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Influence Of Using Context Supportive Of The Area Model On Sixth Grade Students' Performance When Writing Word Problems For Fraction Subtraction And Multiplication

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INFLUENCE OF USING CONTEXT SUPPORTIVE OF THE AREA MODEL ON SIXTH GRADE STUDENTS' PERFORMANCE WHEN WRITING WORD PROBLEMS FOR FRACTION SUBTRACTION AND MULTIPLICATION

by

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ABSTRACT

The purpose of this action research study was to evaluate my own practice of teaching writing word problems with fraction subtraction and fraction multiplication using appropriate context. I wanted to see how focusing my instruction on the use of the area model and manipulatives could develop students' understanding of fractions when writing word problems. I chose this topic because Florida has adopted the Common Core State Standards and will be implementing them in the coming years. These standards encourage the development of deeper understanding of mathematics, including fractions. I hoped this research would give my students the opportunity to make sense of fraction subtraction and fraction multiplication word problems on a deeper level, while giving me insight into my own practice in teaching context within word problems. Through this study, I learned that my students continued to switch the context of subtraction with multiplication within word problems. Students did make clear gains in their writing of fraction subtraction and fraction multiplication word problems. Although there is a limited amount of research on students mixing their context within fraction word problems, this study offers additional insight into a teacher's practice with writing fraction word problems.

This study is dedicated to students everywhere who struggle with writing word problems with subtraction and multiplication of fraction

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I am equally thankful to my grandparents and sister. Without their continued support I would not have finished my thesis. They believed in me, and my ability to write this paper. Although it took longer than expected, they continued to encourage me to move forward. Whenever I felt like giving up they were the ones that told me I could do it and guess what, I did! I am very grateful for the love and support of my family and friends. You were all there for me when I needed you and I will always remember that.

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CHAPTER 1: INTRODUCTION

Rationale

Mathematics standards are changing across the United States in the form of the Common Core State Standards for Mathematics. Florida has recently adopted the Common Core State Standards, and they will be fully implemented by the 2013-2014 school year. The philosophy is that instead of teachers teaching many standards, a mile wide and an inch deep, the emphasis is now being placed upon the students developing a deeper conceptual understanding of mathematics. In the recently published *Curriculum Focal Points for PreKindergarten through Grade 8 Mathematics: A Quest for Coherence*, the National Council of Teachers of Mathematics (NCTM, 2006) states that with a smaller number of standards to focus on students gain extended experience with the core concept skills, which facilitates a deeper understanding and mathematical fluency.

In 2007, Florida adopted new mathematics standards known as the Next Generation Sunshine State Standards (FLDOE, 2007), which were modeled after NCTM's focal points. In fifth and sixth grade, two of the Big Ideas focus on fractions. The first is "Develop an understanding and fluency with addition and subtraction of fractions and decimals" and second "Develop an understanding of and fluency with multiplication and division of fractions and decimals". The focus is narrowed even further within the benchmarks to include that students should use models and be able to explain and justify their answers (FLDOE, 2007). For me the words explain and justify were a wakeup call. Did I understand what they meant and most importantly if asked could my students explain and justify their reasoning in their mathematics problems and would they demonstrate that understanding through writing word problems?

In the last six years of teaching mathematics to fifth and sixth graders, I have begun to notice a trend among my students when it comes to their approach in solving mathematics word problems. From my experience many students can solve an expression by memorizing steps to solve their mathematics problems, but when given word problems that demonstrate the same concept, are unable to determine which operation to perform. Working with the new standards, I wanted to see if using context supportive of the area model would improve sixth graders' performance in writing word problems for fraction subtraction and multiplication.

Area Model

Using manipulatives such as area models in a cooperative group setting can foster a meaningful acquisition and understanding of fraction skills (Krach, 1998). I focused part of my research on using the area models to develop understanding of subtracting and multiplying fractions and the other part on developing word problems based on contextual language for the correct operations.

Krach (1998) describes two approaches to representing fractions with models. The first is using an area model, which can either be circle or rectangles and the second is using a measurement model such as Cuisenaire rods. He states that both models should be used with elementary school students in this order: area model, measurement model.

Subtraction of fractions can be effectively demonstrated with fraction circle manipulatives, by using both a "take away" and "missing addend" approach (Krach, 1998). With the multiplication of fractions Cuisenaire rods can be used, "however, an area model approach may be a more pedagogically effective method for teaching the multiplication of fractions (Krach, p. 21). Students are able to visualize the relationship between fractions with the use of area models, which contributes positively to students' number sense with fractions.

Problem Solving

Students need opportunities to write, discuss, and solve problems (NCTM, 2006). By using students' own experiences and interests to write their story problems in context, students are more engaged and become interested, even excited about sharing their work. "Problem solving lies at the heart of mathematical learning" (Whitin & Whitin, 2008a, p. 426), and in referencing their own illustrations and manipulatives students are able to write word problems.

Whitin & Whitin have noted important features to problem solving, the first being that it engages children in writing their own story problems in context. Students have to reason with which context best exemplifies the mathematics that they are trying to explain. Having to explain and justify their reasoning to their peers, students talk about their work throughout their problem and discuss similarities and differences in their problems. By writing word problems students are required to represent their understanding in several ways, for example, with illustrations, through verbal communication, and students' own writing (Whitin &Whitin, 2008a).

Ultimately through problem solving students are "making thinking and sense making the cornerstones of the classroom community" (Whitin & Whitin, 2008b, p. 432). When students are given the opportunity to explain and justify their work, use reasoning skills, conjecture and defend their work, and even provide alternative perspectives to a problem then they will continue to utilize these skills as it becomes a habit in their mathematical work (Whitin & Whitin, 2008b).

Research Question

After years of teaching computation algorithms to multiplying fractions and experiencing frustration when after completing a unit in mathematics and students were still unable to consistently identify the correct operation for word problems because all I had asked of them was to "Multiply the numerators, multiply the denominators, then simplify", I came to the conclusion that something needed to change to promote conceptual understanding and it needed to begin with me. Through my research, I explored this question:

> Question: How does my practice of focusing on context and the area model for fraction subtraction and multiplication influence student performance when writing word problems for those operations?

Conclusion

With the implementation of the Next Generation Sunshine State Standards for Mathematics I have seen a need for change in my instruction to facilitate a deeper understanding mathematically of multiplying fractions with manipulatives and writing word problems (NCTM, 2006; FLDOE, 2007). Furthermore, my goal is for my students to use manipulatives and the area model to develop a deeper understanding that they can then transfer to the writing of a fraction word problem, which can eventually carry over into dividing fractions as well as other areas of mathematics. I know that my research will help to guide me in becoming a more proficient educator in mathematics and I hope that by encouraging my students to develop these strategies they become deeper thinkers. Finally, my research will be beneficial to the education community who want to help students become better at identifying the context of word problems and how it determines the operation being performed, not just with subtracting and multiplying fractions, but with all mathematics. In Chapter 2, I review literature that supports teaching context supportive of the area model and using manipulatives to develop conceptual understanding of multiplying and subtracting fractions in order to write word problems.

CHAPTER 2: LITERATURE REVIEW

Introduction

Educators have traditionally taught mathematics concepts beginning and ending with algorithms, leaving little room for understanding of why the algorithms work. "It is important to give students ample opportunity to develop fraction number sense and not immediately to start talking about….rules of computation" (Van de Walle, 2006, p. 87). A review of the literature shows that standards and expectations have changed. It is no longer acceptable to introduce fractions with only procedures and say multiply the numerator then multiply the denominator. In order to demonstrate complete understanding explanations need to include mathematical arguments and rationales, not just procedures (Yackel & Cobb, 1996).

It is well documented that many children have difficulty understanding fractions. With mathematics education undergoing reform, the National Council of Teachers of Mathematics (NCTM, 2006) is calling for instruction that supports the development of deeper understanding, reasoning, and proof. In addition to this, a new set of Common Core State Standards is making its way across the United States also promoting a deeper understanding of fractions. According to these standards students should be able to do such things as use "a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$" (Common Core State Standards, 2010, "5.NF.4" para. 1).

Many reasons have been given over the years as to why fractions are probably one of the most serious obstacles to mathematical maturation of children (Behr et al., 1993). One is that the complexities of teaching and learning fractions lie in the fact that fractions comprise a multifaceted construct (Brousseau et al., 2004; Kieren, 1993; Lamon, 2005). Vanhille and Baroody (2002) suggest that first students lack concrete experience necessary in developing conceptual understanding and second they do not understand the multiplicative reasoning that is required of understanding fractions. Lamon states that students have only practiced multiplying and dividing within the context of whole numbers, which only develops a limited understanding of these operations. That true understanding with multiplication and division will only come about by doing more complex problems such as fractions (Lamon, 2005). Students need to build up new ways to think about solving fractions, as the ways that they developed with whole numbers are no longer useful.

Traditional instruction in mathematics has often failed to promote multiplicative reasoning required in understanding fractions (Vanhille & Baroody, 2002). Researchers argue that it is imperative for children to develop the concept that arithmetic is more problem solving and strategic reasoning over simply getting quick answers (Steffe, 1991). In order for students to write subtraction and multiplication word problems in context, they may need to use the area models to give them a visual picture of the story problem they are creating (Whitin & Whitin, 2008b). While writing their word problems students will not be getting a quick solution by multiplying the numerator and denominator, but actually having to problem solve and explain and justify their work through the manipulatives and illustrations.

According to (NCTM, 2006), "each student should be expected not only to present and explain the strategy he or she used to solve a problem but also to analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies." (p. 268)

A mathematics classroom should challenge students to think and reason out their explanations. How can one truly communicate their understanding of a mathematical process without being able to justify their solution or create their own word problem? The purpose of this literature review is to explore context through word problems, the area model and their application to multiplying fractions.

"It is time to shift the emphasis and redefine the goal of fraction instruction in elementary school from learning computational rules to developing fraction operation sense" (Huinker, 2002, p. 72). Educators have traditionally taught mathematics concepts beginning and ending with algorithms, leaving little room for understanding why the algorithms work. A review of the literature shows that standards and expectations have changed. In order to demonstrate complete understanding explanations need to include mathematical arguments and rationales, not just procedures (Yackel & Cobb, 1996).

Context of Word Problems

Most, if not all, important mathematics concepts and procedures can best be taught through problem solving (Van de Walle, 2006). Through problem solving students are learning new mathematical methods to solve problems. They are looking for new meaning in their ideas and they do this actively by making sense of a problem through relationships, analyzing patterns, finding out which methods that work and which don't, justifying results, or evaluating and challenging the thoughts of others (Van de Wall, 2006).

"When students translate real-world contexts to drawings and symbols, they have a better chance of connecting new ideas to their existing knowledge" (Hodges, et al, 2008, p. 81). The problem arises when students are not familiar with the context of a problem. This can cause difficulty in solving a problem if students do not have prior knowledge or experience with it. Therefore it is suggested that when beginning a new task, the teacher should start with context that is familiar to students (Hodges, et al). In the case of fractions, students are familiar with having to take part of a pizza, cake, pie, or candy bar. Therefore these would be appropriate contexts for story problems.

In order to develop this conceptual understanding and reasoning with fractions, students need to have a sense of the operations. Huinker (2002) writes in *Examining Dimension of Fraction Operation Sense*, seven dimensions are needed to develop this sense. According to Huinker students fundamentally need to understand meaning and models of operations, have the ability to recognize and describe real-world situations for specific operations, understand meaning of symbols and formal mathematical language, the ability to translate between real-world connections, oral language and symbolic representations of fractions, be able to understand relationship among operations, have the ability to compose and decompose numbers, and have knowledge of the effects of an operation on a pair of numbers (Huinker). Once students have explored these seven dimensions over time with a problem solving approach they gain a better conceptual understanding of operation sense.

Whitin & Whitin's (2008b) approach to developing operation sense with fractions is to encourage students to represent their thinking in various ways through story problem context. By developing a list of objects and having students create pictures that then translate into stories students are able to create their own word problems. The second step to this is the teacher's problem, once again using "pictures, numbers, and words" students are able to explain their solution by breaking down the problem into these step. Important features noted from the problem solving experience included: engaging children in writing their own story problems, inviting children to talk about their work throughout their problem-solving investigation, inviting children to examine the structural similarities and differences among problems, requiring children to represent their understanding in several ways, and making thinking and sense making the cornerstone of the classroom community (Whitin, 2008). Both Huinker and Whitin's approaches to operation sense are very detailed, specific, and most importantly require time to develop. These steps toward conceptual understanding do not happen overnight and are continually developed as students move from one teacher to another.

A study performed by Jill Drake and Angela Barlow research two questions as it related to problem solving in assessing students mathematical understanding. One, "does the mathematics contained in the problem correctly represent the mathematics called for in the prompt" and two, "is the problem's question appropriate" (Drake et al, 2007, p. 273)? Drake and Barlow found that students could write correct word problems in context, but that they did not always match the expression given to the student or may not be realistic. They both believed that students should be able to demonstrate realistic problems that can occur outside the classroom.

In order to determine understanding they came up with three indicators. The first is that one must decide what to look for in word problems, the second is to determine what will provide insight into students' understanding, and the last was, what indicators show a lack of understanding or mathematical understanding (Drake).

By having students write word problems in context they were able to identify gaps in their understanding and work towards fixing students' misconceptions. Writing word problems keeps students engaged and uses the National Council of Teachers of Mathematics' Process Standards for Communication (2000).

A misconception that students often develop with multiplication is that it means to "make bigger". Faced with a multiplication word problem students begin to realize from context clues and words such as "multiply" and "times" that their answers always tend to be greater. This everyday language that is used with whole numbers then gets applied to fractions, and students become confused as to why their answer is "getting smaller" (Graeber, 1993 p. 2). Graeber suggests that teachers need to help students make sense of "multiplication making smaller" by extending the meaning of multiplication to fractions and using an area model to increase students' chances of seeing that the product of two numbers that are less than one is a number that is smaller than either of the factors.

