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Linking Place Value Concepts With Computational Practices In Third Grade

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LINKING PLACE VALUE CONCEPTS WITH COMPUTATIONAL PRACTICES IN THIRD
GRADE

by

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ABSTRACT

In an attempt to examine student understanding of place value with third graders, I conducted action research with a small group of girls to determine if my use of instructional strategies would encourage the development of conceptual understanding of place value. Strategies that have been found to encourage conceptual development of place value, such as use of the candy factory, were incorporated into my instruction. Instructional strategies were adjusted as the study progressed to meet the needs of the students and the development of their understanding of place value. Student explanations of their use of strategies contributed to my interpretation of their understanding. Additionally, I examined the strategies that the students chose to use when adding or subtracting multidigit numbers. Student understanding was demonstrated through group discussion and written and oral explanations. My observations, anecdotal records and audio recordings allowed me to further analyze student understanding. The results of my research seem to corroborate previous research studies that emphasize the difficulty that many students have in understanding place value at the conceptual level.

I dedicate this thesis to my students who made this research possible. I also dedicate it to

my family and friends who supported me throughout the process.

ACKNOWLEDGMENTS

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CHAPTER ONE: INTRODUCTION

<u>Rationale</u>

It was the end of another school year and once again I had paused to reflect on my mathematics instruction and student learning. I learned the power of reflective practice when I went through the process of National Board Certification, and as I reached the end of another school year, I took time to evaluate what had or had not been accomplished in my mathematics classes. Once again this year, I had found that I was sending students on to fourth grade who were unable to add and subtract with regrouping.

According to the results of the state-mandated criterion-referenced achievement test, the majority of my students were proficient in mathematics last year. This meant that most of my students were performing at or above grade level according to the test. However, a majority of those proficient students could not subtract with regrouping and some could not add with regrouping. This perplexed and frustrated me. What could I have done differently? Why was this a difficult concept for third graders? What did I need to change in order to become more effective at teaching this mathematics skill? Even though I have learned much over the last few years about teaching mathematics through professional development and graduate-level courses, I still have not been as effective as I would like in helping my students master this particular skill. Even with the use of manipulatives and practices learned in my graduate classes, I saw only minimal improvement. I repeatedly worked with my current students at several points throughout this school year, but many students still did not "get it." Something was missing.

While my students have always had problems with adding and subtracting, this year's students seemed to have more difficulty than usual. In previous years, the majority of my students were able to accurately add multidigit numbers using the traditional algorithm and at least half of my students would be capable of subtracting multidigit numbers using the traditional algorithm. However, at the beginning of this school year, less than half of my students could accurately add multidigit numbers using the traditional algorithm and only a very few were capable of subtracting multidigit numbers using the traditional algorithm. As I reflected on the previous school year and analyzed current and previous years' data from various testing sources, it became apparent that many of the children who were unable to add and subtract were also having difficulty in understanding place value. I was sure that the two concepts were linked. Perhaps what my students really needed was more conceptually-based instruction in place value. Would this be enough to improve my students' computation skills? I suspected that still more might be needed. Professional development courses in brain-based learning placed emphasis on making connections between old and new learning and between different aspects of a topic of learning (Jensen, 2000). Additionally, multiple representations of the same content are effective in reaching different types of learners (Wolfe, 2001). I suspected that if I created connections between different representations of number and place value and tied these connections to addition and subtraction, I might see improvement in my students' computational abilities. This became the impetus for my choice of topic for this research.

Purpose

The purpose of this study was to examine my practice of teaching place value in order to determine whether or not my practices affected my students' ability to flexibly use strategies to add and subtract multidigit numbers. I wanted to determine whether or not my students would be able to use more than one strategy to add and subtract multidigit numbers and whether or not they would be able to choose a strategy to use based on effectiveness and efficiency for the given situation. I also wanted to determine whether or not my practices would enable my students to flexibly represent numbers. This flexibility would be demonstrated by my students' ability to manipulate the quantity of ones, tens, and hundreds in a number while conserving the total quantity. Furthermore, I wanted to examine my practice of making connections between place value, number representation, and computation.

Research Questions

My reflection and research reading suggested two questions of study.

1. How does my use of conceptually-based strategies for teaching place value affect my students' ability to flexibly use strategies to add and subtract multidigit numbers?

2. How does my practice of making explicit connections among place value, number representations, and computation affect my students' ability to flexibly use strategies to add and subtract multidigit numbers?

Significance of the Study

Over the last several years, I had begun to focus my mathematics instruction on teaching for conceptual understanding and on making process just as important as

product. Previous research assumes that teaching for understanding leads to learning with understanding which in turn increases "flexibility, transfer and increased learning over time" (Hiebert & Wearne, 1992, p. 99). Furthermore, Hiebert and Carpenter (1992) addressed many issues related to conceptual understanding including the importance of making connections between representations, recognizing patterns and relationships, providing context and supporting and emphasizing social interaction and discussion about mathematical topics.

Teaching and learning with understanding is a multi-faceted topic. Understanding can be defined by the way information is structured and represented as part of an internal network (Hiebert & Carpenter, 1992). A theme that has emerged from many seminal works in mathematics education literature (Fehr, 1955; Hiebert, 1986; Janiver, 1987; Michener, 1978; Polya, 1957; Van Engen, 1949; Wertheimer, 1959) is that "understanding in mathematics is making connections between ideas, facts, or procedures" (Hiebert & Carpenter, 1992, p. 67). Further research has indicated that mathematical ideas start with some form of internal representation within the brain (Hiebert & Carpenter, 1992; Schoenfeld, 1992). As a result children need to be able to communicate this internal representation in such a way that an external representation can be constructed. These external representations then need to be further connected to additional external representations of the same topic or the same type of representations across similar mathematical topics. Additionally, the above researchers discuss similarities and differences between various representations are critical to mathematical understanding.

Hiebert and Carpenter (1992) also discussed the consequences of understanding mathematical concepts. Furthermore, they stated that it is generally accepted by mathematics researchers that students must create their own mathematical understanding rather than receiving it directly from instruction. This makes it imperative that teachers create learning environments that allow students to develop their understanding and create connections in order to expand that understanding. Relationships should come before procedures. Hiebert and Carpenter (1992) also explained that understanding is the result of effective connections both within the brain and with external representations promoting the retention and recall of mathematical knowledge. These connections reduce the amount of information that needs to be remembered since extensive connections have been created and individual pieces of memory, such as the steps to a procedure; do not need to be recalled. Additionally, understanding also enhances transfer. If every new problem required a new solution process, mathematics would be quite difficult. The connections that have been made through understanding allow the student to have a starting point when attempting to solve a new type of problem. Understanding also influences the affective domain in regard to mathematics. When students understand mathematics, they are more positive about their ability to solve new problems and have a decreased negative perception of mathematics (Hiebert & Carpenter, 1992).

I have taught third, fourth, and fifth grades; and each year there were always children who were unable to successfully access computational skills. They also did not have strategies other than the traditional algorithms available to them to complete computational tasks. I believed that children struggled with these skills because they did

not have the conceptual understanding of prerequisite skills and concepts such as place value and number representation in order to be successful with computational strategies. According to the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics,* conceptual understanding is an important factor in mathematics proficiency. Further, foundational concepts like place value and the base-ten number system are keys to making connections between different areas of mathematics (Nataraj & Thomas, 2007; NCTM, 2000).

Wearne, Hiebert and Campbell (1994) described the importance of place value in the following manner.

