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# RESOURCE ALLOCATION AND PRICING IN SECONDARY DYNAMIC SPECTRUM ACCESS NETWORKS

by

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A dissertation submitted in partial fulfillment of the requirements  
for the degree of Doctor of Philosophy  
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## ABSTRACT

The paradigm shift from static spectrum allocation to a dynamic one has opened many challenges that need to be addressed for the true vision of Dynamic Spectrum Access (DSA) to materialize. This dissertation proposes novel solutions that include: spectrum allocation, routing, and scheduling in DSA networks. First, we propose an auction-based spectrum allocation scheme in a multi-channel environment where secondary users (SUs) bid to buy channels from primary users (PUs) based on the signal to interference and noise ratio (SINR). The channels are allocated such that i) the SUs get their preferred channels, ii) channels are re-used, and iii) there is no interference. Then, we propose a double auction-based spectrum allocation technique by considering multiple bids from SUs and heterogeneity of channels. We use virtual grouping of conflict-free buyers to transform multi-unit bids to single-unit bids. For routing, we propose a market-based model where the PUs determine the optimal price based on the demand for bandwidth by the SUs. Routes are determined through a series of price evaluations between message senders and forwarders. Also, we consider auction-based routing for two cases where buyers can bid for only one channel or they could bid for a combination of non-substitutable channels. For a centralized DSA, we propose two scheduling algorithms— the first one focuses on maximizing the throughput and the second one focuses on fairness. We extend the scheduling algorithms to multi-channel environment. Expected throughput for every channel is computed by modelling channel state transitions

using a discrete-time Markov chain. The state transition probabilities are calculated which occur at the frame/slot boundaries. All proposed algorithms are validated using simulation experiments with different network settings and their performance are studied.

*To the soul of my dad*

*To my mom*

*To my brothers and sister*

*To my supportive family, friends, and teachers*

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## CHAPTER 1: INTRODUCTION

Radio spectrum is used as an important resource for wireless services (i.e., mobile telecommunication, TV and radio broadcasts, GPS, maritime navigation, tactical communications). Each service operates on specific portions of the spectrum bands which have been statically allocated for that purpose. For example, 824-849 MHz, 1.85-1.91 GHz, and 1.930-1.99 GHz frequency bands are reserved for licensed cellular and PCS services, whereas 902-928 MHz, 2.40-2.50 GHz, and 5.725-5.825 GHz frequency bands are free-for-all unlicensed bands [1]. In general, radio spectrum allocation and management have traditionally followed a ‘command-and-control’ approach where chunks of spectrum are allocated for specific services under restrictive licenses. The restrictions specify the technologies to be used and the services to be provided, thereby constraining the ability to make use of new technologies and the ability to redistribute the spectrum to higher valued users. In most countries, most of the radio frequencies have been allocated to specific uses and spectrum appears to be a scarce resource within the current regulatory framework. The Federal Communications Commission (FCC) which is an independent agency of the United States government regulates interstate communications by radio, television, wire, satellite, and cable services in all states.

There have been experimental studies that found spectrum utilization is typically time and space dependent and that most parts of radio spectrum are highly underutilized,

and there is a great amount of “white space” or unused bands [2, 3]. As a result, it is intuitive that static spectrum allocation is not the optimal solution toward efficient spectrum access and utilization.

These limitations along with the dis-proportionate and time-varying demand for spectrum have motivated a paradigm shift from static spectrum allocation towards a notion of dynamic spectrum management where secondary networks/users (non-license holders) can ‘borrow’ idle spectrum from primary networks/users (license holders), without causing harmful interference to the latter [4, 5]. This concept of dynamic allocation is referred to as the Dynamic Spectrum Access (DSA) [6, 7]. In DSA, spectrum will be allocated dynamically depending on need of the service providers which in turn depends on end users’ demands in a time and space variant manner [8, 9].

A typical DSA network consists of secondary users who coexist with a network of primary users (spectrum owners). A cognitive node consists of a sensor, a radio, a knowledge database, a learning engine, and a reasoning engine. A cognitive radio is aware of its environment, continuously learns from its surroundings, and adapts its operational parameters. The cognitive radios continuously monitor the presence of primary users and opportunistically access the unused or under-utilized licensed bands. The most important regulatory aspect of these networks is that the secondary nodes must not interfere with primary transmissions. In other words, when the allocated spectrum to a primary user is not fully used, the primary user has the option to sell the unused spectrum to secondary users. One of the open questions is how to stimulate the primary users to share their spectrum with secondary users?

Therefore economic incentives such as spectrum auctions have been used to motivate the primary users to sell parts of the spectrum that they do not need.

In most countries, the competitive behavior among service providers for spectrum was initiated by spectrum auctions held in 2000 and 2001 [10]. Selling spectrum to secondary users not only adds to the revenue of the primary user, it also increases spectrum utilization. Obviously, such spectrum trading includes spectrum selling and buying; thus a new economic model needs to be investigated. In spectrum trading, one of the open questions is how to determine the price for the spectrum being sold? The primaries need to determine how to set the price of their residual spectrum? Such optimal sale price evaluation must be done keeping in mind that i) spectrum is a commodity that cannot be stored for future sale and ii) there are other competing primaries who are also trying to sell their residual spectrum to others. Similarly, a secondary user must determine how much to pay to the primaries? Moreover, after the secondary users buy the spectrum, they must sell the acquired spectrum to others in the form of services. For example, a secondary user may offer to route packets on behalf of others for a certain price. Whenever there is an agreement on the price, a sale is made i.e., bandwidth is sold in the form of a service. In order for a secondary node to do so, it must first acquire some bandwidth from the primary users. The bandwidth acquired by the secondary users is then traded amongst each other as packets get forwarded from one node to another as the packet is routed from the source to the destination. These lead to a new challenge: how can the source (secondary node) choose the intermediate nodes in a route such that cost of the forwarding a packet is minimum and the intermediate nodes

have sufficient capacity to forward a packet? While the above mentioned question is a challenge by itself, what makes it even more difficult is the additional constraints posed by the primary users. Constraints such as the tolerable levels on the signal to interference and noise ratio (SINR) limit the transmission capabilities of secondary users and thus affects the routing path. In the recent past, there have been many propositions for auction-based channel/spectrum allocation; however most of them assume that all channels are identical, and therefore could be priced uniformly. In reality, channels are heterogeneous in the sense that they may vary in their characteristics such as SINR or the expected bit-rate between a transmitter and receiver pair. This would entail that some channels can be considered better than others and therefore can have varied performance outcomes. As a result, secondary users are expected to have a preference over the set of available channels. Moreover, secondary users might want to bid for multiple channels to satisfy their bandwidth requirement.

In the presence of multiple secondary users who try to operate on the same channel(s) that are not used by the primaries, there has to be some policy for them to access the channels. Such contention are resolved by the underlying medium access control (MAC) protocol, wherein the secondary users abide by some transmission policies. MAC protocols affect many network and radio functions such as channel sensing, resource allocation, and spectrum sharing [11]. It is worth mentioning that a channel can be used at multiple locations simultaneously as long as the secondary users using the common channel do not interfere with each other i.e., they are sufficiently apart. To achieve such simultaneous communications, the questions that need to be answered are: what is the best representation of the interference

constraints and what is the most efficient way to schedule the available channel(s) among the secondary users?

### 1.1 Contributions of this Work

This dissertation addresses some of the fundamental challenges that DSA networks face today. We not only propose novel ways for DSA networks to perform better, we quantify the expected performance for a given network setting. Mainly, we focus on channel allocation, spectrum trading, routing, and scheduling in DSA networks.

We propose an auction based spectrum allocation scheme considering the fact that all channels do not offer the same value to a secondary user. The secondary users realize that due to the heterogeneity of the channels, the utility they could get in terms of capacity is different. Hence, we use signal to noise and interference ratio (SINR) as the quality indicator for a channel. Each secondary user defines his SINR based preference of channels from the available ones. These channels sorted in a descending order of preference comprise the preference list. Though these available channels do not interfere with the PUs, there could be interference if two or more secondary users are allocated the same channel. A secondary user places multiple bids for the different channels more the preferred ones and less for the non-preferred ones. The auction is initiated considering the preference list of any secondary user and if there is no conflict with the most preferred channel, then that channel is allocated to the secondary user. If there is a conflict, the bids from the conflicting secondary users

are compared, and the secondary user with the highest bid is allocated that channel. The process continues with the second most preferred channel if the secondary user does not get the most preferred one. If the secondary user gets the most preferred one, the next secondary user is considered and the same process is repeated. This way, the auction tries to maximize the allocation of the preferred channels. To measure the fairness of the allocation process, we compute the achievable bit rate for an allocated channel using Shannon's theory. In order to evaluate the effectiveness of the proposed auction, we conduct simulation experiments for different network scenarios and show to what extent channel preferences are met and what is the corresponding revenue.

Next, we propose PreDA- a Preference based-truthful Double Auction which is a truthful double auction for DSA networks where the buyers have a preference for the channels being put up for auction by the primaries. In PreDA, multiple sellers and multiple buyers participate in the auction that can last for more than one round. A buyer can bid for multiple bands. Considering the fact that all channels do not offer the same value to a buyer, the utility they could get in terms of capacity is different. Hence, we use SINR as the quality indicator for a channel. Each buyer defines his SINR based preferences for the channels from the available ones. These channels sorted in a descending order of preference comprise the preference list. The bids are non-increasing as we go down the channels in the preference list. A bid from a buyer is of the form of a vector that consists of its demand, preference list, and bid list. We use the interference conflict graph to identify sets of users who do not interfere with each other, and hence can transmit on the same channel and at

the same time. Virtual groups are formed with non-conflicting users. Such groups also help translate the multi-unit bids to an equivalent auction with single-unit bids. To the best of our knowledge, PreDA is the first truthful multi-unit double auction that guarantees spectrum reuse by allowing buyers to express their preferences for heterogeneous channels. PreDA achieves the three significant economic properties of truthfulness, individual rationality, and ex-post budget balance. We implement three bid-independent grouping algorithms: i) MAX-SINR, ii) MAX-Degree which maximizes number of buyer groups, and iii) MIN-Degree which minimizes number of buyer groups. Simulation results show that PreDA allocates more bands during the first round, which means most buyers get their most preferred channel. Results also show spectrum re-usability, profit maximization, and fairness.

We address the problem of routing from an economic perspective in a DSA networks. We consider that secondary nodes, scattered in a random fashion, acquire spectrum from the primary users which they then sell to others in the form of services. Thus, the secondary nodes in this network buy and sell bandwidth in order to forward packets for the purpose of routing. On behalf of the seller, we determine the price for per unit of bandwidth. To that end, we propose the selling price as a decreasing *log* function of the residual bandwidth and its cost price. We also compute the interference a seller is exposed to. As for the buyer, in order to achieve a target bit rate, we first back calculate the bandwidth based on the SINR for each seller. The price per hop is computed based on the bandwidth required and unit price for that seller. The total cost of a route is the sum of the prices paid at each hop between the source and the destination. We conduct exhaustive route search techniques and

show how the route price varies for various routes. The final route path is the route that has the minimum total cost.

Then we model the selection of the intermediate nodes in a route using a sealed-bid auction mechanism where the transmitter explores the cost of forwarding a packet by all potential receivers i.e., the forwarders. Message routes are determined through a series of auctions between message senders and forwarders. Similarly, a receiver must determine the optimal bid for it to route packets on behalf of other nodes for a certain price. Whenever there is an agreement by the transmitter (buyer) and the receiver (seller), a sale is made i.e., bandwidth is sold in the form of a (forwarding) service. For computing the bids by the sellers, we find the number of channels a transmitter-receiver pair must use to sustain the bit rate requirement of a given flow. Of course, the data rate on a channel is dependent on the SINR the receiver is exposed to on that channel, and is found using Shannon's law. Thus, the channels are non-substitutable. As for the buyer, the receiver is chosen such that i) it is able to sustain the minimum bit rate requirement, and ii) the price to be paid for the channel(s) is minimum. We consider two cases: i) the seller bids for only one that satisfies the required bit rate and has the minimum price, ii) the seller bids for multiple channels. For second case, all combinatorial options are explored. As finding such sets of channels are NP-hard, we propose a greedy heuristic that finds a reasonable set of channels in  $O(n \log n)$ . To validate the proposed routing scheme, we conduct simulation experiments where we implement route search techniques and show how the route price varies. We also show how the proposed heuristic performs in comparison to the exhaustive search.

Most often, multiple secondary users contend to acquire the bands that are available. Proper scheduling techniques can resolve such contentions. Thus, we propose two slot-based scheduling algorithms. The main goal of the first algorithm is to allocate a channel to the users such that the overall throughput is maximized. We use the SINR between a transmitter-receiver pair to compute the expected throughput on the potential channel which is then used by the scheduler to assign the appropriate channel. Though such SINR-based scheduling schemes maximizes the system throughput, it is unfavorable to users who experience deteriorated SINR for long periods of time. To overcome this drawback, we propose the second algorithm— the main goal of which is to maintain some level of fairness. We use the number of already allocated slots as a (history) index to assign channels. In order to increase the spectral efficiency, we re-use the channel between the slots and also between the users in the same slots that do not have any conflict. Such non-conflicting users are found using the concept of Independent Sets (IS) in the conflict graph. Using simulation experiments, we show the trade-offs of the two schemes and mention how the two could be combined to achieve a desired objective.

Then we go a step further, and propose a queue aware frame-by-frame and slot-by-slot scheduling algorithms for a multi-channel DSA network. Unlike traditional opportunistic scheduling algorithms (which try to maximize the instantaneous throughput), we not only consider the SINR between a transmitter-receiver pair but also consider the number of packets waiting in the queue of the transmitter. That way, we make sure that the secondary users make use of the channels to the best possible extent. This also avoids allocation to

non-backlogged users. We calculate the expected throughput for each channel and for each time slot in a super-frame. We do so by considering the state transitions of the primary channel occupancy, channel quality, and the queue status. The state transitions are modeled using a Discrete-Time Markov Chain (DTMC) process. In order to have collision-free transmissions, we use the interference conflict graph and identify the sets of users who do not interfere with each other, and hence can transmit on the same channel and at the same time (slot). Each set of users is assigned one or more channels *and* one or more time slots based on which set maximizes the overall system throughput. Through extensive simulation experiments, we show how the proposed frame-by-frame and slot-by-slot scheduling perform with respect to many factors.

## 1.2 Organization of the Dissertation

The dissertation is organized as follows. Chapter 2 presents the related work that are relevant to this dissertation. Chapter 3 discusses the basics of auction theory, along with single and double auctions for channel allocation. Economic routing models are presented in chapter 4. In chapter 5, the single channel and multi-channel opportunistic scheduling algorithms are discussed. Chapter 6 presents the simulation model and results. Conclusions are drawn in Chapter 7.

## CHAPTER 2: RELATED WORK

In this chapter, we briefly discuss the relevant works that have been done till date.

### 2.1 Prior Research on Spectrum Allocation

In recent years, there have been many works that have used economic models to study market-like scenarios where spectrum is traded like a commodity considering the temporal and spatial aspects. A general framework for TRuthful doUble Spectrum aucTions called TRUST has been introduced in [12]. TRUST is the first truthful double auction which takes any re-usability-driven spectrum allocation algorithm as the input and applies a novel winner determination and pricing mechanism to determine winning sellers and buyers. TRUST achieves the three economic properties: truthfulness, individual rationality, and ex-post budget balance. TRUST follows the McAfee model [13], which guarantees the three economics properties but does not consider the spectrum re-use. However, TRUST is a single channel auction and assumes that the interference conditions of a buyer are known to the auctioneer. In [14] a DOuble Truthful Auction for dynamic spectrum access (DOTA) was proposed. DOTA provided more flexible spectrum bidding than TRUST by allowing both buyer and seller to request/sell multi-channel. DOTA acheives that by translating multi-unit bids into

an equivalent single-unit bids, and by re-designing the clearing and pricing rules. However, DOTA did not consider the channels heterogeneity.

In [15], the first double auction design for spectrum allocation for single band that explicitly decouples the buyer side and seller side auction design was proposed. The main goal is to achieve truthfulness and budget-balancing. By using the concept of graph partitioning and subgraphs, it reduces the randomness of independent set construction. Moreover, the partitioning provides the ability to compute prices in each subgraph independently, which achieves higher revenue. Finally, a merge strategy combines the auction results. The overall performance is better than TRUST, but it consumes more processing time for partitioning and merging strategies. A double auction called PROMISE was proposed in [16] considers the economic properties, and re-usability. The main feature of PROMISE is profit maximizing without requiring the knowledge of the valuation which uses a technique called cross extraction to compute the group's bid. PROMISE also handles single band with no consideration of heterogeneity. A framework for TRUthful double Multi-Channel Spectrum Auctions (True-MCSA) was proposed in [17], where each seller and buyer have the ability to ask/bid for arbitrary number of channels. Using the virtual buyer group (VBG) concept, splitting and bidding are applied to determine the winners. However, all channels are assumed to be homogeneous and available to all the buyers. While all the above techniques are designed for global markets, a double auction for local markets with and without the knowledge of bid distributions was proposed in [18]. The study in [19] used a portfolio optimization framework to solve the trading decision making problem.

For an online environment, a general framework called Truthful Online Double Auction for Spectrum Allocation (TODA) was proposed in [20]. A complete graph is used as the conflict model among secondary users. Though a complete graph makes the auction design easier, it does not mimic practical scenarios and spectrum reuse. Moreover, TODA only considers homogeneous spectrum. Research in [21] considered the location of the buyers in the auction process and proposed a Location-aware Online Truthful doUble auction Scheme (LOTUS). It introduced the concept of “interference discount”, and uses it to markdown a bid with a wide range of interference. The auctioneer considered all buyers as long as their discounted bid is greater than their opportunity cost. The simulation results show that LOTUS improved system utility and spectrum utilization. In [22] a Truthful double Auction scheme for HEterogeneous Spectrum (TAHES) was proposed that only considers only a single channel. TAHES allowed buyers to explicitly express their preferences for heterogeneous spectrum and achieved the economic properties. Multiple channels with heterogeneity is considered in TDAMH [23]. Though TDAMH is the most complete truthful double auction, it does not consider the preferences for the channels from the buyers’ perspective.

In Table 2.1, we compare the most popular double auctions for DSA networks with respect to the *all three* economic properties, single/multi channel, channel heterogeneity, and buyers’ preferences for channels. Our proposed double auction, PreDA, considers all that TDAMH considers but also incorporates the buyers’ preferences for the channels. Hence, we compare the performance PreDA to that of TDAMH using simulation experiments discussed later.

Table 2.1 Comparison of existing double auctions.

Double Auctions	Truthfulness	Individual Rationality	Ex-post Budget Balance	Multi Channel	Channel Heterogeneity	Channel Preferences
TRUST [12]	✓	✓	✓	×	×	×
DOTA [14]	✓	✓	✓	✓	×	×
PROMISE [16]	✓	✓	✓	×	×	×
AUCTION [15]	✓	✓	✓	×	×	×
TRUE-MCSA [17]	✓	✓	✓	✓	×	×
TODA [20]	✓	✓	✓	×	×	×
LOUTS [24]	✓	✓	✓	✓	✓	×
TAHES [22]	✓	✓	✓	×	✓	✓
TDAMH [23]	✓	✓	✓	✓	✓	×
PreDA	✓	✓	✓	✓	✓	✓

## 2.2 Prior Research on Spectrum Pricing

The research in [25] addresses the problem of spectrum pricing in cognitive radio network using a game theoretic model. The competition is formulated as an oligopoly market in which the firms adjust their prices dynamically to gain the maximum profit. For the primary user, the cost of sharing the spectrum with the secondary user has been calculated as a function of the QoS performance degradation of the primary connections. A utility function is used to obtain the spectrum demand function for the secondary users. The research analyzed the problem as a Bertrand game and obtained the Nash equilibrium to reach the optimal pricing.

Research in [26] analyzes the optimal price setting for the service provider by examining the balance between the subscribers and the secondary buyers in a content-redistribution

network. The behavior between the subscribers and the secondary has been modeled as a hybrid Stackelberg auction game. This kind of analysis can help the service providers reserve their profit and improve the quality of service for end users.

Research in [27] presented two approaches to solve the interference and reliability problems: partitioning approach and conflict graph approach. The proposed dynamic spectrum auction considers both space and time properties of the spectrum. A conflict graph based on measurement-calibrated propagation models was presented in [28], which removed the need for exhaustive signal measurements by interpolating signal strengths using calibrated models. The results show that those estimated conflict graphs improve reliability by reducing the impact of accumulative interference at the cost of some spectrum utilization loss. A real time spectrum auction framework to maximize the auction revenue and spectrum utilization, with conflict-free spectrum allocations was proposed in [29]. It introduced a bidding language, an auction algorithm, and pricing models to control trade-offs between revenue and fairness. It revealed that in order to maximize revenue and spectrum utilization, pricing must be calculated based on local demand and availability of the resources. A novel distributed collusion mechanism to allocate channels in the spectrum pool with graph coloring and bidding theory has been proposed in [30]. The overall utility of both primary and secondary users have been measured via a simulations which showed that the proposed scheme has the overall performance similar to the optimal one. The concept of spectrum micro-trading to enable trading of spectrum on the micro-scale has been discussed in [31]. The study focused on three dimensions: the micro-spatial, micro-temporal,

and micro-frequency scales. The results show that the spectrum utilization can be greatly improved. An auction-based mechanism multi-auctioneer progressive auction for dynamic spectrum access was proposed in [26] where each primary user systematically raises the trading prices and each secondary user consequently decides whether to buy a spectrum band or not. The equilibrium is defined as the state that no seller and bidder would like to deviate from their decision. The results show that the performance is arbitrary close to the global optimal. However, the concept of the spectrum reuse was not considered. Two auctions for mobility and interference support was designed in [32] using two-dimensional bids. The auctions take the and allocate the channel exclusively and non-exclusively, for the single-channel and multiple-channel respectively. As a result, a channel is either exploited or simultaneously reused without interference. Auctions have also been used for routing [33] and scheduling [34] in cognitive radio networks.

### **2.3 Prior Research on Economic Models for Routing**

A unicast routing protocol with selfish nodes has been introduced in [35], where all nodes are considered rational as they always choose a strategy that maximizes their benefit. Each node declares a cost for forwarding a unit of data. As a result, when a node sends data to the destination, it does two main operations: computes the least cost path to the destination, and then computes a fee for each node on this path. This pricing scheme shows that the

profit of each node is maximized when it declares its true cost. Declaration of ‘true’ costs has been addressed in [36].

As far as routing in mobile ad hoc networks (MANET) are concerned, each node has its own authority, so unconditionally forwarding packets for other nodes, cannot be guaranteed. In order to induce nodes to forwarding packets, incentives must be provided. Such a pricing mechanism in the form of dynamic games for routing in MANETs has been presented in [37]. The authors in [38] develop a route selection algorithm with minimum payment, and at the same time guarantees truthful cost reporting by secondary users. This problem has been formulated as finding the least-priced path without and with link capacity constraints. Two algorithms have been proposed: a polynomial-time algorithm to find the truthful low-priced path that efficiently calculates its truthful price, and a payment materialization algorithm to guarantee truthful capacity reporting by secondary users.

In [39], a pricing based routing scheme is proposed that stimulates cooperation among users so that they forward each others packets. Through an iterative price adaptation algorithm, spectrum is allocated for the route selected. A session-based spectrum trading mechanism based on the cross-layer optimization was proposed in [40]. A secondary service provider (SSP) selects the paths considering various parameters including the price of bands, primary activity, flow routing, and link scheduling. A routing algorithm based on spectrum trading and spectrum competition that considers the different QoS levels for unlicensed users is proposed in [41]. Routes are decided based on factors such as user profiles, number of hops, channel identification, neighbor identification, probabilities of idle slots and primary activity.

Though the price for spectrum trade was computed based on what a secondary wants, it does not consider the price already paid to acquire the spectrum from primaries.

An auction-based route allocation (ARA) scheme for routing protocol in mobile ad-hoc networks (MANETs) has been proposed in [42] that prolongs the lifetime of MANETs. In ARA, message routes are determined by auctions between message transmitters and forwarders. The senders have the ability to adjust the route based on the auctions conducted at nodes along the route. Though ARA can prolong the network lifetime, there are unresolved issues, such as bidding strategies and their influences on the ARA. An auction-based incentive scheme called Incentive compatible Auction-based Service Scheme (iPass) to increase cooperation for packet forwarding in MANETs has been proposed in [24]. Each router runs an auction to determine which node should obtain how much bandwidth and at what price. The set of the traffic passing through router are the bidders. Each flow carries a bid representing its willingness to pay to get the resource. The method is based on the generalized Vickrey auction with reserve pricing. iPass has shown to be fair and efficient. The work in [43] studied auction based schemes for multi-path routing in selfish networks. A general model known as optimal auction-based multi-path routing (OAMR) has been developed based on the Vickrey-Clarke-Groves (VCG) auction. Also, a sequential auction-based multi path routing (SAMR) has been proposed. SAMR has different sequencing strategies than OAMR, which reduce the computational time and can run in real-time. Moreover, an algorithm has been applied that assigns the traffic of a request among its available paths and determines the payment. Results show that OAMR or SAMR can reach the destination

node with lower payment-cost ratio, compared to other schemes. For multiple requests with selfish nodes, OAMR can guarantee truthfulness and efficiency, while SAMR is truth-telling and efficient only for the case of a single request.

A combination of both auction and game theoretic based routing algorithm was presented in [44]. Two different ways of bidding have been considered: a modified first and a second-price sealed-bid. Both mechanisms have been proved to work well in terms of general revenue and overall system efficiency. The best route in the proposed algorithm is based on three parameters: total price, amount of compensation, and power consumption. A cross-layer opportunistic spectrum access and dynamic routing algorithm, called ROSA, for cognitive radio networks has been proposed in [45]. The main goal of ROSA is to maximize the throughput and allocate resources efficiently in a cross-layered fashion, by jointly addressing routing, dynamic spectrum allocation, scheduling, and transmit power control. The algorithm maximizes the capacity of links without generating harmful interference to other users while guaranteeing a bounded bit error rate for the receiver. Also, the algorithm gives priority to higher capacity links with a high differential backlog to stabilize the system. ROSA has been shown to outperform simpler solutions for inelastic traffic.

## 2.4 Prior Research on Opportunistic Scheduling

Opportunistic scheduling is a class of scheduling schemes where the physical layer properties (i.e., the wireless channel characteristics) are exploited. For example, in a time division

system, the user with the best channel condition is selected for transmission. Opportunistic scheduling strategy exploits multiuser diversity and maximizes the overall throughput, which might be unfair to users with low long-term SINR. To strike a balance between throughput and fairness, proportional fair (PF) scheduling has been proposed in [46]. Many other scheduling policies have been proposed and analyzed (see [47] and references therein). Most of these scheduling policies are sensitive to flow-level dynamics. For example, in a PF scheduler, a user is selected based on his current average throughput [46].