When looking at multiplying word problems in context you will usually find one of two different ways in which they are written. These two meanings of fraction multiplication can be represented with the words "of" and "times". Although these two words both support the same multiplication rule, they both have very distinct meanings and may be more difficult on a conceptual level (Mick, 1989). In *Two Meanings of Fraction Multiplication,* Mick states that when you ask students to "times" 1/2 x 1/2, this immediately triggers the multiplication-of-whole numbers schema which results in the confusion that when multiplying the product is greater than the factors. This is mainly due to the student's considerable experience with whole number multiplication. As teachers we need to help students modify their present understanding of whole number multiplication to make way for new ideas about fraction multiplication (Mick, 1989). This can begin happening with teachers beginning with 2 of 3, then working with 2 of 1/3 and finally ending with 1/2 of 1/3 to complete the role of whole-number multiplication to fraction multiplication.

With subtraction of fractions students are using “take away” language within context of a problem. There is a difference between the word “more” and the phrase, “how much more”. A student can ask the question, “Who has more?” when they are comparing fractions, but it is not the same as subtraction. Students should be asking, “How much more?” when subtracting or the problem just becomes a comparison one (Lamon, 2005). This attention to detail and the information within the story problem is important to the context of the word problem in order for it to have operation sense.

Overall the educative value in children writing and solving their own story problems can increase when given the opportunity to share and teach their problems with their peers (Winograd, 1991). Students have ownership of their problems, which are at times modeled after their own experiences. This can bring up the question; if students have little experience with fractions in their world can they write fraction word problems in a correct context?

Explaining and Justifying Mathematics

Cobb and Yackel (1996) indicate that social interactions create opportunities for individuals to learn. As students participate in classroom discussion they are able to reflect upon their learning and are able to compare their own results with shared solutions, allowing for interaction amongst the students and brining new ideas, understanding, and reasoning into the classroom. By explaining, justifying, and arguing mathematics in the classroom students are able to develop their own personal understanding. They move beyond the mechanics of a word problem and can begin to reason it out. Instead of just picking random numbers out of problems and trying to determine operation sense, students are able to use common sense and make good judgments working with their peers when they are not driven by procedures (Lamon, 2005).

Through the process of sharing strategies students are able to argue their points, listen to their peers, and talk through their ideas, which are all active ways of cementing their personal understanding of the mathematical concept (Van de Walle, 2006). In making a valid argument, the student takes control of their learning and reinforces their explanation and justification of the solution, making what might have been an abstract concept concrete.

Van de Walle (2006) argues that it is important to give students ample opportunity to have time to develop fraction number sense and not to immediately begin talking about contextual language such as common denominators and other rules of computation. He uses the example of a fifth grader asking why 29 times two-ninths makes the answer go down to explain that students are coming from a whole number background, and become confused when introduced to multiplying with fractions. They immediately want to make generalizations about fractions, but as teachers we need to realize that these ideas about their operations were developed with whole numbers and that students need to build new ideas about fractions.

"Premature attention to rules for fraction computation has a number of serious drawbacks" (Van de Walle, 2006, p. 89). These rules when memorized with little or no conceptual understanding do not develop mastery, and instead prevent students from thinking about the operations and visualizing the meaning of the problem. By working with only the rules students do not develop a means of assessing their results and checking for reasonability, which is evident when they are asking questions such as "Do I need a common denominator?" or "Which number do I need to take the reciprocal of, the first or second?" (Van de Walle, 2006, p.

87). Students are able to become adequately proficient in their operations with fractions by using student-invented methods that they understand and that later can be applied to the formal rules.

In *Teaching Student-Centered Mathematics grades 5-8,* Van de Walle (2006) discusses guidelines that should be kept in mind when developing computational strategies for fractions. The first and foremost is his case for beginning with simple contextual tasks. The problem does not need to be elaborate, just enough that they get the meaning of the operation and the fractions involved. The second is to connect the meaning of fraction computation with whole-number computation, meaning model a simpler problem such as 3 x 4 before asking the students what 2/3 x 1/4 means. The third is to let estimation and informal methods play a role in the development of strategies. It is important to check for reasonability, if 4 x 1 is four then 4 x 1/3 has to be less than four. The last strategy is to explore each of the operations using models. When students are able to interact with each other and manipulate models they are able to build that conceptual understanding needed to take them to the next step of solving problems with algorithms.

Multiplying Fractions using Area Model and Manipulatives

Manipulatives afford students the opportunity to represent fractions using a concrete example for an abstract thought. As students understanding develops over time, they can move from manipulatives, to illustrations, to symbols (Hodges, et al, 2008). The area model can be represented in many ways, some include fraction circles, fraction strips, color tiles, Cuisenaire rods, pattern blocks, and in the case of Robold's method a geoboard (Robold, 2001). When manipulatives are not readily available or if a teacher is ready to move from hands-on practice to a more visual representation of fractions there is always the National Library of Virtual

Manipulatives (Hodges, et al, 2008). For the purpose of my study I will be using fraction circles as well as the rectangle area model to develop students' understanding of fraction subtraction and fraction multiplication.

The NCTM expectations for sixth grade students in regards to fractions calls for understanding of fractions, their meaning, and to be able to compute fluently. For many children, learning fractions is decidedly one of the most complex skills they will encounter in their primary education (Charalambos, 2007). In many classrooms this process may begin with conceptual knowledge, but as the level of difficulty increases many educators begin to resort to teaching through procedure-oriented memory-based instruction, in which terms such as "canceling", "reducing", or "inverting and multiplying" are used quite frequently (Hanselman, 1997), consequently leaving students with little understanding of the reasoning behind the procedures they are using.

One way to develop meaning of concepts and operations in fractions is to use manipulatives and the area model in a cooperative group setting. Models can help clarify ideas that are often confusing and provide students with ways to think about, talk about and explore their reasoning (Van de Walle, 2006). One cannot expect to have manipulatives in the classroom without modeling them with a simpler problem so that students can begin to develop their mathematical ideas. One concern that Van de Walle discusses is the incorrect use of models. If a teacher teaches a structured lesson on how to use them exactly with procedural terms, the models become less of an explore activity to develop strategies and more of a step-by-step process. Little or no reflective thought goes into these procedures and the student is once again only looking for the answer and not looking at their thinking (Van de Walle).

Using the area model to develop students' understanding of fractions is part of my second question. It is said that the area model and Cuisenaire rods "foster meaningful acquisition and understanding of fractional concepts and operations" (Krach, 1998). Krach states that both the area model as well as the measurement model should be used when introducing fractions. With subtraction of fractions there are two ways to demonstrate this operation using the area model. The first is the "take away" method and the second is the "missing addend". Using both of these techniques with the area model provides a visual of the meaning of the operation.

Even though you can use both the area model and the measurement model with multiplying fractions, Krach (1998) suggests the use of the area model as the approach may be more effective for teaching multiplication of fractions. Students are able to easily visualize taking a part of a part with this method, and are less likely to argue the answer. Overall using the area model with multiplying of fractions contributes positively to the continued enhancement of students' number sense (Krach).

Pagni (1999) states that the best interpretation of fraction multiplication is that of a "part *of* something" (Pagni, p. 12). Saying a part of something is more comforting as it has real word application and experience. For example, getting a third of a candy bar or eating half of a pizza are contexts that happen in real life. Therefore when discussing multiplication of fractions we want to look at it from a perspective of one-third *of* two-fifths. Pagni gives instructions on how to create an area model with 1/3 x 2/5 by partitioning vertically into fifths and horizontally into thirds. The overlapping shaded region is the solution to this expression (Pagni). By using real world examples and changing the language being used, we can encourage students to think about the mathematical process of multiplying fractions.

Conclusion

A review of the literature supports the need for instruction with fraction subtraction and fraction multiplication by using visual models and students' creating their own word problems (Whitin & Whitin, 2008b).

Standards and expectations have changed and in order to demonstrate understanding, explanations need to include mathematical arguments and rationales (Yackel & Cobb, 1996). Teachers play an active role in helping their students develop a better understanding of fraction subtraction and fraction multiplication by not immediately jumping into procedures and algorithms (Huinker, 2002).

I was interested in getting my students to use the area model to develop understanding of fraction subtraction and fraction multiplication and then apply that knowledge to the context of word problems. I wanted to explore the research on area models and writing of word problems. Ultimately, I hoped to improve my instruction so that my students would develop their understanding of fractions and be able to demonstrate that understanding by creating their own word problems.

In the next three chapters, I discuss the methodology I chose, the analysis of the data, and my conclusions. My question, "How does my practice of focusing on context and the area model for fraction subtraction and multiplication influence student performance when writing word problems for those operations?" are investigated further in these chapters.

CHAPTER 3: METHODS

Introduction

The topic of my study was exploring context supportive of the area model in order to write word problems for subtraction and fraction multiplication. More specifically, I wanted to know if focusing my practice on teaching students to use manipulatives and the area model was related to their conceptual understanding of the subtle difference between writing a subtraction and a multiplication word problem. In this chapter, I describe the classroom setting and methods used to discover the answers to my questions.

Design of Study

In order to study my own instructional methods in the classroom and how they impacted my students, I conducted a qualitative type of research called Action Research. Action Research is defined as a form of research done by an individual in an attempt to improve one's practice (McNiff, Lomax, & Whitehead, 1996). It was my goal to examine my instructional methods for teaching students to write word problems for subtracting and multiplying fractions in the classroom and how I could improve my instruction in hopes of helping my students be able to conceptually understand this concept by using context and manipulatives such as the area model for deeper understanding. My research question was:

> Question: How does my practice of focusing on context and the area model for fraction subtraction and multiplication influence student performance when writing word problems for those operations?

Setting

School Setting

My school is located within a mid-size district in Florida. We are a magnet elementary school that draws in students who are interested in the Arts such as art, drama, dance, orchestra and band. We go above and beyond the traditional art and music class with these extra classes being offered weekly. The drawback to this scheduling is that students have less academic time in the classroom, which is why we have strict requirements applying to our school. The school admits students from all over the county based on a lottery program. The lottery is where parents who are interested in our school fill out an application and receive a number. During a specified date and time, numbers are drawn and those students are accepted into the school. The only requirements to attend are that students must have an average of a C on their report card in all subjects and if they are being admitted to a fourth through sixth grade class and have Florida Comprehension Assessment Test (FLDOE, 2007) scores that they are a 3 or above on the Reading and Mathematics assessment. We serve a middleclass socioeconomic population with a 94% stability rate with free and reduced lunch being offered in which 8% of students qualify. Of the 437 students at the school, less than 1% are English Language Learners, 8% are enrolled in the gifted program, and 7% are exceptional education (not gifted).

Classroom Setting

My action research was conducted in a sixth grade classroom of twenty-one students. All the parents gave parental consent for their children to be involved in the study; therefore, all twenty-one students participated in my research. These 11-12 year olds were placed heterogeneously in my class by the administration. The class consisted of 14 girls and 7 boys of which one of the students was in the gifted program on consult and another was ELL as her first language was French. This group consisted of 15 Caucasian students, 3 Hispanic students, 1 Asian, 1 Multi racial student, and 1 African American student.

Methods

Preliminary Action

I initially obtained approval to conduct my study through the Institutional Review Board (Appendix A) and my principal approval (Appendix B). During Open House I sent home parental consent forms (Appendix C). All consent forms were returned with permission to participate in the study. I then read the student assent form (Appendix D) to the students and answered any questions they had concerns about. All student assent forms were signed and returned.

Once I gained permission I was ready to begin my first step in exploring my question. Each student was asked to take a pre-test in which they wrote four of their own word problems based on the four number sentences given to the class (Appendix E). This pre-test allowed me to see where students' misconceptions lay in their understanding of basic multiplication and subtraction and how they transferred that knowledge to subtracting and multiplying fraction word problems. I then interviewed each student one-on-one and had them read each of their word problems. Students were ask to explain and justify why they chose to write the problem they did and what specifically about the problem they wrote made it that chosen operation from the number sentence. This interview enabled me to get an idea on how my students were going to be able to communicate their mathematical ideas by explaining their pre-test to me. Each interview lasted approximately two to four minutes and was conducted over three days. From these interviews I made a focus group in which I chose five students to work with who were best able to communicate their mathematical ideas and the process they took in order to solve problems. This decision was based solely on student's ability to communicate their thoughts, regardless of if they were correct or incorrect. I was looking for students who could explain why they wrote the story problems they did and who could hold a mathematical discussion.

Classroom Setup and Discussion

Students were arranged into three main groups in the classroom with mixed ability levels. Students were able to work independently or with each other to share and help with new ideas and strategies. These groups stayed the same throughout the study. Whole group instruction was characterized by both teacher-led and student-led formats. Initially I taught and reviewed explicitly to the students the basic use of circle models with fractions as it related to adding and subtracting fractions and then later on as it applied to multiplying fractions. Students were familiar with the terms "take away" but I also used "compare", which they were not so familiar with. As they became more adept at using the fractions circle manipulatives they developed their own strategies, which I had them share with the whole class. I explained that this year we would work on using context as well as manipulatives such as the area model to gain understanding of subtracting and multiplying fractions. I then posed a question to the students to think about the difference between the operations of subtraction and multiplication. During whole group discussions, students were encouraged to listen to each other's explanations as I would ask different students to volunteer to repeat what their classmate had said and others to elaborate on their responses.

