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## Effects Of A Treatment Using Computer Generation Of Isometric And Orthographic Projections On Middle School Students' Spatial Ability

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EFFECTS OF A TREATMENT USING COMPUTER GENERATION OF ISOMETRIC AND  
ORTHOGRAPHIC PROJECTIONS ON MIDDLE SCHOOL STUDENTS' SPATIAL  
ABILITY

by  
ADAM MICHAEL TRAAS

B.S. University of Central Florida, 2004

A thesis submitted in partial fulfillment of the requirements  
for the degree of Master of Education  
in the Department of Teaching and Learning Principles  
in the College of Education  
at the University of Central Florida  
Orlando, Florida

Spring Term  
2010

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## ABSTRACT

The primary focus of this study examines the effectiveness of the CRIOSAT (Computerized Rotational Isometric and Orthographic Spatial) spatial ability treatment on a random sample of middle school students' (n=137) spatial ability as measured by the Purdue Spatial Visualization Test: Rotations Test (PSVT-ROT) (Guay, 1977). The secondary focus of this study investigates the relationships between mathematical achievement, problem solving preferences, and spatial ability. The secondary focus was tested on a subsample (n=41), with the problem solving preferences measured via the Mathematical Processing Instrument (MPI) (Suwarsono, 1982). Findings indicated no significant gains in spatial ability scores after students' use of the CRIOSAT treatment; while some increases in spatial ability took place in males. Significant positive correlation was identified between mathematics achievement and spatial ability; while conversely, a significant negative correlation was found between mathematics achievement and level of visual problem solving used by students.

This paper is dedicated to all of the important women in my life. I would like to say thank you to my mother Miriam Traas, my wife Genevieve Traas, and my daughter Sophia Traas.

## ACKNOWLEDGMENTS

I would like to express my deepest gratitude and respect for Dr. Erhan Selcuk Haciomeroglu, for all of his guidance and encouragement throughout my graduate studies. This thesis would not have been possible without your help and insight.

I am grateful for having an outstanding defense committee and wish to thank Dr. Janet Andreasen and Dr. Michael Hynes for all of their guidance and support.

I would like to express my deepest thanks to Dr. Juli Dixon, who encouraged me to undertake my graduate studies and for her continuing support throughout.

I would like to thank Richard Lum for undertaking the programming of the treatment. Without all your hard work this thesis would not have been possible.

I am grateful to Marie Pecoraro for all of her assistance in helping me navigate through the paperwork, and scheduling. Your dedication to assisting all of the graduate students is greatly appreciated.

I would like to express my appreciation to my wife, Genevieve, and daughter, Sofia for all of their encouragement, support and patience throughout my graduate work.

Additionally I would like to thank my fellow graduate students, Maura Olvey, Paul Wojcik, and Megan Dufour. You have all been a great source of strength, insight and inspiration these past few years.

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## LIST OF ACRONYMS/ABBREVIATION

<b>2-D</b>	Two-Dimensional
<b>3-D</b>	Three- Dimensional
<b>4-D</b>	Four-Dimensional
<b>CAD</b>	Computer-Aided Design
<b>CAI</b>	Computer-Aided Instruction
<b>CBI</b>	Computer-Based Instruction
<b>CRIOSAT</b>	Computerized Rotational Isometric and Orthographic Spatial Ability Treatment
<b>ESE</b>	Exceptional Student Education
<b>FCAT</b>	Florida Comprehensive Assessment Test
<b>KR-20</b>	Kuder-Richardson 20: a reliability formula
<b>LEP</b>	Limited English Proficient
<b>MCT</b>	Mental Cutting Test
<b>MPI</b>	Mathematical Processing Instrument
<b>MPFB</b>	Minnesota Paper Foam Board Tests
<b>NCTM</b>	National Council of Teachers of Mathematics
<b>PSVT</b>	Purdue Spatial Visualization Tests
<b>ROT</b>	PSVT: Visualization of Rotations
<b>SAT-M</b>	Scholastic Aptitude Test (mathematics portion)
<b>SAT-V</b>	Scholastic Aptitude Test (verbal portion)
<b>SES</b>	Socio-economic Status

## DEFINITIONS OF TERMS

**Analytic Processing** – *see* parcel processing

**Image** - a spatial arrangement that may include mathematical inscription (Presmeg, 2006)

**Imagery** – “the occurrence of mental activity corresponding to the perception of an object, but when the object is not presented to the sense organ.” (Hebb, 1972 as cited in Suwarsono, 1982)

**Concrete Imagery** - single use images that are locked into a specific paradigm (Presmeg, 1986b)

**Dynamic Imagery** –mental imagery that has a level of generalizability for an individual to use

**Gestalt Processing**- the manipulation of mental imagery as a whole object (Bodner & Guay, 1997)

**Isometric Projection** – a three-dimensional perspective view of an object with zero vanishing points

**Mental Image** - recognition or mental reconstruction of previous object stimuli

**Mental Rotation** - the use of *mental images* to rotate an image in two- or three-dimensional space in order to recognize the object in various perspectives (Shepard & Cooper, 1982)

**Orthographic Projection** – a projection of a three-dimensional image whereas the front side and top of the object are viewed perpendicularly to the plane of the relative cubic space in which the object exists.

**Parcel Processing** – the manipulation of mental imagery by mentally deconstructing, or parceling, the object and reconstructing of it in the desired view

**Perspective-taking** - the change in orientation of the observer to the object

**Quasi- 3D** - flat rendition of a three-dimensional object, that are perceived to be three-dimensional (Deregowski, 1979).

**Spatial abilities**- “those mental skills concerned with the understanding, manipulating, reorganizing, or interpreting relationships visually.” (Tartre, 1990, p. 216)

**Spatial orientation** – 1.) the ability to continuously identify an object as a whole regardless of its orientation to the environment (Bodner & Guay, 1997)

2.) the ability to continuously identify an object as a whole regardless of the juxtaposition of the viewer to the object (McGee, 1979; Tartre, 1990)

**Spatial Perception** - the subject being able to “determine spatial relationships with respect to the orientation of their own bodies, in spite of distracting information” (Linn & Peterson, 1985, p. 1482)

**Spatial visualization** - the ability to recall the physical image of an object previously viewed in parts or as a whole as a mental image (Bodner & Guay, 1997)

**Static Imagery** - *see* concrete imagery

**Visual Image** – a mental construct depicting visual or spatial information, and a *visualizer* is a person who prefers to use visual methods when there is a choice. (Presmeg, 2006)

# CHAPTER ONE: INTRODUCTION

## Introduction

Until recently attempts to increase and assess spatial ability have existed primarily in psychological realms (Duesbury & O’Neil, 1996; Pak, Czaja, Sharit, Rogers, & Fisk, 2008; Aflano & Graziano, 2008; Casey, Andrew, Schindler, Kersh, Samper, & Coply, 2008; Hannafin, Truxaw, Vermillion, & Liu, 2008; Sims & Mayer, 2002; Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008) and have been conspicuously absent in mathematics education research. In the last 30 years there has been an increase in the amount of mathematics education researchers (Tartre, 1990; Battista, 1999; Battista, et. al., 1998, Battista & Clements, 1996; Lean & Clements, 1981; Presmeg, 1986; Haciomeroglu, Aspinwall, & Presmeg, 2009, Haciomeroglu, Aspinwall, & Presmeg, 2010) studying this topic and its link to mathematics efficacy. This link has led researchers to explore how students gain and increase spatial ability. According to the National Council of Teachers of Mathematics’ (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, spatial reasoning is “essential for learning” (NCTM, 2006, p.8), however it is rare that spatial ability is explicitly taught and just as rare that there are benchmarks that dictate this teaching. Researchers have identified a myriad of factors that could influence spatial ability and in turn influence mathematics efficacy, such as: gender (Chan, 2007; Kaufman, 2007; Linn & Petersen, 1985; Moè, Meneghetti, & Cadinu, 2009; Sanz de Acedo Lizarraga & García Ganuza, 2003; Walter, Roberts, & Brownlow, 1999; Voyer & Hou, 2006), play related to gender roles (Casey, et. al., 2008; Tluaka, Williams, & Williams, 2008), heredity (Casey, Nuttall, & Perzaris, 1999), mathematics preferences (Haciomeroglu, Aspinwall, Presmeg, Chicken & Bu, 2009), and even experience with technology (Terlecki & Newcombe, 2005; Sims & Mayer, 2002).



Many educators have begun to seek the aid of computer-based materials in instruction. Because of the ambiguous nature spatial ability holds in mathematics curriculum, the educational community needs supplemental material in order to effectively instruct students on varying spatial abilities. Use of manipulatives would be a logical choice in aiding the instruction of spatial abilities; however, the mass and diversity of materials that would be needed, as well as the professional development needed to facilitate instruction could be cumbersome. The goal of this research was to design and test a computer program that will allow for a similar manipulative experience that hands-on materials can offer in a virtual environment; additionally, this program was designed to have little or no reliance upon a teacher.

### Problem Statement

Spatial ability has been historically linked to mathematics achievement (Guay & McDaniel, 1977; Hannafin et al., 2008; Lean & Clements, 1981; Tatsuoka, Corter, & Tatsuoka, 2004; Tolar, Lederberg, & Fletcher, 2009). According to Tatsuoka, Corter, and Tatsuoka (2004) the United States is ranked in the bottom half of the 20 countries studied in both overall mathematics and specifically spatial ability. These data indicate that U.S. schools have a large gap to bridge in worldwide mathematics efficacy and that improving spatial ability is a logical pathway to achieve this. In light of this larger problem, a smaller yet more complex problem is presented: What can educators do to improve spatial ability? Research indicates that utilization of interactive software is an effective way of increasing spatial ability (Duesbury & O'Neil, 1996; Onyancha, Derov, & Kinsey, 2009; Samsudin & Ismail, 2004, Wright et al., 2008).

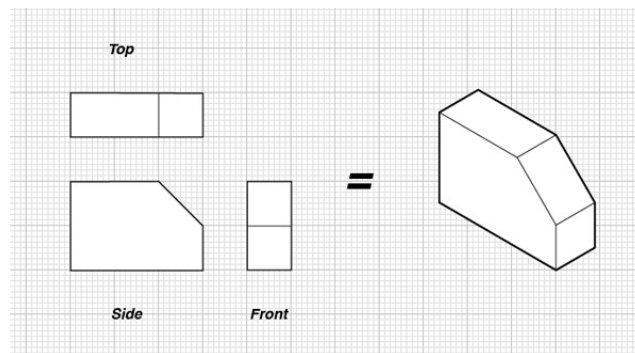
### CRIOSAT

CRIOSAT (Computerized Rotational Isometric and Orthographic Spatial Ability Treatment) is a computer program that has been designed to train students' overall spatial ability

through the manipulation of a pseudo-three-dimensional object and the creation of various views of the object based on its rotation and orientation. CRIOSAT has two main modes:

### Orthographic mode

In this mode students are given an array of cubes that have been placed randomly with a 30% probabilistic density in a  $3 \times 3 \times 3$ ,  $4 \times 4 \times 4$ , or  $5 \times 5 \times 5$  base cube, with the front, side and top labeled, meaning that in each array, every space that could hold a cube has a 30% chance of having one before gravity is applied. Once the figure is randomly generated, it can be manipulated on all three axes by using mouse controls. This freedom in movement is used to allow students to view the figure from any desired angle. Adjacent to the three-dimensional figure, is a set of three two-dimensional arrays which are labeled front, side and top. Students will manipulate the three-dimensional figure into the desired perspective, and then by clicking on empty spaces in the two-dimensional array, fill in what each view looks like. The combination of the front, side, and top views are referred to as an *orthographic projection* of a three-dimensional figure (see Figure 1).



**Figure 1 - Orthographic projection (*Left*) and Isometric Projection (*Right*) of same figure (Walker, 2008)**

## Isometric mode

In this mode the juxtaposition of the three-dimensional area and the orthographic area are the same; however, the task and given information has changed. This mode gives the students a complete *orthographic projection* of the figure and the student must construct the three-dimensional representation of that figure. The figures created for this mode are the same 30% probabilistic density as the previous. Students will construct the three-dimensional versions via two different tools. The first tool that they can use enables them to place blocks directly into the three-dimensional grid. In the grid translucent spheres are located at the centered of each space in which a cube could exist. By clicking on one of these spheres an opaque cube will appear. Below the three-dimensional representation on the screen are two-dimensional arrays which represent the bottom up layer of the three dimensional space. Students may use these arrays to add blocks to their three dimensional representation as well, adding them by layer rather than three-dimensional space.

Instructions detailing the specifics of the isometric and orthographic modes (Appendix A) were given to the students as part of the program. All instructions were determined to be at grade appropriate reading levels for students using the Flesch-Kincaid readability formula (Kincaid, Fishburne, Rogers, & Chissom, 1975). The ratings of the orthographic and isometric mode instructions were 6.6 and 4.6 grade level respectively (Appendices B & C).

*Isometric* and *orthographic projections* are projections that have the primary purpose of representing three-dimensional objects on a two-dimensional plane. *Isometric projections* are projections in which the intersecting edge of the front and side face, making this edge the forward most portion of the object. The bottom edges of the figure, that would be parallel to the ground, are represented at  $30^\circ$  from that parallel, as seen in Figure 1. *Orthographic projections*

conversely represent the same image. Unlike *Isometric projections* which are three-dimensional-like renderings on a two-dimensional plane, *orthographic projections* are a collection of two-dimensional images that show the front, top, and side perspectives of the same figure two-dimensionally, as shown in Figure 1. The relationship between these two types of projections is often used in the design and drafting stages of many products, from chairs to skyscrapers.

### Research Questions

1. Will an interactive computer-based treatment focused on rotational and perspective relationships of three-dimensional affect the spatial ability of students?
2. Is there a correlation between time spent on items from the CRIOSAT and change in spatial ability in students?
3. Does the CRIOSAT affect males' and females' spatial ability differently?
4. How do mathematics problem solving preferences correlate to students spatial abilities?
5. How does mathematics achievement correlate to student spatial abilities?

### Rationale

Research indicates that virtual manipulatives similar to the CRIOSAT treatment can be just as, if not more effective than physical manipulatives in educational situations (Durmuş, Karakirik, 2006). Play with blocks (Casey, et. al., 2008) as well as computer simulated construction of three-dimensional objects (Onyancha, Derov, & Kinsey, 2009) have been shown to increase spatial ability. Increased spatial ability concurrently has been connected to mathematics achievement. While there is extensive research describing the differences in spatial ability based on gender, it is unclear if this gap can be closed by using a focused spatial ability treatment.

By monitoring and tracking pretest and posttest scores, CRIOSAT scores, time spent on each level of the CRIOSAT treatment, the gender of the participants, problem solving preferences of participants, and the mathematics achievement levels of the participant, this researcher aims to show that CRIOSAT has an effect on spatial ability, specifically related to mental rotation. This evidence will aid in laying the groundwork for further spatially related treatments that may prove to increase all facets of spatial abilities and generalized mathematics achievement.

## CHAPTER TWO: LITERATURE REVIEW

### Introduction

Spatial ability is the “skill in representing, transforming, generating, and recalling symbolic, nonlinguistic information” (Linn & Petersen, 1985, p. 1482) and is referred to using varying terminology throughout the existing literature, including: spatial skills (Casey, et al., 2008; Tartre, 1990), spatial abilities (Guay & McDaniel, 1977; Linn & Petersen, 1985; Samsudin & Ismail, 2004), spatial cognition (Sanz de Acedo Lizarraga & García Ganuza, 2003; Tlauka, Williams, & Williamson, 2008), and spatial reasoning (NCTM, 2006); though they are all referring to the same concept. That is:

“In general spatial skills [cognition, reasoning, and abilities] are considered to be those mental skills concerned with the understanding, manipulating, reorganizing, or interpreting relationships visually.” (Tartre, 1990, p. 216)

Spatial ability is an abundantly studied topic (Linn & Petersen, 1985; Voyer, Voyer, & Bryden, 1995) and is often mired in a preponderance of terms used to attempt to disseminate this multifaceted area of research (McGee, 1979; Tartre, 1990). This review of literature will attempt to distill this research in the perspective of the CRIOSAT, by analyzing topics including (a) the nature of spatial ability, which has several paradigms, especially concerning the subcategories and dissemination of terminology regarding specific spatially related tasks (Linn & Peterson, 1985); (b) spatial ability’s connectedness to mathematics achievement (Hannafin, et. al., 2008; Guay & McDaniel, 1977; Smith, 1964), as well as other cognitive skills (Pak, et. al., 2008; Tolar, et. al, 2009) is well documented yet controversial, concept (Friedman, 1995; Hannafin, et. al, 2008); (c) different ways of increasing ability, as there has been sufficient success increasing spatial ability through interactive computer-based treatments (Duesbury & O’Neil, 1996;

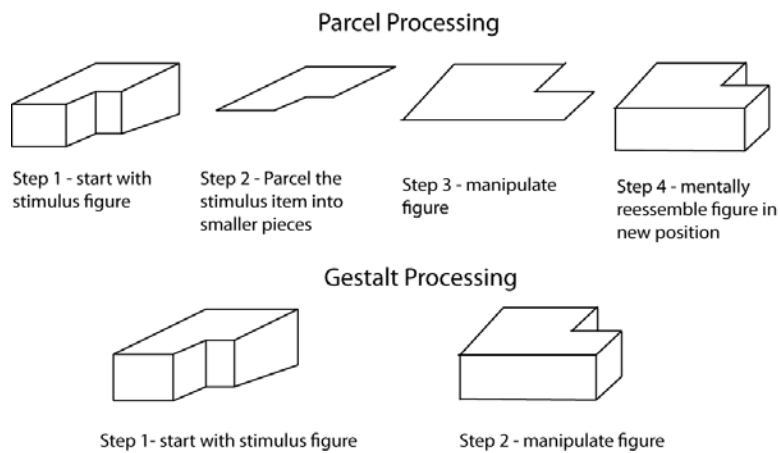
Samsudin & Ismail, 2004; Sanz de Acedo Lizarraga & Garcia Ganuza, 2003; Smith & Olkun, 2005), as well as more traditional manipulative-based methods (Casey, et. al., 2008); (d) an examination of factors that may mitigate the level of spatial ability individuals tend to have, such as gender (Bodner & Guay, 1997; Kaufman, 2007; Linn & Petersen, 1985); (e) and the use of technology in mathematics instruction.

As many studies have shown a connection between spatial ability and mathematics achievement (Guay & McDaniel, 1977; Tolar, Lederberg, & Fletcher, 2008; van Garderen, 2006) is an important topic for study. Spatial ability is however, a very diverse topic. Exploring the many facets of spatial ability is necessary when attempting to increase that ability as in the current study. In the present study the examined areas included: the nature of spatial ability, how it relates to mathematics achievement, methods of improving spatial ability, computer-based instruction in mathematics, and mitigating factors such as gender, mathematics problem solving preferences, and student exceptionalities.

### Spatial Ability

Lohman (1979) defines spatial ability “as the ability to generate, retain, and manipulate abstract visual images” (p.188). In his research, Lohman (1979) identified three main categories of spatial ability: (a) spatial relation, (b) spatial orientation, and (c) visualization. Contrary to Lohman’s (1979) categorization, some researchers analyze spatial abilities as two separate skills, *spatial orientation* and *spatial visualization* (McGee, 1979; Tartre, 1990). Bodner and Guay (1997) describe *spatial orientation* as the ability to continuously identify an object as a whole regardless of its orientation to the environment; while others (McGee, 1979; Tartre, 1990) define it differently, asserting that the objects’ orientation is not in question so much as the change in the relative perspective of the viewer. This disambiguation is partially semantic as both

definitions relate to the recognition of the object from different perspectives; one states that the object moves, while the other states that the viewer moves. Regardless of this difference, both assertions indicate that the perspective of viewing changes, or transforms. This transformation in a three-dimensional object is most often seen in rotations. Within *spatial orientation* it has been demonstrated that manipulating an object holistically in a mental capacity or *gestalt processing*, is significantly more beneficial and effective than *analytic processing* (Battista & Clements, 1996; Bodner & Guay, 1997; Shepard & Cooper, 1982; Linn & Peterson, 1985), which is the manipulation of objects by parceling objects into manageable parts and then reconstructing them mentally (Bodner & Guay, 1996). Due to the parceling of figures in *analytic processing*, it will henceforth be referred to as *parcel processing* in this study. Figure 2 demonstrates the cognitive steps taken by a student in manipulating a figure using both *parcel* and *gestalt processing*.



**Figure 2 - Examples of Parcel (*top*) and Gestalt (*bottom*) processing**

*Spatial visualization* is the ability to recall the physical image of an object previously viewed in parts or as a whole as a *mental image* (Bodner & Guay, 1997). This explanation of *mental imagery* in *spatial visualization* correlates with Hebb’s (1972 as cited in Suwarsono, 1985, p. 270) definition of *visual imagery*, “the occurrence of mental activity corresponding to the perception of an object, but when the object is not presented to the sense organ.” The



specific difference in these definitions is that Hebb is suggesting that a visual stimulus is not necessary to create and maintain an image, while Bodner and Guay (1997) suggest that a previous stimulus was present. While Bodner and Guay's (1997) *mental imagery* is simple recognition or mental reconstruction of previous stimuli and Hebb's (1972 as cited in Suwarsono, 1982) *visual imagery* is the application or new construction of an image used by the visualizer in order to interpret other information, or imagery that constructs "images in the mind" (Lean & Clements, 1981, p. 268), they hold similarities. Haciomeroglu (2010) discusses visual imagery as "a mental construct depicting visual and spatial information" (p. 3), this definition allows for both definitions to combine by leaving out the variable of specific and previous stimulus; hence, this is the definition that will be used in this study.

Through her research in visual imagery, Presmeg (1985) identified 5 kinds of visual imagery: (a) concrete imagery or pictorial imagery; (b) pattern imagery; (c) memory images of formulae; (d) kinesthetic imagery; and (e) dynamic imagery. Among these visual imagery types, some researchers focus on the two most prevalent categories: *dynamic imagery* and *concrete imagery* (Presmeg, 1986a). *Dynamic imagery* is imagery that has a level of generalizability for an individual to use. The malleability of these images to adapt to the problem at hand is what makes them potentially effective for learners to use (Aspinwall, Shaw, & Presmeg, 1997; Presmeg, 1986b). *Concrete imagery* is single use images that are locked into a specific paradigm, which will be referred to as *static imagery* henceforth in this study as it illustrates the counterpoint to *dynamic imagery*. This *static imagery* can "exert a negative influence on the generalization of thinking (Krutetskii, 1969, p. 326)," which can become a hindrance to the learner (Aspinwall, Shaw, & Presmeg, 1997; Presmeg, 1986a; Presmeg, 1986b). For example, many students are introduced to the concept of right triangles that are primarily pointing to the

right as in the picture on the right in Figure 3. This kind of constant reassurance of the idea that right triangles point right can become a hindrance as a static image. When confronted with a right triangle which points to the left on Figure 3, due to their *static image* of what a right triangle is, they will call it a left triangle in some cases. By giving students *dynamic image* examples of right triangle in which all facets of the triangle change (i.e., orientation, size, proportionality of size) with only the measure of one angle as a constant, students are more free to develop a *dynamic image* of what a right triangle is with some static components. Though dynamic images may be preferable, research indicates that when students construct dynamic images it does not ensure that it will not impede their understanding (Haciomeroglu, Aspinwall, and Presmeg, 2010; Haciomeroglu, Aspinwall and Presmeg, 2009).

Researchers (Linn & Peterson, 1985; Lohman, 1979) divided spatial ability into three categories rather than two: *spatial perception*, *mental rotation*, and *spatial visualization*. They defined *spatial visualization* as requiring mental manipulations in a multi-step process, and hence having the possibility of “multiple solutions” (p. 1484). The Mental Cutting test (MCT) (CEEB, 1939), demonstrates *spatial visualization* tasks, as seen in Figure 4. In this item the subject must mentally cut the figure along the indicated plane, rotate the remaining figure, and select the appropriate orthographical view. This *spatial ability* differentiates itself from others primarily by the multiple steps it requires to derive an answer. *Spatial perception* (see Figure 5)



**Figure 3 – A pair of congruent right triangles with different orientations**

is defined as the subject being able to “determine spatial relationships with respect to the orientation of their own bodies, in spite of distracting information” (Linn and Peterson, 1985, p.

1482). An example of this concept (shown in Figure 5), asks the subject to make lines that are framed, align vertically (Witkin, et. al., 1962). This task is in line with the definition by Linn and Petersen, (1985) as the task of vertically aligning the rod is based on one's perspective of viewing, and the frames' relative angle to the rods is distracting to that perspective.

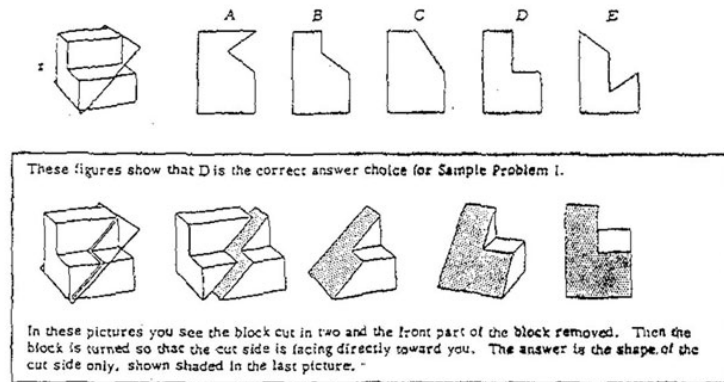


Figure 4 - This Mental Cutting Test (CEEB, 1939) item is an example of a spatial visualization item

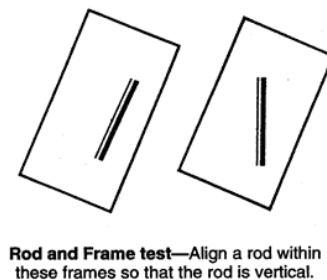
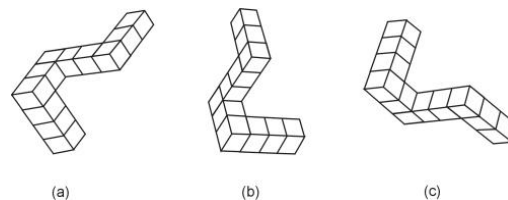


Figure 5 - The Rod and Frame test (Witkin, et. al., 1962) is an example of a spatial perception item.

*Mental rotation* is the use of *mental imagery* to rotate an image in two- or three-dimensional space in order to recognize the object in various perspectives (Shepard & Cooper, 1982). Two-dimensional images can be rotated on their plane in a full 360°; while three-dimensional figures can be rotated about two planes in any combination of 360° each. Due to the flat rendition of these three-dimensional objects, and that they are merely perceived to be three-dimensional they could more accurately be labeled as “quasi-3D” (Deregowski, 1979; Peters &

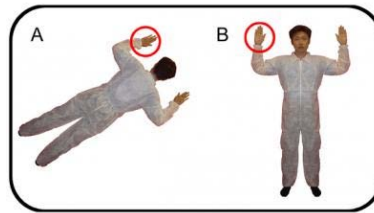
Battista, 2007). In the Shepard-Metzler Rotations test (S-M) multiple three-dimensional figures, which are constructed from ten cubes, are given for subjects at different rotations. The participants must select which figures are the same in construction, but not in position as opposed to a figure which is a reflection of the others, as seen in Figure 6. According to Linn and Petersen (1985), there is some evidence that two-dimensional rotations are cognitively more simplistic in nature than that of three-dimensional figures and that learners may attempt to use inefficient simple strategies when encountering more complex tasks. The amount of time spent on a task seems to be a strong indicator of the complexity level of a rotational task (Linn & Petersen, 1985). Additionally, Shepard and Metzler (1971) noted a direct correlation between angle of rotation and reaction time for identification of paired figures. These two studies lead to the conclusion that angle of the rotation in mental rotational tasks can be used as an indicator as to their relative complexity.