In [48], two optimal schedulers with exponential complexity were proposed. The first one maximized the network throughput while the second one minimized scheduling delay. Then two suboptimal schedulers, referred to as maximum frequency selection (MFS) and probabilistic frequency selection (PFS), were proposed. A queue-aware frame-based opportunistic spectrum scheduling scheme was proposed in [49]. A user uses its queue status and channel conditions to calculate its expected throughput for each channel over the frame. The main goal of the proposed scheduling scheme was to maximize the overall throughput. As a result, the channels are assigned to the users with the highest expected throughput. The proposed scheme tried to handle the fast changing channel conditions by using the queues. However, this technique increased the overhead of the scheduling process since it is slot-based scheduling. Also, there is no re-use of the channel in the frame. Study in [50] also proposed a queuing framework to study the performance of opportunistic spectrum access by cognitive radio users. However, it did not consider the mobility of cognitive radio users across network boundaries. Also, there is no sharing for the frequency. Multi-channel

time slot assignment problem in cognitive radio sensor networks was studied in [51] as an integer linear program where the goal was to minimize the number of slots in the schedule and increase the network throughput offered to sensor nodes. An Opportunistic centralized Time slot assignment in COgnitive Radio sensor networks (OTICOR) was proposed in [52] which also used integer linear programming formulation for the MAC-layer scheduling problem and introduced an optimal schedule. Then a distributed heuristic scheduling schema was proposed which showed very close results to the optimal one in terms of schedule length. However, the results were valid for a small network and did not involve mobility of the nodes. A spectrum allocation algorithm for heterogeneous cognitive radio networks was proposed in [53]. However, the study was done under imperfect sensing, where the sensing time had no effect on the system performance.

Study in [54] proposed a Distributed TDMA based MAC protocol (DTMDD) for data dissemination in multi-hop ad-hoc cognitive radio networks. The objectives of this study were ensuring message reachability and avoiding collision among secondary users. A SU selected the best channel based on an intelligent selection strategy. In [55], a scheduling problem called Maximizing the Number of Satisfied Users (MNSU) was proposed to maximize throughput. Two heuristics were proposed: 1) best first resource assignment and 2) resource assignment with partial backtracking. However, this study focused on centralized cognitive radio network and not a distributed one. Research in [56] provided an opportunistic channel access scheme with channel ordering and periodic sensing. The SUs first select the  $k$  best primary channels out of  $N$ . They sense these  $k$  channels for transmission opportunity,

and then probes the idle channel for channel quality measurement. However, the reward value, which is the average throughput, is calculated based on the values of threshold and transmission period only.

Various overlay spectrum access schemes in cooperative cognitive radio networks were reviewed in [57]. In the overlay techniques, secondary users can access the resources only when primary users are not using them. Similarly, [58] provided a survey of underlay techniques for resource allocation in cognitive radio networks. The underlay model allows both primary and secondary users to access the spectrum simultaneously. Moreover, switching between the overlay and underlay transmission modes is possible. A hybrid overlay-underlay model was proposed in [59].

## CHAPTER 3: AUCTION-BASED CHANNEL ALLOCATION IN DSA NETWORKS

In this chapter, we briefly deliberate the theories that are used in this research. Then discuss auction-based channel allocation techniques.

### 3.1 Basics of Auction Theory

Auction theory is an applied branch of economics which deals with how sellers and buyers trade goods in a market. Oftentimes, the price of the good being traded is uncertain and putting up such goods for an auction reveals the true value of the good. Auction design considers the efficiency, optimal and equilibrium bidding strategies, and revenue. Generally, there are many types of auctions [60]:

- Auctions can be single dimensional or multi-dimensional. In a single dimensional auction, the bidding process focuses on the price offered for a good, while in the multi-dimensional auction, the bidding process considers other things such as quality and timeliness.
- Auctions can be one-sided or two-sided. In one-sided auction, bidders are either buyers or sellers and the main task of the auctioneer is to decide the winning bid. In two-sided

auction both buyers and sellers submit bids and the job of the auctioneer is to match buyers to sellers.

- Auctions can be open-cry or sealed bid. In the open-cry auction every bidder has an ability to hear/know other bids while in the sealed bid only the auctioneer has this right.
- Auctions can be first price or  $k$ th price. In the first price auction, the winner pays its winning bid, while in the  $k$ th price the winner pays the price of the bid that is ranked  $k$ th. The second price auction is a famous example of the  $k$ th price auction, where the winner pays the price bid by the second-highest bidder.
- Auctions can be single-unit or multi-unit of the same commodity. In the single-unit auction, the bidder is allowed to bid for only a single good, either a single item or multiple items collected together. In the multi-unit auction the bidder has an ability to bid for several units together.
- Auctions can be single-item or multi-item (i.e., combinatorial auction). In a combinatorial auction, multiple heterogeneous goods are auctioned at the same time. Also bidders have the option to bid for any combinations of goods, which gives a bidder the chance to check for various bundles of items.
- English auction: It is single dimensional, one-sided and usually sell-side, open-cry, and first price auction. Additionally, the bids are made in ascending order, and the seller might set a reserve price where no sale is made below this price [60, 61].

- Dutch auction: It is a single-dimensional, one sided, open-cry, first price auction. There could be a reserve price. The main difference between Dutch and English auctions is that the bids are descending rather than ascending [62].
- Sealed first-price auction. Bidders have to submit their private, sealed, bids before a deadline. Then auctioneer opens the sealed bids and determines the winner. Normally the winner is a bidder with the highest bid, and the winner pays either the first or the  $k$ th price. The sealed-bid second-price auction is called Vickrey auction. The main advantage of the Vickrey auctions is solving the collusion problem. In this kind of auction, bidders try their best to bid the best estimate of the value of the good because they will pay the second highest bid.

English, Dutch, first-price sealed bid, and second price sealed bid have been shown to generate the same expected revenue using the revenue equivalence theorem (RET) [63].

### 3.2 Single Auction-Based Channel Allocation

We consider a dynamic spectrum access network with multiple secondary users and multiple primary users, randomly scattered over an area. Each primary user owns a spectrum band which represents a channel. We assume that the bandwidth of each such band is identical (e.g., all TV bands in the US are 6 MHz each). However, the price paid by the primary user to acquire their respective band is different which we assume to be a private information known only to the primary buyer.

As far as the secondary users are concerned, they buy the bands from the primaries. However, the quality of the bands varies due to the spatial and temporal reasons— and as a result, a secondary user will be willing to pay more for a better quality channel. A secondary user can easily sort the available channels based on their preference. We use SINR as a metric to determine the goodness of a channel. Thus, each secondary user will have a preference list of channels sorted in a descending order such that preference for channel  $l$  is higher than or equal to the preference for channel  $k$ , for  $l < k$

Secondary user  $i$ , represented by  $SU_i$ , determines a price that he is willing to pay for channel  $j$ , which is uniquely owned by primary user  $j$ , represented by  $PU_j$ . Let that price be denoted by  $p_i^j$ . The prices are also sorted in the descending order such that the highest price is for the most preferred channel and so on. Thus, we have *multi-bids* from the same SU.

We model the spectrum allocation process as a multi-bid auction where PUs are the sellers and SUs are the buyers placing multiple bids. Typically, the auctioneer who controls the auction tries to maximize the revenue. However, other aspects like social welfare might be taken into consideration which might not maximize the revenue.

The utility of  $SU_i$ , represented by  $U_i^s$ , can be calculated as:

$$U_i^s = \begin{cases} v_i^j - p_i^* & \text{if } SU_i \text{ is a winner;} \\ 0 & \text{otherwise.} \end{cases} \quad (3.1)$$

where  $v_i^j$  is the valuation of channel  $j$  by  $SU_i$ , i.e., the maximum price  $SU_i$  is willing to pay for channel  $j$ , and  $p_i^*$  is the bid. The SUs that are able to eventually buy a channel is said to be a winner.

Similarly, the utility of the  $j^{th}$  primary seller, represented by  $U_j^p$ , is defined as:

$$U_j^p = \begin{cases} p_i^* & \text{if } PU_j \text{ is able to sell;} \\ 0 & \text{otherwise.} \end{cases} \quad (3.2)$$

where  $p_i^*$  is the price that the winning SU paid to  $PU_j$  for channel  $j$ . The utility of PU is the summation of all price paid from SU to allocate a channel.

SUs interfere with each other if they use the same channel and are in close proximity of each other. Thus, we use an *interference graph* that gives the conflict of every SU. In the graph, an edge between nodes  $i$  and  $j$  means that they are close to each other and therefore cannot use the same channel at the same time. However, a specific channel can be sold to (and consequently used by) multiple SU simultaneously as long as they do not interfere with each other. Such simultaneous re-use of channels increases the i) spatial utilization of the spectrum and ii) revenue for the primaries. Next, we explain how to calculate the interference and preference list.

### 3.2.1 SINR and Preference List

Each secondary user computes its preference list based on SINR on each channel. The interference experienced by a secondary user on a particular channel is different due to the other users at varying distances who are also using that channel. There are two types of users who are responsible for creating the interferences– the primary and the secondaries. Note, for any channel, there could be only one primary but multiple secondaries that use the same channel.

The interference between secondary user  $i$  and primary user  $j$  on channel  $j$  is calculated as:

$$I_{i,j}^p = \frac{P_p}{d_{i,j}^\alpha} \quad (3.3)$$

where  $P_p$  is the transmit power from the primary  $j$ ,  $d_{i,j}$  is the distance between users  $i$  and  $j$ , and  $\alpha$  is the path loss exponent. We assume all primaries transmit with power  $P_p$ .

The interference between secondary user  $i$  and all other secondary users on channel  $j$  is calculated as:

$$I_{i,j}^s = \sum_{n \in N_i} \frac{P_s}{d_{i,n}^\alpha} \quad (3.4)$$

where  $n \in N_i$  are the neighbors of node  $i$  using channel  $j$ .  $P_s$  is the transmit power from any secondary, and  $d_{i,n}$  is the distance between nodes  $i$  and  $n$ .

As a result, interference perceived at secondary node  $i$  on channel  $j$  from all interfering sources can be calculated as:

$$I_{i,j} = I_{i,j}^p + I_{i,j}^s \quad (3.5)$$

With the interference known, we find the *SINR* for secondary node  $i$  on channel  $j$  as:

$$SINR_{i,j} = \frac{\frac{P_s}{d_{i,t}^\alpha}}{n_0 + I_{i,j}} \quad (3.6)$$

where  $n_0$  is the background noise, and  $d_{i,t}$  is the distance between receiver  $i$  and transmitter  $t$ .

The SINR on all the channels for secondary node  $i$  can be written in the form of a vector:

$$SINR_i = [SINR_{i,1}, SINR_{i,2}, \dots, SINR_{i,K}]$$

where  $K$  is the total number of channels.

The secondary node  $i$  sorts the SINR vector in descending order and uses it as a *preference list* for the channel(s) to be bought.

### 3.2.2 Multi-bid Auction

With the preferences for the channels known, the SUs can bid for the channels. Obviously, the SUs will be willing to pay more for a preferred channel than a non-preferred channel—thus, the bids will be non-increasing for that SU's preference list i.e., the highest bid will be for the most preferred channel and so on.

As for the auctioneer, the first step is to find the interference conflict graph such that the same channel is not allocated to two SUs that interfere with each other. The auctioneer

starts with the preference list of any secondary user  $i$ , which is actually a tuple containing the channel number and the corresponding bid. It checks if the most preferred channel of user  $i$  is also the most preferred channel of any of his neighbor(s). If there is a conflict, the corresponding bids are compared, and the user that has the highest bid is allocated that channel. If user  $i$  wins his most preferred channel, then the auctioneer moves to another user and repeats the process. If user  $i$  does not get the most preferred channel (i.e., it loses to one of his neighbors) then the second most preferred channel is considered for allocation. Again, the bids for that channel is compared from the neighbors and the one with the highest bid gets the channel. As the process continues, node  $i$  might win one of the channels from its preferred list, or it might not win any and remain channel-less.

The allocation of channels via this auction process might result in i) a channel not being allocated to any user, ii) a user not getting any channel, and iii) a channel being shared by multiple non-conflicting users. Here the objective of the auction is not revenue maximization but satisfaction of the users in acquiring their preferred channels.

It is to be noted that most auctions try to maximize the revenue. Likewise, the primaries would like to adopt auction strategies that would maximize their revenue. However, there might be other constraints imposed by the regulatory authority. For example, to induce a competitive market, it is desired that there are at least a certain number of wireless providers in a geographical area so as to have an oligopoly market. Thus, apart from revenue maximization, other aspects are also taken into consideration, which in our case is the preference for bands.

### 3.2.2.1 Fairness

In order to determine whether the SUs are receiving a fair share of the spectrum bands, we used a popular metric– the Jain’s fairness index. First, we compute the achievable bit rate for each SU using Shannon’s capacity:

$$b_i = B * \log(1 + SINR_{i,j}) \quad (3.7)$$

where  $b_i$  is the achievable bit rate for SU  $i$ ,  $B$  is the bandwidth of a channel and,  $SINR_{i,j}$  is the interference for SU  $i$  on channel  $j$ . The bit rate for the whole system can be found by adding the individual bit rates for all  $n$  SUs, i.e.,  $\sum_{i=1}^n b_i$ .

The Jain’s fairness index is obtained as:

$$\mathcal{J}(x_1, x_2, \dots, x_n) = \frac{(\sum_{i=1}^n b_i)^2}{n \cdot \sum_{i=1}^n b_i^2} \quad (3.8)$$

The index varies from  $\frac{1}{n}$  (worst case) to 1 (best case) and it is maximum when all SUs receive the same allocation.

### 3.2.2.2 An Illustrative Example

In Fig. 3.1 we show the conflict graph of a simple network with 6 nodes labelled  $A$  through  $F$ . We consider that there are 8 primaries and hence 8 channels numbered from 1 through

8. For each node, the preference list (shown as ‘PL’ in the figure) of channels and their corresponding bids are also shown. Note, channels 1 and 5 are not in any PL as they conflict with all. Also, the number of channels in the PL varies from node to node.

Suppose the auctioneer starts with node B. The most preferred channel is 6– which happens to be the most preferred channel of nodes A and F as well. Since the bid from node B (i.e., 89) is not the highest among its neighbors, it does not get channel 6. Then its second most preferred channel is considered (i.e., channel 2). Channel 2 is not the most preferred channel for B’s neighbors. As a result, there is no conflict, and channel 2 is allocated to node B. Then the auctioneer moves to the next node, let’s say node A. The most preferred channel for node A is channel 6, which happens to be the most preferred channel of node F as well. Again the bid from node A for channel 6 (i.e., 89) is less than its neighbor F. Then the auctioneer checks the second most preferred channel (i.e., channel 2). Channel 2 has already been allocated to node B. Then the auctioneer checks the third most preferred channel (i.e., channel 8). Node A is allocated channel 8, since there is no conflict among its neighbors. Then the auctioneer moves to the node F. Node F with the highest bid for channel 6 is allocated channel 6. Then the auctioneer moves to the next node, let’s say node C. The most preferred channel is 8, which happens to be the most preferred channel of nodes E and D as well. Since the bid from node C (i.e., 92) is not the highest among its neighbors, it does not get channel 8 or any other channels because there is only one channel in the preference list. Next node is node E with most preferred channel 8, which happens to be the most preferred channel of nodes C and D as well. Since the bid from node E (i.e.,

83) is not the highest among its neighbors, it does not get channel 8, then its second most preferred channel is considered (i.e., channel 7). Node E is allocated channel 7 since there is no conflicts among its neighbors. Node D with the highest bid for channel 8 is allocated channel 8. The outcome of the auction is shown in Table 3.1.

Table 3.1 Auction Results

Node	A	B	C	D	E	F
Channel	8	2	-	8	7	6
Bit Rate	6.598	6.402	0	6.456	5.891	6.679

It can be seen from the allocation that there could a situation where a node does not get any channel e.g., node C. On the other hand, there could be spatial reuse of a channel e.g., channel 8 used by non-conflicting nodes A and D. The Jain's fairness index for this allocation is 0.83.

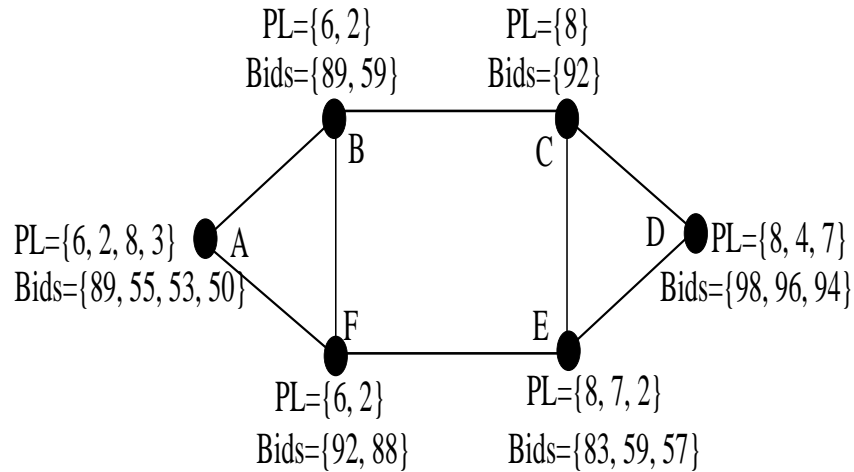


Figure 3.1 Conflict graph with preference list (PL) and bids

### 3.3 Double Auction-Based Channel Allocation

Double auction is different from single auction— here, rational sellers also participate in the auction process. Generally, there are multiple sellers and multiple buyers. In the first stage, sellers submit their ‘asks’ to an auctioneer who at the same time receives ‘bids’ from the buyers. The auctioneer generates market-clearing prices based on all bids, then determines the winners. However, the Impossibility Theorem [64] illustrates that no double auction can *simultaneously* achieve all three economic properties: i) truthfulness, ii) individual rationality, and iii) budget balance while maximizing auction efficiency. As a result, we focus on satisfying these economic properties first, and then try to maximize the efficiency. We propose PreDA— a preference-based truthful double auction for dynamic spectrum access (DSA) networks. We consider a dynamic spectrum access network with  $M$  primary users (i.e., the sellers) and  $N$  secondary users (i.e., the buyers) randomly scattered over an area. Each primary user owns a channel— the bandwidth of channel is identical (e.g., all TV channels in the US are 6 MHz each). Each channel is divided into an equal number of bands  $u$ . The number of *un-used* bands for channel  $i$  is  $u_i$  such that  $0 \leq u_i \leq u$ , i.e., the number of bands to be sold vary from seller to seller. For example, a 6 MHz channel can be divided into 6 bands of 1 MHz each.

As far as the secondary users (SUs) are concerned, they buy bands from the channels that primaries own. However, the quality of the channels varies due to the spatial and temporal reasons— and as a result, a secondary user will be willing to pay more for a better

quality channel. A secondary user can easily sort the available channels based on their preference. We use SINR as a metric to determine the goodness of a channel. Thus, each secondary user will have a preference list of channels sorted in a descending order. However, *there is no preference among the bands from the same channel.*

SUs interfere with each other if they use the bands from same channel and are in close proximity of each other. Thus, we use an *interference graph* that gives the conflict of every SU. In the graph, an edge between nodes  $i$  and  $j$  means that they are close to each other and therefore cannot use the same channel (even if bands are different) at the same time. However, a specific band can be sold to (and consequently used by) multiple SU simultaneously as long as they do not interfere with each other. Such simultaneous re-use of bands increases the i) spatial utilization of the spectrum and ii) revenue for the primaries.

### 3.3.1 Bid notations

A seller  $i$  submits its ‘ask’ denoted as:

$$S_i = \begin{bmatrix} B_i^s & u_i \end{bmatrix} \quad (3.9)$$

where  $B_i^s$  is the ‘asking price’ for each band and  $u_i$  is the number of bands to be sold. Let  $V_i^s$  be the true valuation of each band, and  $P_i^s$  is the actual payment received for selling a band. Thus, the utility for seller  $i$ , represented by  $U_i^s$  is calculated as:

$$U_i^s = \begin{cases} P_i^s - V_i^s & \text{if } i \text{ is able to sell} \\ 0 & \text{otherwise.} \end{cases} \quad (3.10)$$

Similarly, buyer  $j$  submits its ‘bid’ as:

$$B_j = \begin{bmatrix} \begin{bmatrix} B_{j,1}^b \\ B_{j,2}^b \\ \dots \\ \dots \\ B_{j,m}^b \end{bmatrix} & \begin{bmatrix} Ch_1 \\ Ch_2 \\ \dots \\ \dots \\ Ch_m \end{bmatrix} & D_j \end{bmatrix} \quad (3.11)$$

where the column vector  $\begin{bmatrix} B_{j,1}^b, \dots, B_{j,m}^b \end{bmatrix}^T$ ,  $1 \leq m \leq M$ , is the bid list which contains the price the buyer is willing to pay for the bands the corresponding channels for listed in descending order of preference (namely, the preference list) denoted as the column vector  $\begin{bmatrix} Ch_1, \dots, Ch_m \end{bmatrix}^T$ , and  $D_j$  is the number of bands buyer  $j$  wants to buy. To clarify, buyer  $j$  is willing to pay  $B_{j,1}^b$  for a band from the most preferred channel  $Ch_1$ .

If  $V_j^b$  is the true valuation of the band, and  $P_j^b$  is the actual payment made by buyer  $j$ , then the utility,  $U_j^b$ , is calculated as:

$$U_j^b = \begin{cases} V_j^b - P_j^b & \text{if } j \text{ is able to buy} \\ 0 & \text{otherwise.} \end{cases} \quad (3.12)$$

### 3.3.2 SINR and Preference List

Each secondary user computes its preference list based on SINR on each channel, as discussed in 3.2.1 The SINR on all the channels for secondary node  $j$  can be written in the form of a vector:

$$SINR_j = [SINR_{j,1}, SINR_{j,2}, \dots, SINR_{j,M}]$$

Recall,  $M$  is the total number of channels/sellers.

The secondary node  $j$  sorts the SINR vector in descending order and uses it as a *preference list* for the channel(s) to be bought.

With the preferences for the channels known, the SUs can bid for the channels. Obviously, the SUs will be willing to pay more for a preferred channel than a non-preferred channel– thus, the bids will be non-increasing for that SU’s preference list i.e., the highest bid will be for the most preferred channel and so on.

### 3.3.3 Economic Properties

The three economic properties that double auctions typically satisfy are as follows:

1. **Truthfulness:** A double auction is considered truthful such that no seller  $i$  or buyer  $j$  can improve his own utility by bidding untruthfully:  $B_i^s \neq V_i^s$  or  $B_j^b \neq V_j^b$  In some cases, bidders are selfish, i.e, they may declare dishonest valuation, to increase their own prof-

its. Thus, truthfulness is vital to resist market manipulation and guarantee auction fairness and efficiency. In truthful auction, the auctioneer can allocate spectrum efficiently to the buyers who value it the most, and also ensures that bidders' utility will be maximized only by declaring truthfully without knowing the other bidders' bids.

2. **Individual Rationality:** Bidders get non-negative utilities, i.e., no winning seller is paid less than his ask and no winning buyer pays more than bid:

$$P_i^s \geq B_i^s, P_j^b \leq B_j^b \quad \forall \text{ seller } i, \text{ buyer } j$$

3. **Ex-post Budget Balance:** A double auction is considered an ex-post budget balance auction if the auctioneers' profit is non-negative, i.e,  $\Phi \geq 0$ . That means, the total amount paid to the sellers is no more than the total amount received from the buyers:

$$\Phi = \sum_{j=1}^N P_j^b - \sum_{i=1}^M P_i^s \geq 0.$$

### 3.3.4 PreDA

PreDA is a multi-round multi-unit double auction where the buyers place multiple bids for heterogeneous channels. The 5-step auction is controlled by an auctioneer.

#### Step I: Buyer Group Formation

The first step for the auctioneer is to form the buyers' group with a bid-independent method. The bid-independent group formation is helpful to maintain truthfulness and prevent buyers' bid manipulation [12, 23]. The buyers group formation process is modeled as

finding the sets of non-interfering users who can transmit on the *same* channel at the *same* time. Of course, non-interfering users can transmit on different channels at the same time. Therefore, we identify sets of non-interfering users who are placed in non-interfering sets. The problem of finding such non-interfering sets for an arbitrary network is known to be NP-complete.

SUs interfere with each other if they use the same channel and are within the transmission range of each other. Thus, we use an *interference graph* that gives the conflict of every SU. In the graph, an edge between nodes  $i$  and  $j$  means that they are close to each other and therefore can not use the same channel at the same time. However, the channel can be simultaneously used by nodes that belong to different sets since they do not interfere with each other.

We start with the entire network represented by graph  $(\mathcal{G})$ . We start with a node called  $n_1$  and place it in the first non-interfering set, represented by  $G_1$ . (Note,  $G_1$  is a set, not a graph.) The selection of  $n_1$  could be based on any criteria. We discuss and implement three such criteria in section 6.2. All neighbors of  $n_1$  in the interference conflict graph cannot be placed  $G_1$ . Therefore, we remove all such nodes and their edges from  $(\mathcal{G})$  to obtain  $(\mathcal{G}')$ . Note that, no node in  $(\mathcal{G}')$  interferes with  $G_1$ . We follow the same process of finding a node in  $(\mathcal{G}')$  (say  $n_2$ ) and place it in  $G_1$ . We further reduce  $(\mathcal{G}')$  and continue to find nodes that would belong to  $G_1$  till the graph reduces to a null set. This set of nodes belong to non-interfering set  $G_1$ .

With the set of nodes placed in  $G_1$ , we proceed to find the next non-interfering set from nodes that are in  $(\mathcal{G})-G_1$ . We follow the same process and obtain the second non-interfering set,  $G_2$ . Then we consider the remaining nodes in  $(\mathcal{G})-G_1 - G_2$  and continue in the same manner till all nodes belong to some non-interfering set.

## Step II: Virtual Group Formation

Once the groups are formed, we check for the buyers' demand, their preference for channels, and the corresponding bids. In case, a buyer's demand is more than 1 band, then his request cannot be satisfied by simply putting the buyer in one group. As a result, we divide each group into multiple Virtual Groups (VG). The demand of each VG is exactly 1 band for a specific seller. Though there are various methods that have been proposed to create VGs, none consider the channel preference for buyers. Creating the VGs consists of two parts.

*Part I: Bid creation:* The auctioneer starts with a group  $G_1$ , and starts to create a pair of bid for all buyers in  $G_1$ . The pair of bid contains the most preferred channel for a buyer and the corresponding bid. For a buyer  $j$  in  $G_1$ , the pair of bid contains the maximum bid, for the most preferred channel (i.e., the first bid and the first channel). This pair is submitted as a bid for buyer  $j$ , for first round of the auction. Part I is repeated for all buyers in  $G_1$ . The same process is repeated for all the groups. For the second round, the second most preferred channel and its bid is considered. The process continues till the last round.

*Part II: VG creation:* The auctioneer starts to create the VGs for channel/seller  $1 \leq i \leq M$ .

For seller  $i$ :

- If demand for seller  $i$  is 0, there is no VG for  $i$ .
- If there is a demand for seller  $i$ , the buyer with the minimum per unit bid in the group is found and eliminated from any VG for seller  $i$ . It can be noted, [12] did not support the elimination process which guarantees truthfulness, as we charge the winners the bids of the eliminated buyer for seller  $i$ .
- The VGs are created for all buyers who are not eliminated for seller  $i$ . The demand of each VG is 1 unit only.
- The bid of a VG for seller  $i$  is the bid of the eliminated buyer (i.e., the minimum bid) multiplied by the number of buyers in that VG.