Procedures

Once the interviews were conducted, a class discussion was held to review our current classroom norms and expectations. The student's daily routine did not change. During the week in mathematics class students were engaged in three different ways. The first is whole group discussion, then back to groups for small group discussion, and third paired or individual work. A normal mathematics class began with a math warm up problem, followed by discussion of the problem, a five to ten minute review of homework when it was given, thirty minutes of whole group instruction and note taking on mathematical content, twenty minutes of individual or group practices, then ten to fifteen minutes of review with whole group. I used the county's Setting Our Sights on Mathematics pacing guide as well as my research practices with area models to guide my instruction. The textbook and workbook were used for independent practice as well as to provide student remedial work or practice problems when needed.

The students at my school attend because of the additional arts program that we offer which includes dance, drama and extra art and music classes. Students are used to being recorded for varying reasons through the school year and most are quite comfortable around cameras. I set up the video camera in my classroom a week prior to taping at the back of the classroom for the new students to give them a chance to accustom themselves to it. In addition to this, many of the students had never had their conversations audio recorded; therefore, I also introduced audiotapes to group discussions prior to collecting data.

For eight days in which students learned to multiply fractions, I videotaped whole group discussions, and tape-recorded the small focus group. I was observing students to see how clearly they were able to 1) work with the context, 2) use manipulatives and illustrations to demonstrate the area model, and 3) communicate their thinking.

I gave a posttest to the students that paralleled the pretest at the end of the eight day study. The pretest and posttest were designed to include a whole-number subtraction problem and a whole-number multiplication problem to determine basic knowledge of writing a story problem with content they knew. The tests then had a third problem that was subtraction of fractions and a fourth problem that multiplied fractions. The first two problems were the baseline for the story problems. As they were whole numbers, students should have a basic understanding of how to create story problems with subtraction and multiplication. Problems three and four were designed to see how students would carry their knowledge of the operations of subtraction and multiplication over into fractions. Students were encouraged to reread their previous word problems on the pretest and change them according to the new knowledge gained during instruction. This provided students with the opportunity to reflect on changes that may have occurred in their understanding of mathematics.

Instructional Unit

On the first day with subtraction of fractions, I gave each student a set of fraction circles. I asked them to show me how they could use these fraction pieces to model 2/3 – 1/2. Students were given time to explore the manipulatives and to show me their representation of the expression. The class had a long discussion on whether or not the models 2/3 – 4/8 or 4/6 – 4/8 were acceptable as it would essentially give us the same answer just a different meaning. After exploring additional subtraction fraction problems always using the initial idea of taking away 1/2 in the examples, students were asked to draw what they showed with the fraction circles and asked to note any observations about what was happening when they subtracted 1/2 each time. From here we had a class discussion that focused on two things: one, the quantity of 1/2 never changed; and two, the students were always taking the difference or comparing two quantities. I used these observations to drive the remainder of the day's instruction and had students continue to use the fraction models with new problems that did not always involve subtracting one-half.

During the next lesson, which took two days, we worked as a class to write word problems beginning with the day's previous problem 2/3 – 1/2. We reviewed our previous notes and the two observations that were made. I began the lesson using Twizzlers on the document camera and with a student volunteer modeled a subtraction word problem. Unfortunately, at this time it did not occur to me that I was moving from using the area model to a linear model. All I was trying to do was give the students an example of a real world situation by creating a context with something they had experience with, sharing food.

Students were paired up in their groups and given the fraction circle models and instructed to make two fraction subtraction word problems that could be solved using the manipulatives. I then pulled my small focus group of five students and had them also do the same task and listened to their discussions of how they would go about creating their word problems. Once this was done, I alternated between my focus group and the class listening to the discussions and correcting any misconceptions, and clearing up questions students had. After the word problems had been written the class came back together to share with the whole group the problems they created. At this time, I cleared up misconceptions with wordings such as "eating half of it" and using the manipualtives and guiding questions to make my point.

For homework, students were to write a fraction subtraction expression, illustrate it, and write a word problem that corresponded to the problem on an index card. The following class students switched partners and read their problem out loud while their partner used the manipulatives to solve and vice versa. This rotation continued until each group had heard each other's word problems.

Once students had demonstrated that they grasped the meaning of subtraction of fractions and could create word problems we moved on to multiplication of fractions. I began with a simpler problem of 3 x 4 and asked students to discuss the meaning of this problem in their groups. As a whole group we shared our responses. Then I wrote on the board 2/3 x 1/2 and asked students to discuss the meaning of this problem in their groups while I circulated and listened in on their discussions. Going back to the whole class we shared our responses, and students were then asked to show me using the fraction circles on their desks the number sentence. I alternated between me directly modeling and students practicing in their small groups using the fraction circle problems that were given all using 1/2 as the quantity that they had. At the end of this lesson students were asked to write and make some observations about what they noticed throughout the lesson and list any questions they still had.

The following lesson I had students model the same problem of 2/3 x 1/2 with a Twizzler creating a word problem from the situation. I worked with my focus group listening to their responses and guided them as well as the class through developing their word problem. I then had them pull out the fraction circles and we began working with different quantities such as 2/3 of 3/4 and 3/4 of 1/3, illustrating each problem in our notes and creating word problems from the expressions. Student's homework was to write a fraction multiplication word problem on an index card and bring it in the following day.

With the word problems the students made, I introduced the concept of a rectangular area model with a set of laptops. I read aloud students word problems and modeled one using the National Library of Virtual Manipulatives online manipulatives. I then read the remaining students problems. We would discuss whether or not it was a multiplication problem and then solve for the answers using the virtual area model.

In the last lesson, students took a written formative assessment in which they were to write a story word problem for 2/3 – 1/2 and 2/3 x 1/2. When students were done, one by one we went through and assessed each problem and reflected on the errors in our mathematics journal in a two-column format. On the left column we had correct examples of subtraction problems and highlighted why they were correct and on the right side we wrote examples of correct multiplication word problems and highlighted why they were correct. Any misconceptions were noted at the bottom of our notes and called "pitfalls". I first read all the subtraction problems and then all the multiplication problems. After reading a problem, students were to write on their white board if they thought it was subtraction or multiplication and why. This was a great review activity as students were still learning some of the subtle differences between the two operations with fractions.

The final day that I administered the posttest and interviewed my focus group on their thoughts about why some of them changed or did not change their pretest questions, as well as what they could tell me about their posttest word problems.

Table 1: Summary of Lesson Topics

Sequence of Instruction	Mathematical Content
	Pre-test/Interview
Day 1	Introduction of models with subtracting fractions
Day 2	Developing subtracting fraction word problems with area model
Day 3	Continuation of subtracting fraction word problems using students' word problems as examples to reflect upon, work with focus group
Day 4	Fraction circle models with multiplying fractions
Day 5	Developing multiplication fraction word problems with area model
Day 6	Continuation of multiplying fraction word problems and rectangular area model using student work and the National Library of Virtual Manipulatives
Day 7	Written formative assessment using two problems, one fraction subtraction and one fraction multiplication. Review as a class the assessment and discuss the operation sense of subtraction and multiplication word problems that students created
Day 8	Post-Test/Interview

Data Collection

I used several types of data collection during my study including a pre and post test demonstrating understanding of context in word problems, student class work and homework samples, small focus group discussions, informal interviews with students, and observations with field notes. These types of data were used to provide triangulation.

Students' pre-tests provided me with a tremendous amount of information. It gave me a baseline of their initial understanding of subtraction and multiplication with whole numbers. It also pinpointed for me student's misconceptions in subtracting and multiplying fractions so that I could hone in on those specific areas in my instruction.

Students' class work and homework were collected in various ways, including index cards, writing in mathematics journals, and the explore handouts from their practice workbook. The explore handouts were workbook pages from our adopted textbooks series that had a total of six problems. The first three problems students had to find the product of a fraction multiplication number sentence using a model. The remaining three problems were story problems in which students had to represent each situation by drawing a model and then solving.

These resources provided me with the information that I needed in order to see how my students were showing and understanding the differences between subtraction of fractions and multiplication of fractions through their modeling and word problems. Classroom observations also provided me with information regarding students' misconceptions through their explanations and justifications of the strategies they used and how they went about manipulating the fraction circle models and creating their word problems.

Working with a small focus group, I was able to follow the students' trains of thought and the strategies they used as they were developing their understanding of the differences between the two operations as they related to fractions. I was able to ask probing questions for students to clarify and elaborate on their answers, which gave me the insight into their misconceptions and how it stemmed from their beliefs about whole numbers and their application of those rules to fractions.

Data Analysis

Throughout the collection of data, I continually looked for emerging patterns as a means to analyze the data. I used the pretest to determine students' level of ability in writing basic subtraction and multiplication problems with whole numbers and compared that data to their ability to write word problems with subtraction and multiplication of fractions. I looked for the most common contextual differences that students used in the word problems to indicate operation sense as well as how the information changed from the pre-test to the posttest.

The data were categorized into themes such as subtracting fractions with the area model, subtracting fraction word problems in context, multiplying fractions with the area model, multiplying fractions word problem in context, and the differences in students written word problems for subtraction and multiplication of fractions.

Validity

Content validity of the pretest and posttest was upheld by using subtraction and multiplication fraction problems from the textbook and having students be able to identify if they were subtraction or multiplication problems. The expressions chosen for the pretest and posttest was done with my thesis chair who was conducting similar research with preservice teachers. The posttest was administered the day after finishing with the formatives assessment of the subtraction and multiplication word problems.

Summary

The qualitative methodology used in this study provided me with the format to examine my practice of teaching the context supportive of the area model in writing word problems for subtraction and multiplication of fractions.

Interpretations of these data were discussed in Chapter Four, Data Analysis. An analysis of the data revealed the impact of my instruction as my class explored the area model for multiplying fractions to develop a better conceptual understanding of writing word problems using the correct context.

CHAPTER 4: DATA ANALYSIS

Introduction

In the early stages of my action research, I was initially interested in how my students would develop word problems for subtraction and multiplication of fractions in context to demonstrate understanding between the two operations. With the new Common Core State Standards being adopted in Florida, which is promoting a deeper understanding of fractions using "a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15" (Common Core State Standards, 2014, "5.NF.4" para. 1), I felt this would be an interesting subject to focus my attention on in regards to my research. However, although being able to write word problems is important in demonstrating conceptual understanding and offering a visual to students; it is just as important to be able to explain and justify their work using manipulatives and be able to communicate their understanding with each other. Through my qualitative research, I explored this question:

> Question: How does my practice of focusing on context and the area model for fraction subtraction and multiplication influence student performance when writing word problems for those operations?

What did they know?

At the beginning of my study, I wanted to know what my students knew about the operations of subtraction and multiplication and what contextual words they associated with these operations. Students were asked to write four word problems in response to four number sentences given to them on a piece of paper.

The first expression was 5 – 2 and although no one student create the exact story problem as the next, eighteen students used the phrase “how many does he/she have left” in their problem. Out of twenty-one students, eighteen problems were “take away” and only two were comparison problems. One student did not write a correct story problem using the 5 – 2 expression. During his interview, his response to adding to the problem was that he was going for a more elaborate problem which is why he did 10 – 5 – 2. It was safe to say my students could write word problems demonstrating at least one form, “take away”, of subtraction with whole numbers.

The second expression was 3 x 4 and I expected to also see proficiency with the meaning of this expression. Our first lesson at the beginning of this school year covered the standard of multiplying and dividing with decimals. As a class we had already discussed the meaning of multiplication in that it was grouping times quantity and used illustrations and manipulatives to represent three groups of four. This convention had been introduced prior to this lesson that the first factor tells the number of groups and the second the number in each group, which we called the quantity. Therefore, I was not surprised to see that my students could write story problems demonstrating this skill. Out of the seventeen students able to demonstrate a correct multiplication word problem, twelve students followed the conventions of grouping times quantity and five students did not.

For the purpose of my data, I did not count the order of the fraction multiplication expressions in their word problems as incorrect. The purpose of my action research was focused on the area model, as used with fraction circles and the rectangle area model, and the context of the word problems. If students’ conventions were not correct, it was discussed as a class, but it was not counted wrong.

A few observations that I noted as I was reading over the pretest word problems were that students were using the contextual word “times” to make their point that it was a multiplication problem, several wrote a story problem for 4 x 3, placed the number sentence directly in the problem, as well as literally used the phrase “3 groups of 4” as part of their problem.

Although seventeen of my students were able to write a correct multiplication problem similar to the first bulleted problem below, I still had several problems that were incorrect or the meanings were not the same.

Correct Multiplication Word Problems:

- Andrew has 3 marbles. Randy has 4 times the number of marbles Andrew has. How many marbles does Randy have?
- Carley got 3 pieces of candy for each time she walked around the block on Halloween. She walked the block 4 times. How much candy did she get?
- In your class you have 3 groups of 4 people in each of them. How many students are in that class?
- Matt had 3 pencil cases with 4 pencils in each case. How many pencils does he have all together?

Incorrect/Unclear Multiplication Word Problems:

- Tori is doing her homework if one of her questions is 3 x 4 what would her answer be?
- I have 3 boxes that has 4 packs of soccer cards how many packs do I have? (unclear as it does not specifically say four packs each in a box)

The third question on the pretest was 4/5 – 1/2. I had hoped that when given this problem students would have an idea of what this looked like because it was a fifth grade standard. After reviewing the word problems I found that ten students wrote a multiplication problem, indicating that they have a quantity 4/5 and took "half of it". The five students wrote a take away subtraction problem, and six students wrote a multi-step subtraction problem that included multiplication. After reviewing students' interviews, it became clear that they were not consciously aware that they had included multiplication in their problem and just knew their problem was subtraction because of the word "how much is left".

Correct Subtraction Fraction Word Problems:

- Rob has 1/2 a cup of cereal. He needs 4/5 of a cup of cereal according to his diet. How much more cereal does he need?