**Figure 6 - This item from the Shepard-Metzler Rotations Test (Shepard & Metzler, 1971) is an example of a mental rotational item**

Hegarty and Waller (2004) describe a more refined definition of *mental rotation* as opposed to *perspective-taking* similar to the disambiguation of spatial orientation definitions from McGee (1979) and Tartre (1990). Hegarty and Waller (2004) assert that *mental rotations* are the change in an object's orientation to the observer; while, *perspective-taking* is the change in orientation of the observer to the object. In one study by Thakkar, Brugger, and Park (2009), participants are shown the images and told to imagine themselves in place of the person shown

and determine whether the hand circled was their right or left. This demonstrates that the person must mentally alter their personal viewpoint rather than alter their referent of an external figure. While this disassociation does exist Hegarty and Waller (2004) found that there is a strong correlation in a learner's mental rotational ability and *perspective-taking* ability.



**Figure 7 - Perspective-taking task (Thakkar, Brugger, & Park, 2009)**

### Mathematics Achievement

Spatial ability in many cases has been shown to be an indicator of mathematics achievement (Guay & McDaniel, 1977; Hannafin, et. al., 2008; Rhodes & Thompson, 2007; Smith, 1964; Tolar, et.al., 2009). Friedman's (1995) meta-analysis of spatial ability on the other hand indicated that there was a low correlation between these two factors. This discrepancy could be explained via the variety of measures that are used to measure both spatial ability, and more importantly, mathematics achievement. For example, research conducted (Raven, Raven, & Court, 1998; Tolar, et. al., 2009) used the mathematics portion of the Scholastic Aptitude Test (SAT-M) to judge mathematics achievement and found a correlation to spatial ability. It has also been indicated that spatial ability can be a predictor of one's problem solving (Moses, 1977) and reasoning ability (Moses, 1980), which could explain the success on the SAT-M. This assertion is similar to Tolar's et. al. (2009), indicating that the high level of reasoning and problem solving ability found in persons with high spatial ability may be caused by the high correlation found between spatial ability and the Scholastic Aptitude Test verbal portion (SAT-V). Findings

indicated that this correlation was actually higher than the correlation with SAT-M. In regards to the SAT as a measure, Rhode and Thompson (2007) determined that spatial ability is a better predictor of achievement than overall cognitive ability.

While the majority of these studies have been done using high school age students (Tolar, et.al., 2009), there have been studies involving middle school age (Hannafin, et. al., 2008) and elementary age students (Guay & McDaniel, 1977) indicating that spatial ability is a predictor of mathematics achievement across all age ranges. Smith (1964) suggested, that spatial ability may be a good predictor of high performing mathematics students in upper grades because the mathematics conceptualization needed to be high achieving would be present and that spatial ability would not be as good a predictor of low achieving students because low-level mathematics is more operationally based. Guay and McDaniel (1977) however, determined that the predictive nature of spatial ability is equally applicable with regards to high and low achieving mathematics students. Tolar et. al. (2009) also indicated that using spatial ability testing may be a good way to better identify low achieving mathematics students. While spatial ability is not the only mathematics skill that should be examined, when considering other factors spatial ability is one of the main factors that influence overall mathematics achievement (Sherman, 1979).

Spatial ability may also have applications in predicting ability in other areas. Bodner and Guay (1997) indicated that spatial ability correlates to chemistry achievement, because of the way in which students tend to visualize and mentally manipulate molecular structures. As described previously, spatial ability can also be used as a predictor of problem solving ability (Moses, 1977; Tolar et. al., 2009) and reasoning ability (Moses, 1980; Tolar et. al., 2009). Spatial ability has also been shown to predict performance in

“...everyday tasks such as way finding, map reading and computer tasks such as text-reading, spreadsheet usage, map- and computer –based information search tasks...” (Pak et. al., 2008, p. 3045-3046).

Because of the correlation to such a wide variety of skills, spatial ability may need to be treated as a skill set outside of mathematics or reading altogether (Hannafin, 2008).

### Improving Spatial Ability

It has been indicated that spatial ability correlates to overall intelligence (Samsudin & Ismail, 2004), which may lead us to believe that spatial ability is either difficult or impossible to change. While this correlation does exist, sufficient evidence also exists that spatial ability can be increased through traditional methods (Brosnan, 1998; Casey, et. al., 2008; Sprafkin, et. al., 1983), as well as through computer-aided instruction (Duesbury & O’Neil, 1996; Onyancha, Derov, & Kinsey, 2009; Samsudin & Ismail, 2004; Sanz de Acedo Lizarraga & Garcia Ganuza, 2003; Smith & Olkun, 2005).

Conner & Serbin (1977) indicated that there is a correlation between time spent by children simply playing with spatially-related physical manipulatives and performance on some spatial ability tests. While this *play* has been shown to have some implications, Sprafkin, et. al. (1983) indicated that instruction as opposed to play with spatially related manipulatives improved spatial abilities. Block building in reproducing block formations from a stimulus figure has been shown to have a positive effect of mental rotational ability (Brosnan, 1998; Johoda, 1979); while Casey et. al. (2008) determined that adding context, through storytelling, to block building activities increased their effectiveness.

While these physical manipulatives have shown great potential for increasing spatial ability, Durmuş and Karakirik (2006) state in their theoretical framework for virtual

manipulatives, that virtual manipulatives in many cases can be “employed interchangeably” (p. 5) with physical manipulatives, and further that, “Some computer manipulatives may be more beneficial than any physical manipulatives” (p.5). This is partially due to the potential of virtual manipulatives to familiarize learners with real-world applications and multiple representations of mathematical representations.

Using these types of virtual manipulatives and computer-aided instruction has been an area of recent study. Smith & Olkun (2005) indicated that mental rotation skills can be improved with practice using their two-dimensional rotation and reflection program. Duesbury & O’Neil (1996) determined that using a computer-aided design dynamic environment, students could identify the relationship between a two dimensional orthographic view of a three-dimensional figure more easily than when using a static model. Samsudin & Ismail (2004) also developed a program to improved accuracy on spatial tasks. Some researchers have used already existing computer-aided design programs and have noted increases in all forms of figure rotations (Onyancha, Derov, & Kinsey, 2009).

While this planned training can increase spatial ability (Baenninger & Newcombe, 1989), it may seem cumbersome to train each facet of spatial ability separately; however, there is some research asserting that there is transferability of spatial task or that the increase of one specific spatial skill may likely increase other spatial skills (Wright, et. al., 2008). Sanz de Acedo Lizarraga & Garcia Ganuza (2003) demonstrated that an intervention program focused on increasing mental rotation ability had significant transferability in spatial visualization.

There are many mitigating factors to how well one can improve spatial abilities, such as gender (Moè, Meneghetti, & Cadinu, 2009) and age (Samsudin & Ismail, 2004; Ben –Chaim, Lappan, & Houang, 1989). Moè, Meneghetti, and Cadinu (2009) state that, training of spatial



abilities is more effective in women than men. Samsudin and Ismail (2004) suggest that adolescence is the maturation stage for spatial abilities; as Ben-Chaim (1989) specifies that the optimal age for acquiring spatial ability is in ages 11 and 12. These factors are specific to the acquisition or improvement of spatial ability; however, there are further mitigating factors to human predisposition to innate spatial ability skills.

### Computer-Based Instruction in Mathematics

Computer-based instruction (CBI) applications have been studied for many years (Coulsen, 1968) and have generated a preponderance of data in practice (Fletcher-Finn, & Gravatt, 1995; Kulik, 1994; Kulik & Kulik, 1991; Roschelle, Pea, Hoadly, Gordan, & Means, 2000). Summarily, research indicates that CBI learning is an effective form of mathematics instruction (Kulik, 1994; Roschelle, Pea, Hoadly, Gordan, & Means, 2000; Wenglinsky, 1998), showing the effectiveness of increasing reasoning skills (Raghavan, Sartoris, & Glaser, 1997), generalized mathematics instruction skills (Kulik, 1994), positive attitudes (Kulik, 1994), accuracy in mental rotation tasks (Samsudin & Ismail, 2004), spatial visualization skill (Duesbury & O'Neil, 1996), and generalized spatial abilities (Onyanha, Derov, & Kinsey, 2009). Learning gains in mathematics have been augmented by this computer-aided instruction, which when used alongside traditional instruction, yields better results than students only receiving traditional instruction in some basic mathematics skills (Fletcher, Hawley, & Piele, 1999; Traas, 2008).

The effectiveness level of CBI can be affected by the content area being studied as well. Mathematics and science have been shown to be more effective in CBI (Roschelle, Pea, Hoadly, Gordan, & Means, 2000; Selber, 2004 ). This may be due to the high level of reasoning involved in the aforementioned subjects, where CBI has been shown to be more effective in improving

reasoning based skills rather than repetition based skills (Wenglinsky, 1998). In specific relation to spatial ability, computer aided instruction (CAI) has been shown to improve accuracy in mental rotation (Samsudin & Ismail, 2004), spatial visualization (Duesbury & O'Neil, 1996), and generalized spatial ability (Onyancha, Derov, & Kinsey, 2009). Concurrently, mathematics using virtual manipulatives may be more effective than direct face-to-face instruction (Durmuş & Karakirik, 2006; Traas, 2007). Moyer, Bolyard, & Spikell (2002) define virtual manipulatives as “an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p.373). One study indicated that the type of dynamic environment provided by CAI improved reflections and rotations in two-dimensional objects as opposed to the traditional environment; in contrast the difference of environment had little or no significant difference on three-dimensional visualization (Dixon, 1995). Reasons for the effectiveness of computer-based instruction are not as clearly studied and there are differing theories about what these reasons could be (Durmuş & Karakirik, 2006; Roschelle, 2000). Active engagement (Roschelle, Pea, Hoadly, Gordan, & Means, 2000; Smith & Olkun, 2005), interactivity (Smith & Olkun, 2005), and feedback (Roschelle, Pea, Hoadly, Gordan, & Means, 2000; Smith & Olkun, 2005) are all possible factors for the effectiveness of CBI in addition to subject matter (Roschelle, Pea, Hoadly, Gordan, & Means, 2000).

As CBI is a one-on-one interaction, active engagement is common due to this interaction's use of technology (Roschelle, Pea, Hoadly, Gordan, & Means, 2000). Computer based environments make construction accessible and by their nature encourage rapid interaction (Roschelle, Pea, Hoadly, Gordan, & Means, 2000). Research conducted by Roschelle, Pea, Hoadly, Gordan, & Means (2000) also indicates that CBI tend to have more rapid feedback loops available to the learner which differs from the traditional classroom, as feedback may be days or

weeks away from completion of a task. Feedback in mental rotation task have been shown to aid performance (Lohman & Nichols, 1990; Kass, Ahlers & Duggers, 1998), hence some computer treatments have been effective in improving mental rotation ability (Samsudin & Ismail, 2004) In order for CBI to have active engagement, the feedback loop and interactivity must be present in the software (Smith & Olken, 2005).

### Mitigating Factors

Many factors have been identified that may mitigate or predispose humans to differences in spatial ability, such as computer and video game usage (Sims & Mayer, 2002; Terlecki & Newcombe, 2005), participation in sports (Tlauka, Williams, Williamson, 2008), genetic predisposition (Manning & Taylor, 2001; Tlauka, Williams, Williamson, 2008; Williams, et. al., 2000), influence of prenatal testosterone (Sanders, Bereczkei, Csatho, & Manning, J., 2005), preferences in mathematics problem solving (Haciomeroglu, Aspinwall, & Presmeg, 2009; Haciomeroglu, Aspinwall, Presmeg, Chicken & Bu, 2009; Krutetskii, 1969; Moses, 1977; Presmeg, 1985; Suwarsono, 1982), mathematical giftedness (Presmeg, 1986a), influence of learning disabilities (van Garderen, 2006), students being second language learners (Dixon, 1995), and experience playing with blocks in young ages (Casey, et. al., 2008). Though these possible predictors differ greatly, they are all linked by one overarching factor, gender.

### Gender and Related Factors

Sex differences in spatial ability is a major focus of studies conducted, and have spawned several meta-analyses (e.g., Linn & Petersen, 1985; Voyer, Voyer, & Bryden, 1995) to summarize the vast amount of data. Research indicates that sex differences in spatial abilities begin to emerge in early ages (Casey, 2008) to early adolescence (Linn & Petersen, 1985), and that these differences do not diminish with increase in age (Casey, et. al., 2008; Linn & Petersen,

1985; Voyer, Voyer, & Bryden, 1995). Many studies have identified that while there is a gap in general spatial abilities, mental rotation tends to be the greatest area of disparity among genders (Bodner & Guay, 1997; Casey, Nuttall, Pezaris, & Benbow, 1995; Casey, et. al., 2008; Chan, 2007; Johnson & Meade, 1987; Kaufman, 2007; Levine, Huttenlocher, Taylor, & Langrock, 1999; Linn & Petersen, 1985; Voyer, Voyer, & Bryden, 1995). Linn & Petersen (1985) suggest that this may be caused by females' tendency to use inefficient *analytic* or *parceling* processing when practicing mental rotational tasks rather than *gestalt* thinking, specifically stating that, there is a "propensity of females to select less efficient or less accurate strategies" (p. 1492). These findings are mirrored in a longitudinal study by Fennema and Carpenter (1998), which unveiled that there was no difference in students' mathematics abilities based on gender; however, there was a distinct difference in the types of strategies used. Males tended to use more abstract solution strategies, while females tended to use more concrete solution strategies. This 'propensity' of females to use less efficient solution strategies could help to explain findings that indicate that the type of distracter used in rotational items does not cause variation in the gender effect (Voyer & Hou, 2006). Interestingly, these findings are not completely consistent; as some studies identify no significant or consistent gender difference with some stimuli (Casey, et. al., 2008; Sanz de Acedo Lizarraga, & Garcia Ganuza, 2003).

Some researchers suspect that prenatal testosterone may play a role in this gender effect (Sanders, Bereczkei, Csatho, & Manning, J., 2005). Tlauka, Williams, & Williamson (2008) continue in this vein, suggesting that *in utero* testosterone levels may cause a predisposition to sports activities and have an effect on finger-length ratio. The ratio of the second (i.e. - pointer) and fourth (i.e. - ring) fingers has been well studied in their possible correlation to spatial abilities (Manning & Taylor, 2001; Sanders, Bereczkei, Csatho, & Manning, J., 2005; Tlauka,

William, & Williamson, 2008; Williams, et. al., 2000). Male participants in professional sports tend to have both higher spatial abilities and a lower finger-length ratio (Manning & Taylor, 2001). Tlauka, William, and Williamson (2008) found that women who chose to participate in sport related activities had lower finger length-ratios and mental rotation abilities. While the finger-length ratios in humans may seem to be an esoteric factor, there is mounting evidence that there is a correlation between the factors of finger-length ratios, prenatal testosterone levels, propensity to involve oneself in physical activities, and spatial abilities. These correlations infer that there is a biological predisposition on the part of men to outperform women in spatial tasks.

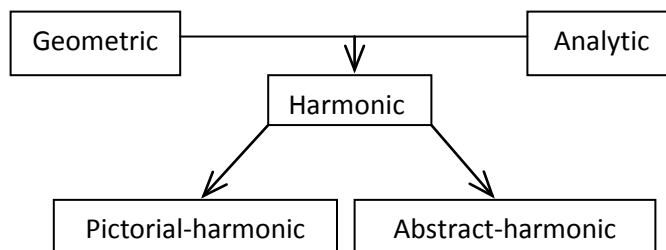
Casey et. al. (2008) suggested that while these differences take hold at an early age, it may be caused by lack of access on the part of females to spatial activities at young ages; as many boys tend to spend more playtime engaged in block-type play than girls. Chan (2007) also indicates that experience during childhood with spatial orientation related tasks could especially influence mental rotations in females. This kind of nurture over nature thinking helps support assertions by Linn & Petersen (1985) that sex differences in spatial abilities emerge during early adolescence. As toy exposure becomes diminished in a technological age, others have identified that exposure and experience with computers and video games may play a role in both predicting spatial ability (Tlauka, William, and Williamson, 2008) and improving it (Sims & Mayer, 2002; Terlecki & Newcombe, 2005). Playtime with computer and video games may be a newly emerging factor in the spatial ability gender effect (Sims & Mayer, 2002; Terlecki & Newcombe, 2005), since the amount of experience with spatially-related computer and video game stimuli is more prevalent in males (Sanford & Madill, 2006). Regardless of the nature versus nurture argument, there is evidence that the external stimuli of computer/video game use (Sims & Mayer, 2002; Terlecki & Newcombe, 2005), sports involvement (Tlauka, William, &

Williamson, 2008), and block play (Casey, et. al., 2008) all have the potential to impact or exacerbate the gender effect in spatial ability. Some researchers suggest that if there is a continuation of segregation in some of these tasks the gender divide in spatial performance will continue to grow (Terlecki & Newcombe, 2005). Fortunately, Ferrini-Mundy(1987), found that there is a potential in spatial ability treatments to increase both the tendency and ability with which women visualize solid objects in college age calculus students. This research is important to keep in mind, as it aids in demonstrating that certain treatments may be able to override previous experiences and genetic predispositions in cases related to spatial visualization.

### Mathematical Preferences

Student thinking and preferences in problem solving as it relates to visualization has been a well studied area in mathematics education (Haciomeroglu, 2007; Haciomeroglu, Aspinwall, Presmeg, Chicken & Bu, 2009; Krutetskii, 1976; Moses, 1977; Stylianou, 2002; Suwarsono, 1982). Richardson (1977) and Walter (1963) identified three categories of problem solvers: 1.) verbalizers, 2.) visualizers, and 3.) mixers. Walter (1963) asserted that there is a need for an instrument to accurately identify a person's category of mathematical thinking. Krutetskii (1976) worked on a similar concept, identifying verbal-logical, and visual-pictorial as the two major categories of mathematical processing types. While people generally lean toward one or the other they are still able to use the other in limited perspective. Students that are visual and are unable to understand an analytic understanding may have difficulties in problem solving (Haciomeroglu, 2007). Haciomeroglu (2007) indicates that the same is true of analytic students who cannot understand the visual domain; that both groups will have difficulties in problem solving if they are over-reliant upon either type of processing. Learners that have a processing preference that is visual-pictorial are referred to as *geometric* learners, while those who prefer

using verbal-logical processing are considered *analytic* (Krutetskii, 1976). Students who are balanced in preference are called *harmonic* (Krutetskii, 1976) as shown in Figure 8. Of harmonic thinkers, Krutetskii (1976) identified two subtypes: abstract-harmonic and pictorial-harmonic.



**Figure 8- Mathematical processing preference categorization.**

Walter's (1963) call for testing the indicated categories of mathematical thinking went partially unanswered; however, Moses (1977) developed an instrument that could be used primarily in elementary realms and Suwarsono (1982) developed the Mathematical Processing Instrument (MPI). Later Presmeg (1985) used existing items from Kordemsky (1972) Krutetskii (1976) and Suwarsono (1982) and refined them for use with high school algebra students. Searching for a better instrument to identify preferences in calculus students, Haciomeroglu, Aspinwall, Presmeg, Chicken, and Bu (2009) used the frameworks of Krutetskii (1976), Suwarsono (1982), and Presmeg (1985) in order to create the Mathematical Processing Instrument for Calculus (MPIC).

Though the study of visual imagery usage and preferences in problem solving tasks is important, there is no conclusive evidence on its effect on spatial ability. While there is some evidence indicating that there exists a relationship between imagery and spatial ability (Clements & Battista, 1992; Lin, 1979), there is also research that indicates that there is not necessarily a correlation between them (Carey, 1915; Clements, 1984). Concurrently some research indicates that use of visual imagery tends toward higher spatial ability scores (Barrett, 1953), while others

indicate that verbal students tend to outperform visual students in mathematics and spatially related tasks (Lean & Clements, 1981) or that spatial ability can be used as an indicator for problem-solving performance (Moses, 1977; Moses, 1980); while more recent research asserts that harmonic thinking may prove best for understanding certain concepts (Haciomeroglu, Aspinwall, Presmeg, Chicken, & Bu, 2009). These differing opinions on the topic may be colored by each individual's conception of what math is or should be. The overarching math community is inclined to believe that visual concepts are not to be considered truly mathematics (Dreyfus, 1991; Guzman, 2002). These pervasive attitudes and the mathematics education community's reluctance to derivate instruction to visual learners (Haciomeroglu, Aspinwall, Presmeg, Chicken, & Bu, 2009) may be leading to the reluctance on the part of learners to use visual imagery. This reluctance on the part of the learner may however be more intuitive, as seen in students inclination toward the *analytic*, even when instruction focuses upon the visual (Vinner, 1989); though this reluctance is changing more recently (Stylianou, 2002).

### Exceptionalities

Studies focusing on exceptional education students' spatial and visualization abilities are an understudied facet amidst the overarching topic of spatial ability (Montague, Bos, & Doucette, 1991). Van Garderen (2006) investigated the amount of types of imagery used in problem solving between high-achieving, average-achieving, and students with learning disabilities (LD). She disaggregates the types of imagery used by each group into two categories based on sophistication of image: *pictorial imagery* and *schematic imagery* (Hegarty & Kozhevnikov, 1999). *Pictorial imagery* is described as "images that encode the visual appearance of objects of persons described" (van Garderen, 2006, p. 497); whereas, *schematic imagery* is described as "images that encode the spatial relations described in a problem" (van



Garderen, 2006, p. 497). Van Garderen (2006) indicates that schematic imagery is a more sophisticated type of visualization than pictorial. This assertion may be based on research that shows a correlation between the use of schematic imagery and success in mathematics (Hegarty & Koxhevinikov, 1999). High and average-achieving students have been shown to use visualization techniques more often than LD students (Montague, Bos, & Doucette, 1991). Similarly gifted students use visual imagery more often than LD students (van Garderen, 2002). When visual image types of gifted and LD students were compared, the gifted students used more *schematic imagery*; whereas, the LD students primarily relied upon *pictorial imagery* (van Garderen, 2002).

Mathematical giftedness as discussed by Krutetskii (1969) and Presmeg (1986a) is less about official definitions of giftedness status and more about proclivity toward and efficacy in mathematics. According to Presmeg (1986a) most mathematically gifted students are not visualizers. She lists several reasons for this phenomenon divided into internal and external factors. Internal factors are generally directed around students finding images difficult to apply because of the amount of *static images* that students have. External factors discussed are that:

- (a) Textbooks are usually written to focus on *analytic* strategies and hence do not foster or support the thinking of *harmonic*, and *visual* students.
- (b) Teachers of *harmonic* or *analytic* backgrounds do not tend to promote the *visualization* strategies and in some cases devalue them.
- (c) Additionally, teachers that do use *visualization* themselves may not understand how to appropriately instruct others on the use of their *visualizations*; because of this, these teachers may also assert the *analytic* approach and in some cases devalue it, suggesting that *analytic* strategies are generally preferred in mathematics.

Students classified as Limited English Proficiency (LEP) have been rarely focused upon in relation to spatial ability learning. Dixon (1995) focused on these students, using LEP status and instructional environment as variable. Dixon indicates that within the same instructional environment LEP and English Proficient (EP) students did not perform significantly differently in spatial ability tasks. Though the growth differentiation of the LEP and EP students was not significant, the increase in visualization in all students that experienced a dynamic constructivist learning environment was significant. Dixon continues by suggesting that because visual preference correlates with mathematics achievement, that “effort should be focused on improving the visualization skills of LEP students” (p. 123).

## CHAPTER THREE: METHODOLOGY

### Design of study

This research is classified as experimental research because of the use of random sampling techniques and a control group. The specific type is a pretest-posttest control group (Key, 1997), in which the experimental group is given a pretest, the desired treatment, then a posttest; whereas, the control group is given the pretest, during the treatment time the control is given either no treatment or normal activity, then given the posttest. Key (1997) states, that if this design is properly carried out, it can be very effective and control many threats to validity.

### Participants

Sampling was randomized. Release and voluntary participation forms were passed out to all 1,159 students in the participating school for all levels through mathematics classrooms and collected by the teachers of those classrooms, then passed on to the researcher for compilation. The total amount collected was 161. This group was split into experimental and control groups randomly, by entering the forms into a spreadsheet and marking every other participant as control. Due to student mobility, student attendance, or teacher cooperation, some participants did not complete integral portions of the study (i.e., pretest or posttest). Due to this lack of completion, these students were taken out of the study, leaving a remaining working sample of 63 participants in the experimental group and 74 participants for the control, for a total of (n<sub>1</sub>=137), from 64 mathematics classes (21 sixth grade, 22 seventh grade, 21 eighth grade). The discrepancy in experimental and control size was primarily due to lack of continued cooperation on the part of participating teachers. Of the sample, 41% were males, 59% were females, 49% sixth grade, 22% seventh grade, and 29% eighth grade students.

The teachers selected to administer all portions of the CRIOSAT were selected from a convenience sample. There were 12 mathematics teachers who were administering the treatment. This sample includes all mathematics teachers in the school. This sampling technique is justified primarily because the teachers are simply facilitators and do not interact with the treatment or instrument materials, yielding no opportunity for corruption of results.

A subsample of students was used to analyze the mathematics preferences. This subsample was a convenience sample of the previous larger sample based on participating teacher volunteerism to provide the sample base. By volunteering to be part of the subsample, the teacher agreed that all students that they had participating in the first part of the study would be participants in the subsample. This subsample was originally 60 participants, due to participants failing to return the packets the working subsample was ( $n_2=41$ ). Of this subsample, 44% were male and 56% were female; while 46% were from the original control group and 54% were from the original experimental group. This subsample spanned all grades and ability levels, as per class placement.

#### School setting

The school selected for study is a sub-urban middle school located in Central Florida. The school was selected to reflect the total population of the county in which the study was conducted. As a middle school the school serves students in grades six through eight. All research was conducted through mathematics classrooms of which there are 16 grade regular education, 4 advanced mathematics, and 1 pre-algebra class(es) in 6<sup>th</sup> grade; 17 regular education, 4 advanced mathematics, and 1 algebra class(es) in 7<sup>th</sup> grade; and 15 regular education, 4 advanced mathematics, 1 algebra, and 1 geometry class(es) in 8<sup>th</sup> grade. This school is identified as a Title I (Elementary and Secondary Education Act of 1965).

The school has a racially diverse population including: 23 % White, 65% Hispanic, 8% Black, 2.5% Asian/Pacific Islander, and 2.2% are identified as Multicultural. 69% of students are on free or reduced lunch and 29% of the school population was Limited English Proficient (LEP).

