. They conducted a study known as the "Pilot Twelve-Country Study" (IEA, 2011) to explore the educational achievements of thirteen year old students from twelve countries. This study unveiled findings of feasible testing across nations and the ability to compare the educational quality through assessments. This group was known as the International Association for the Evaluation of Educational Achievement, IEA (IEA, 2011). The First International Math Study was conducted in 1964 between twelve countries that involved thirteen year old students and graduating students. The results of these tests throughout the years are what began the concern of American education and how the US educational system did not appear to be teaching American children the same content as their contending countries. The math wars began.

A variety of different standards and curriculums have been created sporadically for the last 50 years hoping that each new one will enhance the quality of teaching. "American educators have been concerned with the educations standards of public schools since the common school system was established in the 19th century" (Miyamoto, 2008, p. 27). Not having consistent objective measurement tools and standardized tests were hindrances in schools and a change was needed. Standards or "norms" were soon created by "men of scientific ideals and scientific training" (Miyamoto, p. 36). In 1980, President Ronald Reagan created the *A Nation at Risk Educational Reform* report that began the standards race for America. The report suggested that four years of English, three years each of science, math and social studies along with a half a year of technology science be included in America's educational curriculum (NCEE, 1983). Eleven years later, high expectations were set to improve the quality of math and science with recommended measures to be used in tracking the progress towards baseline goal (Blank et al., 1992).

The national studies reported that the United States had a decline in mathematics and science scores compared to other countries and there was a shortage in quality teachers in these fields. The states' policy makers decided to raise the standards for teacher preparation, mandate teacher tests for certifications, develop curriculum guidelines, and statewide assessments (National Governors Association, 1986).

In the late 80s, President George Bush and his administrative team decided to call upon the states to develop standards for students to be measured and assessed in a standards-based reform. So government officials and professional educators began to meet and create principles, curriculums, and assessments that would play key roles in meeting goals and hopefully shaping the performance of students as higher scores among international assessments. Along with the states trying to create assessments, in 1995, The IEA sponsored Trend in International Mathematics and Science Study (TIMSS) created by various educators in mathematics and science (Plomp, 1996). The assessments were for 3 groups: (1) third and fourth grade, (2) seventh and eighth grade, (3) graduating year of students. Unfortunately, the results from this study showed that the United States was among the lowest countries in performance among mathematics and science.

To America's astonishment of its low test placement in an international race of grades, the next President, Bill Clinton, addressed these assessment outcomes by stating a possible solution in his 1997 State of the Union Address that

> "Every state should adopt high national standards, and by 1999, every state should test every 4th grader in reading and every 8th grader in math to make sure these standards are met" (Clinton, 1997).

In 1997, the "high" standards for the United States educational system that President Bill Clinton alluded to were adopted by only thirty-one states at first but soon grew to forty-nine states within five years. The states' standards varied significantly and the level of proficiency for the students were different as well (Ross, 2010). Again, a call for uniform standards was soon to prevail but a no "one size fit all" system was available. To heed the call of President Clinton's concern for education, Florida created Sunshine State Standards (SSS) in 1998 (FDOE, 1998).

Prior to SSS, the state had competency exams such as the High School Competency Test (HSCT) which allowed educators to measure the level of mastery in English and mathematics.

Though this assessment was the initial attempt by Florida to have accountability statewide, it was not in alignment with the SSS. This assessment was phased out when the Florida Comprehensive Assessment Test (FCAT), was piloted in 1995 (FDOE, 2014) and HSCT was finally discontinued in 1998.

Standardized Testing in Florida

The new criterion-referenced FCAT was administered to students in grades three through eleven to test mathematics, reading, science, and writing. Passing the FCAT was a crucial criterion for graduating high school. Unfortunately, high school students who were passing their classes but not passing the FCAT became a concern of educators. Not only did the educators show concern of this growing epidemic throughout states, but President Barack Obama also expressed his concern in 2009. *The Recover and Reinvestment Act* of 2009 became a new and improved “Race to the Top” Program with financial incentives to states who could create and implement new standards to help with the country’s low scores in these international assessments (Obama, 2009). With the decision to revisit the standards, Florida created the Next Generation Sunshine State Standards (NGSSS) in 2009 (FDOE, 2014). These standards included End-of-Course (EOC) assessments to overrule the passing of the FCAT for graduation.

The FCAT was administered for the last times during fall of 2014 and again in spring of 2015. EOC assessments will replace the graduation requirements along with a new assessment team, Partnership for the Assessment of Readiness for College and Careers (PARCC), as the summative assessments for the most recently adopted standards, Common Core State Standards (FDOE, 2014). Starting in the 2014-2015 school year, Florida students will take computer-based PARCC assessments in literacy, English, and mathematics to gauge the child’s readiness of college and/or career in efforts to assist the parents and teachers to customize the educational needs of a under prepared student (FDOE, 2014). Florida again changed the name of their revised standards to *Mathematics Florida Standards (MAFS)* since the State Board of Education approved the decision on February 18, 2014 (FDOE, 2014) and have adopted to align the state's standards with the Common Core Standards. According to Florida's CCSS timeline (FDOE, 2014), by the school year 2014-2015, the full implementation for all content areas will be in place and computerized assessments through PARCC will commence. In grades K-8, the Florida Standards Assessment (FSA) will be the end of year assessment that measures English for grades 3-11, mathematics for grades 3-8, and includes end of course assessments for high school mathematics classes such as Algebra 1, Geometry, and Algebra 2 (FSA,2015).

Teachers are required to implement the Florida Standards and are evaluated on their success via testing of the students through assessments that align with the standards (FDOE, 2014). Universities are now required to realign their methods courses in order to produce qualified educators to teach according to the Florida Standards. Teacher preparation is going to be more rigorous and veteran teachers will need more professional development to keep up with the changing standards. Wise and Darling-Hammond (1984) believe that increasing the standards for teachers, but not increasing the pay, would make most good teachers leave the profession. Even though certification will become more stringent, the scores on teacher competency exams have not been found to correlate to teacher performance (Wise & Darling-Hammond, 1984). True evaluation of teachers is not just ten minutes of an administrator in the back of a classroom, and these researchers believe "Remote controlled classrooms" will not be the end all to lower achievement scores (Wise & Darling-Hammond, 1984).

Pre-Service Teaching

Using microteaching to provide simulation in a teaching environment is "a useful tool for pre-service teachers' professional development" (He &Yan, 2011, p. 301). The authors define microteaching as a short time of teaching focusing on "one particular aspect of a teaching technique" (p. 291) used to simplify a complex teaching process. Pre-service teachers learn how to teach a complex topic and then videotaped during their turn to teach. Reflection and feedback are used to view and discuss strengths and weaknesses (He & Yan, 2011). Elementary education majors should also observe classrooms of different grade levels so that they may reflect on teaching styles and techniques that may or may not be successful because different grade levels require different techniques. Tait (2006) describes pre-service courses as "an important role to play in helping new teachers prepare to teach math well" (p.2).

There is also a lack of connection to theoretical and practical experiences for pre-service elementary education teachers. "Moseley et al. (2007) investigated the knowledge of fractions of 7 Japanese and 6 American experienced elementary education teachers to find that the American teachers focused on the part-whole sub-construct (procedural) while Japanese teachers taught the underlying sub-constructs (conceptual)" (Van Steenbrugge et al., 2014, p.142). The ultimate goal is to improve the mathematical ability of the children in American elementary schools so they are more "successful contributors to democratic society" (Langlotz, 2000, p.2). Novice teachers feel confident entering into the classroom after graduating college, but soon find this self-efficacy decreased when they begin to teach in their own classrooms (Tait, 2006).

In other countries such as Finland, teachers are prepared with three years of normal school. However, to set the bar to a higher standard, the accreditations of the teacher education reform act of 1979 became a master's degree requirement for employment in the educational field. This demand gave rise to the teacher being considered a higher paid, respectable profession on the same level as a doctor or lawyer (Tucker, 2011). Comparatively, in the United States, elementary education teachers only need a bachelor's degree to be hired to teach (FDOE, 2014).

Curriculum Resources

Textbook publishers create books and supplemental material that are aligned with the new standards, hoping that sales will flourish across the nation (King & State Higher Education, 2011). Even though textbooks may have great explanations and examples, teachers make the final decisions about how to complete the mathematical tasks at hand. Textbooks with readability level too high or confusing could be considered a challenge to use in a classroom especially for children with literacy deficiencies. As the population of diverse students grow, the range in learning levels will call for the need of adequately developed textbooks (Sood & Jitendra, 2007). The study conducted by Sood and Jitendra (2007) discovered that there is a "need to improve mathematics textbook instruction" especially for teachers "who may not have deep understanding of the content" (p.155).

Two instructors, Massey and Riley (2013) state that "Mathematics textbooks play a critical role shaping instruction and the ways students and teachers use strategies" (p. 577). They also strongly believe that reading is a major part of mathematics textbooks and the ability to understand what is written is a metacognition factor for teachers (Massey & Riley, 2013). Mathematic textbooks are designed with pictures and many formulas but more so a different vocabulary that most books. Not understanding the language patterns that are different than narrative patterns is sometimes the underlying problem of not understanding the mathematics displayed in the textbook (Massey & Riley, 2013). Thus, pre-service teachers must have a depth of conceptual knowledge of mathematics in order to understand the complex language. A lack of this kind of understanding can contribute to possible misinterpretation of the textbook terms.

Instructional Strategies

Conceptual knowledge versus procedural knowledge is also a factor that can affect the mathematical instruction of elementary education teachers. Ma (1999) documents an in-depth study of Chinese and American teachers' differences of conceptual understanding and performance of teaching. Those teachers that were more procedural in their deliverances did not understand the mathematics as thorough as the teachers who used conceptual approaches with real-world applications. Common Core Standards require teachers to change the design of teaching fractions as not only parts of a whole but to also think about fractions as distinct values on number lines (Heitin, 2014).

In the Mathematics Florida Standards (MAFS), each concept in the different domains has four levels of cognitive complexity (FDOE, 2014). The first level is "Recall" and involves recalling simple facts, information, and/or procedure. The second tier is "Skill/Concept" which contains using information or conceptual knowledge to complete two or more steps. The third rank of cognitive complexity is called "Strategic Thinking." This level encompasses reasoning, developing plans or sequences of events with sometimes more than one possible answer. The highest tier of cognitive complexity is "Extended Thinking." Level 4 comprises of investigative thinking through processes of multiple conditions or steps to a problem. As the grade level and content increases, so does the expected cognitive complexity (Webb, 2005).

Teachers will have to teach how to think about fractions as not only area of a visual object but to also think about how that value plays a part on the number line in correlation to other values which will help in understanding number sense in mathematics aligned with the cognitive complexities. Teachers who do not understand or enjoy teaching mathematics will spend significantly less time teaching the subject (Sloan, 2010). Spending less time on mathematical concepts such as fractions can lead to students having more difficulties with higher level mathematics involving fractions and possible math anxiety (Sloan, 2010). If the teachers do not understand the cognitive complexity of the fractional problems they are expected to teach, then the math anxiety could continue to find its way into the classroom. "Students often develop math anxiety in schools, frequently as a result of learning from teachers who are themselves anxious about their mathematical abilities" (Finlayson, 2014, p. 101).

Math Anxiety

In the late 1970s, Sheila Tobias wrote a book *Overcoming Math Anxiety* that stemmed from her observations at the university which focused on women who avoided math classes due to their lack of confidence in their ability to complete mathematical tasks (Tobias, 1978). The definition of math anxiety is typically the feelings found in the affective domain: panic, helplessness, paralysis and disorganization of thoughts usually aroused during a time of mathematical calculation (Tobias, 1978). Math anxiety has continued to grow into not only the cognitive domain of the students but also the teachers. If elementary education teachers have math anxiety while teaching mathematics, these teachers could pass on the anxiety to their students (Finlayson, 2014). Teachers need to know if they have math anxiety and learn how to cope with it so that it is not transferred to their students nor displayed in the classroom.

The Model Design

Project Scope and Stakeholders

The proposed model informed by the pilot study conducted will be designed to measure the understanding of number sense and fractions of pre-service elementary education teachers. Additionally, the model will be designed to strengthen pre-service elementary teachers' knowledge of mathematics. It will focus on the improvement of abilities to teach fractions and their operations. The model will also be used to discover the level of math anxiety, if it exists, that the pre-service teachers have and if there is a relationship between the level of math anxiety and the level of mathematical ability in teaching fractions. The stakeholders will be the pre-service teachers in one methods class at a large metropolitan university in central Florida.

The significance of this model is that it addresses the possible reasons for misconceptions of these primary but essential mathematical concepts, and seeks to deepen knowledge of teaching techniques. "Remembering rules and mastering standard procedures rather than demonstrating comprehensive understanding of mathematical ideas and procedures" (Luo et al., 2011, p.165) are reasons why pre-service elementary educators are weak in their ability to complete operations involving fractions. Accurately measuring students' knowledge is important when dealing with misconceptions, e.g. mixing prior knowledge that is not accurate with current concepts being taught (Durkin & Rittle-Johnson, 2014). Not having proper fraction knowledge interferes with the learning process of other mathematical concepts (Durkin & Rittle-Johnson, 2014). The rationale is to establish a more in-depth delivery of these mathematical concepts as they relate to the Florida Standards required by pre-service elementary teachers.

Basis of the Model

A pilot study will be completed to determine specific components to inform the model design. This Dissertation in Practice will describe the process and findings from the pilot study. The study presented in this Dissertation in Practice will address the following areas of pre-service teachers' practice:

1. teaching methods that provide instructional strategies for procedural learning of fractions according to CCSS (FDOE, 2014);
2. teaching methods that provide instructional strategies for conceptual learning of fractions according to CCSS (FDOE, 2014);
3. measuring math anxiety levels of pre-service teachers using the Mathematics Anxiety Rating Scale Shortened Version (MARS-S)

Teaching Methods

Different methods of teaching fractions will be shared with the pre-service teachers using the standards from MAFS and videos of teaching concepts. The videos will be from National Council of Teachers of Mathematics (NCTM) YouTube Channel and Educational Week (NCTM, 2015; EdWeek, 2014). Also, the researcher, an instructor in higher education, will demonstrate procedural and conceptual strategies for teaching fractions of the two most missed problems on the fractions worksheet distributed to the pre-service teachers during the pre-test phase of the project. Discussion of the concepts needed for certain fraction problems will be included throughout the project.

Math Anxiety Measures

The anxiety level of teachers in classrooms can be transferred to their students. For example, according to research conducted at University of Chicago, math anxiety from a female elementary educator can transfer to female students in a way of confirming a stereotype of girls not being good at mathematics (Math Anxiety, 2010). Since most elementary education majors are female, the transfer of this math anxiety could actually be higher than studies show (Math Anxiety, 2010). It is important for teachers to understand this phenomenon. The proposed model will include the shortened version of the Mathematics Anxiety Rating Scale (MARS-S) which will allow participants to know if an anxiety level exists and to what degree.

Richardson and Suinn (1972) developed a 98-item questionnaire constructed to include real-world and academic situations to stimulate math anxiety of the participant. A five-point Likert scale of one (lowest) to five (highest) to represent the potential math anxiety rate of each question listed. This inventory of questions is called the Mathematics Anxiety Rating Scale Shortened Version (MARS-S) and has been validated in several studies to demonstrate a positive correlation with dislike to mathematics, anxiety self-report, the length of time this anxiety has existed for the participants, and test anxiety (Brush, 1978). If there is a high degree of anxiety, the model will propose the use of coping strategies that can positively impact teacher performance.

Documentation

Before beginning the pilot, IRB consent forms will be distributed to all of the pre-service teachers involved in the research. Because the participants are over the age of 18, the consent forms will not need to be signed but each participant will receive a copy prior to participating.

Observations made of the pre-service elementary education teachers during the lessons will be documented and examined for various teaching methods and mathematical language usage. Participants' responses, feedback, and reflective papers will also be documented. The model will include participants' reflective responses, feedback, pre-post test results from both the MARS-S and FCAT fraction worksheets (FDOE, 2014), and the performance assessments.

The pilot will demonstrate a process for understanding procedural and conceptual fraction knowledge. The intended outcomes will be for pre-service teachers to:

1) learn how to conceptualize the teaching of fractions,
2) increase their self-efficacy about teaching fractions, and
3) become aware of math anxiety if it is present.

Implementation

The researcher intends to determine if, indeed, pre-service elementary teachers lack understanding of how to work with fractions, which is essential learning for students at the elementary level. In order to determine what is needed in terms of teacher learning to ensure deep understanding, the pilot study will include a small group of pre-service elementary teachers in a reading methods class in Spring 2015. Due to time constraints of presenting the Dissertation in Practice, the model will be created and completed during the summer semester of 2015. However, classes of elementary schools in the United States are not in session during summer months, therefore it will not be possible to execute the model after it is created. The plan for implementation would be for further research to be conducted after the Dissertation in Practice is complete.

Data Tools

A major part of this model includes gathering information that will inform the design. Each participant will be given a letter for an identification for security reasons and all material will be kept in a folder securely in the office of the researcher. The first type of data to be collected to inform the model will the computational skills and knowledge of how to complete fraction problems grades 3-6. This data will be documented as pre-model scores of the pre-service teachers when given the Florida Comprehensive Assessment Test (FCAT) Test Item Specifications for grades 3-6 and in that order of grade level. These scores will be documented at the beginning of the model design, and the same FCAT questions will be given again at the end of the semester but in a different sequence of grade level. The scores of pre- and post-intervention will be collected and analyzed for improved scores of correct answers involving fraction computation. The level of mastery is important when teaching mathematics and the feedback from the FCAT test items will be significant for self-confidence in their content knowledge of the pre-service teachers. Consequently, due to the randomness of the post-test order of problems, the scores could be impacted in a negative correlation.

The second most important data piece of the pilot will be the pre-and post-test of the MARS-S. A measurement tool such as the MARS-S is used to determine if there are anxiety levels in the participants. This exam will be distributed on the second day of contact and again on the last day of contact. The data will remain anonymous and will be displayed in a table of repeated measures via SPSS to reveal any changes from inception to current anxiety levels.

Summary

Since the Florida Standards have been adopted for full implementation in the state of Florida, elementary to high school educators will need to teach more rigorously and in-depth than before. According to the TIMMS, mathematics average scores from 1995 to current, the United States needs improvement in the realm of number sense. Elementary education teachers will have to be “highly qualified” but, may be “highly anxious” about teaching mathematics with the cognitive complexity of the domains begin addressed in each grade and concept taught. The preparation of elementary education teachers needs to be more thorough in the understanding of mathematical concepts. Compared to other countries, teaching more is not always the best way. Teaching more in-depth is better when teaching fractions to develop a deeper understanding. Since the math wars, standards and textbooks have changed with the times in an attempt to enhance the quality of teaching. Textbooks will need to be carefully considered to improve deep understanding of the content since it plays a critical role in shaping curriculum design and strategies. Subsequently, the conceptual knowledge of the mathematics will need to be addressed in the textbooks in a readable language for the users to better understand the material.

Pre-service teachers need to be aware of their own weaknesses in mathematics, especially in number sense such as fractions. They should be aware of how needed areas of improvement affect their teaching and lesson planning. The comparison of elementary school scores on last year’s FCAT fractions domain to a fraction problem on a college mathematics placement exam showed that the percentage of conceptual growth of understanding was minimal (51.6% to 66.4%) considering the differences in the levels of the students. From the data given, it appears that students in college are not mathematically prepared. Mathematics is a hierarchy of learning concepts that build from and relate to each other. Pre-service teachers need to understand effective strategies for teaching fractions to elementary level students so that the level of conceptual understanding grows as the students progress to the next level of learning.

With the lack of connection between theoretical and practical experiences for pre-service elementary education teachers, this model will demonstrate teaching techniques used to simplify complex teaching processes. Designed to strengthen the pre-service teachers' knowledge of fractions and help those understand math anxiety if it exists, this model will utilize audio taping of pre-service teachers during a performance assessment, assess pre-service teachers' math anxiety with the MARS-S, and utilize microteaching as tools to improve the participants' teaching abilities. As stated by Kilpatrick et al. in the 2001 book *Adding It Up: Helping Children Learn Mathematics*, the teachers need to know "the mathematics they teach", "understand the concepts correctly", and to "improve their capacity to use it" (pp. 370-372).

CHAPTER 2: DETAILS AND RATIONALE FOR THE MODEL

Description and Meaning

This dissertation in practice focuses on the need for professional development of procedural and conceptual knowledge in teaching fractions. Although certified educators have the credentials to teach in grades kindergarten through sixth grade, those practitioners may be deficient in their ability to teach mathematics procedurally and conceptually, specifically fractions, due to their lack of conceptual knowledge, their moderate to high math anxiety level, and/or a combination of both. Most elementary school teachers "possess a limited knowledge of mathematics, including the mathematics they teach" (Kilpatrick et al., 2001, p. 372). The model that is presented in this dissertation resulted from a pilot study with pre-service teachers that focused on determining their level of knowledge pertaining to the teaching of fractions both procedurally and conceptually. Additionally, the pilot measured their levels of math anxiety and their ability to demonstrate procedural and conceptual teaching of fractions. The model will provide the framework for future reference to improve educational practices in teaching mathematics aligned to Common Core Standards.

Procedural and Conceptual Knowledge of Mathematics

The purpose of the pilot is to inform pre-service teachers of conceptual and procedural methods of teaching fractions. According to Kilpatrick et al. (2001), conceptual understanding and procedural fluency are two of the five strands discussed in his book *Adding It Up: Helping Children Learn Mathematics* needed to learn mathematics successfully. Conceptual understanding is defined as "the comprehension of mathematical concepts, operations, and relations" and procedural fluency is known as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001, p. 5). Conceptual method of teaching is giving the "why" and not just the "how" to complete a problem. For example, conceptual teaching addition of fractions is to explain why the denominators need to be the same and why the numerators only are summed during the same time of exposure to the procedural problem solving. Very few mathematical problems are needed to demonstrate the conceptual design of teaching. To demonstrate conceptual knowledge, the teacher must be aware of the type of problems taught and the level of difficulty for each. The concepts can be shown through just a few examples but more in depth when taught conceptually. Educational specialists define conceptual understanding as the "connected web of knowledge" (Stohlmann et al., 2015, p. 4) that allows the procedural concepts to be more understood if learned first. According to Stohlmann (2015), "robust conceptual understanding can build meaning for procedural knowledge" (p. 4).