> Understanding place value involves building connections between key ideas of place value – such as quantifying sets of objects by grouping by ten and treating the groups as units – and using the structure of the written notation to capture this information about groupings. Different forms of representation for quantities, such as physical materials and written symbols, highlight different aspects of the grouping structure. Building connections between these representations yields a more coherent understanding of place value (p. 274).

An additional aspect of place value includes understanding numbers and their various representations as well as number words (Fuson & Burghardt, 2003; Hiebert & Carpenter, 1992; Hiebert & Wearne, 1992; Jones et al., 1996). The concept of place value is critical to developing number sense and is part of the infrastructure needed to effectively apply addition, subtraction, multiplication and division strategies and algorithms (Nataraj & Thomas, 2007). Other research has shown that the connection between place value and computation needs to be explicitly taught for young children

(Hiebert & Wearne, 1992). The concept of place value is complicated and incorporates many elements which can cause students difficulty and create misconceptions. Some of these elements include understanding the difference between the face value and the complete value of a digit within a multidigit number and the ability to manipulate the quantity of ones, tens, and hundreds within a multidigit number while maintain the complete value of the number.

One way to teach the concept of place value is the use of concrete materials. Various studies (Hiebert & Carpenter, 1992; Raphael & Wahlstrom, 1989; Sowell, 1989) have shown that the use of concrete materials produces mixed results when it comes to conceptual understanding. Many other factors come into play with the use of concrete materials to teach mathematical concepts such as place value. Children may have an understanding of the representation of number through the use of concrete materials, but they may not have transferred that understanding to other representations. Additionally, the external representation of the concrete materials may not connect to the student's current internal representation of place value. For example, if a child's internal representation of numbers only uses units of one, the use of tens units with base-ten blocks to represent numbers would be in conflict with the child's current internal representation. This conflict may cause the child to have difficulty using base-ten blocks to represent numbers until new connections and internal representations can be constructed. These aspects, as well as how well the concrete materials contextually match the concept of place value, play an important role in the effectiveness of the use of concrete materials (Hiebert & Carpenter, 1992). Also, the use of base-ten blocks does not ensure that children will develop the ability to trade units (e.g., ten ones for one ten).

Some children will cover up blocks while others will count all of the blocks as single units rather than groups of ten (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

Another facet of teaching place value involves the use of addition and subtraction of multidigit numbers as a way to teach the base-ten number concepts that are essential to the effective use of traditional algorithms. This does not mean that procedures should be taught to children who have not developed a strong sense of place value. These types of problems can be used as the context to develop the understanding of base-ten. When children are allowed to develop their own procedures and discuss how they used the procedures, they become increasingly proficient in base-ten understanding (Carpenter, et al., 1999).

A further aspect of mathematics understanding and specifically the understanding of place value involves the culture of the mathematics classroom and the enculturation of mathematics that occurs outside the school setting. Schoenfeld (1992) believes that enculturation is critical to mathematical understanding. Most of the beliefs that we carry as individuals come through interaction with others. This includes our beliefs about mathematics. If parents dislike mathematics and claim lack of proficiency, then children will often take on these same beliefs regardless of their own particular skill. The opposite is equally true. Children often come to school with preconceived ideas about their ability to "do math."

The culture of the mathematics classroom plays a role in children's understanding of mathematics. Sociomathematical norms, those social norms that apply specifically to mathematics such as what constitutes a sophisticated mathematical solution, should be established within the mathematics classroom by the teacher and students (Cobb &

Yackel, 1996). Discussions should focus on students' methods and ideas, not the performance of individual students. Students should be allowed to choose their own methods for solving problems and then share them with others. Mistakes are not just mistakes, but opportunities for learning, and the correctness of an answer or procedure is determined by the logic of the mathematics (Hiebert, et al., 1997). The culture of the mathematics classroom is just as critical to the development of mathematical understanding as the content that is taught.

The mathematics classroom culture is strongly affected by the beliefs, both spoken and unspoken, of the teacher. The beliefs that students bring to the classroom also affect the classroom culture. This implies that there are a multitude of mathematics classroom cultures. While the students bring their mathematical beliefs and understandings into the classroom, the teacher can guide the students toward a classroom culture that will promote strong mathematical understandings (Nickson, 1992; Schoenfeld, 1992). As the nature of mathematics instruction has changed in recent years, so too has the nature of the mathematics classroom. Students need to be given the opportunity to discuss with one another their thinking about mathematics. Teachers need to expand their knowledge of the content and current research in mathematics instruction. As mathematics educators, we need to realize that mathematics instruction involves not just a particular procedure for a particular problem, but also the "hidden social messages in what we do and the power of their influence on the young people we teach" (Nickson, 1992, p. 111).

<u>Summary</u>

While I have been able to produce proficient mathematics students according to state assessment scores, the existing body of research and the NCTM *Principles and Standards for School Mathematics* (2000) document tell me that this is not enough. Strong ties between conceptual understanding for computation and conceptual understanding for place value are important in producing a truly proficient, fluent and flexible mathematics user. "Students who memorize facts or procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile" (NCTM, 2000, p. 19). The learning of many of my students was fragile in this particular area. For me, that must change.

The initial classroom assessments of my current students indicated that their understanding of place value was very weak. Most knew the place value positions, but only about half the students knew the value of a digit in a given number (e.g., the value of the 3 in 365 is three hundred). About one-third of my students could add using the traditional algorithm but did not understand that they were trading ones for tens or tens for hundreds when they were using regrouping. Only a few students could subtract with regrouping and again, those students did not understand the implications of regrouping. The students were limited in their ability to flexibly represent numbers. Our core mathematics program had given the students some practice in creating other names for numbers such as using number words or drawing a picture of a group of objects to represent a number, but the students did not understand that $300 + 60 + 5$ is the same as 365. Expanded notation is one way to help students understand place value. The

evaluation of my students this year was little different than it had been for students in previous years.

Therefore, I chose to look at how I taught place value concepts and computation, evaluated research regarding the teaching of place value and computational practices, and looked for connections between what I did, what I learned, and how my students learned. The lack of understanding of place value has persisted in the students I have taught. In order to address this lack of understanding, I chose to pursue this action research regarding the conceptual understanding of place value and its impact on the flexible use of numbers and computational strategies.

In chapter two, I will explore research related to place value, computation and sociomathematical norms. Chapter three details the methodology that I chose to use. Chapter four explains my findings and chapter five addresses conclusions that have been made from this research.

CHAPTER TWO: LITERATURE REVIEW

Introduction

"Generally, the arguments assume that teaching for understanding induces learning with understanding and learning with understanding has both short- and long-term benefits such as flexibility, transfer, and increased learning over time" (Hiebert & Wearne, 1992, p. 99). There appears to be no one single approach to teaching mathematics that accomplishes this purpose. Making connections between multiple representations such as concrete, pictorial, verbal, and symbolic supports students' understanding of mathematical concepts such as place value (Fuson et al., 1997; Hiebert & Wearne, 1992). Further, the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) document stresses that recent research supports the important role that conceptual understanding plays in the "knowledge and activity of persons who are proficient" (p. 19).

When teachers teach for understanding, their students are able to flexibly utilize their understanding in novel situations. This understanding and flexibility allow students to function more effectively in a constantly changing world. Whatever mathematics content has been choosen as critical for instructional purposes, the overriding goal should be to teach that content for understanding (Hiebert et al., 1997).