Incorrect Subtraction Fraction Word Problems:

- Dave has 4/5 of a pizza left. He had a friend come over. His friend ate 1/2 of what was left. How much pizza is left? (16 students wrote similar problems)

The final expression was 1/3 x 3/4. I did not know what I would get going into this question as the students had never multiplied fractions before and this was their first exposure to this type of question. I found after analyzing the questions that I had a wide range of word problems with only three students who were able to correctly write a fraction multiplication word problem.

I made several observations after looking over the pretest. The first was that most students indicated that 1/3, which was the grouping, was the quantity in their word problem. For example one student wrote, "Phile has drank 1/3 cup of Gatorade. Julie has drank ¾ times as much. How much water has Julie drank"? Although I counted this as a correct word problem, the convention of multiplication was incorrect. Twelve students were able to demonstrate in 3 x 4 the correct convention for multiplication, as three groups of four, yet were unable to apply this whole number concept to fractions. My second observation was that students wanted to use the word "multiply" or "times" to tell the reader that this was the operation that should be performed. The last observation that was noted was that several of the students did not create story problems, but instead wrote a sentence with the expression written directly into it or the word problem did not make sense.

Correct Fraction Multiplication Word Problem:

- In Merritt Island there is a very big park. It is 1/3 of a mile by 3/4 of a mile. What is the area of the park?
- Sophi is doing a project on rice if she has 1/3 cup of rice, and she has to multiply it by 3/4 how much rice will Sophi have?

Incorrect Fraction Multiplication Word Problems:

- Alex had a homework question and he didn't understand it. This is his problem: 1/3 x 3/4. What is the answer to his problem?
- Jerry collects coins. He had 1/3 of the amont of mico. Mico gave jery 3/4 of his quarter collection. What did the persentage of the amount jery has of mico's?

In addition to the pretest I sat down with each student and interviewed him or her individually about his or her word problems. Most were confident with their whole-number story problems. They gave clear examples of story problems and most did not fall back on writing 3 x 4 as an expression in the problem itself. When it came to the fraction story problems, students were once again confident that subtracting one-half meant to take “half of” the first quantity, which was demonstrated in their problems. With the multiplication fraction problems many students did not feel their problems were correct, but were unable to explain why other than they just did not understand the number sentence. This left me with the task of correcting their misconception of taking a half of a quantity for subtraction, which at times became an arduous task as students knew how to find common denominators to subtract using the algorithm from fifth grade mathematics.

Subtraction with the Area Model

For my very first lesson with subtraction using the fraction circle manipulatives, I had students place them on their desk and explore a little. Some students organized them by color and denominator and others stacked them on top of each other showing equivalency. I began with a remedial discussion about fractions since the students have been exposed to them since third or fourth grade. I asked students to look at the fraction circles in front of them and discuss within their groups, “What is a fraction”? As I circled the room I observed students manipulating their fraction pieces in order to explain their responses to their group. These discussions mostly ranged from “part of a number” to “a numerator and a denominator” to “a number less than one”. It appeared that overall students were comfortable being able to show and give an informal definition for fraction.

Therefore I asked them to use their manipulatives to illustrate 2/3 – 1/2. As I once again circled the room I noticed students placing two one-third pieces in front of them and a half piece. With a few confused looks students raised their hands and asked if they needed to find a common denominator to subtract. I restated my directions and told them they could do what they needed to do with the fraction circles in order to show me 2/3 – 1/2. At this point the students began placing sixth or twelfth pieces over the two-thirds, then placed the half piece over those. Three students became lost and stated they couldn't remember how to take away one-half from two-thirds, in which I proceeded to have them show me using their fraction circles and asking them guiding questions such as:

- Could you make an equivalent fraction to two-thirds that you can take one-half from?
- How would you go about subtracting or taking away one-half with your equivalent fraction?

At this time students were able to give me an answer of 1/6, while others gave me 2/12. This began a very in depth discussion of the meaning of equivalent fractions. While the majority of the class chose to pick sixth pieces to cover their two-thirds a few students chose the twelfth pieces. As a class we reviewed the meaning of equivalency of a fraction and that although it is an equal amount the meaning was different, whereas the sixth pieces were larger than the twelfths we still had the same quantity left over after subtracting one-half.

In order to demonstrate a pattern for the students I had them continue to use their fraction circles to subtract a half using a few problems:

- 3/4 – 1/2
- 5/8 – 1/2
- 11/12 – 1/2
- 5/6 – 1/2

Here was a response from our discussion on 3/4 – 1/2 in which students modeled using their fraction pieces. Students were consistent in their responses on how to take away one-half from three fourths and used one of two methods. They either made the one-half into an equivalent fraction or in the case of this problem two-fourths equaled a half and they were able to just pull those two pieces away.

> Teacher: I have three-fourths of a piece of pizza. How can I take away one-half of this pizza?
>
> Student 1: You find how many fourths are in a half, and you take half away.
>
> Student 2: You could also place the one-half piece over the three-fourths and take away the part that is covering it.

Once students became proficient with showing me using their manipulatives how to subtract one-half from a fraction I asked began introducing other examples that did not involve one-half. I then had them illustrate an example of their choosing in their math journals and to record their observations of the last few problems that they did.

As a whole group, I had the class share their examples and observations. Several students had bulleted in their journal that no matter what fraction they started off with they always took the same quantity away, which was the one-half. A second observation that was made during the sharing of examples was that they were always subtracting two different quantities trying to see how much more there was of one than the other.

Subtracting Fractions within Context

Once students had a grasp using the fraction circles to model subtracting one-half, I moved the class onto writing story problems to support the area model representation that was on their desk or in their notes. I modeled the first story problem for the students using the previous problem of 2/3 – 1/2 on the document camera with Twizzlers. Using 2/3 of a Twizzler and 1/2 of a Twizzler, I posed this problem to the students, “Johnny has 2/3 of a Twizzler and I have 1/2 of a Twizzler. How much more does Johnny have than me”?

With this problem on the board I asked students to turn to their neighbor and create a similar problem using the same number sentence, but different situation. At this time I moved over to my focus group of students that I had picked based on their ability to communicate their thoughts and ideas mathematically. I immediately noticed that one of the students had opened her bag of fraction circles and placed the corresponding pieces on her desk so that she could have a visual. As she began talking to the group she placed the one-half piece on top of the two-thirds and stated, “Johnny has 2/3 of a pizza and Bob eats 1/2 of it, how much pizza does he have left”.

For clarification, I asked her what she meant by “half of it”. She picked up the one-half piece and showed it to me and said, “He eats this half of the pizza”. Therefore I asked her a guiding question. “If we share the two-thirds pizza in your problem, wouldn’t we each get an equal share”? She nodded her head in affirmation, but was not making the connection that her problem stated she was splitting two-third into equal parts, but her models showed a completely different image. I then questioned the group, “Where is the error that we are making in the problem”? I placed the one-half piece next to the two-thirds and asked the group, “Show me using your fraction circles, half of two-third”. One out of the five students placed sixth pieces on top of the two-thirds and separated two from the group, telling me that half of two-thirds was two-sixths, while the remaining five students said half of two-thirds was one-third.

I took my one-third piece and placed it next to the one-half piece and asked the students if these two pieces represented the same quantity. All five students indicated no they were not equal. So I restated the first girl’s problem, “Johnny has 2/3 of a pizza and Bob eats 1/2 of it, how much pizza does he have left? Did Bob eat half the pizza?" The students responded with a no. He didn't eat half of it, because half was one-third. I explained that what was throwing them off was that the problem had the phrase "how much pizza does he have left".

Due to the student's choice of words she had a multi-step subtraction problem with multiplication in it. We were only focusing on a single step word problem and the direction that this student was going implied she was performing multiplication, even though she did create a subtraction word problem. She did not realize this and it was not until I began analyzing my data that I noticed it as well.

I restated my original Twizzler problem to the students and had them word a similar problem using Johnny and Bob's pizza. A second student spoke up and used his fraction circle placing the two-third to the far left and one-half to the far right of the table and said, "Johnny has 2/3 of a cheese pizza, Bob has 1/2 of a pepperoni pizza, who has more pizza"? The group agreed that this was a subtraction problem because you "take away one-half from the two-thirds" and get the answer one-sixth. After reviewing this response, it was actually a fraction comparison problem because the student did not indicate "how much more" in his response.

As I brought the whole class back together to share their word problems with each other, I heard two additional students that I called upon who also had a multiplication fraction problem. As a class we modeled the problem similarly to what I had done in my group with the guiding questions and illustrated the examples in our mathematics journals.

I had not planned on covering multiplying fractions at this time, but I seemed to have begun addressing it even as we were discussing fraction subtraction word problems. I tried not to use the term multiplication yet, as I wanted them to build a good foundation of subtracting fraction word problems, but I found that every time I had to correct their misunderstanding of taking "half of it" that inevitably I was discussing multiplying fractions with the students.

For homework that evening, students were to create their own fraction subtraction word problem on an index card and bring to class the next day to share with students in class. Out of twenty-one students, thirteen wrote a comparative problem, three students wrote a take-away problem, and five students wrote a fraction multiplication problem. Although a total of sixteen students were able to write subtraction problems, three of them either came up with an incorrect answer or asked the reader to solve for the incorrect fraction. For example, this student compared pizza to pizza and asked how much more did Julia have than Kate, but then gave an incorrect response of 2/3 instead of 1/6. (Figure 1)

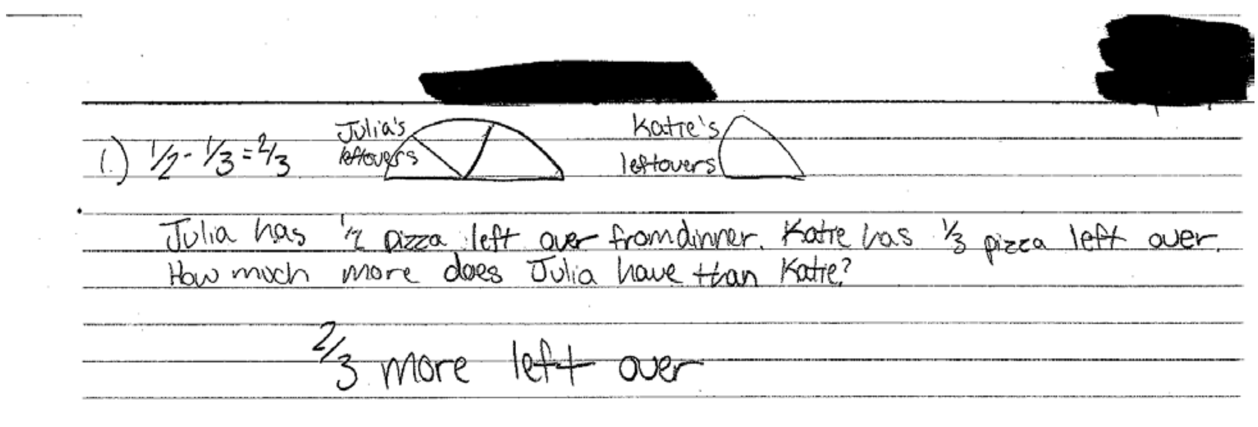


Figure 1: Fraction Subtraction Word Problem

Figure 2 demonstrates a correct fraction subtraction word problem. In this problem the student indicates that Maggie is taking away 1/8 of a chocolate bar from Jesse, and wants to know how much of Jesse's chocolate bar is left? The word problem was correct as it related to the expression. The only comment that I made to the student was that their model should have had equal lengths.

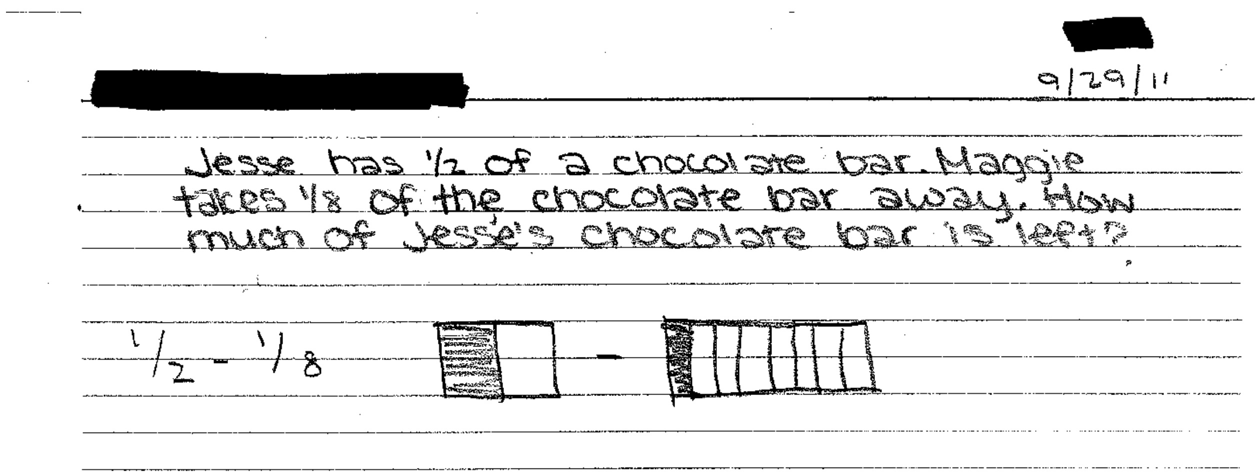


Figure 2: Fraction Subtraction Word Problem

Two correct examples below were comparing flatbread pizzas, and cups of sugar and chocolate. In Figure 3 the student writes a comparative statement between two different pizzas and in Figure 4 a student compares 3/4 cup of sugar to 2/3 cup of chocolate, asking the question, “How much more sugar do I have then chocolate?”