Procedural method of teaching is simply showing step-by-step how to complete a problem. For instance, if someone were to teach addition of fractions procedurally, s/he would demonstrate step-by-step how to work the problem without explaining the how and why of the process. Teaching mathematics only procedurally is considered to be the less effective strategy and does not allow the students to have a full grasp of the conceptual idea of the problem in order to transfer knowledge of the process to higher level mathematics (Stohlmann, 2015). The purpose of teaching mathematics is for the students to learn the material in such a way that retention of concepts is established and transferrable to the next level of mathematics. With that in mind, the students continue to build on the knowledge and hopefully will not need remediation math classes as in the case of the students found in this study. Van Steenbrugge et al. (2014)

found that the level of teacher education (first-year vs third-year pre-service teachers) had no impact on teachers' ability to explain procedural rationale or conceptual meaning of fractional problems which was found to be still higher on procedural knowledge than the conceptual knowledge. The level of knowledge of fractions is the beginning point to discover what the pre-service teachers do or do not understand. The assessment of their fractional content knowledge is required to determine the mathematical difficulties these participants may exhibit (Van Steenbrugge et al., 2014).

The Pilot

The four participants enrolled in an elementary education reading methods course at a central Florida university participated in this pilot study. These four participants represent the largest pre-service program at the university--elementary education. There were five sessions with the participants that lasted approximately one hour each. The study was conducted in a classroom setting at the university during the last hour of a reading methods course.

Session 1

In session 1, the participants were thoroughly informed of the study by their instructor, insured that the participation had no bearing on their grade in the methods course, and were given the IRB approved consent forms. In order to determine their skills at working with fractions, the participants were provided fifteen fraction questions as a pre-test directly obtained from Florida Department of Education website (FDOE, 2014; see Appendix E). The participants were given thirty minutes to complete these fifteen questions. Three of the participants finished the FCAT pre-test in twenty minutes, but Participant D required thirty minutes for completion. The reason for FCAT questions from FDOE website is for commonality of typical fraction problems found in the classroom. The questions were sample items readily available for anyone's use in a classroom or preparation for FCAT testing. Each question directly pertained to fractional operations only.

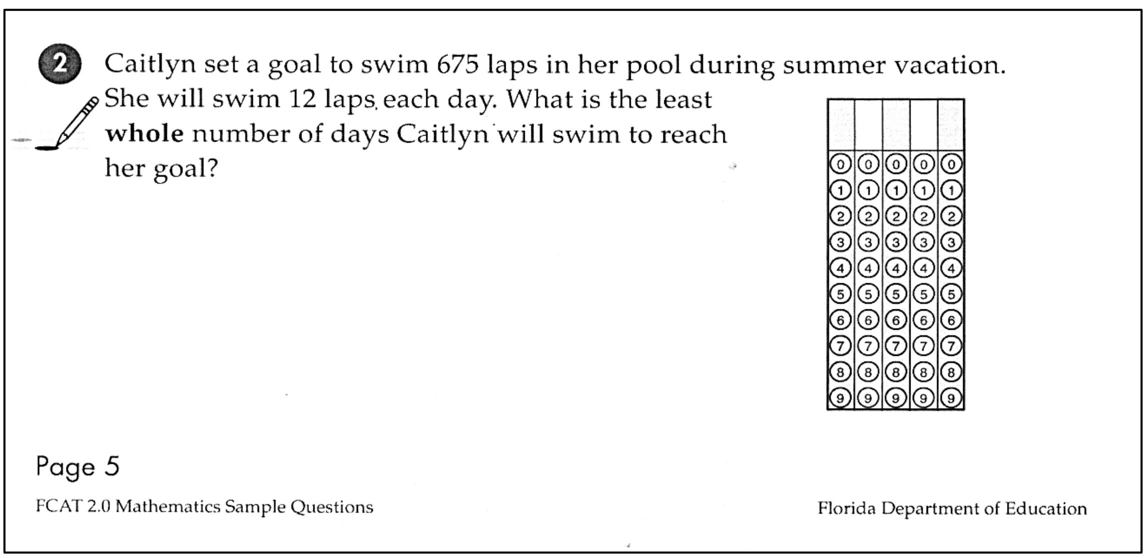
These fraction questions ranged from Grade 3 level to Grade 6 level and were arranged in order of grade level when given as the pre-test. The researcher has found in her own classroom that students tend to do better on assessments when the mathematical material is in order of simplest to more difficult problems. When the level of difficulty is randomly designed or shuffled, the students seem to have varied scores. To continue with this notion, the researcher decided to investigate the same phenomenon in the model. Hence the FCAT fraction worksheet post-test, even though same problems, were in a randomized order of grade level. For example, the first page was a sixth-grade problem and the next page was a third-grade problem. The third page was a sixth-grade problem while the fourth page was a fourth-grade problem. No two same grade levels were back to back in the page order. The order of the problems were random and not in order of difficulty according to grade level. The results of the post-test and the comparison will be discussed later. The participants' scores on the pre-test FCAT fraction worksheet ranged from perfect score to missing three problems. Table 5 represents the scores of the participants.

Table 5: Pre-test FCAT Fraction Worksheet Scores

Participant	Scores
A	15 out of 15 correct (100%)
B	13 out of 15 correct (86.7%)
C	12 out of 15 correct (80%)
D	15 out of 15 correct (100%)

Note: These scores are based on one point per correct answer with no partial credit.

There were four problems from third grade, three from fourth grade, two from fifth grade, and five problems from sixth-grade level on the FCAT 2.0 Mathematics Sample Question worksheets. The FDOE website had sample mathematics problems for all grades, but grade 3 is when the fraction domain is first introduced (FDOE, 2014). Also, elementary education teachers when certified to teach have the teaching range of kindergarten to grade six. Therefore, the fraction problems from Grade 3 to Grade 6 were the only problems chosen for the assessment to align to the certification grade span. Two of the participants achieved a perfect score while the other two participants understood the elementary level fraction problems at a "B" (80-89%) level. The problems that were answered incorrectly were from Grade 5 and Grade 6. The two problems that were missed the most were taught procedurally and conceptually to the participants during Session 2 (see Figures 1 and 2).



2 Caitlyn set a goal to swim 675 laps in her pool during summer vacation. She will swim 12 laps each day. What is the least **whole** number of days Caitlyn will swim to reach her goal?

Page 5

FCAT 2.0 Mathematics Sample Questions

Florida Department of Education

Figure 1: Grade 5 FCAT fraction problem

15 Mr. Madsen worked 49 hours last week at his job. He spent $\frac{1}{5}$ of this time in meetings and $\frac{1}{3}$ of this time talking to customers on the phone. Which method would provide the most reasonable estimate of the total number of hours Mr. Madsen spent in meetings and talking to customers on the phone at his job last week?

F. multiply $\frac{1}{4}$ by 50

G. multiply $\frac{1}{2}$ by 50

H. multiply $\frac{1}{5}$ by 50 and add $\frac{1}{3}$ to the product

I. multiply $\frac{1}{5}$ by $\frac{1}{3}$ and multiply the product by 50

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FCAT 2.0 Mathematics Sample Questions Florida Department of Education

Figure 2: Grade 6 FCAT fraction problem

The first problem missed the most was a fifth-grade level concept involving base ten and fractions. This problem uses division of multi-digit whole numbers fluently and checking the reasonableness of the results and is denoted in the Common Core Standards as *Big Idea 1 (Category 1): Develop an understanding of and fluency with division of whole number* (FDOE, 2014). Fractions are whole numbers with a division symbol separating them. This particular problem involved dividing 675 by 12 to achieve an answer of 56.25. It is a division estimate problem interpreting the division solution of a multi-digit divisor. The interpretation of the directions would be to write the answer as the next whole number rounded up, i.e. 57. Both of the participants who answered this problem incorrectly answered the problem as 56 and did not comprehend or misunderstood the “whole number” element in the directions. No partial credit was considered even though the mathematics procedurally was shown and properly performed.

Similar to a student taking the FCAT with this problem as a question, gridded answers did not receive partial credit.

The second problem missed the most was a sixth-grade level concept of *Category 1: Fractions, Ratios/Proportional Relationships, and Statistics* (FDOE, 2014). This decimal estimate problem involved the participant's understanding the whole number 49 estimated as 50 in the multiple choice selections. The correct method of choice depended on the knowledge of addition of fractions $\frac{1}{5}+\frac{1}{3}$, round that answer $\frac{8}{15}$ to the nearest fraction $\frac{1}{2}$, and multiply that fraction by 50. Participant B showed no work for the answer given and just circled choice "I". Similarly, Participant C circled choice "F" with no work shown. Both incorrect answers with no work shown reflect neither knowledge of how to complete the procedural nor the conceptual concepts required to answer the problem correctly as "G". Furthermore, Participant C missed a third problem that was open-ended and involved multiple steps to complete. It pertained to translating a percentage to a fraction, adding two fractions, and multiplying a whole number by the summed fraction. As per the work shown, her error was due to working in decimals instead of fractions. She translated the fraction into an incorrect decimal which incurred the final error of her answer. Considering that problem was only missed once by one person, it was not considered a most missed question. After the FCAT pre-test was completed by all participants, the session ended. The first session is table 6.

Table 6:Summary of Session 1

Process	Rationale	Time	Materials Used
Fraction Pre-test	Discover content knowledge level of fractions	30 minutes	FCAT 2.0 Worksheets involving fractions for grades 3-6 (Appendix E)

Session 2

The second vital component of the model is to determine the level of math anxiety the participants may possess. Math anxiety is defined as the lack of confidence in one's ability to complete a mathematical task (Tobias, 1978; Richardson & Suinn, 1972). In an elementary education setting, math anxiety can lead to less time spent on the subject (Rayner et al., 2009; Sloan, 2010) and can surface when teaching the subject (Tooke & Lindstrom, 1998). Unfortunately, students can develop it as a result from teachers who demonstrate the anxiety (Finlayson, 2014) especially in same gender situations such as female students from female teachers (Blazer & Miami-Dade, 2011).

Also, students have been found to have math anxiety as early as first or second grade due to timed testing situations (Commentary Online, 2012). Research shows pre-service teachers stating their dislike of mathematics or feelings of inability to complete difficult mathematical tasks as some of the reasons they choose to teach young children because of the mathematics being considered lower levels than middle or high school mathematics (Lake & Kelly, 2014). This avoidance of solving mathematical problems is a sign of math anxiety and inadequate ability to teach the mathematics can be a potential contributor to math anxiety in the students (Blazer & Miami-Dade, 2011). Lake & Kelly (2014) found that helping pre-service educators recognize their feelings and having awareness of their level of math anxiety has a direct correlation to how they teach mathematics.

One of the types of math anxiety assessments is the Mathematics Anxiety Rating Scale (MARS). The MARS was created in 1972 by Richardson and Suinn as an instrument that explored issues relating to academic situations and everyday life in respect to mathematical tasks (Richardson & Suinn, 1972). It has been used for research and clinical studies since 1972. It contains 98 items with a Likert scale ranging from score of (1) for a “not at all” response to a (5) for a “very much” response. For the original MARS, scores could range from a 98 (score of 1 for all 98 items) to a 490 (score of 5 for all 98 items) with the higher score correlating to the higher level of math anxiety the participant exhibits. They discovered through various test-retest situations, there is a negative correlation between anxiety and mathematical ability (Richardson & Suinn, 1972).

Due to the time restraints, this study involved the revised and shortened version of the original MARS called the MARS-S. The copyright holder of the MARS-S was contacted via e-mail and 100 copies of the scale was obtained with permission to use for this study. The MARS-S is a 30-itemed Math Anxiety Rating Scale copyrighted in 1999 with the same reliability and validity as the original. High internal consistency due to a Cronbach alpha of .96 and test-retest reliability of .90 ($p < .001$) confirms that the shortened version is comparable to the longer version of 1972 (Suinn & Winston, 2003).

In session 2, utilizing the MARS-S, the four participants engaged in this 30-itemed questionnaire to determine their level of math anxiety. They were given fifteen minutes to answer thirty questions. See Appendix B for the full list of questions in the MARS-S. A factor analysis by Baloglu (2010) revealed a structure of five factors according to the questions posed: (1) Mathematics Test Anxiety, (2) Mathematics Course Anxiety, (3) Application Anxiety, (4) Social Anxiety, and (5) Computation Anxiety. The questions found in each factor are found in Table 7.

Table 7: Factor Analysis of MARS-S Questionnaire

Factors	Associated Questions
Mathematics Test Anxiety	1, 2, 3, 4, 5, 6, 9, 11, 12, 15
Mathematics Course Anxiety	7, 8, 10, 13, 14
Application Anxiety	18, 19, 20, 23, 24, 25, 26
Social Anxiety	21, 22, 28, 29, 30
Computation Anxiety	16, 17, 27

Note: Adapted from Baloglu, M. (2010). *An investigation of the validity and reliability of the adapted mathematics anxiety rating scale-short version (MARS-SV) among turkish students* Springer.

Much like the scale for the original assessment, MARS-S has a Likert scale representation of the emotional designation for the participant's fear or apprehension of the question posed: (1) for a "not at all" response, (2) for "a little", (3) for "a fair amount", (4) for "much", and (5) for a "very much" response. The lowest possible total score is a 30 (score of 1 for all 30 items) and a highest feasible score of 150 (score of 5 for all 30 items). Typically, according to Suinn & Winston (2003), a percentile of 75% (approximately a raw data score of 78) would be a significantly high score and may indicate potential math anxiety that needs to be addressed.

If a student received a cumulative score at or above the 75^{th} percentile, that student was considered to have an elevated level of math anxiety. The participants' scores ranged from 25% to 78%. Table 8 represents participants' scores on the MARS-S pre-test.

Table 8: Pre-test MARS-S Ratings

Participant	Examinee's Ratings (raw points)
Participant A	55
Participant B	63
Participant C	**81**
Participant D	46

Note: The MARS-S is a shorted version of the 95 questionnaire created in 1972 by Richardson and Suinn. The 5-point Likert Scale ranges from 1-not at all to 5-very likely. Copyright permission granted.

The scores of Participants D (46) and A (55) appear to represent very little math anxiety while Participant B (63) has a minimal level that could be considered borderline. Participant C is considered to have a significant score (**81**) that indicates math anxiety exists according to the MARS-S anxiety criteria noted above. Of the questions posed from the MARS-S, there were three that rendered a mean score of "3" or higher. A score of "3" on any question represents "a fair amount" of apprehension or fear. The three questions are found in Table 9.

Table 9: Significant Questions from Pre-test MARS-S

Question	Mean	Standard Deviation
#1: Taking an examination(final) in a math course	3	.8165
#5: Thinking about an upcoming math test five minutes before	3	.8165
#9: Being given a "pop" quiz in a math class	4	1.1547

Note: Excerpt from Suinn & Winston (2003)Mathematics Anxiety Rating Scale Shortened Version. Copyright permission.

The significance of these three questions is they all reflect a testing environment such as a final exam, math test immediately upcoming, and a surprise exam (pop quiz). It appears that the participants are most fearful of an assessment design in mathematics and that fear could be considered testing anxiety rather than mathematics anxiety. Subsequently, testing anxiety has been shown to be related to math anxiety (Dew et al., 1984).

After the MARS-S pre-test was collected in the second session, the two most missed FCAT fraction problems were taught procedurally and conceptually to the participants. The participants were distributed their scored FCAT pre-test worksheets in the designated folders assigned to each participant. The participants were able to review their answers on the FCAT worksheet but not keep the material considering the FCAT post-test would be the exact same problems just in a different order of difficulty. The reason for the change in the order of difficulty is discussed later in the document. After the participants were given the time needed to review their answers, the FCAT pre-test was returned to the designated folders and returned to the exclusive possession of the researcher.

To begin the conceptual and procedural discussion of the two most missed problems, a participant read the Grade 5 level problem found as Figure 1.

> Participant D: "Caitlyn set a goal to swim 675 laps in her pool during summer vacation. She will swim 12 laps each day. What is the least whole number of days Caitlyn will swim to reach her goal?"
>
> Researcher: "The key number that you would be showing your students is the total amount of 675. Then you would want to explain the next significant value of 12 laps per day. Therefore, 12 laps equals 1 day. Now the question is how many days will equal 675 laps? There are several ways you could display this problem. Of course, you would have to know what the answer is first. For the kinesthetic learners, you could have a card that states 12 laps to represent a single day. You could group the students together so many could bring their cards together and collaborate with multiples of 12 such as 12, 24, 36, and so on. They would add 12 together so many times to equal as close to 675 or some would simply multiple 12 by a number to get close to 675. However, make sure the students understand that the question says whole number and they will not reach 675 exactly. They will go under or above that number, but not obtain it exactly. It is an assumption that whoever is looking at this problem, say on a test, knows to round up due the words "whole number". The problem states "what is the least whole number of days" which signals the rounding up concept. If a person sees the answer 56.25 and decides to round down, then concept of least amount required is not understood and that quarter of a day is lost in translation. When I looked at everyone's work on this particular problem, everyone displayed the procedural design of 675 divided by 12. This means that you understand conceptually division of multi-digit numbers. However, what happens when you have a student who doesn't understand this concept? You show them with the cards through multiples of 12 and visually the division design of 12 dividing into 67 first, then subtraction of the values 67 and 60 with remainder 7, dropping the 5 to create 75. As you show them the multiples with the cards and the visual of multi-digit division, the concepts begin to intertwine for conceptual understanding of the procedure. The actual answer is 56 and three-twelfths which is a mixed fraction. You could demonstrate reducing fractions which is called equivalent fraction such as one-fourth but it is not necessary for this problem. The question is asking for a whole value and you have to include the 0.25 in your rounding process. Therefore, to include all of the answer, you will have to round up to 57 which is the correct answer. It will take 57 days for Caitlyn to swim her summer goal of 675 laps at a rate of 12 laps per day."
>
> Participant C: "I missed that word whole. I was thinking round to the nearest decimal and rounded down."

> Researcher: “Yes, you could miss the concept of ‘least whole number of days” by thinking you were to round down but you have to include the fractional piece. That is the conceptual understanding of this fractional word problem.”
>
> Participant B: “I did the same thing. I read it as round to the nearest rather than rounding up.”
>
> Researcher: “Understanding the reading material is part of the difficulties in teaching the mathematics. You have to explain to your students the wording that may mean ‘round up’ or ‘round down’. The idea of what the question is asking needs to be discussed before moving on to the actual mathematics procedurally. For this problem, you have to include the fractional portion of the day to include that one-fourth of a day needed to meet the goal. There is another way to show this problem. You can use proportions. If you say ‘one day is to twelve laps, then how many days to 675 laps’, then the problem becomes an algebraic proportion with an unknown such as ‘x”. Most of the time, we use ‘x’ as the unknown representative of the variable. With the fractional proportion, you would cross multiply to start the solving process. So, let’s cross multiply to get 1 times 675 on the left side of the equation and then ‘x’ times 12 for the right side. To finish, you would divide both sides by 12 to get the ‘x’ by itself which becomes the same process and what you did originally. However, you still have to understand to round the answer up rather than down due to the nature of the problem. There are several ways to demonstrate the problem procedurally but explain the process conceptually as you go. Format is another hurdle you have to overcome because this problem is a grid question and the students need to have gridded it correctly to get full credit. Any questions?”(Personal communications, April 13, 2015)

Since there were no questions for the first most missed problem, the discussion continued to the second most missed problem. A different participant read the Grade 6 level problem found as Figure 2.

> Participant B: “Mr. Madsen worked 49 hours last week at his job. He spent one-fifth of this time in meetings and one-third of this time talking to customers on the phone. Which method would provide the most reasonable estimate of the total number of hours Mr. Madsen spent in meetings and talking to customers on the phone at his job last week?”
>
> Researcher: “Again, there is a key word that helps you when figuring out the problem. What is the key word? It is not in bold writing so you have to think about it.’
>
> Participant C: “There is no bold, I looked.”
>
> --pause--

Participant D: "Estimate?"

Researcher: "Yes, estimate is the key word. There are trigger words just like what we saw prior—rounding up meant to look for the next whole value. Estimate is another trigger word in mathematics to say you are not going to do exact or accurate computation, but round estimate the answer. Sometimes you will have those students who have to use exact calculating or precision and they will need to be prepared to estimate since you do see these type of problems. Estimation is a very important part of our life—we estimate how much we may need at a grocery store or for a budget. So, let's start with the key factors of 49 hours, one-fifth of the time is meetings, and one-third of the time is phone. Do you agree those are key elements of the problem? --pause-- This problem is about estimating not calculating accurately. Another way to help your students is to look at the answers and realize that none of the key factors are located in the answer. Sometimes kids will just look at the answers in a multiple choice and try to guess by comparing the key factors with the choices. Let's look at the answers. Fifty is in all of the answers and that is the estimated value for 49. So trying to use the answers as the tool to guess is a wrong way to look at it and the students may guess wrong if they don't know how to do the problem. You have to add the two fractions involved, one-fifth and one-third, and bring them together in order to complete this problem. Then the question is asking 'How much of this time was used?' That statement would mean you need to multiply the sum of the fractions times fifty. You want to know what portion or how much time of the approximately fifty hours was used for meetings and phone. So I am showing you the actual procedure of how to add the fractions and I use this in my classroom. I stack the fractions vertically as such and ask my students for the equivalent fractions needed to add one-third and one-fifth. So what would be the equivalent fractions and why?"

Participant D: "Fifteen"

Researcher: "What do you mean by fifteen?"

Participant D: "Fifteen is the common denominator between three and five."

Researcher: "Yes, it is. How did you get that?"

Participant D: "Because three and five both go into fifteen."

Researcher: "You are correct, but explain how you arrived at that answer. What would be the equivalent fractions?"