### Step III: Winner Determination

Let  $VG_1, VG_2, \dots, VG_V$  represent  $V$  groups formed in Step II for round  $r$ . Any virtual group  $VG_v$  has  $B_v = |VG_v|$  buyers. Suppose, the bid of the eliminated buyer  $j$  is  $B_j^b = B_{min}$ . The virtual group bids  $\pi_v$  is:

$$\pi_v = B_{min} \times B_v \quad (3.13)$$

In this step, the auctioneer determines the winning VGs and the winning sellers. Similar to any traditional double auction, all VGs are sorted in a non-increasing order based on their bids, (i.e.  $\pi_1 \geq \pi_2 \geq \dots \geq \pi_V$ ). While all sellers are sorted in a non-decreasing order based on their asks, (i.e.  $B_1^s \leq B_2^s \leq \dots \leq B_M^s$ ). Ties can be broken arbitrarily.

The auctioneer starts from the maximum VG bids and tries to match it to the suitable seller, and continues to do so till the last VG. No seller sells for anything less than the asking price.

#### **Step IV: Payment Calculation**

All buyers in the winning group  $VG_i$  equally share the payment  $\pi_v$  of winning, while each winning seller  $i$  gets exactly its asking price  $B_i^s$ . The difference between  $\pi_v$  and  $B_i^s$  goes to the auctioneer. No charge or payment is made to the losing sellers and buyers. Thus, the auctioneer's profit is:

$$\Phi = \sum_{v=1, i=1}^{V, M} (\pi_v - B_i^s) \quad (3.14)$$

#### **Step V: New Demand Calculation**

The auctioneer calculates the new demand for all winning buyers and winning sellers (since they still might have demands to be fulfilled) by subtracting the number of allocated unites from the original demands. In case there is no more demand for any buyer or seller, that buyer or seller is removed and does not participate in further auction rounds. Else, they participate in the next round with the new demands.

After Step V, The auctioneer starts the new round of the auction from Step II. The auction stops when either demands are met or there are no more bands to be sold.

### 3.3.5 An Illustrative Example

We demonstrate the working of PreDA using a toy example. We consider 6 sellers whose asking price for the bands to be sold are shown in Table 3.2. There are 9 buyers whose conflicts are shown in Fig 3.2. Their bid list, preference list, and demands ( $D$ ) are shown in Table 3.3.

Table 3.2 Sellers' Asks

Seller	Asking Price	Units
1	\$5	3
2	\$6	2
3	\$3	1
4	\$7	2
5	\$5	5
6	\$3	2

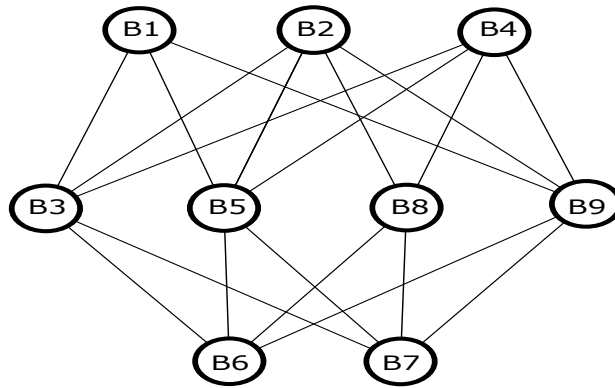


Figure 3.2 Buyers' Conflict Graph

**Step I:** For the Group formation, we start with  $B1$  in the first group called  $G_1$ . Nodes that do not conflict with  $B1$  are included in  $G_1$  i.e.,  $G_1 = (B1, B2, B4, B6, B7)$ . Similarly, we get the second group as  $G_2$  i.e.,  $G_2 = (B3, B5, B8, B9)$ . Since all the buyers have been accommodated in a group, there will be no more groups.

Table 3.3 Buyers' Bids

Buyers	Bid List	Preference List	Demand
B1	[\$5,\$2,\$1]	[1,3,6]	2
B2	[\$7,\$6,\$5]	[1,6,4]	3
B3	[\$4,\$3]	[2,6]	2
B4	[\$5,\$3,\$2]	[3,6,4]	2
B5	[\$8,\$7]	[3,2]	2
B6	[\$4,\$3,\$2]	[4,3,2]	1
B7	[\$10,\$5,\$2]	[1,4,3]	1
B8	[\$7,\$6]	[2,6]	1
B9	[\$8,\$6,\$5]	[2,6,5]	2

**Step II:** To start with (i.e., Part 1), the bid of each buyer in  $G_1$  is created. The bid for  $B1$  is  $(\$5, 1)_2$  because \$5 is the highest bid, the most preferred is channel 1, and the demand is 2. Similarly, the bids for  $B2, B4, B6, B7$  are  $(\$7, 1)_3, (\$5, 3)_2, (\$4, 4)_1, (\$10, 1)_1$ , respectively. For  $G2$  the bids are  $(\$1, 2)_2, (\$8, 3)_2, (\$7, 2)_1, (\$8, 2)_2$ .

To create the Virtual Groups (i.e., Part 2), we start with  $G_1$  for all channels. As  $B1$ ,  $B2$ , and  $B7$  bid for Ch 1, the minimum bid (from  $B1$ ) is eliminated. Thus,  $B2$  and  $B7$  form virtual group  $VG1$ . The corresponding bid is \$10 which is 2 times the minimum bid (i.e., the bid of  $B1$ ) as shown in Table 3.4. As  $B7$  had a demand of 1 band, its requirement has been met. However,  $B2$  needs 2 more bands (as only 1 band has been met out of the 3 it demanded for). Thus,  $VG2$  and  $VG3$  are created with  $B2$  only. The bids for both are \$5. The same process is repeated for all the channels that are in demand for  $G1$ . Note, since there is no demand for Ch 2, Ch 5, and Ch 6, they do not appear in virtual groups of  $G_1$ . We do the same for  $G_2$ .

Table 3.4 Step II, Part II

Group	Channel	VG	VG member	Bid
G1	Ch1	VG1	{B2,B7}	5*2=\$10
		VG2	{B2}	\$5
		VG3	{B2}	\$5
	Ch3	VG4	{B4}	\$5
		VG5	{B4}	\$5
	Ch4	VG6	{B6}	\$4
G2	Ch2	VG7	{B8,B9}	1*2=\$2
		VG8	{B9}	\$1
	Ch3	VG9	{B5}	\$8
		VG10	{B5}	\$8

**Step III:** The virtual groups are sorted in a descending order based on the bids as shown in Fig. 3.3. The sellers ‘asks’ (see Table 3.2) are sorted in ascending order. Starting with the virtual group from the left, we find the matches over *all* sellers that can satisfy the price for the corresponding channel. The matchings are shown by the arrows.

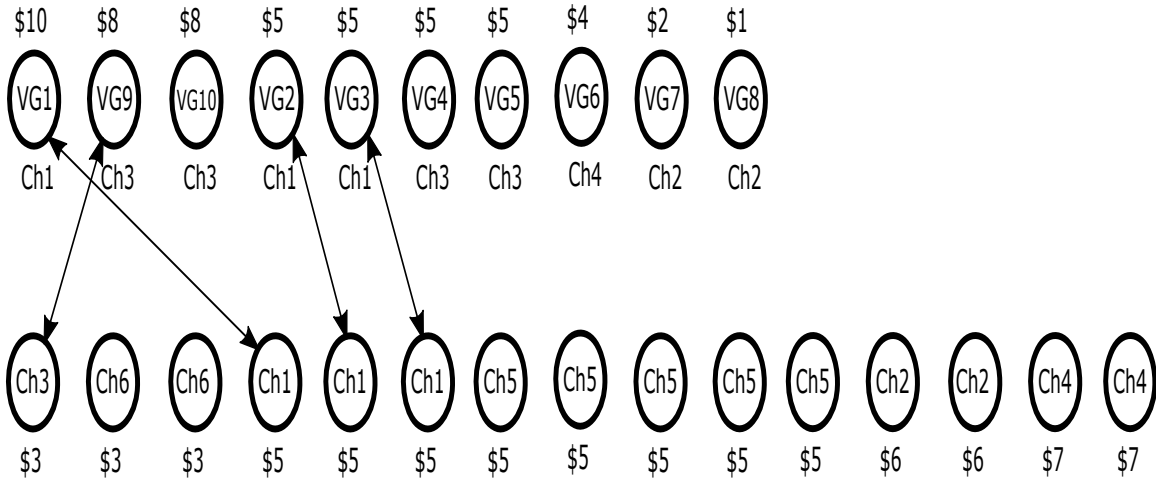


Figure 3.3 Step III: Winner Determination

**Step IV:** For each match, the payment to the seller, payment by the buyer, and the auctioneer profit are calculated as shown in Table 3.5.

Table 3.5 Step IV: Payment Calculation

Winners	Payment to seller	Payment of buyer	Auctioneer profit
VG1-Ch1	\$5	B2 pays \$5 B7 pays \$5	\$5
VG9-Ch3	\$3	B5 pays \$8	\$5
VG2-Ch1	\$5	B2 pays \$5	\$0
VG3-Ch1	\$5	B2 pays \$5	\$0

**Step V:** The demands are updated based on what fraction of the demand has been met. For example, the demand of  $B1$  remains at 2 as it did not get any band, where as the demand of  $B2$  is 0 as it received the required 3 bands. The demand of  $B5$  is reduced to 1 as it received 1 band of the required 2. The updated demands are shown in Table 3.6. The asking price for the sellers and the remaining bands to be sold are shown in Table 3.7.

Table 3.6 Buyers' Updated Demands

Buyers	New Demands
B1	2
B2	0
B3	2
B4	2
B5	1
B6	1
B7	0
B8	1
B9	2

Since not all the buyers won their most preferred channel, we still allow them to participate in the auction for further rounds for them to win their next preferred channel.

Table 3.7 Sellers' Updated Units

Seller	Asking Price	Units
1	-	0
2	\$6	2
3	-	0
4	\$7	2
5	\$5	5
6	\$3	2

The same steps are followed but with the demand that has not been made and the residual bands to be sold. The stopping criteria is discussed in Section 6.2.

### 3.3.6 Proof of Economic Properties

**Theorem 1.** *PreDA is individual rational for sellers and buyers.*

*Proof.* As per our definition of a “winning” seller, the seller  $i$  is paid  $P_i^s$  which is equal to the asking price  $B_i^s$ , else there is no sale from that seller. Thus, individual rationality is maintained for the seller. As for the buyers, the bid of the eliminated buyer (i.e., the minimum bid,  $B_{min}$ ), is used as the bid for all the buyers in the winning virtual group. Thus, no winning buyer  $j$  pays more than his original bid as:  $P_j^b \leq B_j^b$ , satisfying buyer's individual rationality.  $\square$

**Theorem 2.** *PreDA is ex-budget balanced.*

*Proof.* Based on the sorting process of the PreDA, all  $VGs$  are sorted in non-increasing order based on their bids. Thus, the results of winner determination must be as:  $\pi_v \geq P_i^s$ , where  $\pi_v$  is the bid of the winner  $VG_v$ ,  $P_i^s$  is the ask of the winner seller  $i$ . As a result,  $\sum_{v=1}^V \pi_v - \sum_{i=1}^M P_i^s \geq 0$ . However, the summation of the winning virtual groups ( $\sum_{v=1}^V \pi_v$ ) is equivalent to the summation of the winning buyers ( $\sum_{j=1}^N P_j^b$ ), so:

$$\sum_{j=1}^N P_j^b - \sum_{i=1}^M P_i^s \geq 0$$

□

**Theorem 3.** *PreDA ensures the truthfulness of the sellers and the buyers.*

*Proof.* We prove truthfulness of the auction by showing that the expected utility does not increase for the sellers or the buyers by deviating from the true ask or the bid values.

**For Sellers:** Let us consider that the asks by the sellers are uniformly distributed between the lowest  $a_l$  and highest  $a_h$  values. (The assumption on the density function does not matter; a uniform density makes it easy to prove.) Suppose the true ask is  $a_t$  and  $a'_t$  is an untrue ask. **Case 1:**  $a'_t < a_t$

With a lower asking price, the utility of the winning seller cannot increase as the sellers get no more than the asking price.

**Case 2:**  $a'_t > a_t$

For a seller  $i$  to win, the payment  $P_j^b$  from a buyer  $j$  has to be more than or equal to the asking price  $B_i^s$ . Let the probabilities of winning with  $a_t$  and  $a'_t$  be  $P_1$  and  $P_2$  respectively:

$$P_1 = P\{a_t \geq P_j^b\} = \frac{a_h - a_t}{a_h - a_l} \quad (3.15)$$

$$P_2 = P\{a'_t \geq P_j^b\} = \frac{a_h - a'_t}{a_h - a_l} \quad (3.16)$$

As  $a'_t > a_t$ ,  $P_2 < P_1$ . Thus, the expected utility ( $P_2 \times P_j^b$ ) does not increase from ( $P_1 \times P_j^b$ ). So, no seller can improve its utility by asking untruthfully.

**For Buyers:** Same as the sellers side, the bids by the sellers are uniformly distributed between the lowest  $b_l$  and highest  $b_h$  values. Suppose the true bid is  $b_t$  and  $b'_t$  is an untrue bid.

**Case 1:**  $b'_t > b_t$

With a higher bid, the utility of the winning buyer can not increase, as the buyers payment is equal to the  $B_{min}$ . Moreover, the process of group formation is bid-independent, which is helpful to maintain truthfulness and prevent buyers' bid manipulation, as mentioned before.

**Case 2:**  $b'_t < b_t$

The buyer  $j$  can still win by bidding less than or equal to the  $b_t$ , as long as its bid is greater than or equal to the  $B_{min}$ . However, even with the winning with  $b'_t$  the utility of the winning buyer can not increase since the actual payment is equal to the  $B_{min}$ .

In case,  $b'_t$  is less than the  $B_{min}$ , the buyer definitely loses as the buyer with the minimum bid is eliminated from the virtual group.

Thus, in all cases, bidding other than the true values can not increase the utilities of the buyer. □

---

**Algorithm 1** Spectrum Auction

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**Require:**

```
1: Create the CR Network and denote as  $G=(V; E)$ 
2: Initialize all PUs and SUs
3: Compute Conflict Graph
4: for all Users do
5:   Calculate Users' Interference
6:   Calculate Users' SINR
7:   Generate Users' Preference List
8:   Generate Users' Biding List
9: end for
10: Start Auction
11: while Till all users are considered do
12:   Initializing all Flags
13:   Get User's Neighbor (Conflict Nodes)
14:   if Assignedflag == 0 then
15:     for all User Preferences do
16:       Check First Preference of the user
17:       if There is only one neighbor and its a PU then
18:         User assigned 1st preference (as PU's channel cannot be in the list)
19:         Calculate the Revenue for the PU
20:         Change user's status to assigned user
21:         Auction Round Done, Exit While
22:       end if
23:       if There is a Conflict with another SU and with higher bid than current user's
         bid then
24:         Set Conflict Flag
25:         Move to the next preference, Exit For
26:       end if
27:       if There is a Conflict with already signed SU then
28:         Set Conflict Flag
29:         Move to the next preference, Exit For
30:       end if
31:     end for
32:     if AllConflictFlags == 0 then
33:       User assigned current preference
34:       Do steps 19-21
35:     end if
36:   end if
37: end while
```

---

## CHAPTER 4: ECONOMIC MODELS FOR ROUTING IN DSA NETWORKS

In this chapter, we investigate the routing problem in DSA networks from an economic perspective. In order for secondary nodes to route packets, they must first acquire spectrum from primary users which they then sell to others in the form of various services, including routing of packets. Thus, bandwidth acquired by secondary users are traded amongst each other as packets get routed. We model the selection of the intermediate nodes in a route using two different methods: i) a selling price mechanism, and ii) a sealed-bid auction mechanism. The transmitter explores the cost of forwarding a packet by all potential receivers i.e., the forwarders. Similarly, a receiver must determine the optimal price for it to route packets on behalf of other nodes for a certain price. The route with total price is minimum and sustain the transmitter's requirement is chosen as a final path.

### 4.1 Pricing-Based Routing

When a secondary (source) node wants to route packets to a destination node, it must pay all the intermediate nodes that relay the packets to the destination node. Without such payments, the intermediate nodes will have no incentive to relay the packets of others as that would cause resource depletion. Obviously, the source node must seek a path that: i)

minimizes the total payment to the intermediate nodes and ii) is able to support the target bit rate as requested by the source node.

**The routing problem:** (from an economic perspective) is to find a path (or a set of paths) from a source node  $s$  to a destination node  $d$ , that has sufficient capacity to route a given flow, and for which the total payment that needs to be made to relaying nodes is minimized.

To solve the routing problem, we have two decision problems at hand. First, the relay nodes that sell (also called sellers) the bandwidth must determine the price they must ask the node that wants to route packets. To do so, the seller must consider the price it already paid to the primary user to acquire the bandwidth and how much of it has been already sold (i.e., used by others). Second, the buyer (i.e., the source node) must determine the path(s) that can sustain its data rate and at the same time minimizes the sum of the payments to the intermediate nodes involved in the route.

#### 4.1.1 Computation of offered price

We assume that the cost per bandwidth (in Hertz) that the secondary node  $i$  pays to the primary node is  $p_{i_{cost}}$ ; thus for acquiring a bandwidth of  $B_i$ , the cost is  $B_i \times p_{i_{cost}}$ . We assume that node  $i$  retains the rights for the spectrum bought indefinitely. Now, secondary node  $i$  sells portions of the acquired bandwidth to various other secondary nodes based on what

their demands are and what prices they are willing to pay. The chunks of spectrum thus sold are also for an indefinite time.

We make three assumptions: i) we assume that node  $i$  is somehow able to predict how much bandwidth ( $B_i$ ) it must acquire from the primaries, ii) that node  $i$  is able to sell the entire spectrum acquired, and iii) all sales are made one-time i.e., there is no spatial or temporal reuse of the spectrum chunks. Assumptions (i) and (ii) guarantee that the nodes are able to generate at-least  $B_i \times p_{i_{cost}}$  as revenue.

To sell the spectrum, secondary node  $i$  must advertise the optimal price— a higher price will drive buyers to other sellers and a lower price will result in a loss. We argue that the advertised/offered price is a function of the fraction of the residual bandwidth (i.e., part of the bandwidth that has not yet been sold), which is represented by  $B_{i_{res}}$ , where  $0 \leq B_{i_{res}} \leq 1$ . We employ the commonly used logarithmic function used for market-driven bandwidth allocation [14] to determine the price offered by node  $i$  and represent it as:

$$p_i = p_{i_{cost}}(1 - \log(B_{i_{res}})) \quad (4.1)$$

This conforms to the fact that when  $B_{i_{res}} = 0$ ,  $p_i$  is infinity (i.e., there cannot be any sale) and when  $B_{i_{res}} = 1$ ,  $p_i = p_{i_{cost}}$ . The offered price as given in equation (4.1) is shown in Fig. 4.1. The value of  $p_{i_{cost}}$  used is 1. We choose the  $\log$  function because it decreases quickly from infinity as  $B_{i_{res}}$  increases. Moreover, the  $\log$  function is mathematically tractable, strictly concave, and continuously differentiable.

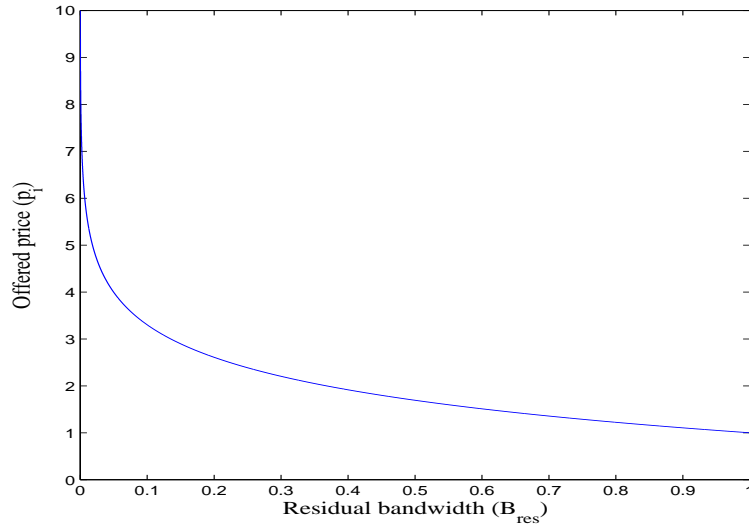


Figure 4.1 Offered price as a function of residual bandwidth

#### 4.1.2 Determination of noise exposure

Alongside the price offered, the seller must also reveal the *quality* of the bandwidth that it is selling, i.e., it must let the buyer know what kind of bit-rate it can achieve with the bandwidth being sold. It is to be noted that for a given bandwidth, the maximum bit rate attainable depends on the signal to noise ratio (SNR). Thus, the seller which would be receiving the packet from the buyer (i.e., transmitter) must compute and truthfully declare the noise and interference it is exposed to due other nearby nodes. The buyer then, based on the inter-nodal distance, computes the SNR and the corresponding bit rate.

We assume that the noise exposure of seller node  $i$  is due to all its neighbors/interferers. Two nodes are considered neighbors if they are in the transmission range of each other and thus can hear each other. We assume that the bi-directional links are symmetric. As a result,

noise perceived at node  $i$  from all its neighboring nodes can be calculated as:

$$n_i = n_0 + \sum_{j \in N_i} \frac{P_j}{d_{i,j}^\alpha} \quad (4.2)$$

where  $n_0$  is the thermal noise,  $N_i$  is a set of neighbors of node  $i$ ,  $P_j$  is the transmit power from node  $j$ ,  $d_{i,j}$  is the distance between nodes  $i$  and  $j$ , and  $\alpha$  is the path loss exponent.

#### 4.1.3 Price for buyer

Given the noise obtained from seller  $i$ , a transmitter  $t$  computes the SNR for node  $i$  as:

$$SNR_t = \frac{P_t/d_{i,t}^\alpha}{n_i} \quad (4.3)$$

where the transmitter  $t$  transmits with power  $P_t$  and the distance to the receiver  $i$  is  $d_{i,t}$ .

Let us suppose, the transmitter  $t$  wants to achieve a bit rate of  $b_t$ . In order to do so, it must buy bandwidth from a seller. Now, the amount of bandwidth to be bought from seller  $i$  would depend on the SNR between nodes  $t$  and  $i$  as obtained in equation (4.3).

Let  $B_{t,i}$  be the bandwidth required by transmitter  $t$  to achieve a bit rate of  $b_t$  with receiver node  $i$ . It is well know that the maximum achievable bit rate using a bandwidth of  $B_{t,i}$  can be found using Shannon's capacity as:

$$b_t = B_{t,i} \times \log\left(1 + \frac{P_t/d_{i,t}^\alpha}{n_i}\right) \quad (4.4)$$

Alternatively, the transmitter, in order to meet its bit rate requirement, must buy bandwidth of  $B_{t,i}$  from node  $i$  as given by

$$B_{t,i} = \frac{b_t}{\log(1 + \frac{P_t/d_{i,t}^\alpha}{n_i})} \quad (4.5)$$

As different sellers exhibit different prices (as given by equation (4.1)), a buyer can compute the prices for the different sellers and choose the seller for which the cost will be minimum. Thus, the payment to be made by transmitter (buyer)  $t$  is:

$$pay_t = \arg \min_i (B_{t,i} \times p_i) \quad (4.6)$$

#### 4.1.4 Minimum Cost Route

The cost of the overall path between the source  $s$  and the destination  $d$  is the sum of the payment made at each hop. Thus, for path  $k$  between  $s$  and  $d$ , the total cost is

$$cost_k = \sum_{t \in \text{nodes in path } k} pay_t \quad (4.7)$$

The source node chooses the route for which the sum of the prices to the destination node is minimum. That is, picks path  $k$ :

$$\arg \min_i (cost_k) \quad (4.8)$$

The algorithm for min-cost route determination is shown in Algorithm 2.

---

**Algorithm 2** Routing Algorithm

---

**Require:**  $G=(V; E)$ ; Source  $s$ ; destination  $d$  randomly among all nodes

```

1:  $TotalPrice \leftarrow 0, Steps \leftarrow 1, Avg(s) \leftarrow 0$ 
2: while true do
3:    $Sequence[step] \leftarrow$  Current Node
4:   Get Neighbors of the Current Node
5:   Check
6:     1. Knows the route to the destination?
7:     2. Receive multiple copies of same request? Cycle?
8:     3. Disconnected node?
9:   if 1,2,3 then
10:    break
11:   end if
12:   for all Neighbors  $j \in N_i$  do
13:     $OriginalSource \leftarrow SourceNode.ID$ 
14:     $NextNodeID \leftarrow CurrNeighbors[NextGeneration]$ 
15:     $CurrPrice \leftarrow$  Price between  $OriginalSource$  and
16:     $CurrNeighbors[NextGeneration]$ 
17:     $TotalPrice = TotalPrice + CurrPrice$ 
18:     $NextNode = CurrNode$ 
19:     $Sequence[Steps] = NextNode \rightarrow ID$ 
20:     $Steps = Steps + 1$ 
21:     $Routing(NextNode, DestinationNode)$ 
22:   end for
23: end while

```

---

## 4.2 Auction Based Routing

We consider a cognitive radio network with multiple secondary cognitive radio nodes randomly scattered over an area as mentioned in section 4.1. We do not consider how the primaries determine the price of the spectrum when selling to secondaries. Such price func-

tions are usually logarithmic in nature as they are mathematically tractable, strictly concave, and continuously differentiable. Interested readers can refer to [65] for spectrum price determination by the primaries.

We consider a multi-channel system where the secondary nodes can potentially use a subset of the  $K$  channels. Each channel has a bandwidth of  $b$ . Note that, the achievable capacity (i.e., bit rate) on a channel depends on the SINR the receiver is exposed to on that channel. Hence, the channels are non-substitutable. We assume that a source node has a bandwidth requirement of  $b_{req}$ . Thus, the source node must seek a path that is able to support this target bit rate. While there might exist multiple paths that can cater to the bit rate requirement, we find the one that minimizes the total payment to the intermediate nodes.

For the purpose of routing, each intermediate node plays the role of both a seller and a buyer. If we consider any single hop in a route, the transmitter is the buyer which wants to buy channels from a potential receiver which is the seller. Since there could be multiple next hop receivers for a transmitter, the transmitter decides to buy the channel from the seller which has the minimum bid and is also able to sustain the bandwidth requirement. The winning seller (receiver) becomes the buyer (transmitter) for the next hop, and the process continues till the destination node becomes the final receiver.

A buyer sorts the bids in an ascending order and disregards any bid that is higher than the *reserve price*. Then, it chooses the one with the minimum bid. The reserve price is the maximum price that a buyer is willing to pay. Using such a reserve price eliminates the

number of competitors which would not win anyway. Of course, the choice of the reserve price is critical— a low reserve price might lead to elimination of all competitors, whereas, a high reserve price might lead to too many sellers to deal with.

Note, when a seller computes the bid, it takes into consideration the bit rate requirement. A seller not able to satisfy the bit rate requirements on any of the channels will simply not make a bid. Thus, a bid is always valid in terms of supporting a specific bit rate.