In teaching for understanding, several aspects of both the mathematics classroom and instructional practices need to be examined. Within each classroom, there exists a culture of mathematical learning. This culture varies from classroom to classroom depending on the mathematical beliefs of the teacher, and the prior formal and informal

learning of mathematics by the students. The term *norms* was introduced to describe the expectations and procedures that exist within a classroom (Cobb, Wood, Yackel, & McNeal, 1992; Yackel & Cobb, 1996). When these norms apply specifically to mathematics, they are called *sociomathematical norms*. As teachers and students interact about mathematics or other content, these norms can become what is called "taken-as-shared" understandings that make up the classroom culture (Cobb, Yackel & Wood, 1992; Lopez & Allal, 2007).

In addition to teaching and learning for understanding and classroom culture, the research on specific practices related to the instruction of place value and its connection to the knowledge of adding and subtracting multidigit numbers needs to be examined. A review of the literature shows that there are several paths to the development of the concept of place value and how well this conceptual understanding connects to the knowledge of adding and subtracting multidigit numbers. All of these aspects taken together provide the opportunity to increase to students' understanding of mathematics in such a way that they will be able to flexibly solve problems.

<div align="center">Mathematical Community</div>

Bowers, Cobb, and McClain (1999) approached the task of understanding what goes on in the mathematics classroom from a social constructivist perspective. Yang and Cobb (2007) stated that, "mathematical activity is inherently social and cultural in nature" (p. 27). This view summarized the of the results of previous research studies (Brown, Collins & Duguid, 1989; Greeno, 1991; Lerman; 1996, Schoenfeld; 1987; Sfard, 1994). Yang and Cobb also view "students' mathematical interpretations, solutions, explanations, and justifications not merely as individual acts but, simultaneously, as acts

of participation in collective or communal classroom processes" (p. 26). This sense of the dual nature of mathematical participation is shared by many other researchers (Cobb & Yackel, 1996; Lampert, 1990; Lopez & Allal, 2007; Simon, 1995; Voigt, 1995; Yang & Cobb, 2007). Additionally, Lopez and Allal (2007) stress that,

> Learning through participation in a classroom community entails two interconnected processes: the appropriation by the students of the norms, beliefs, practices, tools, and artefacts that are elaborated collectively; the contribution of the students to the elaboration of these norms, beliefs, practices, tools, artefacts. Knowledge is constructed in the classroom through the transactions among the members and in particular through the negotiation of the meaning attributed to the activities undertaken (p. 252).

One aspect of the mathematics classroom culture is social norms. These norms are the expectations of classroom participation that are negotiated by both the teacher and the students (Bowers et al., 1999; Cobb & Yackel, 1996; Dixon, Andreasen, & Stephan, in press; Lopez & Allal, 2007). These norms could include "explaining interpretations and solutions, attempting to make sense of explanations given by others, indicating understanding or nonunderstanding, and questioning alternatives when a conflict in interpretations becomes apparent" (Bowers et al., 1999, p. 27).

The next aspect of the mathematics classroom culture is sociomathematical norms. These norms are specific to the mathematical activity. Some examples of sociomathematical norms include "what counts as a different mathematical solution, a sophisticated mathematical solution, an insightful mathematical solution, and an acceptable mathematical explanation" (Bower et al., 1999, p. 27). These norms are

strongly tied to both the students' and the teacher's beliefs and values about mathematics. As these norms are renegotiated when children progress in their understanding, they help to further develop those beliefs and values (Bowers et al., 1999; Cobb & Yackel, 1996; Lopez & Allal, 2007).

The third aspect of the mathematics classroom culture is the actual mathematical practices. These practices evolve and change and students develop greater understanding of a mathematical practice. These practices can become *taken as shared* understandings. Some examples of taken as shared understandings can include situations where a justification is no longer necessary because the class has internalized the understanding, when students have consensus about what constitutes different solution, or an understanding of symbolic representation that no longer requires explicit explanation (Bowers et al., 1999; Lopez & Allal, 2007). An example would be that number words for two-digit numbers have a tens and ones component, and it is no longer necessary to justify this with an explanation.

Another view of mathematical community is that of *mathematical enculturation*. Yang and Cobb (1995) state that mathematical enculturation is an "interactive process that is carried out within the constraints of sociocultural practices and that results in the active recreation of mathematical ways of knowing" (p. 3). Beyond the culture of the classroom, there exists the culture of the community. How parents and caregivers interact with children in regard to mathematics even before they enter school may play as important a role in mathematical understanding as do the norms and practices of the classroom (Yang & Cobb, 1995). The symbols and tools that are utilized to understand mathematics play an active role in student and teacher understanding of mathematical

content. These aspects need to be taken into consideration whenever comparisons are made between the mathematics learning of one culture and another or even between one mathematics classroom and another within the same culture (Yang & Cobb, 1995).

All of the above referenced research argued that mathematical learning is social in nature. Students bring to the classroom knowledge of mathematics that is based in part on how parents and caregivers view mathematics and the family's own cultural practices (Cobb & Yackel, 1996). The teacher also contributes to the culture of learning mathematics with the practices and expectations that are used in the classroom. These norms, whether social or sociomathematical, contribute to the depth of understanding that children develop about mathematical content, especially the understanding of place value.

<u>Place Value Instruction</u>

While classroom norms are integral to the creation of conceptual understanding, many researchers agree that students often have a difficult time acquiring a deep conceptual understanding of place value and multidigit number sense (Fuson & Briars, 1990; Jones et al., 1996; Nataraj & Thomas, 2007; Varelas & Becker, 1997). Research indicates that this difficulty can persist into middle school (Cawley, Parmur, Lucas-Fusco, Kilian, & Foley, 2007; Kamii, 1986; Resnick & Omanson, 1987). There are many factors that influence this difficulty in acquiring place value understanding.

According to Cawley et al., (2007), place value is a significant mathematics concept that is generally presented at a surface level in most mathematics classrooms. The researchers believed that many teachers neglect to help students develop the concepts related to place value such as the foundation of the number system, estimation and rounding, the use of alternative representations including expanded notation, the

conservation of number within alternative representations, and the ability to interpret the oral and written number systems. Children who have a poor understanding of the concept of place value also demonstrate difficulties with algorithmic procedures. If this lack of understanding is not corrected, the gap widens as the children are expected to handle more complex algorithms. Additionally, it has been suggested that children who have an idea of the history of numbering systems and spend a considerable amount of time in two-digit numeration as well as using problem solving, estimation, alternative algorithms, and multiple representations of numbers as context for understanding place value develop a deeper conceptual understanding of place value (Cawley et al., 2007; Hiebert & Wearne, 1992; Nataraj & Thomas, 2007).

The ability to generalize a mathematical pattern or concept is critical to effective mathematical understanding. This is especially true of the concept of number. The concept of place value, or a positional numbering system, is key to developing number sense and transferring that understanding to other aspects of mathematics such as algebra. Additionally, *representational versatility*, the ability to move between representations of the same concept or to utilize these representations for new concepts, developed students' flexibility in using mathematical strategies in novel situations. Students with representational versatility had a greater repertoire of strategies to access and apply to novel problems (Cawley et al., 2007; Hiebert & Wearne, 1992; Nataraj & Thomas, 2007).

When children are presented with new mathematical representations, they need to "construct a mental model that reflects the structure of that concept" (Jones et al., 1996, p. 311). Children also need to internalize the concept of a group of tens (ones, tens, hundreds, etc.) as a counting unit. This internal structure is critical to the development of

multidigit number sense (Jones, et al., 1996). Multiple representations can include the use of manipulatives, written numerical representations, number words, and oral representations. Research has shown that there are multiple paths of instruction that lead to deep conceptual understanding of place value (Fuson et al., 1997).