Participant D: "To add fractions, you need to have the same denominator. I thought of what number both three and five could go into and came up with fifteen. Then one-fifth would become three-fifteens and one-third would become five-fifteenths."

Researcher: "Excellent answer. Now why do we have to have the same denominator?"

--pause—

Participant D: "Because that is how I was taught. I don't really know."

Researcher: "Ok. The reason why you need to have the same denominators is because you are adding fractions of unlike denominators. Like a puzzle, they do not fit together neatly. If you could imagine a pie with one-third and another with one-fifth, then how much do you have total? Hard to answer because they can be drawn pretty as a snug picture but there is no math to explain the answer. Portions have to have the same pieces to fit together like a puzzle. One-third and one-fifth have to have the same portions to be able to bring them together. You create equivalent fractions by multiplying each fraction by the number one. One is considered the multiplicative identity because I can multiply anything in the world by one and it doesn't change the value. Now, one can be of any design except using zeros. It can be one over one, two over two, three over three, and so on. For one-fifth, I need to multiply by the one that looks like three over three. For the fraction one-third, I need to multiply by the one that looks like five over five. Now I have equivalent fractions three fifteenths and five fifteenths. I can add these fractions because they have the same portions—fifteenths. So I add how many I have which is five plus three. My answer is eight-fifteenths. Strangely enough, that number is nowhere to be found in my answer selections. Remember the key word 'estimate'? It applies here as well. What is eight-fifteenths an estimate of?"

Participant D: "one-half"

Researcher: "Yes, one-half. So now let's look at the answer. Remember, the problem is not to be solved but to pick the correct process that would allow us to solve the problem. What do you think the answer is?"

Participant D: "G"

Researcher: "That's right. G is the answer. Multiply one-half by fifty is the correct answer. Any questions?" (Personal communication, April 13, 2015)

There were no other questions asked regarding the explanation of the second most missed problem from the FCAT pre-test therefore session two was concluded. The summary of session 2 is found in table 10.

Table 10: Summary of Session 2

Process	Rationale	Time	Materials Used
Anxiety Pre-test	Discover math anxiety level of participants if it exists	15 minutes	Mathematics Anxiety Rating Scale Shortened Version (Appendix B)
Discussion of two most missed problems from FCAT 2.0 pretest results	To teach procedural and conceptual understanding of questions missed	40 minutes	FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Figure 1, Figure 2)

Session 3

In the third session, the participants viewed three videos collected from the National Council of Teachers of Mathematics Channel website (see Table 11). All videos were previewed and selected prior to showing. Several videos were previewed for selection but only those that demonstrated procedural and conceptual teaching designs were chosen.

Table 11: NCTM Channel and EdWeek Videos

Title and Session Viewed
Mathematics in the Early Grades (Session 3)
Developing Mathematical Skills in Upper Elementary Grades (Session 3)
Mathematical Foundations for Success in Algebra (Session 3)
Building Conceptual Understanding in Mathematics (Session 3 and 4)
Preparation for Higher Level Mathematics (Session 4)
Approach to Fractions seen as Key Shift in Common Standards (EdWeek, Session 4)

Note: All videos are copyright permission via YouTube online.

The first video, *Mathematics in the Early Grades*, was published online in April, 2015. It began with demonstrating a student explaining to his teacher and the rest of the class how he found an answer to an addition problem. When he finished his explanation, the teacher asked if anyone else wanted to share how they deduced the answer to the problem on the board. Another child came to the board and started to show her way of thinking. Dr. Douglas H. Clements, Professor and Kennedy Endowed Chair in Early Childhood Learning from the University of Denver, explained his rendition of how people think mathematically.

> We are in-born with a mathematical sense, a number sense, and that is something very young kids come with. We can build on that conceptually right from the beginning. One of the things that people often ask me is, when I am asked about the Common Core Standards for young children is 'Mathematics isn't that very abstract?', and 'Why are we pushing it down on kids?', and 'It just doesn't feel developmentally appropriate for kids to be doing all this math in early years.', but it's a misunderstanding largely of what's the nature of mathematics. Mathematics is abstract. It's an abstraction, but children from very early age show signs of being able to work with mathematics and work with mathematical abstractions. As soon as a kid can say two doggies and two chairs and recognize and use that term to describe the quantity in both those very different situations, they are making an abstraction (Clements, 2015).

The video continued to interview a second-grade teacher from Philbrick Elementary School, Erk Berg, who explained how he tries to move students' understanding from concrete objects (what they know) to a picture, and then to just numbers. Another interview but of a first-grade teacher, Jennifer Kiederer Lawrence, at Warren Elementary School stated her beliefs of Common Core Standards being very developmentally appropriate because "they build on foundational skills that students may need to know" (Lawrence, 2015). Paraphrased, she states simple addition, say in first grade, is a building block to draw pictures of the concept, write equations of the same concept, or skip counting to get to the answer. All variations are appropriate at the first-grade level. Showing different ways to get to the same idea is the basis for using Common Core Standards.

Dr. Douglas H. Clements (2015) continued with the idea that CCSS were not meant as standards at first but as learning path trajectories and stories of how kids think and learn about mathematics through the grades. He explained that these standards are not only ones the children can handle and learn, but enjoy learning at the same time.

The next interviewed teacher was a first-grade instructor from Winthrop School named Brian Gaines. He informed the viewers how he likes to focus on the number ten system and all the different strategies to get to the number ten. Erik Berg comes back to make a point of how CCSS shows story problems that allow kids to make movies in their mind about the problem. The next teacher in the video was Michele Glynne who is a second-grade teacher at Beethoven School and demonstrated a teaching concept of asking students in a circle group setting the different ways to solve a story problem.

Dr. Douglas E. Clements described the different ways to demonstrate problem types pictorially with addition and subtraction equations. He stated the interpretation of the problem is the challenge. He declared that kids who can answer all those different styles of problems are more powerful thinkers than those who have not been challenged or exposed to the variety of problem solving.

Dr. Francis (Skip) Fennell, a L. Stanley Bowlsbey Professor of Education in Graduate and Professional Studies from McDaniel College, continued with the storyline of mathematics of arithmetic historically has not changed at all but linking the concepts together is the new design of teaching. Again, Jennifer Kiederer Lawrence noted that children need to be able to build off of what they know and become strategic thinkers. Erik Berg made a statement about parents thinking it is ok for students not to understand the mathematics because everyone in the household had the same problems growing up. He informed the viewers that thinking that way about reading is not acceptable because everyone is expected to read and why should anyone think that way regarding mathematics.

Brian Gaines commented about persuading parents to help the students show their work so instructors can know the way the students are thinking in order to help with any misconceptions. Clements advised listeners to be active parents by asking children to explain different ways to view the problems at hand and just to have conversation about the mathematics. A parent, Karen Wontan, discussed her relationship with her daughter and how she tries to interact mathematically with her daughter in real-life situations such as grocery shopping. Erik Berg enlightened the listeners that parents need to communicate with the teachers so the instructors can inform them of the why and how concepts are being taught to their children.

Dr. Douglas E. Clements stated he wants kids to explore and think about mathematics. "The more kids talk about mathematics, the better they get at reading and literacy along with the conceptual understanding of the mathematics" (Clements, 2015). Jason Barnett, principal of Warren Elementary School, talked about how he discusses at home with his children all the work they bring home and how crucial it is to know where they are at mathematically. This video concluded with Jennifer Kiederer Lawrence making a bold statement concerning the number sense that kids in her classroom have now is so far beyond what children were just a few years ago because they have a solid foundation.

This video was chosen because of its connection to showing work in mathematical calculations, positive comments about the CCSS and the abstraction of mathematics, and the different designs of teaching mathematics conceptually. Showing work allows the instructor to get inside of the thinking process of the student. Visual misconceptions and/or errors of the procedural steps of a problem while solving can assist the instructor in correcting the thinking process and demonstrating the proper conceptual idea. In exchange, the student will learn the correct process and hopefully, regain proper knowledge of how to correctly complete the math problem at hand. The comments about the CCSS and how mathematics is abstract allowed the participants to get more views of professional educators and their understanding of how important CCSS is in the teaching process because it is comprised of learning patterns and story lines for the students. The different teaching designs are crucial for different styles of learning and different levels of abilities. Most elementary mathematical concepts can be taught through pictorial images but then explained in the abstraction of numbers as well. The connection of the graphics to the number system helps students in the primary grades connect to what they already know pictorially, and then connect the concepts to the abstract level needed for the progression in conceptual learning of mathematics. This video was approximately twelve minutes long and seemed appropriate to begin with considering it discussed kindergarten through second grade.

After the first video was shown, the researcher presented the second video of choice. The second video viewed was *Developing Mathematical Skills in Upper Elementary Grades* and was also published in April, 2015. It began with Leah McKetty, principal at Winthrop Elementary School, who spoke about how parents need to have high expectations for their kids in elementary school so they can be prepared for middle and high school challenges. Dr. Jim Pellegrino is the co-director of Learning Sciences Research Institute and a distinguished professor of psychology and education at the University of Illinois. He engaged the viewer in the concepts of CCSS and what they mean.

> The CCSS are trying to get to the core of what that kind of knowledge is in the area of mathematics. What do kids really need to understand about the nature number? What do they need to really understand about ratio and proportion? Not just can I solve a fractions problem or this kind of fraction problems and give you the answer, but do I understand what a fraction is? Do I understand it terms of relationships among quantities? (Pellingrino, 2015).

Dr. Francis (Skip) Fennell implicated that anyone who has been in the field of teaching for any amount of time can see that there are many concepts in the CCSS that are the same as the former standards. “The significant and noticeable change is there are fewer standards and kids should have an opportunity to truly understand the mathematics they are learning” (Fennell, 2015). Fennell said there is no rush now to try to run through so many topics since the standards are pretty much cut in half than what it was and teachers can “dig deep” into the concepts now. Teachers now can explain concepts more thoroughly rather than just procedurally so students can’t say they have no idea how they got the answer to a problem. He believes that was the case for many kids for many decades. While he was speaking, various videos of classroom teaching sessions are playing with teachers speaking to kids, children speaking to each other, and different classroom settings.

Fennell continued by reminding the viewers that many different math councils have found that many students for generations before this one never understood thoroughly “those funny numbers” called fractions. In the research he spoke of, there was a survey of over 1000 algebra teachers whom were asked what one concept would you really want your students to truly understand before they enter your algebra class. “Overwhelming the most consistent response from these surveyed teachers was they would like their students to know fractions thoroughly” (Fennell, 2015). Karen Wontan, a parent, described the process in which her daughter and she complete the homework every night. Karen said that her daughter would rather be told what the answer is but instead Karen has her daughter talk out the problem in several ways to truly understand what the question is asking. “Communication and talking through the problem is key to understanding what is known, needed, and to be discovered” (Wontan, 2015).

Dr. Cathy Seeley, Senior Fellow at Dana Center at the University of Texas and past president of NCTM, made a comment about the support needed from parents to encourage the learning and to talk to the children about what they learn in lieu of trying to teach them the work being brought home. A classroom interactive lesson began with the voice over of Dr. Francis (Skip) Fennell interjecting the idea of how CCSS brings the mathematical concepts together rather than making them separate entities. He went on to say that even though learning the rote memorization tables of multiplication is not set aside as a separate idea, it is integrated into learning other concepts and is the building block for higher levels of learning such as fractions, number sense, and algebra.

Lisa Nguyen, a fifth-grade teacher at Kenny Elementary School, spoke about how number lines help her students conceptualize numbers such as whole numbers, decimals, and intertwined values found all through the number line system. She believes using the number line helps the students connect the decimal number concepts to the whole number values on the number line rather than think they are separate entities that have nothing to do with the other numbers.

Dr. Francis (Skip) Fennell shared his relationship with his grandkids in respect to their mathematics homework. He compared the rehearsal of math to the rehearsal of a musical instrument or sports--the more you rehearse, the better you get at it. He went on to state homework is that rehearsal and needs to be in the home of every kid. Erik Berg made a profound comment by stating that many generations of people knew certain steps of math but really didn’t understand how to do the problems, for example, in algebra. Berg (2015) finished the video with stating “as educators, we should feel that every kid can learn and do math at higher levels.” This video was approximately eight minutes long.

The decision to use this video was because of the correlation of procedural and conceptual learning of fractions goes hand in hand with what Dr. Fennell proclaims. Erik Berg's statements correlate to the idea of procedural knowledge as not being the "knowing why or how" we do the problem but simply the robotic steps taken habitually like those similar problems shown by the instructors. This video braids the importance of this model with the ideals of the professionals speaking about CCSS and how teaching fractions conceptually is most important when effective learning is to take place in the classroom.

After the second video was complete, the third video was played. The third video observed from the NCTM channel on YouTube was called *Mathematical Foundations for Success in Algebra* and was published in April, 2015. This video began with a teacher in a classroom discussing with her class an algebraic problem on the overhead projector. Interposed over her lecture was Dr. David Bressoud, a former president of the Mathematical Association of America, former chair of the Advanced Placement Calculus Development Committee, and a DeWitt Wallace professor of mathematics at Macalester College. He proposed the reasons for deficiencies found in algebra and calculus classes.

> What we do see in colleges is a lot of students who have been rushing through the earlier preparatory material and lacking the foundation that they need in order to succeed in that calculus class. They are lacking the skills in algebra and often they are lacking the kind of expertise they should picked up in middle school, grades six through eight in ratios and proportions. I see a lot of calculus students who are still weak in those areas. And yes, they've memorized lots of procedures but unless you really understand what you are doing and you have that foundation, once you get to that fast pace of college and university mathematics, you are really going to stumble (Bressoud, 2015).

Kristen Simms, a mathematics eighth grade teacher at Pine Grove Middle School, discussed how teaching algebraic concepts in eighth grade math classes allows the students to enter algebra I with a strong foundation. A secondary mathematics resource teacher for Howard

County Public Schools, Jenny Novak informed the viewer that the CCSS concentrates on the earlier grades for a solid foundation of skills such as ratio and proportion relationships, expressions, and equations that help build the algebra knowledge. Another instructor of middle school eighth grade math and algebra teacher at Traverse City East middle School, Jane Porath agreed that CCSS allows teachers to build on the basic skills such as fractions, basic facts, procedural fluency, and decimals. She wants her students to be fluent in these skills when they reach eighth grade or beyond. Jane is also on the board of directors for NCTM.

Dr. Cathy Seeley made a point of saying, according to CCSS, a lot of algebra has been intermingled into the concepts before seventh and eighth grade. She feels that having that strong preparation during middle school grades of algebraic thinking, understanding ratios/proportions, and using proportional reasoning will allow the students to be better prepared for high school. Angela Purpura, a mathematics teacher at Kentwood High School, commented that she prepares her students to be their own thinkers so that they are prepared to enter college to think about real world problems that they may encounter. Jenna DeMario is a mathematics instructional support teacher at Mayfield Woods Middle School. She explained an exponential function problem she uses in her classroom as it relates to a real world situation involving money. She explained how she wants her students to investigate the ideas rather than just be told the outcome. Damitra Newsome, a mathematical instructional support teacher at Lake Elkhorn Middle School, talked about how she finds value in technology used in the mathematics classroom to help boost the students' abilities to go beyond simple calculations. She wants her students to reason, explain, justify why a solution is better than other options in math problems. She believes students should be able to tackle real world problems.

Dr. Solomon Friedberg, a James P. McIntyre professor of mathematics and chair at Boston College, stated

> If students can develop understanding of standard algorithms at the elementary level, they can understand why they multiply multi-digit numbers, for example, the way they do, then that understanding will serve them very well when they go off to multiply polynomials as they learn algebra because they will recognize that the steps are basically the same. So when we develop good understanding at the elementary level, we give students a fantastic foundation to succeed in algebra (Friedberg, 2015).

The current president of NCTM and former mathematics director at Pittsburgh Public Schools, Dr. Diane Briars, informed the viewers how she tries to build conceptual understanding through investigation of algebraic and real world problems while demonstrating the procedural background for the problems involved. She believes the students will be able to have procedural fluency for various styles of problems previously discussed. This video was almost seven minutes in length.

This particular video seems to trail after the concepts from the two previous videos that discussed topics and ideas from kindergarten through elementary grades and now to middle and high school years. Those speakers reiterated that there is weakness in ratio and proportion knowledge in college students and these concepts need to be conceptually taught in the early years of education. The middle school instructors also reminded the viewer that fraction knowledge is essential in the higher level mathematics such as algebra. The common statement among many was real world problems need to be discovered and discussed in the classroom rather than just procedurally shown. Understanding how to complete the mathematics and/or algorithms at an earlier age will enhance the students' abilities to conquer more difficult mathematical problems in high school and college.

The last video shown during session three is titled *Building Conceptual Understanding in Mathematic.* This video began with Ann Marie Varlotta, a middle school math instructional support teacher in Howard County Public Schools. She stated how conceptual understanding is very important because we need to understand "the why and the how" we are doing something. If students memorize the procedures, skills, or facts but they don't understand the reasoning involved, they will not know when or how to apply the knowledge unless the situation is identical to what they have memorized. Bill Barnes, coordinator of secondary mathematics for Howard County public schools, explained that the county has created the rigors of teaching mathematics as a three-legged stool with the three legs representing: (1) procedural fluency, (2) conceptual understanding, and (3) application as the three legs of the stool. He commented that mathematics prior to CCSS was taught mostly procedurally.

At this moment, the video was stopped due to time already allocated for the previously viewed online videos and time needed for participant feedback. The reason for this action was similar to Ambrose (2004) whom also included participant feedback in her research since "written responses of individuals can be used to provide insights into their beliefs and interpretations" (p. 58) rather than just accept a Likert scale with a rubric that has limitations within the concept of the question offered. The participants were given a sheet of paper with an image of a three-legged stool and asked three questions (1) What's the difference between conceptual and procedural? (2) What are your 3-legged rigor steps to teaching? (3) What is a problem you did not understand how to do (misunderstood) but now you know how to do it? This inquiry handout can be found in Appendix C and was used for feedback purposes of ideas presented in the videos viewed during session three. The responses are found in table 12.

Table 12: Responses to Questions from Videos

Participant	Q1: What's the difference between conceptual and procedural?	Q2: What are the 3-legged rigor steps to teaching?	Q3-What is a problem you did not understand how to do (misunderstood) but now you know how to do it?
A	Conceptual understanding is knowing how/why to do something. Procedural is understanding the process necessary to solve the problem.	1-Understand how to solve a problem. 2-Understand why a problem is solved the way it is, or if more ways are possible for it to be solved. 3-Understand how to take solving a problem in a tangible manner and now do it abstractly.	Converting fractions to decimals and percentage.
B	Conceptual is more broad mathematics (is a concept such as multiplication). Procedural is more of the method behind answering a problem. (ex. ? x ? = ?)	1- What is it? 2-How to do it. 3-Application! (most important step for every subject) (how do I use it?)	I have always struggled with understanding the "why" behind different procedures in Statistics, but I've been helping my mom with her work and it seems to be "clicking".
C	Conceptual is understanding the overall idea of a method or concept. Procedural is understanding how to complete the particular method.	1-Introduction of topic/skills 2-Practice 3-Application/ testing of knowledge	How to compute a percentage of a number Ex: 20% of 125
D	Procedural is the formula or steps used to solve Conceptual is the understanding or reasoning of why the numbers or equation arrive at such answer	1-What are we looking for? 2-How do I solve? 3-Why do I solve?	$a^2 + b^2 = c^2$ to figure out the diagnal of a television

Note: The three questions were concepts found in the NCTM videos viewed during session three. These responses are verbatim of the written words from the participants. No changes have been made to the original script.

The first question, *What's the difference between conceptual and procedural process of teaching fractions?* was posed for insight on the participants' beliefs on these two important notions of teaching. The purpose of this model is to inform pre-service teachers of conceptual and procedural methods of teaching fractions. Knowing how the participants define these methods can allow the researcher to discuss any misconceptions about conceptual and procedural teaching. Participant A wrote "Conceptual understanding is knowing how/why to do something. Procedural is understanding the process necessary to solve the problem." These words sound very much like the ones spoken from Erik Berg in the second video when he said the idea of procedural knowledge as not being the knowing why or how we do the problem but simply the robotic steps taken habitually like those similar problems shown by the instructors. There is a slight misunderstanding that procedural is understanding the process. Procedural is simply being able to go through the algorithmic motions of how to complete a problem, but that does not necessarily mean one understands the process.

Participant B wrote "Conceptual is more broad mathematics (is a concept such as multiplication). Procedural is more of the method behind answering a problem. (ex. ? x ? = ?)" It appears that participant B remembered the statement from Dr. Solomon Friedberg in the third video when he discussed multiplication of problems. This participant believed conceptual is broad mathematics and did not really answer the question posed. Saying procedural is more of the method behind answering a problem is somewhat correct due to it is just rigorously displaying the steps but more so not explaining how the steps are derived.

Participant C answered the first question with "Conceptual is understanding the overall idea of a method or concept. Procedural is understanding how to complete the particular method." This comment was getting closer to the idea of conceptual as the understanding of a method but it was also meant to explain what is going on behind the scenes of the problem. Again, using the word "understanding" for procedural is an overused thought since procedural is really just the performance of a problem. It does not necessarily mean the person understands the problem.

Participant D's perceptions of question one was "Procedural is the formula or steps used to solve. Conceptual is the understanding or reasoning of why the numbers or equation arrive at such answer." This participant reflected back to the third video when Dr. David Bressoud discussed students just memorizing procedures and formulas rather than knowing why they use those particular steps. Participant D also understood the meaning of conceptual as how the answer develops and the reason(s) for the end result. Participant D answered the question correctly and understood the difference between conceptual and procedural. However, later in the performance assessment, Participant D fell short in presenting a fraction lesson with both concepts included.

The second question asked "What are the 3-legged rigor steps to teaching?" comes directly from the third video viewed when Bill Barnes discussed his ideas of the rigors of teaching mathematics as a three-legged stool with components of (1) procedural fluency, (2) conceptual understanding, and (3) application as the three legs of the stool. He commented that mathematics prior to CCSS was taught mostly procedurally. The question was used to spark the ideals of the participants, and used for them to think about what are the three key theories of teaching mathematics in their perspective.