#### **4.2.1 Bid Computation by Seller**

The objective of the bidder is to sell a bundle (i.e., a set) of channels that satisfies the bit rate requirement. Since there could be numerous such bundles, the seller must offer the bundle for which the cost will be minimum. The buyer gets multiple offers from multiple sellers and chooses the seller who offers the minimum price for its bundle. As for the buyer, it does not matter what the composition of the bundle is i.e., whether the bundle is composed of 1 channel or multiple channels. All it knows for sure is that the offered bundle will satisfy the bit rate requirement.

Each seller calculates its bid based on three parameters: i) signal to interference and noise ratio on a specific channel, ii) achievable capacity on each channel, and iii) the required bandwidth or the number of channels needed to satisfy the buyer's bit rate requirement.

#### 4.2.1.1 SINR on each channel

As we know for a given bandwidth, the maximum capacity achievable depends on the SINR. As a result, the seller must determine the SINR on each channel, before estimating the achievable capacity on each channel. Note, each receiver is exposed to different levels of noise and interference on every channel based on their distances from various transmitters. For example, receiver  $i$  is interfered on channel 1 only by the transmitters which use channel 1. Similarly, receiver  $i$  will be interfered on channels 2 and so on.

As a result, interference perceived at node  $i$  on channel  $j$  from all its neighboring nodes can be calculated as:

$$I_{i,j} = \sum_{n \in N_i} \frac{P_n}{d_{i,n}^\alpha} \quad (4.9)$$

where  $n \in N_i$  are the neighbors of node  $i$  using channel  $j$ .  $P_n$  is the transmit power from neighbor  $n$ ,  $d_{i,n}$  is the distance between nodes  $i$  and  $n$ , and  $\alpha$  is the path loss exponent.

The SINR for node  $i$  on channel  $j$  and for transmitter  $t$  is :

$$SINR_{i,j}^t = \frac{\frac{P_t}{d_{i,t}^\alpha}}{n_0 + I_{i,j}} \quad (4.10)$$

where  $P_t$  is the transmit power of the transmitter,  $d_{i,t}$  is the distance between node  $i$  and transmitter  $t$ , and  $n_0$  is the background noise.

#### 4.2.1.2 Capacity on each channel

The seller can use the Shannon's theory to calculate the capacity on each channel between itself and transmitter  $t$ . The capacity on channel  $j$  for seller  $i$  for transmitter  $t$  given as

$$c_{i,j}^t = b \times \log(1 + SINR_{i,j}^t)$$

where  $b$  is the bandwidth of each channel. The capacity on all  $K$  channels can be written as in the form of a vector:

$$C_i^t = [c_{i,1}^t, c_{i,2}^t, \dots, c_{i,K}^t]$$

#### 4.2.1.3 Channel bundle

With the capacities on every channel known, the seller can offer to sell the channels for a price. Let us consider two cases: i) when the seller can sell only *one* channel and ii) when the seller can sell multiple channels. For case ii), we can have two sub-cases based on whether the prices for every channel is the same or different.

##### Case i) Seller sells only one channel

The buyer buys one channel that satisfies the bit rate requirement of  $b_t$ . The seller identifies the channels that can meet the requirement. It is possible that none of the channels meet the requirement. In that case, there is no route through node  $i$ . Among the valid channels,

the seller picks the one with the minimum price. This is the bidding price by seller  $i$ .

Case ii) Seller sells multiple channels

When this scenario, we consider two sub-cases where all the channels have the same price and when the channels have varying prices.

1. *All channels have fixed price:*

For every seller, we arrange the expected capacity of each channel in a descending manner. Given the bit rate requirement of the buyer ( $b_t$ ), we check how many channels would be required. If the best channel (with respect to the capacity obtained) can sustain  $b_t$ , then just that channel would suffice. Else, we check if the best two would suffice. If not, we consider the best three and so on. If we assume best  $k$  channels are required to meet the demand of  $b_t$ , then the offered price is  $Kp_i$ , where  $p_i$  is the price of each channel by seller  $i$ .

2. *All channels have varying prices:*

Since the price for every channel by seller  $i$  is different, we have to check for all possibilities of channel bundles that could satisfy the bit rate. Among all valid bundles, seller  $i$  picks the one that has the least cost i.e., the sum of cost of all the channels in that bundle is minimum. In order to find the least-cost bundle, all combinatorial possibilities need to be explored, which we discuss next.

### 4.2.2 Combinatorial Auctions

Advantages of the exploring all combinatorial possibilities for the auction bid include higher efficiency, better fairness, and lower transaction costs. On the other hand, combinatorial auctions are considered NP-hard because the auctioneer has to calculate the result of the winner determination problem (WDP) to solve the revenue-maximization or cost-minimization in the auction.

The upper bound of computing for single item auctions is  $O(|N||M|)$ , where  $N$  is the number of items and  $M$  is the number of buyers or sellers. This is because, finding the minimum or maximum is a simple task of searching for the lowest/highest bids. In combinatorial auctions, the auctioneer must test *all* bid combinations with the intention of finding the optimal bid.

The number of feasible combinations is:

$$\sum_{q=1}^M \binom{M}{q} = 2^M - 1 \quad (4.11)$$

where  $\binom{M}{q}$  is the number of feasible combinations with  $q$  item with  $m$  being the total number of items. The complexity of combinatorial actions is between  $o(M^{M/2})$  and  $O(M^M)$  [66].

There are usually three broad approaches to solve the WDP depending on how good the solution quality would be and what the allowed computation time is. These are: i) *deterministic methods* where the optimal solution is found by searching (enumerating) the entire search space explicitly or implicitly, ii) *heuristic methods* where the aim is to find a

trade-off between the solution quality and the calculation effort; thus, global optimal are not guaranteed, and iii) *equilibrium methods* where bidders are allowed to bid simultaneously on bundles of items in multiple rounds which reduces the algorithmic and communication complexity of the auction by using temporal price and other information from the equilibrium process. First, we mention the exhaustive scheme where *all* possible routes are considered. Though theoretically they provide the most optimized route, they are practically infeasible due to their intractability. Then, we discuss the proposed heuristic which uses the few lowest *unit cost* of the bandwidth provided by each channel. Our argument of using the unit cost is based on that fact that unit costs are good indications of total costs.

#### 4.2.2.1 Exhaustive scheme

Here, the idea is to search in all possible routes from the source to the destination with all possible bid options considered at each hop. The route with overall minimum payment is chosen as the optimal one. For the single channel case, only the channels that satisfy the bit rate requirement are considered and the least cost one is the only channel to be considered at each hop between a transmitter and potential receivers. This reduces the complexity; however, for the multi-channel case, none of the channels can be ignored as all channels can potentially be a part of the least-cost bundle.

#### 4.2.2.2 $O(n \log n)$ Heuristic

In order to reduce the computational time and the complexity, we propose to consider the unit price for the achievable capacity on every channel being sold by the receiver. The channels are sorted in an ascending order with respect to their unit price. Starting with the least-cost (unit price wise) channel, we consider one additional channel at a time and check if the channels considered so far meet the bit rate requirement. If not, we consider the next channel and so on.

**An Illustrative Example:** Let us consider an example where seller  $i$  has 8 channels to sell as shown in Table 4.1. Though all the channels have the same bandwidth, the capacities are different because the SINR varies on different channel as was discussed in section 4.2.1.2. The total price of the capacity on each channel is also shown. We get the unit price by dividing the total price by the capacity.

Table 4.1 An illustrative example

Channel	1	2	3	4	5	6	7	8
Capacity (Mbps)	4.9	1.0	2.3	5.1	3.8	2.0	5.5	6.2
Total price	10	2	3	11	7	4	11	12
Unit price	2.04	2	1.30	2.16	1.84	2	2	1.93

Now suppose, a required capacity by a buyer is 5 Mbps. Each of the channels 4, 7, and 8 satisfy this requirement. Thus, if only one channel is considered then the minimum cost is 11. However, if we bundle channels 2, 3, and 6, we see that they satisfy the requirement with

the minimum cost of 9. Of course, finding such bundle requires us to consider all possible bundles that satisfy the requirement of 5 Mbps.

Table 4.2 Sorted with unit price

Channel	3	5	8	2	6	7	1	4
Capacity (Mbps)	2.3	3.8	6.2	1.0	2.0	5.5	4.9	5.1
Total rice	3	7	12	2	4	11	10	11
Unit price	1.30	1.84	1.93	2	2	2	2.04	2.15

For a quicker solution, we sort the channels based on the unit prices in an ascending order. Any sorting algorithm, such as the merge sort with time complexity  $O(n \log n)$ , can be used. The result is shown in Table 4.2. The idea is to start with the least unit price channel and check if it satisfies the bit-rate requirement of 5 Mbps. Here, channel 3 does not. Then the least two unit price channels are considered i.e., channels 3 and 5. These two provide a total capacity of  $2.3 + 3.8 = 6.1$  Mbps which is more than 5 Mbps. Thus, the channel bundle is found which has a cost of  $3 + 7 = 10$ . Note that, this heuristic need not guarantee the lowest possible price for the bundle.

#### 4.2.2.3 The Routing Algorithm

So far, we presented how the next hop will be determined based on the least cost channel bundle where the seller's role is to submit its bid in a sealed-bid form, reflecting its ability and willingness to forward. The buyer's role is to execute a local sealed bid auction to determine the winner of the auction. The winner is the bidder with the lowest bid. We

assume that the bidder reveals the true price and does not know about other bidders' price. This per hop phenomenon is repeated at each forwarding node. A seller in a particular hop becomes the buyer in the next till the destination is reached. The total price of each route is accumulated through the repeated bidding process over all the hops. The route with minimum payment is chosen as an optimal solution. The proposed auction based algorithm is shown in Algorithm 3.

---

**Algorithm 3** Routing Algorithm

---

**Require:**  $G=(V; E)$ ; Source  $s$ ; destination  $d$  randomly among all nodes

```
1:  $TotalPrice \leftarrow 0$ ,  $Steps \leftarrow 1$ ,  $BiddingFlag \leftarrow 0$ ,  $NumberValidPaths \leftarrow 0$ ,  $Average(s) \leftarrow 0$ 

2: while true do
3:    $Sequence[step] \leftarrow$  Current Node
4:   Get Neighbors of the Current Node
5:   Check
6:     1. Received multiple copies of same request? Cycle?
7:     2. Disconnected node?
8:   if 1,2 then
9:     break the routing and choose another  $s$  and  $d$ 
10:  end if
11:  if  $d$  is a child of Current Node then
12:     $NumberValidPaths = NumberValidPaths + 1$ 
13:    Calculate the overall bidding/payment
14:    Calculate the Average for all variables
15:    Check, is it the optimal path?
16:  end if
17:  for all Neighbors  $j \in N_i$  do
18:     $OriginalSource \leftarrow SourceNode.ID$ 
19:     $NextNodeID \leftarrow CurrNeighbors[NextGeneration]$ 
20:     $Currbid \leftarrow$  bid from  $CurrNeighbors$  to  $OriginalSource[NextGeneration]$ 
21:    1. calculate SINR for each channel, based on the transmitter
22:    2. calculate achievable capacity for each channel, based on the transmitter
23:    3. if Case 1 then
24:      Seller bids for the channel that satisfies the bit rate requirement and has
25:      the minimum price
26:    4. else {Case 2}
27:      calculate unit price for each channel
28:      sort channels by mergesort
29:      search for channel(s) satisfies the bit rate
30:    5. end if
31:  if Is there a bidding value?  $BiddingFlag == 1$  then
32:     $TotalPrice = TotalPrice + Currbid$ 
33:     $NextNode = CurrNode$ 
34:     $Sequence[Steps] = NextNode \rightarrow ID$ 
35:     $Steps = Steps + 1$ 
36:     $Routing(NextNode, DestinationNode)$ 
37:  end if
38: end for
39: end while
```

---

## CHAPTER 5: OPPORTUNISTIC SCHEDULING IN DSA NETWORKS

In case of multiple secondary users who try to operate on the same channel(s), it is the responsibility of the MAC protocol to manage the policy for them to access. However, multiple secondary users can use the same channel simultaneously, as long as they do not interfere with each other. Such constraints are usually represented by a *conflict graph*, where the nodes represent the SUs and an edge represents interference between the nodes that share the edge. In this chapter we present opportunistic scheduling algorithms for DSA networks. The first part of this chapter discusses algorithms that schedule one channel among multiple SUs. The second part discusses a multi-channel scheduling algorithm.

### 5.1 Single Channel Opportunistic Scheduling

We consider a dynamic spectrum access network with multiple secondary users randomly scattered over an area, and one primary user. The primary user owns a spectrum band which represents a channel. Secondary users contend for that channel to transfer their packets and perform other functions.

We consider a time-slotted system where a super-frame is a repetitive structure and is composed of  $k$  slots. We assume all the users to be synchronized. At the beginning of each super-frame, all users sense the channel to determine the status of the primary user. If the channel is idle, the slots in that superframe are allocated to the users. The sensing process also allows the users to determine their signal to interference and noise ratio (SINR) which we use as a metric to determine the goodness of a channel. Of course, SINR varies from one user to another due to spatial and temporal reasons. We represent  $SINR_{i,j}$  as the SINR for user  $i$  channel  $j$ .

In order to maximize throughput and to increase overall spectrum efficiency, multiple SUs can be assigned to the same time slot. At the same time, in order to insure collision free communication, all the nodes in the neighborhood of the allocated node  $i$  should be silent or must transmit on a different time slot. We tackle this situation by finding the *independent sets* ( $IS$ ) of the  $SUs$  based on the *interference graph*, which is then used by the scheduling algorithm to allocate the slots.

### 5.1.1 Scheduling using Independent Sets

With the randomly deployed network, each node will experience varying levels of SINR on the channel when vacant by the primary, as mentioned before. The objective of the proposed scheduling scheme is to allocate time slots to those nodes that experience high SINR—implying better throughput. However, the channel can not be allocated to nodes that

are interfering even if they have high SINR. As a result, we identify sets of non-interfering nodes and allow them to transmit on the same time slot. This problem of identifying such sets is similar to the problem of finding independent sets for the given network which is NP-complete [67].

An independent set contains all SUs such that they do not interfere with each other. SUs interfere with each other if they use the same channel and are within the transmission range of each other. Thus, we use an *interference graph* that gives the conflict of every SU. In the graph, an edge between nodes  $i$  and  $j$  means that they are close to each other and therefore can not use the same channel at the same time. However, the channel can be simultaneously used by nodes that belong to different independent sets since they do not interfere with each other. Such simultaneous re-use of channel increases the spatial utilization of the spectrum bands.

#### 5.1.1.1 Finding Independent Sets

As mentioned before we try to maximize the throughput and also address the fairness issue. As a result, we propose two algorithms to find the independent sets for each goal.

##### Algorithm 1: Maximizing the Throughput

We start with the entire network represented by graph  $G$ . We find the receiving node that can achieve the highest SINR given its corresponding transmitter. We call that node  $n_1$

and place it in the first independent set, represented by  $IS_1$ . All neighbors of  $n_1$  in the interference conflict graph cannot be placed  $IS_1$ . Therefore, we remove all such nodes and their edges from  $G$  to obtain  $G'$ . Note that, no node in  $G'$  interferes with  $IS_1$ . We follow the same process of finding the node with the highest SINR in  $G'$  (say  $n_2$ ) and place it in  $n_1$ . We further reduce  $G'$  and continue to find nodes that would belong to  $IS_1$  till the graph reduces to a null set. All the nodes placed thus are non-interfering with each other and can transmit on same slot. This set of nodes belong to independent set  $IS_1$ .

With the set of nodes placed in  $IS_1$ , we proceed to find the next independent set from nodes that are in  $G - IS_1$ . We follow the same process and obtain the second independent set,  $IS_2$ . Then we consider the remaining nodes in  $G - IS_1 - IS_2$  and continue in the same manner till all nodes belong to some independent set.

**An Illustrative Example:** Consider the network of 11 nodes as shown in Fig. 5.1. Suppose the SINR of the nodes labelled from  $A$  to  $K$  are sorted from the highest to the lowest—rank 1 is for the highest SINR and so on. Fig. 5.1, Fig. 5.2, and Fig. 5.3 show the steps of finding the independent sets for algorithm 1. Thus, the nodes in network  $G$  can be partitioned into 3 independent sets with 7, 2, and 2 nodes in each respectively.

#### Algorithm 2: Maximizing the Fairness

Though Algorithm 1 maximizes the throughput, it does not guarantee that all nodes are allocated an equal share of the time slots. Some will get more time slots than others. In order to be fair with respect to the number of slots allocated, we use the history of the allocation for making future scheduling decisions. Each node maintains a counter that contains the

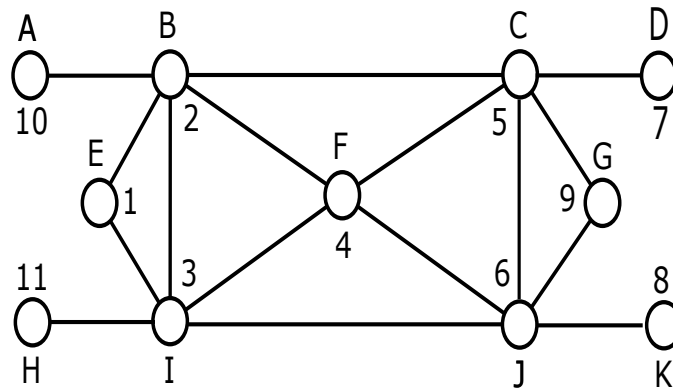


Figure 5.1 Network:  $G$ ;  $IS_1 = [E, F, D, K, G, A, H]$

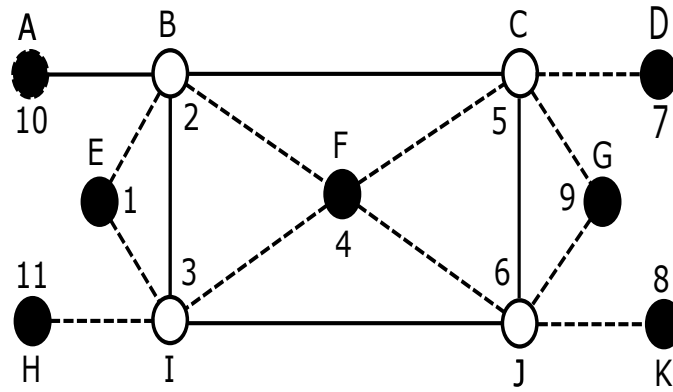


Figure 5.2 Network:  $G' = G - IS_1$ ;  $IS_2 = [B, J]$

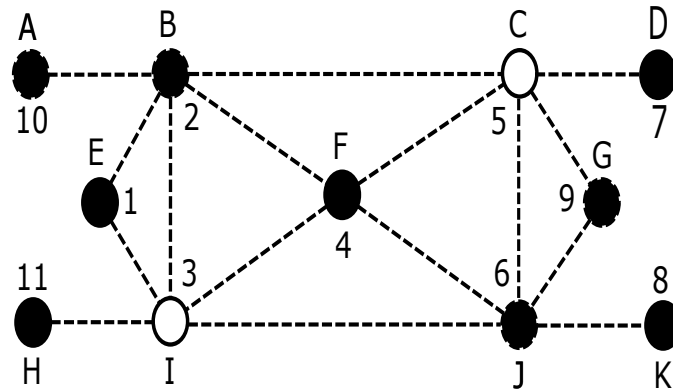


Figure 5.3 Network:  $G'' = G - IS_1 - IS_2$ ;  $IS_3 = [I, C]$

number of time slots already allocated to it so far. Generally, we use the same technique as in Algorithm 1 to create the independent sets, but this time we start with the node that has the lowest number of allocated time slots allocated so far and place it in  $IS_1$  and remove its neighbors and their edges from the network  $G$  to form  $G'$ . Then, we follow the same process of finding the node with the second lowest number of allocated time slots in  $G'$  and place it to  $IS_1$  and so on till the network reduces to a null set. We follow the same process to create the second independent set,  $IS_2$ , and continue in the same manner till all nodes belong to some independent set.

**An Illustrative Example:** Again, we use the same network of 11 nodes as shown in Fig. 5.4. Nodes are labelled from  $A$  to  $K$  and the adjoining numbers represent the number of slots already allocated to each node till the current super-frame. Fig. 5.4, Fig. 5.5, Fig. 5.6, and Fig. 5.7 show the steps of finding the independent sets for algorithm 2. Thus, the nodes in network  $G$  can be partitioned into 4 independent sets with 5, 3, 2, and 1 nodes in each respectively.

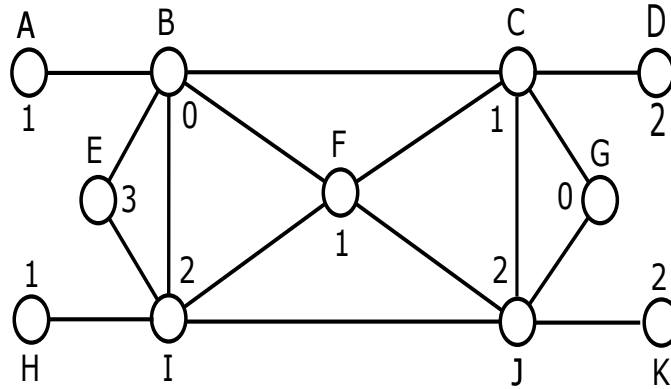


Figure 5.4 Network:  $G$ ;  $IS_1 = [B, G, H, D, K]$

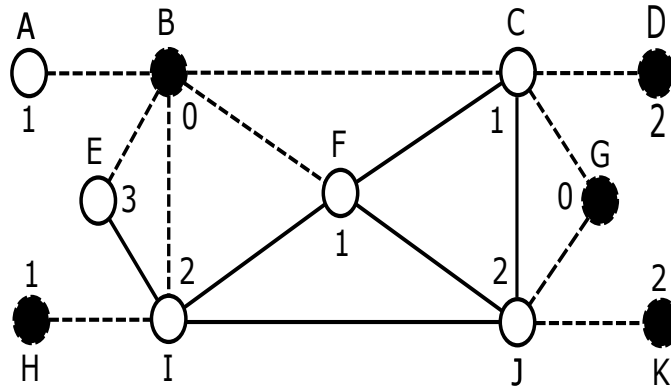


Figure 5.5 Network:  $G'=G-IS_1$ ;  $IS_2 = [A, C, I]$

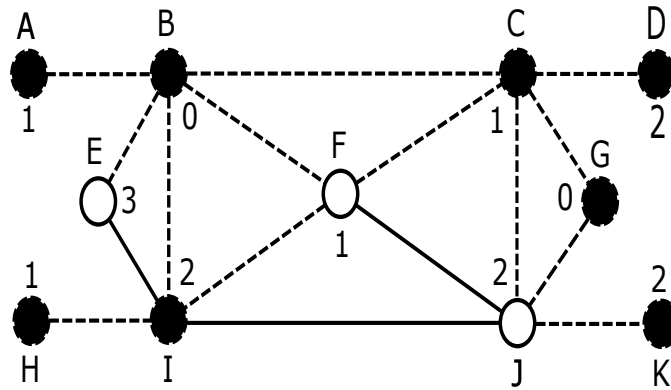


Figure 5.6 Network:  $G''=G-IS_1-IS_2$ ;  $IS_3 = [F, E]$

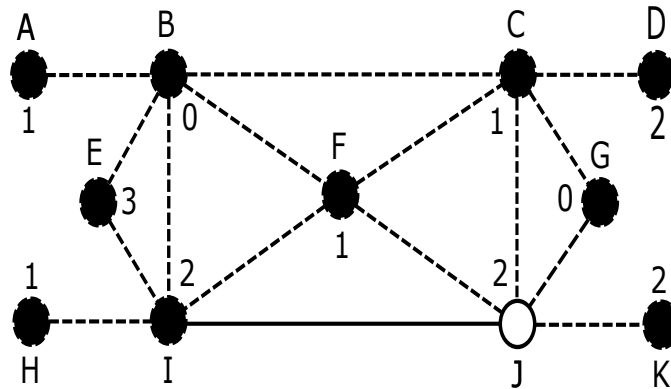


Figure 5.7 Network:  $G'''=G-IS_1-IS_2-IS_3$ ;  $IS_4 = [J]$

### 5.1.1.2 Allocation Process

For a randomly distributed network, it is likely that the first independent set will have the maximum number of nodes followed by the second independent set and so on. Though there could be cases where this observation does not hold true, we still use this argument as is true in *most* cases for a Poisson distributed network.

We assign the first time slot of the super-frame to  $IS_1$ , the second time slot to  $IS_2$  and so on. Two cases might arise. The number of time slots in a super-frame, ( $s$ ), could be equal to or more than the number of independent sets, say  $k$ . In that case, all the independent sets can be assigned a time slot. If on the other hand, the number of time slots in a super-frame is less than the number of independent set, then not all sets can be assigned time slots, and thus, we will have to somehow select the ones that will be assigned.

**Case 1:** When  $k \leq s$

When  $k = s$ , each independent set is uniquely assigned one slot within a super-frame. When  $k < s$ , some of the independent sets can be assigned more than one time slots in a super-frame. In order to identify which independent sets should be allocated more than once, we use the sets that have the highest SINR. Once the  $k$  independent sets are assigned to  $k$  time slots, there are  $s - k$  time slots that remain. We start with  $IS_1$  and then  $IS_2$  and continue in that fashion till all the remaining slots are assigned. If needed, the process of allocating the additional slots has to be repeated for more than once. At the end of the allocation, none of the time slots will remain unassigned.

For Algorithm 2, we follow the same process but start with the independent set that has the most number of un-allocated nodes.

**Case 2:** When  $k > s$

When  $k > s$ , some of the independent sets will not get a time slot within the current super-frame.

Using a counter, we keep track of the number of slots allocated to all users irrespective of the independent set they belong to. This counter is used to create the independent sets for Algorithm 2 which ensures fairness between users.

In case there is a tie between two independent sets such as the total number of allocated time slots in the  $IS_i$  is equal to the total number of allocated time slots in the  $IS_j$ , the set with more number of nodes is allocated. Fig. 5.8 shows an example of the two previous cases.

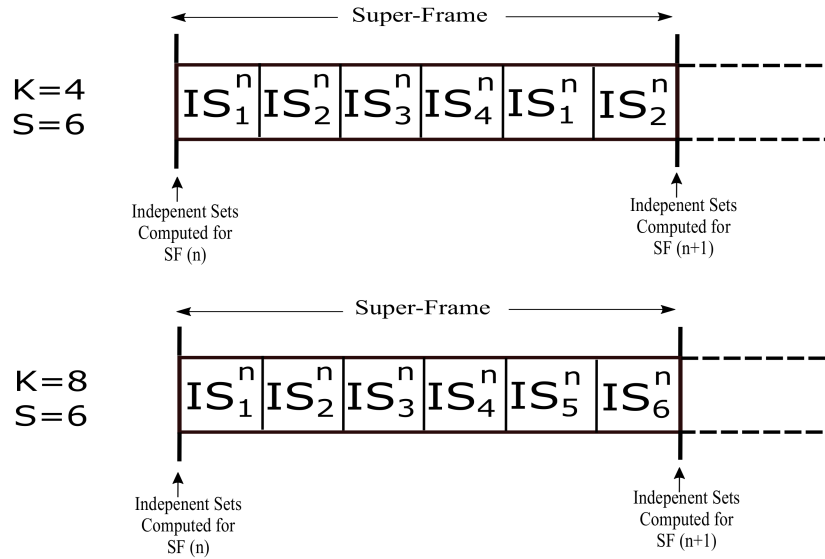


Figure 5.8 Allocation Process: Top figure shows,  $IS_1$  and  $IS_2$  get two slots in the super-frame. Bottom figure shows,  $IS_7$  and  $IS_8$  do not get any time slot.