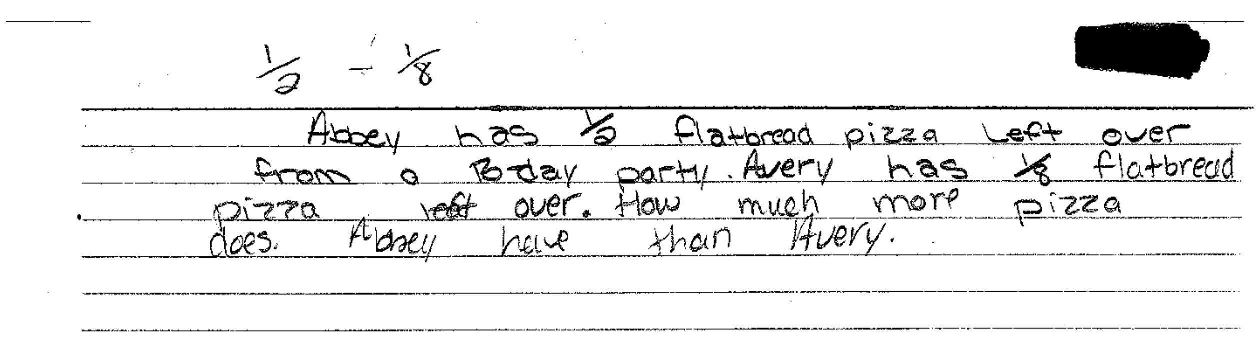


Figure 3: Fraction Subtraction Word Problem

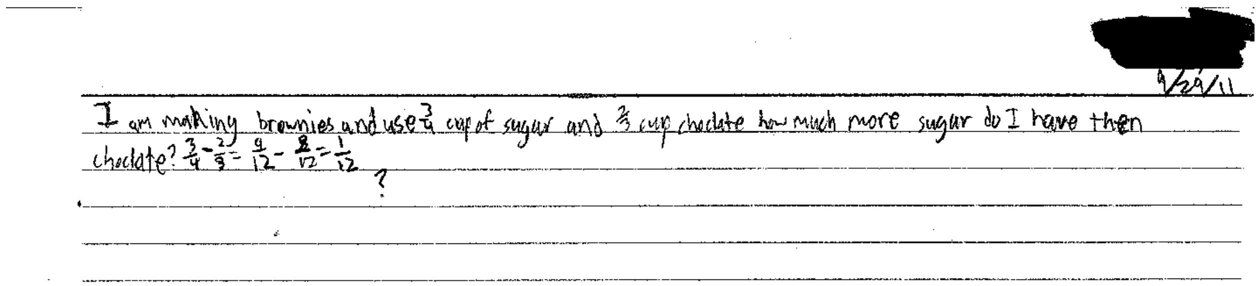

[redacted]
9/29/11
I am making brownies and use 3/4 cup of sugar and 2/3 cup choclate how much more sugar do I have then choclate? $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$
?

Figure 4: Fraction Subtraction Word problem

These index cards showed some improvement in the class from the pre-test. The class went from five students who could write a fraction subtraction word problem to sixteen students who could write a subtraction word problem. At this point I hoped that with more practice and some experience with multiplying fractions that they would be able to make better connections between the two and the use of the phrase “half of it”.

Multiplication with the Area Model

As we shifted instruction from fraction subtraction to fraction multiplication with the area model students seemed to pick up the multiplication much quicker. Students had some minimal exposure to the concept of taking a part of a number when I was correcting mistakes with fraction subtraction word problems. At this point I wanted them to explore with the fraction circle pieces more in depth to get them to see that when they used specific context in word problems such as “one-half of it” they were multiplying fractions.

Therefore I began my lesson much like I did with subtraction of fractions and wrote a simpler problem on the white board (3 x 4) and asked students to discuss the mathematical meaning of this number sentence. Students told me that they had three groups of four. When I asked them to illustrate an example on their white boards they were able to do so, drawing three circles with either the number four, four tally marks, or four dots in the three circles. Out of twenty-one students I had three students who wanted to tell me it was four groups of three. Those students were still having difficulty remembering which the grouping number was.

My next step was to place 2/3 x 1/2 on the board next to 3 x 4, which had the words "3 groups of 4" written underneath it. Students were asked to talk to their neighbor about the meaning of this fraction expression. When students shared their responses with the class they used the terms "2/3 group of 1/2", because those were the words that we had used to describe the whole-number multiplication. When I asked them would they be able to show me what it looked like, I received many confused expressions. One of my more advanced students said they had been taught that a fraction was a part of a whole; therefore we were multiplying parts. This student also happens to be the only one in class whose pretest had four correct responses.

Students pulled out their fraction circles and started with placing a half, because that was the quantity that we had. I then asked them to divide into groups of three. This confused them because they heard the words divide and thought we were multiplying. I reworded my directions and the specific wording I was using as they were not familiar with the word divide being applied to multiplication in this sense. I again asked how I would go about "splitting or dividing" one-half into three groups. Students proceeded to place the sixth pieces on top of the one-half piece. I asked them to look at the problem and tell me how many groups of the three did we make? The consensus was two groups of three, because that was what 2/3 represented, and our answers were two-sixths or one-third. There was still confusion about the word division as it applied to multiplying fractions, therefore for the next several problems I continued to use the student's language of "splitting" the fractions and then I would come back to the contextual term of dividing once they had the conceptual understanding of multiplying fractions.

For me, I was at a place where I was pleased that they knew we were taking groups of a number. So we moved on to another example, 3/4 x 1/2 in which I had students place one-half in front of them and asked them to show me how to take three-fourths of one-half. With my fraction circles representing one-half on the document camera I began questioning my students on how to take three-fourths of one-half. Initially one of my students wanted to place another three-fourths down and was confused, but when I reworded my question and asked her if I could split or share half of a pizza, she understood the direction I was going with the problem. It made sense that you can split half a pizza into smaller segments as they have had to do this before.

Teacher: In this problem 3/4 x 1/2, what quantity do I have?

Student: You have 1/2.

Teacher: So if I have half of a pizza, which we are going to represent with our pink one-half fraction tile, what does the 3/4 mean?

Student: It means you have 3/4 a group.

Teacher: Can I take 3/4 of a number?

Student: Yes, you can place three fourth pieces on the table.

Teacher: So you are saying that I have one-half piece of pizza and I'm going to add an additional three-fourths pieces to the one I have?

Student: Ummm, no?

Teacher: Can you split or divide your one-half pizza into fourths? Show me using your fraction pieces, dividing one-half into fourths.

The students placed the different fraction pieces on top of the one-half in order to see which ones would fit completely over the one-half and have four groups. As I walked around, I had to guide a few students on how to divide their fraction into groups. I instructed the students to look back at the problem 3/4 x 1/2 and asked them how many of the four groups are we going to take. The class's response was three. My question was, "three what"? They looked back down at their fraction pieces and stated three-eighths.

Multiplying the fractions using examples of pizza and cake seemed to make it easier as they could all envision what splitting pizza or birthday cake would look like. Therefore with the rest of our problems for that day I would verbally place them in a word problem that had to do with pizza or cake.

At the end of this lesson students were asked to write in their mathematics journals any observations or questions they still had about multiplying fractions. Fourteen students responded that they still were initially confused when I said the word divide, as their immediate response was to think about a division symbol. It was only when they physically took the circle pieces and split them into groups that they saw the division happening. I was particularly taken with one student's writing, "Most of what Ms. Friske did was confusing until I got to practice with the pieces of fractions, even when I get it with the pieces I still don't get it when she talks about it or rites the problem on the board, I need the pieces." This was a turning point for me. I really had to think when I gave my formative and summative assessments for this material was I going to continue to let them use the manipulatives. Normally, I would give them to the students for a few days, and put them away back in the cabinets after we explored with them. I never took the time to reflect and think that because this was new knowledge that it would take more than a week or two to be mastered. Therefore, they should have access to the manipulatives any time they need, as some may still need that concrete example in front of them to make sense of a problem.

Multiplying Fractions within Context

Linear Model

As we moved onto multiplying fractions within word problems, I brought out the Twizzlers again as the students were able to make sense of a real word situation in order to create their story problems. I wrote the number sentence 2/3 x 1/2 on the board and each student received a Twizzler this time. They all worked in small groups as I guided them with questions. During this time I worked with my focus group, listening to their word problems and observing their manipulation of the Twizzler.

I began by asking the class to show me indicating with one finger in front of their chest that two-thirds is the quantity they have or with two fingers that one-half is the quantity. In my focus group four out of five students indicated our quantity was one-half. Out of the whole class I had three students who told me two-thirds was our quantity. I had one of the students from my focus group explain to the class why they chose one-half. He said, “Well you see, the two-thirds is the grouping number and the one-half is what we have.” I followed up with my question and asked him to explain with an example how he knew this. “Yesterday we did the problem on our white boards. I remember drawing three circles and placing four dots in each circle to show 3 x 4.” I illustrated his example on the board as he was talking then I asked the students did they agree or disagree with a show of hands. Students agreed with his response and I let them cut the Twizzlers in half, keeping one-half in front of them and placing the other half aside.

Students were instructed to create the beginning of their word problem with the fraction piece in front of them. The students all began their word problem similar to this, “Jamal had one-half of a Twizzler.” As a class we moved onto the next part of our number sentence, the group number two-thirds. I questioned my students, “My grouping number is two-thirds, how many times am I going to split or divide my one-half piece, please discuss within your groups?” My entire focus group agreed that they would split it three times. When I asked for an explanation, I received this collective response.

Students: Well two-thirds means you have three groups but can only take two of them.

Teacher: What do you mean I can only take two of them?

Student: Can I draw it on the board, it helps me explain?

Teacher: Yes

Student draws a linear fraction and divides it into three groups shading in two of them.

Student: This bar is the whole piece that you have, but it is split into three pieces, and the two is how many pieces we are going to take.

Student: Show me with your Twizzler how you are going to take two-thirds of the piece in front of you.

As I was walking around the class I saw students cutting their one-half piece into thirds and placing two of them to the side to indicate this is what they had when they were done. I asked my class, “So my answer to this problem is two-thirds, right?”, as I wanted to see what their response was to placing the two-thirds to the side. I was quite surprised when most answered together at once “NO”! They knew that it was just part of the entire piece. As I was making my way back to my focus group I had them explain and justify their answer to their group. My focus group was arguing between the answer two-fourths and two-sixths. One student believed it was two-thirds and when I asked her to clarify, this was her response.

Student 1: Well, I took the two-thirds from the one-half that the problem said to do, and I have two out of four pieces on my table.

Teacher: Explain the four pieces to me?

Student 1: There are three of these pieces, (points to thirds), and there is this piece, (points to one-half)

Student 2: But that piece isn’t split into thirds, you can’t add it to the thirds pieces like that.

Teacher: Do you all agree with this response?

Students: Yes!

Teacher: Show me and Student 1 with your Twizzler how you got the answer two-sixths.

Student 3: You have to split the other one-half into thirds as well and see how many total pieces you have. (Begins cutting)

Student 3: I have six pieces and we had the two from our answer, so it is two-sixths, and that simplifies to one-third.

The students could explain and justify their response with their manipulatives. I had them finish their original problem, in which “Jamal had one-half of a Twizzler”.

Teacher: So let’s finish our word problem, “Jamal had one-half a Twizzler”.

Student 3: Jamal had one-half a Twizzler and ate two-thirds of it. How much does he have left?

This statement that Student 3 made was not corrected at the time as I still had the misconception that this was a correct answer. It was not until I was analyzing the data that it was brought to my attention that the student was giving a subtraction problem 1/2 – (2/3 x 1/2). What I should have done was have the student reword their answer and point out that in the example he gave the context of “how much does he have left” was subtraction and a better way to phrase this would have been to ask, “What fraction of the Twizzler did Jamal eat”?

Area Model

The students in the class all had similar problems to this one just with different names. Once they had finished with the Twizzler I practiced a few more problems moving away from the one-half quantity and began to use some other examples with the fraction circles. Here were a few that we practiced with and the corresponding word problems that the students created.

- 3/4 x 2/3 John had two-thirds of a coke and he spilt three-fourth of it. How much coke does he have left?
- 1/2 x 2/3 Luna has two-thirds birthday cake left over from her party. I come over the next morning and eat half of it for breakfast. How much is left?
- 1/2 x 1/4 Bill has one-fourth of a Three Musketeers and nibbles at a half of it. How much does he have left?

A trend that continued to show up throughout my data was that students were modeling the same words I was using. Since I had begun to overuse an incorrect phrase “how much is left”, several students began repeating this misconception in their writing. Throughout much of the remaining data, when there was a class discussion students would use this phrase. Yet, when I asked for them to illustrate their work and give a word problem I was able to get a few correct responses, which showed in the posttest results.

Since most students had written a multiplication problem for their subtraction problem on their pretest this went very quickly. They were all comfortable in giving me word problems in which someone ate part of another part.

Students were instructed to write any additional observations or question they had in their mathematics journal at the end of our lesson and if they wanted to, share with the class. I had a student that was perceptive enough that he noticed every problem we did could be solved with our fraction circles, but what about the problem 1/5 x 2/3? How did we solve these types of problems? My response was that they were going to create a word problem on an index card for homework and we would solve it using a rectangular area model the next day.