### Classroom setting

Class size was based on normal class placement determined by school administration, which is limited to an average class size of 22 students. After the sample was obtained, there was no analysis done on the racial, or age demographics as these data trends are not the focus of the current study. Ages in the sample are estimated to be between 11 and 16 based on total school enrollment, while racial groups that are likely represented are White, Hispanic, Black, Asian/Pacific Islander, those identified as Multicultural. The study was not limited to any specific mathematics program and hence likely has some amount of LEP and Exceptional Education students. No special characteristics were considered in the selection of participating students, from using a random sampling approach.

### Variables

Different variables were defined in relation to each of the research questions and will be discussed per each:

**Research Question 1.)** Will an interactive computer-based treatment focused on rotational and perspective relationships of three-dimensional affect the spatial ability of students?

The independent/manipulated variable in identification of any effect of the computer treatment is the identification of students into the control and experimental groups. The focus of

the dependant/outcome variable in determination of the effectiveness of the treatment was the difference value from pretest to posttest.

**Research Question 2.)** Is there a correlation between time spent on items from the CRIOSAT, and spatial ability in students?

The independent variable used to identify any correlation between time spent on the designed treatment and change in spatial ability was the average time each student spent on the treatment; while the dependant/outcome variable was the change in scores between the pre- and post-tests. Deeper analysis into time spent focused on separation the average time spent in the treatment into the average time spent on the isometric and orthographic modes by themselves, while maintaining the dependant/outcome variable regarding the pre- and posttest differences.

**Research Question 3.)** Does the CRIOSAT affect males' and females' spatial ability differently?

The primary independent/manipulated variables in the determination of the effects of the treatment on are the gender of the participant in both the control and experimental groups; while the primary dependant/outcome variable was the change in pre- and posttest scores. As a secondary focus, each of the aforementioned was analyzed with the additional independent/manipulated variable of the time spent on each mode of the treatment.

**Research Question 4.)** How do mathematics problem solving preferences correlate to students spatial abilities?

The primary independent/manipulated variable in determining a correlation between spatial ability and problem solving preference was the analytic-visual (hereafter referred to as AVA-VIS) score based on the MPI, this scale from +10 to -10 indicates visual preferences on the high end of the scale and analytic preferences on the low end, as the primary dependant/outcome

variable was the pretest scores. Taking a deeper look into this question, increase in test scores was used to determine if spatial ability changes due to treatment despite problem solving preferences.

**Research Question 5.)** How does mathematics achievement correlate to student spatial abilities?

The primary independent/manipulated variable in determining a correlation between mathematics ability was student scores on a state standardized test, using a primary dependant/outcome variable of pretest scores from a spatial ability test. Additionally, student mathematics performance as seen by teachers and standardized test scores was used compared with pretest spatial ability scores. These teacher perception score were determined through interviews with teachers in which they were asked about each participating students to identify the mathematics achievement on a scale of 1 to 5 in which five is considered high-achieving and one is considered low achieving. Standardized testing score are from the Florida Comprehensive Assessment Test (FCAT).

As this is a study involving human participants as well as teachers that have individual styles, extraneous variables were expected. These extraneous variables were intended to be limited by decreasing teacher influence through the use of CBI. Though this attempt at control of extraneous variable was taken, there are always uncontrollable facets of human study. Similarly to Lean and Clements (1981), “student motivation, work habits, teaching and language competence were not measure in the present study” (p. 296), as they were not the focus of the current study. All of which could have had an effect on student performance of the measured variables.

## Instruments

### ROT

The instrument used as pre- and post-tests to determine change in spatial ability was the Purdue Visualization of Rotations (ROT) test (*see* Appendices D and E). The ROT was developed as a part of the Purdue Spatial Visualization Test Battery (Guay, 1976). This test was developed by Guay (1976). The ROT was designed to assess one's spatial ability using *gestalt processing*. Use of the ROT is considered appropriate by Bodner and Guay (1997), stating:

“It can be used as the basis for evaluating courses developed to enhance students' spatial skills. It can be used to probe students' perceptions of computer-based activities that require them to perceive three-dimensional structures from two-dimensional representations on a computer screen” (p. 14).

The ROT consists of 30 test items. Each item shows a type of rotation using an example figure, then a new figure and five answer choices from which the participant selects the view that mirrors the rotation in the example (*see* Figure 7). The ROT has a 15 minute time limit. The time limit is imposed because it increases the use of *gestalt processing*. While this test was administered during class time, the test items were scanned into CRIOSAT and administered via computer to participants, so that there was no involvement from teachers and the time limit was strictly enforced, as the computer logged students of the ROT after the prescribed amount of time.



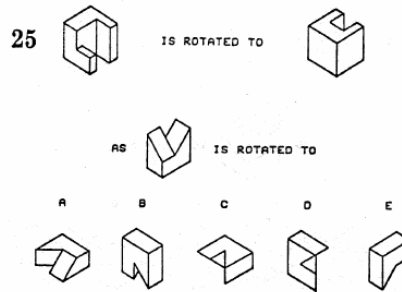


Figure 9 - Sample problem from the ROT (Guay & Bodner, 1997).

To show construct validity of the ROT, Guay (1976) compared it with the Shepard-Metzler test (S-M) (Shepard & Metzler, 1971) and the Minnesota Paper Form Board tests (MPFB) (Likert & Quasha, 1941). When compared to the S-M tests, the ROT yielded results typically indicative of very beneficial results ( $r = 0.61, p < 0.001$ ); whereas, the comparison with the MPFB tests yielded results that are likely to be useful ( $r = 0.25, p < 0.01$ ). Further study of these tests (Guay, McDaniel, & Angelo, 1978), indicated that the stronger correlation to the S-M tests was partially due to the S-M tests and ROT being more conducive to gestalt processing, while the MPFB tests used little gestalt and more analytic processing.

The reliability of the ROT was calculated using Kuder-Richardson (KR-20) and split-half analyses. Reliability testing was done on samples of 757, 850, and 1273 science students. KR-20 results were 0.80, 0.78, and 0.80 respectively, which are considered moderate to good reliability (Frary, 2008). The S-H reliability was calculated using the samples. The S-H results were 0.83, 0.80, and 0.85 respectively, or good reliability (Frary, 2008).

### MPI

The instrument that was used to determine mathematics processing preferences was the Mathematical Processing Instrument (MPI) (Appendix F) developed by Suwarsono (1982). The MPI was originally designed consisting of 30 questions per test. Two tests were developed by

Suwarsono (1982). Fifteen of the questions are designated as Part I consisting of word problems in which multiple solutions could be used to determine the correct answer. The other fifteen questions from Part II, which is a corresponding set of questions.

The grading method that Suwarsono (1982) used in the MPI is as follows:

An attempt using a visual method, whether the answer given is correct or incorrect: a visuality score of 2 is allocated;

An attempt which does not give any indication of method both on the worksheet and on the questionnaires, whether the answer is correct or incorrect or no answer is given: a visuality score of 1 is allocated;

An attempt using a nonvisual method, whether the answer given is correct or incorrect: a visuality score of 0 is allocated. (p. 151)

Lean and Clements (1981) used the MPI as well however they scored students based on the following scale:

+ 2 if the correct answer was obtained *and* reasoning was based on a diagram (drawn by the pupil) or on some ikonic visual image (constructed by the pupil);

+ 1 if an incorrect answer was obtained *and* reasoning was based on a diagram or on some ikonic visual image;

0 if no answer was given to a question *or* the pupil could not decide which method he used;

- 1 if an incorrect answer was obtained *and* reasoning was based on a verbal-logical method which did not involve a diagram or the construction of an ikonic visual image;

- 2 if a correct answer was obtained and reasoning was based on a verbal-logical method which did not involve a diagram or the construction of an ikonic visual image. (p. 283-284)

The rationale that Lean and Clements (1981) cited for the difference in scaling resulting in a positive/negative scale was based in how they viewed incorrect responses in regards to preference. They indicated that “persons who give incorrect responses are not confident that the methods they have used are appropriate,” (p. 284) hence assigning  $\pm 1$  to these responses. The lower on this scale that students score, the more analytically they problem solve; conversely the higher students score, the more visual they are. Statistical testing of the MPI demonstrates reliability using Cronbach’s alpha ( $\alpha = 0.87$ ; Suwarsono, 1982). The validity of the MPI has been questioned by some claiming that there is “No clear relationship between the degree of visuality and students’ performance on either mathematics or spatial ability tests has been found.” (Blajenkova, Kozhevnikov, & Motes, p. 241) While the validity of the MPI has been questioned partially due to the use of a Rasch model in its construction, which does not use a sample for calibration, Lean and Clements (1981) determined that there is validity by using Spearman’s rank-order. In this testing they determined that there is validity of this instrument ( $\rho = .90$ ). Lean and Clements (1981) continued testing the validity of the instrument by interviewing students based on answers to the MPI and grading them as analytic or visual, and compared that to the standard grading of the instrument. This validity check resulted in an extremely strong correlation.

For the purposes of this study, five questions of the MPI Test 1 were administered to participants (questions 1, 2, 3, 5, and 10). Grading of these was done using Lean and Clements’s (1981) described above.

## Procedures

In preparation for research, school principal permission was obtained (Appendix G) a research proposal was filed with the IRB of both the county (Appendix H) in which the study was conducted and the University of Central Florida (Appendix I) due to the researcher's link to the institution. Parent consent (Appendix J) and a child assent form (Appendix K) were sent home in a packet with each participant detailing what the research consisted of and the confidentiality measure.

Confidentiality was maintained by assigning each student a random number that linked the participant to the data from the treatment and testing. Participant names were not collected. Knowledge of participant computer login information and assigned number were limited to the researcher.

The identified participants' progress was tracked through the instructional treatment over the course of two months per group (control and experimental). Participants engaged in CRIOSAT activities one day a week during mathematics class for the entire 50 minute class period in the experimental group after the initial testing, while the control group was subjected to the pretest only. The posttest was administered to control and experimental groups at the end of the two month period.

Participants logged into the treatment for the first time using a user selected identification number. Since the user selects their own used ID, the information could not be linked to participant data as to preserve anonymity. The treatment itself included instructions built in for participants that may have difficulty had with the content. Teachers were explicitly instructed that they were to only place students onto computers and how much time they were spending on the treatment, but not to aid in the completion of any task.

The treatment took participants through various tasks and levels of difficulty (Appendix A). Participants were instructed to complete each task. If a task was completed correctly, the treatment moved them to the next task. If a task was completed incorrectly, the treatment offered support to aid the participant in the task. After this aid, a new task of the same difficulty was supplied. When a participant had completed 5 tasks, correctly or incorrectly, in any given difficulty level, the treatment moved on to the next difficulty level.

The MPI was administered to a subsample of students four weeks after the completion of the treatment. The MPI was distributed by participating teachers to the participants in the form of a packet containing the following: an information page, Part I word problems, and Part II questionnaire. These packets were sent home with participants to complete. The participants were given one week in which to complete the contents of the packet, at which point they returned them to the participating teacher.

### Data Analysis

The first research question asked whether the treatment given to students had an effect on spatial ability. Using this question a null hypothesis was developed stating:  $H_0$  = there is no affect of the treatment on student spatial ability. In testing this hypothesis, the change in scores on the PSVT-ROT from pre- and posttest score of both the experimental and control groups were determined. Based on these changes in spatial ability scores descriptive statistics were calculated and an ANOVA was conducted, in order to determine significance of findings.

In order to address the second research question pertaining to correlations between time spent on CRIOSAT and change in spatial ability, data associated with time was broken into the following categories for analysis: (a) total time spent on treatment versus change in ROT score, (b) time spent on isometric portion of the treatment versus change in ROT score, and (c) time

spent on isometric portion of the treatment versus change in ROT score. This yielded the following null hypotheses: (a)  $H_0$  = there is no correlation between total time spent on treatment and change in ROT score, (b)  $H_0$  = there is no correlation between time spent on isometric portion of the treatment and change in ROT score, and (c)  $H_0$  = there is no correlation between time spent on orthographic portion of the treatment and change in ROT score, respectively. In testing each of these hypotheses, an ANOVA was conducted to determine the significance of the finding. To determine the strength of the correlation between factors, a correlation coefficient was found ( $r^2$  was used as correlation coefficient based on Pearson's  $r$ ). Descriptive statistics for each subset of data were also calculated. These tests were limited to participants in the experimental group as students in the control group did not spend any time using the treatment.

In analyzing whether or not the CRIOSAT affected males' and females' spatial ability differently, the following null hypothesis was developed:  $H_0$  = CRIOSAT does not affect males' and females' spatial ability differently. The difference measured in association with this question was the change in ROT scores from pre- and posttest. To determine significance of findings, an ANOVA was conducted. The means of the experimental males, experimental females, control males, and control females were calculated for comparison. The difference of means was then calculated, between control and experimental groups of both males and females to determine if there was an average difference in the change of score and as to whether this change was constant between genders.

The fourth research question of the present study was based outside of the use of the designed treatment. Investigations about the nature of the relationship between problem solving preferences and student spatial abilities were examined in order to gauge the need of treatments similar to the one developed for the present study. The subsample previously described was used

for analysis comparing the posttest scores of the ROT and the ANA-VIS scores from the MPI. Similarly to previous analysis, an ANOVA was conducted to determine the significance of the data collected. To determine a correlation coefficient for the null hypothesis,  $H_0 =$  there is no correlation between mathematics problem solving preference and student spatial abilities, the Pearson's  $r$  was calculated and represented as an  $r^2$  value was used for interpretation. In further analysis connecting the fifth research question of the present study and mathematics preferences, another ANOVA and  $r^2$  correlation coefficient were calculated between mathematics achievement in students and ANA-VIS scores from MPI.

The final research question presented in this study related to the determination of correlation between mathematics achievement and spatial ability. Mathematics ability was determined on a scale of one to five based on an average composite of teacher interviews and standardized test scores, while the spatial ability was measured using the posttest scores of the administered ROT. The null hypothesis that was developed in testing this research question was,  $H_0 =$  There is no correlation between mathematics achievement and students' spatial abilities. The same protocols used for analyzing correlational data for other similar questions in present study were used here. An ANOVA was used for determining significance of results and Pearson's  $r$  was calculated for use as an  $r^2$  value.

## CHAPTER FOUR: DATA ANALYSIS

### Effectiveness of treatment

In reference to the first research question, determining if the treatment had an effect on spatial ability the differences in pre- and posttest PSVT-ROT were analyzed for the experimental (n = 63) and control groups (n = 73). The ROT contained 30 questions and was analyzed as number correct from 0-30. Descriptive statistics indicate that the increase in spatial ability was  $\bar{X} \approx .21$  and  $\bar{X} \approx .68$  in the experimental and control groups respectively. In the experimental group  $\sigma \approx 3.53$  with  $\sigma \approx 4.11$  in the control, from pretest to posttest. The one-way ANOVA conducted indicates that there is no significant effect on spatial ability due to use of treatment at the  $p < .05$  level [ $F(1,134) \approx .52$ ,  $p \approx .47$ ]. This lack of significance suggests that the null hypothesis ( $H_0$  = there is no affect of the treatment on student spatial ability) must be accepted.

### Time on treatment

Primary analysis regarding the second research question using descriptive statistics show the means of  $\Delta$  in ROT ( $\bar{X} \approx .21$ ), total time spent using CRIOSAT ( $\bar{X} \approx 57.8$ min), time spent in isometric mode ( $\bar{X} \approx 34.4$ min) and time spent in orthographic mode ( $\bar{X} \approx 23.4$ min), each with the respective standard deviations of  $\sigma \approx 3.53$ ,  $\sigma \approx 33.58$ ,  $\sigma \approx 26.84$ , and  $\sigma \approx 11.99$  (see Table 1). Analysis of correlational relationships between time spent on treatment yielded highly significant results at the  $p < .01$  level [ $F(1, 124) \approx 183.2$ ,  $p < .0001$ ] from comparison of total time spent using CRIOSAT and  $\Delta$  of pre-and posttest scores. A linear regression between data sets demonstrated that there was a negative correlation between factors (see Figure 10). The correlation coefficient indicates that there is little or no correlation ( $r^2 = .0009$ ) between factors though the linear regression leaned toward the negative. This first analysis indicate that the null hypothesis,  $H_0 =$



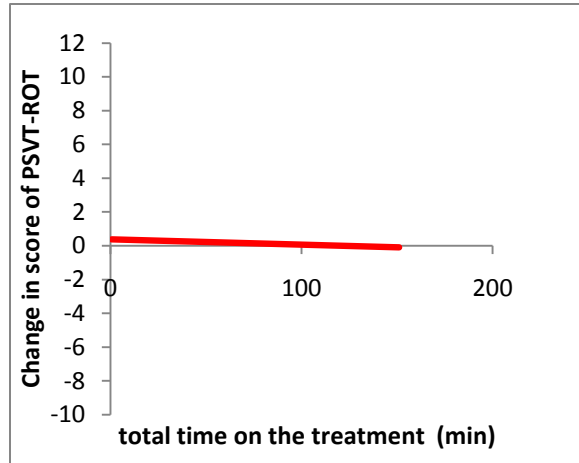


Figure 10 -Total treatment time vs. change in PSVT

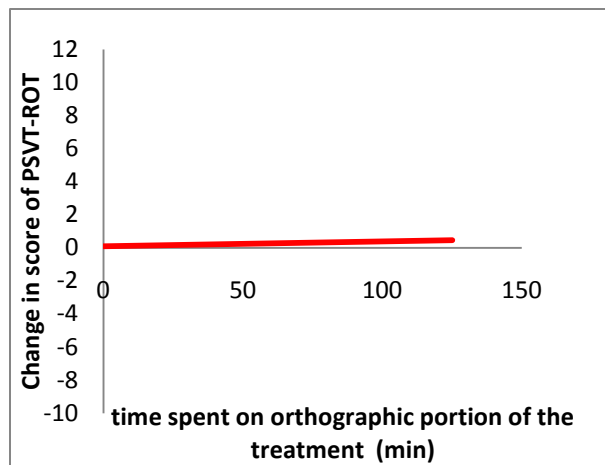


Figure 11 - Isometric time vs. change in score on PSVT

there is no correlational strength between total time spent on treatment and change in ROT score, must be accepted. Further analyses related to the time spent on different modes of the treatment were conducted to determine if only specific portions of the treatment had lack of correlation. When comparing the time spent on the isometric portion of the treatment was compared to the  $\Delta$  of pre-and posttest scores, similar significance was found as in total time. The data were highly significant at the  $p < .01$  level [ $F(1,124) \approx 100.61$ ,  $p < .0001$ ].

Unlike the analysis of total time, the isometric mode times versus the  $\Delta$  ROT yielded a positive linear regression, as seen in Figure 11; however, the r-squared value indicates that there

is little or no strength to this correlation ( $r^2=.0005$ ). This comparison indicates that the second null hypothesis related to the time spent on the isometric mode,  $H_0 =$  time spent on isometric portion of the treatment and change in ROT score, must also be accepted. Tests between the time spent on orthographic mode and  $\Delta$  ROT again indicated a high significance to the data at the  $p<.01$  level [ $F(1,124)\approx 216.28$ ,  $p<.0001$ ]. Similar to the linear regression in the total time versus  $\Delta$  ROT comparison, the linear regression calculated in this comparison indicates a negative correlation (see Figure 12); however, unlike other comparisons related to total time, there is a stronger correlation ( $r^2=.0178$ ). While the correlation coefficient indicated is magnitudes stronger than previous time related analyses, this is still a weak correlation. While this correlation is not strong, when we compare this with the mean time spent in this mode ( $\bar{X} \approx 23.4\text{min}$ ). This leads to ambiguity related to the null hypothesis,  $H_0 =$  time spent on orthographic portion of the treatment and change in ROT score, to the extent that this cannot be accepted or rejected.

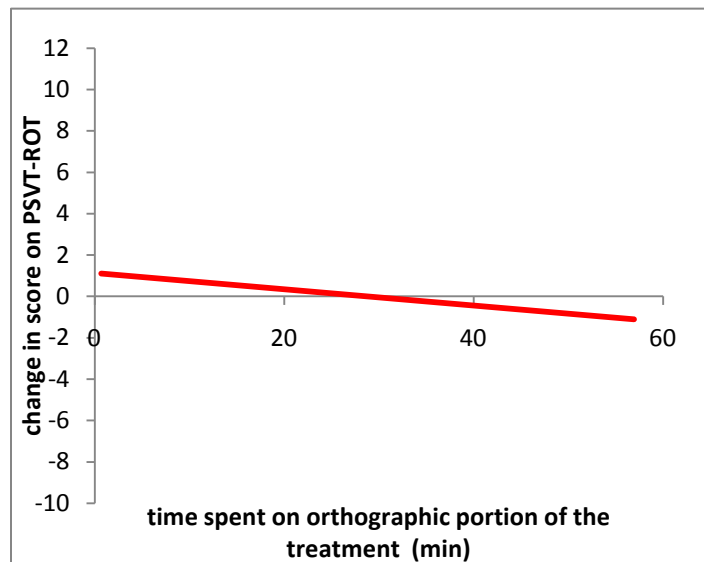


Figure 12 - Orthographic time vs. change in score on PSVT

### Gender analysis

The third facet of analysis relates to the determination of the designed treatment and its effects and differences in effects dependent upon gender. Groups of participant were broken into the subgroups of experimental male, experimental female, control male, and control female for analysis. The data used for statistical analysis were the  $\Delta$  ROT scores. The means of the  $\Delta$  ROT scores were ( $\bar{X} \approx -.03$ ), ( $\bar{X} \approx .12$ ), ( $\bar{X} \approx 1.10$ ), and ( $\bar{X} \approx -.13$ ), for the experimental female, experimental male, control female, and control male respectively, as seen in Table 1. As with the experimental versus control analysis, the ANOVA indicated that the current analysis was not statistically significant at the  $p < .05$  level [ $F(3, 137) \approx .88$ ,  $p \approx .45$ ]. To determine the effect of the

Primary Study			
	$\bar{X}$	$\sigma$	$\sigma^2$
Total pre (n=137)	8.62	4.45	19.77
experimental pre (n=63)	8.33	4.39	19.31
male (n=32)	8.60	4.79	22.95
female (n=31)	8.06	4.00	16.00
control pre (n=74)	8.89	4.51	20.32
male (n=25)	10.88	5.76	33.16
female (n=49)	7.92	4.50	20.25
Total post (n=137)	8.98	4.88	23.84
experimental post (n=63)	8.35	4.63	21.47
male (n=32)	8.66	5.48	30.00
female (n=31)	8.03	3.62	13.12
control post (n=74)	9.58	5.07	25.66
male (n=25)	10.75	5.99	35.93
female (n=49)	9.00	4.50	20.25
Treatment time	52.76	36.01	1297.01
Level 1	8.84	5.65	31.93
Level 2	11.73	8.64	74.67
Level 3	5.72	4.74	22.43
Level 4	10.60	10.76	115.84
Level 5	6.77	5.85	34.26
Level 6	9.10	16.80	282.14

**Table 1- Descriptive Statistics**

treatment two values were calculated from the mean scores of each group,  $\Delta_{\text{female}}$  and  $\Delta_{\text{male}}$ . The  $\Delta_{\text{female}} \approx -1.13$ , which show that the control females averaged about 1.13 points increase more than the experimental group from pretest to posttest, or that the control group did 3.78% better on the ROT posttest on average than the experimental group after treatment. The  $\Delta_{\text{male}}$  was calculated in the same way showing a difference of .24 between the experimental and control groups. This shows that on average the males in the experimental group increased by .24 points, or .81%. Again due to the lack of significance determined the null hypothesis,  $H_0 = \text{CRIOSAT}$  does not affect males' and females' spatial ability differently, must be accepted.

Problem solving preferences and spatial ability

Examining the relationship between students spatial abilities, as measured by the ROT, and their mathematics problem solving preferences, as measured by the shortened 5 question version of the MPI on a scale of -10 to +10, an ANOVA revealed that there was a highly significant relationship between factors at the  $p < .01$  level [ $F(1, 80) \approx 92.76, p < .0001$ ]. A linear regression of the plotted data indicate that there is a positive correlation between factor (as shown in Figure 13), indicating that as the preference to solve using visual methods increased so

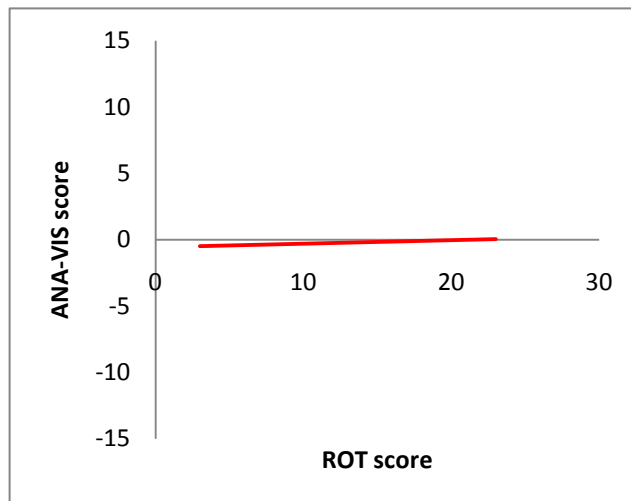


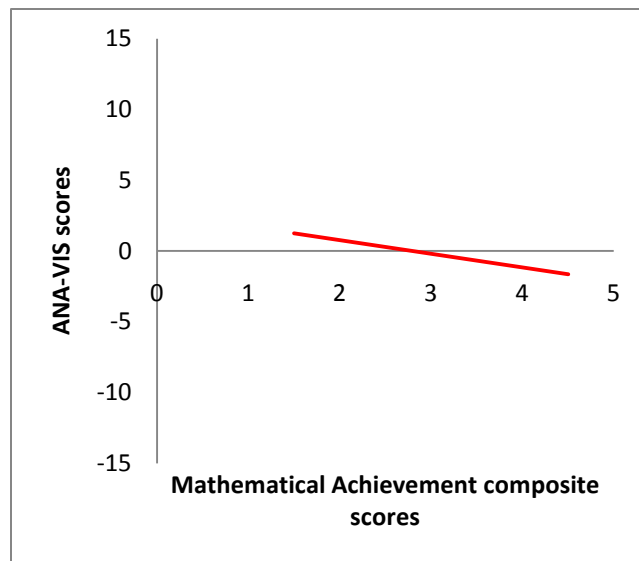
Figure 13 – Level of visuality compared to spatial ability

did spatial ability. The strength of this correlation however was slight ( $r^2=.0007$ ), indicating that there is little or no correlation between these factors. This result allows for the acceptance of the null hypothesis,  $H_0$  = there is no correlation between mathematics problem solving preference and student spatial abilities.

Analysis of correlation between mathematics achievement and problem solving preferences was done in the following three stages each consisting of an ANOVA and linear regression: (a) analysis between mathematics achievement composite scores and ANA-VIS score, (b) analysis between mathematics achievement as indicated by teacher perceptions and ANA-VIS scores, and (c) analysis between standardized test scores and ANA-VIS scores. Standardized test scores were measured using the mathematics portion of the FCAT (Florida Comprehensive Assessment Test) in which students are given a score from 1-5, where a score of 3 indicates that students are on grade level in mathematics. Teacher perceptions of students' mathematics achievement scores were obtained by interviewing teachers about students. The teachers rated students using a scale of one through five, where one is low achieving and five is high achieving. The composite scores aligned to each participant were calculated by using the average of the standardized test scores and the teacher perception score as they both use similar scales.