Research into various strategies for promoting the conceptual understanding of place value covers many of the aspects previously mentioned. The research of Heibert and Wearne (1992) indicates that there is a strong correlation between the type of instruction used in the classroom and students' flexible use of strategies. However, there is little agreement on which type of instruction is most beneficial. Studies of the use of manipulatives have produced evidence of varying degrees of success with their use. There are many issues predicated upon the success or lack of success with the use of manipulatives in producing deeper understanding of place value (Cawley et al., 2007; Nataraj & Thomas, 2007). Additionally, Cawley, et al. suggests that the use of manipulatives that are contextually relevant to the concept being taught, in this case place value, are essential to developing understanding. However, they believe that the significance of the use of manipulatives is not clear. Additionally, the use of manipulatives required an extensive use of memory by the student. Manipulatives have to be moved and the student must remember what was done with the materials. However, Nataraj and Thomas (2007) believe that representational versatility, or the ability to move between representations of the same concept or to utilize representations for new concepts, develops students' flexibility in using mathematical strategies in novel situations.

Another aspect of place value that should be taken into consideration when planning instruction for conceptual understanding is symbolic representation. Varelas and Becker (1997) believe that at least some of the difficulty that children experience with place value is that they "fail to differentiate between the face value of each symbol in a number and the complete value of the same symbol" (p. 265). For example, when children are given a number such as 37, they will indicate that the 7 represents 7 objects and the 3 represents 3 objects rather than 30 objects. A child identifies only the face value of each digit rather than the complete value of the digit. This difficulty is related to the symbols used to represent numbers and their quantities, Varelas and Becker conducted research regarding the semiotic, or symbol, aspects of place value. They developed a system that is between the written and concrete representations called FVCV (face value, complete value). In their research, they used two-color chips that had the face value written on one side and the complete value on the other side. All of the upper sides of the chips would be the same color and had digits written on them. The lower sides of the chips were colored the same, but in a color that was different than the upper side. The lower side contained the value of the digit on the upper side. For example, a chip with a 3 on the upper side, could have 3 (ones), 30 (tens), 300 (hundreds), etc., written on the lower side for the complete value. The results of the study indicated that the FVCV system did improve student differentiation between face value and complete value and that each complete value representation added up to the complete value of the number. The majority of the place value research deals with grouping and regrouping in which children need to understand the powers of ten and that a number such as 17 can be conceptualized as 17 individual units or one unit of ten and seven individual units.

Children may be able to understand this aspect of number quantities and may be able to perform operations such as addition and subtraction, but this does not mean that they understand place value. Additionally, children may be able to utilize base-ten materials to represent numbers and perform computation functions and still not have a conceptual understanding of place value.

In addition to the semiotic aspects of place value, Fuson and Briars (1990) have suggested that the irregularities of the English number naming system may contribute to difficulty with place value. They cite this as one reason for the difference in performance of American children and those of Asian countries. There are many irregularities with the English naming system when compared to that of Chinese, Burmese, Japanese, Korean, Thai, and Vietnamese. The English word for 13 is thirteen. In the Asian languages it would be said "one ten three." In English, the number 57 is fifty-seven. In the Asian languages it would be said "five ten seven." The number naming system in the above languages has place value implicit within it. English speaking children "must construct named-value and positional base-ten conceptual structures for the words and the marks and relate these conceptual structures to each other and to the words and the marks" (Fuson & Briars, p. 180). This creates a greater cognitive load and interferes with a child's conceptual understanding of place value. Understanding of these irregularities in the English number naming system makes it important that teachers find ways to support the construction of "ten-structured conceptions" (Fuson & Briars, p. 181).

Another aspect of place value instruction that should be taken into consideration is the instructional context. The instructional context that was utilized within the research addressed by this paper was previously utilized by Bowers, Cobb, and McClain (1999).

This instructional sequence, known as the candy factory, was designed "to support students' development of increasingly sophisticated place value understandings and their construction of personally meaningful algorithms for adding and subtracting three-digit numbers" (p. 30). The objective was that students would perceive numbers in such a way that they would flexibly utilize computational algorithms because they had an understanding of number, not because they had to recall the steps of a calculational process.

In the candy factory, the students would come to understand that the candy can be packaged in a variety of ways that includes powers of 10; ten pieces is equivalent to one roll, ten rolls are equivalent to one box, etc. In the learning trajectory, it was anticipated that regardless of how the candy was packaged, the total number of pieces of candy remained the same. After using unifix cubes, the authors proceeded to utilize a series of computer-based microworlds that moved students from the concrete stage to the representational stage of development of the mathematical concept of place value. Situations which naturally involved addition or subtraction would then be introduced, such as putting newly made candy into the store room or removing candy to fill an order placed by a customer (Bowers et al., 1999).

While all students showed an improved understanding of place value through the use of the candy factory, there was wide divergence in that understanding. Some children still relied heavily on verifying that the total number of candies remained constant when the packaging was changed by recounting the candy. Some also utilized counting strategies, rather than place value position, to identify the total number of candies or finding new totals in addition and subtraction situations (Bowers, et al., 1999).

Hiebert and Wearne (1992) point out that the students in their study developed their understandings of place value over the course of an entire school year and the variety of levels of understanding indicated that place value is not "all or nothing" (p. 113). There seems to be a continuum of understanding when it comes to place value and it is not necessary to understand all of it in order to utilize it. According to this study, there was no indication of a direct link between conceptual understanding of place value and procedural ability. The authors believed that understanding of place value increased students' flexible use of strategies. Finally, the authors also indicated that there are many other factors that need to be investigated such as pedagogy and content in order to obtain a more detailed picture of how children learn with understanding and apply that understanding (Hiebert & Wearne, 1992).

A multitude of elements make up the effective instruction of place value. Some of these elements include: students' development of number sense, students' facility with multiple representations of numbers, the semiotic aspects of place value, and the specific instructional practices that are used in the classroom. These aspects of place value instruction and understanding are strongly connected to the instruction and understanding of multidigit computation.

Multidigit Addition and Subtraction

If the traditional instruction of place value has been done without any degree of depth (Cawley et al., 2007), traditional instruction of computation has often not been much improved. The tradition for teaching multidigit addition and subtraction is to teach a standard procedure such as the regrouping alogorithm, and expect that all students will utilize the procedure correctly. When students make mistakes, teachers frequently make

little attempt to analyze why the mistakes are made and simply reteach the procedure and provide more practice (Fuson et al., 1997). Additionally, the instruction of multidigit addition and subtraction is often extended over several grades when compared to the same instruction in other countries such as Japan, China, and Russia (Fuson & Briars, 1990). Fuson and Burghardt (2003) reported that much of the research on multidigit instruction has focused on two-digit numbers. Procedures that work well for two-digit numbers do not always generalize to larger numbers. Baroody (1999) suggested that there is a need for children to develop an understanding of the interconnected relationship between addition and subtraction. The results of his study indicate that this relationship is not obvious to young children and is not easily taught. While the inverse relationship between addition and subtraction seems to be difficult to attain, it is an important aspect of number sense and carefully structured activities that allow children to discover this concept over time are part of the foundation that children need to add and subtract multidigit numbers.