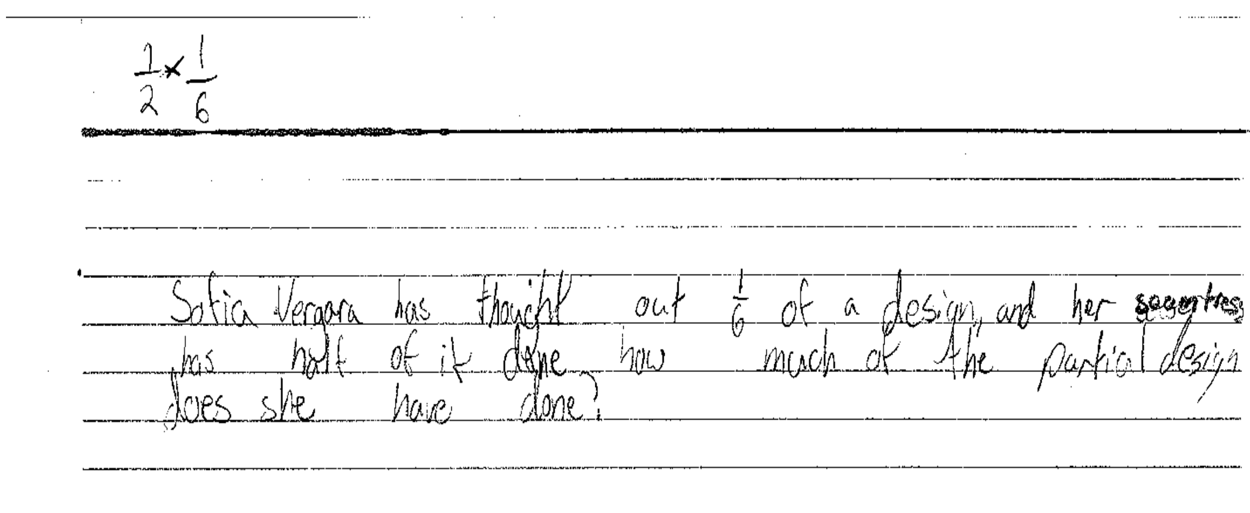
Rectangular Area Model

Students came in the next day with their word problems that they had created for homework. Students were instructed to write a fraction multiplication expression, to give a model if they could, and to write a word problem from their expression. Out of twenty-one students, fifteen of them wrote a fraction multiplication word problem. Of the six remaining students, five of them wrote a multi-step subtraction problem and one student's work was unclear.

Each student had a laptop on his or her desk and we logged into the National Library of Virtual Manipulatives (NLVM), clicked on 6-8 and scrolled down the page until we saw "Fraction – Rectangular Multiplication". Figure 5 is the first problem that I read from the student's homework. The student wrote, "Sofia Vergara has thought out 1/6 of a design, and her seamstress has half of it done how much of the partial design does she have done"? Notice the one word "partial" changes the context of this problem. This student's word problem was not a completely correct problem, as the student should have said, "how much of the whole design does she have done", instead of, "how much of the partial design does she have done". By inserting the word partial the context of the word problem and its meaning has changed, because we are looking for how much of the whole design has been finished.

For each problem I asked students if they believed this was a multiplication or subtraction word problem. For Figure 5 they indicated it was multiplication because, "how much of the partial design" was an indicator in their eyes that they were finding a part of part. This was a trend that I noticed would occur in students' work. They would either insert a word or phrase that would change the meaning such as in Figure 5 or leave out one word or phrase that made the difference in their word problem. For example, one student wrote, "Christian has 1/2 of a granola bar. Grady ate 1/8 of it. How much did he eat"? I know from the student's response that Grady ate 1/8 of the granola bar. The student needed to be more specific with the information given. The reader does not know what "it" is in the problem. Is it the whole granola bar, or just the one-half that Christian has? Another similar problem in which the student was not specific enough, "Jerry has 1/3 of a chocolate chip cookie. For lunch, Stacy eats 5/6 of it. How much did she eat"? Again, the same questions can be asked, how much did she eat of the whole cookie or just the one-third that Jerry had. With just a little more information and specificity in students' word problems they could have been clearer and correct.

I began modeling for students using the National Library of Virtual Manipulatives website with their word problems. First they would draw vertically the quantity or how much they had. In the design word problem it was one-sixth. As I was illustrating one-sixth on my computer they were following along on theirs. I moved the sliding bar to represent splitting one-sixth in half horizontally, to represent taking half and explained to the student the answer was the shaded area where both one-sixth and one-half overlapped each other. We continued to share additional problems discussing them and using our computers to model the rectangular area model.


$\frac{1}{2} \times \frac{1}{6}$

Sofia Vergara has thought out $\frac{1}{6}$ of a design, and her [illegible] has half of it done how much of the partial design does she have done?

Figure 5: Fraction Multiplication Word Problem

For the next problem Figure 6, I instructed the students to visualize their favorite chocolate bar and to solve the problem using their computers as I read it aloud to them. As a class we agreed that the fraction multiplication problem was correct and had a correct illustration as it matched theirs on the computer after solving.

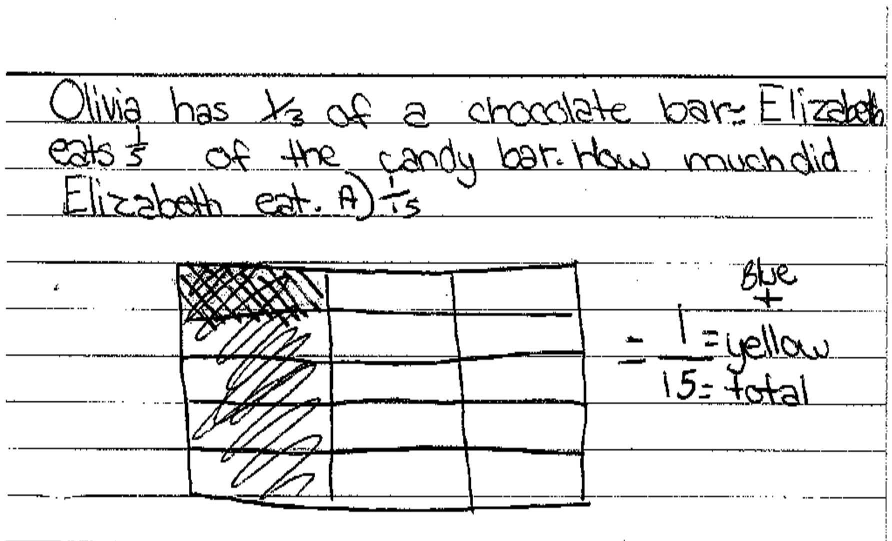


Figure 6: Fraction Multiplication Word Problem

Another word problem shown in Figure 7 had a correct illustration, but once again demonstrated a subtraction problem 1/2 – (1/4 x 1/2), because I had used this language while I was instructing the students. The difference between Figure 6 and Figure 7 was "how much did the student eat" and "how much is left". Six out of twenty one students wrote a similar problem for their homework, which was almost a third of my class. This difference in context made a difference in the operational sense of the word problem and changed it from multiplication to subtraction.

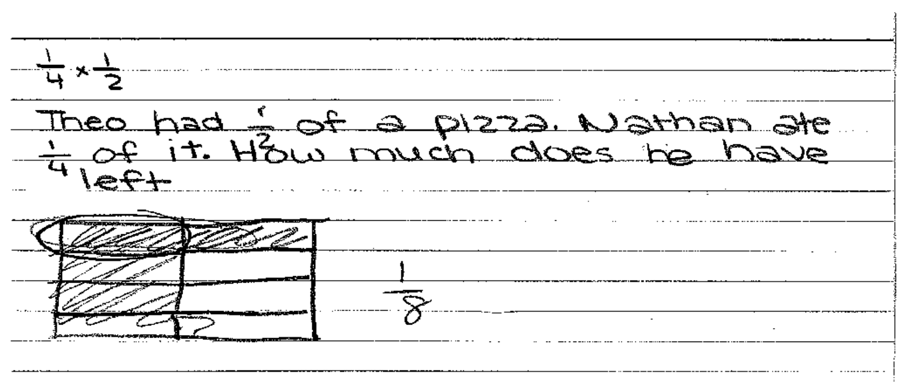


Figure 7: Fraction Multiplication Word Problem

A final trend that I saw within students' work was how they would take a fraction of an object that we would not normally split into parts. For example two-thirds of a book can be read, but you do not typically place two-thirds of a book in three separated boxes. The most popular examples that students used in their word problems included candy, pizza, pie and cake. These items can easily be split into parts, but on a few occasions I would get problems in which students would try to take parts of objects that are not normally split. Although this did not occur with most students, it happened in several instances and was worth mentioning. For example, one student wrote, "I have 2/3 of a box and it can carry 5/6 of a plane. How much does it carry"? Another student example was, "I have 1/3 boxes and 3/4 baseballs cards in each. How many cards do I have"? Although this is a correct fraction multiplication problem that the student wrote, one can ask the question does the word problem need to make sense? The student can use the rectangular area model to find an answer, but in the real world do we collect parts of baseball cards and does this make a difference in teaching students to write word problems in context?

Differences in Subtraction and Multiplication of Fractions

For eight days my class had been working on using the area model to develop conceptual understanding of differences between writing a fraction subtraction and fraction multiplication word problem. Students were given a short written formative assessment in which they were to demonstrate knowledge of these two operations as they apply to word problems.

Out of twenty-one students nineteen were able to write a correct subtraction word problem. The two students who did not write a subtraction problem wrote a multiplication problem. Twenty students wrote a multiplication word problem, but of those twenty, nine of them switched the grouping and quantity. I was a little shocked with this as the students have been consistent throughout their lessons on which number was the grouping and which was the quantity that they had.

After students had finished with the assessment I placed three examples one after another on the document camera, making sure not to show student names. As a class we had a discussion about each and students found their errors and made note of misconceptions in their mathematics journal.

Notice that in Figure 8 the student wrote a correct fraction subtraction and fraction multiplication word problem, but it begins to look like the student is using an algorithm to solve both. I had not yet introduced an algorithm to the class, and I did not draw attention to correct or incorrect notations of this at this time. When I asked the student to explain what she was doing she stated, “Well I was beginning to see a pattern. So far every time we have found the answer, it is the same answer as if I were to multiply the numerator and denominator. It has worked for me, so I am checking my answer this way”. This was an important discussion; because several other students at this time also spoke up that they had seen the same pattern. Students were beginning to develop their own strategies and using them to solve fraction multiplication problems.

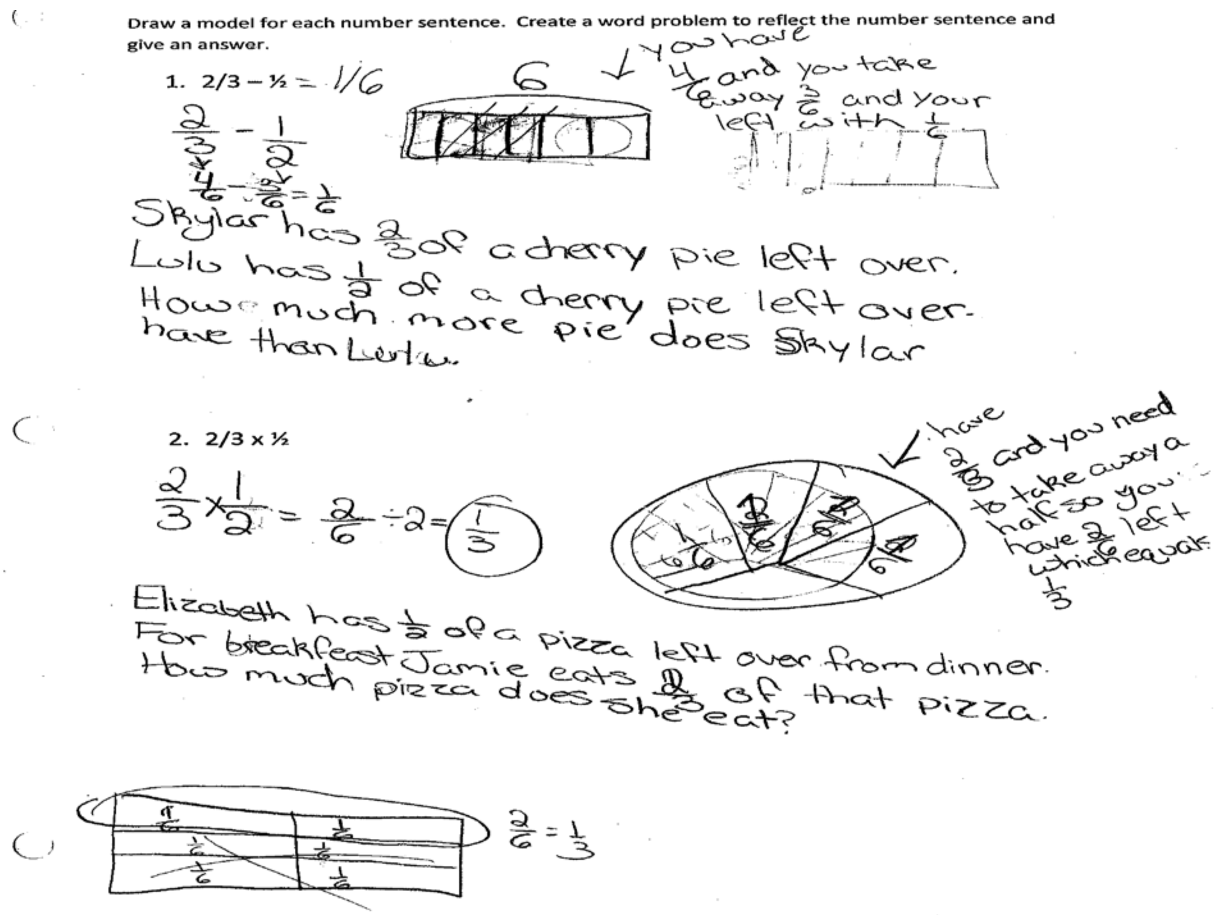


Figure 8: Formative Assessment

In Figure 9, the student was able to demonstrate a correct fraction subtraction problem, but it did not match their expression of 2/3 – 1/2. Instead the student wrote a word problem for 1/3 – 1/2, because they asked “How much more did abbey eat than avery”? This should be written, “How much more was leftover”? This is exactly was Drake and Barlow (2007) was discussing in their article. Students are able to write word problems, but do they match the expression given. In this case the word problem was not incorrect, but it did not match the expression. The fraction multiplication word problem was correct, but it was brought up whether or not the illustration was completely correct because the student did not shade one-half all the way across the row. Even though I had been using language that was incorrect such as, "how much is left" with fraction multiplication word problems, this student was still able to write a correct problem.