In the analysis between composite mathematics achievement scores, and ANA-VIS scores the ANOVA results indicate a high significance at the  $p<.01$  level [ $F(1, 80)\approx 18.3$ ,  $p<.0001$ ]. The linear regression of these factors shows a negative correlation (see Figure 14) indicating that as visual preferences increase mathematics achievement decreases; however, the strength of this correlation is weak in this case ( $r^2=.0199$ ). Also significant at the  $p<.01$  level, is the data relating the teacher perception of student mathematics achievement [ $F(1,80)\approx 20.18$ ,

$p < .0001$ ]. The regression of these factors shows a positive correlation (see Figure 15) indicating that as students become more visual in their problem solving preferences, teachers perceive them to achieve more highly in mathematics. This correlation is however nearly as weak ( $r^2 = .0166$ ) as the correlation between composite achievement scores and ANA-VIS scores.



**Figure 14 - Level of visibility vs. mathematical achievement**

The final analysis of mathematics achievement and problem solving preferences holds similar significance to the other factors analyzed [ $F(1,80) \approx 16.29$ ,  $p < .001$ ], while having a much stronger correlation coefficient ( $r^2 = .476$ ). The regression of these factors shows a negative relationship between standardized test scores and students with visual problem solving preferences, as seen in Figure 16. As the visual preference of the student increase, the standardized test scores decrease.

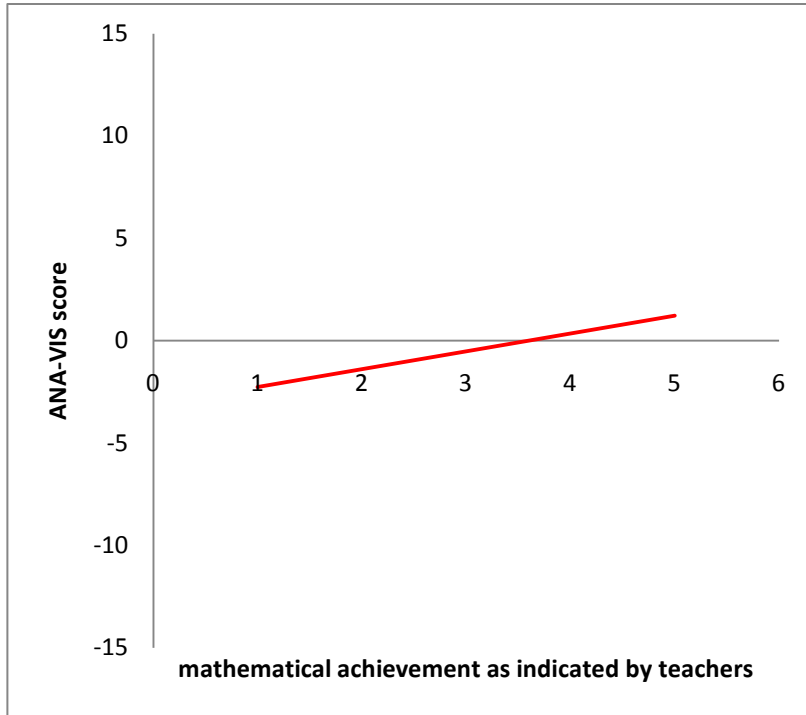


Figure 15 - Level of visibility vs. teacher perception of mathematics achievement

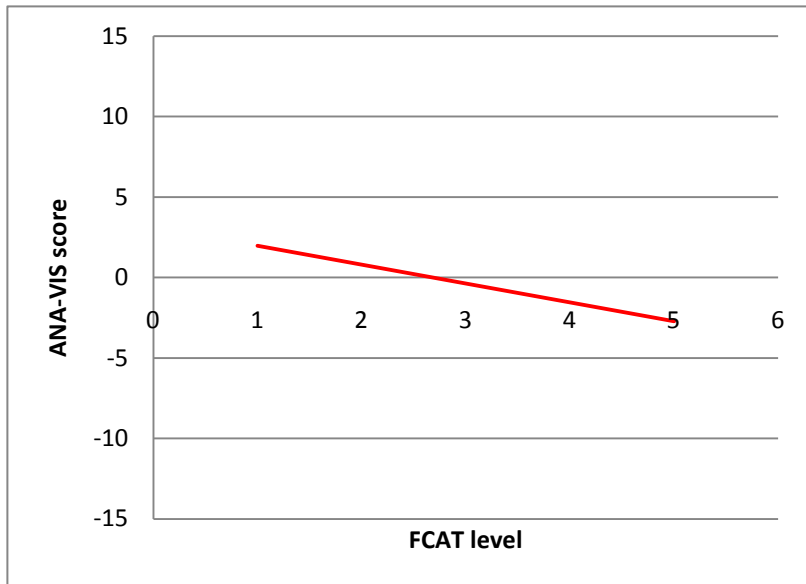


Figure 16- Level of visibility vs. FCAT scores

Mathematics achievement and spatial ability

The final research question regards the analysis of correlation between mathematics achievement and student spatial abilities. In the analysis of this correlation, an ANOVA was

used to determine significance. The correlation of these two factors was found to be highly significant at the  $p < .01$  level [ $F(1, 282) \approx 166.29, p < .0001$ ]. From this level of significance, the null hypothesis,  $H_0 =$  there is no correlation between spatial ability and mathematics achievement in students, must be rejected. Since there is a significant correlation between factors, strength of the correlation of factors needed to be determined. This determination was made using a linear regression and an  $r^2$  value. The linear regression of factors shows that there is a positive correlation between factors (see Figure 17), meaning that as spatial ability scores increased in participants the mathematics achievement scores also increased. The strength of this correlation is substantial ( $r^2 = .143$ ) enough to be noted but not considered strong. While this analysis showed positive correlation (see Figure 18), other analyses of data need be considered.

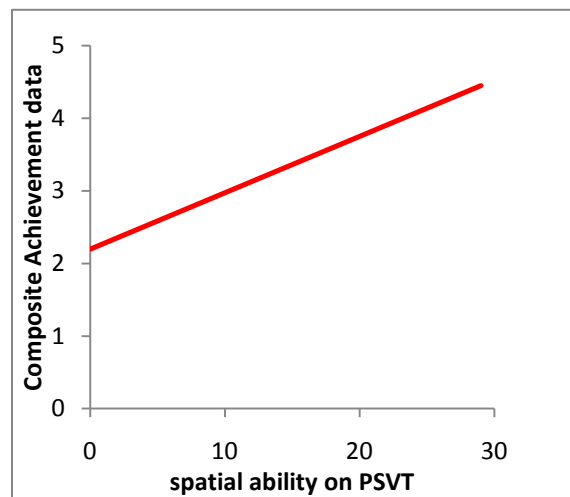
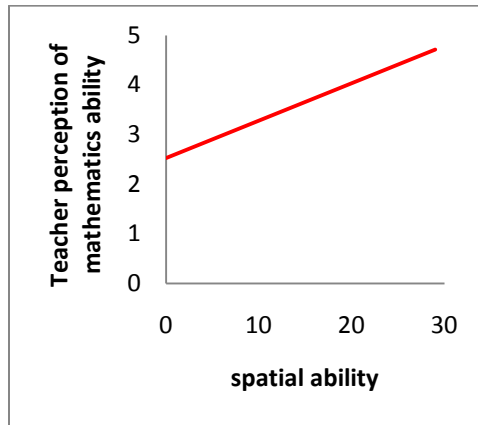


Figure 17 – Mathematics achievement comparison with spatial ability

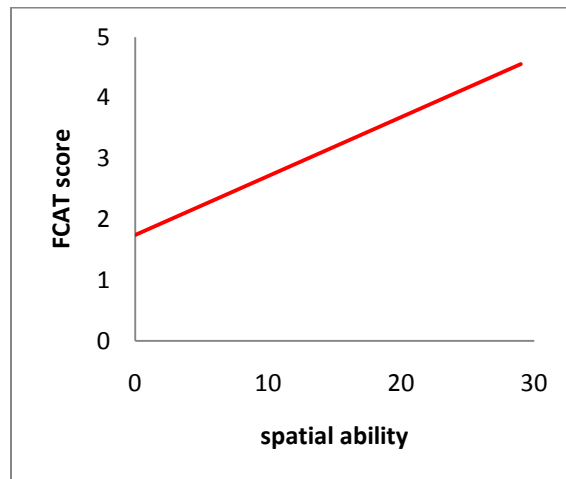
The prior analysis used composite scores for mathematics achievement from standardized tests and teacher perceptions. For subsequent analysis, each individual factor of the composite score was compared to spatial ability. In analysis between teacher perception of mathematics achievement and spatial ability, the ANOVA unveiled similar results in significance [ $F(1, 266) \approx 174.9, p < .0001$ ], however the correlational strength was weaker ( $r^2 = .1117$ ) than the composite score. Similar analysis between standardized test scores not only showed a high level



of significance [ $F(1, 272) \approx 236.12, p < .0001$ ], but in this case a stronger positive correlation (see Figure 19) was found ( $r^2 = .184$ ).



**Figure 18 – Mathematics Achievement (teacher perception) compared to spatial ability**



**Figure 19 – Mathematics achievement (standardized test) compared to spatial ability**

Deeper analysis was conducted by dividing the subsample into groups of high and low achieving students. This divide utilized the composite mathematics achievement score, scores below 3 were considered low achieving, while those above 3 were considered high achieving. The data were further categorized by gender (descriptive statistics of subsample seen in Tables 2 through 4). Correlation coefficients were then found for the relationships of these groups between student visuality and spatial ability. In low achieving male students, a very strong

( $r^2=.716$ ) negative correlation between these factors was found; while, in low achieving female students a weak ( $r^2=.0077$ ) positive correlation, as seen in figure 20.

Subsample Study			
	$\bar{X}$	$\sigma$	$\sigma^2$
Teacher indicated achievement	3.27	0.74	0.55
Standardized test achievement	2.93	0.93	0.87
Achievement composite	3.10	0.74	0.54
ANA-VIS score	-0.29	5.02	25.21

**Table 2 – Subsample basic demographics**

Subsample Descriptive Statistics						
	spatial ability			Total ANA-VIS		
	$\bar{X}$	$\sigma$	$\sigma^2$	$\bar{X}$	$\sigma$	$\sigma^2$
female (n=23)	8.09	3.60	12.99	1.00	4.91	24.09
high achieving (n=8)	10.25	3.45	11.93	-0.25	5.68	32.21
low achieving (n=8)	7.63	2.33	5.41	0.50	3.85	14.86
male (n=18)	10.39	5.67	32.13	-1.94	4.80	23.00
high achieving (n=9)	12.67	6.93	48.00	-2.44	5.68	32.21
low achieving (n=3)	9.33	2.08	4.33	-3.33	4.16	17.33

**Table 3 – Subsample descriptive statistics by gender for analysis of high and low achieving groups**

Subsample Descriptive Statistics						
	spatial ability			Achievement Avg		
	$\bar{X}$	$\sigma$	$\sigma^2$	$\bar{X}$	$\sigma$	$\sigma^2$
female (n=23)	8.09	3.60	12.99	2.98	4.91	24.09
visual (n= 8)	6.88	3.72	13.84	2.94	0.50	0.25
analytic (n=4)	7.75	1.50	2.25	3.38	0.75	0.56
male (n=18)	10.39	5.67	32.13	3.25	-1.94	3.78
visual (n= 2)	15.00	15.56	242.00	3.50	0.71	0.50
analytic (n=7)	9.71	4.79	22.90	3.50	0.58	0.33

**Table 4 - Subsample descriptive statistics by gender for analysis of visual and analytic groups**

In high achieving students, the correlation of ANA-VIS scores and spatial ability in females was similar to their low achieving counterparts, having a weak ( $r^2=.0182$ ) positive correlation, as seen in figure 20. Contrary to the findings of low achieving male students, the

findings in high achieving males indicates a moderately weak ( $r^2=.2349$ ) positive correlation between factors.

Further analysis using ANA-VIS scores as a basis for participant division was conducted. Students were divided into two categories, analytic and visual and then again by gender. After the group division correlation analysis was done between composite mathematics achievement scores and spatial ability scores, an ANA-VIS score of -10 to -4 was categorized as analytic, -3 to 3 was

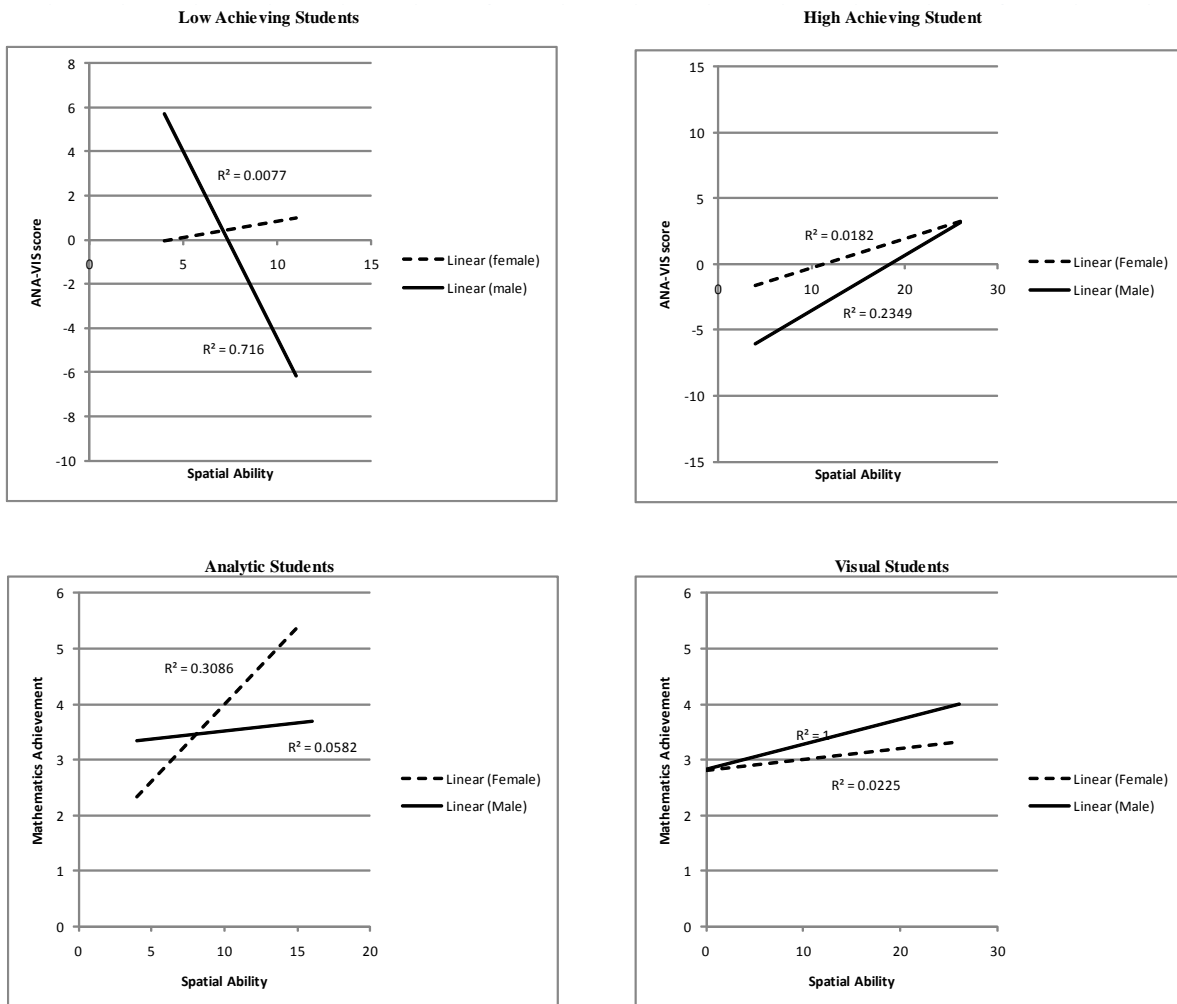


Figure 20 – Analysis between gender, mathematics achievement, spatial ability, and strategic preferences.

categorized as having no preference, and 4 to 10 categorized as visual preference. Using this, 27%, 51%, and 22% of participants were found to have analytic preferences, no preferences, and visual preferences in problem solving respectively.

Females identified as analytic had a moderately strong ( $r^2=.3086$ ) positive correlation between these factors; while, the visual females had a much weaker ( $r^2=.0225$ ) positive correlation between these factors. Males categorized as analytic had a weak ( $r^2=.0582$ ) positive correlation between factors. Visual male however, had a perfect ( $r^2=1$ ) positive correlation between factors (see Figure 20). Explanation for this perfect correlation is necessary due to the exceedingly uncommon result. The correlation strength is due to there being only two males that were categorized as visual.

## CHAPTER FIVE: CONCLUSIONS

### Discussion

As no significance can be determined for the effect of the treatment on spatial ability, this researcher cannot state if the treatment was significantly effective. These results suggest that the CRIOSAT has no effect on student spatial ability. To explain some of the possible reasons for these results, outside factors must be considered. As in Lean and Clements' (1981) study, student motivation, work habits, teaching and language competence were not measured, and are all factors that can influence the effectiveness of the treatment; but of the utmost importance in the results of the present study, is contact time with the treatment. Since the treatment tested was computer-based in nature rather than computer-assisted instruction, participants who did not have sufficient time, were not given enough time by participating teachers, or did not want to spend sufficient time using the treatment, may not have learned the intended content. Teachers were asked to have students in the experimental group on the treatment for at least an hour per week for an 8-week period; however, when the average time participants spent on the treatment is calculated for the duration of the study the result is  $\bar{X} = 57.78$  min. These data illustrate that while students were intended to interact with the treatment for 8 hours over the study period, the average participant was only in contact with the treatment for about one hour.

Duesbury and O'Neil (1996) also found no statistically significant relationships between a similar, yet not dynamic, treatment and change in mental rotations, which is in agreement with the findings of this study. Duesbury and O'Neil (1996) did indicate however that even in studies that demonstrated positive effects on spatial abilities, there is little evidence that demonstrates a long lasting effects of these treatments if any. While they indicated that the rotation and ability to manipulate a figure and comparing it to the 2D representation of a figure may be

instructionally beneficial; however the findings of the current study indicate that this may not be the case. Contrary to Duesbury and O'Neil's (1996) results suggesting that there may be no long lasting effects to such treatments, Ben-Chaim, Lappan, and Houang's (1988) study tested students using similar stimuli and found that retention can occur in spatial ability training. The evidence related to effective spatial ability training and retention paired with the evidence examining the relationship between spatial ability and mathematics achievement, indicate that finding effective ways to improve spatial ability is still an important endeavor.

Items from the CRIOSAT were based in the framework of Coulsen (1968), specifying that "an item of instructional material (1) containing information and a question or problem requiring a response, is displayed (2) to the student (3). The response (4) is evaluated (5), and a feedback message (6) is automatically transmitted to the student. A new item is then selected (7)." (p. 141). Furthering this line of thought, Sanz de Acedo and Garcia (2003) indicated that feedback must be provided after each item to affect future performance. Though this methodological approach to feedback was used, it did not seem to affect future performance. Some participants improved during the early stages of the CRIOSAT and continued to improve; however, those who did not improve early on had little or no improvement throughout. As the items designed for CRIOSAT were randomly generated, there was some difficulty in determining what types of feedback would be given. Feedback interpretation by participants may be a factor in lack of spatial increase.

Time that participants spent in contact with the CBI was measured to help describe reasons for any change in spatial ability. Students in the current study had an average of one class period of contact time with the treatment. Hannafin, Truxaw, Vermillion, and Lui (2008) used a CAI (computer assisted instruction) to determine effects of geometry skill and indicated

that it was unclear as to whether additional time with the instruction would have had an impact on achievement. Further, they indicated that other studies showed that students involved with CAIs needed less time to master a task than with traditional instruction (Kulik & Kulik, 1991 as cited in Hannafin, Truxaw, Vermillion, and Lui, 2008). While the stimuli of the ROT items and of the CRIOSAT are not congruent, research regarding time of tasks as it relates to transference in spatial ability tasks applies. Wright, Thompson, Ganis, Newcombe, and Kosslyn (2008) stated that “transference effects occurred after approximately 7 hours of practice across a 3-week period.” (p. 770) and that concurrently 12 to 14 hours of practice to transfer non-practiced items (Terlecki, Newcombe, & Little, in press as cited in Wright, Thompson, Ganis, Newcombe, & Kosslyn 2008). As the current study required transference, the time that was spent by the participants may not have been sufficient.

Gender differences in relation to spatial ability are well documented indicating that females tend to have lower spatial ability than males (Battista, 1990; Ferrini-Mundy, 1987; Lean & Clements, 1982; Linn & Peterson, 1990; Voyer, Voyer, & Bryden, 1995). These findings are contrary to those of the current study, wherein it was determined that there is no significant difference between males and females in spatial ability. This decrease in the gender gap may be from a change in dynamic between play interactions in male and females; as Chan’s (2007) study indicates that the role of spatial experience had a stronger effect on spatial ability than gender. The increase in females interacting in rendered 3D environments in video games may be a factor in decreasing this spatial divide. The current results indicate that there is no spatial superiority for either gender follows Linn and Peterson’s (1990) results which suggest that there is a diminishing difference between the genders as it relates to spatial ability and gives some evidence to the assertions of Sanz de Acedo and Garcia’s (2003) who state that, “there is no

scientific justification to continue to believe that men's performance is superior to women's in spatial issues." (p. 278)

The most interesting and significant findings of this study related to the secondary foci, correlations of spatial ability (measured by the ROT), mathematics achievement (measured by teacher interviews and standardized test scores), and mathematics problem solving preferences (measured by the MPI). While an intuitive avenue of thought would lead to the conclusion that students who have a greater grasp of spatial ability would have a tendency to solve problems using visual means, this study indicates that this is not true. To the contrary significant evidence is presented that there is no correlation strength between these factors. In fact students with higher spatial ability tend to use analytic strategies rather than visual. This finding indicates that while students may have spatial ability they do not use the ability they have or at least that they do not rely upon that ability when solving problems similar to findings in other research (Lean & Clements, 1981; Presmeg 1986a). It is possible that these students have decided that though they can solve problems by visual means, the students have found the visual means of solving problems to be inefficient or unreliable. These strategies have been shown to be unreliable in the use of both static and dynamic images used to solve problems in the absence of analytic thinking (Haciomeroglu, Aspinwall, & Presmeg, 2010).

While this study shows a significant negative correlation between problem solving strategies toward visualization and standardized testing, it also showed a significant positive correlation between visual preferences and teacher indicated mathematics achievement. This finding is interesting, considering how teachers are denoting student achievement and how standardized test scores are measuring the same factor are incongruous; however, the implications gleaned from this incongruity are difficult to extract. The nature by which teachers



perceive mathematics achievement and how it is judged by standardized tests are not necessarily the same. Similarly, student performance on the MPI and mathematics achievement on standardized tests correlate; however, because the stimuli are not the same and in many cases not similar at all, this correlation cannot be used as a predictor of mathematics achievement on standardized tests. Not surprisingly as the level of student visuality increased performance on standardized tests decreased. This indicates that some students who are highly visual tend to not use this ability in problem solving. Again, overreliance upon visual strategies can be inaccurate (Haciomeroglu, Aspinwall, & Presmeg, 2010).

A much stronger and consistent correlation between factors was found between spatial ability and mathematics achievement, which is in agreement with Booth and Thomas (1999), but contrary to the findings of Tartre (1990). A moderately strong correlation between these factors was found in the present study. Unlike the correlations relating problem solving preferences and mathematics achievement, the correlation between spatial ability and mathematics ability was much more consistent when analyzing both teacher perceptions and standardized measures of mathematics achievement, indicating that students with spatial ability may not prefer to use it. This is demonstrated by the composite low correlation of visual preference and spatial ability that can be derived from the positive correlation of mathematics achievement and spatial ability combined with the negative correlation between visual preferences and spatial ability.

This correlation of spatial ability and mathematics achievement is similarly found in elementary school students, (Guay & McDaniel, 1977) middle school students, (Ben-Chaim, Lappan, & Houang, 1988) algebra students, (Tolar, Lederberg, & Fletcher, 2008), and middle school students with varying exceptionalities (van Garderen, 2006). While Guay and McDaniel (1977) found, that overall there were positive correlations among elementary students'

mathematics achievement and spatial ability, not all measures of spatial ability were consistent in this correlation. The surface development task used by Guay and McDaniel (1977) showed the greatest inconsistency in correlation ranging from negative to strongly positive coefficients is the most closely related to the types of stimuli that were given in CRIOSAT. The inconsistency with their findings along with the lack of significance in change in spatial ability from the current study, suggest that this skill may be one that is not solely synonymous with others identified as spatial ability. The skill needed for interpretation of surface nets compared to their 3D rendering may require more specialized skills than just mental rotation. Tolar et.al (2008) indicated that while spatial ability did correlate to algebra achievement, it was not one of the prevalent factors; however, they did discover that spatial ability was in fact a prevalent factor in determining success on the SAT-M. These findings are parallel to those in the current study, indicating that mathematics achievement in the classroom may not necessarily be predicted by a student's spatial ability; however, success on standardized tests may be predicted.

When students were divided into high and low achieving categories, it was found that the correlation between student visuality and spatial ability among females in both groups had similar correlations. This correlation indicated that as both high and low achieving females became more visual in the problem solving strategies that they used as their spatial ability increased. This indicates that mathematics achievement does not change this relationship in females. It also indicates that as female students become more efficacious in spatial ability, they tend to use those abilities more while problem solving. Though there is no significant difference between males and females relating to spatial ability in the current study, the difference in visual preferences is still apparent. High achieving males followed the same patterns as both groups of females. These results suggest that students play to their own personal strengths relating to

visuality and spatial ability. Results are based on the most available data in this study. Contrary to their high achieving counterparts, low achieving males had a strong tendency to use less visual strategies as their spatial ability increased. These data indicate that low achieving males tend to not follow strengths that they have.

### Implications

While much of the presented data from the secondary foci of the study hold an implicit value, the primary focus also holds implications of its own. As with Duesbury and O'Neil (1996), the current study had no significant effect on spatial ability. Both studies share distinct similarities in stimuli used. This concurrent finding explicates that treatments of this type do not have an effect on overall spatial ability as it relates to mental rotation, implying that these types of treatments are not useful in training spatial ability. While this may be true, it has been shown that other treatments on spatial ability are effective for instruction, retention (Ben-Chaim, Lappan, & Houang, 1988) and transference (Wright, et. al. 2008) to varying spatial skills, hence devising such a treatment to train spatial ability is still a goal worth pursuing.

The implications of the secondary foci showed even more interesting findings. These investigations examined the correlation of factors related to spatial ability, mathematics achievement, and problem solving preferences. The correlation found between spatial ability and standardized test scores combined with Tolar's et. al. (2008) results indicate that spatial ability may be used as a general indicator for mathematics achievement on standardized tests. With further research using larger samples, this implication may lead to the use of spatial ability as a major determination gauge in predicting success on standardized tests. Spatial ability also had a positive correlation to teacher perception of mathematics achievement; however, similar to Tolar's et. al. (2008) study this correlation was not as prevalent as the aforementioned. This

implies that students with high spatial ability may both exhibit behaviors that are preferable in mathematics classrooms as well as hold a proclivity to excel in the content area in testing situations. Additional training for teachers would be beneficial as it relates to training spatial ability in students, since increasing this ability may lead to increased mathematics achievement.