The difficulties that children encounter in understanding place value can lead them to "use for a long time unitary conceptual structures for two-digit numbers as counted collections of single objects or as collections of spoken words" (Fuson & Briars, 1990, p. 181) rather than seeing the numbers as collections of tens and ones. This can delay the understanding of adding and subtracting multidigit numbers. Children will often view each digit in a multidigit number as a single entity rather than as a component of a larger number. This perception can cause a profusion of errors when children need to add or subtract multidigit numbers with regrouping. Children need to make connections between written number words and numeral notation and give meaning to

these connections in order to effectively use and understand the base-ten number system and add and subtract multidigit numbers (Fuson & Briars, 1990).

Much of the literature regarding the teaching of multidigit addition and subtraction supports the use of innovative instructional settings and techniques to develop children's conceptual understanding and flexible use of strategies (Bowers et al., 1999; Fuson, 1990; Hiebert & Wearne, 1992, 1996; Kamii, Lewis & Livingston, 1994; Sowder & Schappelle, 1994; Steffe, Cobb & von Glaserfeld, 1988). Fuson and Briars (1990) utilized a learning/teaching approach to assist students in making connections between base-ten written numerals, the English number words, and adding and subtracting multidigit numbers. Students' actions were constrained through the use of a calculating board (see Figure 1) that included a place value chart, space to utilize base-ten blocks, and space for numerical representation.
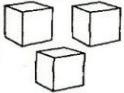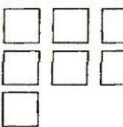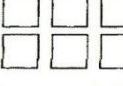
Figure 1: Fuson & Briars Calculating Board for an Addition Problem

When suing the board, practices included: a) immediately connecting the base-ten blocks to the written marks; b) connecting words that represented the place value, the base-ten block designation (e.g. longs for tens, flats for hundreds, etc.), and the numeral words; c) allowing students to move from concrete representation to written form as they were

comfortable doing so; d) closely monitoring students when they moved to the written form to make sure they were not practicing errors; e) utilizing four-digit numbers for addition and subtraction; f) incorporating place value instruction into addition/subtraction instruction after the first few days; and g) using a trade first algorithm for subtraction that involved regrouping. The trade first algorithm was used in place of the traditional algorithm because it eliminated the need to continuously switch between regrouping and subtracting. The students in the study demonstrated significantly more competence adding and subtracting multidigit numbers than did students who received a traditional form of instruction. Additionally, the students from the study were able to correctly determine whether or not a problem had been solved accurately. This learning/teaching approach shows promise as one way to help students achieve conceptual understanding of multidigit addition and subtraction while developing competence with a specified algorithm.

Hiebert and Wearne (1992) used an instructional approach built upon connections between external representations in order to build internal mental models that would increase the development of conceptual understanding of both place value and addition and subtraction computational practices. The use of various representations (physical, pictorial, verbal, and symbolic) was sequenced for the instruction. Story problems were used to represent quantities and the actions on quantities. Discussions took place in the classroom as both students and teachers explained their solution strategies. Students who received this instructional approach showed more versatility in their use of strategies when compared with students who were taught using a traditional approach that followed the instruction outlined in the textbook. Many students were able to extend their

understanding to regrouping of addition problems without explicit instruction in regrouping. However, many students continued to have difficulties with subtraction which would indicate that there are additional cognitive needs for students when attempting to master subtraction. The researchers were unable to determine a cause and effect relationship between the instructional techniques and conceptual understanding. Hiebert and Wearne believe that understanding evolves over time and is not an "all or nothing" type of understanding. In other words, a child does not need a complete understanding of place value in order to be able to grasp computation concepts, and computation can help develop the understanding of place value.

Fuson and Burghardt (2003) studied the use of small groups and teacher support of problem solving and reflection. The researchers concluded that: a) cooperative learning groups and manipulatives must be used with care and consideration in order to promote conceptual learning; and b) conceptual learning "promotes adaptive expertise and flexibility" (p. 268) when compared to rote learning of algorithms. Students were taught how to work in cooperative learning groups, and roles were assigned and rotated. An adult supervised each group and prompted students to make connections between representations in order to decrease errors made on computation tasks. In spite of the beginning instruction on working in cooperative groups, some groups had difficulty,the groups that worked well together and had the support of an adult to help them reflect on their invented methods were able to produce significant and sound procedures for solving multidigit addition and subtraction problems. The use of cooperative groups was shown to increase students' flexible use of strategies as they shared the different ways that they invented procedures. However, a problem emerged when students did not write down the

numerical representations of the strategies they were using. When they were required by

the instructor to connect their base-ten blocks to written notation by writing down the

numerical representations, the students were better able to understand traditional

algorithms for adding and subtracting rather than just being able to recite a procedure.

One significant finding was that the students' invented methods for adding and

subtracting were often conceptually or procedurally superior to the traditional algorithm.

As in the Hiebert and Wearne (1992) study, subtraction proved to be more problematic

than addition. The lack of commutativity of subtraction and the need to look at multidigit

numbers one column at a time with the traditional algorithm may contribute to this

difficulty. An inquiry approach to teaching multidigit computation is a viable route to

conceptual understanding when it is partnered with teacher feedback and manipulatives

such as base-ten blocks.

Fuson et al. (1997) reported on their work with four separate instructional

methods for promoting conceptual understanding of multidigit addition and subtraction.

The authors investigated children's conceptual understanding of multidigit numbers and

how the children used these conceptions to add and subtract. Each of the projects was

designed to assist children in understanding number concepts and operations. Carpenter,

Fennema and Franke utilized Cognitively Guided Instruction (CGI), Hiebert and Wearne

used Conceptually Based Instruction (CBI), and Human, Murray and Olivier utilized the

Problem Centered Mathematics Project (PCMP). Karen Fuson's project was Supporting

Ten-Structured Thinking (STST). All of the projects encompassed a conceptual problem-

solving approach and did not teach use of a single algorithm. Children were allowed to

spend time working out the problems with their own invented procedures and discussing

their solutions methods. A classroom environment was created in each project that encouraged the expectation that students were capable of generating their own solution strategies and did not have to have one "right" way.

In the CGI classrooms, students worked with word problems and used manipulatives to model their problem solving strategies. Place value understanding is not taught as a separate concept, but is allowed to develop through the use of problem solving. Students are allowed to operate at their level of understanding and progress at their own rate. In the CBI project, the sequence of instruction followed that of the textbook in order to permit a control comparison. Instruction progressed from place value to combining groups to addition and subtraction. Base-ten blocks were used as well as problems in various contexts. Students discussed their solution strategies. The PCMP classrooms were composed primarily of counting activities that promoted both depth and breadth of understanding. The counting activities promoted ways to solve addition and subtraction problems, and the students were given problems that coordinated with their level of developmental understanding. The STST project moved children from "a single accessible and generalizable strategy" (Fuson et al., 1997, p. 135) to the invention of their own strategies. Connections were made between word, numeral, and manipulative or pictorial representations. Students also used base-ten blocks to move from single-digit addition and subtraction to adding and subtracting four-digit numbers (Fuson et al., 1997).

The conceptual structures that children developed varied across all four projects. The structures were generally related to the instructional method used, with the CGI project children showing the most variability in the structures used. In the CGI

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classrooms, the students were not given any one specific way to solve problems. Students were allowed more leeway in this project to develop their own methods than in the other projects. Most children across all of the projects demonstrated an understanding of multidigit addition and subtraction. The conceptual structures that children bring to the classroom from their previous experiences seem to influence how they interpret and use new structures. In general, children will build new structures based on what they already know. This can enhance the understanding of new structures, or it can cause misconceptions or overgeneralizations. This idea ties in with the ideas related to the mathematics understanding that children bring to the classroom from their own families and outside school experiences. Commonalities to all of the project conceptual supports included problem-based situations and discourse regarding problem solution strategies. This is strongly related to my own study in that my students came to my classroom without prior experience explaining solution processes and with an instructional reliance on procedures rather than understanding. In chapters four and five, I make note that students' knowledge of the regrouping procedure for addition seemed to interfere with their ability to use alternative strategies for addition.