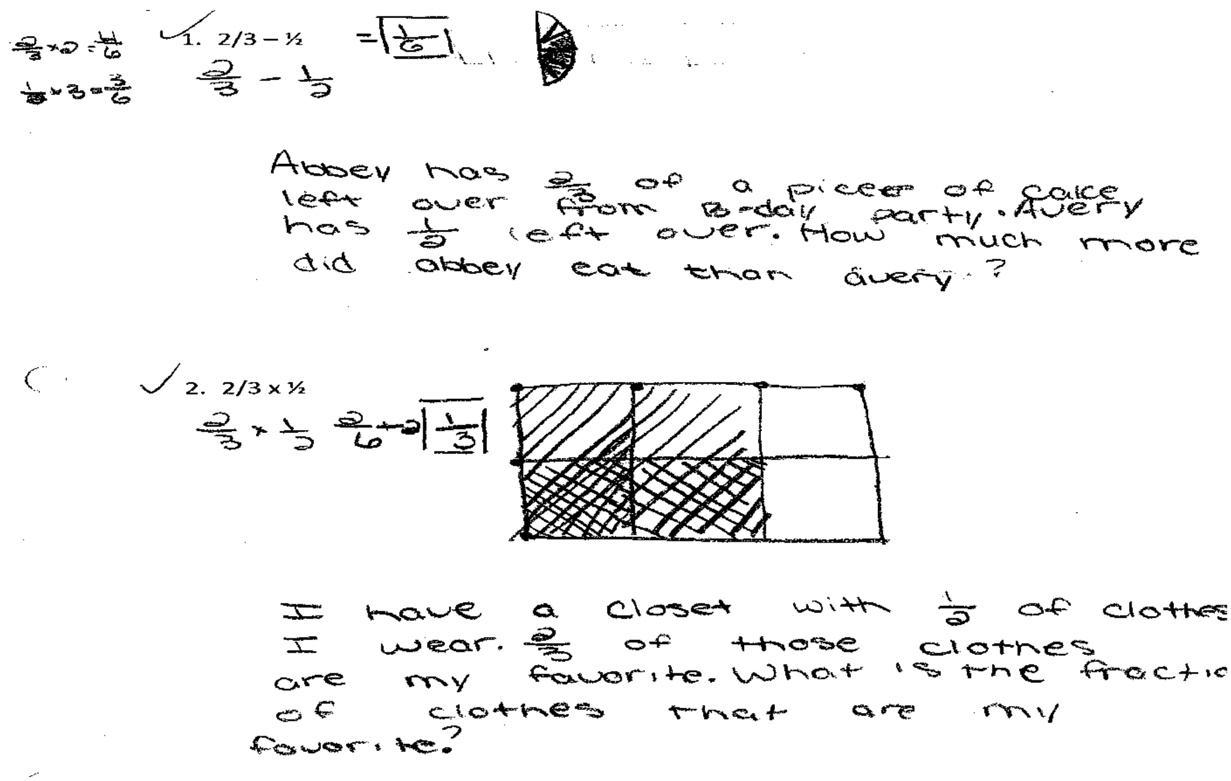


Figure 9: Formative Assessment

Figure 10 illustrates an example of the misconception that I projected onto my students teaching fraction multiplication. Once again, I continued to use the phrase "how much is left", which is why the student's were writing subtraction problems for their multiplication problems. Here the student wrote, "Abbey woke up and found 1/2 a pizza in her frige. She ate 2/3 of it. how much is left"? This problem is a fraction subtraction problem and would be written like this 1/2 – (2/3 x 1/2). As this was a misconception on my part, I noticed this trend within my students work. In addition to this the subtraction problem was a straight fraction comparison problem, because it just asks, “Who has more”? It does not ask for how much more.

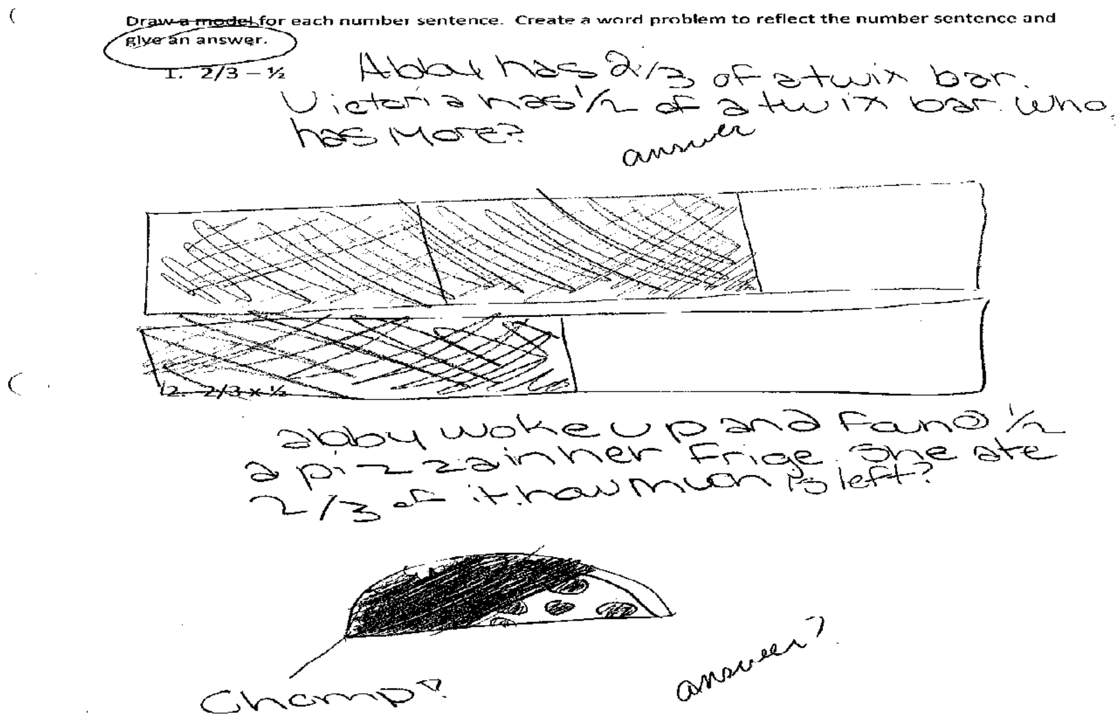


Figure 10: Formative Assessment

<u>Posttest</u>

After eight days of modeling fractions, writing fraction word problems, practicing and reflecting on the differences between subtraction and multiplication of fractions it was time to take the posttest. Students were given the opportunity to rewrite their original pretest word problems and make any corrections that they had seen in their work.

Table 2: Pretest Results

Problems	5-2	3 x 4	4/5 – 1/2	1/3 x 3/4
Incorrect	1	4	11	18
Correct	20	17	10	3

As you can see in Table 3, which is the corrected pretest results, students were able to recognize their mistakes with their previous word problems and correct them. Even though at times throughout my lesson I used a context supporting subtraction when discussing fraction multiplication, fourteen students were able to demonstrate on their posttest a correct fraction multiplication problem. The remaining seven problems all used the phrase “how much do I have left”, which created subtraction problems.

In comparing the pretest to the posttest, students made clear growth in a short period of time. Whereas in the pretest eleven out of twenty-one students wrote an incorrect fraction subtraction word problem on the posttest eighteen students were able to write a correct problem. On the pretest eighteen students wrote an incorrect fraction multiplication problem and on the posttest fifteen students wrote a correct fraction multiplication problem. This could have had the potential of increasing even more if I had corrected my misconception early on.

Table 3: Corrected Pretest Results

Problems	5-2	3 x 4	4/5 – 1/2	1/3 x 3/4
Incorrect	0	1	3	6
Correct	21	20	18	15

Table 4: Posttest Results

Problems	7-4	5 x 3	3/4 - 1/2	2/3 x 1/2
Incorrect	0	0	4	7
Correct	21	21	17	14

I did not correct student's pretest as I wanted to see if they could identify their mistakes and correct them on their own. In hindsight, this may have helped them out more in having the opportunity to correct their mistakes if they knew what they were.

Changes that I noticed from the pretest to the posttest included the use of the illustrations on some of the students' work. In the pretest, none of the students drew a picture at all. Although I wanted to give students the opportunity to use the fraction circle manipulatives if they wanted to, since I did not give them the manipulatives with the pretest I chose not to change my procedures when giving the posttest. With that being said, five students drew illustrations with their subtraction and multiplication word problems. It was noted that each of these students had correct word problems.

In addition to the illustrations, I saw that although my class had spent the whole first quarter of the school year discussing meanings of multiplication I still had students who would switch their grouping and quantity numbers. Only two students corrected this convention in the 3 x 4 word problem and five missed the meaning of 5 x 3 on the posttest. These were not the same students making the mistakes, but different ones each time. I did not know if it was a lack of focus on the problem, if it was a memorization error, or if they really were confused.

Based on the posttest and final interview of the focus group, all students felt that the multiplication of fractions was easier than the subtraction of fractions. When asked what about it was easier one student replied, "It is easier to picture the last slice of pizza and having to share it with your sister and brother, the only part that I still forget sometimes is which number goes first in the problem." This was the consensus with most of the group. Even after taking the posttest and moving on into multiplying mixed numbers with my class, I was still asked from time to time, "Which number goes first?"

Summary

Data were collected through a pretest, a posttest, student class work, homework samples, small focus group discussions, informal interviews with students, and observations with field notes. The data revealed that students had come to sixth grade with basic understanding of subtraction and multiplication of whole numbers, and were able to write word problems demonstrating this operational sense.

Students were unfamiliar with how to write fraction subtraction and fraction multiplication word problems with the correct operational sense within context. Throughout the study, many demonstrated the ability to write fraction subtraction word problems, but there were still a few students who were writing straight fraction comparison problems.

A trend that emerged was students writing a fraction multiplication problem and attaching the context, "how much is left" to it, which resulted in it being a multistep problem including both fraction multiplication and fraction subtraction. This was a result of my own misconceptions with this context in writing fraction multiplication word problems. In class I used language such as "how much is left" when speaking with the students. This resulted in the phrase being used in their writing, which changed their fraction multiplication problem into a multistep problem including multiplication and subtraction.

A second trend that showed in the data was students' specificity in writing their word problems. When they were not using the phrase "how much is left" they would leave out an important word that indicated they wanted to find how much of the whole pizza or cake was eaten, not just how much of “it” was eaten as the reader does not know what “it” was referring to.

A final trend that was noticed was students writing fraction multiplication problems for objects that we do not normally take parts of. For example students would take a part of a box, baseball card, a plane, etc. Although this trend was only with a few students and did not occur often it was enough that it brought up the question, should mathematics make sense?

Overall, despite the errors that I made within my class many students were able to demonstrate a correct fraction subtraction and fraction multiplication word problem in context.

Finally the pretest and posttest scores revealed an increase in the number of students that could write a fraction subtraction word problem and a fraction multiplication word problem in context. The final chapter of this study explains the results of the study, implications, limitations, and recommendations.

CHAPTER 5: CONCLUSION

Introduction

As I began my action research study, I sought to explore whether or not students could transfer the meaning of subtraction of fractions and multiplication of fractions using the area model to writing word problems. My research question was:

> Question: How does my practice of focusing on context and the area model for fraction subtraction and multiplication influence student performance when writing word problems for those operations?

In this chapter I review the results of my study in relation to the literature. I also discuss implications, limitations, and recommendations for further research.

Results

Overall, I learned that students could develop a deeper understanding of fractions, and write fraction subtraction and fraction multiplication word problems in context. Given enough time and my ability to correct my misconception, I believe students' results would have been better. Researchers argue that it is important to give students time to develop understanding of fractions and to not immediately begin with procedures and rules to compute (Van de Walle, 2006). My students were forced to think about the differences between the operations of subtraction and multiplication as they applied to fractions, and to learn they cannot rely solely on their prior knowledge of whole number context when solving fraction problems. Researchers also support the use of manipulatives and a problem solving approach, such as the area model in developing conceptual understanding of fractions (Van de Walle, 2006; Huinker, 2002; Whitin, 2008a; Krach, 1998).

Focusing my instruction on the area model and context within word problems created an atmosphere for students to explore their misconceptions between fractions and whole numbers. It allowed them to see visual differences between subtraction and multiplication of fractions and that changing the wording or scenario of a word problem results in a change in the operation being performed. As a result, this produced several positive outcomes. Students gained confidence in themselves as they were able to explain and justify their reasoning with the area model as well as through writing their own word problems. This was evident in their excitement in pulling out the manipulatives each day, and their eagerness to share their word problems with the class. Students' communication skills were strengthened verbally as well as through writing. All of these findings correlate to the belief of the importance that students need to explain and justify their work in order to make connections and develop an understanding of the skills being learned (Cobb & Yackel,1996).

A trend that I noticed only after analyzing my data was that students were writing fraction multiplication problems, and then adding the phrase "how much was left". This contextual difference changed the problem from a multiplication problem to a multistep problem that included both multiplication and subtraction. As I reviewed my classroom videos I noticed that I was using this language and had my own misconceptions about writing fraction multiplication word problems. I passed this language onto the students through my teaching. Although several students picked up on this language and replicated it, I was pleased to see that I still had many students who wrote alternative problems that were correct fraction multiplication word problems.

Research done by Rule and Hallagan (2006) on preservice elementary teachers explaining multiplication and division by fractions has also suggested that teachers do not have a deep understanding of these concepts. Rule and Hallagan instructed teachers to illustrate and create a story problem from a list of preselected fractions. They found that after these activities most teachers improved, but their knowledge was still incomplete as the teachers were switching the division operation with multiplication. In my case, I was switching context within subtraction and multiplication, which was misleading for my sixth grade students.