The lack of a significant difference in spatial ability between genders also holds the implicit property that the gender gap is diminishing, and that it may not be necessary to focus studies regarding spatial ability on gender as much as they have been in the past. This concept is mirrored in the suggestions by Sanz de Acedo Lizzaraga and Garcia Ganuza (2003). The current research further indicates that regardless of gender, students with increased spatial ability, mathematics achievement increases as well. Because of these findings further research should focus on the nature of and instruction techniques for spatial ability; as well as, focus on factors influencing their performance for visual and analytic strategies.

### Limitations

Limitations of the current study primarily regard fidelity to the study on the part of teachers and students. While many participating teachers and students made efforts to maintain fidelity regarding the amount of time spent using the treatment, the lack of fidelity was still apparent based on the average times that students spent interacting with the treatment. From the data collected it cannot be determined whether participants partook in fewer sessions of the treatment or shorter sessions than originally intended for students, for the effective use of this treatment, as time per level was collected but not time per login. Related to the time spent on tasks is the overall time frame in which the study took place. An eight week period in which participating teachers were asked to have students engage the treatment, totaling eight hours of student contact time may not be enough. If the window for the participation was increased along

with contact time, participating teachers may be better able to place students in contact with the treatment for adequate time periods.

Another limiting factor may have been the mode of instruction. While CBI (computer-based instruction) has been shown to have significant increases in spatial ability (Onyanha, Derov, & Kinsey, 2009) and mental rotation tasks (Samsudin & Ismail, 2004), it may not be the most effective mode of instruction. Computer assisted instruction (CAI) may have been a more effective mode as significant gains in spatial ability has also been found in students using manipulative (Casey, et. al., 2008) based instruction and paper/pencil instruction of the manipulation of three-dimensional arrays (Battista, & Clements, 1996).

Without further instruction, tasks associated with this treatment may have become too difficult as well. It is possible that the lower levels would have benefitted from more robustness and practice or a more detailed instructional process. It is evident from the data that the most difficult questions had a very low rate of correctness. Only 8.7% of the 46 students who attempted questions in level 6 got any questions correct. This demonstrates that by the time participants reached level 6 there may not have been enough time for students to invest in the simpler fundamental problems. It would be suggested that upon using CRIOSAT further this limitation be changed by removing  $5 \times 5 \times 5$  isometric creating tasks until more practice has taken place or possibly take them out all together.

### Suggestions for Future Research

Suggestions for further research should be centered on the limitations of the current study. Research using the CRIOSAT should be conducted using time as a focused variable to a larger degree. These studies should focus on using CRIOSAT in a similar time frame, a year-long treatment, and in a longitudinal perspective spanning 6<sup>th</sup> through 8<sup>th</sup> grade participants.

This type of research would have a greater capacity for determining a correlation between time spent on CRIOSAT and change in spatial ability.

A separate study using multiple modes of instruction in relation to CRIOSAT would also be beneficial to determine which mode is most effective when instructing students on similar stimuli. A fully hands-on inquiry instructional pattern should be compared to a CBI approach and a mixed mode approach. While the evidence that each mode is effective in facets of spatial ability training exists, determining which mode is most effective in training students to transfer images back and forth between three-dimensional arrays and orthographic views are important for increasing a facet of spatial ability that can lead to increased engineering prowess, as these skills are used by a variety of engineers.

The level of problem difficult and appropriateness of tasks should also be analyzed. While the randomization of the tasks makes this difficult, studies should attempt to discover what sets of randomized figures are more conducive to students in varying age ranges, increasing their spatial abilities through this treatment. In addition to analyzing the task difficulty, the method by which spatial ability is measured should be looked into as well. While the ROT (Guay, 1976) is a very reliable instrument for measuring mental rotation, it is suggested that spatial ability in future studies regarding CRIOSAT use multiple measures in assessment.

Qualitative analysis of students using CRIOSAT should also be conducted. This researcher suggests primarily that this be a case study of students that are indicated by pretesting as having high, medium, and low spatial ability. These students should be interviewed during and after the treatment. This type of study will help to indicate areas in which CRIOSAT may be deficient, as well as provide insight about students' thinking during the treatment.

Any of the suggested research should also add a component regarding retention. Each should give a pre- and posttest as well as another testing period well after the treatment period to determine the lasting effects of treatment.

## **APPENDIX A: CRIOSAT DIFFICULTY LEVELS**



Each of the levels will have about 5-10 exercises.

### **Level 1**

**Given:** Randomly Generated isometric projection of Cubes in a  $3 \times 3 \times 3$  space. Each side of each cube will be color coded. Isometric will be able to be manipulated for a  $360^\circ$  view in all directions.

**Students:** Create all three orthographic views of same object. Squares will be created in corresponding colors to those of isometric projection.

### **Level 2**

**Given:** Randomly Generated orthographic projection of squares in a  $3 \times 3$  space.

**Students:** Create isometric view of same object using cubes. Isometric will be able to be manipulated for a  $360^\circ$  view in all directions.

### **Level 3**

**Given:** Randomly Generated isometric projection of Cubes in a  $4 \times 4 \times 4$  space. Each side of each cube will be color coded. Isometric will be able to be manipulated for a  $360^\circ$  view in all directions.

**Students:** Create all three orthographic views of same object. Squares will be created in corresponding colors to those of isometric projection.

### **Level 4**

**Given:** Randomly Generated orthographic projection of squares in a  $4 \times 4$  space.

**Students:** Create isometric view of same object using cubes. Isometric will be able to be manipulated for a  $360^\circ$  view in all directions.

### **Level 5**

**Given:** Randomly Generated isometric projection of Cubes in a  $5 \times 5 \times 5$  space. Each side of each cube will be color coded. Isometric will be able to be manipulated for a  $360^\circ$  view in all directions.

**Students:** Create all three orthographic views of same object. Squares will be created in corresponding colors to those of isometric projection.

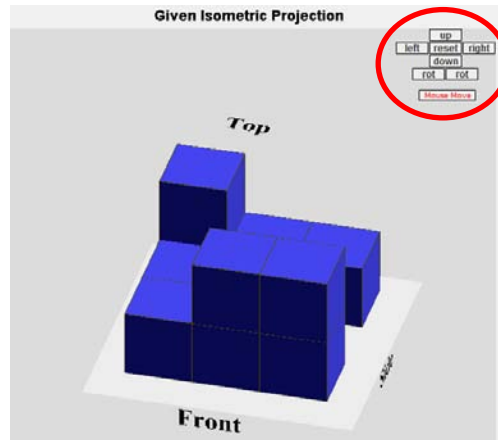
### **Level 6**

**Given:** Randomly Generated orthographic projection of squares in a  $5 \times 5$  space.

**Students:** Create isometric view of same object using cubes. Isometric will be able to be manipulated for a  $360^\circ$  view in all directions.

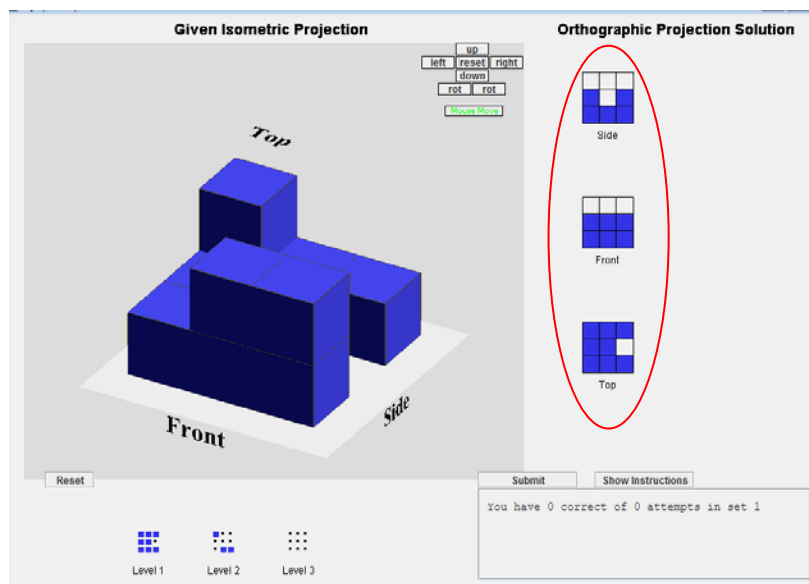
**APPENDIX B: CRIOSAT INSTRUCTIONS FOR LEVELS 1,3, & 5**

In this stage of the CRIOSAT treatment you will be given a three dimensional object. With this object you will have to create the front, top, and side views of the object. You may use the control buttons provided to move the object and see it from different sides. The controls are located to the upper right portion of the three dimensional object, as seen below.



The buttons labeled up, down, left and right will turn the object in the indicated direction. The buttons labeled “rot” will rotate the object. The button labeled “reset” will put the object back into starting position. The button labeled “mouse mouse” will allow you to click on the object and move it however you wish; if the “mouse move button is green then it is turned on, red means off.

The goal is to move the object however you need to in order to make the different views. Below is an example of a correct solution for a top, front, and side view.



You enter your answer by clicking on the empty boxes to the right of the three dimensional figure. One click will place a block in an empty space and a second click will make that block disappear. When you have finished your solution for all three views, click on submit. When you click submit, it will

show you **your solution**, the **correct solution** and **whether or not you got the item correct**. It will also show you what parts are correct and incorrect.

**Orthographic Projection Solution**

Side

Front

Top

Side

Front

Top

■ Correct  
■ Missing  
■ Extra

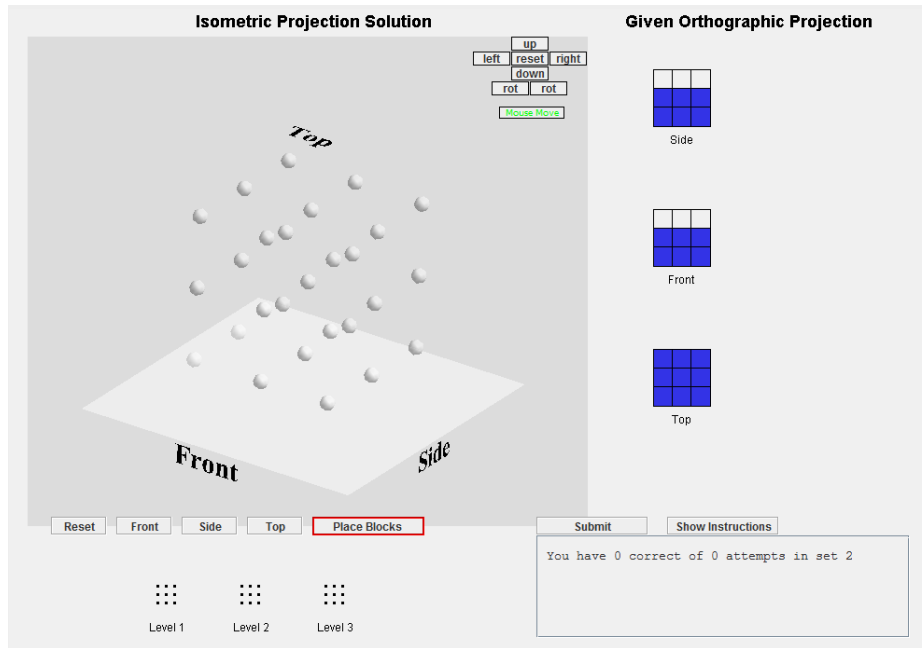
Incorrect.

When you see the correct solution, you may move the object still to see how you got the answer correct or incorrect. Then Click the “next question” button to continue. When you have completed enough items you will move to the next level.

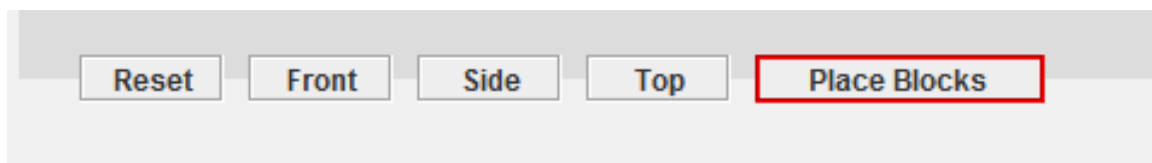
If you need help at any time, click on the “Show instructions” button.

## **APPENDIX C: CRIOSAT INSTRUCTIONS FOR LEVELS 2, 4, & 6**

In this stage of the treatment you will be asked to do the opposite of the last stage. You are given the front, top, and side views. You will have to create the three dimensional view. In the screen below you can see that the front, top, and side view that are given and the blank space provided for the three dimensional figure that you are to create.



The controls in the upper right work the same as in the last level. There are some new features though. In this level there are two ways to place the cubes where you want them. First look at the buttons along the bottom, which are labeled “reset”, “front”, “side”, “top”, and “place blocks”. These buttons are seen below.

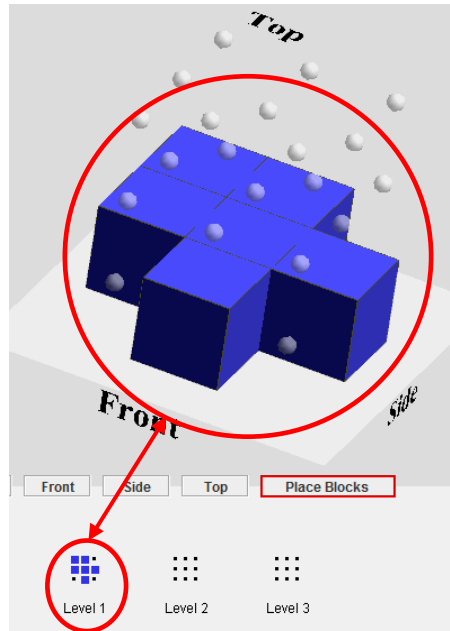


The reset button moves the three dimensional object to a starting position. The front button will show you what your three dimensional object looks like from the front. The side and top buttons shows your object from those angles. The button labeled place blocks, switches between place blocks and remove blocks. This button tells you whether you can place new cubes or remove ones that are already there.

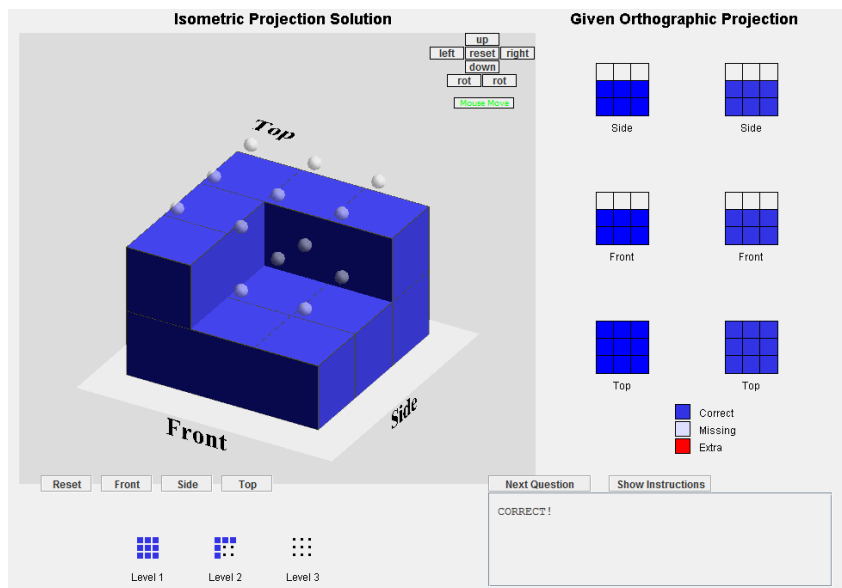
You can place cubes by clicking on the transparent spheres, as seen below. When you click on a sphere a cube will appear in its place.



You can also place cubes by clicking on the squares at the bottom of the screen. Each layer of the object is separated and labeled. By clicking on the square once a cube will appear. If you click on the square a second time the cube will be removed. You can tell that a cube has been placed because the square you click on will enlarge. View the image below to see how this works.



When your front, side, and top views look the same as the given views, click on the submit button. This will show you if you are correct and what you got correct or incorrect, as seen below.



When you have seen your answer and compare it to the correct answer, click the next question button. This will give you a new item. If you need to read these instructions again, click on the show instructions button.



**APPENDIX D: PSVT: ROT (PURDUE SPATIAL VISUALIZATION TEST OF ROTATIONS)**



Purdue  
Spatial Visualization  
Tests



Roland B. Guay

VISUALIZATION OF  
ROTATIONS

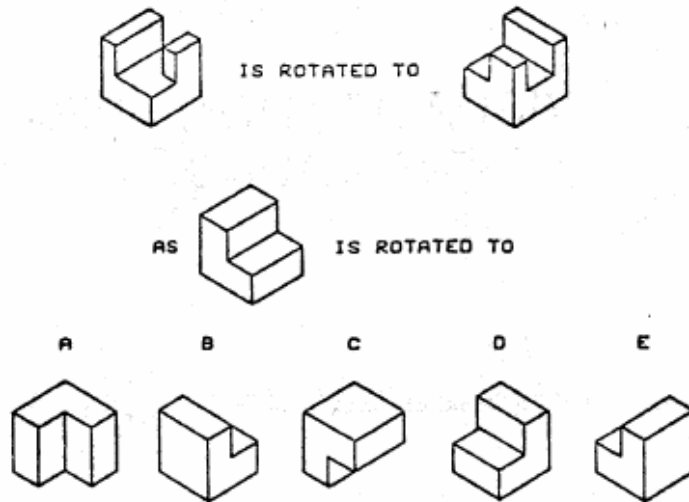
Do NOT open this booklet until you are instructed to do so.



Do NOT make any marks in this booklet.  
Mark your answers on the separate answer card.

### Directions

This test consists of 30 questions designed to see how well you can visualize the rotation of three-dimensional objects. Shown below is an example of the type of question included in the second section.



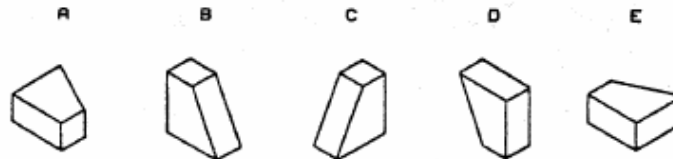
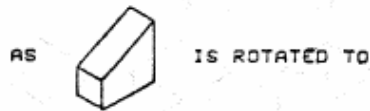
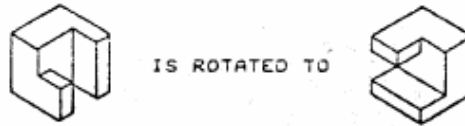
You are to:

1. study how the object in the top line of the question is rotated;
2. picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner;
3. select from among the five drawings (A, B, C, D, or E) given in the bottom line of the question the one that looks like the object rotated in the correct position.

What is the correct answer to the example shown above?

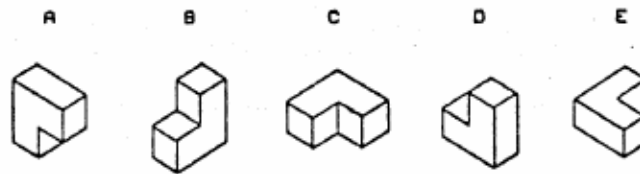
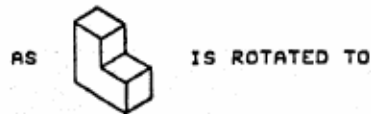
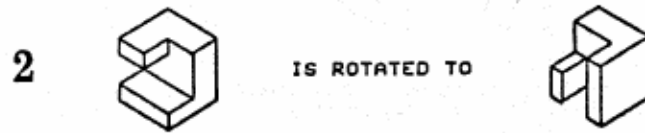
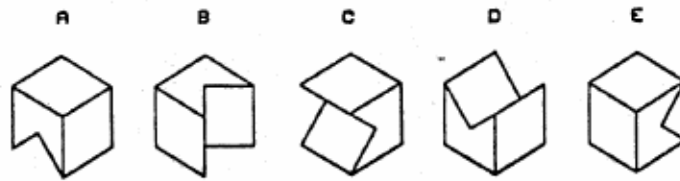
Answers A, B, C, and E are wrong. Only drawing D looks like the object rotated according to the given rotation. Remember that each question has only one correct answer.

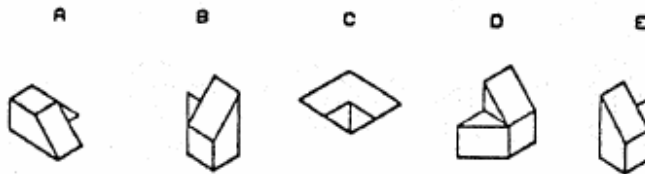
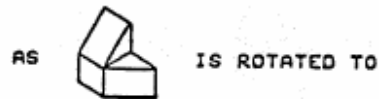
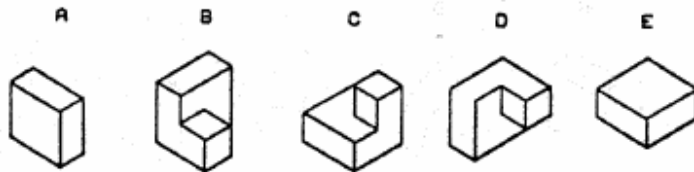
Now look at the next example shown below and try to select the drawing that looks like the object in the correct position when the given rotation is applied.

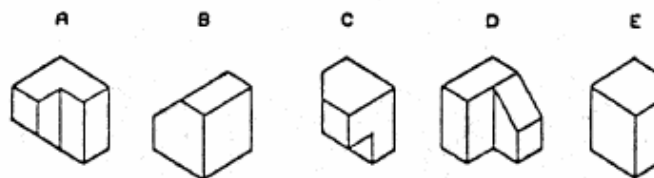
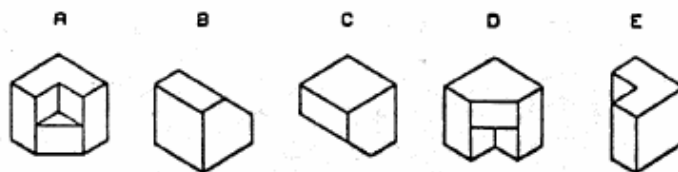
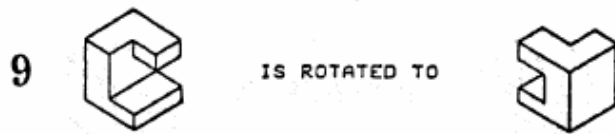


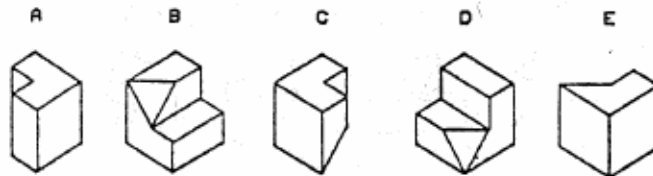
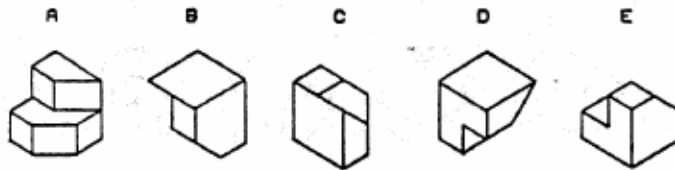
Notice that the given rotation in this example is more complex. The correct answer for this example is B.

Do NOT make any marks in this booklet.  
Mark your answers on the separate answer card.  
You will be told when to begin.