Bowers, Cobb, and McClain (1999) made use of an instructional strategy called the candy factory. This strategy was previously noted in the section regarding place value and had computational aspects that were somewhat unique. The candy factory became a real world context for addition and subtraction as more candy was added to the shop by the candy makers or candy left the shop as customers bought candy. The teacher and students initially acted out the scenarios packing and unpacking boxes and rolls of candy in order to fill customer requests. The students moved from acting out the

situations, to using a microworld that simulated the same practices, to recording transactions on an inventory sheet. The actions of packing and unpacking boxes and rolls of candy seemed to help students develop a better understanding of the more traditional regrouping method.

Multidigit computation is just one extension of the understanding of place value. Place value informs the understanding of computation, and computation can inform the understanding of place value (Hiebert & Wearne, 1992). The different instructional methods identified in this paper, CGI,STST, CBI, candy factory, etc., have all shown success in developing conceptual understanding of computation. For the purposes of my research, I chose to focus on the candy factory as the context for my instruction since I had used it during one of my graduate courses.

Summary

Effective mathematical instruction starts with the establishment of a classroom community through the use of norms. Norms are the processes and expectations that are developed by the teacher and the students and lead to effective mathematical communication. Discourse among students is one element that helps to encourage and develop conceptual understanding of mathematics. Many researchers have incorporated social and sociomathematical norms into their research as important elements to the overall development of conceptual understanding (Bowers et al., 1999; Cobb & Yackel, 1996; Fuson & Burghardt, 2003; Jones et al., 1996; Nickson, 1992; Yang & Cobb, 1995).

Place value is a foundational concept to other mathematical topics such as algebra, addition, and subtraction (Cawley et al., 2007; Hiebert & Wearne, 1992; Nataraj & Thomas, 2007). Place value is also a concept that can take years to fully develop.

Some children do not acquire an extensive understanding of place value until middle school (Cawley et al., 2007). From the research discussed in this chapter, it would seem that effective instruction of place value should be a priority in mathematics classrooms. It is not enough for students to know place value positions or the value of a digit within a number; but students need to be able to apply place value understanding in other areas of mathematics such as computation.

There are many strategies that effectively teach conceptual understanding of multidigit addition and subtraction (Bowers et al., 1999; Fuson et al., 1999). An understanding of place value helps to promote an understanding of computation, and an understanding of computation can further enhance the understanding of place value (Hiebert & Wearne, 1992). Additionally, the learning of traditional algorithms before gaining conceptual understanding can interfere with the development of conceptual understanding (Cawley et al., 2007). This would indicate that while the instruction of traditional algorithms is an essential component of mathematics curriculum, the teaching of traditional algorithms should be delayed until conceptual understanding has developed.

Place value can be a difficult topic for children to fully understand. It takes time and repeated exposures in multiple contexts in order to gain a full understanding of place value. The elements of classroom community, the instruction of place value, and the instruction of computation are strongly related to one another in the development of students' conceptual understanding of place value. Additionally, research regarding computation also makes reference to both norms and place value. All three aspects should be explicitly connected in the classroom to allow students to develop the deepest possible understanding of place value.

CHAPTER THREE: METHODOLOGY

Introduction

Action research is a form of research that addresses what takes place within a classroom. The purpose of action research is to solve a problem or gather information in order to inform practice (Fraenkel & Wallen, 2006). In this chapter I will describe the setting, the procedures and the methodology that I utilized in addressing my research questions. I investigated the following:

1. How does my use of conceptually-based strategies for teaching place value affect my students' ability to flexibly use strategies to add and subtract multidigit numbers?

2. How does my practice of making explicit connections among place value, number representations, and computation affect my students' ability to flexibly use strategies to add and subtract multidigit numbers?

Design of the Study

According to the authors of *Action Research for Teachers, Traveling the Yellow Brick Road,* action research is focused on the action that a teacher takes in response to a situation and the desire to make changes in the classroom for the betterment of student learning (Holly, Arhar & Kastan, 2005). My own reflective practice had shown me that computation was a problem for my students. Additional investigation through classroom assessment indicated that my students did not have conceptual understanding of place value. Therefore, I conducted an action research study with a group of six girls that met with me before school for math tutoring. This allowed me to collect information on my

students' learning of place value and how my students used this learning to understand multidigit computation with addition and subtraction.

<center>Setting</center>

My school is a diverse, urban, low socio-economic school with 75.3% of the students on free or reduced lunch. The school is within a large urban school district in the Southeastern United States. There are 729 students in grades prekindergarten through grade five. Of these students, 22.9% are African-American, 55.1% are Hispanic, 17.0% are Caucasian, 0.7% are Asian and 3.6% are listed as Other.

My action research was conducted with a convenience sample of six third-grade girls who were available to come thirty minutes before school started three days per week in order to participate in mathematics tutoring. Four of the girls were Caucasian and two were Hispanic. The girls were eight and nine years old and four of them were on free or reduced lunch. Three girls came from my morning mathematics class and three were from my afternoon mathematics class. Previous experience has shown me that girls whose mathematical ability is below grade level tend to participate less in whole class discussions of mathematics as compared to girls or boys with average to above average mathematical ability. When these girls are in small groups consisting of both boys and girls, they continue to participate less in group discussions than do boys. I had also previously conducted tutoring groups of all girls and found that the girls not only participated more frequently and openly in discussions, but also showed more improvement than girls who were in mixed gender tutoring groups. In addition to my own observations, research has shown that boys and girls learn mathematics differently.

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The context in which mathematics is presented can affect the performance of girls (Geist & King, 2008; Zohar & Gershikov, 2008).

According to the beginning-of-the-year district benchmark assessments of mathematics ability, all six girls were performing below grade level in mathematics. These assessments assess students' ability to solve problems based on state benchmarks. The benchmarks assessed are at the third-grade level and may or may not have been taught at the time of the assessment. Half of the state benchmarks are assessed at the beginning of the year covering the mathematical strands of number sense, measurement, geometry, and data analysis. In December, a second test is given covering the same benchmarks as the first test as well as an additional test that covers the rest of the state benchmarks including the mathematical strands of number sense, measurement, geometry, algebraic thinking, and data analysis. The data from these tests are then analyzed at the district level, and schools are provided with individual expected student performance data on the annual state-mandated benchmark assessment. These tests have been analyzed by Dr. Henry May at the Consortium for Policy Research in Education (CPRE) at the University of Pennsylvania and found to have a reliability estimate between .78 and .86, depending on the test. This reliability approaches that which you would expect with commercially prepared tests. To get a higher degree of reliability, more items would need to be added to the test (*Psychometric Report, 2008)*. At the time of this action research, district benchmark mathematics assessments indicated that Kaitlyn, Crystal, and Mary were predicted to perform *at grade level* on the state-mandated annual benchmark assessment; Kelsie was predicted to perform *below grade level,* and Annie and Kelly were predicted to perform *significantly below grade level* on

the state-mandated annual benchmark assessment. Calculations to determine these predictions were based on the number of correct responses and the difficulty of each problem that was answered correctly. For this reason, while Kelsie had a higher percentage of correct responses, (52%), than did Crystal, (48%), her predicted outcome was lower since she had a lower number of difficult problems answered correctly (see Table 1).