Participant A listed her three rigors as (1) Understand how to solve a problem, (2) Understand why a problem is solved the way it is, or if more ways are possible for it to be solved, and (3) Understand how to take solving a problem in a tangible manner and now do it abstractly. All three answers sounded very similar to the same design of Bill Barnes' three legged stool. Procedural fluency is knowing how to solve a problem, conceptual understanding is understanding why a problem is solved the way it is, and application is understanding how to take a problem and relate it to other ideas.

Participant B replied to this second question with (1) What is it? (2) How to do it, and (3) Application! (most important step for every subject-how do I use it?). The first answer did not make sense to a theory of teaching mathematics. To ask "What is it?" is not categorized into any mathematical field of teaching. Answer two sounded like a procedural fluency model and again, could have been just paraphrased like answer three, application, from Bill Barnes' design. The two answers that made sense in mathematical teaching were again answers two and three, but could have been influenced by Bill Barnes' clip.

Participant C answered question two with (1) Introduction of topic/skills, (2) practice, and (3) application/testing of knowledge. Introduction of topic or skills is the same as completing a problem with the procedural knowledge of how to do a math problem. Conceptualization of the skills may be part of what Participant C was trying to say, but it is not clear. Practice is a certainly a procedural design due to continued practice will allow students to have fluency in how to do a problem. Unfortunately, time constraints in a classroom do not allow all situations to be discussed. Practice could be taken as extended classwork that is taken home, i.e. homework. Practice was definitely a concept brought up in the videos as a key element in procedural fluency. Application and testing of knowledge are two different procedures which means Participant C gave four answers to question two. Application is the extension of conceptual learning in order to see if transfer of knowledge occurs through applying what one learns to other questions. Testing is an action taken such as assessment of retention of that conceptual learning.

Participant D wrote her three answers to question two as if a mathematics problem were being asked: (1) What are we looking for?, (2) How do I solve?, and (3) Why do I solve? These answers are not theories of teaching principles and apparently, participant D misunderstood the question. This response sounded more like what a teacher would ask the class every time s/he posed a mathematical problem probing for systematic steps to solving that particular math problem.

Question three was enquired for reflection from the participants of their mathematical ability now as an adult. Sometimes an enlightenment occurs for a mathematical concept that was misunderstood as a child or adolescent and now is clear of how to procedurally perform it. Participant A felt more confident converting fractions to decimals and percentages. Participant B stated she struggled with understanding conceptual procedures in statistics, but now understands statistics due to helping a parent with work involving these type of calculations. Participant C felt more comfortable computing a percentage of a number such as 20% of 125. Participant D gave a story behind the answer of knowing how to use Pythagorean Theorem now. This person had to purchase a television and did not understand the dimension description of it, for example a 65 inch television. With using the mathematics of $a^2 + b^2 = c^2$, this participant stated she now understands that the diagonal is not the length or the width but the actual diagonal across the television. After the questions were answered by all and the feedback papers were collected, the session concluded. Table 13 is the summary of session 3.

Table 13: Summary of Session 3

Process	Rationale	Time	Materials Used
Viewing Videos	To emphasis the importance of procedural and conceptual teaching of mathematics (especially fractions)	45 minutes	NCTM Videos (Table 11)
Participants' Responses	Discover viewpoints of participants in reference to ideas found in the videos	15 minutes	Response Questions (Appendix C)

Session 4

In the fourth session, the participants continued viewing *Building Conceptual Understanding in Mathematics*. After Bill Barnes explained the three-legged stool idea, Gail Burrill, a Michigan State University academic specialist and member of Advanced Placement Calculus Development committee, suggested that students need to understand how to do problems conceptually and procedurally so they can make sense of why they work a problem the way they do and if it is sensible.

Leah McKetty talked about conceptual teaching such as borrowing in a subtraction problem needs to be explained thoroughly rather than students just crossing out numbers. She said that students need to understand what is the place value of the number you are borrowing from and where did it come from. Connie Henry, an academic response team manager for mathematics K-5 for Boston public schools, gave an example of adding two multi-digit values together, for example 199 + 199, in a standard algorithm by aligning them vertically and adding the positional digits together with the carrying value involved procedurally and robotically. She also discussed how this problem could be reconstructed flexibly as 200 + 200 with the removal of the overage, 2, and this kind of thinking needs to be encouraged.

Linda Ruiz Davenport, the director of K-12 mathematics for Boston Public Schools, verbally displayed the example of twelve divided by three is really asking how many threes are in the amount of twelve. This design of dividing whole values makes sense and could be the connection to explain the concepts of fraction division problems such as one-half divided by one-fourth. Linda commented that to know the concept behind the problem helps explain the validity of the rule for dividing fractions and why when dividing two fractions does the answer become larger than both original values.

Jennifer Kiederer Lawrence addressed the concept of applying a formula or algorithm only is not how a student grasps or understands the whole problem. She talked about teaching key words as an important part of solving problems but to make sure it is not just a quick fix of shortcuts when focusing on the key words involved. She made a point of teaching rules only doesn't allow the students to really think about the problems.

Dr. Solomon Friedberg ended the video with stating that there are many ways to learn mathematics, such as calculus, by memorizing a set of rules and specific problems you have already solved but that doesn't allow you to transfer the knowledge in a way you can use it for many different problems. He testified that CCSS from kindergarten to high school is created for students to enable usage of mathematics in new problems they haven't encountered before by applying principles and concepts of the computational skills they have developed to work and solve the new problem. He believes this design of standards is what students need to succeed in college. The video was almost six minutes long.

The choice of viewing this video was due to being tied closely even to the title of the dissertation in practice. Conceptual understanding of mathematics is the heart and soul of being successful in mathematics throughout one's educational career. Having the basics taught procedurally and conceptually so that transferring the knowledge of each previously learned concepts can occur into new mathematical problems is the ultimate goal for a student to be successful in mathematics, *Preparation for Higher Level Mathematics* was the second video viewed during session four and was created in April, 2015. Dr. David Bressoud believes that the preparation for college has changed predominantly because of the amount of acceleration of high school students who are trying to get into advanced courses earlier in their high school career. He stated that the largest advancement in enrollment is the amount of students in calculus classes in high school. Dr. Bressoud commented the students are in such a rush to complete calculus in high school that they lack the foundational mathematics necessary to succeed in the calculus classes in college.

Gail Burrill stated she is a fan of CCSS because it allows students to have the opportunity to get a solid foundation that will enable them to progress to calculus classes. Her experiences established the need for students to obtain the essential foundations needed before they enter into higher level math classes such as calculus, and she believes that the CCSS will provide this groundwork for the students so they are not looking at their college professors dumbfounded.

Dr. Bressoud informed the viewers that there is a strong national concern for needing more engineers and scientists and a high need to prepare students who are mathematically literate in the mathematical sciences. He believes we are losing many promising students. Jenny Novak commented that she likes the CCSS for its strong foundation of modeling and statistics. Novak is seeing a “deeper treatment” (2015) of statistics beginning in the middle schools than have ever been seen before. She believes this progression will allow a growth in more careers that involve statistics and it will support the research that is being conducted. Dr. Bressoud concurred with Jenny Novak in the growing development of statistics from the CCSS and more research will be erupting from these newly inspired mathematicians who understand statistics. From his experience and contacts through his various positions in the realm of mathematics, Bressoud (2015) has found that there is a common concern of professors confessing that the students need to be “explorers of mathematics” to succeed. He supports the CCSS in reference to the conceptual understanding being taught more and pleased to announce more mathematicians are strong supporters as well of the CCSS. This video was approximately five and a half minutes long.

The emphasis in this video that corresponds to this model was that the conceptual understanding of mathematics truly is the necessity for higher level mathematics learning. One of the major reasons for why students do not succeed in math classes in college is due to not having the solid foundation of the basics such as fractions.

The last clip shown to the participants came from Educational Weekly and was called *Approach to Fractions seen as Key Shift in Common Standards.* The speaker, Zachary Champagne, is an assistant researcher at Florida State University for STEM research. He discussed CCSS design of fraction instruction for third grade in relation to how it has been previously taught. Pictorial images of a fraction such as two-eighths would be a rectangle divided into eight equal portions with two portions shaded. The denominator of the fraction represented the total equal pieces and the shaded portion represented the numerator or how many parts we have. This design of teaching is called representing “part of a whole” or “area model” (Champagne, 2015).

Common Core Standards now expands on that design by including the fraction on the number line and thinking about it as a value on the number line which has been missing in the traditional teaching but is very important in later mathematics. Fractions are taught as area and as a specific number on the number line in CCSS. Equivalent fractions are being shown in graphical images as well as portions on a number line. For example, three-fourths is the same as twelve-sixteenths whether it is drawn as sixteen equal squares with twelve shaded portions or sixteen tick marks on a number line with a significant position at the twelfth mark. Both items can be redesigned to show three-fourths in turn aiding the students taught to see the equivalent fractions.

The distinct difference between previous traditional teaching and the new CCSS teaching is fractions are numbers and should be seen as such in respect to a number line. This conceptual understanding of fractions is "critical for their future success in mathematics" (Champagne, 2015). The video was approximately three and a half minutes long.

This Edweek video coincided with this model and the purpose of teaching fractions conceptually as well as procedurally attributably the CCSS requirements. After the videos were viewed for the fourth session, a question was distributed to participants for feedback on each participant's personal thoughts on the topic viewed. Considered question four and probably the most important reflective response related to this model, see Appendix D, the participants answered "What is a major difference between the way fractions were taught to us and the way Common Core State Standards require teachers to teach it?" after viewing the video that actually discussed this topic. Table 14 displays the responses of the participants.

Table 14: Question four responses

Participant	Response
A	Fractions were taught mainly using rules that you memorized. There were shading activities, and often it was related to money. Now, it is taught using a number line and a deeper understanding of a fraction being a number.
B	N/A-absent
C	The new Common Core State Standards incorporates a number line. Students are encouraged to think of a fraction as a number and not just a fraction. When I was taught fractions, we only thought of them as fractions or part of a whole.
D	I went to a Catholic school (K-8) in New York, over 30 years ago. I was taught fractions very similar to the Common Core State Standards of today. The teachers were strict and we had to break everything down and be able to explain why. We also used the ruler to understand fractions. I think it was very beneficial because I have a good understanding of fractions.

Note: The question was a reflective feedback in reference to the EdWeek video viewed. These responses are verbatim of the written words from the participants. No changes have been made to the original script. Participant B was absent during session four.

Participant A and C reflected back to the EdWeek video and the comments that Zachary Champagne made in regards to how fractions were taught then and now. Participant A remembers fractions as they relate to money and memorizing rules. Participant C remembers fractions as just a part of a whole and not as a significant number itself. Participant D doesn't state any differences because it appears that the way fractions were taught to this person was the same as the design of teaching is executed now. However, rather than using a number line, Participant D remembers using a ruler instead. Notice that Participant D makes a significant comment in relationship to attitude regarding ability to understand fractions, "I have a good understanding of fractions". This participant feels very confident in understanding fractions. This declaration is an important indication of confidence and self-efficacy involving computation of fractions which was evident in the demeanor and comments given during the sessions. Incidentally, the performance assessment of Participant D reveals this understanding of fractions is strong but only procedurally. After the participants finished answering this question and all papers were collected, session four was dismissed. The summary of session 4 is found in table 15.

Table 15: Summary of Session 4

Process	Rationale	Time	Materials Used
Viewing Videos	To emphasize the importantce of procedural and conceptual teaching of mathematics (especially fractions)	30 minutes	NCTM Video and EdWeek Video (Table 11)
Participants' Responses	Discover viewpoints of participants in reference to ideas in the videos	15 minutes	Response Questions (Appendix D)

Session 5

Session five began with the distribution of the same FCAT questions given previously as the FCAT pre-test, but the questions were arranged in a random order of level of difficulty (See Appendix E). The participants were given 25 minutes to complete the problems. The surprising phenomenon mentioned earlier is the inability to cognitively process the problems in the same design as completed before. When the problems were arranged in order of lowest grade level three to highest grade level six, the participants worked the pre-test in order of the level of mathematics learned in an educational setting--least difficult to most difficult such as third-grade math, fourth- grade math, fifth-grade math, and then sixth-grade math. However, when the order of difficulty was randomly distributed as such in the FCAT post-test, the students had difficulty remembering how to complete the problems. Consecutive order of learning sometimes interferes with the cognitive processing of concepts within the basis of how to complete each problem separately when the problems are not arranged in the same chronological order (Rohrer, 2012). Interleaved practice (intertwined conceptual learning) is not a fundamental design of teaching, but rather blocked practice (one concept at a time) of the same concept is the mathematical design taught today (Rohrer, 2012). Rohrer's research on comparing interleaved practice versus block practice of mathematical problems revealed that the critical skill of identifying what kind of problem and which concept needed is appropriate was more prevalent in the interleaved practice (Rohrer, 2012). Table 16 displays the comparison of the participants' FCAT pre and post test results.

Table 16: Comparison of results for FCAT Pre- Post Test Scores

Participant	Pre-test Scores	Post-test Scores
A	15 out of 15 correct (100%)	14 out of 15 correct (93.3%)
B	13 out of 15 correct (86.7%)	14 out of 15 correct (93.3%)
C	12 out of 15 correct (80%)	11 out of 15 correct (73.3%)
D	15 out of 15 correct (100%)	15 out of 15 correct (100%)

Note: These scores are based on one point per correct answer with no partial credit.

Participant A did not keep the perfect score and missed the fifth-grade level problem that was most missed. It appears that this person did not grasp the fundamental concept of "key words" and overlooked the "whole number" concept.

Participant B's results showed an improvement on understanding how to complete the fraction problems. The most missed question from fifth grade was still not comprehended and this participant answered it incorrectly same as before.

Participant C showed the most significant change but in a negative sense due to missing more problems in the post-test than in the pre-test. Considering this participant also had the highest level of math anxiety according to the MARS-S, the mathematics assessment anxiety could have been a factor of why more problems were missed during the post-test. One of the questions missed by Participant C was the grade five most missed question. Similar to Participant B, this person forgot to round up to the nearest whole number.

The second question missed by Participant C was labeled 19 for grade five level (see Appendix E). The work was shown and was partially correct but a fraction was missing in the calculation. The five and seven-eighths, the three and one-fourth, and the fifteen-sixteenths were changed to equivalent fractions of the same denominator (16), but the fifteen-sixteenths was overlooked in the calculation and not included in the sum. That mistake led to an incorrect answer thus an incorrect choice. Participant C chose “D” as the answer when the correct solution was “B”. The next problem missed by this participant was labeled 4 for grade six level. The problem involved either (1) changing a percentage to a fraction, adding that fraction to another fraction, and then multiplying the fractional sum to the total value listed, or (2) changing a fraction to a percent, adding that percent to the other percent listed, and multiplying the decimal value to the total value listed. Participant C chose to take the second design of calculation by changing the two-fifths to a decimal. Unfortunately, the participant changed two-fifths to 0.45 which is incorrect. Therefore the answer was incorrect in the final calculation and the open ended question should have been answered as 350 votes rather than the incorrect answer given of 300 votes.

The last problem answered incorrectly by Participant C was labeled 10 for Grade 3 level. There were shaded rectangles representing three and two-ninths and the responder had to pick which improper fraction multiple-choice answer was equivalent to three and two-ninths. The participant chose answer “H = twenty-nine ninths” rather than the correct answer “F= twenty-nine fourths”. It could have been a mistake of oversight or possibly a misconception of translating a mixed fraction to an equivalent improper fraction.

Participant D showed no change in ability to complete the problems correctly. This participant also showed the least amount of mathematics anxiety and continued to make comments of self-efficacy during the session.

After the FCAT 2.0 worksheet post-test was completed and collected, the MARS-S post-test was distributed to the participants. They were given fifteen minutes to answer the thirty questions inventory. Found in Table 17, the comparison of the results of the pre-test and the post-test MARS-S ratings shows a significant difference for Participant C and Participant D.

Table 17: Comparison of Results for the Pre- Post-test MARS-S Ratings

Participant	Pre-test Examinee's Ratings (raw points)	Post-test Examinee's Ratings (raw points)	Difference
Participant A	55	49	-6
Participant B	63	57	-6
Participant C	81	85	+4
Participant D	46	48	+2

Note: The MARS-S is a shorted version of the 95 questionnaire created in 1972 by Richardson and Suinn. The 5-point Likert Scale ranges from 1-not at all to 5-very likely. Copyright permission granted.

Utilizing Baloglu's five factors (2010) for the questionnaire, table 17 displays the dispersion of the differences of scores from pre- to post-test of the MARS-S ratings for each participant according to the associated questions. The values in bold are significant due to an increase of two or more Likert scale points in the difference between pre- and post-test responses of the MARS-S questionnaire. Question nine refers to the feelings toward being given a pop quiz in a math class. The scores show that Participant B and D both feel calmer about this event occurring than they did prior to the study. On question sixteen, Participant B also reduced the anxiety from the score of three (a fair amount) to a one (not at all) in regards to dividing a five digit number by a two digit number in private with pencil and paper. Participant A appears to have reduced in anxiety when observing a pre-test score of fifty-five to a forty-nine. It appears that the Mathematics Test Anxiety (questions 4, 5, and 6) were less of a concern in the post-test for Participant A.

Participant B decreased also in the potential math anxiety found in testing and in the computation section of the questionnaire. Participant C appears to have acquired a feeling of more anxiety across the board of all five factors. Surprisingly, Participant D increased in anxiety in the mathematics testing section even though this participant continued to express verbally the confidence in calculation of fractions and ability to complete mathematical tasks. Table 18 displays the difference in response values of each MARS-S question for each participant.

The comparison of results for the means and standard deviations is found in Table 19 and is denoted by the specific questions that rated a mean score of three or higher. The pre- post-test anxiety mean increased on Question 1 from M=3 to M=3.25. Taking an examination in a math course seems to be more of a concern in the post-test scores compared to the pre-test scores. Question 2 became a concern in the post-test and increased from M=2.75 to M=3 among the participants. Again, referencing an exam, question 2 regarding thoughts of an upcoming math test one week prior was a concern for the participants

Table 18: Dispersion of MARS-S Values

Factors		A	B	C	D
		Participants			
Mathematics Test Anxiety	1				+1
	2			+1	
	4	-1		+1	
	5	-1	-1	+1	
	6	-1	-1		+1
	9		**-2**	+1	**-2**
	11		+1		
	12		-1		
	15				+1
Mathematics Course Anxiety	7			+1	
	8		+1		
	10		-1	+1	
	14			-1	-1
Application Anxiety	18			+1	
	19	-1		-1	
	24			-1	
	26	-1	-1		
Social Anxiety	22	-1	+1	-1	
	28			+1	
	29			+1	
	30			+1	
Computation Anxiety	16		**-2**	-1	
	17			-1	
Differences from Pre-test		-6	-6	+4	+2

Note: Adapted from Baloglu, M. (2010). *An investigation of the validity and reliability of the adapted mathematics anxiety rating scale-short version (MARS-SV) among turkish students* Springer. The numbers represent the question found in the MARS-S. The bold values show a significant change.

Question 5 changed from M=3 to M=2.75 and seems to be of slightly less concern which seems peculiar considering it is in regards to thinking about an upcoming math test five minutes before an exam rather than a week prior. Question 9 exhibits the most significant change with a decline from M=4 to M=3.25. It appears that the math anxiety of “five minutes before” or immediately surprised with a “pop quiz” is no longer the deepest concern of the participants. Rather, taking an examination in a math course and thinking about an upcoming math test one week prior are stronger issues with the participants with reference to the positive increase of means.

Table 19: Comparison of results from Pre- Post-test MARS-S

Question	Pre-test Mean	Standard Deviation	Post-test Mean	Standard Deviation
1: Taking an examination(final) in a math course	3	.8165	3.25	.5
2: Thinking about an upcoming math test one week before	2.75	1.5	3	1.4142
5: Thinking about an upcoming math test five minutes before	3	.8165	2.75	.9574
9: Being given a "pop" quiz in a math class	4	1.1547	3.25	.5

Note: Excerpt from Suinn & Winston (2003). Mathematics Anxiety Rating Scale Shortened Version. Copyright permission.

Performance Assessment

After the MARS-S post-test was completed, the researcher took a copy of each of the problems from the FCAT 2.0 worksheet pre- post-tests and folded them in half several times so that no one could determine what question was written on it and they all looked uniform. The two most missed questions that had been procedurally and conceptually discussed during session two were omitted from the selection. The folded papers were placed in a pile in front of the participants who were sitting in a rectangle arrangement facing each other. The participants were asked to pick a folded paper and take a few minutes to look over the problem they had chosen. After they were given time to review the problem they had previously encountered twice already from the FCAT pre- and post-test assessments, the participants were asked to volunteer to present the problem as procedurally and conceptually as they knew how on the board to the rest of the participants.

Rittle-Johnson & Koedinger (2002) believes it is important to intertwine procedural and conceptual instruction for students to develop a firm understanding of procedural knowledge which leads to improvements in conceptual knowledge. Unfortunately, "there is little guidance for how to integrate context into conceptual and procedural instruction" (Rittle-Johnson &

Koedinger, 2002, p. 971), hence the reason for potential teachers to have more experience in attempting to teach procedurally and conceptually.

The first person to volunteer was Participant A. The problem displayed in Figure 3 was the same question missed during the FCAT post-test for Participant C and therefore, advantageous for the audience to experience.

Sam is mailing some items to his brother. Before he went to the post office, he weighed the items. The table below shows the weight of each item.

ITEMS TO MAIL

Item	**Weight**
Computer paper	$5\frac{7}{8}$ pounds
Laptop computer	$3\frac{1}{4}$ pounds
Recipe book	$\frac{15}{16}$ pounds

Between which two weights is the total weight of all three items?