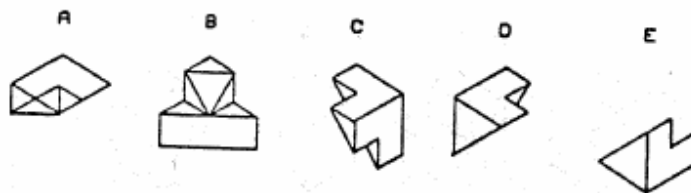
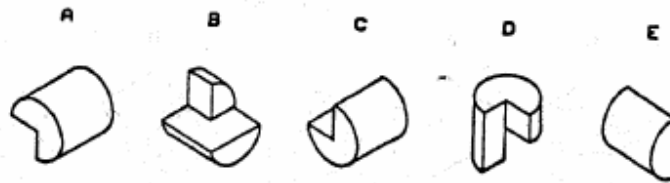


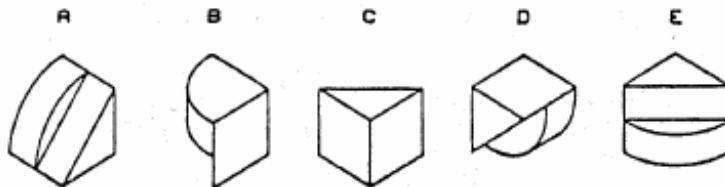
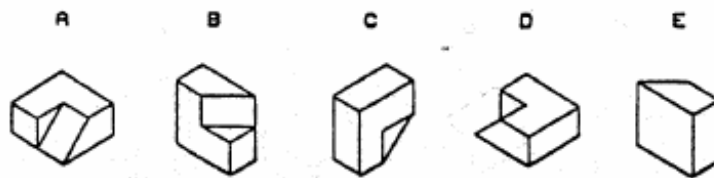


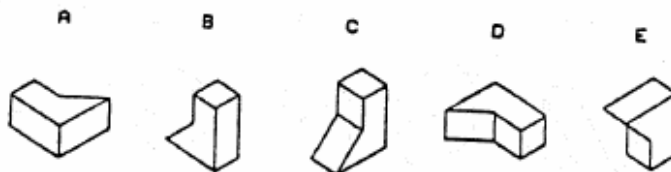
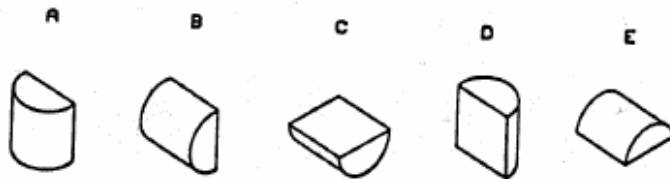


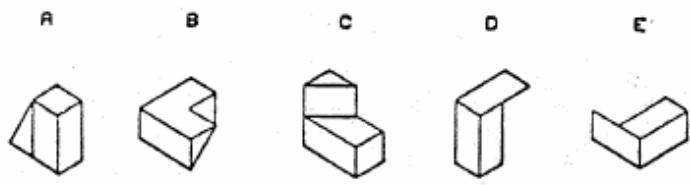
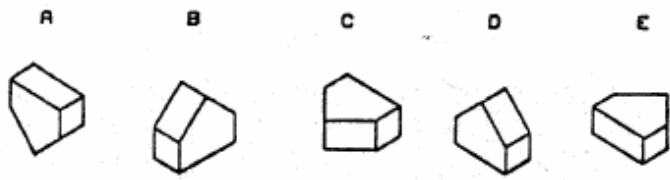
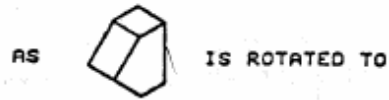


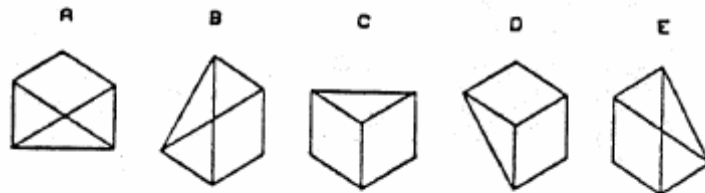
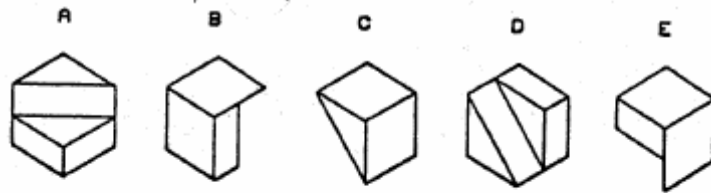


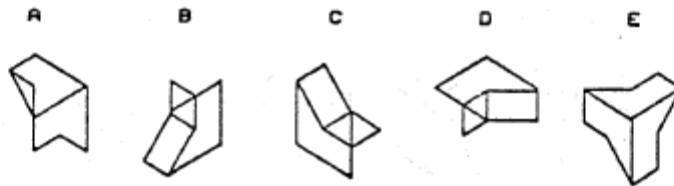
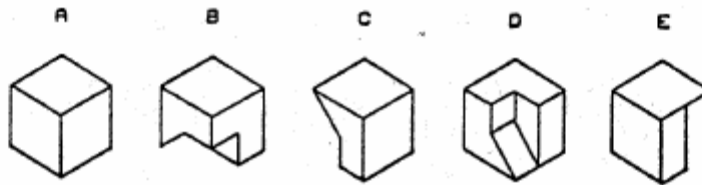


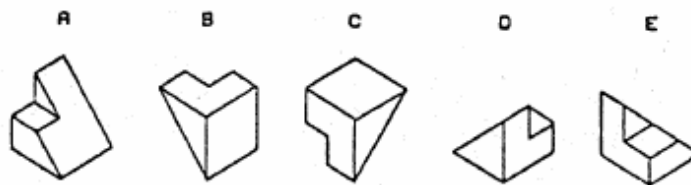
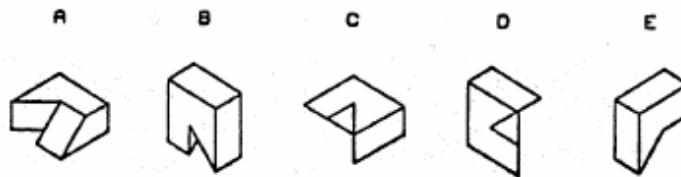


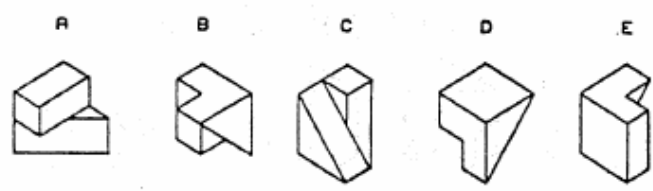
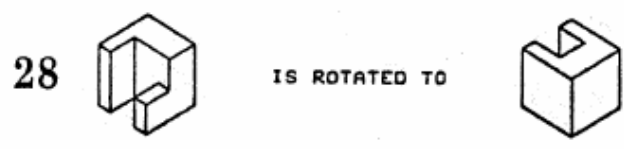
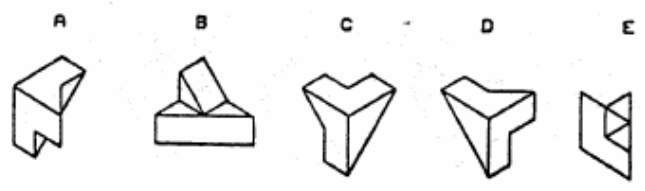
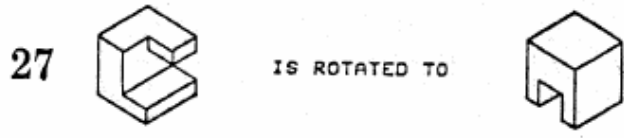




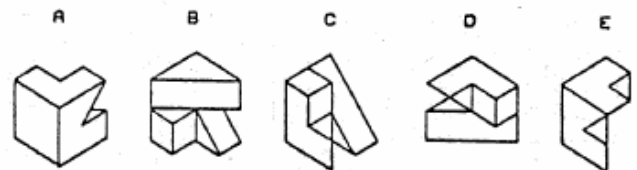
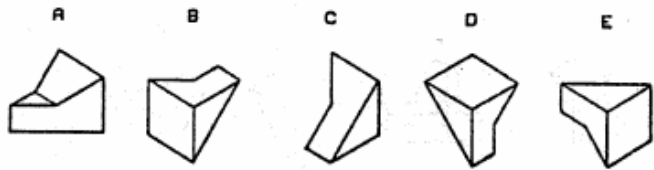












**APPENDIX E: PURDUE SPATIAL VIZUALIZATION TEST/ TEST OF  
ROTATION ANSWER KEY**

## Key

1. A
2. D
3. A
4. B
5. E
6. C
7. C
8. D
9. B
10. E
11. B
12. A
13. C
14. D
15. B
16. E
17. A
18. E
19. D
20. E
21. B
22. C
23. A
24. E
25. B
26. C
27. D
28. D
29. E
30. E

## **APPENDIX F: MATHEMATICAL PROCESSING INSTRUMENT**

## MATHEMATICAL PROCESSING TEST I

### IMPORTANT:

1. Do not write on this problem sheet. Write your solutions on the solution sheet provided.
2. For each problem, you are required to explain your working as much as you possibly can.
3. You are required to attempt all problems, including those you find difficult.

### **PROBLEM 1:**

John is taller than Mary. John is shorter than Jane. Who is the tallest?

### **PROBLEM 2:**

Two years ago Mary was 8 years old. How old will she be in five years from now?

### **PROBLEM 3:**

Two families held a party. Three members of the first family and five members of the second family attended the party. Each of the members of the first family shook hands with each of the members of the second family. How many handshakes were there altogether?

### **PROBLEM 4:**

On one side of a scale there is a 1-kg weight and half a brick. On the other side there is one full brick. The scale is balanced. How many kg does the brick weigh?

### **PROBLEM 5:**

Altogether there are 8 tables in a house. Some of them have four legs and the others have three legs. Altogether they have 27 legs. How many tables are there with four legs?

### **PROBLEM 6:**

One morning a boy walked from home to school. When he got half way, he realized that he had forgotten to bring one of his books. He then walked back to get it. When he finally arrived at school, he had walked 4 km altogether. What was the distance between his home and school?

### **PROBLEM 7:**

A girl had eleven plums. She decided to swap the plums for some apples. Her friend, who had the apples, said: 'For every 3 plums, I will give you an apple.' After the swap, how many apples and how many plums did the girl have?

### **PROBLEM 8:**

Tim was given 79 one-cent coins by his mother. At a shop he exchanged his one-cent coins for more valuable coins, so that now he got the smallest number of coins giving the value of 79 cents. How many fifty-cent coins, twenty-cent coins, ten-cent coins, five-cent coins, and two-cent coins did he get at the shop?

**PROBLEM 9:**

Only four teams took part in a football competition. Each team played against each of the other teams once. How many football matches were there in the competition?

**PROBLEM 10:**

If the time is 8 o'clock in the morning, what was the time 9 hours ago? (Make sure you include a.m. or p.m. as part of your answer.)

**PROBLEM 11:**

A mother is seven times as old as her daughter. The difference between their ages is 24 years. How old are they?

**PROBLEM 12:**

Three quarters of a vegetable garden is occupied by potatoes. The remaining part (4 hectares) is occupied by cabbages. What is the area of the whole garden, in hectares?

**PROBLEM 13:**

At each of the two ends of a straight path a man planted a tree, and then every 5 meters along the path (on one side only) he also planted another tree. The length of the path is 25 meters. How many trees were planted on the path altogether?

**PROBLEM 14:**

A balloon first rose 200 m from the ground, then moved 100 m to the east, then dropped 100 m. It then traveled 50 m to the east, and finally dropped straight to the ground. How far was the balloon from its starting point?

**PROBLEM 15:**

Donny's height was 150 cm. One day he swam in a swimming pool, and when he stood upright in the water there was 28 cm of his body which was above the surface. How deep (in cm) was the water at that time?

## MATHEMATICAL PROCESSING TEST II

### IMPORTANT:

1. Do not write on this problem sheet. Write your solutions on the solution sheet provided.
2. For each problem, you are required to explain your working as much as you possibly can.
3. You are required to attempt all problems, including those you find difficult.

### **PROBLEM 1:**

Dave has more money than Carol, and Mike has less money than Carol. Who has the most money?

### **PROBLEM 2:**

In an athletics race Johnny is 10 m ahead of Peter, Tom is 4 m ahead of Jim, and Jim is 3 m ahead of Peter. How many meters is Johnny ahead of Tom?

### **PROBLEM 3:**

A track for an athletics race is divided into three unequal sections. The length of the whole track is 450 meters. The length of the first and second sections combined is 350 m. The length of the second and third sections combined is 250 m. What is the length of each section?

### **PROBLEM 4:**

Jack, Chris, and Karen all have birthdays on the 1st of January, but Jack is 1 year older than Chris, and Chris is 3 years younger than Karen. If Karen is 10 years old, how old is Jack?

### **PROBLEM 5:**

One day John and Peter visit a library together. After that, John visits the library regularly every two days, at noon. Peter visits the library every three days, also at noon. If the library opens every day, how many days after the first visit will it be before they are, once again, in the library together?

### **PROBLEM 6:**

Two children were given some money by their father. The total amount of money was twelve dollars. The first child received twice as much as the second child. How much did each of them receive?

### **PROBLEM 7:**

One day a third of the potatoes in a storeroom were taken out of it. If 80 kg of potatoes were left in the storeroom, how many kg of potatoes were in the storeroom at first?

### **PROBLEM 8:**

Some sparrows are sitting in two trees, with each tree having the same number of sparrows. Two sparrows then fly from the first tree to the second tree. How many sparrows does the second tree then have more than the first tree?

### **PROBLEM 9:**

At first, the price of 1 kg of sugar was three times as much as the price of 1 kg of salt. Then the price of 1 kg of salt was increased by half its previous price, while the price of sugar was not changed. If the price of salt is now 30 cents per kilogram, what is the price of sugar per kg?

**PROBLEM 10:**

After a pedestrian travelled half of his journey, he still had to travel 4 km more to complete the journey. What was the length of his whole journey, in km?

**PROBLEM 11:**

Mr. Jones traded his horse for two cows. Next he traded the two cows, and for each cow, he got three pigs. Then, he traded the pigs, and for each pig, he got 3 sheep. Altogether, how many sheep did Mr. Jones get?

**PROBLEM 12:**

A saw in a sawmill saws long logs, each 16 m long, into short logs, each 2 m long. If each cut takes two minutes, how long will it take for the saw to produce eight short logs from one long log?

**PROBLEM 13:**

How many ways can 30 dollars be paid to a person if the money must be in 5-dollar and 2-dollar notes only, and the person must get some 5-dollar notes and some 2-dollar notes. (For each possible solution, summarize your answer by saying how many 2-dollar notes and how many 5-dollar notes the person would get.)

**PROBLEM 14:**

A tourist travelled some of his journey by plane, and the rest by bus. The distance that he travelled by bus was half the distance he travelled by plane. Determine the length of his entire trip if the distance that he travelled by plane was 150 km longer than the distance he travelled by bus.

**PROBLEM 15:**

A straight path is divided into two unequal sections. The length of the second section is half the length of the first section. What fraction of the whole path is the first section?



**MATHEMATICAL PROCESSING QUESTIONNAIRE I**

Name: ----- Male/Female

Date of birth: -----

School: -----

Form/Grade: -----

## **IMPORTANT:**

On this questionnaire you are asked to consider how you did the mathematical processing problems that you were recently asked to do. Every problem is accompanied by two or more possible solutions.

1. For every problem, you are required to indicate which solution, among all the solutions presented, is the one that you used, or is very similar to the one that you used, when you first attempted the problem.

It does not matter whether you got the right or wrong answer, or whether you completed the solution or not, as long as your method of solution is very similar to any of the solutions presented on this questionnaire, you are asked to tick the box which corresponds to that solution.

2. If for any of the problems you think that none of the solutions presented is the one that you used, or is very similar to the one that you used, you are asked to explain, in the space provided, the method that you used when you first attempted the problem. Explain your solution as clearly as you possibly can.

Even if you did not get the correct answer to the problem, you are still asked to state, in writing, your method in attempting the problem.

## MATHEMATICAL PROCESSING QUESTIONNAIRE I

### **PROBLEM 1:**

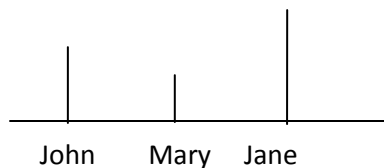
John is taller than Mary. John is shorter than Jane. Who is the tallest?

#### **Solution 1:**

To answer this question, I imagined a picture of the three children in my mind. From this picture I could 'see' that Jane is the tallest of the three.

#### **Solution 2:**

I drew a diagram representing the three children.



From the diagram it could be seen that Jane is the tallest of the three children.

#### **Solution 3:**

I found the answer to this question simply by drawing conclusions from the two statements in the questions.

The two statements are: 'John is taller than Mary' and 'John is shorter than Jane'.

The second statement can be changed into another statement with the same meaning:

'John is shorter than Jane' → 'Jane is taller than John' (because 'taller' is the opposite of shorter').

Therefore, the two statements become: 'John is taller than Mary' and 'Jane is taller than John'.

Or, if the order is reversed: 'Jane is taller than John' and 'John is taller than Mary'.

Conclusion: Jane is taller than Mary. Therefore, Jane is the tallest.

#### **Solution 4:**

I solved the problem by eliminating the shorter person in each statement in the problem.

First statement: 'John is taller than Mary'. In this statement 'Mary' is crossed out because Mary is the shorter person.

Second statement: 'John is shorter than Jane'. In this statement 'John' is crossed out because he is the shorter person.

Jane is the only person who is left. So she is the tallest person.

#### **I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 2:**

Two years ago Mary was 8 years old. How old will she be in five years from now?

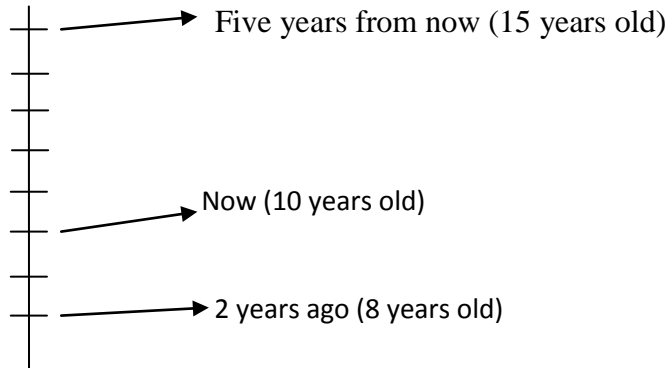
**Solution 1:**

I solved this problem in this way:

Two years ago, she was 8 years old. Now, she is 10 years old. Thus, five years from now she will be 15 years old.

**Solution 2:**

I solved this problem by drawing a diagram which represents Mary's age.



In the diagram it can be seen that five years from now Mary will be 15 years old.

**Solution 3:**

I used the same method as for Solution 2, only I drew the diagram 'in my head' (and not on paper).

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 3:**

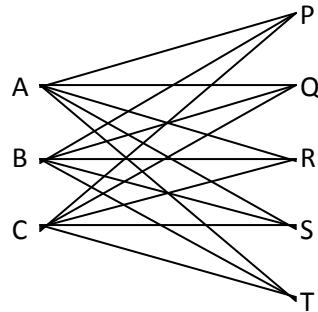
Two families held a party. Three members of the first family and five members of the second family attended the party. Each of the members of the first family shook hands with each of the members of the second family. How many handshakes were there altogether?

**Solution 1:**

I solve this problem by imagining all the handshakes and counting them in the mind. I found 15 handshakes altogether.

**Solution 2:**

I solve the problem by drawing a diagram of the handshakes and then counting them:



There were 15 handshakes altogether.

**Solution 3:**

I used a method like Solution 2, only I drew the picture 'in my mind' (and not on paper).

**Solution 4:**

I solved this problem by listing all the hand-shake pairs and then counting them.

First family:	Second family:	Hand-shake pairs:		
A	P	AP	BP	CP
B	Q	AQ	BQ	CQ
C	R	AR	BR	CR
	S	AS	BS	CS
	T	AT	BT	CT

I found 15 hand-shakes pairs. Thus there were 15 hand-shakes altogether.

**Solution 5:**

I solved the problem by using the following reasoning:

Each member of the first family shook hands five times with the member of the second family. Since there were 3 members in the first family, the number of handshakes altogether =  $3 \times 5$  handshakes = 15 handshakes.

**I did not use any of the above methods.**

I attempted the problem in this way:

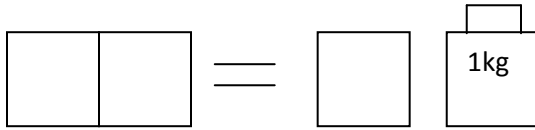
**PROBLEM 4:**

On one side of a scale there is a 1-kg weight and half a brick. On the other side there is one full brick. The scale is balanced. How many kg does the brick weigh?

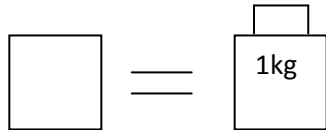


**Solution 1:**

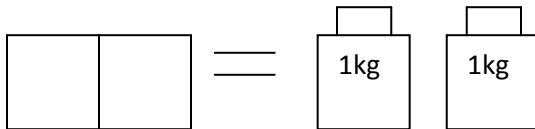
I solved the problem by drawing a diagram representing the objects.



Therefore



And



Thus the weight of one full brick is 2 kg.



**Solution 2:**

I solved this problem by using symbols and equations:

$$\begin{aligned} 1 \text{ full brick} &= 2 \text{ halves of a brick} \\ &= \square \square \\ \text{Thus } \square \square &= \square + 1 \text{ (1 = 1-kg weight)} \\ \square &= 1 \\ \square &= 2 \end{aligned}$$

Thus the weight of one full brick = 2 kg.



**Solution 3:**

In order to solve this problem, I imagined the scale and the objects on the two sides of it (half a brick, one 1-kg weight, one full brick).

The scale is balanced; this means that the weight of half of a brick plus one 1-kg weight equals the weight of one full brick. As one full brick equals two halves of a brick, it also means that the weight of one half of a brick equals 1-kg weight. Therefore, the weight of one full brick = 1 kg + 1 kg = 2 kg.



**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 5:**

Altogether there are 8 tables in a house. Some of them have four legs and the others have three legs. Altogether they have 27 legs. How many tables are there with four legs?

**Solution 1:**

I solved the problem by trial and error:

If the number of tables with four legs were ...	Then, the number of tables with three legs would be ...	So the total number of legs would be ...
1	7	25 (NO)
2	6	26 (NO)
3	5	27 (YES)

Thus there are three tables which have four legs (and five tables with three legs).

**Solution 2:**

I solved this problem using symbols and equations:

Suppose the number of tables with 4 legs =

Then the number of legs altogether =  $(4 \cdot \text{} ) + 3(8 - \text{} )$

This is equal to 27; thus

$$4 \cdot \text{} + 3(8 - \text{) = 27$$

$$4 \cdot \text{} + 24 - 3 \text{} = 27$$

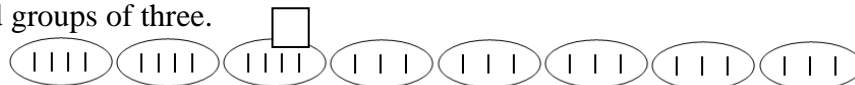
$$\text{} + 24 = 27$$

$$\text{} = 3$$

Thus the number of tables with 4 legs = 3 (and the number of tables with 3 legs = 5).

**Solution 3:**

To solve this problem I drew a picture of the tables' legs, and then grouped them into groups of four and groups of three.



A group of four legs represents a table with four legs. A group of three legs represents a table with three legs. From the picture it can be seen that there are 3 groups of legs with four legs each, and 5 groups of legs with three legs each. Thus there are 3 tables with four legs (and 5 tables with three legs).

**Solution 4:**

I solved this problem by drawing the tables. First, I drew them as if all tables had three legs only, then I kept on adding a leg to tables until the total number of legs reached 27.



I found there are 3 tables with four legs (and 5 tables with three legs).

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 6:**

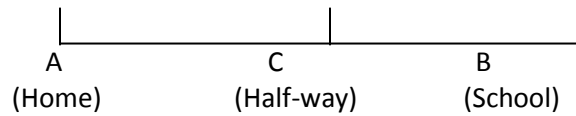
One morning a boy walked from home to school. When he got half way, he realized that he had forgotten to bring one of his books. He then walked back to get it. When he finally arrived at school, he had walked 4 km altogether. What was the distance between his home and school?

**Solution 1:**

To solve this problem, I imagined the route travelled by the boy that morning. When he finally arrived at school, he had walked twice the distance between home and school. This was equal to 4 km, so the distance between home and school was 2 km.

**Solution 2:**

I drew a diagram representing the route between his home and school.



The distance covered by the boy was AC, then CA, then AB. This means that when he finally arrived at B (school) he had walked twice the distance between his home and school. This was 4 km, so the distance between his home and school was 2 km.

**Solution 3:**

I solved this problem by using symbols and equations.

Suppose the distance between home and school =

Then half the distance =  $1/2 \cdot$

The total distance travelled that morning

$$= 1/2 \cdot \text{} + 1/2 \cdot \text{} + \text{}$$

$$= 2 \cdot \text{}$$

This was equal to 4 km. Thus  = 2 km, which was the distance between his home and school.

**I did not use any of the above methods.**

I attempted the problem in this way:



**PROBLEM 7:**

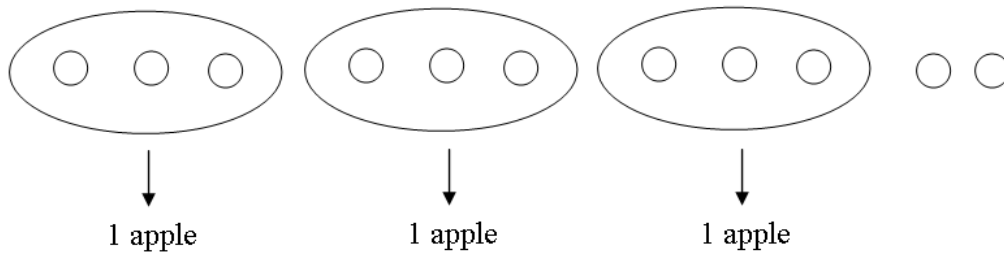
A girl had eleven plums. She decided to swap the plums for some apples. Her friend, who had the apples, said: 'For every 3 plums, I will give you an apple.' After the swap, how many apples and how many plums did the girl have?

**Solution 1:**

The number 11 can be separated into 3, 3, 3, 2. Every three plums were swapped for an apple; so after the swap she had 3 apples and 2 plums.

**Solution 2:**

I solved this problem by drawing the plums, and then separating the plums into groups containing 3 plums each:



In the picture it can be seen that after the swap, the girl had 3 apples and 2 plums.

**Solution 3:**

I solved this problem by imagining the plums and the swap. I could 'see' in my mind that after the swap the girl had 3 apples and 2 plums.

**Solution 4:**

I solved this problem in this way:

11 divided by 3 gives 3, remainder 2. Thus after the swap, the girl had 3 apples and 2 plums.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 8:**

Tim was given 79 one-cent coins by his mother. At a shop he exchanged his one-cent coins for more valuable coins, so that now he got the smallest number of coins giving the value of 79 cents. How many fifty-cent coins, twenty-cent coins, ten-cent coins, five-cent coins, and two-cent coins did he get at the shop?

**Solution 1:**

I solved this problem by separating the number 79 into 50's, 20's, 10's etc, so far as this is possible.

$$\begin{aligned} \text{Thus, } 79 &= 50 + 29 \\ &= 50 + 29 + 9 \\ &= 50 + 29 + 5 + 4 \\ &= 50 + 29 + 5 + 2 + 2 \end{aligned}$$

Thus, 79 one-cent coins can be exchanged for one 50-cent coin, one 20-cent coin, one 5-cent coin, and two 2-cent coins.

**Solution 2:**

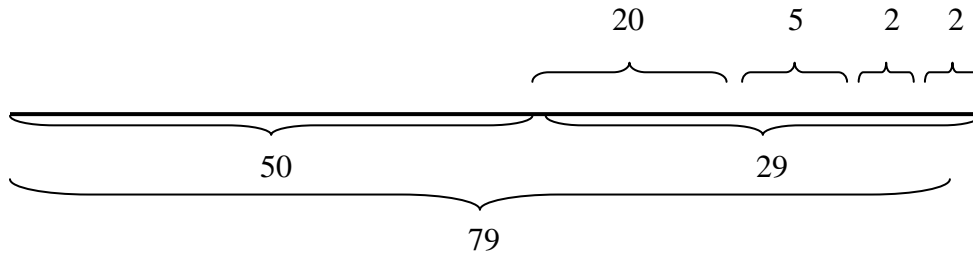
I solved this problem by imagining the 79 one-cent coins, and then trying to 'arrange' those coins into several groups each containing 50 one-cent coins, 20 one-cent coins, etc. I found that those coins can be arranged into:

One group containing 50 one-cent coins, one group containing 20 one-cent coins, one group containing 5 one-cent coins, and 2 groups containing 2 one-cent coins each.

Thus, 79 one-cent coins can be exchanged for one 50-cent coin, one 20-cent coin, one 5-cent coin, and two 2-cent coins.

**Solution 3:**

I solved this problem by drawing a line which represents the money that Tim got from his mother. Then, I divided the line into sections, one of which containing 50 units, another 20 units, another 5 units, and two others each 2 units.



Each section represents a coin. Thus 79 one-cent coins can be exchanged for one 50-cent coin, one 20-cent coin, one 5-cent coin, and two 2-cent coins.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 9:**

Only four teams took part in a football competition. Each team played against each of the other teams once. How many football matches were there in the competition?

**Solution 1:**

I solved this problem by using the following reasoning:

Each team played against each of the three other teams once. As there were 4 teams, there would be  $4 \times 3$  matches, or 12 matches, altogether.

But in that way each match had been counted twice.

So the correct answer was =  $12 \text{ matches} / 2$   
= 6 matches.

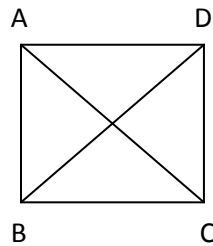
**Solution 2:**

I solved this problem by listing all the match pairs and then counting them.

The teams were A, B, C, D. The match pairs were AB, AC, AD, BC, BD, and CD. There were 6 matches.

**Solution 3:**

I solved this problem by drawing a diagram representing the matches and then counting the matches as shown in the diagram.



There were 6 matches altogether.

**Solution 4:**

I did this problem like the method in solution 3, but I drew the pictures 'in my head' (and not on paper).

**Solution 5:**

I solved this problem by using the following reasoning:

As there were 4 teams, in each round there could be only 2 matches. Altogether there were 3 rounds since each team had to play each of the three other teams once — this could be done in 3 rounds.