Table 1: Beginning-of-the-Year and Mid-Year District Benchmark Assessment Results

Student Pseudonyms	Number Sense		Measurement		Geometry		Algebraic Thinking		Data Analysis		Overall Assessment	
Kelly	15%	36%	20%	50%	0%	50%	N/A	33%	0%	20%	12%	36%
Kaitlyn	23%	71%	40%	75%	0%	50%	N/A	44%	25%	80%	24%	72%
Annie	23%	36%	20%	25%	33%	50%	N/A	22%	50%	20%	28%	32%
Crystal	23%	36%	60%	50%	0%	100%	N/A	22%	25%	60%	28%	48%
Kelsie	31%	36%	20%	100%	67%	50%	N/A	33%	0%	60%	28%	52%
Mary	31%	50%	20%	50%	67%	100%	N/A	55%	0%	80%	28%	60%

Scores of 0% to 29% indicate that the student *needs much improvement*. Scores between 30% and 39% indicate that the student *needs improvement*. Scores 40% and above indicate that the student is performing *on target*.

At the beginning of the school year, only Kelly was able to consistently add and subtract multi-digit numbers with regrouping. Mary was able to consistently add with regrouping and Kelsie, Crystal, Annie, and Kaitlyn could not consistently add or subtract with regrouping. None of the girls demonstrated a conceptual understanding of place value and Annie could not name place value positions with any consistency. By the time this action research started in January, all the girls had learned to reliably add multi-digit

numbers using the traditional algorithm, but still had little concept of the meaning of regrouping.

Methods

Data Collection

After receiving approval from my principal and the Institutional Review Board (IRB), I obtained permission from all the parents/guardians of my students to participate in the study. Additionally, I read and explained the assent form to my students and received agreement from them to take part in the research. Students were chosen by me to be part of a before-school tutoring group in order to increase their mathematics proficiency. Student work samples and journals were collected and reviewed throughout the study to assess student progress related to the concepts taught. Pseudonyms were used on all student work. A teacher journal was also kept with anecdotal records and observations of students and how they worked. Audio tapes were made of lessons or parts of lessons when new concepts were introduced or when it was necessary to address misconceptions that had arisen in previous sessions. Final interviews were also audio taped. The audio tapes were made so that I could have a more accurate record of student discussions for analysis. These audio tapes were then transcribed. A key of pseudonyms was used throughout the study to protect student confidentiality, and audio tapes were destroyed after they were transcribed.

Procedures

From the beginning of the school year, I had worked with all my students on place value concepts and alternative algorithms for solving multidigit addition and subtraction

problems. The girls who were chosen to be part of my tutoring group were all experiencing difficulty with mathematics concepts, particularly in place value and number sense. Analysis of their performance predictions for the state-mandated benchmark assessment indicated that the girls were capable of improving their outcome with additional instructional assistance. Kelly was the only exception; she had a 52% prediction of being in the lowest performance level. However, her performance in class indicated to me that she was capable of performing to a higher standard. I began this action research project in January, meeting with the six girls three mornings a week, outside of regular class time.

At our first meeting, I asked the girls questions about their understanding of place value. For example: (a) What do you think or know about place value? (b) Why do we learn place value? (c) In addition to what you have already told me, what else is place value good for? From there, I introduced the girls to the candy factory (Bowers, Cobb & McClain, 1999), and we called it "Cuffel's Candy Shop." Actual candy pieces were used to represent ones. Rolls of candy with ten pieces in each roll were created to represent tens. Boxes of candy were created with ten rolls to represent hundreds. The context for problem solving was an actual candy factory/store that made candy and sold it to customers. The candy could be packaged and sold in pieces, rolls, or boxes. The students were asked to find different ways to present one hundred pieces of candy for sale. The students created an inventory sheet to track the different ways that one hundred pieces of candy could be packaged for sale.

I created a projected order of instruction for this research project (see Table 2).

Table 2: Intended Order of Instruction

Week	Instructional Content
Week 1	Have students explain their understanding of place value and how place value is used. Introduce Cuffel's Candy Shop and have students practice finding different ways to package "candy" for specified numbers.
Week 2	Transition from using candy to base-ten blocks. Give students a group of ones blocks and have them find a way to count them all. Explain counting method.
Week 3	Allow students to investigate ways to add groups of base-ten blocks using the context of Cuffel's Candy Shop. Continue to work with students on developing their ability to explain and justify their choice of solution methods both orally and in writing.
Week 4	Add numerical designation to addition problems with base-ten blocks.
Week 5	Continue to practice with addition problems using base-ten blocks, transitioning to pictorial representations.
Week 6	Add subtraction problems within the context of Cuffel's Candy Shop. Make use of base-ten blocks and numerical designation.
Week 7	Continue to practice addition and subtraction problems, allowing students to choose how to represent the problems.
Week 8	Transition from invented methods to traditional algorithms.

However, the content of each lesson was adjusted based on my perceived needs of the students' performance in prior sessions. During the fourth session, base-ten blocks were introduced and were connected to the candy packaging. The students were given candy factory problems to solve involving addition of multidigit numbers. Work continued with counting of either base-ten blocks or candy pieces and adding in numerical representations in order to connect the manipulatives to the number representations.

After one month of sessions, the students were questioned again with the same questions that were used at the first session to see if understanding had changed. The students were also assessed regarding their ability to count base-ten blocks. Each girl was given a number of base-ten blocks that was based on my observations of their conceptual understanding to this point. The students continued to work with counting of base-ten blocks or candy packages, adding multidigit numbers with and without manipulatives and solving problems in the context of the candy factory. In late February, the students were given a two-digit plus two-digit addition problem to solve without manipulatives and were told they could not use regrouping. After this session, one session focused on explicitly explaining the nature of regrouping and how it connected to place value and counting numbers.

We continued to work with problem solving in the context of the candy factory, utilizing counting of total amounts of candy, adding new amounts of candy, and subtracting amounts of candy when customers made purchases. Students were expected to explain all work in all sessions. I asked probing questions to further access student understanding of their work and to further their practice of providing complete justifications of their problem solving processes. Some sessions were audio recorded in order to better preserve students' explanations of their work. At the end of the action research project, individual interviews were conducted in order to assess any changes in conceptual understanding of place value and multidigit addition and subtraction.

Data Analysis

Data were collected for this study through student work samples, student journals, teacher observations, teacher journal, and audio recordings of group work and interviews

which were then transcribed and analyzed. Tapes were kept under lock and key until

transcribed and then were destroyed. A key of pseudonyms was utilized. The key was

kept on a password protected laptop computer to which only the principle investigator

had access. The key was destroyed at the end of the study. Student journals included

written explanations of work that was performed during each session. Student work

included specific problem solving situations and the work that each student did to solve

the problem. The teacher journal provided anecdotal records of critical events as well as

teacher impressions of students' thinking and progress. Triangulation of data was made

possible through the use of multiple data collection tools: recorded group work and

interviews, student work, and journals, as well as my anecdotal notes and observations.

This allowed emergent themes to be verified through more than one source.

Limitations

One limitation to this study was the size and nature of the group used. There were

only six students and they were all girls. Additionally, the girls were all of a similar

proficiency level in mathematics. A second limitation was the restricted amount of time

that I was able to work with these students. We were only able to meet for thirty minutes,

three mornings a week, for eight weeks.