### 5.1.2 Metrics

In order to measure the performance of the proposed scheduling algorithm 1 and algorithm 2 in single channel environment, we use three metrics: throughput, number of slots allocated, and fairness among users.

#### Throughput

Let us consider a system with  $n$  users. The MAC super-frame has  $s$  slots,  $s > 0$ , and consider  $T$  such super-frames where  $T$  is a large number.

We define and study the throughput in three different ways:

**Definition 1:** Average throughput of user  $i$  per slot denoted by  $t_i$  is defined as:

$$t_i = \sum_{k=1}^T \sum_{j=1}^s \frac{t_{i,j,k}}{s \times T} \quad (5.1)$$

where,  $t_{i,j,k}$  is the throughput of user  $i$  on slot  $j$  at super-frame  $k$ .

**Definition 2:** Average throughput per slot due to all users denoted by  $S_{slot}$  is given

as:

$$S_{slot} = \sum_{i=1}^n \sum_{k=1}^T \sum_{j=1}^s \frac{t_{i,j,k}}{s \times T} = \sum_{i=1}^n t_i \quad (5.2)$$

**Definition 3:** Average throughput per slot /per user is denoted by  $S_{slot/user}$  is given

as:

$$S_{slot/user} = \frac{\sum_{i=1}^n t_i}{n} = \frac{S_{slot}}{n} \quad (5.3)$$

### Number of Slots Allocated

Let us denote the total number of slots allocated to user  $i$  as  $c_i$  for  $T$  super-frames. Obviously, the total number of slots is  $s \times T$ .

The fraction of slots allocated to user  $i$  denoted by  $f_i$  is given as:

$$f_i = \frac{c_i}{s \times T} \quad (5.4)$$

The average fraction of slots allocated to all users denoted by  $\bar{f}$  is given by:

$$\bar{f} = \frac{\sum_{i=1}^{i=n} \frac{c_i}{s \times T}}{n} = \frac{\sum_{i=1}^{i=n} f_i}{n} \quad (5.5)$$

### Fairness among users

In order to determine whether the secondary users are receiving a fair share of the spectrum band, we resort to the popularly used Jain's fairness index. Recall, the total number of slots allocated to user  $i$  is  $c_i$ .

Jain's fairness index is defined as:

$$\mathcal{J}(c_1, c_2, \dots, c_n) = \frac{(\sum_{i=1}^n c_i)^2}{n \sum_{i=1}^n c_i^2} \quad (5.6)$$

The index varies from  $\frac{1}{n}$  (worst case) to 1 (best case).

### 5.1.2.1 Optimal Size of Super-frame

So far, we use  $s$  as a variable denoting the number of slots in each super-frame. Having a small  $s$  does not allow the opportunity for all users to be allocated; however, higher throughput is obtained per slot on the average. On the other hand, a large  $s$  allows the opportunity for more users to be allocated; however, a lower throughput is obtained per slot on the average. Thus, we seek to find an optimal value of  $s$  that would strike a balance between the average number of users assigned in each super-frame and average number of users assigned in each slot.

We define  $n_{j,k}$  as the number of users assigned on slot  $j$  of super-frame  $k$ . Thus, the average number of users assigned in each super-frame denoted by  $n_{sf}$  is given by:

$$n_{sf} = \frac{1}{T} \sum_{k=1}^T \sum_{j=1}^s n_{j,k} \quad (5.7)$$

The average number of users assigned in each slot denoted by  $n_{slot}$  is given by:

$$n_{slot} = \frac{1}{s \times T} \sum_{k=1}^T \sum_{j=1}^s n_{j,k} = \frac{n_{sf}}{s} \quad (5.8)$$

It can be noted that  $n_{sf}$  is an increasing function and  $n_{slot}$  is a decreasing function as  $s$  increases. We take the linear combination of the two to get

$$n = \alpha \times n_{sf} + (1 - \alpha) \times n_{slot} \quad (5.9)$$

where  $\alpha$  is a weighing factor.

## 5.2 Multi-channel Opportunistic Scheduling

Now, we consider a dynamic spectrum access network with multiple secondary and multiple primary users. There are  $U$  primary users and each owns an unique channel— thus, there are  $U$  channels. Secondary users rely on the primaries' channel(s) for their communication needs. We consider a time-slotted system where a super-frame is composed of  $N$  slots. We assume all the secondary users are synchronized through some beacon signaling over some common control channel. Though this widely used assumption is open to criticism, nevertheless we make use of a out-of-band signaling for secondary users' synchronization. At the beginning of each super-frame, all secondary users sense the channels to determine the status of the primary channel occupancy. If any primary channel is idle, that channel can potentially be used by the SUs in that super-frame. The sensing process also allows the secondary transmitter-receiver pairs to determine their SINR on each channel which we use as a metric to determine the goodness of each channel. Due to the spatial and temporal reasons, the SINR varies from user to user and also from channel to channel.

Let PU  $u$  use channel  $u$ . SINR at SU  $v$  on channel  $u$  is denoted as  $SINR_{v,u}$ . Note, the interference on channel  $u$  is from the PU and the SUs using channel  $u$  (as in (3.5)). SUs may or may not have packets to transmit. Backlogged packets reside in their queues. Irrespective of the packet handling process, packet arrivals are independent for each user and

follow batch Bernoulli process (BBP) with rate of  $\lambda$  packets per unit time. Packet lengths are exponentially distributed with mean  $l$  bits.

We consider a cellular-type network with a centralized scheduler which has the ability to compute or learn the SU locations; thus it can compute the interference graph. Each SU  $v$  reports  $SINR_{v,u}$  and its current queue length to the scheduler. Knowledge of  $SINR_{v,u}$  enables the scheduler to calculate the expected throughput of SU  $v$  on channel  $u$ .

Though the channel with the highest SINR is supposed to be the best channel for user  $v$ , allocating the best channel to user  $v$  need not maximize the throughput if user  $v$  does not have enough packets in the transmission queue. Thus, apart from  $SINR_{v,u}$ , the current queue size must also be taken into consideration. For example, it might be better to assign a channel to a user who can make better use of the channel, i.e., have enough packets waiting to be transmitted.

The objective of the scheduler is to schedule on frame-by-frame basis, i.e., assign slots in a frame to SUs that maximize the throughput while avoiding any conflict. It may be noted that the PU channel, SINR, and queue status can change in every slot – thus making the decision at the beginning of a frame non-optimal. Therefore, it is important to consider how and with what probability these parameters change. Such considerations do not guarantee optimal solutions, nevertheless, the solutions would be close to optimal, i.e., the solution we would have gotten if we considered slot-by-slot scheduling.

To capture the conflicts (interference) among the SUs, i.e., when they are within each other's transmission range, we use an *interference graph*, where an edge between nodes  $v$

and  $u$  indicates they are close-by and hence cannot simultaneously use the same channel. However, the channel can be simultaneously used by nodes that are non-conflicting. Such re-use of channel increases the spatial utilization of the spectrum bands.

### 5.2.1 Primary Channel Occupancy

We assume that the channel occupancy by primary users follows a two-state Markov process, and the process is identical and independent for each channel. The channel status could be 0 or 1 depending on the primary activity, where 0 means ‘idle’ and 1 means ‘busy’. The channel status in slot  $n$  is denoted by  $C_n$ . Similar channel occupancy model was used in [50] and [49]. The state transition matrix  $P(C_{n+1} | C_n)$  is defined as:

$$P(C_{n+1} | C_n) = \begin{pmatrix} P(0 | 0) & P(1 | 0) \\ P(0 | 1) & P(1 | 1) \end{pmatrix}$$

where  $P(\acute{i} | i)$  represents the transition probability from state  $i$  in the current slot to  $\acute{i}$  in the next slot. We consider two states,  $(i, \acute{i} \in \{0, 1\})$ . Also, with  $U$  channels from the same number of PUs, we denote the state transition matrix conforming to channel  $u$  where  $u \in \{1, \dots, U\}$  as  $P(C_{n+1}^u | C_n^u)$ . As mentioned before, we assume that all secondary users sense the channels at the beginning of the each super-frame to determine the status. As a result, the transitions probabilities are known to the SUs.

### 5.2.2 SINR Levels

The calculation methods of  $SINR_{i,j}$  is mentioned in subsection 3.2.1. To avoid any ambiguity, starting from this section till the end of multi channel scheduling, we calculate the interference the interference between SU  $v$  and PU  $u$  on channel  $u$  is calculated as [68, 50]:

Though  $SINR_{v,u} \in (SINR_{min}, SINR_{max})$ , where  $SINR_{min}$  and  $SINR_{max}$  are the minimum and maximum SINR values respectively, we quantize  $SINR_{v,u}$  as:

$$SINR_{v,u} \in \{S_0, S_1, S_2, \dots, S_{S-1}\} \quad (5.10)$$

where  $S$  is the number of quantization levels. With a little abuse of notation for ease of presentation, we write (5.10) as:

$$SINR_{v,u} \in \{0, 1, 2, \dots, S-1\} \quad (5.11)$$

We model the channel quality as a Markov process with  $S$  states. The state transition matrix is given by  $P(s_{n+1}^{u,v} | s_n^{u,v})$  for every channel  $u$  associated with user  $v$ . The probability that the channel state changes from quality  $j$  in slot  $n$  to quality  $\acute{j}$  in slot  $n+1$  is  $p(s_{n+1}^{u,v} = \acute{j} | s_n^{u,v} = j)$ , where  $j$  and  $\acute{j} \in \{0, 1, \dots, S-1\}$ .

### 5.2.3 SU Queues

A common assumption in the literature is that all SUs have back-logged traffic i.e., they have sufficient number of packets waiting in the output queue to be transmitted. This need not be true in all cases where some of the SUs might be lightly loaded and might even have empty queues at certain times. Thus, it becomes necessary to check if a SU has enough packets in the output queue to fill a slot, or at-least most of it. For example, it does not make sense to allocate a channel to a SU who does not have any packets to transmit even if the SU has favorable channel conditions.

We assume that packet arrivals follow the batch Bernoulli process (BBP) where up to  $P$  packets might arrive in a slot. We represent the process as:  $\alpha = [\alpha_0, \alpha_1, \dots, \alpha_P]$ , where  $\alpha_i$  is the probability that  $i$  packets arrive in a slot. One may think of a BBP as a type of a superposition of independent and identically distributed Bernoulli process [69].

The average arrival rate,  $\lambda$ , can be calculated as:  $\lambda = \sum_{i=0}^P i \times \alpha_i$ . The number of packets in the queue are identically and independently distributed. For any user, the packets are served in a first come first serve manner.

### 5.2.4 Scheduling using Non-Interfering Sets

Based on the PU channel status, SINR, and SU queue, we propose a scheduling technique that maximizes the expected throughput. We also consider the interference conflict graph

to identify the SUs who can be assigned the same set of channels to maximize the channel re-use spatially as well as temporally.

#### 5.2.4.1 Expected Throughput

We calculate the expected throughput of each user on each channel for each slot as  $T_n(u, v)$  where  $v$  is the user,  $u$  is the channel and  $n$  is the slot. Motivated by [46], we present a state vector  $X_n^{u,v} = [s_n^{u,v}, q_n^u]$  for  $n = 1, \dots, N$ , where  $s_n^{u,v}$  is the channel quality (i.e., SINR) for channel  $u$  for the user  $v$  in slot  $n$ , and  $q_n^u$  is the queue status of user  $v$  at the beginning of slot  $n$ . We assume that the status of a channel  $u$  in slot  $n$  is idle, that means  $c_n^u = 0$  and there is no PU activity and any SU can use channel  $u$  in slot  $n$ . For simplicity,  $X_n^{u,v}$  is written as  $X_n$ . The probability distribution for the state vector  $X_n$  in slot  $n$  is denoted as:

$$\Pi_n = [\Pi_{(s_n=i, q_n=k)}] \quad (5.12)$$

where  $i \in \{0, \dots, S-1\}$  and  $k \in \{0, \dots, Q\}$ , where  $Q$  is the maximum queue length. It represents the probability that the values of the states  $s_n$  and  $q_n$  are  $i$  and  $k$ , respectively.

The transitions among the states occur at the slot boundaries with probability  $P(x_{n+1} | x_n)$ , also written as  $P(s_{n+1} = \acute{i}, q_{n+1} = \acute{k} | s_n = i, q_n = k)$ , where  $\acute{i} \in \{0, \dots, S-1\}$ , and  $\acute{k} \in \{0, \dots, Q\}$ . All previous states variables are discrete values and modeled with Markov chain as above.

As a result, we can model the evaluation of the state vector  $x_n$  using a discrete-time Markov process with the state transition matrix as:  $P(X_{n+1} | X_n) =$

$$\begin{pmatrix} P(0,0|0,0) & P(0,1|0,0) & \cdots & P(S-1,Q|0,0) \\ P(0,0|0,1) & P(0,1|0,1) & \cdots & P(S-1,Q|0,1) \\ \vdots & \vdots & \ddots & \vdots \\ P(0,0|S-1,Q) & P(0,1|S-1,Q) & \cdots & P(S-1,Q|S-1,Q) \end{pmatrix}$$

The transition probability is written as:

$$\begin{aligned} P(\acute{i}, \acute{k} | i, k) &= P(s_{n+1} = \acute{i}, q_{n+1} = \acute{k} | s_n = i, q_n = k) \\ &= \frac{P(s_{n+1} = \acute{i}, q_{n+1} = \acute{k}, s_n = i, q_n = k)}{P(s_n = i, q_n = k)} \\ &= \frac{P(q_{n+1} = \acute{k} | s_n = i, q_n = k, s_{n+1} = \acute{i})}{P(s_n = i, q_n = k)} \times P(s_n = i, q_n = k, s_n + 1 = \acute{i}) \\ &= P(q_{n+1} = \acute{k} | s_n = i, q_n = k, s_{n+1} = \acute{i}) \times P(s_{n+1} = \acute{i} | s_n = i, q_n = k) \end{aligned} \tag{5.13}$$

We can reformulate eqn. (5.13) as:

$$P(\acute{i}, \acute{k} | i, k) = P(q_{n+1} = \acute{k} | s_n = i, q_n = k) \times P(s_{n+1} = \acute{i} | s_n = i) \tag{5.14}$$

since the states variables of  $s_{n+1}$  and  $q_{n+1}$  are independent from one another. This is because the queue length also depends on the arrival process which is no way related to the channel quality that governs the packet departure process.

The value of  $P(s_{n+1} = \acute{i} | s_n = i)$  is calculated as discussed in section 4.2.1.2, while the value of  $P(q_{n+1} = \acute{k} | s_n = i, q_n = k)$  is calculated based on the queue status,  $q_{n+1}$ , which

depends on i)  $q_n$ , ii) the number of arriving packets at the queue,  $a_n$ , in slot  $n$ , and iii) the number of packets departing,  $d_n$ , in slot  $n$ . Thus, we get:

$$q_{n+1} = q_n - d_n + a_n \quad (5.15)$$

Though a good SINR would allow many packets to be transmitted from the queue, but an SU might not have that many packets. Thus, the number of packets that can be transmitted is limited by the channel capacity or the current queue length. If  $t_n$  denotes that maximum number of packets that can be potentially transmitted as per Shannon's law, then the number of packets departing in slot  $n$  is given as:

$$d_n = \min(q_n, t_n) \quad (5.16)$$

The expected throughput  $T_n(u, v)$  on any channel  $u$  and SU  $v$  in slot  $n$  is calculated from equations (5.12) and (5.16) as:

$$T_n = \sum_{i,k} \Pi_{(s_n=i, q_n=k)} d_n \quad (5.17)$$

For each channel  $u$ , the expected throughput  $T_n(u, v)$  for all SUs  $M$  in all slots  $N$  can be written in one  $M \times N$  matrix. Similarly, for each SU, the expected throughput  $T_n(u, v)$  on all channels  $U$  in all slots  $N$  is written in one  $U \times N$  matrix.

#### 5.2.4.2 Finding Non-Interfering Sets

With randomly deployed nodes, each SU experiences varying SINR on vacant PU channels. The aim of the proposed scheduling is to allocate time slots to the SUs which experience high SINR and have sufficient queued packets to transmit.

For allocating channels to the SUs that offers the best throughput, we recall that the same channel cannot be allocated to users that are within the interfering range of each other. However, multiple non-interfering users can simultaneously transmit on the *same* channel. Therefore, we identify sets of non-interfering users which we call *Non-Interfering Sets* (NIS). The problem of finding such non-interfering sets in an arbitrary network is known to be NP-complete.

In addition to grouping the non-conflicting users, we try group them in a set such away that they would also maximize the system throughput. Thus, we propose an algorithm to find the NISs that also maximize the throughput.

**Algorithm:** As mentioned before, we have an expected throughput matrix for each channel for each user for each slot. We start with the entire network represented by  $G$ . We find the user (represented by ‘node’ in a graph) that can achieve the highest expected throughput for first slot. We call that node  $n_1$  and place it in the first non-interfering set, represented by  $NIS_1$ . All neighbors of  $n_1$  in the interference conflict graph cannot be placed  $NIS_1$ . Therefore, we remove all such nodes and their edges from  $G$  to obtained  $G'$ .

Suppose, the throughput of  $n_1$  was maximized for some channel, say  $c_1$ . We follow the same process of finding the node with the highest expected throughput for first slot for the same channel ( $c_1$ ) in  $G'$  (say  $n_2$ ) and place it in  $NIS_1$ . Note that, no node in  $G'$  interferes with  $NIS_1$ . We further reduce the  $G'$  and continue to find nodes that would belong to  $NIS_1$  till the graph reduces to a null set. Thus, all the nodes belonging to non-interfering set  $NIS_1$  do not interfere with each other and can transmit on channel  $c_1$  in first slot.

With channel  $c_1$  allocated to  $NIS_1$ , we find the second non-interfering set from the remaining channels  $U - c_1$ , where  $U$  is the total channel set. We find the channel from  $U - c_1$  that offers the maximum throughput. Suppose, that channel is  $c_2$ . We continue as before to find  $NIS_2$  which is allocated  $c_2$ .

Note, it may happen that  $NIS_1$  and  $NIS_2$  can have some or all of the nodes common. However, since the two sets use different channels, having common node(s) between the two non-interfering sets is allowable. Continuing similarly, we get  $U$  non-interfering sets – each having a unique channel. Note that, some nodes may not belong to any non-interfering set.

### **An Illustrative Example:**

Let us consider a network with 6 nodes labeled  $A$  through  $F$  and 3 channels, as shown in Fig. 5.9. The expected throughput on each channel in a slot is shown next to a node. For example,  $C$  has the expected throughput on channels 1, 2, and 3 respectively 17, 24, and 13.

Using the proposed algorithm, we form the non-interfering sets. With 3 channels, the non-interfering sets formed are:  $\{B, D\}$ ,  $\{B, E\}$ , and  $\{A, C\}$ . It may be noted that,  $B$

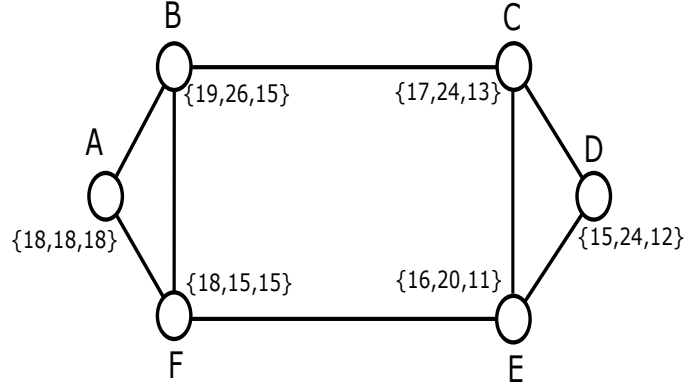


Figure 5.9 Finding the Non-interfering Sets.  $NIS_1 = (B, D)$  uses channel  $c_1$ ;  $NIS_2 = (B, E)$  uses channel  $c_2$ ;  $NIS_3 = (A, C)$  uses channel  $c_3$ .  $B$  belongs to two NISs while  $F$  belongs to none.

belongs to two non-interfering sets, whereas  $F$  does not belong to any. Though  $F$  is not allocated any channel during this slot, it may be assigned channel(s) in the other slots in the super-frame.

### 5.2.4.3 Allocation Process

Channels are allocated to the nodes on slot-by-slot basis. The number of available channels are allocated to the equal number of non-interfering sets. In the first slot,  $NIS_1$  is allocated channel  $c_1$  that maximizes its throughput. Then, channel  $c_2$  is allocated to the  $NIS_2$ , and so on, until all channels are allocated to all the non-interfering sets. The same process is repeated for all slots in a super-frame. As a result, a node may be assigned more than one channel in the same slot. Also, multiple nodes are allocated the same channel. Thus, the process increases the overall throughput.

### 5.2.5 Metrics

In order to measure the performance of the proposed algorithm for scheduling in multi-channel environment , we use five metrics: throughput, number of slots allocated, fairness among users, delay, and blocking probability.

#### Throughput

The system is considered to have  $M$  users and  $U$  channels. We consider a large period of time consisting of  $T$  super-frames (each consisting of  $N$  slots) and quantify throughput during this time. Obviously, the total number of slots is  $N \times T$ . Setting  $T \rightarrow \infty$  gives the steady state average.

We define and study the throughput in three different ways.

*Definition 1:* Average throughput of user  $v$  per slot for all channels is denoted by  $t_v$  is defined as:

$$t_v = \sum_{l=1}^T \sum_{k=1}^N \sum_{u=1}^U \frac{t_{v,u,k,l}}{n \times T} \quad (5.18)$$

where,  $t_{v,u,k,l}$  is the throughput of user  $v$  on channel  $u$  in slot  $k$  of super-frame  $l$ .

**Definition 2:** Average throughput per slot due to all users and all channels denoted by  $S_{slot}$  is given as:

$$S_{slot} = \sum_{v=1}^M \sum_{l=1}^T \sum_{k=1}^N \sum_{u=1}^U \frac{t_{v,u,k,l}}{N \times T} = \sum_{v=1}^m t_v \quad (5.19)$$

**Definition 3:** Average throughput per slot/per user is denoted by  $S_{slot/user}$  is given as:

$$S_{slot/user} = \frac{\sum_{v=1}^m t_v}{M} = \frac{S_{slot}}{M} \quad (5.20)$$

### Number of Slots Allocated

Denote the total number of slots allocated to user  $v$  for all channels as  $c_v$  for  $T$  super-frames.

The fraction of slots allocated to user  $v$  is given as:

$$f_v = \frac{c_v}{N \times T} \quad (5.21)$$

The average fraction of slots allocated to all users is:

$$\bar{f} = \frac{1}{M} \sum_{v=1}^{v=M} f_v \quad (5.22)$$

### Fairness among users

Just like section 5.1.2, we also use JFI for this algorithm.

### Delay

The overall system delay is obtained by absorbing the Markov chain. Packets arrive at the queues of all SUs with an arrival rate of  $\lambda$  packets per unit time. The queue length varies based on the number of packets that arrive and the number of packets that depart

the queue. The delay incurred by a packet is the difference between the time when it enters and leaves the queue. We denote  $D_{p,v}$  as the delay for packet  $p$  for user  $v$ . Average system delay is obtained as:

$$\text{Avg. System Delay} = \frac{\sum_{v=1}^M \sum_{all\ p} D_{p,v}}{N_p} \quad (5.23)$$

where  $N_p$  is the total number of packets sent from all users.

### **Blocking probability**

It is defined as the probability of a user not getting a slot for transmission. In other words, the blocked users are the ones who do not belong to any non-interfering set. We find the long term fraction of those users.

## CHAPTER 6: SIMULATION MODEL AND RESULTS

In this chapter, we discuss the simulation models, experiments and corresponding results of our work. In order to evaluate the performance of proposed mechanisms, we conducted extensive simulation experiments in C++ and MATLAB on UNIX and Windows based platforms. Our intention was to generate and test situations that represent the real world scenarios as closely as possible.

Our simulation study is broadly divided into six parts. In section 6.1 and in section ??, we discuss the results of auction-based channels allocation. In section 6.3, we present the results of the proposed pricing-based routing mechanism. In section 6.4, we show the results of using auction theory for routing. The results for opportunistic scheduling techniques are discussed in sections 6.5 and 6.6.

Unless otherwise stated, we considered  $N$  nodes randomly scattered over a  $100 \times 100$  square grid. All nodes are assumed to have the same transmission range. By using different values of the transmission range, we could get different topologies— from sparsely connected to densely connected.

### 6.1 Single Auction-Based Channel Allocation

We consider up to 200 nodes which were randomly scattered over the network. We fixed the number of primaries to 8. The transmit power of a primary PU was 3.010 dBW, while the transmit power of a secondary was 0 dBW. Two nodes are considered neighbors if their mutual distance is within the transmitting range of 10m, 20m, and 40m. Nodes calculate the interference on each channel based on the interference received from their neighbors on all channels. Based on the interference, nodes calculate the SINR on each channel and generate the preference list. The background noise,  $n_0$ , is assumed to be -29.999 dBW. The size of the preference list is different for each node as it is dependent on the location of the nodes. The price of each channel is assumed to be uniformly distributed between 1 and 100.

In Fig. 6.1, we show the total number of nodes that are assigned channels for three different values for the transmitting range  $R$ . As expected, higher transmitting range means more interference to other nodes which decreases the chances of using a channel for more than one node. Also, the ability to satisfy nodes with first preference is decreased.

In Fig. 6.2, we show what fraction of nodes get the most preferred channel. With more number of nodes trying to acquire their most preferred channels, the chances of acquiring their most preferred is the least. However, if nodes are willing to settle with the lower preferred nodes, then more nodes could be allocated. In fact, we show what fraction get within a specific choice as the cumulative distribution function. Though nodes do not necessarily get their most preferred channels, the overall channel utilization and the total revenue increase.