Therefore there were  $3 \times 2$  matches, or 6 matches, altogether.

**I did not use any of the above methods.**

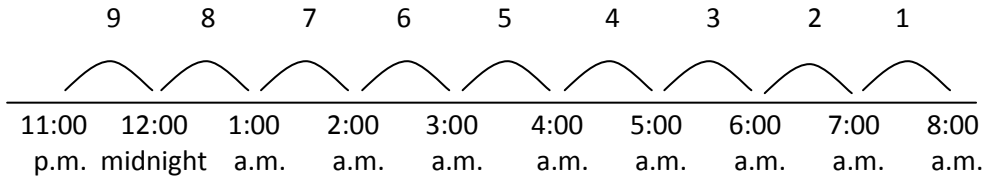
I attempted the problem in this way:

**PROBLEM 10:**

If the time is 8 O'clock in the morning, what was the time 9 hours ago? (Make sure you include a.m. or p.m. as part of your answer.)

**Solution 1:**

I solved this problem by drawing a line representing the time. In the diagram it can be seen that if the time now is 8 o'clock in the morning, 9 hours ago it was 11:00 o'clock at night; that is 11:00 p.m.



**Solution 2:**

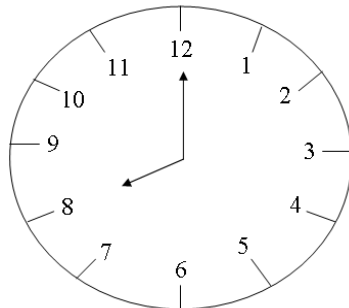
I used the same solution as for Solution 1, only I drew the diagram 'in my head' (and not on paper).

**Solution 3:**

I did not imagine any picture, but I solved this problem by merely 'counting back' 9 hours from 8:00 a.m.

**Solution 4:**

I solved this problem by drawing a clock face (or by looking at my own watch, or a clock).



Using this clock face I could work out the time 9 hours ago, which was 11:00 p.m.

**Solution 5:**

I used the same method as for Solution 4, only I drew the clock face 'in my mind', and did not look at, or draw a clock face.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 11:**

A mother is seven times as old as her daughter. The difference between their ages is 24 years. How old are they?

**Solution 1:**

I solved this problem merely by trial and error:

Daughter's age:	Mother's age:	
2 years	26 years	No
3 years	27 years	No
4 years	28 years	Yes

Thus daughter's age = 4 years, and mother's age = 28 years.

**Solution 2:**

I solved the problem in this way:

Suppose daughter's age =  years.

Thus, mother's age =        years.

The difference between their ages =       years.  
= 24 years

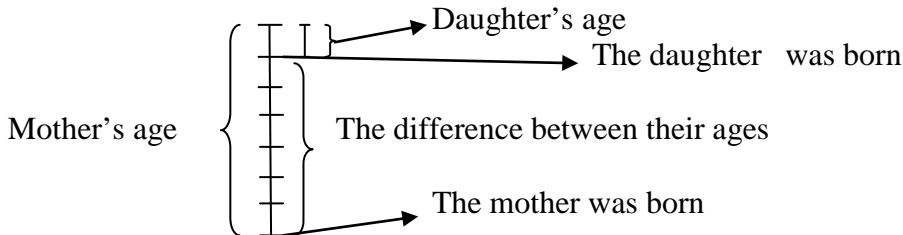
Thus = 4.

So, daughter's age = 4 years.

Mother's age = (4 + 4 + 4 + 4 + 4 + 4 + 4) years.  
= 28 years.

**Solution 3:**

I solved the problem by drawing a diagram representing their ages:



In the diagram it can be seen that the difference between their ages is represented by a line segment which consists of 6 equal parts. This difference = 24 years. Thus, each part represents 4 years. Daughter's age is represented by a line segment which consists of one part only. This means that daughter's age = 4 years. Mother's age is represented by a line segment which consists of 7 parts. Thus mother's age = 28 years.

**Solution 4:**

I used the same method as for Solution 3, only I drew the diagram 'in my head' (and not on paper).

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 12:**

Three quarters of a vegetable garden is occupied by potatoes. The remaining part (4 hectares) is occupied by cabbages. What is the area of the whole garden, in hectares?

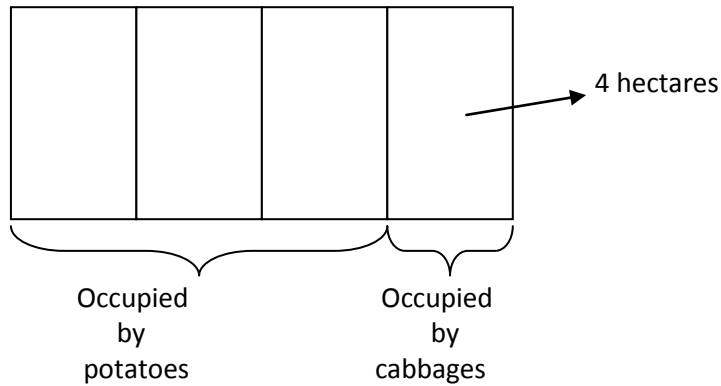


**Solution 1:**

I solved the problem in this way:

The part occupied by cabbages =  $1 - 3/4 = 1/4$  of the whole garden. This is 4 hectares.

Therefore, the area of the whole garden is  $4 \times 4$  hectares, i.e. 16 hectares.



In the picture it is clear that the area of the whole garden =  $(4 + 4 + 4 + 4)$  hectares.  
= 16 hectares.



**Solution 2:**

I used the same method as for Solution 2, only I drew the diagram 'in my head' (and not on paper).



**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 13:**

At each of the two ends of a straight path a man planted a tree, and then every 5 m along the path (on one side only) he also planted another tree. The length of the path is 25 m. How many trees were planted on the path altogether?

**Solution 1:**

I solved the problem in this way:

Every 5 m along the path a tree was planted. This means that the path was divided into  $25/5 = 5$  equal parts. Every part corresponded to one tree, but at one of the two ends of the path, the part corresponded to two trees. Therefore the number of trees was:

$$\begin{aligned} &= (4 \times 1) + (1 \times 2) \\ &= 4 + 2 \\ &= 6 \end{aligned}$$

**Solution 2:**

I solved the problem by imagining the path and the trees, and then counting the trees in the mind. I found there were 6 trees on the path.

**Solution 3:**

I solved the problem by drawing a diagram representing the path and the trees, and then counting the trees.



I found 6 trees.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 14:**

A balloon first rose 200 m from the ground, then moved 100 m to the east, then dropped 100 m. It then traveled 50 m to the east, and finally dropped straight to the ground. How far was the balloon from its starting point?



**Solution 1:**

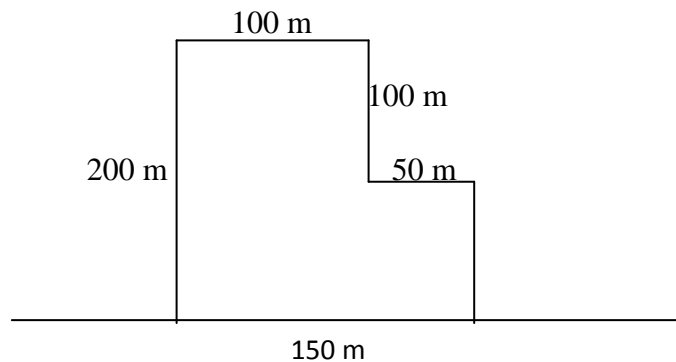
To solve this problem, I imagined the path taken by the balloon, and then worked out the distance between the starting and the finishing places, I found the distance was:

$$= 100 \text{ m} + 50 \text{ m} = 150 \text{ m}$$



**Solution 2:**

To solve this problem, I drew a diagram representing the path taken by the balloon, and then worked out the distance between the starting and finishing places.



The distance was =  $100 + 50 = 150\text{m}$ .



**Solution 3:**

In order to solve this problem, I noticed only the information in the problem which was important for the solution. That is, I only noticed: 'moved 100 to the east', and 'then travelled 50 m to the east again'. Therefore, the distance between the starting and the finishing places was  $100 \text{ m} + 50 \text{ m} = 150 \text{ m}$ .

(I did not draw or imagine any picture at all).



**I did not use any of the above methods.**

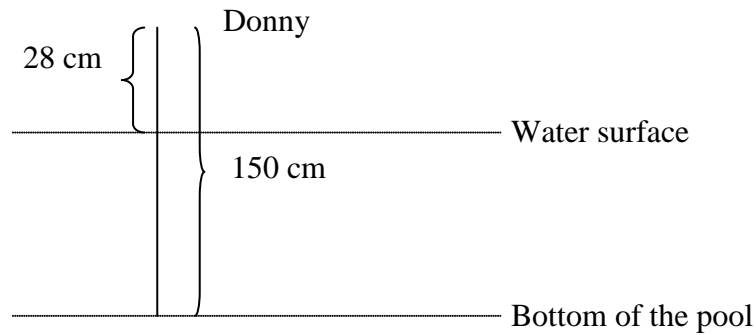
I attempted the problem in this way:



**PROBLEM 15:**

Donny's height was 150 cm. One day he swam in a swimming pool, and when he stood upright in the water there was 28 cm of his body which was above the surface. How deep (in cm) was the water at that time?

**Solution 1:**



I found the depth of the water at that time = 122 cm.

**Solution 2:**

To solve this problem, I imagined Donny and the water. I could 'see' in my mind that part of Donny's body which was below the surface was 122 cm. Thus, the depth of the water at that time was 122 cm.

**Solution 3:**

To solve this problem, I only noticed the information in the problem which was important for the solution. That is, I only noticed:

'Donny's height was 150 cm' and 'there was 28 cm of his body which was above the surface'. From this information I could conclude that part of Donny's body which was below the surface was =  $150 \text{ cm} - 28 \text{ cm}$   
= 122 cm.

Thus the depth of the water at that time was 122 cm.

(I did not draw or imagine any picture at all).

**I did not use any of the above methods.**

I attempted the problem in this way:

**MATHEMATICAL PROCESSING QUESTIONNAIRE II**

Name: ----- Male/Female

Date of birth: -----

School: -----

Form/Grade: -----

## **IMPORTANT:**

On this questionnaire you are asked to consider how you did the mathematical processing problems that you were recently asked to do. Every problem is accompanied by two or more possible solutions.

1. For every problem, you are required to indicate which solution, among all the solutions presented, is the one that you used, or is very similar to the one that you used, when you first attempted the problem.

It does not matter whether you got the right or wrong answer, or whether you completed the solution or not, as long as your method of solution is very similar to any of the solutions presented on this questionnaire, you are asked to tick the box which corresponds to that solution.

2. If for any of the problems you think that none of the solutions presented is the one that you used, or is very similar to the one that you used, you are asked to explain, in the space provided, the method that you used when you first attempted the problem. Explain your solution as clearly as you possibly can.

Even if you did not get the correct answer to the problem, you are still asked to state, in writing, your method in attempting the problem.

## MATHEMATICAL PROCESSING QUESTIONNAIRE II

### **PROBLEM 1:**

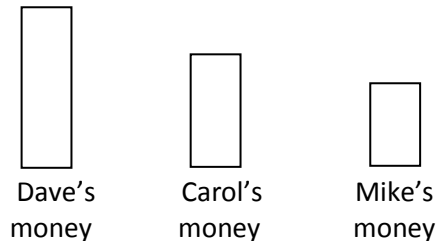
Dave has more money than Carol, and Mike has less money than Carol. Who has the most money?

#### **Solution 1:**

I solved this problem by imagining each person's money. I could 'see' in my mind that Dave had the most money.

#### **Solution 2:**

I solved this problem by drawing a diagram representing the money.



From the diagram it could be seen that Dave has the most money.

#### **Solution 3:**

I solved this problem by using examples. Suppose Dave has 35 dollars; Carol has 25 dollars (as Dave has more money than Carol); and Mike has 20 dollars (as he has less money than Carol).

From these examples it can be seen that Dave has the most money.

#### **Solution 4:**

I solved this problem by eliminating, in each statement in the problem, the person who has less money (as we only want the person who has more money in each statement). That is:

'Mike has less money than Carol' means that 'Mike' is crossed out since Mike is the person who has less money.

'Dave has more money than Carol' means that 'Carol' is crossed out since Carol is the person who has less money.

The only person who is left is Dave. So He has the most money.

#### **Solution 5:**

I solved this problem merely by drawing conclusions from the sentences in the problem.

'Dave has more money than Carol' → 'Dave has more money than Carol'

'Mike has less money than Carol' → 'Carol has more money than Mike' (Since the opposite of 'less' is 'more').

Conclusion: Dave has more money than Mike. Thus, Dave has the most money.

#### **I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 2:**

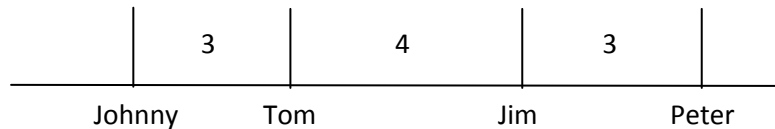
In an athletics race Johnny is 10 m ahead of Peter, Tom is 4 m ahead of Jim, and Jim is 3 m ahead of Peter. How many meters is Johnny ahead of Tom?

**Solution 1:**

To solve this problem, I imagined the four people in my mind, and then worked out the distance between Johnny and Tom. I found the distance is 3 m. So Johnny is 3 m ahead of Tom.

**Solution 2:**

I solved this problem by drawing a diagram representing the four people, and then working out the distance between Johnny and Tom.



I found the distance between Johnny and Tom is 3 m. So Johnny is 3 m ahead of Tom.

**Solution 3:**

I solved this problem merely by drawing conclusions from the sentences in the problem:

'Tom is 4m ahead of Jim'  
'Jim is 3m ahead of Peter'

Conclusion: Tom is 7m ahead of Peter.

'Johnny is 10 m ahead of Peter'  
'Tom is 7 m ahead of Peter'

Conclusion: Johnny is 3 m ahead of Tom.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 3:**

A track for an athletics race is divided into three unequal sections. The length of the whole track is 450 meters. The length of the first and second sections combined is 350 m. The length of the second and third sections combined is 250 m. What is the length of each section?



**Solution 1:**

I solved this problem by imagining the track for the race and then working out the length of each section.

The length of the first and the second sections combined is 350 m, so the length of the third section must be 100 m (since the length of the whole track is 450 m).

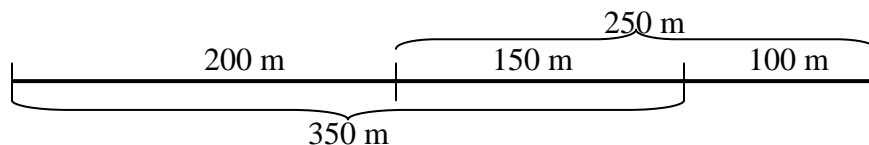
The length of the second and the third sections combined is 250 m, so the the length of the first section must be 200 m.

Since the length of the first section is 200 m, and the length of the third section is 100 m, the length of the second section is 150 m.



**Solution 2:**

To solve this problem, I drew a diagram which represents the track and then worked out the length of each section.



The length of the first section is 200 m, the section section is 150 m, and the third section is 100 m.



**Solution 3:**

To solve this problem I drew conclusions from the information in the problem only, and did not imagine or draw any picture at all.

That is:

A track is divided into 3 unequal sections.

The length of whole track is 450 m.

The length of the first and the second sections combined is 350 m.

Conclusion: The length of third track =  $450 - 350 = 100$  m .

The length of the second and the third sections combined is 250 m.

Conclusion: The length of first section =  $450 - 250 = 200$  m.

And the length of the second section =  $450 - 200 - 100 = 150$  m.



**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 4:**

Jack, Chris, and Karen all have birthdays on the 1st of January, but Jack is 1 year older than Chris, and Chris is 3 years younger than Karen. If Karen is 10 years old, how old is Jack?

**Solution 1:**

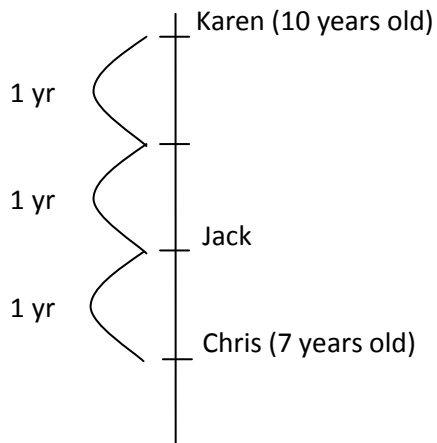
I solved the problem in this way:

Chris is 3 years younger than Karen. Karen is 10 years old.  
Therefore, Chris is 7 years old.

Jack is one year older than Chris.  
Therefore, Jack is 8 years old.

**Solution 2:**

I solved this problem by drawing a diagram that represents their ages:



From the diagram it can be seen that Jack is 8 years old.

**Solution 3:**

I used the same method as for Solution 2, only I drew the line 'in my head', and not on paper.

**I did not use any of the above methods.**

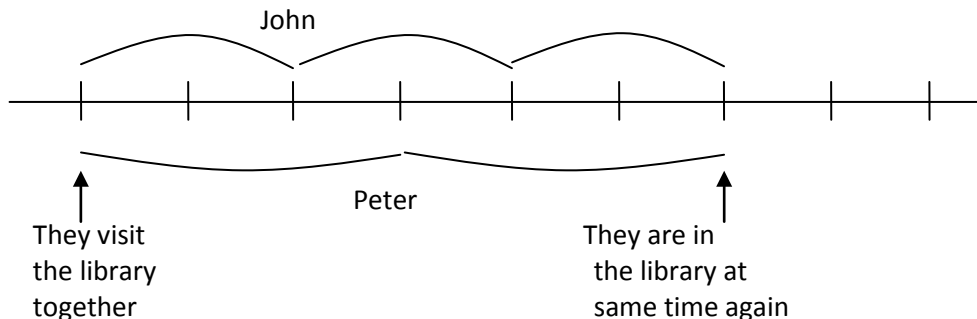
I attempted the problem in this way:

**PROBLEM 5:**

One day John and Peter visit a library together. After that, John visits the library regularly every two days, at noon. Peter visits the library every three days, also at noon. If the library opens every day, how many days after the first visit will it be before they are, once again, in the library together?

**Solution 1:**

I solved this problem by drawing a diagram representing the days after they first visit the library.



From the diagram it can be seen that, once again, they will be in the library together six days after the first visit.

**Solution 2:**

I used the same method as for Solution 1 only I drew the diagram 'in my head' (and not on paper).

**Solution 3:**

I solved this problem by using examples. Suppose they first visit the library together on Monday. Then after that, John will visit the library on Wednesday, Friday, Sunday, Tuesday, etc., and Peter will visit the library on Thursday, Sunday, Wednesday, etc. This means that on Sunday they will be in the library at the same time again. From Monday to Sunday there are 6 days. This means that, once again, they will be in the library together six days after the first visit.

**Solution 4:**

I solved this problem by saying in my mind that after the first day, John will visit the library on the third day, the fifth day, the seventh day, etc.; and Peter, after the first day, will visit the library again on the fourth day, the seventh day, etc. So on the seventh day they will be in the library at the same time again. From the first day to the seventh day there are 6 days. So, once again, they will be in the library together six days after the first visit.

**I did not use any of the above methods.**

I attempted the problem in this way:



**PROBLEM 6:**

Two children were given some money by their father. The total amount of money was twelve dollars. The first child received twice as much as the second child. How much did each of them receive?

**Solution 1:**

I solved this problem by trial and error:

First child:	Second child:	
\$6	\$6	No
\$7	\$5	No
\$8	\$4	Yes

So the first child received \$8, and the second child \$4.

**Solution 2:**

I solved this problem by using equations.

Suppose the second child's share =

Thus, the first child's share =

and  +  = 12

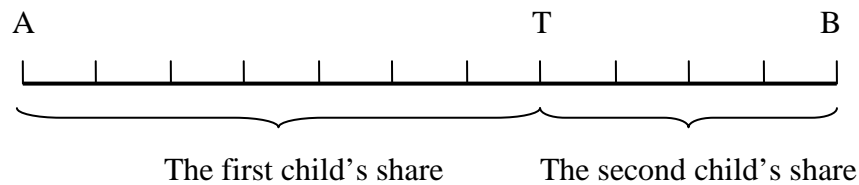
= 12

= 4

Thus, the second child received \$4, and the first child received \$4 + \$4 = \$8.

**Solution 3:**

I solved the problem by drawing a diagram which represented the money.



Then I determined the point T so that the length of AT = twice the length of TB. In the diagram it can be seen that the first child received \$8, and the second child \$4.

**Solution 4:**

I used the same method as for Solution 3, only I drew the diagram 'in my mind' (and not on paper).

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 7:**

One day a third of the potatoes in a storeroom were taken out of it. If 80 kg of potatoes were left in the storeroom, how many kg of potatoes were in the storeroom at first?

**Solution 1:**

I solved the problem in this way:

One third of the potatoes were taken out, so two-thirds of the potatoes were left in the storeroom. This means that the amount of potatoes left was twice the amount taken out. It was given that the amount left was 80 kilograms, so the amount taken was 40 kg. Thus the amount of all the potatoes in the storeroom at first was  
 $= 80 \text{ kg} + 40 \text{ kg} = 120 \text{ kg}.$

**Solution 2:**

I solved the problem using symbols and equations.

Suppose the amount of potatoes at first was  kg.

The amount taken out  $= 1/3 \cdot \square$

The amount left  $= 2/3 \cdot \square$

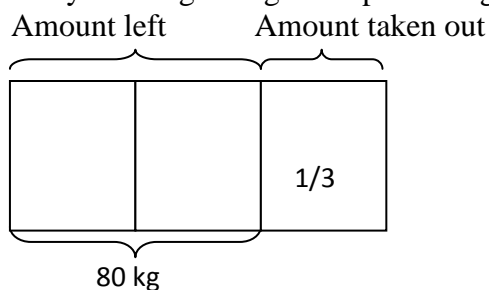
Thus,  $2/3 \cdot \square = 80$

and  $\square = 120$

Thus, the amount of potatoes at first = 120 kg.

**Solution 3:**

I solved this problem by drawing a diagram representing the potatoes:



From the diagram it can be seen that the amount of all the potatoes at first was  
 $= 80 \text{ kg} + 40 \text{ kg} = 120 \text{ kg}.$

**Solution 4:**

I used the same method as for Solution 3, only I drew the diagram 'in my mind' (and not on paper).

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 8:**

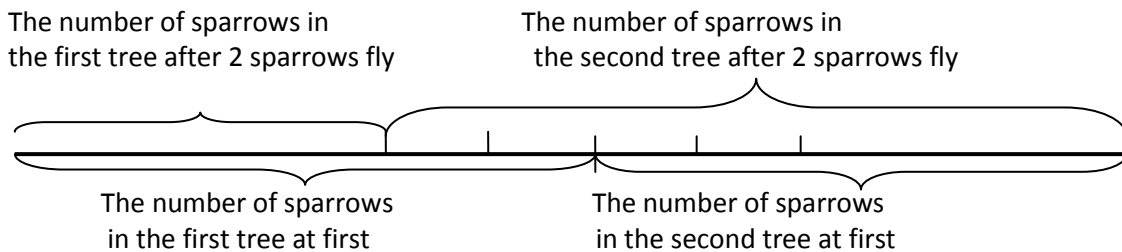
Some sparrows are sitting in two trees, with each tree having the same number of sparrows. Two sparrows then fly from the first tree to the second tree. How many sparrows does the second tree then have more than the first tree?

**Solution 1:**

To solve this problem, I used the following reasoning: First there is the same number of sparrows in each tree. Then 2 sparrows fly from the first tree to the second. This means that now the number of sparrows in the first tree is two less than the number before, while the number of sparrows in the second tree is two more than the number before. This means that now the second tree has 4 more sparrows than the first.

**Solution 2:**

In order to solve this problem, I drew a diagram representing the number of sparrows in the two trees:



From the diagram it can be seen that now the second tree has 4 more sparrows than the first.

**Solution 3:**

I used the same method as for Solution 2, only I drew the diagram 'in my head' (and not on paper).

**Solution 4:**

I solved this problem by using examples.

Suppose at first there are 8 sparrows in each tree. After the 2 sparrows fly from the first tree to the second, the number of sparrows in the first tree becomes 6, and the number of sparrows in the second tree becomes 10. So now the second tree has 4 more sparrows than the first.

**Solution 5:**

I solved this problem by using symbols.

Suppose the number of sparrows in each tree at first = . Then 2 sparrows fly from the first tree to the second.

Thus the number of sparrows in the first tree is now ( - 2), and in the second tree ( + 2).

$$\begin{aligned} \text{The difference in the number of sparrows now is } & (\text{input} + 2) - (\text{input} - 2). \\ & = \text{input} + 2 - \text{input} + 2 = 4 \end{aligned}$$

**I did not use any of the above methods.**

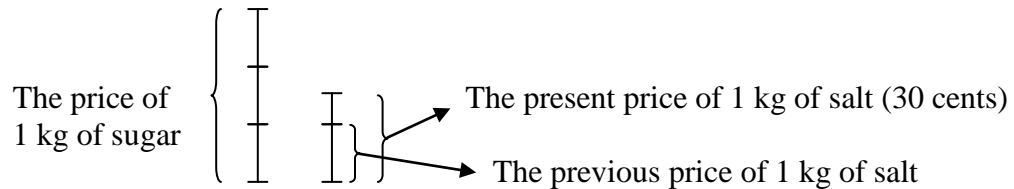
I attempted the problem in this way:

**PROBLEM 9:**

At first, the price of 1 kg of sugar was three times as much as the price of 1 kg of salt. Then the price of 1 kg of salt was increased by half its previous price, while the price of sugar was not changed. If the price of salt is now 30 cents per kilogram, what is the price of sugar per kg?

**Solution 1:**

I solved this problem by drawing a diagram which represents the prices of the sugar and the salt:



In the diagram it can be seen that after the price of 1 kg of salt was increased, the price of 1 kg of sugar was twice the price of 1 kg of salt.

As now the price of 1 kg of salt is 30 cents, and the price of 1 kg of sugar is 60 cents.

**Solution 2:**

I used the same method as for Solution 1, only I drew the diagram 'in my head' (and not on paper).

**Solution 3:**

I solved the problem in this way.

The price of 1 kg of salt is now 30 cents. This is 1 and 1/2 times the previous price. Thus the previous price was 20 cents per kg. This means that the price of sugar is  $3 \times 20$  cents, or 60 cents, per kg.

**Solution 4:**

I solved the problem using symbols and equations.

Suppose the price of 1 kg of salt previously =  cents.

Thus the price of 1 kg of sugar =  $3 \cdot$   cents.

Now, after the increase, the price of 1 kg of salt =  $1 \frac{1}{2} \cdot$   cents.

This means that the price of 1 kg of sugar is twice the present price of 1 kg of salt. As the price of 1 kg of salt now = 30 cents, the price of 1 kg of sugar =  $2 \times 30$  cents = 60 cents.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 10:**

After a pedestrian travelled half of his journey, he still had to travel 4 km more to complete the journey. What was the length of his whole journey, in km?