Ⓐ between $9\frac{1}{2}$ and 10 pounds

Ⓑ between 10 and $10\frac{1}{8}$ pounds

Ⓒ between $10\frac{1}{4}$ and $10\frac{1}{2}$ pounds

Ⓓ between $10\frac{1}{2}$ and 11 pounds

Figure 3: Participant A's Selection for Performance Assessment
Note: Image is cropped from original design FCAT mathematics sample question. Copyright permission granted from FDOE for reprint.

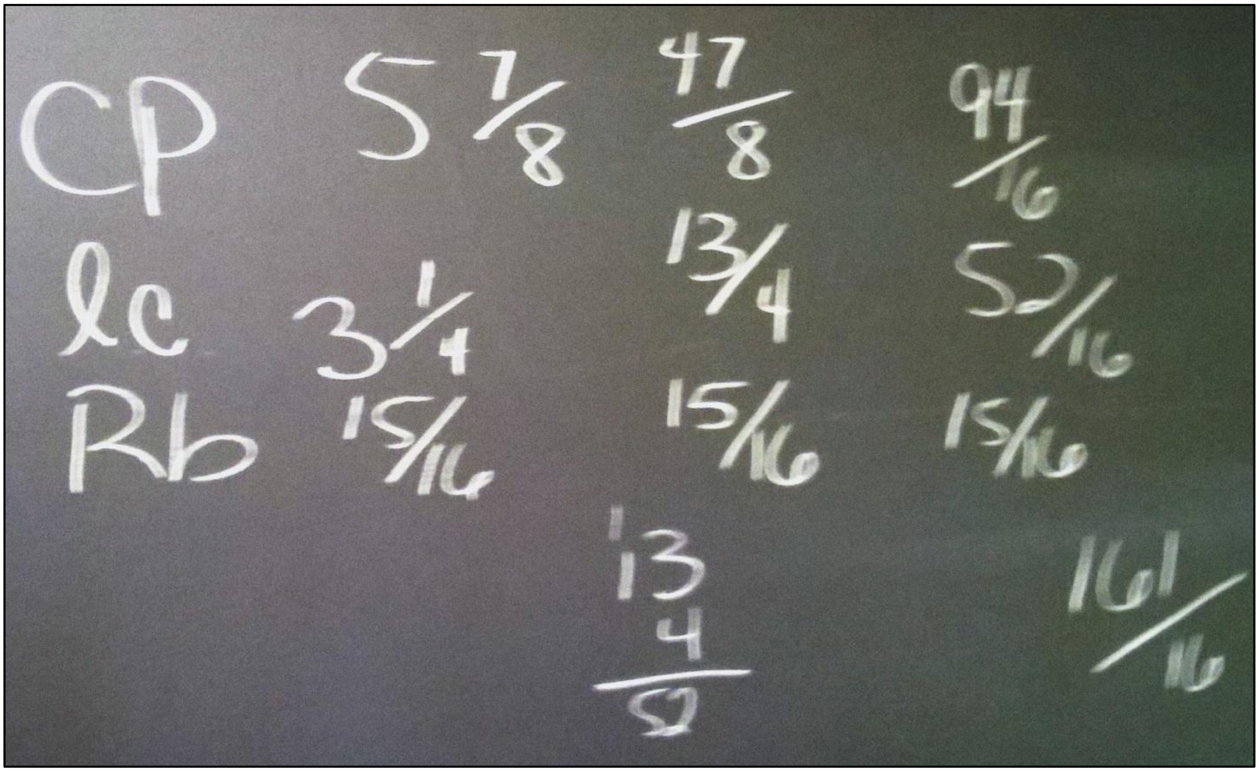


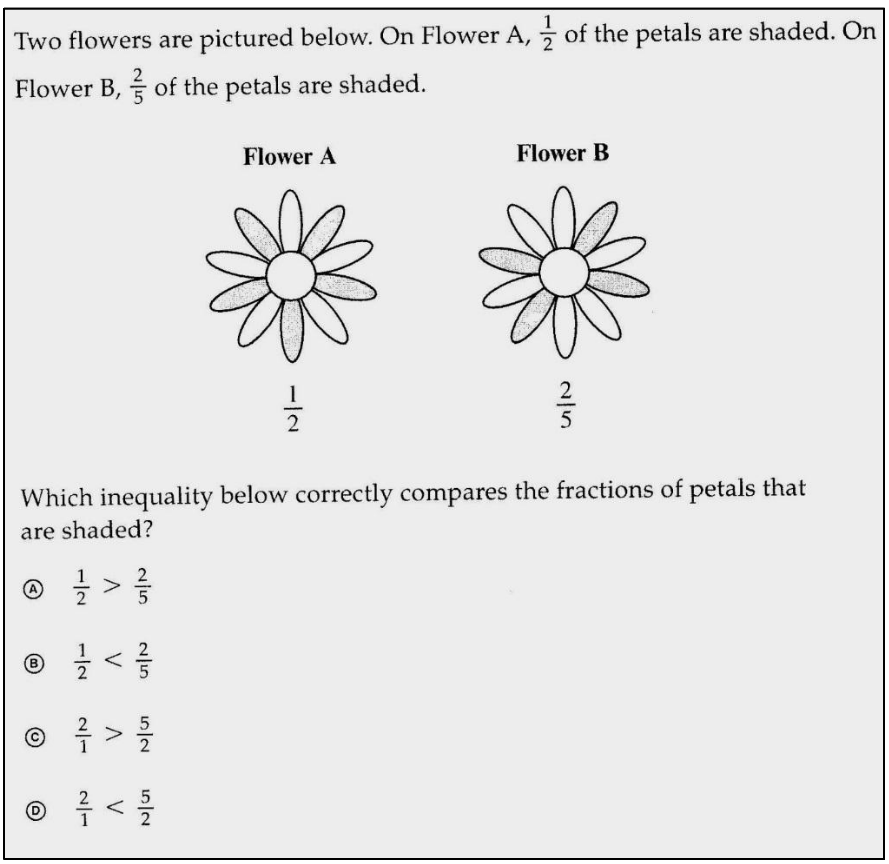
Figure 4: Image of Participant A's Board work

Figure 4 is the captured image of Participant A's work whom began by writing each item from the question on the chalkboard as initials: "CP" to represent computer paper, "lc" to represent laptop computer, and "Rb" for recipe book. Then she proceeded to write the mixed fraction that corresponds to each item: five and seven-eighths, three and one-fourth, and fifteen-sixteenths respectively. She decided to change the fractions to improper fractions but mistakenly called them mixed fractions rather than improper fractions. She caught her uttered mistake and corrected herself. She said that she had to multiply the whole number by the denominator and add it to the numerator. She talked through the calculations needed by saying "five times eight plus seven to make forty-seven over eight", "three times four makes twelve plus one to get thirteen over four", "and then fifteen-sixteenths". She did state a conceptual rule for fractions by saying "you can't add numbers that have not like denominators so I then made them into common denominators". She made a mistake again by stating "to do that you have to multiply forty seven times eight", but this time she did not recognize her oral error. Her calculations of changing all the denominators to sixteen were correct.

As Participant A continued to state each step she made a comment, "This is a lot of work as I am saying it." She also said that we were making her nervous and she didn't even know if her work was right. She then stopped and looked at her work on the board and confirmed verbally that it was right. She had some difficulty with the mental multiplication but showed her work (13 times 4 equals 52) on the bottom of the board. She continued to say she added the tops to get one hundred sixty-one over sixteen. "If you divide one hundred sixty-one by sixteen, you get ten with a remainder of one-sixteenths. If you look on here, you see the different options." She proceeded to convey her way of thinking by reading the four choices for answers and the one that was closest to her answer.

What Participant A showed the viewers was a verbal walk through of how she completed the problem. She did not discuss what "the total weight" meant that lead to her needing to add the fractions. She did not explain the concept of changing mixed fractions to improper, instead just showed the others procedurally how to do it. She did not conceptually explain why all fractions have to have the same denominator when adding and how it is truly the concept of multiplying times "one" but in an equivalent fraction design. She did talk through the addition of numerators but again missed explaining why this process is key to adding fractions (and not adding the denominators). Her final explanation of dividing fractions (161 / 16) was not displayed visually but yet disclosed orally when she quickly shifted her attention to the answer choices in the problem. She did pose one conceptual idea of not being able to add fractions that have unlike denominators, but she did not elaborate on the "why" of this very important fractional concept. Her performance was traditionally procedural and was a demonstration of-her thoughts on how to complete the problem.

The second volunteer for the performance assessment was Participant B. Figure 5 is an image of the grade level 3 problem and no one missed this question in either pre or post- FCAT worksheets. This problem involves recognizing the shaded portions of Flower A as one-half and the shaded portions of Flower B as two-fifths. It pertains to understanding inequalities and comparing two fractions with knowledge of the inequality symbols. The question is asking which statement choice is correct in comparing the fractions.


Two flowers are pictured below. On Flower A, $\frac{1}{2}$ of the petals are shaded. On Flower B, $\frac{2}{5}$ of the petals are shaded.

Which inequality below correctly compares the fractions of petals that are shaded?

Ⓐ $\frac{1}{2} > \frac{2}{5}$

Ⓑ $\frac{1}{2} < \frac{2}{5}$

Ⓒ $\frac{2}{1} > \frac{5}{2}$

Ⓓ $\frac{2}{1} < \frac{5}{2}$

Figure 5: Participant B’s Selection for Performance Assessment
Note: Image is cropped from original design FCAT mathematics sample question. Copyright permission granted from FDOE for reprint.

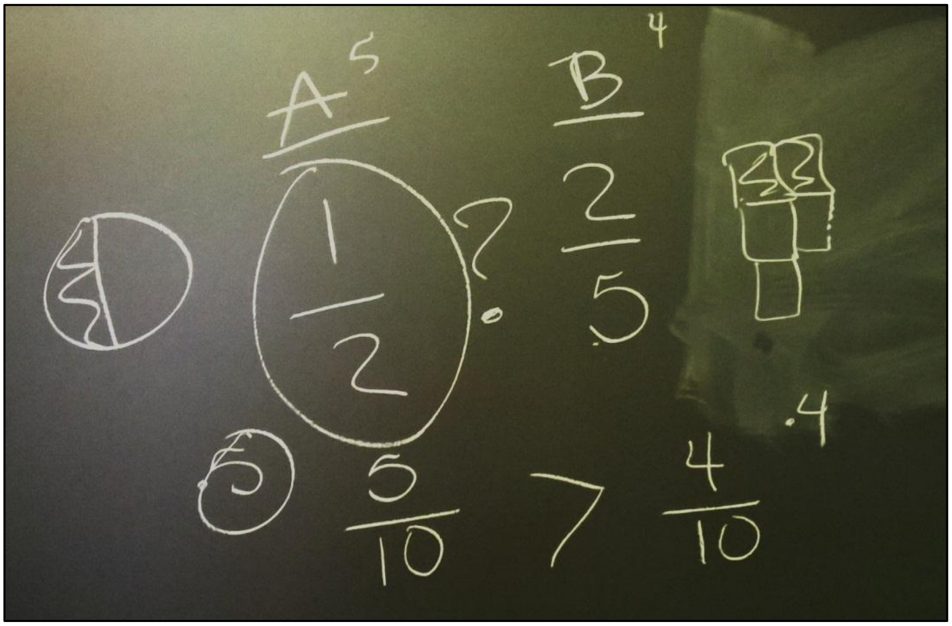


Figure 6: Image of Participant B's Board work

Notice in Figure 6 that Participant B decided not to duplicate the images because she said "we are trying to figure out which one is bigger." She labeled the one-half as "A" and two-fifths as "B" to represent the flower images in the problem. She conveyed "you could just look at the problem and decide one half is bigger." She told the viewers that "you might automatically decide one-half is bigger since it has five petals shaded and two-fifths only has four petals shaded. If you had different shading where you didn't have the exact number of petals, you wouldn't be able to just decide that." She continued with her statement by drawing and shading half of a circle to pictorially represent one-half (on the left of Figure 6).

Participant B continued by drawing five squares and shading two of the squares to represent the fraction two-fifths (on the right of Figure 6). She said that "we are going to just use the numbers to decide which one is bigger." The one conceptual idea she used in her explanation was having common denominators to compare fractions. She stated "we are going to make our denominators the same because we can't really compare fractions if they are not the same denominators." She conveyed two and five go into ten so we make the denominator ten. She divulged into changing the fractions into equivalent fractions by orally walking the viewers through her steps of changing five into ten and then doing the same to the top. She explained the same procedures of how to change the four-tenths conversion from two-fifths. "You multiply two by five and do the same thing to here (pointing at the numerator) and you get four." She said "and somehow looking at it we can see this one is bigger (circling the one half) which makes one half bigger." She continued with changing the fractions into decimals and telling the viewers "you know one half equals point five" and "this (referring to two-fifths) is equal to point four so the point five is bigger."

Clearly the problem chosen by Participant B was a simple one because it allowed the observers to visualize the fractions but the explanation was very procedural with one glimpse of conceptual knowledge. She did not recognize the problem itself was already illustrated with ten petals for both flowers and the shaded petals were the same as the written fractions she changed equivalently. The statement of comparing fractions with common denominators was a conceptual idea but how she changed the equivalent fractions was missing. Again, the concept of multiplying the fractions times the multiplicative identity, 1, allows the viewers to better understand equivalent fractions conceptually. Instead, she demonstrated the thought orally but did not show the work involved. Her procedural knowledge of how to complete the problem was evident but explaining how to change fractions to decimals was not. She called the equivalent decimals using the word "point" rather than the proper enunciation. For example, 0.4 is called "four-tenths" not "point four." The language used expresses misconception of a decimal that is truly a fraction written in a decimal format.

The third volunteer to complete the performance assessment was Participant D. Figure 7 is an image of the grade level 4 chosen problem and, incidentally, no participant missed this question from either pre- or post-test FCAT worksheets.

A store had sweaters on sale for 75% off of the original price. Which of the following is equivalent to 75%?

Ⓕ $\frac{1}{4}$

Ⓖ $\frac{1}{2}$

Ⓗ $\frac{3}{4}$

Ⓘ $\frac{5}{7}$

Figure 7: Participant D's Selection for Performance Assessment
Note: Image is cropped from original design FCAT mathematics sample question. Copyright permission granted from FDOE for reprint.

Being a simple multiple choice question of just changing a percentage to a fraction, this particular question would appear to be not difficult enough to demonstrate the level of knowledge for Participant D considering this participant has verbalized her self-efficacy in mathematics especially fractions. However, after she completed the task of attempting to teach it to the other participants, she stated how hard it is to teach fractions rather than just do them. Participant D's board work is located in Figure 8 displaying where she began by writing the four possible choices of the answer selections: 1/4, 1/2, 3/4, and 5/7. She continued with reading the question again and re-stating "the 75% off of the original price will be 100%." She wrote "100%" to the right of the fractions previously written and disclosed that "seventy-five means seventy-five of the one hundred percent." She stated "we have seventy-five hundredths and we want to make it a smaller fraction so what number can go into seventy-five and one hundred?" She wrote the fractions twenty-five twenty-fifths beside the seventy-five hundredths and then another fraction, three-fourths, to the right of the previously written fraction. She informed the viewers the answer was "three- fourths" and circled that fraction as the choice in the first written set of fractions.

Participant D continued by saying "a better way to visualize it is to think of one hundred as a dollar with quarters in the dollar." She drew the rectangle below the written 100% and separated it into four equal parts with the number 25 written in each smaller rectangle. She expressed how she sees three of four quarters in a dollar and marks three of the smaller rectangles. She verbalized how the "whole thing is one hundred percent of our whole dollar and we have four quarters, then three-fourths. The left over quarter is one-fourth which makes one hundred percent."

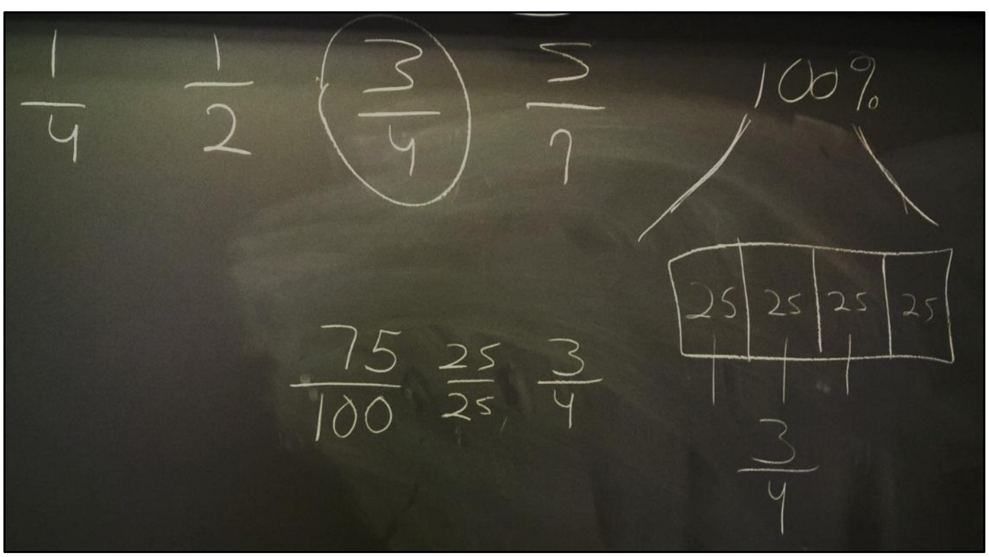


Figure 8: Image of Participant D's Board work

Beginning with the answers rather than explaining the question first is not demonstrating conceptual understanding of the problem. Working backwards from the answers to the problem is a very procedural design of potential guessing of the answer. The image of the dollar was a significant visual aid and many may be able to relate due to the current currency of the United States. She should have begun by explaining an original price being one hundred percent of the sweater. However, the explanation of seventy-five percent off of an original price was not demonstrated. Her work and thought process of reducing the seventy-five hundredths fraction to three-fourths was correct. This process was procedural since she did not explain the purpose of the fraction twenty-five twenty-fifths (conceptual idea of "1") written on the board nor why she wrote three-fourths.

The last volunteer to complete the performance assessment was Participant C. Figure 9 is an image of the chosen problem and again, no participant missed this question in either pre- or post-test FCAT worksheets. Unfortunately, for the participant with the highest level of math anxiety, this question was one of the hardest problems from the FCAT worksheets due to all the mathematical concepts needed to complete it.

Mr. Nichols wants to store $25\frac{1}{2}$ cups of stew in containers. Each container holds a maximum of $1\frac{1}{2}$ cups of stew. What is the minimum number of containers Mr. Nichols needs to hold all the stew?

A. 9
B. 17
C. 25
D. 51

Figure 9: Participant C's Selection for Performance Assessment

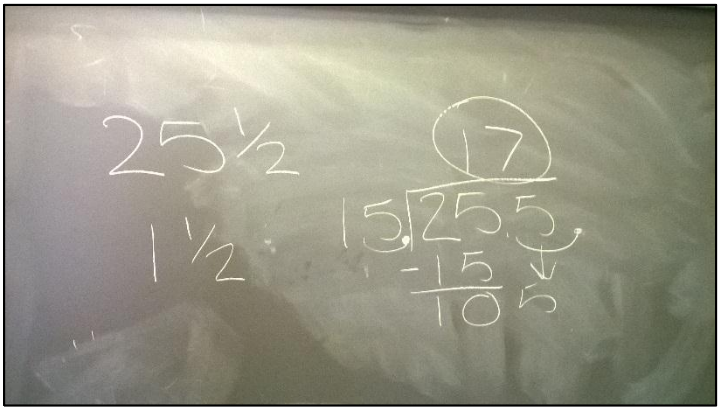


Figure 10: Image of Participant C's Board work
Note: Image is cropped from original design FCAT mathematics sample question. Copyright permission granted from FDOE for reprint.

"We have twenty five half cups of stew in containers and each container holds a maximum of one and a half cups of stew. So we are looking for the minimum numbers of containers that this person can hold all the stew. So basically I know we are going to have to divide twenty five and a half by one and a half." began Participant C as she wrote the two mixed fractions on the chalkboard. She disclosed with the viewers that she gets really confused with dividing fractions and decimals. She chose to change the mixed fractions to decimals and attempt to divide. She told the observers that she knows what the answer is but gets confused with the operation of dividing. Participant D assisted her by saying "you have to move the decimal over and then move it over for the other because what you do to one side you do to the other." Participant C, surprised with the procedure, exclaimed "You move it over? That's all you do?" Participant D nodded and said "Now you divide fifteen into twenty five." Participant C stopped and said "I don't know how to do this." The researcher told the participant to just "show us what you know" and to keep going. Participant C replied "Ok." She continued to attempt the division with the assistance of Participant D walking her through the division of multi-digit values. Participant C realized that the problem should have started with fifteen going into twenty five and wrote the number one over the divisor bar. She proceeded to write the subtraction of fifteen and computed the difference of ten. She did not remember to bring the five down to be joined with the ten already written but did as Participant D said when told the procedure. “So fifteen into one o five?” proclaimed Participant C as she pondered what the quotient would be. She commented that she needed a calculator and should factor out. Participant D replied with “seven” and Participant C says “Seven? That was really fast math.” Participant D said “How I did it was seven times ten is seventy and seven times five is thirty-five. Thirty-five plus seventy is a hundred and five.” Participant C thanked Participant D for the assistance and continued with the problem by writing the seven over the divisor bar. Participant C circled the answer 17 and commented that “now she knows.” She also confided that she “had not done a problem like this in…” but stopped mid-sentence. Participant D felt compelled to share that she did not do the problem like this but rather drew a picture with one and a half in it with trying to find out how many of those pictures made twenty-five and a half. Participant C continued with her statement of not seeing a problem like this since fifth grade.