Another trend was attention to specific context within the fraction multiplication word problems. Students would not specify when asking their question, "how much of it was eaten" whether they were referring to the whole piece or a specific piece. This caused a bit of confusion within the word problem, and with an extra word could have made the difference in their word problems.

One last trend that occurred a few times throughout the study was students' ability to write word problems on objects that can be split easily. Most students wrote about pizza, pies, and cake, but a few students chose to take two-thirds of a box, three-fourths of a coin, five-sixths of an airplane, etc. This brought up the question, should mathematics make sense? According to Van de Walle (2004), basic mathematics should make sense. Students should come to the belief that they are capable of making sense of their mathematics, and that teachers should stop telling and begin to let students make sense of their own work. That brings the question back, did these students understand what they were writing or were they mimicking a pattern that they had seen develop in class. In the Common Core State Standards (2014) Standards for Mathematical Practice it states that students should make sense of problems and persevere in solving them, and attend to precision. With a slightly different context in the way a question is asked in a word problem, “How much do we have leftover” and “What fraction of the pie did he eat” students may have written more correct word problems. Only through doing this study and observing my own practice would I have picked up on these trends and misconceptions that I had about fraction word problems.

If I had not spent time teaching students fractions with the area model, and having them write word problems to represent those fractions, students may never have gained the opportunity to look at fraction operations within context. This provided them with an experience to build upon and can be a guiding factor in solving future fraction word problems. In the past, I have solely taught multiplication of fractions with procedures, never making the connection that students were using their knowledge of whole-number operations when approaching fractions. This was indeed a valuable lesson for me that has already impacted how I teach mathematics.

Implications

Since research indicated that learning to subtract and multiply fractions should be introduced with simpler problems (Van de Walle, 2006), manipulatives representing the area model, and story problems, perhaps more attention should be paid to these strategies.

Teachers across all grade levels should allow their students to have manipulatives at hand for each and every lesson, to allow students the chance to develop that deeper meaning. The Next Generation Sunshine State Standards and the Common Core State Standards, require that teachers change their instruction to be more meaningful (FLDOE, 2007; Common Core State

Standards, 2014). Teaching fractions with the area model before introducing procedures and having students develop word problems to demonstrate correct operational sense is one way to do this. It fosters the idea that mathematics is more than just writing down and following steps, but a deeper thinking that requires problem solving and reasoning skills. While the strands of fractions cannot be mastered in a short time, they can definitely be developed through use of the area model and the creation of story word problems.

Because of the new mathematics standards and the adoption of the Common Core State Standards (2014), both the fifth and sixth grade curricula are centered on fractions. The NGSSS pertaining to subtraction and multiplication of fractions for fifth and sixth grade are as follows:

Next Generation Sunshine State Standards:

MA.5.A.2.1: Represent addition and subtraction of decimals and fractions with like and unlike denominators using models, place value and properties.

MA.5.A.2.2: Add and Subtract fractions and decimals fluently and verify the reasonableness of results, including in problem situations.

MA.6.A.1.1: Explain and justify procedures for multiplying and dividing fractions and decimals.

MA.6.A.1.2: Multiply and divide fractions and decimal efficiently.

MA.6.A.1.3: Solve real-world problems involving multiplication and division of fractions and decimals (FLDOE, 2007).

Common Core State Standards:

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem (Common Core State Standards, 2014).

These standards cannot be achieved without looking deeper into fractions. Students in fifth and sixth grade should be exposed to the models often when they work with fractions, not just as an introductory lesson. This will not only give them the confidence in using additional strategies such as solving a simpler problem, using manipulatives, and writing a word problem, it will also lead them on a path to future success in mathematics. This study gives valuable insight to fifth and sixth grade teachers who will be teaching subtracting and multiplying fractions with the area model for deeper meaning in the coming years.

Limitations

Two major limiting factors in this study included time and students' prior knowledge. With so many skills to teach prior to the state's standardized testing, time is of the essence. I spent more time on multiplying fractions by fractions than is recommended by both my county's pacing guide as well as the textbook. In addition to this, in order to discuss the differences between subtracting and multiplying fractions I had to go back and re-teach subtracting fractions, which is a fifth grade standard. Even though I spent more time on a skill that only takes up three pages in my textbook, I feel like it was still not enough. I eventually had to move forward due to the realization that I still had so many more mathematics skills to teach, like multiplying and dividing mixed numbers. If I wanted to catch up to my pacing and still use the models and creation of word problems that I had begun implementing in my class, I would need to keep going and continue to spiral this knowledge within other strands.

Another factor was student's prior knowledge. As students move up through the grades we expect them to come to us with knowledge of the meaning of multiplication and in this case subtraction of fractions. I believe that the number of times I continued to review the meaning of multiplication with my students shows that we as teachers are still teaching rote memorization of facts which affect future teachers and their lesson in mathematics. I found at times I was teaching strategies for multiplication that are taught in third and fourth grade, which took up a significant amount of time. This prior knowledge is essential for students to have in order to build onto with new skills in each grade level.

Research supports the notion that conceptual understanding of fractions is developed over time (Van de Walle, 2006). If I had been able to devote more classroom instruction time to students' prior knowledge, I believe the students would have benefited more.

Another factor to consider is the population of my study. My students generally come from homes in which parents support their students' learning. These students are involved within their communities and participate in the Arts classes that are offered in my school. These children have an advantage over other students who may not have the same level of support at home or the same level of activity within their schools.

Recommendations

Although there are numerous studies on using area models with fractions and problem solving, I was unable to find any specific one that looked at the relationship of fraction subtraction word problems and multiplication word problems that I conducted in my study. With the education system continuing to undergo reform, teaching is no longer the same as it once was. Educators need to have access to research to improve their practice of teaching students. There have been a few studies that were devoted to writing story word problems (Van de Walle, 2006; Whitin, 2008b), but there is still much to be learned about how students transfer knowledge of subtracting and multiplying fractions to writing word problems within the correct context.

If I were to do this study again, I would teach multiplying fractions first, then subtraction of fractions. After reviewing the pretest and noticing how many students wrote multiplication problems correctly regardless of the meaning, it was at least clear that students were capable of writing them. I should have taken this knowledge and immediately directed it to the correct area model. Instead students kept trying to explain their subtraction as multiplication and only saw a clear difference when we began using the area model with multiplying fractions.

Summary

I began my action research to learn more about how my sixth graders used context in their word problems that supported a specific operation such as subtracting and multiplying fractions. I wanted to know that if I focused my instruction on working with the area model and developing students' conceptual understanding of the differences between subtracting and multiplying fractions could they write word problems to reflect those operations. While fractions are a challenging skill for most students and even adults, using the area model to solve fractions did make it easier for students and many of them continued to draw illustrations when they did not have the use of the manipulatives.

I chose this study topic because of the reform-taking place in mathematics throughout my state with the Next Generation Sunshine State Standards, and the country with the Common Core State Standards. Since the new standards are calling for a deeper understanding as well as the use of visual models and real-world word problems, I realized teaching fractions through step-by-step procedures would not develop students' understanding of the meaning of fractions. Rather than focus on phrases such as "multiply the numerator", "multiply the denominator", "find a common denominator", etc., I sought to give my students the experience to explore fractions with fraction circle manipulatives through discussion and eventually the creation of their own word problems. Students demonstrated adequate knowledge of creating their own word problems, but still have room to improve. They developed a deeper understanding of the area model and could write a multiplication fraction problem, but needed more practice with the meaning of multiplication as it related to the order in which the factors came in the expression.

It is crucial for students to develop a solid foundation of fractions in the elementary and middle school years. By giving students an opportunity to look deeper into the meaning and differences of subtraction and multiplication of fractions, it lead students on a path to better understanding the skills that follow.

On a personal note, I can still remember making it to high school and never thinking or being asked to think about the meaning of fractions, much less the context of a word problem and its subtle differences. While procedures do have a place in mathematics, it is only after students have developed understating. I often wonder how differently I would have thought of mathematics if I was only given the opportunity to explore using manipulatives and given the opportunity to synthesize information by creating my own word problems. I have learned just as much and in some cases more, because I am an adult and can make more connections with fractions in my life. It has made a difference in how I not only approach fractions, but all other skills introduced in sixth grade. To all teachers, teaching with just procedures to solve a problem may not be the best way. It is time to help our students develop a deeper understanding of fractions and it begins with us.

APPENDIX A: INSTITUTIONAL REVIEW BOARD (IRB) APPROVAL

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Exempt Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Monica L. Friske**

Date: **September 15, 2011**

Dear Researcher:

On 9/15/2011, the IRB approved the following activity as human participant research that is exempt from regulation:

Type of Review:	Exempt Determination
Project Title:	Affects of using context supportive of the area model on sixth graders performance writing word problems for fraction multiplication.
Investigator:	Monica L Friske
IRB Number:	SBE-11-07842
Funding Agency:	
Grant Title:	
Research ID:	N/A

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these changes affect the exempt status of the human research, please contact the IRB. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Signature applied by Joanne Muratori on 09/15/2011 12:49:50 PM EDT

IRB Coordinator

APPENDIX B: PRINCIPAL APPROVAL

[redacted]

July 17, 2011

Dear Ms. Friske,

Thank you for informing me about your continued pursuit toward the completion of your masters program. I was most impressed with the conversation that we had about your thesis project. I was equally impressed with how you went about to change the emphasis slightly based on additional "live" research from your own teaching practice and by comparing the work of those regarded as experts in the field of mathematics.

I wish you the best as you continue to compile your data that supports your thinking and hope that you will eventually share what you have learned with me and others so that we reap the benefit from your findings. My sole and only intent in expressing such is that as eductors, we assist students in purposeful and meaningful ways that will allow them to grow and to be successful. I believe this is what and why we do what we do as educators.

If you need any further assistance, please let me know.

Sincerely,

[redacted]

[redacted]

APPENDIX C: PARENTAL CONSENT FORM

Parental Consent Form

September 20, 2011
Dear Parents,

Hello! I am writing to request permission for your child to participate in a research study that I am conducting in our classroom at [redacted] this year. I am currently a student in the Lockheed Martin Academy at the University of Central Florida, working towards a Master's Degree in K-8 Mathematics and Science Education. This research project is being conducted as a part of my educational requirements.

My research will focus on students understanding of subtracting and multiplying using contextual clues in word problems. I will be conducting this research during our regularly scheduled math instruction, starting in mid-September and lasting through October. All students will be instructed using our county math curriculum, with additional methods designed to enhance their learning of math concepts. Research activities include: one small focus group, occasional tape and audio recording of student/teacher interactions.

There are no anticipated risks, only potential benefits from participation in a study designed to increase math understanding in our classroom. The identities of the students will be kept confidential in discussions with my advisor as well as the final research report. Student names will be removed from work samples, and student names will be changed in any written documentation. I will occasionally be using video and voice recording of students' responses and discussions. These recordings will only be heard by my advisor and myself, and will be destroyed at the conclusion of the study.

Participation is NOT mandatory, and your student's grades will not be influenced in any way regardless of your decision. Please know that you also have the right to withdraw your student from the study at any time. Unfortunately, I cannot offer any compensation, but I will be happy to share the results of the research with you once it has concluded.

If you have any questions, please feel free to call me at [redacted] You may also contact my faculty advisor, Dr. Juli Dixon, [redacted]. Any questions or concerns about participant's rights may be directed to the UCF Office of Research and Commercialization. [redacted].

If you do consent for your child to participate in this study, please sign and return this form to school as soon as possible. Please remember, there will be NO negative effects on your child's grades or treatment in the classroom if you decide not to consent.

Thank you,

Ms. Monica L. Friske

_________________ Yes, I have read the project description provided above (initial)
_________________ Yes, I give permission for my child ____ to participate in Ms. Monica L. Friske's research project (initial).
Parents Signature:____

UCF University of Central Florida IRB
IRB NUMBER: SBE-11-07842
IRB APPROVAL DATE: 9/15/2011

APPENDIX D: STUDENT ASSENT FORM

Child Assent Template for use with children 7 to 17 years of age

My name is Monica L. Friske. I am doing a research project on my ability to teach you to multiply and subtract fractions in word problems. The purpose is to see how well I can help you to understand and explain your math work. I am interested in how students like you are able to write word problems that show the difference between multiplying and subtracting fractions. This research is part of my studies at the University of Central Florida.

As a way to study this, I would like to video tape you explain your answers during whole group class time as well as audio record one small focus group of 5-6 students. There will be times in which I make observation notes and collect your class work to use towards my research. At the end of the activity, I will give you a similar post test and have you show what you learned by writing new word problems.

Only Dr. Juli Dixon, my professor at UCF, and I will see the pre/post tests, video tapes, audio recordings, and observation notes. I will destroy the research notes, video, and audio recordings at the end of the study. All names will be changed so that nobody will know it was you in my study.

This will not affect your grade if you decide you don't want to do this. You can stop at any time and you do not have to answer a question if you do not want to. If you do not want to take part in this study, your teacher will give you another activity to do. You will not be paid for doing this. You will not get extra credit for doing this. Would you like to take part in this research project?

_______ I want to take part in Ms. Friske's research project.

____ __________

Student's Signature Date

____

Student's Printed Name

APPENDIX E: PRETEST/POSTTEST

Pretest: Word Problems

Name: ________________________ Date: ________________

Directions: Write a word problem demonstrating the meaning of each number sentence below.

1. 5 – 2

2. 3 x 4

3. 4/5 – 1/2

4. 1/3 x 3/4

Posttest: Word Problems

Name: ________________________ Date: ________________

Directions: Write a word problem demonstrating the meaning of each number sentence below.

1. 7 – 4

2. 5 x 3

3. 3/4 – 1/2

4. 1/2 x 2/3

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