**Solution 1:**

I solved this problem by using the following reasoning:

Since the pedestrian had travelled half of his journey, he still had to travel another half of the journey. This was equal to 4 km. This means that the length of the whole journey was 8 km.

(I did not draw or imagine any picture at all).

**Solution 2:**

I used symbols and equations to solve this problem.

Suppose the length of the whole journey =  km.

He had already travelled  $1/2 \cdot$

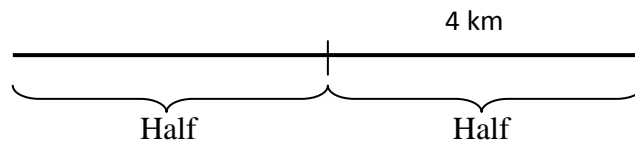
$$\text{Thus, } 1/2 \cdot \text{  } = 4$$

$$\text{  } = 8$$

The length of the whole journey is 8 km.

**Solution 3:**

In order to solve this problem, I drew a diagram representing the journey:



From the diagram it can be seen that the length of the whole journey was 8 km.

**Solution 4:**

I used the same method as for Solution 3, only I drew the diagram 'in my head' (and not on paper).

**I did not use any of the above methods.**

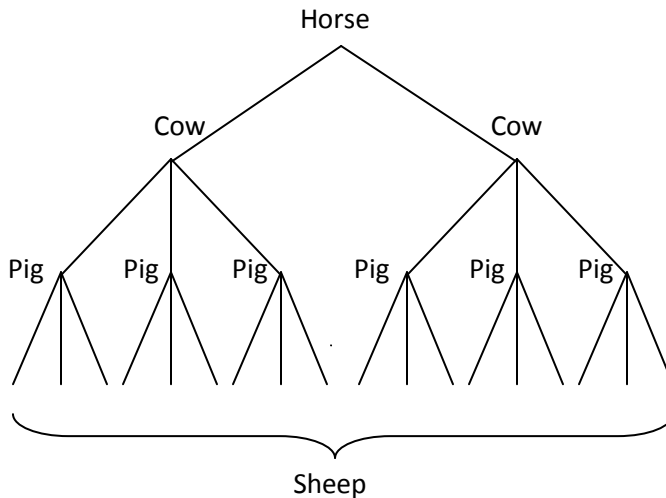
I attempted the problem in this way:

**PROBLEM 11:**

Mr. Jones traded his horse for two cows. Next he traded the two cows, and for each cow, he got three pigs. Then, he traded the pigs, and for each pig, he got 3 sheep. Altogether, how many sheep did Mr. Jones get?

**Solution 1:**

In order to solve this problem, I drew a diagram representing the animals:



From the diagram it can be seen that M. Jones got 18 sheep.

**Solution 2:**

I used the same method as for Solution 1, only I drew the diagram 'in my head' (and not on paper).

**Solution 3:**

I solved the problem using the following reasoning:

Mr. Jones traded his horse for 2 cows. Next he traded the two cows, and for each cow, he got three pigs. This means that he got  $2 \times 3$  pigs, or 6 pigs. Then he traded the pigs, and for each pig, he got three sheep. This means that he got  $6 \times 3$  sheep, or 18 sheep.

(I did not draw or imagine any picture at all).

**Solution 4:**

The number of sheep that Mr. Jones got

$$= 1 \times 2 \times 3 \times 3$$

$$= 18$$

He got 18 sheep.

**I did not use any of the above methods.**

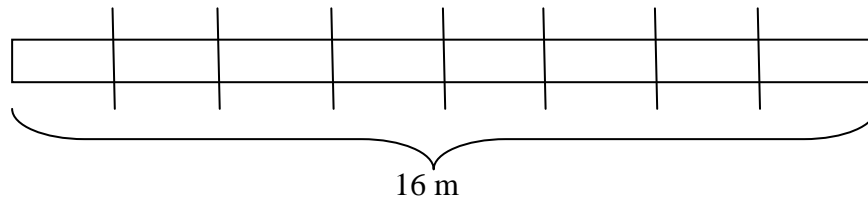
I attempted the problem in this way:

**PROBLEM 12:**

A saw in a sawmill saws long logs, each 16 m long, into short logs, each 2 m long. If each cut takes two minutes, how long will it take for the saw to produce eight short logs from one long log?

**Solution 1:**

To solve this problem, I drew a diagram showing the long log being cut into small logs.



In the diagram it can be seen that 7 cuts are needed to produce 8 short logs from one long log. Thus it will take  $7 \times 2$  minutes, or 14 minutes, to produce 8 short logs from one long log.

**Solution 2:**

I solved this problem by imagining one long log and the cuts needed to produce the short logs. I could 'see' in my mind that 7 cuts are needed to produce 8 short logs from one long log. Thus it will take  $7 \times 2$  minutes, or 14 minutes, to produce the 8 short logs.

**Solution 3:**

I solved the problem using the following reasoning:

If the long log were more than 16 m long, one would need 8 cuts to produce 8 short logs, each 2 m long, from that long log.

But the long log is only 16 m long, so the last cut is not needed. So one will only need  $(8 - 1)$  cuts, or 7 cuts. As each cut takes 2 minutes, 7 cuts will take  $7 \times 2$  minutes, or 14 minutes.

**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 13:**

How many ways can 30 dollars be paid to a person if the money must be in 5-dollar and 2-dollar notes only, and the person must get some 5-dollar notes and some 2-dollar notes. (For each possible solution, summarize your answer by saying how many 2-dollar notes and how many 5-dollar notes the person would get.)

**Solution 1:**

I solved the problem by guessing the combinations of 5-dollar and 2-dollar notes which add up to 30 dollars?

5 5 5 5 5 2 2	= NO
5 5 5 5 2 2 2 2 2	= YES
5 5 5 2 2 2 2	= NO
5 5 2 2 2 2 2 2 2 2 2	= YES
5 2 2 2 2	= NO

Thus there are only two ways: 1) Four 5-dollar notes and five 2-dollar notes, and 2) Two 5-dollar notes and ten 2-dollar notes.

**Solution 2:**

I solved the problem by using this reasoning: The number of 5-dollar notes should be such that the rest of the money is a multiple of 2. This means that the total amount composed by the 5-dollar notes can be 10 dollars or 20 dollars. Then the rest of the money which is composed by the 2-dollar notes can be 20 dollars or 10 dollars. Therefore, there are two ways in which the money can be paid: 1) 10 dollars composed by 5-dollar notes, and 20 dollars by 2-dollar notes. This means that there are two 5-dollar notes and ten 2-dollar notes, and 2) 20 dollars composed by 5-dollar notes, and 10 dollars by 2-dollar notes. This means that there are four 5-dollar notes and five 2-dollar notes.

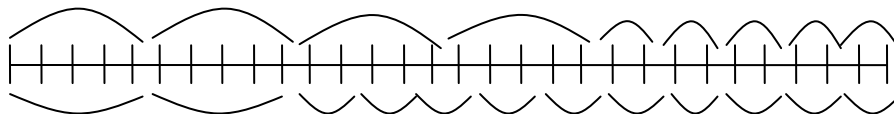
**Solution 3:**

I solved the problem by using this reasoning:

The number of 2-dollar notes should be such that the rest of the money is a multiple of 5. The rest of the solution then is similar to Solution 2.

**Solution 4:**

I solved the problem by drawing a diagram representing the money. The diagram is a line consisting of 30 equal parts. To solve the problem I tried to arrange the line into combinations of line segments consisting of 5 parts and 2 parts.



In the diagram it can be seen that there are two ways to make such arrangement:

- 1) Four line segments which consist of 5 parts each and five line segments which consist of 2 parts each.
2. Two line segments which consist of 5 parts each and ten line segments which consist of 2 parts each.

This means that there are 2 ways in which 30 dollars can be paid using 5-dollar and 2-dollar notes: 1) Four 5-dollar notes and five 2-dollar notes, and 2) Two 5-dollar notes and ten 2-dollar notes.

**Solution 5:**

I used the same method as for Solution 4, only I drew the diagram 'in my head' (and not on paper).



**I did not use any of the above methods.**

I attempted the problem in this way:

**PROBLEM 14:**

A tourist travelled some of his journey by plane, and the rest by bus. The distance that he travelled by bus was half the distance he travelled by plane. Determine the length of his entire trip if the distance that he travelled by plane was 150 km longer than the distance he travelled by bus.

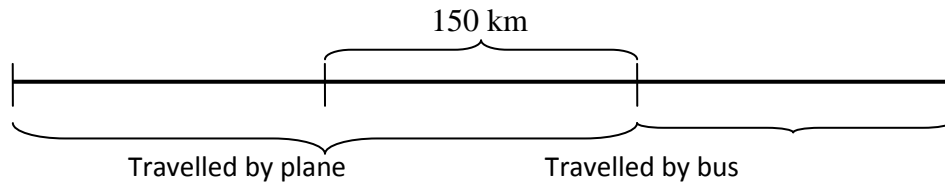
**Solution 1:**

To solve this problem, I divided the journey into three equal sections, two sections being travelled by plane, one section by bus. The difference in the distance travelled by plane and that travelled by bus was one section. This was equal to 150 km. Thus the length of the whole journey was  $3 \times 150$  km, or 450 km,

(I did not draw or imagine any picture at all).

**Solution 2:**

I solved this problem by drawing a diagram of the journey.



In the diagram it can be seen that the difference between the distance travelled by plane and that travelled by bus was one section. It was equal to one section. In the diagram it is also clear that the length of the whole journey was 450 km.

**Solution 3:**

I used the same method as for Solution 2, only I drew the diagram 'in my head' (and not on paper).

**I did not use any of the above methods.**

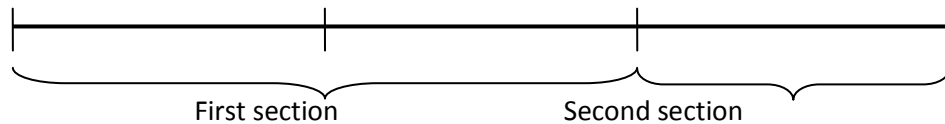
I attempted the problem in this way:

**PROBLEM 15:**

A straight path is divided into two unequal sections. The length of the second section is half the length of the first section. What fraction of the whole path is the first section?

**Solution 1:**

I solved this problem by drawing a diagram representing the path:



From the diagram it can be seen that the first section is two-thirds ( $2/3$ ) of the whole path.

**Solution 2:**

I used the same method as for Solution 1, only I drew the diagram 'in my head' (and not on paper).

**Solution 3:**

As the length of the second section is half the length of the first section, the path can be divided into three equal parts. The first section contains two parts, and the second section one. Thus the second section is two-thirds of the whole path.

(I did not draw or imagine any picture at all.)

**Solution 4:**

I solved this problem by using examples. Suppose the length of the first section is 50 m, then the length of the second section is 25 m, as the length of the second section is half the length of the first. The length of the whole path then will be 75 m; This means that the first section (50 m) is two-thirds of the whole path.

**I did not use any of the above methods.**

I attempted the problem in this way:

TYPES OF SOLUTIONS  
IN  
MATHEMATICAL PROCESSING QUESTIONNAIRE I

V = Solution by a visual method

N = Solution by a nonvisual method

For the definitions of visual and nonvisual methods, see pp. 128-129

Problem 1

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = N

Problem 6

Solution 1 = V  
Solution 2 = V  
Solution 3 = N

Problem 11

Solution 1 = N  
Solution 2 = N  
Solution 3 = V  
Solution 4 = V

Problem 2

Solution 1 = N  
Solution 2 = V  
Solution 3 = V

Problem 7

Solution 1 = N  
Solution 2 = V  
Solution 3 = V  
Solution 4 = N

Problem 12

Solution 1 = N  
Solution 2 = V  
Solution 3 = V

Problem 3

Solution 1 = V  
Solution 2 = V  
Solution 3 = V  
Solution 4 = N  
Solution 5 = N

Problem 8

Solution 1 = N  
Solution 2 = V  
Solution 3 = V

Problem 13

Solution 1 = N  
Solution 2 = V  
Solution 3 = V

Problem 4

Solution 1 = V  
Solution 2 = N  
Solution 3 = V

Problem 9

Solution 1 = N  
Solution 2 = N  
Solution 3 = V  
Solution 4 = V  
Solution 5 = N

Problem 14

Solution 1 = V  
Solution 2 = V  
Solution 3 = N

Problem 5

Solution 1 = N  
Solution 2 = N  
Solution 3 = V  
Solution 4 = V

Problem 10

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = V  
Solution 5 = V

Problem 15

Solution 1 = V  
Solution 2 = V  
Solution 3 = N

TYPES OF SOLUTIONS

IN

MATHEMATICAL PROCESSING QUESTIONNAIRE II

V = Solution by a visual method

N = Solution by a nonvisual method

Problem 1

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = N  
Solution 5 = N

Problem 6

Solution 1 = N  
Solution 2 = N  
Solution 3 = V  
Solution 4 = V

Problem 11

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = N

Problem 2

Solution 1 = V  
Solution 2 = V  
Solution 3 = N

Problem 7

Solution 1 = N  
Solution 2 = N  
Solution 3 = V  
Solution 4 = V

Problem 12

Solution 1 = V  
Solution 2 = V  
Solution 3 = N

Problem 3

Solution 1 = V  
Solution 2 = V  
Solution 3 = N

Problem 8

Solution 1 = N  
Solution 2 = V  
Solution 3 = V  
Solution 4 = N  
Solution 5 = N

Problem 13

Solution 1 = N  
Solution 2 = N  
Solution 3 = N  
Solution 4 = V  
Solution 5 = V

Problem 4

Solution 1 = N  
Solution 2 = V  
Solution 3 = V

Problem 9

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = N

Problem 14

Solution 1 = N  
Solution 2 = V  
Solution 3 = V

Problem 5

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = N

Problem 10

Solution 1 = N  
Solution 2 = N  
Solution 3 = V  
Solution 4 = V

Problem 15

Solution 1 = V  
Solution 2 = V  
Solution 3 = N  
Solution 4 = N

**APPENDIX G: PRINCIPAL SUPPORT LETTER**

Orange County Public Schools

ORLANDO, FLORIDA 32829

8/7/2009

Orlando, FL 32829

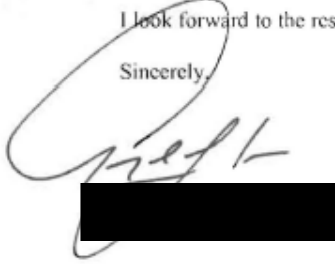
Adam Traas,

I am pleased to support to your research to develop a computerized spatial ability treatment at the Orange County Public Schools' [redacted] location. I understand that the program will be focused on increasing student spatial orientation skills through an interactive and dynamic computer-based environment.

As principal of [redacted], I am excited that this proposal affords us an opportunity to make strides toward exploring and augmenting students' mathematical thinking and ability, by developing this treatment with [redacted] students and faculty. It will blend closely with our efforts to increase mathematical efficacy through the most effective and inventive ways.

I look forward to the results of this research, as we implement this ambitious project.

Sincerely,



[redacted]

Orange County Public Schools

"The Orange county School Board is an equal opportunity agency

**APPENDIX H: ORANGE COUNTY PUBLIC SCHOOLS RESEARCH REQUEST  
FORM**



Submit this form and a copy of your proposal to: <b>Accountability, Research, and Assessment</b> P. O. Box 271 Orlando, FL 32802-0271	<b>Orange County Public Schools</b>  <b>RESEARCH REQUEST FORM</b>	<b>Your research proposal should include:</b> <ul style="list-style-type: none"> <li>• Project Title</li> <li>• Purpose and Research Problem</li> <li>• Instruments</li> <li>• Procedures and Proposed Data Analysis</li> </ul>	
Requester's Name <u>Adam Traas</u>		Date <u>7-24-09</u>	
Address <u>16605 Cedar Crest Dr</u> <u>Orlando, FL</u> <u>32828</u> <small>Street City, State Zip</small>		Phone <u>407-923-2467</u>	
Institutional Affiliation <u>University of Central Florida</u>			
Project Director or Advisor <u>Dr. Erhan Selcuk Haciomeroglu</u>		Phone <u>(407) 823-4336</u>	
Address <u>University of Central Florida 4000 Central Florida Blvd. Orlando, FL 32816-1250</u>			
Degree Sought: (check one) <input type="checkbox"/> Associate <input type="checkbox"/> Doctorate <input type="checkbox"/> Bachelor's <input type="checkbox"/> Not Applicable <input checked="" type="checkbox"/> Master's <input type="checkbox"/> Specialist			
Project Title: <u>Development of a Spatial Ability Treatment Using Computer Generation of Isometric and Orthographic Projections</u>			
<b>ESTIMATED INVOLVEMENT</b>			
PERSONNEL/CENTERS	NUMBER	AMOUNT OF TIME (DAYS, HOURS, ETC.)	SPECIFY/DESCRIBE GRADES, SCHOOLS, SPECIAL NEEDS, ETC.
Students	500	8-12 class periods	6-8 grade in [redacted]
Teachers	12	8-12 days	6-8 grade teachers
Administrators	0	0	na
Schools/Centers	1	8-12 days	[redacted]
Others (specify)	0	na	na
Specify possible benefits to students/school system: <u>Program developed will be given for free usage to OCPS. Students could increase spatial ability and hence math scores.</u>			
<b>ASSURANCE</b>			
Using the proposed procedures and instrument, I hereby agree to conduct research in accordance with the policies of the Orange County Public Schools. Deviations from the approved procedures shall be cleared through the Senior Director of Accountability, Research, and Assessment. Reports and materials shall be supplied as specified.			
Requester's Signature <u>[Signature]</u>		<b>RECEIVED JUL 28 2009</b>	
Approval Granted: <input checked="" type="checkbox"/> Yes <input type="checkbox"/> No		Date: <u>8-7-09</u>	
Signature of the Senior Director for Accountability, Research, and Assessment <u>[Signature]</u>			

NOTE TO REQUESTER: When seeking approval at the school level, a copy of this form, signed by the Senior Director, Accountability, Research, and Assessment, should be shown to the school principal who has the option to refuse participation depending upon any school circumstance or condition. The original Research Request Form is preferable to a faxed document.

**APPENDIX I: UNIVERSITY OF CENTRAL FLORIDA IRB ACCEPTANCE**



University of Central Florida Institutional Review Board  
Office of Research & Commercialization  
12201 Research Parkway, Suite 501  
Orlando, Florida 32826-3246  
Telephone: 407-823-2901, 407-882-2012 or 407-882-2276  
[www.research.ucf.edu/compliance/irb.html](http://www.research.ucf.edu/compliance/irb.html)

### Notice of Exempt Review Status

From: UCF Institutional Review Board  
FWA00000351, Exp. 10/8/11, IRB00001138

To: Adam M. Traas

Date: August 14, 2009

IRB Number: SBE-09-06350

Study Title: Development of a Spatial Ability Treatment Using Computer Generation of Isometric and Orthographic Projections

Dear Researcher:

Your research protocol was reviewed by the IRB Vice-chair on 8/14/2009. Per federal regulations, 45 CFR 46.101, your study has been determined to be minimal risk for human subjects and exempt from 45 CFR 46 federal regulations and further IRB review or renewal unless you later wish to add the use of identifiers or change the protocol procedures in a way that might increase risk to participants. Before making any changes to your study, call the IRB office to discuss the changes. A change which incorporates the use of identifiers may mean the study is no longer exempt, thus requiring the submission of a new application to change the classification to expedited if the risk is still minimal. Please submit the Termination/Final Report form when the study has been completed. All forms may be completed and submitted online at <https://iris.research.ucf.edu>.

The category for which exempt status has been determined for this protocol is as follows:

1. Research conducted in established or commonly accepted educational settings, involving normal educational practices, such as:
  - (i) research on regular and special education instructional strategies, or
  - (ii) research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.

The IRB has approved a consent procedure which requires participants to sign consent forms. Use of the approved, stamped consent document(s) is required. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Subjects or their representatives must receive a copy of the consent form(s).

All data, which may include signed consent form documents, must be retained in a locked file cabinet for a minimum of three years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained on a password-protected computer if electronic information is used. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

On behalf of Joseph Bielitzki, M.S., DVM, UCF IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 08/14/2009 10:41:31 AM EDT

IRB Coordinator

## **APPENDIX J: PARENT CONSENT FORMS**

Permission to Take Part in a Human Research Study



*Development of a Spatial Ability Treatment Using Computer Generation of Isometric and Orthographic Projections*

*Informed Consent from a Parent for a Child in a Research Study*

Principal Investigator(s): *Adam Traas*

Faculty Supervisor: *Dr. Erhan Haciomeroglu, PhD*

Sponsor: *University of central Florida, College of Education, Department of Teaching and Learning Principals*

Investigational Site(s): [REDACTED]

**Introduction:** Researchers at the University of Central Florida (UCF) study many topics. To do this we need the help of people who agree to take part in a research study. You are being asked to allow your child to take part in a research study which will include about 500 people in Liberty Middle School. Your child is being invited to take part in this research study because he or she is enrolled in Liberty and is currently enrolled in a mathematics course there. Please read this form and sign for your child to take part.


The person doing this research is Adam Traas of the University of Central Florida and is a mathematics teacher himself at [REDACTED] Because the researcher is a master's degree student he is being guided by Dr. Erhan Haciomeroglu, a UCF faculty supervisor in the Department of Teaching and Learning Principles.

**What you should know about a research study:**

- This letter will explain this research study to you.
- A research study is something you volunteer for.
- Whether or not you take part is up to you.
- You should allow your child to take part in this study only because you want to.
- You can choose not to take part in the research study.
- You can agree to take part now and later change your mind.
- Whatever you decide it will not be held against you.

1 of 4

Parent for a Child in a Non-medical Research Study  
Version.#

University of Central Florida  
Participant Initials  
 University of Central Florida IRB  
IRB NUMBER: SBE-09-06350  
IRB APPROVAL DATE: 8/14/2009

IRB #: [IRB Number]  
Traas

- Feel free to ask all the questions you want before you decide.
- Participation in this study will not affect your child's grade

**Purpose of the research study:** The purpose of this study is to test a new computer program aimed at increasing spatial ability in students. Spatial ability is very important more math and critical thinking skills to develop that many students in the United States lack.

**What your child will be asked to do in the study:** As part of this study your child will be asked to participate in a pretest to assess his or her current spatial abilities. Some students who are selected randomly will be asked to then use the computer program one day a week in their math class for about eight weeks. After this time, students will take a posttest to determine what change in spatial ability has occurred. The entirety of the study is scheduled for completion before winter break.

**Location:** This research will be conducted at Liberty Middle School and all portions will happen during the normal school day.

**Time required:** We expect that you will be in this research study for about eight weeks for one 45-50 minute session a week.

**Benefits:** We cannot promise any benefits to you, your child, or others from your child taking part in this research. However, possible benefits include but are not limited to your child increasing their spatial ability, critical thinking skills, and mathematical ability. These skills are important, not only for achievement in school, but also in many jobs they may seek later in life.

**Compensation or payment:** There is no compensation, payment or extra credit for your child's part in this study

**Confidentiality:** We will limit your personal data collected in this study. Efforts will be made to limit your child's personal information to people who have a need to review this information. We cannot promise complete secrecy. Organizations that may inspect and copy your information include the IRB and other representatives of UCF.


**Anonymous research:** This study is anonymous. That means that no one, not even members of the research team, will know that the information your child gave came from him or her.

**Study contact for questions about the study or to report a problem:** If you have questions, concerns, or complaints, or think the research has hurt your child talk to: Adam Traas, Graduate Student, UCF Lockheed Martin Academy, College of Education, (407) 923-2467 or Dr. Haciomeroglu, Faculty Supervisor, Department of Teaching and Learning Principles at (407) 823-4336 or by email at [erhansh@mail.ucf.edu](mailto:erhansh@mail.ucf.edu).

2 of 4

Non-Medical Research Consent Form  
Version.#

University of Central Florida  
Participant Initials \_\_\_\_\_

 University of Central Florida IRB  
IRB NUMBER: SBE-09-06350  
IRB APPROVAL DATE: 8/14/2009

IRB #: [IRB Number]  
Traas

**IRB contact about you and your child's rights in the study or to report a complaint:** Research at the University of Central Florida involving human participants is carried out under the oversight of the Institutional Review Board (UCF IRB). This research has been reviewed and approved by the IRB. For information about the rights of people who take part in research, please contact: Institutional Review Board, University of Central Florida, Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901. You may also talk to them for any of the following:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.
- You want to talk to someone besides the research team.
- You want to get information or provide input about this research.

**Withdrawing from the study:** You may decide not to have your child continue in the research study at any time without it being held against you or your child. If you decide to have your child leave the research, it can have negative effects on the results of the study. If you decide to have your child leave the study, contact the investigator so that the investigator can remove the student and all data collected from the student immediately.

The person in charge of the research study or the sponsor can remove your child from the research study without your approval. Possible reasons for removal include refusal of the student to participate in the computer treatment.

3 of 4

Non-Medical Research Consent Form  
Version.#

University of Central Florida  
Participant Initials \_\_\_\_\_



University of Central Florida IRB  
IRB NUMBER: SBE-09-06350  
IRB APPROVAL DATE: 8/14/2009

IRB #: [IRB Number]  
Trace:

Your signature below indicates your permission for the child named below to take part in this research:

<hr/>	
Name of participant	
<hr/>	
<hr/>	<hr/>
Signature of first parent or guardian	Date
<hr/>	
<b>DO NOT SIGN THIS FORM AFTER THE IRB EXPIRATION DATE BELOW</b>	
<input type="checkbox"/> Parent	
<input type="checkbox"/> Guardian (See note below)	
<hr/>	
Printed name of first parent or guardian	
<hr/>	
<hr/>	<hr/>
Signature of second parent	Date
<hr/>	
Printed name of second parent	
<hr/>	
Printed name of person obtaining consent and assent	

My signature and date indicates that the information in the consent document and any other written information was accurately explained to, and apparently understood by, the participant or the participant's legally authorized representative, and that informed consent was freely given by the participant or the legally authorized representative. **Note on permission by guardian:** An individual may provide permission for a child only if that individual can provide a written document indicating that he or she is legally authorized to consent to the child's general medical care. Attach the documentation to the signed document.



**APPENDIX K: CHILD ASSENT FORM**



*Child Assent Template for use with children 7 to 17 years of age*

My name is Adam Traas. I am doing a research project on Spatial ability in middle school students I am interested in finding ways to improve math and spatial skills. This research is part of my studies at the University of Central Florida.

As a way to study this, I would like to have some students take a pretest on spatial abilities and use a newly developed computer program. At the end of the activity, I will ask you to take a posttest to see how you improved.

Only Dr. Haciomeroglu, my professor at UCF, and I will see the test scores. No names will be used so that nobody will know it was you in my study.

This will not affect your grade if you decide you don't want to do this. You can stop at any time. If you do not want to take part in this study, your teacher will give you another activity to do. You will not be paid for doing this. You will not get extra credit for doing this. Would you like to take part in this research project?

\_\_\_\_\_ I want to take part in Mr. Traas's research project.

\_\_\_\_\_  
Student's Signature

\_\_\_\_\_  
Date

\_\_\_\_\_  
Student's Printed Name



University of Central Florida IRB  
IRB NUMBER: SBE-09-06350  
IRB APPROVAL DATE: 8/14/2009

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