Participant C had the highest math anxiety score on the MARS-S pre test and post test. Her anxiety level actually increased throughout the study which was verbally demonstrated in the performance assessment with the comment of “I don’t know how to do this.” However, she had completed it correctly on both pre- and post-test of the FCAT worksheets. Being asked to show her work and verbally talk it through to others began the onset of the anxiety, which might have caused her cognitive processing to slow down. It seemed she knew how to do the problem but could not convey it during the performance assessment. She could have converted the mixed fractions to improper fractions and used the rule of division for a much easier route of completion. However, under the pressure of others watching, she attempted to approach the problem by reverting back to a familiar comfort zone of decimals. Unfortunately she could not think clearly of how to divide decimals either. Participant D was confident enough to assist Participant C with the mathematics but did not explain why the movement of the decimals took place. After Participant C sat down, the researcher asked the participants if they knew why the decimals moved during division of decimals. No one could answer the conceptual question. The researcher continued to explain the idea of decimal fractions and multiplying by one in a design of 10/10. With that concept of the multiplicative identity, any number can be adjusted equivalently no matter if it was a decimal or fraction design. Participant C's anxiety did not allow her to complete the problem on the board and may interfere with her teaching abilities later as an educator. Table 20 is the summary of session 5.

Table 20: Summary of Session 5

Process	Rationale	Time	Materials Used
Fraction Post-test	Discover increase of content knowledge level of fractions in relation to pre-test	20 minutes	FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Appendix E)
Anxiety Post-test	Discover decrease of math anxiety level of participants in relation to pre-test results	15 minutes	Mathematics Anxiety Rating Scale Shortened Version (Appendix B)
Performance Assessment	Discover pre-service teachers' abilities to teach fractions procedurally and conceptually	35 minutes	Problems from FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Figures 3, 5, 7, 9)

Summary

The purpose of the pilot is to inform pre-service teachers of conceptual and procedural methods of teaching fractions. The FCAT 2.0 Mathematics sample third-grade to sixth-grade question worksheets for pre- and post-tests allow for the assessment of the fractional content knowledge required to determine if there are any difficulties these participants may exhibit (Van

Steenbrugge et al., 2014). Most elementary school teachers "possess a limited knowledge of mathematics, including the mathematics they teach" (Kilpatrick et al., 2001, p.372). The shortened version of the Mathematics Anxiety Rating Scale (MARS-S) created in 1972 by Richardson and Suinn as an instrument that explored issues relating to academic situations and everyday life in respect to mathematical tasks (Richardson & Suinn, 1972) was used for a pre- and post-test to determine if mathematics anxiety existed. In an elementary education setting, math anxiety can lead to less time spent on the subject (Rayner et al., 2009; Sloan, 2010) and can surface when teaching the subject (Tooke & Lindstrom, 1998). Math anxiety was evident in Participant C during the performance assessment more so than any other participant reflecting consistency with the higher math anxiety rating of this particular participant's post-test of the MARS-S. Helping pre-service educators recognize their feelings and having awareness of their level of math anxiety has a direct correlation to how they teach mathematics (Lake & Kelly, 2014). Exposing the level of math anxiety that these participants appear to have can help encourage them to spend more time learning the fractional material and possibly gaining conceptual knowledge of the mathematics.

The videos viewed from the National Council of Teachers of Mathematics Channel website and Educational Week supported teaching practices aligned to Common Core Standards and how important it is to teach mathematics procedurally and conceptually. The feedback from the participants provided insight into the beliefs and interpretations of various concepts related to teaching (Ambrose, 2004). The performance assessments finale bestowed the most evidence that pre-service elementary education teachers have difficulty teaching fractions procedurally with very little conceptual knowledge. Teaching mathematics only procedurally is considered to be the less effective strategy and does not allow the students to have a full grasp of the conceptual idea of the problem in order to transfer knowledge of the process to higher level mathematics (Stohlmann et al., 2015). Procedural teaching is also changing to more conceptual teaching in order to align with the standards of the Common Core (FDOE, 2014). Even though there were only four participants in this pilot study, the range of significant math anxiety and moderate to low procedural knowledge of fractions (Participant C) to low math anxiety with high self-efficacy (Participant D) validates the argument of needing more research similar to this pilot. The model informed by this pilot will provide the framework for future reference to improve educational practices in teaching mathematics aligned to Common Core Standards. Chapter 3 includes a comparison of the anticipated outcomes versus the actual outcomes. Table 21 is the overview of the five sessions of the pilot study.

Table 21: Overview of Pilot

Session	Process	Rationale	Time	Materials Used
One	Fraction Pre-test	Discover content knowledge level of fractions	30 minutes	FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Appendix E)
Two	Anxiety Pre-test	Discover math anxiety level of participants if it exists	15 minutes	Mathematics Anxiety Rating Scale Shortened Version (Appendix B)
	Discussion of two most missed problems from FCAT 2.0 pretest results	To teach procedural and conceptual understanding of questions missed	40 minutes	FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Figure 1, Figure 2)
Three	Viewing Videos	To emphasize the importance of procedural and conceptual teaching of mathematics (especially fractions)	45 minutes	NCTM Videos (Table 11)
	Participants' Response	Discover viewpoints of participants in reference to ideas found in videos	15 minutes	Response Questions (Appendix C)
Four	Viewing Videos	To emphasize the importance of procedural and conceptual teaching of mathematics (especially fractions)	30 minutes	NCTM Video and EdWeek Video (Table 11)
	Participants' Responses	Discover viewpoints of participants in reference to ideas in the videos	15 minutes	Response Question (Appendix D)
Five	Fraction Post-test	Discover increase of content knowledge level of fractions in relation to pre-test	20 minutes	FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Appendix E)
	Anxiety Post-test	Discover decrease of math anxiety level of participants in relation to pre-test results	15 minutes	Mathematics Anxiety Rating Scale Shortened Version (Appendix B)
	Performance Assessment	Discover pre-service teachers' abilities to teach fractions procedurally and conceptually	35 minutes	Problems from FCAT 2.0 Worksheets involving fractions for Grades 3-6 (Figures 3,5, 7, 9)

CHAPTER 3: MODEL ANALYSIS

Model Goals and Expectations

The purpose of this Dissertation in Practice was to inform pre-service elementary education teachers of conceptual and procedural methods for teaching fractions. The intended outcomes for the pilot were for pre-service teachers to:

1) learn how to conceptualize the teaching of fractions,
2) develop self-efficacy about teaching fractions, and
3) become aware of math anxiety if it is present.

The targeted audience were four participants enrolled in an elementary education reading methods course at a central Florida university. The benefits for the targeted audience from this pilot were providing teaching methods for instructional strategies for procedural and conceptual learning of fractions according to the Common Core Standards (FDOE, 2014), and to determine if the participants possessed mathematics anxiety according to the Mathematics Anxiety Rating Scale Shortened Version (MARS-S). The four participants had not experienced a math methods course in their program and the information obtained was not biased or construed by previous knowledge that could have been gained from a math methods course.

Conceptualization of Teaching Fractions

Very common misconceptions are that school mathematics for elementary education is easy to teach and all teachers understand the mathematics they have to teach (Van Steenbrugge et al., 2014). When a teacher does not understand the mathematics in a lesson, s/he should take the extra time needed to truly understand the concepts underlying the math problems involved. However, fractions are considered abstract and a difficult subject to learn. There are numerous reasons for this lack of conceptual knowledge as a result of not being taught conceptually in the adolescent years, interference in prior knowledge of natural numbers, and developing little procedural knowledge of fractions which leads to incorrect calculations (Van Steenbrugge et al., 2014; Ma, 1999). Not having prior conceptual knowledge can be associated with less procedural knowledge that may lead to calculations errors such as needing to keep common denominators when multiplying fractions (Hecht, 1998; Van Steenbrugge et al., 2014).

The pilot study included an FCAT 2.0 pre- and post-knowledge of fractions test. The results of the pre-test FCAT fraction problems disclosed a few mistakes for Participant B and C. These participants (along with Participant C) missed the same two questions warranting a presentation on how to teach these two problems procedurally and conceptually. When one of the same questions were missed during the post-test, the pilot revealed that more discussion was needed in the model that addressed concepts similar to those problems. One demonstration was not enough to gain insight to the type of problems these participants missed. Therefore, the model should include more procedural and conceptual examples of fraction problems for a possible better understanding of how to complete problems similar to the most missed questions of the FCAT 2.0 worksheet.

The use of NCTM videos during sessions three and four gave the participants insight into different perspectives of teaching conceptually from elementary grades to college. The need for conceptual knowledge of teaching fractions was a common thread throughout the videos and was expressed as a difficult subject to teach. The participants did gain knowledge from the information included in the videos as reflected in their responses on the questions from session three and four. The participants were expected to gather information regarding teaching conceptually and increase a perception of the importance of fraction knowledge in higher level mathematics.

Video Viewing

The response questions were given to the participants to reveal what knowledge of conceptual and procedural teaching was gained from observing the NCTM videos. Participant A achieved some insight when stating that procedural teaching involves the process necessary to solve a problem, but did not completely understand that procedural knowledge does not automatically mean one understand the procedures (Kilpatrick et al., 2001). This participant did, however, gain information about conceptual knowledge when she stated that conceptual understanding is knowing the how or the why to do something (mathematically).

Similar to Participant A, the response from Participant B pertaining to procedural teaching is the method behind answering a problem was on target. Nevertheless, she did not understand or gain comprehension of conceptual teaching due to stating it is a more broad mathematics such as multiplication. The element in one of the videos watched prior to this participant feedback, *Mathematical Foundations for Success in Algebra*, had a comment from Dr. Solomon Friedberg about understanding standard algorithms such as multiplying multi-digit numbers. Participant B could have become confused with the thought of his statement being a definition of conceptual teaching. Stopping the video and discussing what Dr. Friedberg was saying could be a possible change to the model so that the participants do not misunderstand what he was trying to convey.

Participant C answered the procedural versus conceptual question very comparable to Participant B. The only participant that answered the question "What is the difference between conceptual and procedural?" correctly was Participant D. She stated that procedural is defined by the formulas or steps used to solve and conceptual is the understanding or reasoning why the answers are as such (Kilpatrick et al., 2001).

NCTM Videos

The videos were important in conveying the necessity of understanding the mathematics at not only the procedural level, but more importantly, the conceptual level according to the standards that have been adopted by most of the United States (CCSS, 2014). Teachers are required to re-learn the mathematics in order to understand the concepts at a deeper level and to acquire some self-efficacy before stepping foot in a classroom full of elementary level students. There has been research conducted on the depth of conceptual knowledge in mathematics for pre-service elementary educators and continues to be investigated (Alexander & Ambrose, 2010; Alibali et al., 2009). More exploration of procedural and conceptual knowledge of pre-service teachers would be advantageous in math methods courses through the use of different grade level math problems similar to the ones found in this model. Also, educational leaders and researchers in mathematics education should organize and host faculty development workshops in content specific fields such as fractional operations.

Self-Efficacy

The participants verbalized their abilities during the pilot study by either making comments that ranged from "I don't like math and that is why I can't do it" to "I am great at fractions" (Personal Communication, 2015). The pilot study allowed the participants freedom to express their feelings in regards to mathematics. Their comments were similar to those heard from the researcher's previous experiences in mathematics discussions within the classrooms taught throughout the researcher's teaching career. This comfort level of candidness was appropriate and desired in the pilot. Prior to the pilot, the researcher expected the participants to have feelings of inadequacy or dislike towards mathematics due to the consistent and similar comments made by most people the researcher comes in contact with in her own classroom. However, it was quite refreshing to hear statements such as "I like math. I am good at fractions." from Participant D which were not expected (Personal Communication, 2015). Participant D also made comments regarding how hard she had to work in her required college mathematics classes which helped her gain the confidence she needed.

Performance Assessment

The confidence however seemed to decreased for Participant D when challenged with the task of teaching a fraction problem to the rest of the participants during the performance assessment in session five. The comments from Participant D were "It is harder to teach." and "It is a lot easier to just do the problems than teach them" (Personal Communication, 2015). Even though she had confidence she could teach the fraction problem due to her confidence in completing the question herself, she soon found herself losing efficacy like many other novice teachers when her skills were put to the challenge of conceptual explanation (Tait, 2006).

Participants A and B demonstrated procedural fluency in their scores from the FCAT pre- and post-tests, but neither verbally declared having a positive attitude and/or confidence in their mathematics abilities. Still when shown how to complete the two most missed problems from the FCAT worksheet pre-test, the overall consensus from the participants was a sense of efficacy when they had stated how the problems discussed were not difficult. The ability to procedurally complete the problem was quite different than having to show it conceptually as seen in the performance assessment in session five.

Participant C never felt confident in the fraction work due to the statements of "I can't do fractions" and "I have never been good at math" (Personal Communication, 2015). The pilot was designed to shed light on any negative or positive feelings and to help build confidence in teaching fractions procedurally and conceptually. Even though it did open the awareness door of math anxiety for the participants, the pilot did not help build confidence in teaching fractions. Tait (2006) states "efficacy beliefs appear to increase during university course work, then decline when novice teachers are confronted with the realities and complexities of teaching" (p. 4).

Awareness of Math Anxiety

Pre-service teachers have an important role in their learning how to teach mathematics to children, but sometimes they may experience math anxiety while performing mathematical tasks (Tait, 2006). The third outcome for this pilot was to provide awareness of math anxiety if it were present. To measure the level of math anxiety, the Mathematics Anxiety Rating Scale Shortened Version (MARS-S) was used as a pre-test and then again as a post-test to determine if math anxiety existed and/or reduced from session one to session five. The MARS was created in 1972 by Richardson and Suinn as an instrument that explored issues relating to academic situations and everyday life in respect to mathematical tasks (Richardson & Suinn, 1972). It has been used for research and clinical studies since 1972. It contains 98 items with a Likert scale ranging from score of (1) for a “not at all” response to a (5) for a “very much” response. For the original MARS, scores could range from a 98 (score of 1 for all 98 items) to a 490 (score of 5 for all 98 items) with the higher score correlating to the higher level of math anxiety the participant exhibits. They discovered through various test-retest situations, there is a negative correlation between anxiety and mathematical ability (Richardson & Suinn, 1972).

Due to the time restraints, this study involved the revised and shortened version of the original MARS. The MARS-S is a 30-itemed math anxiety rating scale copyrighted in 1999. Much like the scale for the original assessment, MARS-S has a Likert scale representation of the emotional designation for the participant’s fear or apprehension of the question posed: (1) for a “not at all” response, (2) for “a little”, (3) for “a fair amount”, (4) for “much”, and (5) for a “very much” response. The lowest possible total score is a 30 (score of 1 for all 30 items) and a highest feasible score of 150 (score of 5 for all 30 items). Typically, according to Suinn & Winston (2003), a percentile of 75% (approximately a raw data score of 78) would be a significantly high score and may indicate potential math anxiety that needs to be addressed. If a student received a cumulative score at or above the 75th percentile, that student was considered to have an elevated level of math anxiety.

The researcher anticipated mathematics anxiety to exist prior to beginning the pilot due to the researcher’s experience in the classroom. The participants’ levels of math anxiety from the MARS-S ranged from very low (Participant D, 46) to an elevated level (Participant C, 81). The full range of minimal math anxiety to a prominent level was expected and experienced in this pilot study. What was not expected was the increase of mathematics anxiety ratings from both Participant C (81 to 85) and Participant D (46 to 48) as shown on the MARS-S post-test scores.

The mathematics anxiety level, if existed, was expected to decrease for all participants but only transpired for Participants A and B. This increase of scores on the post-test could have been from the acknowledgement of having to complete a performance assessment the same day as the given MARS-S post-test and the elevated level of math anxiety was shown when answering the MARS-S inventory. Sometimes math anxiety increases from low to moderate levels when pre-service teachers are confronted with the realities of having to teach mathematics (Tait, 2006). "When faced with a math task, math anxious individuals tend to worry about the situation and its consequences. These worries compromise cognitive resources, such as working memory" (Maloney & Beilock, 2012, p. 404).

According to Verkijika and DeWet (2015), about 93% of Americans experience some form of math anxiety and it is very important to identify those with high math anxiety to try to help them build confidence. Because of the moderately high score according to the MARS-S scale for Participant C, the pilot confirmed math anxiety existed among one participant, but was not significantly high among the others. The exact occurrence or events that led up to Participant C's math anxiety was not disclosed, but could be an element for the model that would help identify the factors and possibly help the participants cope with the lack of confidence in mathematical performance.

Limitations

There were only four participants in this pilot study which could be a limitation for the model. If there had been more participants, the results could have changed to contain more or less problems missed in the FCAT 2.0 pre- and/or post-test, a different range of math anxiety levels (potentially no one with a level above the minimum of 75%), and possibly participants who did demonstrate conceptual teaching in the performance assessment. All participants were female but had there been any males, the scores of the pre- post-tests or performance assessments could have been different with less math anxiety or abilities to teach conceptually. According to Finlayson (2014), males tend to perform better in mathematics and have less math anxiety.

The time frame was a limitation due to only being held for five sessions lasting an hour each. Had there been more sessions or if the sessions were longer, the procedural and conceptual understanding of fractions could have been explained more in-depth with more problems and/or deeper discussions regarding these concepts. This deeper indulgence possibly could have decreased the math anxiety post-test scores and/or the decreased the amount of missed problems on the FCAT post-test worksheet. It also could have informed the pre-service teachers with more conceptual design that may have been demonstrated in their performance assessment. Another limitation was the implementation of the model. Because of the pilot being conducted during spring semester which was the last semester of the researcher's doctoral program, the model created from this pilot could not be implemented. However, it will be during the fall semester with the permission of a local elementary school in hopes of further advancement of the model.

The Model

Teachers need to improve their math skills since Common Core Standards require the mathematics topics to be taught both procedurally and conceptually\. The proposed model will be designed to teach needed math content and teaching skill as well as measure the level of math anxiety. The combination of pedagogical content, assessment of teaching performance, and assessment of math anxiety is intended to assess the level of procedural and conceptual knowledge in mathematics that the participants possess, allow the participants to experience teaching mathematics prior to entering a classroom, and obtain metacognition of their teaching styles. The proposed model will measure the level of math anxiety if it exists and incorporate coping strategies for those participants. The model is found in Table 22.

Table 22: The Proposed Model

Session	Process	Rationale	Time	Materials Used
One	Fraction Pre-test	Discover content knowledge level of fractions	30 minutes	Fraction Problems that span from Grades 3-6
	Anxiety Pre-test	Discover math anxiety level of participants if it exists	15 minutes	Mathematics Anxiety Rating Scale Shortened Version (Appendix B) or equivalent
	Anxiety Questionnaire	To begin an awareness of the timeline of the participants' anxiety if it exists	15 minutes	Questions similar to ones found in Finlayson (2014)
Two	Discussion of the most missed problems from the fraction pre-test	To teach procedural and conceptual understanding of questions missed	40 minutes	Problems answered incorrectly in Fraction Pre-test
Three	Viewing Videos	To emphasize the importance of procedural and conceptual teaching of mathematics (especially fractions) with appropriate pauses for clarity of ideas found in videos	45 minutes	Videos aligned with CCSS
	Participants' Response	Discover viewpoints of participants in reference to ideas found in videos	15 minutes	Response Questions referencing concepts from videos viewed
Four	Fraction Post-test	Discover increase of content knowledge level of fractions in relation to pre-test	15 minutes	Same questions used in Fraction pre-test from Session One
	Anxiety Post-test	Discover decrease of math anxiety level of participants if existent in relation to pre-test	15 minutes	Mathematics Anxiety Rating Scale Shortened Version (Appendix B) or equivalent (same as those used in Session Two)
Five	Performance Assessment	Discover pre-service teachers' abilities to teach fractions procedurally and conceptually	35 minutes	Same questions used in Fraction pre-test from Session One

CHAPTER 4: IMPLICATIONS AND RECOMMENDATIONS

This Dissertation in Practice describes the process and findings from the pilot study in order to create a model for professional development of procedural and conceptual knowledge in teaching fractions. The model will provide the framework for future reference to improve educational practices in teaching mathematics aligned to Common Core Standards. The study presented in this Dissertation in Practice addressed the following areas of pre-service teachers' practice:

1) teaching methods that provide instructional strategies for procedural learning of fractions according to CCSS (FDOE, 2014);

2) teaching methods that provide instructional strategies for conceptual learning of fractions according to CCSS (FDOE, 2014);

3) measuring math anxiety, if it exists, in the pre-service teachers using the Mathematics Anxiety Rating Scale (MARS).

Procedural and Conceptual Learning

Fraction Knowledge Assessment

The experiences teachers provide in a classroom will shape their students' future learning and feelings toward mathematics (NCTM, 2014). To be effective, teachers should build procedural fluency that enhance the conceptual understanding over time with the purpose of building knowledge that allows the students to use in higher mathematics (NCTM, 2014). Additionally, to be effective, the teacher needs to have conceptual knowledge of the mathematics, especially fractions, and not just simply be able to compute the problems at hand (Dixon et al., 2014). The FCAT pre-test fraction worksheet was the beginning of the pilot study and informed the model that a similar fractions problem worksheet would be the initial data collection for the procedural and conceptual knowledge of the pre-service teachers. Having a base level of procedural knowledge will allow the pre-service teachers to know if they understand the mathematics at hand.

<u>Workshops</u>

The researcher attended a workshop sponsored by the Regional Educational Laboratory, which presented current research on fractional concepts conducted in Macon, Georgia on May 21, 2015. This workshop presented current research on fractional concepts conducted in elementary classrooms along with conceptual designs of how to teach fraction/decimal problems. It provided deep, significant learning activities involving fractions that are necessary for conceptual understanding and were aligned with Common Core Standards. When students are shown problems that are not in the traditional procedural design, but yet in a full conceptual format, they likely gain the conceptual knowledge (Hiebert et al., 1997) needed to keep up with the demands of college level mathematics.