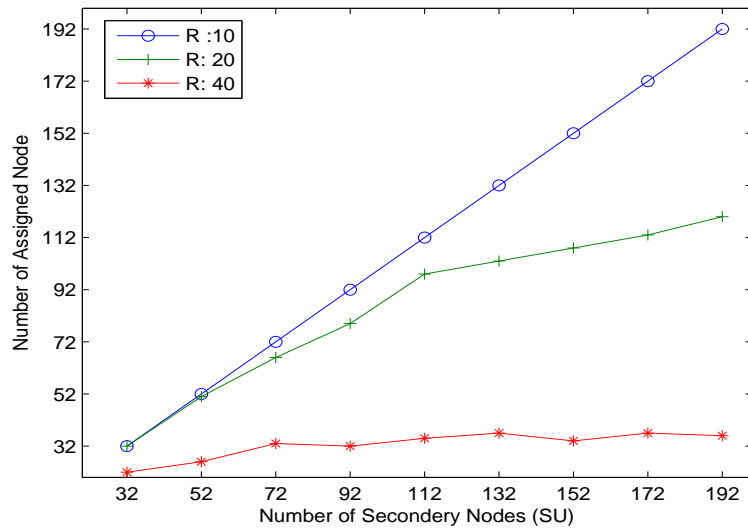


Figure 6.1 Number of nodes with channels assigned

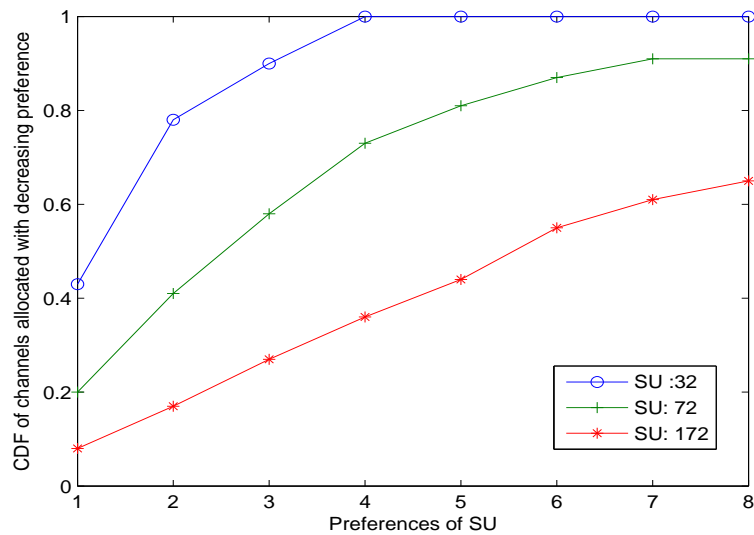


Figure 6.2 CDF of channels allocated with decreasing preference

Fig. 6.3 shows the effect of transmitting range on the total number of SUs that eventually get a channel assigned. Higher transmitting range leads to more interfering nodes resulting in less nodes with channels assigned.

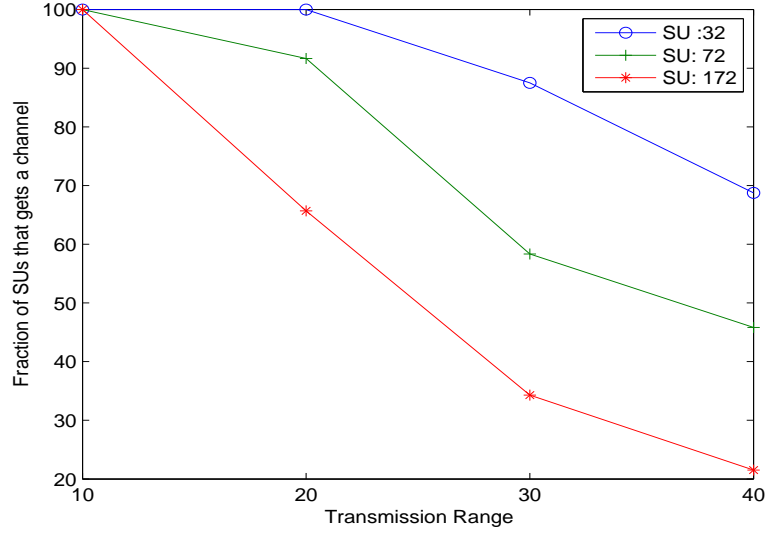


Figure 6.3 Effect of transmission range (R) on channel assignment

In Fig. 6.4, we show the revenues generated by each primary when the number of SUs were 32, 72, and 172. The revenues by each is based on the total number of nodes, conflict between nodes, and position of the PUs. It is worth mentioning that the position of the PUs also affect their revenue. Some PUs located close to many SUs will lead to increased interference; thus reduced SINR. As result, SUs will have a lower preference for the channels of that PU. Moreover, the positions of the SUs also affect the total revenue. If many SUs are located near to each other then i) they would interfere with each other and ii) they would likely to have the same preference for channels. Consequently, the chances of assigning the

preferred channels to all decreases; thus the revenue generated is less. Fig. 6.5 shows the total revenue generated by all the primaries with increasing SUs.

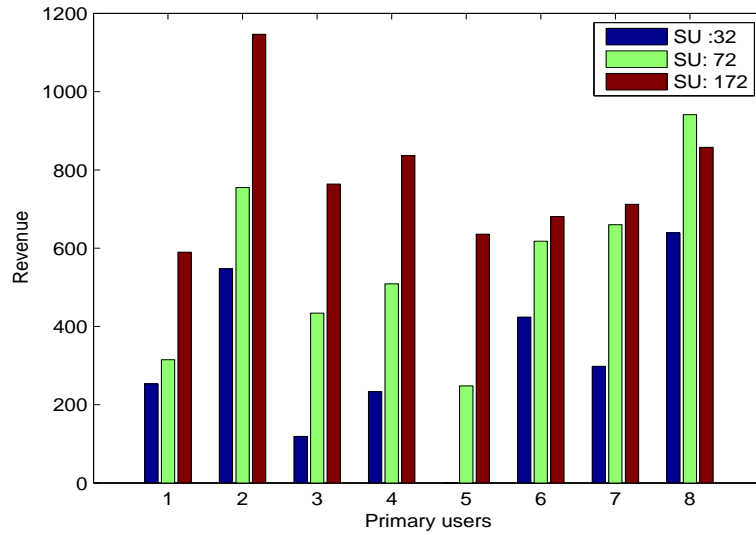


Figure 6.4 Revenues for each primary

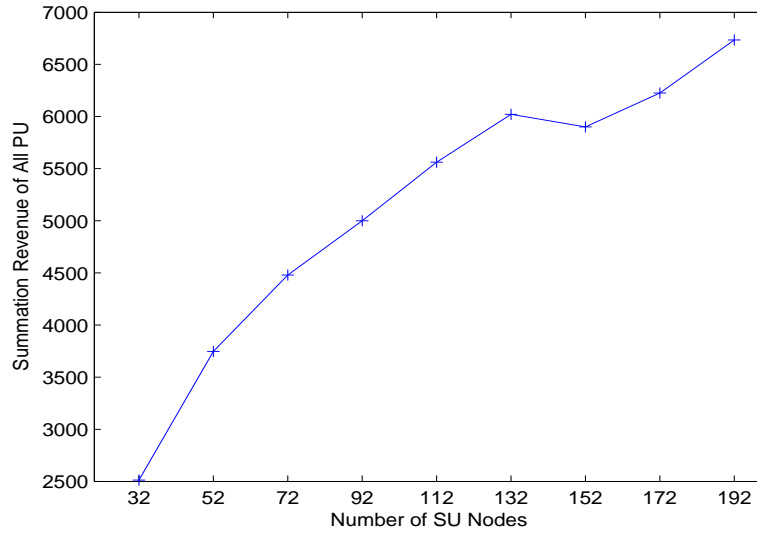


Figure 6.5 Total revenue for all primaries

Fig. 6.6 shows the fairness of the proposed algorithm based on Jain's fairness index. The overall fairness of the algorithm decreases when the number of secondary users and transmission range increase. More number of secondary users and larger transmission range lead to more number of unassigned users, as shown in Fig. 6.1. As a result, the overall fairness decreases. On the other hand, when the number of secondary users increase within the small transmission range ( $R = 10$ ), there are no unassigned nodes and the fairness of the proposed method is close to perfect (0.99).

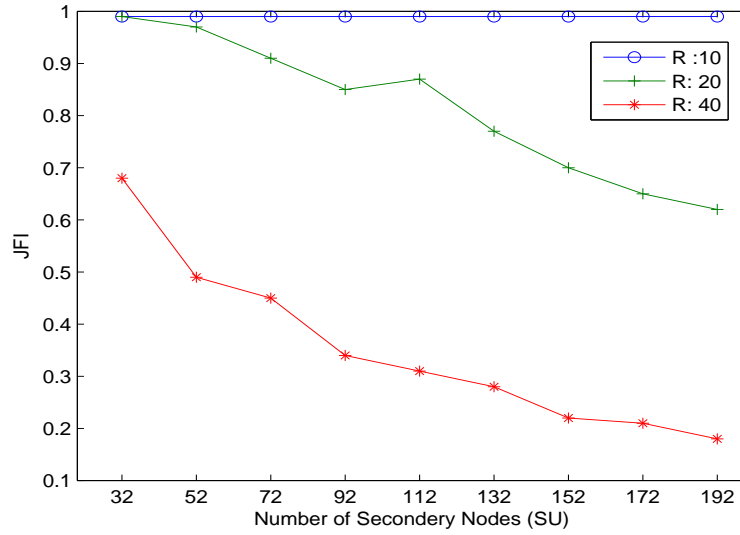


Figure 6.6 Jain's Fairness Index

## 6.2 Double Auction-Based Channel Allocation

We consider up to 30 sellers and 50 buyers. Each auction was conducted for four rounds for reasons discussed later. The transmit power of a primary PU was kept at 2 W (3.010 dBW),

while the transmit power of a secondary was kept at 0 dBW. Two nodes are considered neighbors if their mutual distance was within 20m from each other. All nodes calculated the SINR for each channel and generated the preference list– the size of which varied based on the location of the nodes. The background noise,  $n_0$ , is assumed to be -29.999 dBW. The price of each band was assumed to be uniformly distributed between 1 and 100. The number of bands to be sold by each seller was randomly chosen between 1 and 6, while the demands of buyers are randomly chosen between 1 and 3 bands.

Since the method of grouping of conflict-free buyers is an important issue, we explore three different ways, namely MAX-SINR, MIN-Degree, and MAX-Degree. For MAX-SINR, the node with the highest SINR is assigned to the first group. Similarly, for MIN-Degree (MAX-Degree) the node with the minimum (maximum) number of neighbors was assigned to the first group.

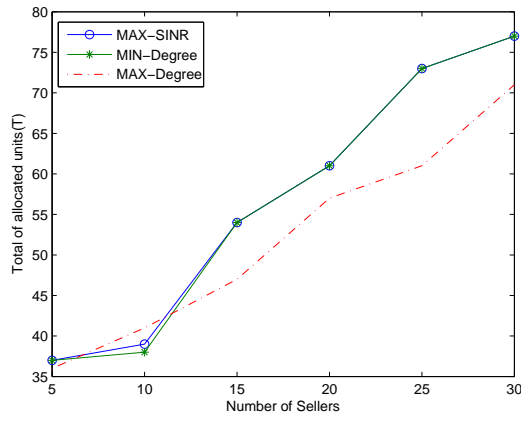
Fig 6.7 shows how i) total number of allocated bands ( $T$ ), ii) allocated bands per seller ( $U$ ), and iii) Jain's fairness index vary as the number of sellers increases. With more sellers, the buyers had more options to buy from and hence could acquire more bands as shown in Fig. 6.7(a); however, due to competition and interference the number of bands sold on average by a seller decreased as shown in Fig. 6.7(b). Fig. 6.7(c) shows the Jain's fairness index (JFI) for the three algorithms which determines whether the buyers receive a fair share of the spectrum.

Generally, the performance of MAX-SINR, and MIN-Degree are better than MAX-Degree. MAX-SINR and MIN-Degree give fewer number of groups with larger number of

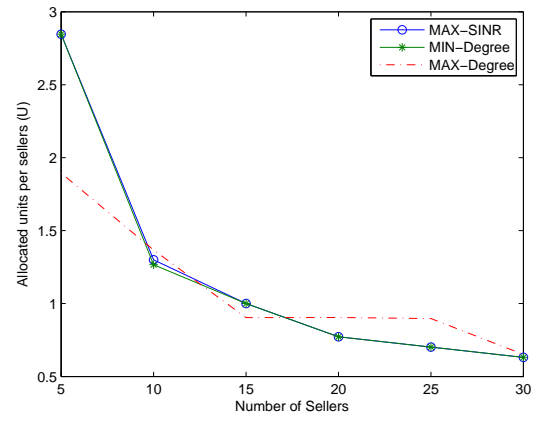
buyers in one group which indicates that the number of buyers in one VG is large. As a result, the total number of successfully auctioned units ( $T$ ) is higher when using MAX-SINR or MIN-Degree, than MAX-Degree. Also, MAX-SINR and MIN-Degree perform better than MAX-Degree in terms of ( $U$ ) and fairness. However, MAX-SINR and MIN-Degree show almost similar performance because the node with highest SINR is usually the node with minimum number of neighbors. Thus, the starting node for both these methods are most often the same. However, MAX-SINR is slightly better than MIN-Degree in term of fairness, so we use MAX-SINR for the rest of results.

Generally, when there are more number of the sellers, the auctioneer's revenue and sellers' revenue increase as shown in Fig. 6.8. Moreover, with more auction rounds, the overall revenue increases as there could be buyers who win in subsequent rounds– but the number of buyers who win in  $(r + 1)$  th round is usually lower than the number winning in the  $r$ th round. For the experiments, there is not much different between third and fourth rounds– so we stop after three rounds for the remaining experiments.

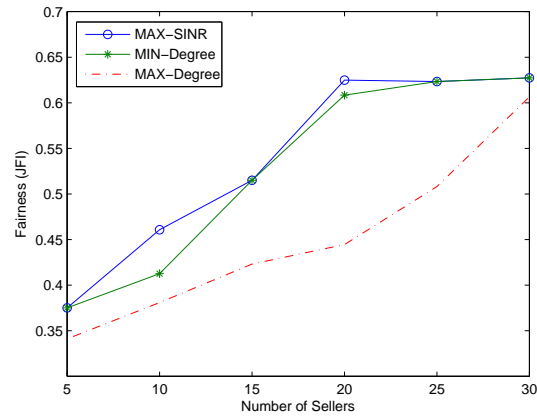
It is to be noted that the auctioneer's revenue is higher than of the sellers because of virtual grouping. The bid of a VG is the number of the buyers in that VG multiplied by the minimum bid of the eliminated buyer. So, the difference between the asking price of the seller and the actual bidding is more which goes to the auctioneer. With increasing number of the *buyers*, the auctioneer's and sellers' revenue increase as show in Fig. 6.9. The number of sellers was kept at 10.



(a)

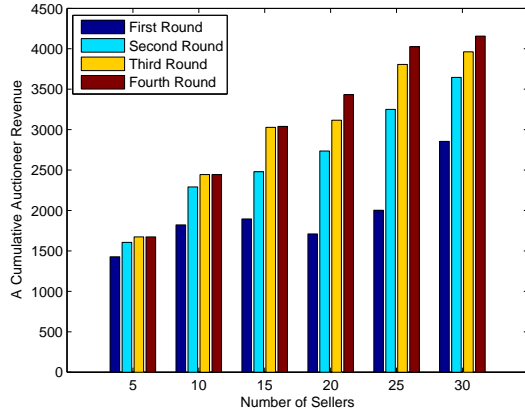


(b)

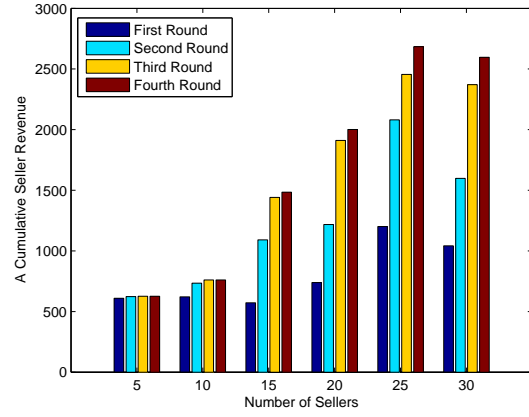


(c)

Figure 6.7 Comparisons of grouping methods

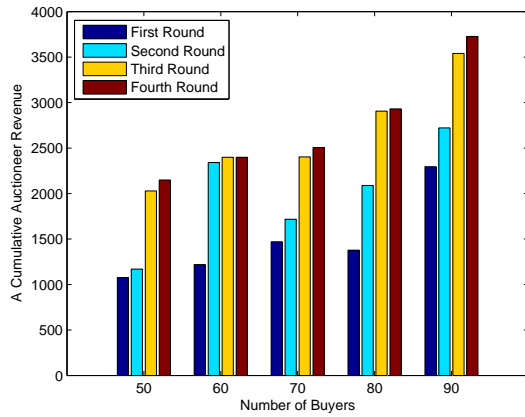


(a) Auctioneer's Revenue

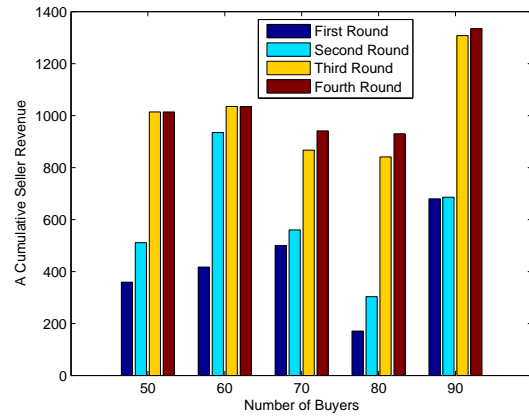


(b) Sellers' Revenue

Figure 6.8 Auctioneers' and Sellers' Revenue with increasing number of sellers



(a) Auctioneer's Revenue



(b) Sellers' Revenue

Figure 6.9 Auctioneers' and Sellers' Revenue with increasing number of buyers

Fig. 6.10 shows that the proposed method is successful in satisfying more buyers in the first round with their most preferred channel. The number of winning buyers decreases within more rounds, because of the decreasing number of available units to sell. However, with more number of sellers, the number of winning buyers increases and the ability to satisfy them with their preferred channels also increases. As mentioned earlier, most of the allocations are made within the first three rounds.

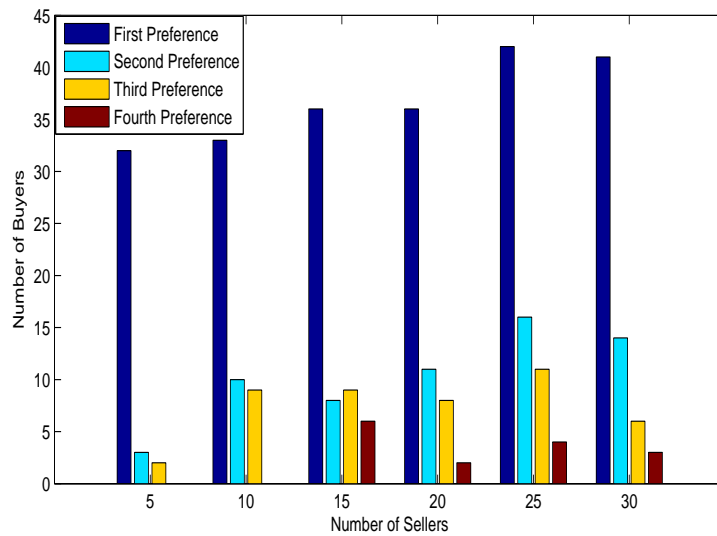


Figure 6.10 Number of satisfying buyers with their preference, with increasing number of sellers and auction' rounds

We compare our scheme with TDAMH as it based on almost same criteria as was shown in Table 2.1. There are two ways to do the grouping in TDAMH: i) maximize the number of groups, and ii) minimize the number of groups. These two methods are equivalent to our MAX-Degree and MIN-Degree. Thus, we compare MAX-Degree with TDAMH-maximize number of groups in terms of following metrics: i) total number of allocated bands, ii) auctioneer revenue, iii) sellers revenue, and iv) fairness. Since we want to have a fair com-

parison, we use the same network topology such that the SINR for all buyers remain the same. Moreover, we allow any buyer to bid for any seller.

The comparisons are shown in Figs. 6.11, 6.12, 6.13, and 6.14. As expected, total number of allocated bands, auctioneer revenue, sellers revenue, and fairness for both methods increase with more number of sellers. However, PreDA performs better than TDAMH since it does the allocation considering the preference for channels by the buyers. As a result, the total number of allocated bands and the revenue for PreDA is more than in TDAMH. The fairness is also higher for PreDA.

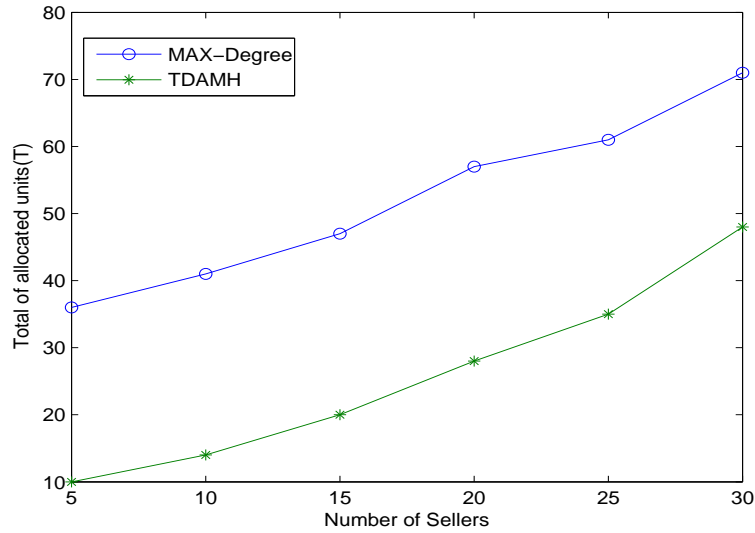


Figure 6.11 Comparison in terms of total allocated bands (T)

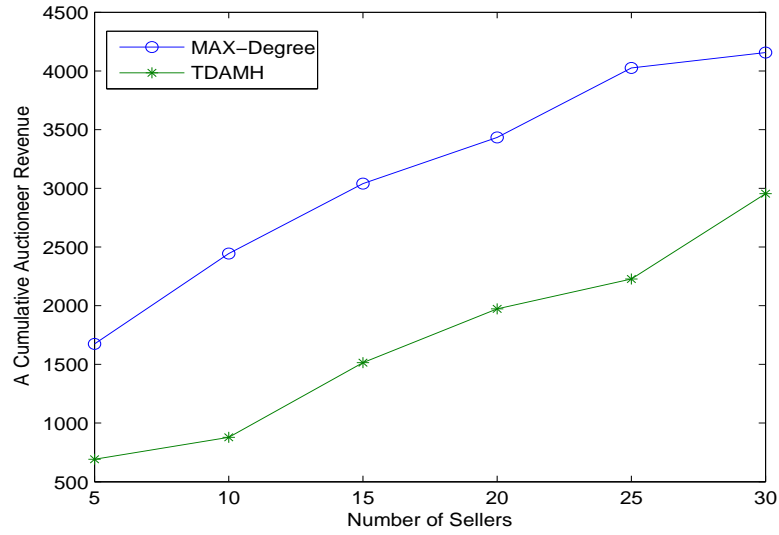


Figure 6.12 Comparison in terms of auctioneer revenue

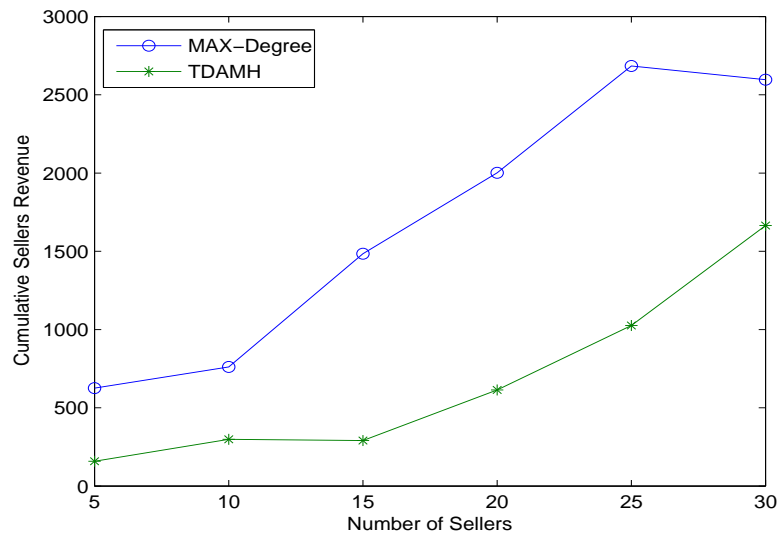


Figure 6.13 Comparison in terms of sellers revenue

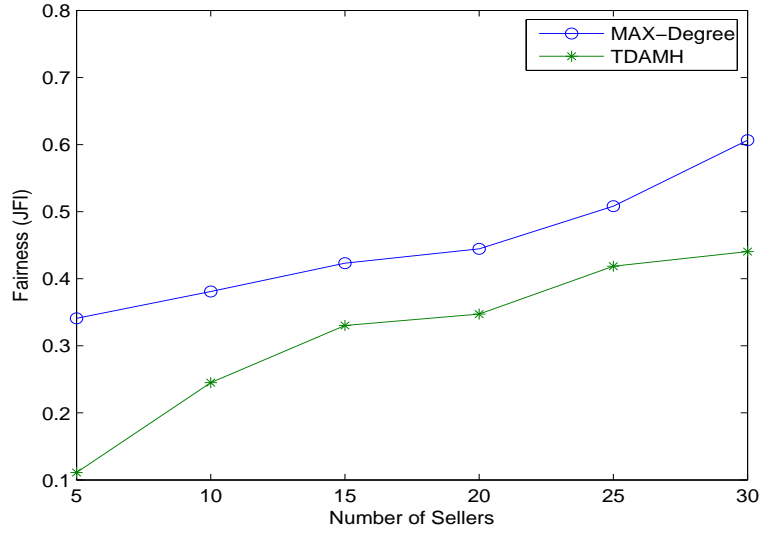


Figure 6.14 Comparison in terms of fairness (JFI)

### 6.3 Pricing-Based Routing

We assume that secondary nodes acquire different amounts of bandwidth from the primary user by paying different prices. The cost prices ( $p_{cost}$ ) are assumed to be uniformly distributed between 0 and 1. Also, nodes are cooperative in the sense that they will forward packets when paid the right amount. We consider that there are up to 40 nodes. The price that a node pays to the primary for acquiring spectrum is assumed to be uniformly distributed between 1 and 100. Bandwidth requirements for the source nodes are assumed to be 2 KHz. Two nodes are considered neighbors if their mutual distance between them is within 20 m, i.e., transmission range is 20 m. The buffer of each node can hold upto 100 packets beyond which packets are tail-dropped.

### 6.3.1 Route Selection Implementation

We implement the proposed routing scheme to find possible routes between the source and destination nodes. Here, the objective is to find the best relay node at every hop. However, such greedy selection might not always yield a valid path to the destination. Thus, only a subset of the paths will be valid. By *valid*, we mean the paths that can satisfy the bit-rate requirements of the source node. Out of all possible valid routes, we are able to identify the one that is the most optimal in terms of the price paid. As the path loss exponent ( $\alpha$ ) has an impact on signal propagation and hence the transmitter-receiver distance for correct decoding, we use two values of  $\alpha$  (2 and 3).

To find average values, we execute the simulation runs for random topologies. For a given topology, we randomly chosen (without repetition) 200 source-destination pairs, if there are that many. For each source-destination pair, we calculate the average for all variables for all valid paths. For continue to do this for 10 different topologies, with all other parameters remaining the same. We eliminate any observation, that may arise from a skewed topology.