The Importance of Self-Efficacy

If self-efficacy as related to teaching mathematics is deficient, the teacher may have a tendency to teach the problem devoid of conceptual depth and only teach the procedural design of the memorized steps remembered from days of learning as an adolescent (Tait, 2006). Teachers that teach procedurally will find themselves wanting to move away from this design of how they were taught as an elementary school student (Thrift & Ortiz, 2007) and re-learn the mathematics conceptually in order to keep up with the needs of Common Core Standards

(Heitin, 2015). Teachers should also be assessed on their own procedural and conceptual knowledge of mathematics (Drake & Barlow, 2007; Whittin & Whittin, 2008) with the intention of obtaining awareness of their own strengths and weaknesses in their knowledge of mathematics, especially fractions. The model would allow this process when utilizing the FCAT (or similar) mathematics worksheets. Unfortunately, it is not easy to accomplish the gauging of conceptual knowledge (Fennema et al., 1996; Tirosh, 2000), but using a pre-test set of fraction problems from a variety of grade levels would be a place to start for at least the procedural side of it. For thirty-two pre-service teachers participating in a research project similar to this model, Rayner et al. (2009) used a paper and pencil assessment for fraction procedural and conceptual knowledge known as the Knowledge of Fractions Assessment (KFA). Van Steenbrugge et al. (2014) also utilized a paper and pencil test corresponding to elementary school level fraction computation with the intention to assess 290 pre-service teachers procedural and conceptual knowledge of fractions as the beginning stage of a research study.

Performance Feedback and Reflection

Microteaching and hosting performance assessments during methods courses would allow the pre-service teachers a chance to demonstrate their teaching abilities while the audience provides feedback of any evidence of conceptual teaching. The feedback from the peers and faculty would be a step in the right direction toward knowing how one teaches. Feedback from peers could assist the pre-service teachers in becoming reflective practitioners. Included with microteaching and/or performance assessments could be some sort of reflective papers. Reflection would focus on refining their lesson planning to include better conceptual designs of teaching. Maloney and Beilock (2012) believe that expressive writing can give the “opportunity to re-evaluate the stressful experience in a manner that reduces the necessity to worry altogether” (p.405). Reflective writings should be included in the model and could involve pre-determined questions that are found relative to observing videos and/or feelings towards mathematics.

A Mathematics Anxiety Measurement

MARS-S

Using the Mathematics Anxiety Rating Scale Shortened Version for the measurement tool of math anxiety in the pre and post test was helpful in identifying potential math anxiety in the participants. When the anxiety levels ranged from minimal anxiety to potentially significant math anxiety, the model was informed that the MARS-S was an adequate tool to measure potential math anxiety levels. There are other math anxiety questionnaires/inventories such as personally designed questionnaires used from previous research studies (Tait, 2006), the Revised Mathematics Anxiety Rating Scale (RMARS) used by Rayner et al. (2009), the Mathematics Anxiety Rating Scale for Adults (MARS-A) utilized in a math anxiety reduction in pre-service educators research project by Tooke (1998), or the standard 98- questionnaire originally designed as the MARS (Sloan, 2010) used as a pre/post instrument for measuring math anxiety of 72 pre-service elementary educators. The model using the MARS-S helped identify if math anxiety existed among the participants similar to the investigation by Brunye et al. (2013), but those researchers also used other measurement tools for measuring perceptions, thoughts, feelings, and other psychosomatic factors with the aim of teaching the participants coping mechanisms for math anxiety.

Possible modifications could be understanding the antecedents of math anxiety (Maloney & Beilock, 2012) through open discussion with the participants of math anxiety, personal written reflections and/ or expressive writings with guiding questions about attitude and past mathematical performance, and a more in-depth exploration of math anxiety conducted by trained professionals. New York City based institutions are implementing math anxiety reducing techniques by brushing up on their basic mathematics in after school meetings with teachers and offering workshops on math anxiety (Heitin, 2015). These teachers are needing to improve their math skills since the Common Core Standards are requiring the topics to be taught conceptually as well as procedurally.

Reflective Writings

A potential modification of the model would be to include an informative session on math anxiety and coping techniques for reducing math anxiety. Reflective writings with possible questions pertaining to math anxiety could be: 1) Do you know if you have math anxiety? 2) Do you know what math anxiety is? 3) Have you taken a math class in the past that made you feel anxious or nervous?, 4) Do you feel confident in completing simple mathematical tasks such as tips at restaurants or calculating percentages off at clothing stores?, 5) Have you ever taken a course that helped you overcome insecurities about teaching mathematics? Finlayson (2014) created a survey to allow the pre-service elementary teachers an opportunity to recount their experiences with math anxiety with the purpose of finding strategies to overcome math anxiety. The survey included questions such as

> (1)Have you ever had math anxiety? If so, at what grade level did you first experience it?
> (2) If yes, please describe your math anxiety situation.

(3) What are the causes of your math anxiety? What do you think caused math anxiety?

(4) What strategies have you used to help overcome math anxiety?

(5) What strategies would you suggest as future teachers to help your students overcome math anxiety? (p. 103)

A reflective summary of feelings toward mathematics would be a recommendation for further work involved for this model.

The Model in Action

Potential research could be a longitudinal study of the four participants as they move through their math methods course and teaching career. Following their progress and assisting them with their deficiencies in teaching procedurally and conceptually could possibly aid in their reduction of math anxiety and improve their confidence in teaching fractions. Using the model at the elementary school level with in-service teachers could be valuable to help teachers learn procedural and conceptual knowledge of teaching fractions. In-service teachers may not be aware of their math anxiety and the awareness of potential anxiety could be beneficial since sometimes a teachers' math anxiety can influence the development of students' math anxiety (Maloney & Beilock, 2012).

Recommendations

The purpose of this dissertation in practice was to inform pre-service teachers of conceptual and procedural methods of teaching fractions. The proposed model of professional development focused on procedural and conceptual teaching of fractions, as required by Common Core Standards. The proposed model would include the following key components:

1) FCAT 2.0 fractions worksheet (pre- and post) or something equivalent spanning the elementary grade levels in difficulty,

2) A mathematics anxiety ratings scale,

3) Reflective writings in reference to mathematics anxiety,

4) Videos addressing the importance of procedural and conceptual teaching of mathematics,

5) Questions about the topics discussed in the videos with pauses for clarity of ideas mentioned in the videos,

6) Video-taped performance assessment similar to microteaching.

A further developed model could also include possible coping techniques if math anxiety exists. Possible enhancements to the model would include microteaching and video taping of participants during a performance assessment for constructive feedback. Peer reviewing a lesson plan involving fractions created by the participants could potentially improve the model.

A recommendation for the video viewing would be to stop the video after a significant statement and discuss the meaning with the participants. Content specific workshops taught by mathematics educators and focused on specific mathematical concepts such as fractions or decimals with hands-on activities could enrich the model's design. No matter what improvements or changes that could be made to the model, further research is needed to help pre-service teachers become aware of the challenges of teaching and potential anxieties that they may be experiencing and passing on to their students (Finlayson, 2014).

APPENDIX A: IRB APPROVAL LETTER

University of Central Florida

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Deborah M. Edwards** and Co-PI: **Carolyn W. Hopp**

Date: **April 02, 2015**

Dear Researcher:

On 04/02/2015, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review:	Exempt Determination
Project Title:	Teaching fractions procedurally and conceptually to pre-service elementary education teachers
Investigator:	Deborah M. Edwards
IRB Number:	SBE-15-11150
Funding Agency:	
Grant Title:	
Research ID:	N/A

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Signature applied by Patria Davis on 04/02/2015 03:08:47 PM EDT

IRB Coordinator

Page 1 of 1

APPENDIX B: MARS-S

NAME ______________________ Total Score ____________

MATHEMATICS ANXIETY RATING SCALE: SHORT VERSION

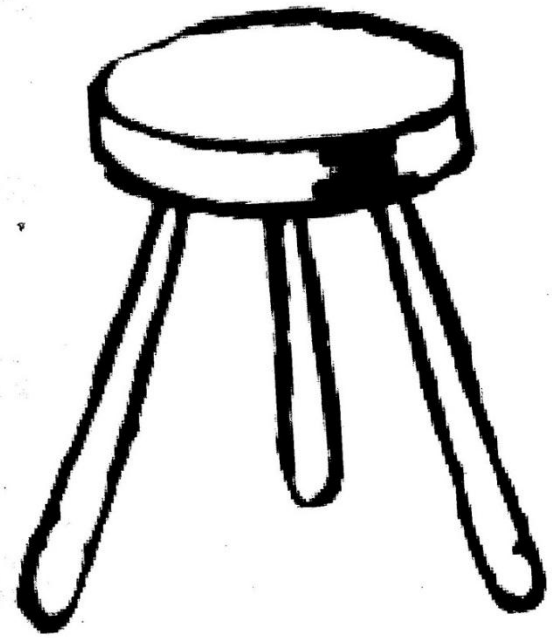
The items in the questionnaire refer to things that may cause fear or apprehension. For each item, place a check in the box under the column that describes how much you are frightened by it nowadays. Work quickly but be sure to consider each item individually.

	Not at all	A little	A fair amount	Much	Very much
1. Taking an examination (final) in a math course.	☐	☐	☐	☐	☐
2. Thinking about an upcoming math test one week before.	☐	☐	☐	☐	☐
3. Thinking about an upcoming math test one day before.	☐	☐	☐	☐	☐
4. Thinking about an upcoming math test one hour before.	☐	☐	☐	☐	☐
5. Thinking about an upcoming math test five minutes before.	☐	☐	☐	☐	☐
6. Waiting to get a math test returned in which you expected to do well.	☐	☐	☐	☐	☐
7. Receiving your final math grade in the mail.	☐	☐	☐	☐	☐
8. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	☐	☐	☐	☐	☐
9. Being given a "pop" quiz in a math class.	☐	☐	☐	☐	☐
10. Studying for a math test.	☐	☐	☐	☐	☐
11. Taking the math section of a college entrance exam.	☐	☐	☐	☐	☐
12. Taking an examination (quiz) in a math course.	☐	☐	☐	☐	☐
13. Picking up the math text book to begin working on a homework assignment.	☐	☐	☐	☐	☐
14. Being given a homework assignment of many difficult problems which is due the next class meeting.	☐	☐	☐	☐	☐
15. Getting ready to study for a math test.	☐	☐	☐	☐	☐

	Not at all	A little	A fair amount	Much	Very much
16. Dividing a five digit number by a two digit number in private with pencil and paper.	☐	☐	☐	☐	☐
17. Adding up 976 + 777 on paper.	☐	☐	☐	☐	☐
18. Reading a cash register receipt after your purchase.	☐	☐	☐	☐	☐
19. Figuring the sales tax on a purchase that costs more than $1.00.	☐	☐	☐	☐	☐
20. Figuring out your monthly budget.	☐	☐	☐	☐	☐
21. Being given a set of numerical problems involving addition to solve on paper.	☐	☐	☐	☐	☐
22. Having someone watch you as you total up a column of figures.	☐	☐	☐	☐	☐
23. Totaling up a dinner bill that you think overcharged you.	☐	☐	☐	☐	☐
24. Being responsible for collecting dues for an organization and keeping track of the amount.	☐	☐	☐	☐	☐
25. Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car going at different speeds.	☐	☐	☐	☐	☐
26. Totaling up the dues received and the expenses of a club you belong to.	☐	☐	☐	☐	☐
27. Watching someone work with a calculator.	☐	☐	☐	☐	☐
28. Being given a set of division problems to solve.	☐	☐	☐	☐	☐
29. Being given a set of subtraction problems to solve.	☐	☐	☐	☐	☐
30. Being given a set of multiplication problems to solve.	☐	☐	☐	☐	☐

APPENDIX C: THE THREE LEGGED STOOL

What's the difference between conceptual and procedural?



What are your 3-legged rigor steps to teaching?

What is a problem you did not understand how to do (misunderstood) but now you know how to do it,

APPENDIX D: QUESTION 4 HANDOUT

Teaching Fractions Conceptually and Procedurally to Pre-service Elementary Education Teachers (video day)

What is a major difference between the way fractions were taught to us and the way Common Core State Standards require teachers to teach it?

APPENDIX E: FCAT 2.0 FRACTIONS WORKSHEETS POST-TEST

____________________________Student Name

The following questions are a compilation of FCAT 2.0 mathematics test questions from Florida Department of Education's website for Grades 3 through 6. Please read the questions that do not have a strike-through mark and answer them to the best of your ability. There is no partial credit and any question skipped will be considered incorrect. You may not use a calculator.

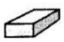
Grade 5 FCAT 2.0 Mathematics Reference Sheet

Area

Rectangle	$A = bh$
Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2}bh$ or $A = (bh) \div 2$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$ or $A = h(b_1 + b_2) \div 2$

KEY	
b = base	A = area
h = height	B = area of base
w = width	V = volume
	$S.A.$ = surface area

Volume of Rectangular Prism



$V = bwh$ or

$V = Bh$

Surface Area of Rectangular Prism

$S.A. = 2bh + 2bw + 2hw$

Customary Conversions 1 foot = 12 inches 1 yard = 3 feet 1 mile = 5,280 feet 1 mile = 1,760 yards 1 acre = 43,560 square feet	**Customary Conversions** 1 cup = 8 fluid ounces 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts 1 pound = 16 ounces 1 ton = 2,000 pounds
Metric Conversions 1 centimeter = 10 millimeters 1 meter = 100 centimeters 1 meter = 1000 millimeters 1 kilometer = 1000 meters	**Metric Conversions** 1 liter = 1000 milliliters 1 liter = 1000 cubic centimeters 1 gram = 1000 milligrams 1 kilogram = 1000 grams
Time Conversions 1 minute = 60 seconds 1 hour = 60 minutes 1 day = 24 hours 1 week = 7 days 1 year = 365 days 1 year = 52 weeks	

Note: Metric numbers with four digits are presented without a comma (e.g., 9960 kilometers). For metric numbers greater than four digits, a space is used instead of a comma (e.g., 12 500 liters).

Grades 6–8 FCAT 2.0 Mathematics Reference Sheet

Area

Rectangle	$A = bh$
Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	$A = \pi r^2$

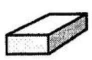
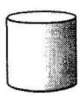
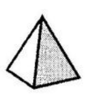
KEY

b = base	A = area
h = height	B = area of base
w = width	C = circumference
d = diameter	V = volume
r = radius	P = perimeter of base
ℓ = slant height	$S.A.$ = surface area

Use 3.14 or $\frac{22}{7}$ for π.

Circumference

$C = \pi d$ or $C = 2\pi r$

		Volume/Capacity	Total Surface Area
	Rectangular Prism	$V = bwh$ or $V = Bh$	$S.A. = 2bh + 2bw + 2hw$ or $S.A. = Ph + 2B$
	Right Circular Cylinder	$V = \pi r^2 h$ or $V = Bh$	$S.A. = 2\pi rh + 2\pi r^2$ or $S.A. = 2\pi rh + 2B$
	Right Square Pyramid	$V = \frac{1}{3}Bh$	$S.A. = \frac{1}{2}P\ell + B$
	Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$ or $V = \frac{1}{3}Bh$	$S.A. = \frac{1}{2}(2\pi r)\ell + B$

Sum of the measures of the interior angles of a polygon $= 180(n - 2)$

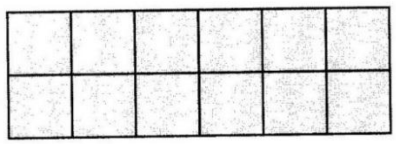
Measure of an interior angle of a regular polygon $= \dfrac{180(n-2)}{n}$

where:

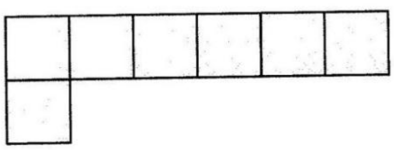
n represents the number of sides

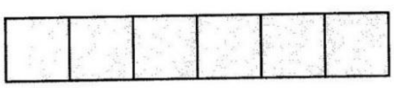
16 The diagram below represents Akeem's chocolate bar.

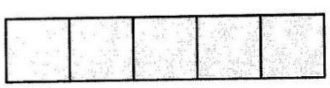
Akeem's Chocolate Bar

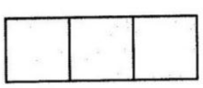


Akeem gave $\frac{1}{4}$ of his chocolate bar to his sister and saved the rest for himself. After dinner, he ate $\frac{1}{3}$ of the part he saved for himself. Which of the following represents the part of Akeem's chocolate bar that he had left?

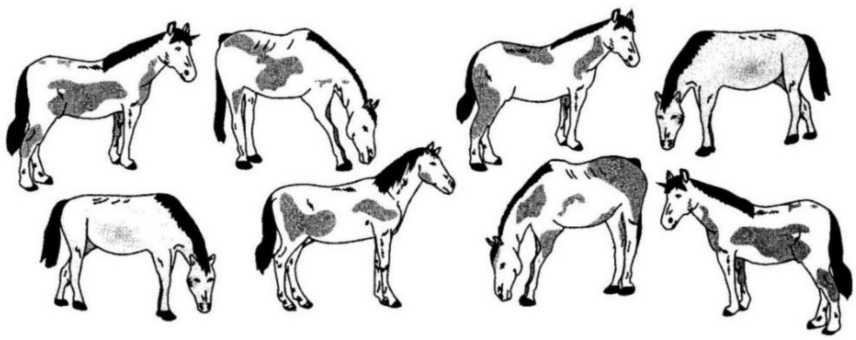
A. 

B. 

C. 

D. 

5 Leia visited a horse ranch. She noticed that $\frac{6}{8}$ of the horses were spotted, as shown below.



Which fraction is equivalent to $\frac{6}{8}$?

Ⓐ $\frac{1}{2}$

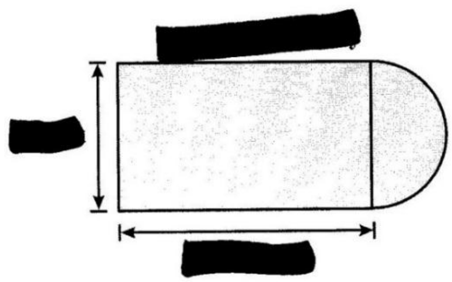
Ⓑ $\frac{1}{4}$

Ⓒ $\frac{2}{3}$

Ⓓ $\frac{3}{4}$

[redacted]

9 [redacted]



[redacted]

10 Mr. Nichols wants to store $25\frac{1}{2}$ cups of stew in containers. Each container holds a maximum of $1\frac{1}{2}$ cups of stew. What is the minimum number of containers Mr. Nichols needs to hold all the stew?

A. 9

B. 17

C. 25

D. 51

11 [redacted]

13 [redacted]

14 A store had sweaters on sale for 75% off of the original price. Which of the following is equivalent to 75%?

Ⓕ $\frac{1}{4}$

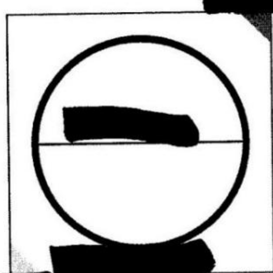
Ⓖ $\frac{1}{2}$

Ⓗ $\frac{3}{4}$

Ⓘ $\frac{5}{7}$

1 [redacted]

14 [redacted]



[redacted]

[redacted]

15 Mr. Madsen worked 49 hours last week at his job. He spent $\frac{1}{5}$ of this time in meetings and $\frac{1}{3}$ of this time talking to customers on the phone. Which method would provide the most reasonable estimate of the total number of hours Mr. Madsen spent in meetings and talking to customers on the phone at his job last week?

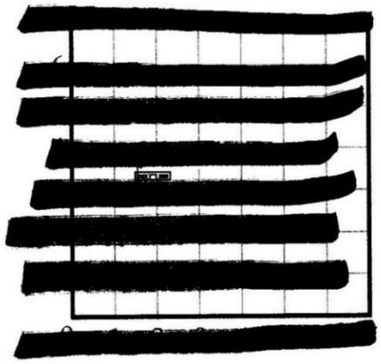
F. multiply $\frac{1}{4}$ by 50

G. multiply $\frac{1}{2}$ by 50

H. multiply $\frac{1}{5}$ by 50 and add $\frac{1}{3}$ to the product

I. multiply $\frac{1}{5}$ by $\frac{1}{3}$ and multiply the product by 50

[redacted]



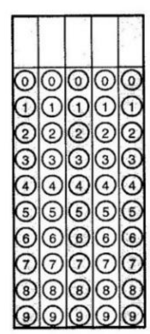
[redacted]

Ⓐ [redacted]

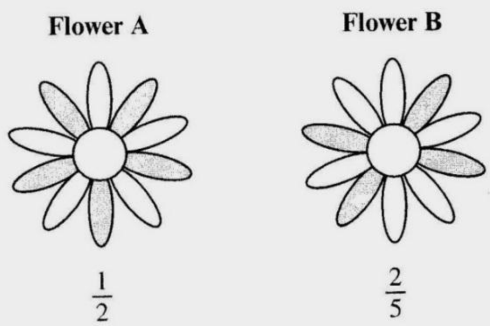
Ⓑ [redacted]

[redacted]

2 Caitlyn set a goal to swim 675 laps in her pool during summer vacation. She will swim 12 laps each day. What is the least **whole** number of days Caitlyn will swim to reach her goal?



7 Two flowers are pictured below. On Flower A, $\frac{1}{2}$ of the petals are shaded. On Flower B, $\frac{2}{5}$ of the petals are shaded.



Which inequality below correctly compares the fractions of petals that are shaded?

Ⓐ $\frac{1}{2} > \frac{2}{5}$

Ⓑ $\frac{1}{2} < \frac{2}{5}$

Ⓒ $\frac{2}{1} > \frac{5}{2}$

Ⓓ $\frac{2}{1} < \frac{5}{2}$

19 Sam is mailing some items to his brother. Before he went to the post office, he weighed the items. The table below shows the weight of each item.

ITEMS TO MAIL

Item	Weight
Computer paper	$5\frac{7}{8}$ pounds
Laptop computer	$3\frac{1}{4}$ pounds
Recipe book	$\frac{15}{16}$ pounds

Between which two weights is the total weight of all three items?

Ⓐ between $9\frac{1}{2}$ and 10 pounds

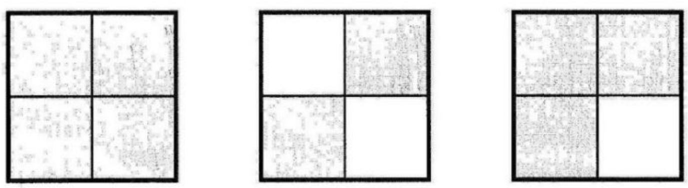
Ⓑ between 10 and $10\frac{1}{8}$ pounds

Ⓒ between $10\frac{1}{4}$ and $10\frac{1}{2}$ pounds

Ⓓ between $10\frac{1}{2}$ and 11 pounds

[redacted]

9 Rosalyn drew three figures and shaded parts of each figure.



Which mixed number is represented by the shading of the three figures above?

Ⓐ $2\frac{1}{4}$

Ⓑ $2\frac{3}{4}$

Ⓒ $3\frac{1}{4}$

Ⓓ $3\frac{1}{2}$

8 At their closest points, the distance between Alaska and Russia is 2.5 miles. Which of the following is equivalent to 2.5 ?

Ⓐ $2\frac{1}{5}$

Ⓑ $2\frac{5}{10}$

Ⓒ $2\frac{1}{25}$

Ⓓ $2\frac{5}{100}$

9 Denisse's puppy weighs 9.8 pounds. Which of the following fractions is closest to 9.8 ?

Ⓕ $9\frac{1}{2}$

Ⓖ $9\frac{3}{4}$

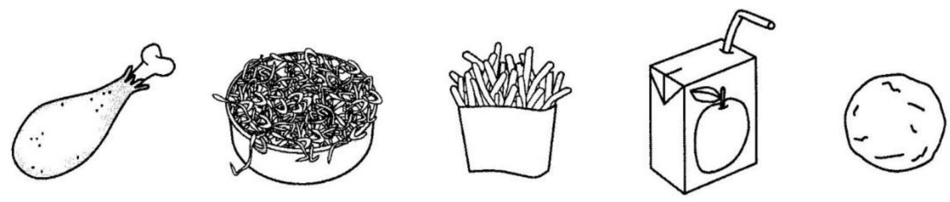
Ⓗ $9\frac{1}{8}$

Ⓘ $9\frac{2}{10}$

[redacted]

3 After eating lunch, Samantha compared the number of calories in each food item she ate to the total number of calories in her lunch. The picture below shows an approximate portion of the total calories for each food item she ate.

Samantha's Lunch



Chicken	Cole slaw	French fries	Juice	Oatmeal cookie
0.29	0.2	$\frac{1}{4}$	$\frac{1}{10}$	16%

Which of the following lists shows the numbers in order from least to greatest?

A. $\frac{1}{10}$, 16%, 0.2, $\frac{1}{4}$, 0.29

B. 16%, $\frac{1}{4}$, $\frac{1}{10}$, 0.2, 0.29

C. 0.29, 0.2, $\frac{1}{10}$, 16%, $\frac{1}{4}$

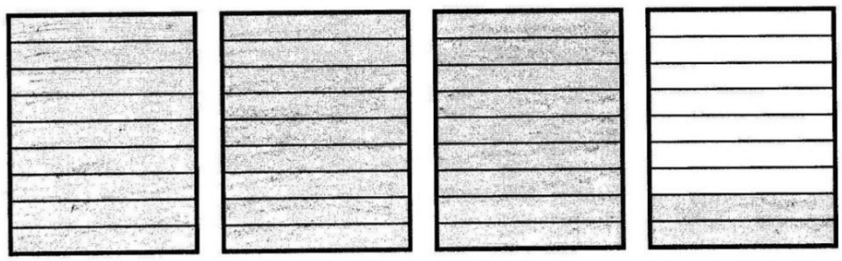
D. 0.2, $\frac{1}{4}$, 16%, $\frac{1}{10}$, 0.29

4 A television show announcer asked viewers to vote online for their favorite commercial. The viewers could choose from one of three commercials. The results of their votes are shown below.

- A total of 1,000 votes were cast.
- The first commercial received 25% of the votes.
- The second commercial received $\frac{2}{5}$ of the votes.

The third commercial received the remaining votes. What was the total number of votes for the third commercial?

10 In the picture below, $3\frac{2}{9}$ of the figures are shaded.



Which fraction is equivalent to $3\frac{2}{9}$?

Ⓕ $\frac{29}{4}$

Ⓖ $\frac{29}{7}$

Ⓗ $\frac{29}{9}$

Ⓘ $\frac{29}{36}$

FLORIDA DEPARTMENT OF EDUCATION
www.fldoe.org

The Florida Department of Education and its test contractors currently employ strategies to protect the environment in the production and destruction of FCAT 2.0 materials. The Department encourages schools and districts to recycle nonsecure FCAT 2.0 interpretive publications after use.

540176 1 2 3 4 5 A B C D E Printed in the USA ISD4546

REFERENCES

Alexander, C. M. & Ambrose, R. C. (2010). Digesting student-authored story problems. Mathematics Teaching in Middle School, 16(1), 27-33.

Alibali, M. W., Brown, A. N., Stephens, A. C., Kao, T. S., & Nathan, M. J. (2009). Middle school students' conceptual understanding of equations: Evidence from writing story problems (WCER Working Paper No. 2009- 3). Retrieved from http://www.wcer.wisc.edu/publications/workingPapers/papers.php.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientation to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education*, *7*, 91-119

Andersen, F. O. (2010). Danish and Finnish PISA results in a comparative, qualitative perspective: How can the stable and distinct differences between the Danish and Finnish PISA results be explained? *Educational Assessment, Evaluation and Accountability*, *22*(2), 159-175. doi: http://dx.doi.org/10.1007/s11092-010-9095-x

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775-785. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1036929&site=eds-live&scope=site; http://dx.doi.org/10.1111/desc.12155

Baloglu, M. (2010). *An investigation of the validity and reliability of the adapted mathematics anxiety rating scale-short version (MARS-SV) among turkish students* Springer. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=edsjsr&AN=edsjsr.23421485&site=eds-live&scope=site

Blank, R. K., Dalkilic, M., & Council of Chief State School Officers, (1992). Washington, DC State Education Assessment Center. *State policies on science and mathematics education 1992*.Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED350163&site=eds-live&scope=site

Blazer, C., & Miami-Dade County, P. S. (2011). *Strategies for reducing math anxiety. information capsule. volume 1102*.Research Services, Miami-Dade County Public Schools. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED536509&site=eds-live&scope=site

Brunye, T.T., Mahoney, C.R., Giles, G.E., Rapp, D.N., Taylor, H.A., & Kanarek, R.B. (2013). Learning to relax: Evaluating four brief interventions for overcoming the negative emotions accompanying math anxiety. *Learning and Individual Differences*. 271-7. Doi:10.1016/j.lindif.2013.06.008

Brown, G., & Quinn, R. J. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher, 62*(4), 28-40. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ765838&site=eds-live&scope=site; http://www.aamt.edu.au/shop/entire_catalogue/australian_mathematics_teacher

Brush, L. R. (1978). *A validation study of the mathematics anxiety rating scale (MARS)* Educational and Psychological Measurement. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ191599&site=eds-live&scope=site

Charter School A. (2013). FCAT test results for academic year 2013-2014 for 3rd grade students.

Charter School A. (2013). Moving forward 2014-2015 retrieved from school's website.

Charter School A. (2013). Philosophical views and mission statements retrieved from school's website.

Charter School A. (2014). Personal communication with school's principal.

Clinton, B. (1997). State of the union address: February 4, 1997. The Washington Post Company, Retrieved from http://www.washingtonpost.com/wp-srv/politics/special/states/docs/sou97.h

Commentary online. (2012). *Education Week, 31*(36), 26-26. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eft&AN=77943356&site=eds-live&scope=site

Complete, C. A. (2012). *Remediation: Higher education's bridge to nowhere*.Complete College America. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED536825&site=eds-live&scope=site

Council of Chief State School Officers (CCSSO) & National Governors Association (NGA). (2010). Common core state standards - mathematics. Retrieved from http://www.corestandards.org/the-standards/mathematics.

Dew, K. H., Galassi, J. P., & Galassi, M. D. (1984). Math anxiety: Relation with situational test anxiety, performance, physiological arousal, and math avoidance behavior. *Journal of Counseling Psychology, 31*(4), 580-583. doi:10.1037/0022-0167.31.4.580

Dixon, J.K., Andreasen, J.B., Avila, C.L., Bawatneh, Z., Deichert, D.L., Howse, T.D., & Turner, M.S. (2014). Redefining the whole: Common errors in elementary preservice teachers self-authored word problems for fraction subtraction. *Investigations In Mathematics Learning, 7*(1), 1-22.

Drake, J. M., & Barlow, A. T. (2007). Assessing students' levels of understanding multiplication through problem writing. *Teaching in the Middle School, 19*(3), 156-163.

Durkin, K., & Rittle-Johnson, B. (2014) Diagnosing misconceptions: Revealing changing decimal fraction knowledge, Learning and Instruction http://dx.doi.org/10.1016/j.learninstruc.2014.08.003

EducationWeek website (2014). *Approach to fractions seen as key shift in common standards*. Published November 10, 2014. http://www.edweek.org/ew/articles/2014/11/12/12cc-fractions.h34.html?qs=Approach+to+Fractions+seen+as+Key+Shift+in+Common+Standards

Edwards, D. (2014). Personal communication of remedial math exam results for a Daytona Beach university during fall semester of 2014 classes.

Fennema, E., Carpenter, T. P., Franke, M. L., Levo, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematical instruction. *Journal for Research in Mathematics Education*, *27*(4), 403-34.

Finlayson, M. (2014). Addressing Math Anxiety in the Classroom. *Improving Schools*, *17*(1), 99-115.

Florida Consortium of Public Charter Schools. (2014). Laws on public charter schools retrieved from http://floridacharterschools.org/schools/law_and_compliance/

Florida Department of Education (1998). *Sunshine state standards*. (1998). Tallahassee, FL.

Florida Department of Education (2012) Start a charter school in Florida. Office of Independent Education and Parental Choice. November 2012. Retrieved from http://www.floridaschoolchoice.org/information/charter_schools/files/How_to_Start_Charter_School_in_Florida.pdf

Florida Department of Education (2014). Retrieved from http://fldoe.org

Geiger, V., & Galbraith, P. (1998). Developing a diagnostic framework for evaluating student approaches to applied mathematics. *International Journal of Mathematical Education in Science & Technology*, *29*(4), 533. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=tfh&AN=933120&site=eds-live&scope=site

Florida Standards Assessments (2015). Retrieved from http://www.fldoe.org/accountability/assessments/k-12-student-assessment/fsa.stml

He, C., & Yan, C. (2011). Exploring authenticity of microteaching in pre-service teacher education programmes. *Teaching Education*, *22*(3), 291-302. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/logi

n.aspx?direct=true&db=eric&AN=EJ950003&site=eds-live&scope=site; http://dx.doi.org/10.1080/10476210.2011.590588

Hecht, S.A. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology, 90*(3), 545-59.

Heitin, L. (2014). Approach to fractions shifting under common core. Ed Week Update. Retrieved November 15, 2014, from http://www.edweek.org/newsletter/daily-update/2014/11/14/email/ewupdate.html?cmp=ENL-EU-TOPNAV

Heitin, L. (2015). Curriculum Matters. What two schools are doing about teachers' math anxiety. Retrieved from http://blogs.edweek.org/edweek/curriculum/2015/06/teachers_math_anxiety_brief_new_york.html?cmp=ENL-EU-NEWS3

Hiebert, J., Carpenter, T. P., Fennema, E., Fushon, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). Making sense: Teaching and learning mathematics with understanding. Portsmouth, NH: Heinemann.

International Association for the Evaluation of Educational Achievement. (2011). Retrieved from http://www.iea.nl/.

Kastberg, D., Roey, S., Ferraro, D., Lemanski, N., Erberber, E., National Center for, E. S., & Westat, I. (2013). *U.S. TIMSS and PIRLS 2011 technical report and Users guide. NCES 2013-046*. National Center for Education Statistics. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED545101&site=eds-live&scope=site

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up. [electronic resource] : Helping children learn mathematics* Washington, DC : National Academy Press, c2001. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=cat00846a&AN=ucfl.033216554&site=eds-live&scope=site; https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://site.ebrary.com/lib/univcentralfl/Doc?id=10038695

King, J. E., & State Higher Education, E. O. (2011). *Implementing the common core state standards: An action agenda for higher education*. State Higher Education Executive Officers. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED540255&site=eds-live&scope=site

Lake, V. E., & Kelly, L. (2014). Female preservice teachers and mathematics: Anxiety, beliefs, and stereotypes. *Journal of Early Childhood Teacher Education*, *35*(3), 262-275. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1035873&site=eds-live&scope=site; http://dx.doi.org/10.1080/10901027.2014.936071

Langlotz, P. C. (2000). *The odyssey of an American school system*. Pride Enterprises.

Luo, F., Lo, J., & Leu, Y. (2011). Fundamental fraction knowledge of preservice elementary teachers: A cross-national study in the United States and Taiwan. *School Science and Mathematics, 111*(4), 164-177. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ927224&site=eds-live&scope=site; http://dx.doi.org/10.1111/j.1949-8594.2011.00074.x

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers understanding of fundamental mathematics in China and the United States*. Mahway, NJ: Lawrence Erlbaum Associates.

Maloney, E.A., & Beilock, S.L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends In Cognitive Sciences*, *16(8*), 404-406. Doi: 10.1016/j.tics.2012.06.008

Manly, M., Ginsburg, L., & National Institute, f. L. (2010). *Algebraic thinking in adult education*. National Institute for Literacy. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED512294&site=eds-live&scope=site

Martin, A. (2011). Growing charter schools in Volusia, Flagler earn praise, raise concern. Daytona Beach News Journal article published August 21, 2011. Retrieved from http://www.imagineschools.org/2011/08/growing-charter-schools-in-volusia-flagler-earn-praise-raise-concerns

Massey, D., & Riley, L. (2013). Reading math textbooks: An algebra Teacher's patterns of thinking. *Journal of Adolescent & Adult Literacy, 56*(7), 577-586. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1013158&site=eds-live&scope=site; http://dx.doi.org/10.1002/JAAL.187

Math anxiety (2010). *Science & Children, 47*(7), 8-10. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=ofm&AN=508142345&site=eds-live&scope=site

Miyamoto, K. (2008). The origins of the standards movement in the United States: Adoption of the written test and its influence on class work. *Educational Studies in Japan: International Yearbook,* (3), 27-40. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ842865&site=eds-live&scope=site; http://www.soc.nii.ac.jp/jsse4/kikansi-e.html

Moseley, B. J., Okamoto, Y., & Ishida, J. (2007). Comparing US and Japanese elementary school teachers' facility for linking rational number representations. *International Journal of Science and Mathematics Education, 5*(1), 165-185. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ924406&site=eds-live&scope=site; http://dx.doi.org/10.1007/s10763-006-9040-0

National Academy of Sciences - National Research Council, Washington, DC Mathematical Sciences Education Board. (1996). *Mathematics and science education around the world: What can we learn from the survey of mathematics and science opportunities (SMSO) and the third international mathematics and science study (TIMSS)?* (). Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED401126&site=eds-live&scope=site

National Center for Educational Statistics (1989). Retrieved from http://nces.ed.gov/pubs91/91191.pdf

National Council on Excellence in Education (1983). A nation at risk: The imperative for educational reform. Washington, D.C.: U.S. Government Printing Office.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author.

National Council of Teachers of Mathematics You tube Website (2015). *Building conceptual understanding in mathematics* Published April 10, 2015. *https://www.youtube.com/watch?v=W1eLt0Dz8Fk*

National Council of Teachers of Mathematics You tube Website (2015). *Developing mathematical skills in upper elementary grades.* Published April 10, 2015. https://www.youtube.com/watch?v=7sMNxaBVkCQ

National Council of Teachers of Mathematics You tube Website (2015). *Mathematical foundations for success in algebra.* Published April 7, 2015. *https://www.youtube.com/watch?v=Dg6BalrPe5I*

National Council of Teachers of Mathematics You tube Website (2015). *Mathematics in the early grades,* Published April 10, 2015. https://www.youtube.com/watch?v=x666A-UIikw

National Council of Teachers of Mathematics You tube Website (2015). *Preparation for higher level mathematics.* Published April 7, 2015. *https://www.youtube.com/watch?v=gnmCWNwlUSU*

National Governors' Association (1986) Time for Results: The Governors' 1991 Report on Education, Washington, DC: National Governors' Association, Center for Policy Research and Analysis.

Obama, B. (2009). Race to the Top education reform. Retrieved from http://www.whitehouse.gov

O'Connor, J.(2014). State Impact: A reporting project of npr reporting stations-Florida. Message posted to http://stateimpact.npr.org/florida/tag/charter-schools

Plomp, T. (1996). Statement by IEA Chairman about the significance of TIMSS and IEA. Boston College, Lynch School of Education, November 20, 1996

Rayner, V., Pitsolantis, N., and Osana, H..(2009). Mathematics anxiety in preservice teachers: Its relationship to their conceptual and procedural knowledge of fractions." *Mathematics Education Research Journal (21)3*. 60-85. EBSCO <https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ883873&site=eds-live&scope=site>.

Richardson, F. C., & Suinn, R. M. (1972). The mathematics anxiety rating scale: Psychometric data. *Journal of Counseling Psychology*. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ069707&site=eds-live&scope=site

Rittle-Johnson, B., & Koedinger, K. R. (2002). *Comparing instructional strategies for integrating conceptual and procedural knowledge.* Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED471761&site=eds-live&scope=site

Rohrer, D. (2012). Interleaving helps students distinguish among similar concepts. *Educational Psychology Review, 24*(3), 355-367. doi:10.1007/s10648-012-9201-3

Ross, E. (2010, May). The common core standards: Looking back and looking ahead. National Association of State Boards of Education, (17, 5).

Schmidt, W. H. (2012). At the precipice: The story of mathematics education in the United States. *Peabody Journal of Education, 87*(1), 133-156. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ971398&site=eds-live&scope=site; http://dx.doi.org/10.1080/0161956X.2012.642280

Sloan, T. R. (2010). A quantitative and qualitative study of math anxiety among preservice teachers. *Educational Forum, 74*(3), 242-256. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ886943&site=eds-live&scope=site; http://www.informaworld.com/openurl?genre=article&id=doi:10.1080/00131725.2010.483909

Sood, S., & Jitendra, A. K. (2007). A comparative analysis of number sense instruction in reform-based and traditional mathematics textbooks. *Journal of Special Education, 41*(3), 145-157. doi:10.1177/00224669070410030101

Soto-Johnson, H., Iiams, M., Oberg, T., Boschmans, B., & Hoffmeister, A. (2008). Promoting preservice elementary teachers' awareness of learning and teaching mathematics conceptually through “KTEM”. *School Science & Mathematics, 108*(8), 345-354. doi:10.1111/j.1949-8594.2008.tb17849.x

Stohlmann, M., Moore, T., Cramer, K., & Maiorca, C. (2015). Changing pre-service elementary Teachers’ beliefs about mathematical knowledge. *Mathematics Teacher Education and Development, 16*(2), 4-24. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1052599&site=eds-live&scope=site

Suinn, R. M., & Winston, E. H. (2003). The mathematics anxiety rating scale, a brief version: Psychometric data. *Psychological Reports, 92*(1), 167-173. doi:10.2466/PR0.92.1.167-173

Sunshine state standards (1998). Tallahassee, FL: Florida Dept. of Education, 1998. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=cat00846a&AN=ucfl.029816693&site=eds-live&scope=site

Tait, M. (2006). The influence of pre-service mathematics methods instruction on math anxiety and implementation of reform-based approaches and strategies. *Ontario Action Researcher, 9*(2) Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ814917&site=eds-live&scope=site; http://www.nipissingu.ca/oar/archive-Vol9No2.htm

Thrift, M.L., & Ortiz, E. (2007). Memorable menu math: The journey from teaching procedurally to teaching conceptually. *Teaching Children Mathematics, (1)*58.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of students' conceptions: The case of division of fractions. Journal for Research in Mathematics Education, 31(1), 5-25.

Tobias, S. (1978). *Overcoming math anxiety*. New York : Norton, c1978.

Tooke, D. J., & Lindstrom, L. C. (1998). Effectiveness of a mathematics methods course in reducing math anxiety of preservice elementary teachers. *School Science and Mathematics,* (3), 136. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=edsgao&AN=edsgcl.20748781&site=eds-live&scope=site

Tucker, M. S. (2011). *Surpassing Shanghai : An agenda for American education built on the world's leading systems* Cambridge, MA : Harvard Education Press, c2011.

Van Steenbrugge, H., Lesage, E., Valcke, M., & Desoete, A. (2014). Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge? *Journal of Curriculum Studies, 46*(1), 138-161. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1029411&site=eds-live&scope=site; http://dx.doi.org/10.1080/00220272.2013.839003

Verkijika, S.F., & DeWet, L. (2015). Using a brain-computer interface (BCI) in reducing math anxiety: Evidence from South Africa. *Computers & Education*, 113. Doi: 10.1016/j.compedu.2014.10.002

Webb, N. L. (2005). Webb Alignment Tool. Wisconsin Center of Educational Research. University of Wisconsin-Madison. http://www.wcer.wisc.edu/WAT/index.aspx.

Whittin, P., & Whittin, D. J. (2008). Learning to solve problems in the primary grades. Teaching Children Mathematics, 14(7), 426-32.

Wiener, R., Institute, A., & Council of Chief State,School Officers. (2013). *Teaching to the core: Integrating implementation of common core and teacher effectiveness policies.* ().Aspen Institute. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED542704&site=eds-live&scope=site

Wise, A. E., & Darling-Hammond, L. (1984). *Teachers' views of educational policies and teaching : Collected papers* Washington, D.C.] : Rand Corporation, 1984]. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=edsgpr&AN=gpr000214804&site=eds-live&scope=site; http://catalog.gpo.gov/fdlpdir/locate.jsp?ItemNumber=466-A-3&SYS=000214804

Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science (Sage Publications Inc.), 23*(5), 492-501. Retrieved from https://login.ezproxy.net.ucf.edu/login?auth=shibb&url=http://search.ebscohost.com/login.aspx?direct=true&db=sph&AN=76332204&site=eds-live&scope=site