### 6.3.2 Results

In Fig. 6.15, we show how the average number of hops varies with increasing number of nodes in the network. It is to be noted that with more neighboring nodes, a transmitter chooses a relay node that is relatively closer than others. This is because shorter transmitter-

receiver distances will yield higher SNR and higher thus capacity. With higher capacity, the payment to be made by the buyer (transmitter) to the seller (receiver) will be low. Thus, the intermediate nodes choose shorter hops (distance-wise) resulting in more hops per route. As expected, with a lower path loss (i.e.,  $\alpha = 2$ ) the number of hops is more than higher path loss (i.e.,  $\alpha = 3$ ).

As discussed earlier, not all possible paths are valid. In Fig. 6.16 we show the absolute number of valid path and in Fig. 6.17 we show it as a fraction of all possible paths. As expected, more paths are obtained with  $\alpha = 2$  than  $\alpha = 3$ .

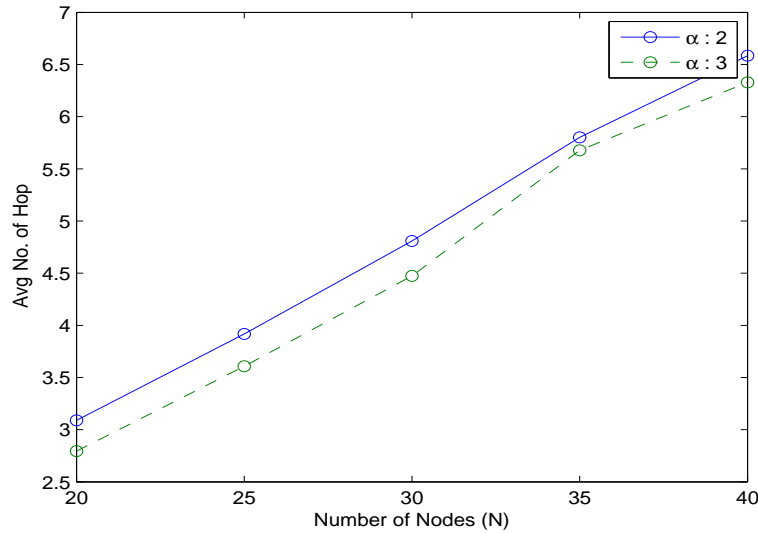


Figure 6.15 Average no. of hops

Next, we show how the cost of routes vary with increasing number of nodes. In particular, we show the average optimal price in Fig. 6.18. In fact, the route with the optimal price is eventually chosen as the route for data transfer. In Fig. 6.19, we simply

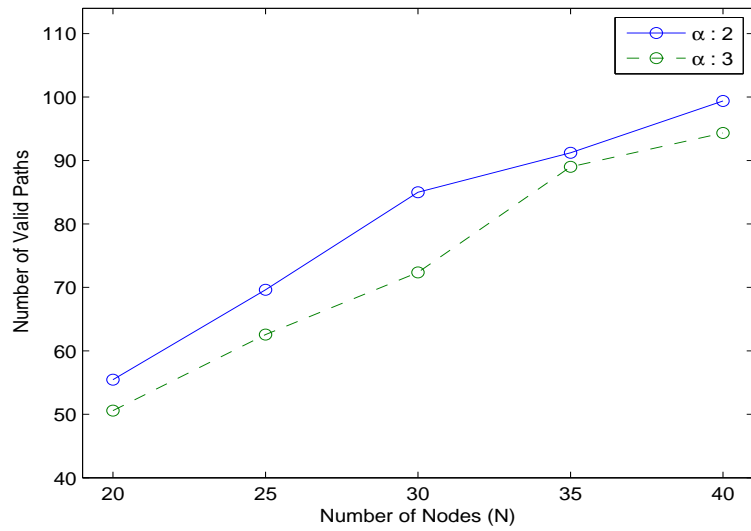


Figure 6.16 No. of valid paths

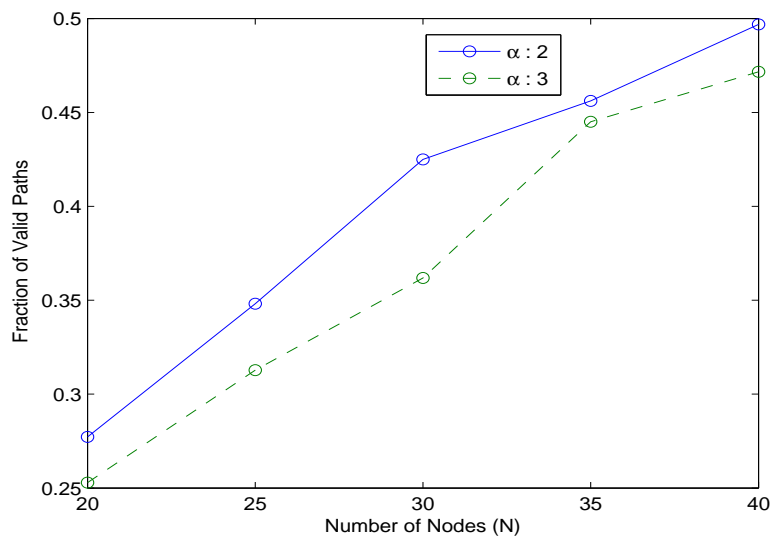


Figure 6.17 Fraction of valid paths

divide the optimal price by the corresponding hop count to get the average cost per hop. It is evident that the cost is more for  $\alpha = 3$  than  $\alpha = 2$ .

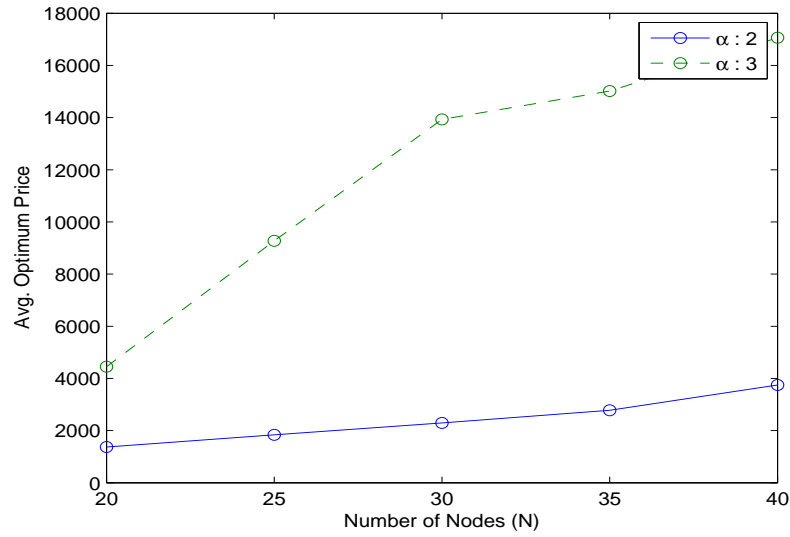


Figure 6.18 Avg. optimal price

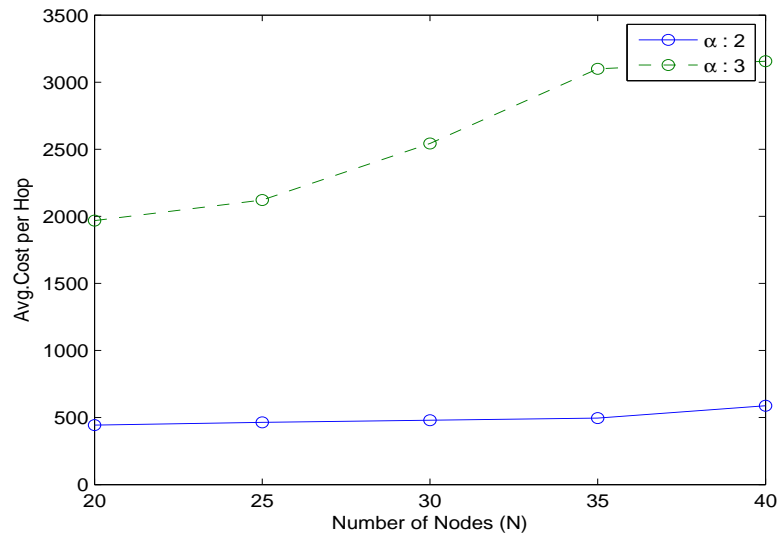


Figure 6.19 Average cost per hop

## 6.4 Auction-Based Routing

We consider up to 40 nodes which were randomly scattered over a network. All nodes had the same transmitting range of 20 m. Thus, two nodes are considered neighbors if their mutual distance between them is within 20 m. Nodes calculate the interference on each channel based on the interference received from their neighbors on all channels. All channels have a bandwidth of 2 KHz. Required bit rate is assumed to be 2 Kbps. The transmit power is 0 dBW. Also, the background  $n_0$  noise is assumed to be -30 dBW. The price of each channel is assumed to be uniformly distributed between 0 and 1. The price that a node pays to the primary for acquiring spectrum for each channel is assumed to be uniformly distributed between 1 and 100.

We implement the proposed routing scheme to find all possible routes between the source and destination nodes, and discover the most optimal one. Optimal path means the path with the minimum total payment. However, for some source-destination pairs there could be paths that are not valid. An invalid path means a path that cannot satisfy the bit rate requirement of the source node.

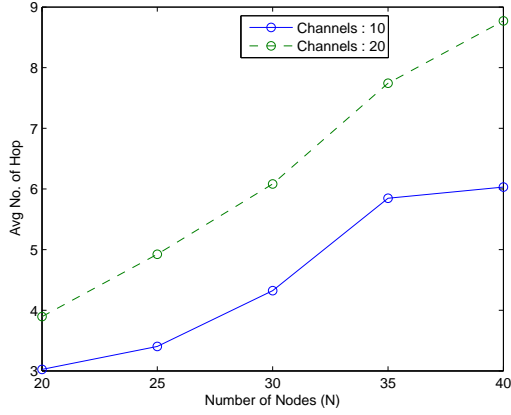
As the number of channels has an impact on data availability and capacity, we study their impact using two different values of the available channels (10 and 20). In order to get a trusted result, we run the simulation for different topologies and calculate the average values. For each topology, we randomly choose (without repetition) 200 source-destination pairs. For each source-destination pair, and for all valid paths, we calculate the average

for all variables. Then we repeat the experiments for 10 different topologies, with all other parameters remaining the same. We discuss our results as per the two cases we discussed in chapter 4

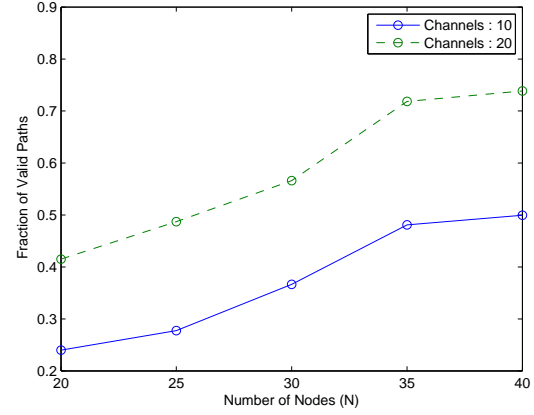
#### **6.4.1 Case 1: Seller sells only one channel**

As number of nodes in the network increases, the average number of hops in the optimal route also increases as shown in Fig. 6.20(a). With more neighboring nodes, a transmitter chooses a closer node than others. This is because shorter transmitter-receiver distances yields higher SINR and thus higher capacity. With higher capacity, the payment to be made by the buyer (transmitter) to the seller (receiver) will be low. Thus, the intermediate nodes choose shorter hops (distance-wise) resulting in more hops per route. With more number of channels the number of hops are also more.

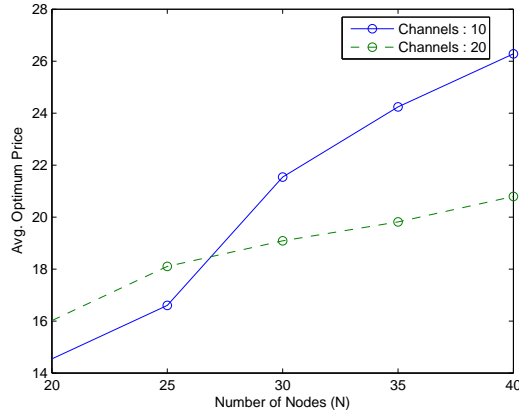
As mentioned before, not all possible paths are valid. Fig. 6.20(b) shows the fraction of valid paths over all possible paths. With more number of channels more paths are found. Fig. 6.20(c) shows how the cost of optimal routes changes with increasing number of nodes and number of channels.



(a) Average no. of hops



(b) Fraction of valid paths



(c) Avg. optimal price

Figure 6.20 Routing with only one channel per hop

#### 6.4.2 Case 2: Seller sells multiple channels

In this case, we increase the bit rate requirement of the buyer which necessitates the seller to sell a bundle of channels to meet the demand. We also consider that the price of each channel varies (fixed price for all channels is just a special case). With higher bit rate and more available channels, there are more valid paths between source and destination as

shown in Fig. 6.21. The results are shown for four bit rates: 60, 80, 100, and 120 Kbps. The average number of hops is shown in Fig. 6.22. Optimal price increases with higher bit rate and decreases with number of channels as shown in Fig. 6.23. Fig. 6.24 shows the result of optimal price with increasing bit rate requirement. As expected, a higher demand in bit rate implies more cost. Also, fewer number of available channels results in increased cost.

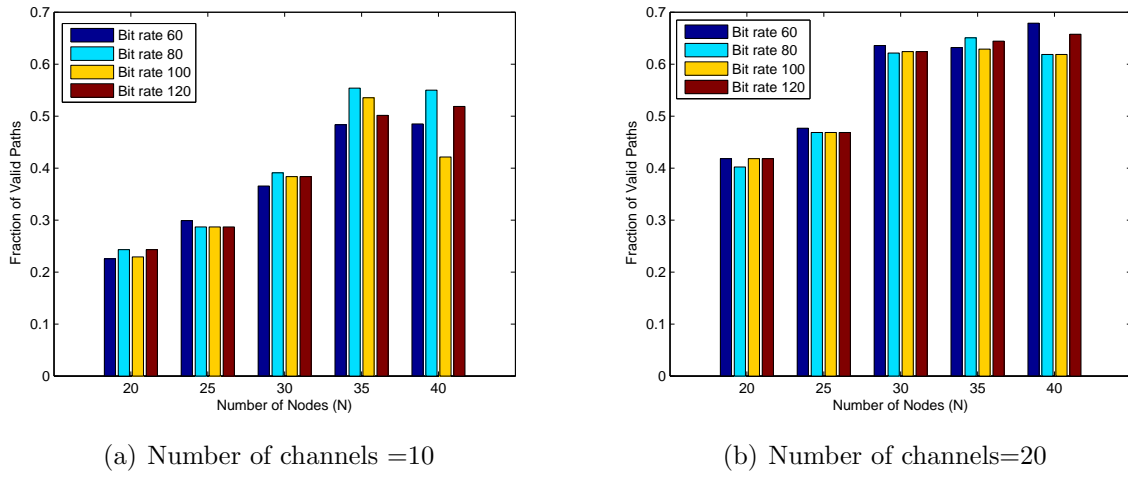
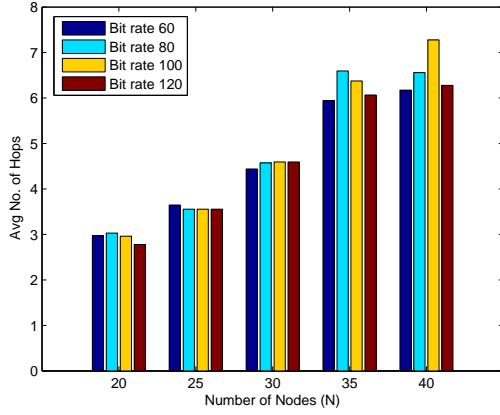


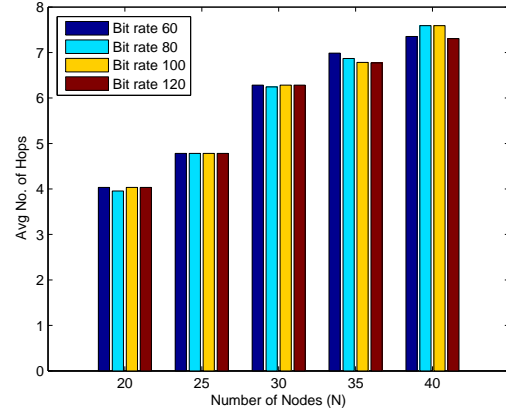
Figure 6.21 Fraction of valid paths with multiple channels

## 6.5 Single Channel Scheduling

We consider up to 160 nodes which were randomly scattered over the network. Two nodes are considered neighbors if their mutual distance is within the transmitting range of 20m. The SINR between the primary node and any secondary node on the channel is assumed to be uniformly distributed between 0 dBW and 20 dBW.

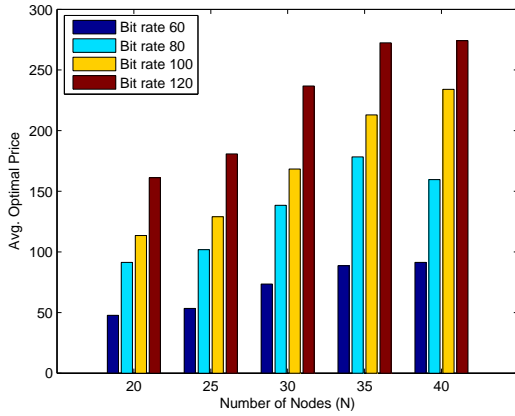


(a) Number of channels =10

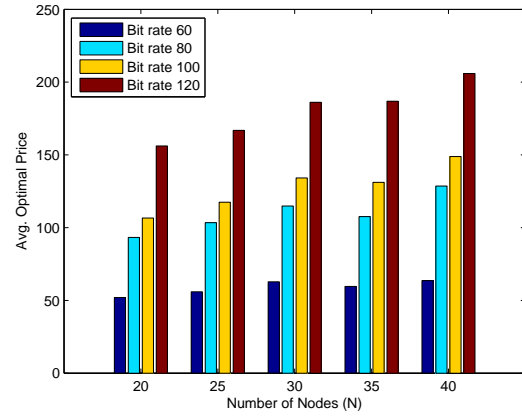


(b) Number of channels=20

Figure 6.22 Average no. of hops with multiple channels



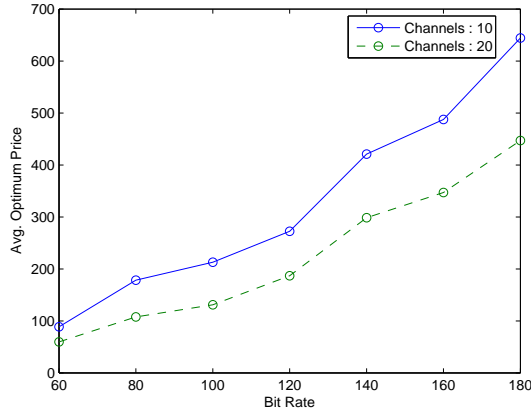
(a) Number of channels =10



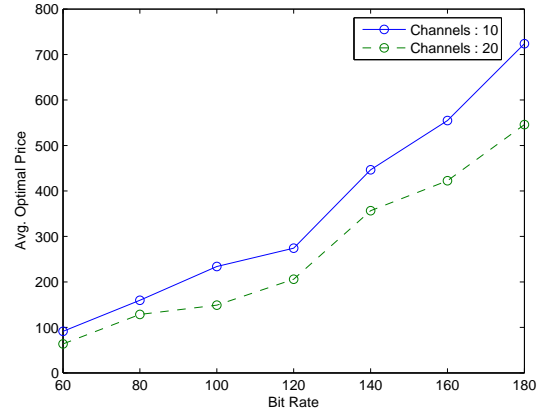
(b) Number of channels=20

Figure 6.23 Avg. optimal price with multiple channels

At the beginning of each super-frame, the proposed scheduling algorithms create the independent sets. Number of independent sets and users in each set are different based on the SINR of the users which changes dynamically. We use both algorithms to assign slots



(a) N=35



(b) N=40

Figure 6.24 Avg. optimal price with increasing bit rate requirement

to the users and obtain the i) throughput per slot for all users, ii) throughput per user per slot iii) fractions of slots allocated, and iv) Jain's fairness index.

In Fig. 6.25, we show the average throughput per slot due to all users for  $s = 4$ . While, in Fig. 6.26, we show the average throughput per user per slot. More number of users means more demand of throughput. Overall, the proposed algorithms have the ability to satisfy more users with the concept of independent sets and reuse of the same channels within the same super-frame, without any conflict. Thus, the system's throughput is increased with more number of users. On the other hand, the average throughput per user per slot is decreased with more number of users. This is because of the size of the independent sets. More users lead to larger independent sets, and as a result the available bandwidth in each slots is divided among more users, which decreases the average throughput per user per slot.

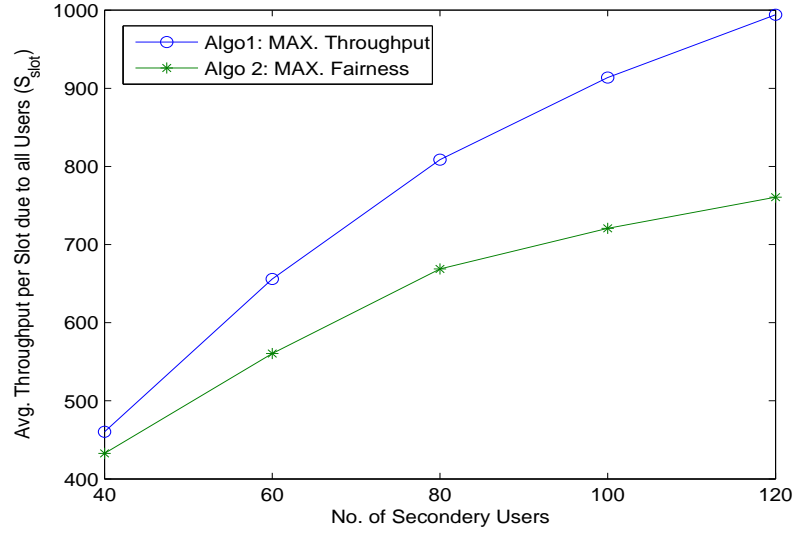


Figure 6.25 Avg. throughput per slot due to all Users ( $S_{slot}$ )

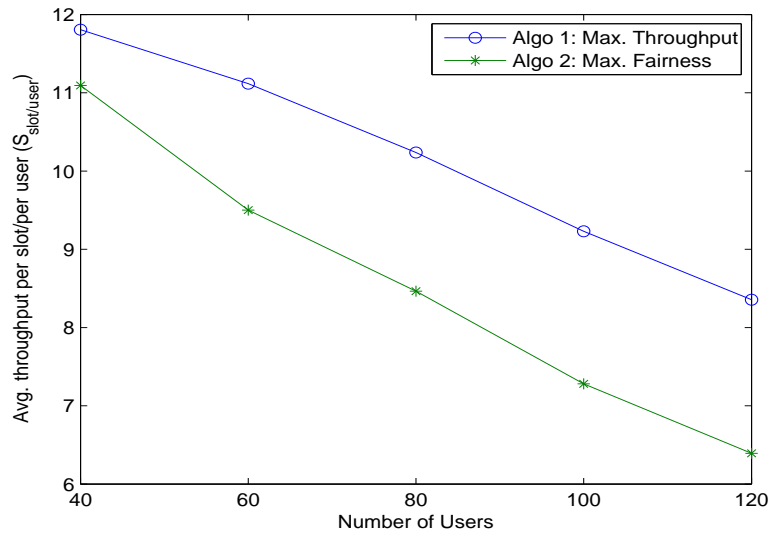


Figure 6.26 Avg. throughput per slot /per user ( $S_{slot/user}$ )

The main goal of the first algorithm is to maximize the throughput, while the main goal of the second algorithm is to maximize the fairness, which is number of allocated slots.

In Fig. 6.25 and In Fig. 6.26 we show that first algorithm achieves more throughput than the second one. While in Fig. 6.27 we show that the second algorithm allocates the slots more fairly among users than the first one. The minimum value of the fairness index for the second algorithm is 0.925 while it is 0.878 for the first one. However, for both, fairness decreases with more number of users but not lower than 0.87 for 120 users.

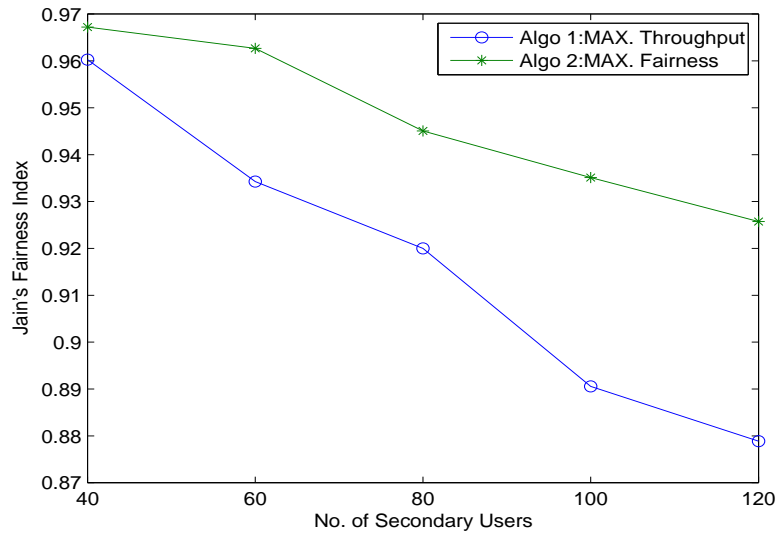


Figure 6.27 Jain's Fairness Index

In Fig. 6.28 we exhibit the average fraction of slots allocated to all users for increasing number of users. With more users, the fraction of slots allocated decreases as the available bandwidth is shared among them. However, the proposed algorithms manages to assign slots to all users as shown in Fig. 6.29.

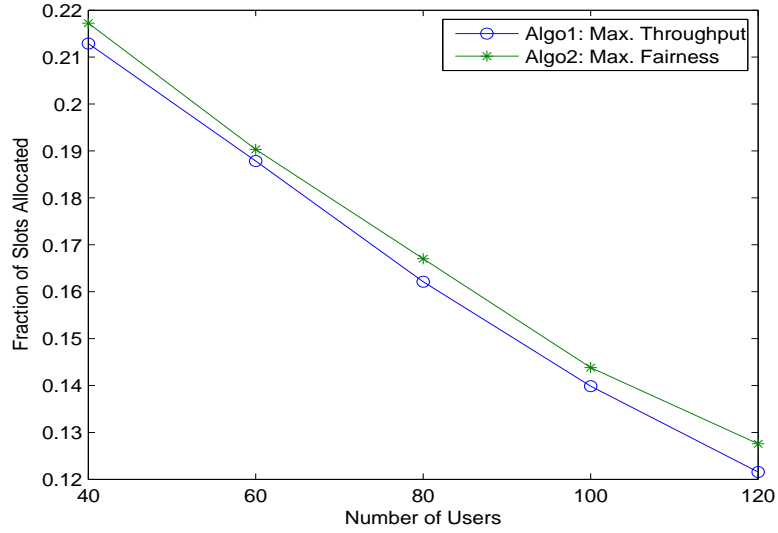


Figure 6.28 Avg. Fraction of Slots Allocated

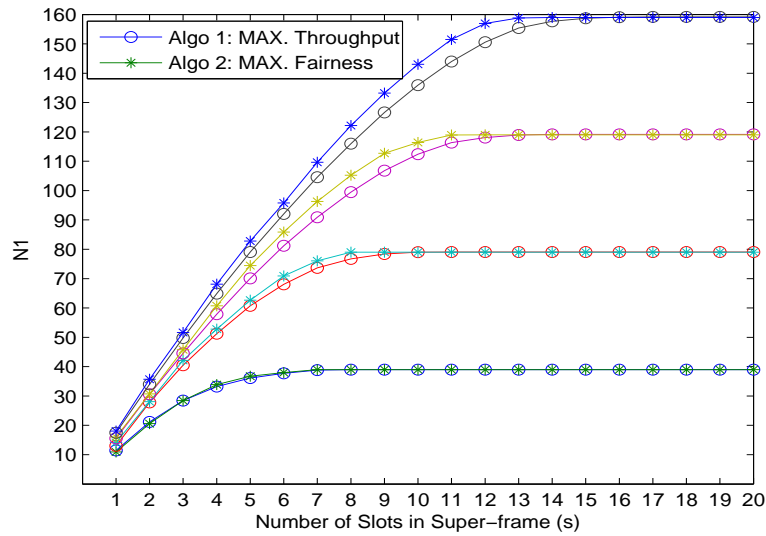


Figure 6.29 Avg. number of assigned user in super-frame ( $n_{sf}$ )

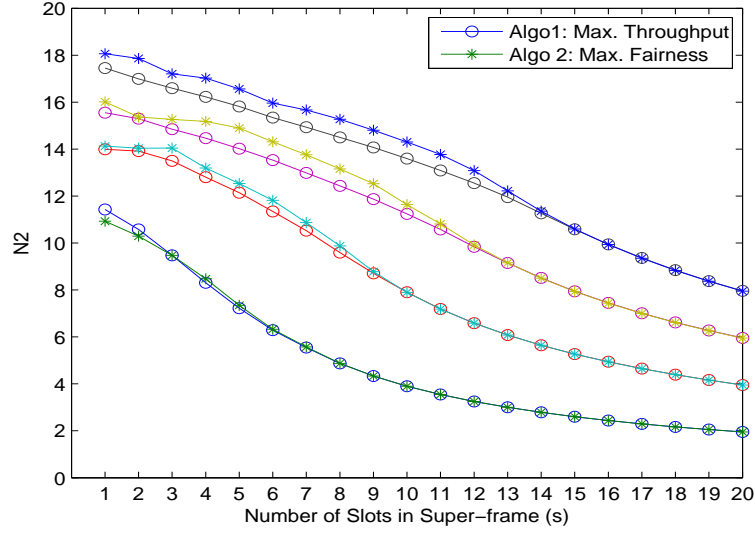


Figure 6.30 Avg. number of assigned user in slots ( $n_{slot}$ )

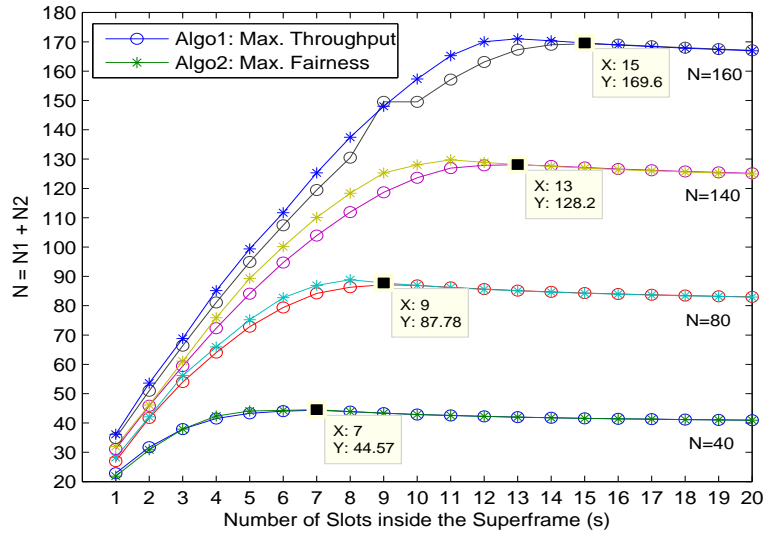


Figure 6.31 The combination of ( $n_{sf}$ ) and ( $n_{slot}$ )

In Fig. 6.29, we show the effect of increasing the number of slots on the average number of assigned user in each super-frame. As expected, we get an increasing function. and in Fig. 6.30, we shows the effect of increasing the number of slots on the average number of assigned user in each slot. As expected, we get a decreasing function.

In order to find the optimal size of super-frame, we vary the number of slot per super-frame from 1 to 20. Using  $\alpha = 0.5$ , we show the combined effect of  $n_{sf}$  and  $n_{slot}$  in Fig. 6.31. As can be seen, the optimal size of super-frame is varies between 7 to 15 based on the number of users.

For  $n = 80$ , the optimal value for  $s$  from Fig. 6.31 is found to be 9. This is also verified by Fig. 6.32 that shows the Jain's fairness index to attain the maximum around  $s = 9$ .

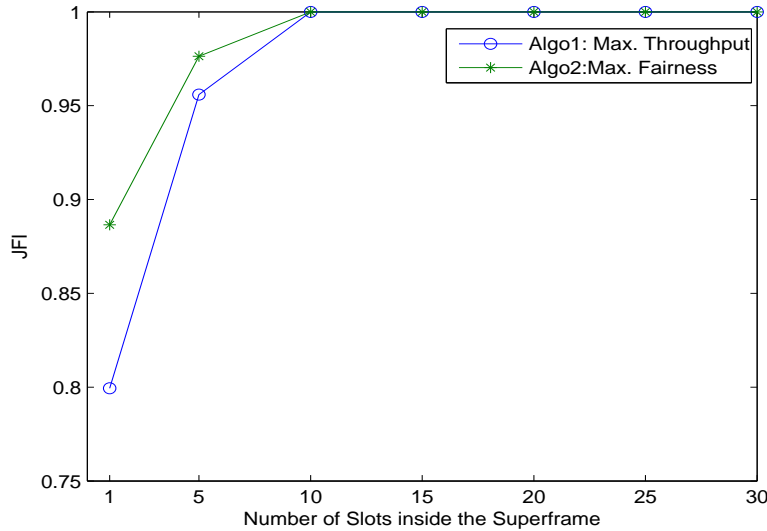


Figure 6.32 Jain's Fairness Index

## 6.6 Multi-Channel Scheduling

In this section, we first evaluate the performance of our proposed queue-aware scheduling strategies under the standard performance metrics, namely, throughput, user-level fairness, blocking probability, and system delay. Subsequently, we conduct comparative performance evaluation with respect to the conventional strategy, namely, queue-unaware scheduling.

We consider up to 120 nodes which were randomly scattered over the network. Number of channels are varied from 5 to 20, each with a bandwidth of 2 MHz. Two nodes are considered neighbors of each other if their mutual distance is within the transmitting range of 20m. The SINR for a transmitter-receiver pair is assumed to be uniformly distributed between 0 dBW and 20 dBW. Packets arrive to the queues of users following the batch Bernoulli process with an average arrival rate of 250 packets every unit time, unless otherwise specified.

### 6.6.1 Performance of Queue-Aware Scheduling Strategies

We study the performance of the proposed scheduling in two different ways depending on how the non-interfering sets are created: i) at the beginning of each super-frame, which we refer to as frame-by-frame scheduling, and ii) at the beginning of each slot, which we refer to as slot-by-slot scheduling. For the simulations, we consider a frame to consist of 10 slots.

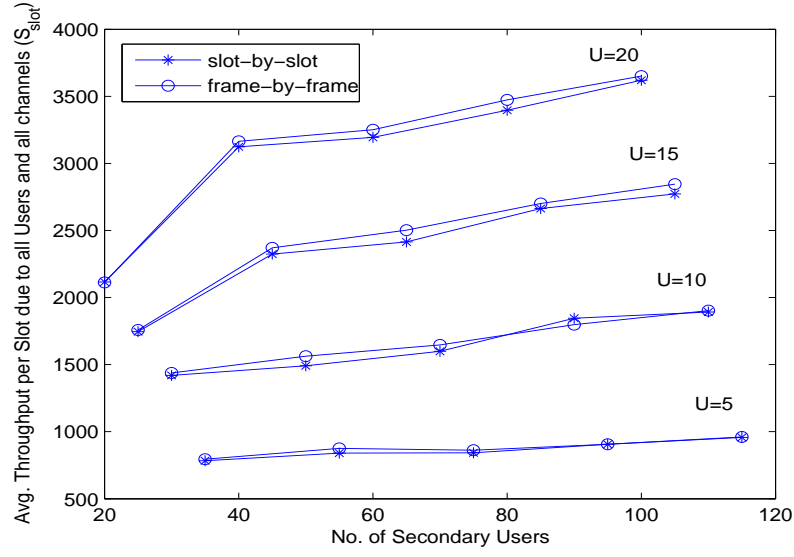


Figure 6.33 Avg. Throughput per Slot due to all Users and all Channels ( $S_{slot}$ )

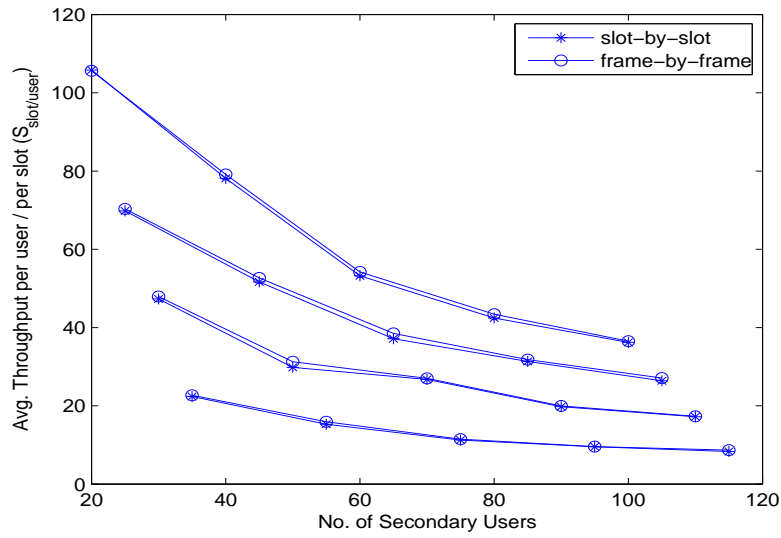


Figure 6.34 Avg. Throughput per slot/per user ( $S_{slot/user}$ )

In Fig. 6.33, we show the average throughput per slot due to all users and all channels, ( $S_{slot}$ ). In Fig. 6.34, we show the average throughput per user/per slot due to all users and all channels, ( $S_{slot/user}$ ). As expected, with more users, the total system throughput increases. However, the throughput of each user decreases with increasing number of users. More users lead to many non-interfering sets, and as a result the available bandwidth in each slot is divided among more users, which decreases the average throughput per user per slot.

Overall, the proposed algorithm has the ability to satisfy more users and also attain higher throughput via the use of the Markov model discussed earlier. Also, the ability to use the same channel in the same slot by multiple users allows better spatial reuse of the channels.

As expected, the performance of slot-by-slot scheduling is better than frame-by-frame scheduling. This is primarily because the scheduler is able to make many short-term decisions with less error than making decisions for all the slots in a frame. Of course, this enhanced performance comes at the cost of more frequent schedule cycles.

Generally, the proposed methods have an ability to satisfy more users and yield more system throughput compared with [50] and [49]. Since in [49], a channel is assigned only to one user, there is no reuse of the channel within the super-frame. In [50], a channel is assigned only to one user in slot, so there is no re-use of the channel within the slot.

In Fig. 6.35, we exhibit the average fraction of slots allocated to all users for increasing number of users. With more users, the fraction of slots allocated decreases as the available

bandwidth is shared among them. More number of channels mean more slots allocated to the users. Again, both scheduling methods manage to assign slots to all users.

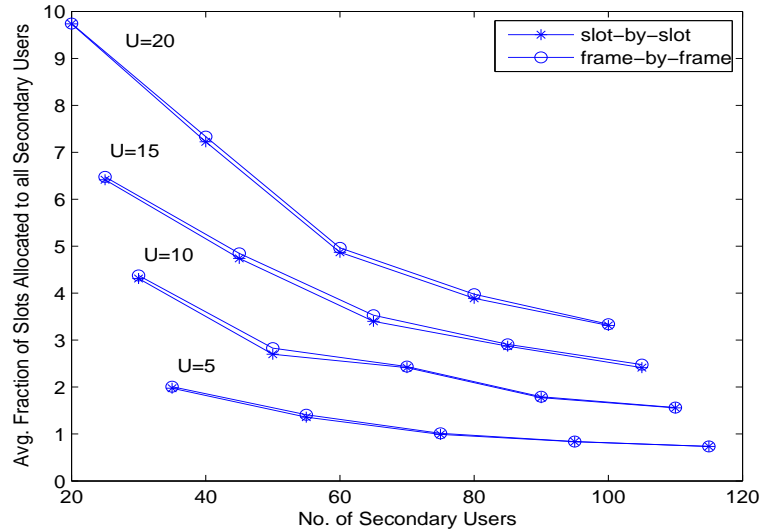
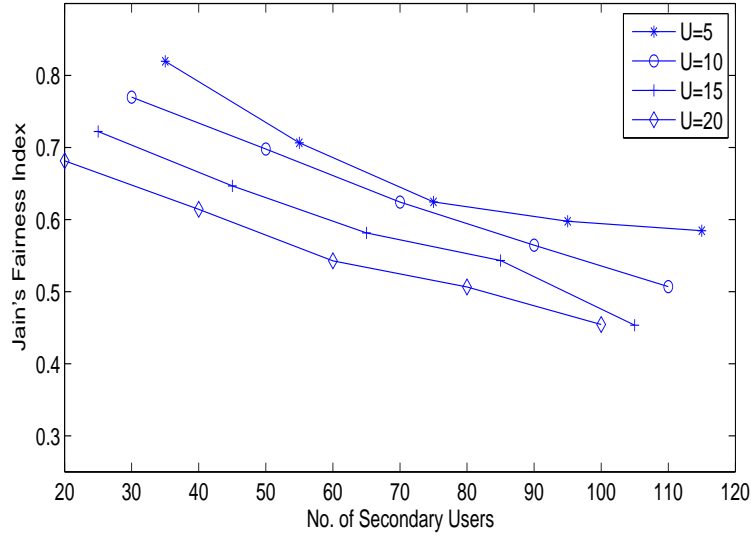


Figure 6.35 Avg. Fraction of Slots Allocated

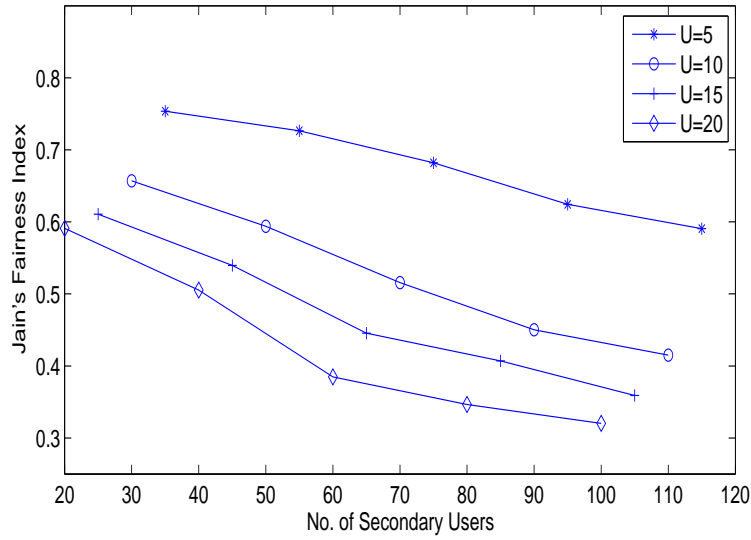
With the objective of maximizing the throughput, users with low expected throughput are not assigned any slots. Thus, with more number of users the fairness decreases. We also show the effect of the number of channels on fairness in Fig. 6.36

In Fig. 6.37, we show the blocking probability when  $\lambda = 30$  and number of channels being 15. With more number of users the blocking probability increases. The blocking probability is slightly better for slot-by-slot scheduling as non-interfering sets are calculated after every slot allowing more users to be accommodated.

In Figs. 6.38(a), 6.38(b), and 6.38(c) we show the delay performance with varying number of available channels, arrival rates, and number of users. In Fig. 6.38(a),  $\lambda = 30$  and number of channels =15, while in Fig 6.38(b),  $\lambda$  is varied between 10 packets per unit time



(a) slot-by-slot



(b) frame-by-frame

Figure 6.36 Jain's Fairness Index

to 30 packets per unit time, number of channels was set at 15 with number of users being 80. In Fig 6.38(c), number of channels are varied between 5 to 20, with  $\lambda = 30$ , and number of users as 80. As expected, with more users and higher packet arrival rates, the average

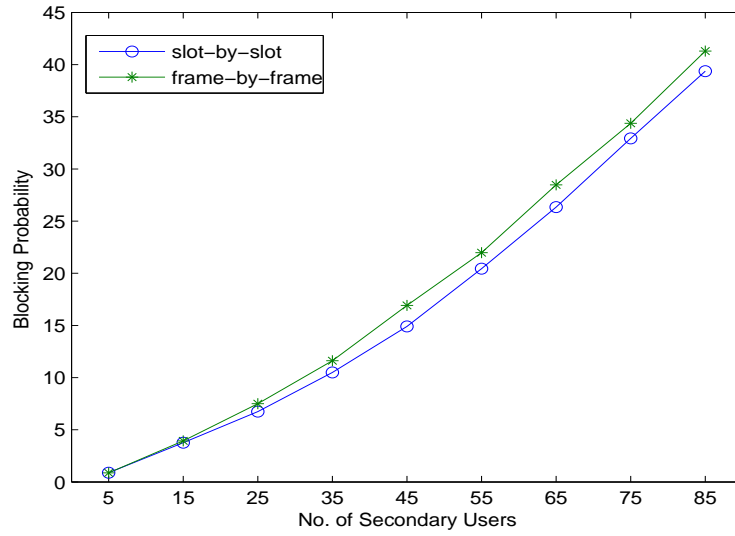
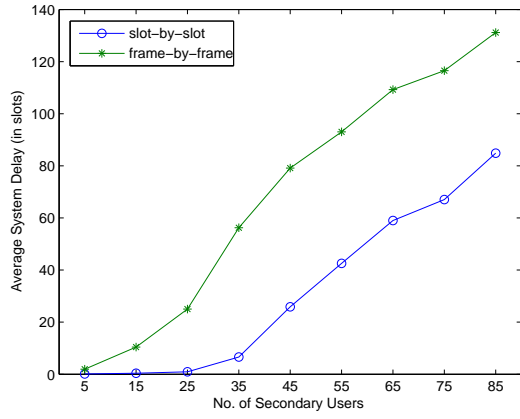


Figure 6.37 Blocking Probability

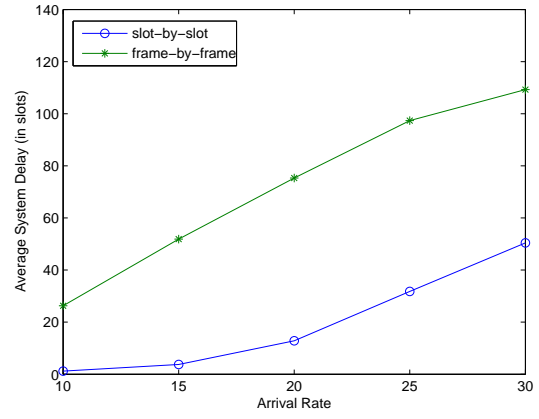
system delay increases, while with more channels the average system delay is decreases. It has to be noted that the better delay performance of the slot-by-slot scheduling comes at the cost of more frequent scheduling.

### 6.6.2 Performance Comparison with Queue-Unaware Approach

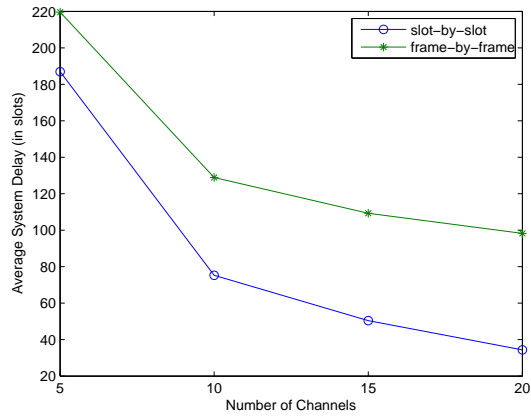
We now compare the queue-aware scheduling with queue-unaware scheduling in terms of the total number of allocated slots to all SUs and the throughput each gets. Since our performance studies have clearly demonstrated that the overall performance with slot-by-slot allocation granularity is better than the frame-by-frame allocation, in comparative study of our proposed queue-aware allocation with queue-unaware allocation we only consider slot-by-slot allocation strategy.



(a)



(b)



(c)

Figure 6.38 Average System Delay (in slots) with a) varying number of SUs; b) varying arrival rates; and c) varying number of channels

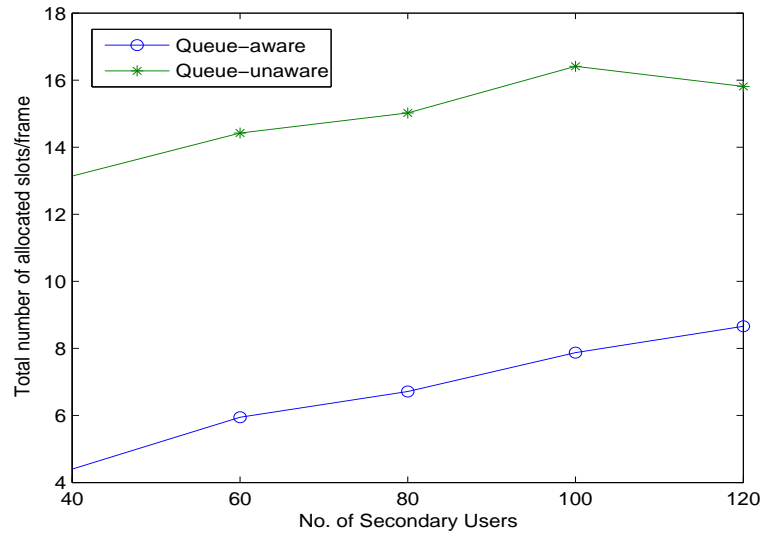


Figure 6.39 Comparison of total number of slots allocated.

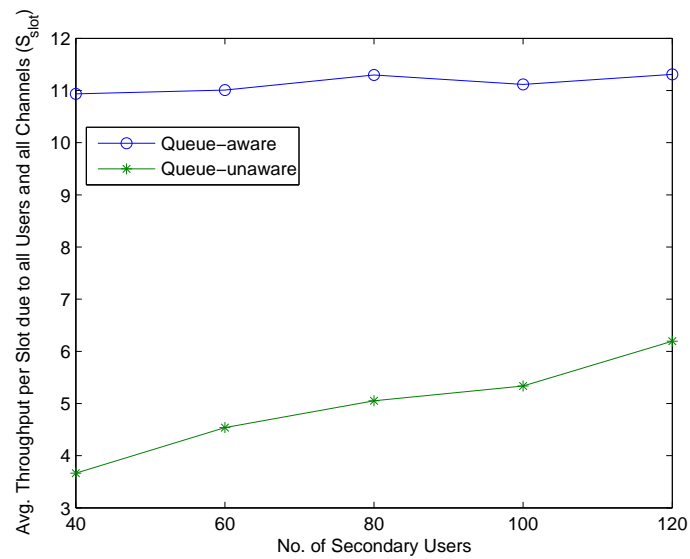


Figure 6.40 Comparison of average system throughput per slot.

When the queue status is not accounted for scheduling purposes, the scheduler might allocate slots to the SUs who do not have queued packets, thereby wasting resource. This is shown in Fig. 6.39, where the queue-unaware scheduler unnecessarily allocates more slots than the queue-aware scheduler. On the other hand, the queue-aware scheduler only assigns channels/slots to the users with packets in their queue. As a result, the system throughput is better, as shown in Fig. 6.40.

## CHAPTER 7: CONCLUSIONS

To realize the paradigm shift from static spectrum allocation to dynamic spectrum access, a host of problems must be solved to make DSA a reality. In this dissertation, we addressed some of the problems that arise due to spectrum trading and resource allocation (allocation, routing and scheduling) in DSA networks from an economic perspective.

We started by considering a randomly deployed network where primary and secondary users co-exist on the same spectrum bands and trade those bands amongst each other. We proposed an auction-based allocation algorithm in a distributed multi-channel DSA networks, where secondary users place multiple bids for the channels. The bids from a secondary user are based on the quality of a channel as indicated by the SINR of that channel. The bids are more for channels with higher SINRs than that with low SINRs. The proposed auction scheme allocates channels based on the users preferences, considering the interfering users. The proposed scheme guarantees conflict free allocation, channel reuse, and fairness.

Next, we proposed the PreDA— a preference-based truthful double auction for dynamic spectrum access DSA networks, where secondary users place multiple bids, and primary users place asks price for the bands of the channels. As the previous single auction-based allocation, the bids from a secondary user are based on the SINR. The proposed auction scheme allocates channels based on the users preferences, considering the interfer-

ing users. The proposed multi-bid auction tries to assign channels to the buyers with the highest payment among the potential buyers, while guaranteeing the truthfulness by eliminating the buyer with minimum bid. Also, the proposed auction assures other economic properties. Moreover, we used the concept of virtual groups to transform multi-unit bids to the single-unit bids. The proposed scheme guarantees conflict free allocation, channel reuse, and fairness.

For pricing and routing, we presented a *log*-based pricing scheme for the seller that is based on its cost price and the amount of unsold bandwidth. Using the signal to interference and noise ratio, we computed the bandwidth that a buyer must buy in order to achieve the desired bit rate. The total cost of a route is the sum of the prices paid at each hop between the source and the destination.

We proposed an auction-based routing algorithm where channels are traded between transmitters and receivers based on the outcome of a sealed-bid auction. Each forwarding node runs an auction among its neighbors, and explores all possible channel bundles and routes between source and destination. We consider two cases where the seller bids for only one channel or bids for multiple channels. In order to minimize the searching time and the overhead, we proposed a heuristic based on unit price to find the near-optimal combination of channels.

We proposed two scheduling algorithms for allocating time slots to secondary users. Both algorithms utilized the channels by allowing multiple conflict-free secondary users to use the same channel in the same slot. For maximizing throughput and achieving fairness, we

used the SINR and number of allocated slots to find the independent sets. In case of multi-channel allocation, we proposed a queue aware frame-by-frame and slot-by-slot scheduling. We used the primary channel occupancy, channel quality, and the queue status to create the discrete-time Markov chain to estimate the maximum expected throughput for each channel on every slot. We maximized the spatial and temporal re-use of channels by allowing multiple conflict-free secondary users to use the same channel on the same slots.

All theoretical proposition were validated through extensive simulation exterminates. Results demonstrate how our proposed algorithms performed under different radio and network settings.

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