Summary

Action research was the most appropriate methodology for this study since I

desired to inform my teaching practice in order to improve student learning in regard to

place value and multidigit addition and subtraction. By changing the instructional

sequence as needed during the actual research, I was better able to meet the needs of each

student in the group in regard to their understanding of place value. Chapter four

provides a thorough analysis of the data, and chapter five identifies the conclusions that I

have drawn from this data.

CHAPTER FOUR: DATA ANALYSIS

Introduction

The purpose of this study was to examine my practice of using conceptually-based teaching practices in order to promote conceptual understanding of place value and increase third grade students' flexibility in representing numbers and in adding and subtracting multidigit numbers. As mentioned in chapter two, there are multiple paths to conceptual understanding of mathematical content. The methods addressed in chapter two have all been shown to have merit in producing conceptual understanding of both place value and multidigit computation (Bowers, Cobb, & McClain, 1999; Fuson et al., 1997; Fuson & Burghardt, 2003; Fuson & Briars, 1990; Varelas & Becker, 1997). As previously noted, an understanding of place value develops over time and during that time there is a continuum of understanding (Jones et al., 1996). It is not necessary to have a complete understanding in order to utilize place value concepts to flexibly represent numbers or to invent methods for adding and subtracting multidigit numbers (Hiebert & Wearne, 1992). In light of this, I chose to move through place value topics quickly at the beginning of the study and transition to computation topics as rapidly as possible so that the instruction of place value would inform understanding of computation and instruction of computation would inform place value at the same time. While I had planned a course of instruction, this course fluctuated on a weekly and sometimes daily basis as I attempted to meet the needs of my students and to gain insight into their understanding. Misconceptions often cropped up at the end of a session and I knew that

they needed to be addressed before proceeding further in the course of instruction or even before continuing with the same lesson.

As I reviewed my data, several themes began to emerge. Even though I did not explicitly teach social or sociomathematical norms throughout the study, these types of norms eventually began to surface within the group of girls. As various norms came to light, I encouraged their development and allowed the girls to negotiate much of it on their own. A second theme that became apparent was that the girls tended to stay with a single way of counting and did not seem to be inclined to change methods even as they discussed different ways of counting. This aspect was highlighted by the fact that when given the choice to choose any computation method, they all chose traditional regrouping. A third theme that surfaced was the instability of their application of place value. While the girls appeared to maintain their understanding of place value and how it could be used for computation, their stability with this understanding came into question when the same types of lessons were conducted within a different setting. The difficult nature of teaching place value (Cawley, Parmar, Lucas-Fusco, Kilian, & Foley, 2007; Nataraj & Thomas, 2007) became much more apparent as I analyzed my girls' interactions and progress toward understanding.

Progression of Norms Development

Initial Responsiveness

When this action research started, the social norm of explaining answers to mathematical problems had already been established within my classroom. The girls in my research group knew that they were expected to explain their answers and solution processes. However, since these girls rarely participated in whole class discussions, it

took time for them to become comfortable explaining answers within this particular group. My observations indicated that the girls were unsure of their ability to explain as well as unsure if their answers were correct. Their manner when explaining answers was hesitant and full of pauses and looks to me for assurance that they were saying the "right" thing. They were often easily confused within their explanations, and this often produced embarrassment and a lack of willingness to either continue or start over. The girls were often unsure of the vocabulary that was necessary to give an effective explanation. Both written and verbal explanations followed a similar pattern. The girls often referred back to their written explanations or looked at the manipulatives to support their verbal explanations.

On one such occasion, each girl was given a set of Cuffel's Candy Shop candy packaging (boxes, rolls, and pieces) and asked to find the total number of pieces of candy. The girls were asked to document how they counted the candy and obtained a total. They first wrote down their explanation and then explained verbally to the group. An example of one student's written response, Kelsie, follows.

> Kelsie's written response: Answer 214. I got my answer because you have one box then count by tens and then count by ones and that's how you get 214.

Kelsie initially recounted the same response verbally. I asked her probing questions to draw out a more thorough explanation. I was careful to use appropriate mathematical vocabulary in order to encourage its use by the girls. By asking questions and providing vocabulary, Kelsie was able to give a more thorough response as indicated below.

> Instructor: I have some questions for you. How did you know that one box was equal to 100 pieces?

Kelsie: Um, there are, no, yeah, 100 pieces in a box.

Instructor: Okay. What did you do when you counted the rolls by tens?

Kelsie: I looked at my rolls and I counted 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, um, let me start over. 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 (pauses and looks to instructor in order to count one more ten).

Instructor: 110.

Kelsie: Yeah, 110.

Instructor: What did you do next?

Kelsie: I kept counting the ones, 111, 112, 113, 114.

Instructor: What are the ones the same as?

Kelsie: The ones are the pieces.

Instructor: Then what did you do?

Kelsie: Um, I . . . (looks at manipulatives). I forget.

Instructor: Let's think about what you have said. You said that one box was 100 pieces and your rolls and pieces totaled 114. Now you need to tell me the total amount of all of the pieces.

Kelsie: (talking quietly to herself) 100 and 114 . . . 114 and 100 is, yeah! 214.

Instructor: 214 what?

Kelsie: 214 pieces.

Explanations continued to require support for some time. The support took the form of scaffolded questioning and providing mathematical vocabulary as needed. When counting quantities of candy or base-ten blocks, the girls were encouraged to write it down as they went in order to more fully remember the methods they employed while counting.

Further Progress

Three weeks into the sessions, all of the girls except Kelly had become more confident in their ability to explain their answers and seemed to have less of a need to be "right." Kelly still became easily flustered if she made a mistake in her explanation such as counting tens as hundreds. This was a persistent problem for Kelly that had not entirely gone away by the end of the study. However, while the girls were more comfortable giving their explanations, they were still completely accepting of others' explanations, even if they did not make sense or were not clear. One example of this is the day I had the girls count an unknown quantity of ones cubes. Each girl was given a handful of ones cubes and asked to count them any way that they desired in order to find the total number of cubes. Kaitlyn and Annie made groups of five and then counted by fives plus the extras that did not make a group of five. Mary used the same procedure, but made groups of ten. Crystal made groups of ten, but then counted by ones to get the total. Kelsie made groups of nine and attempted to add by nines using her fingers, but was unable to arrive at a total. She would repeatedly lose track of the number of cubes she had already counted. Each girl explained how they arrived at the total number of cubes to the rest of the group. Kelsie admitted that she had been unable to come up with a total. When I asked the other girls what they thought of the way Kelsie had chosen to count the cubes, they were all very supportive of her and said that it was just another way to count. Nothing was said about the fact that she could not get a total or that it might have been easier to count by a different number. When asked, Kelsie said she just "decided to count by nines."

It was not until two weeks later when the task was to solve a two-digit plus two-digit problem that any questioning of answers or methods done by others in the group came up. The girls were told that they could use any method to add the numbers except the traditional algorithm. The problem was 38 + 57. The correct answer is 95. When Crystal came up with an answer of 815, Kelsie spoke up and said that she did not understand Crystal's answer. She said that it "did not make sense." Annie had the same solution as Crystal, and from that point a lively discussion ensued about why the others thought these answers did not make sense and what Crystal and Annie should do to correct the problem. Unfortunately, the discussion focused on the idea that if the problem was regrouped, the answer would not be 815, it would be 95. Instead of using an understanding of place value to talk about repackaging 15 ones into 1 ten and 5 ones and then combining the 1 ten with the 8 tens, they focused on a "regrouping or traditional algorithm" mistake. Suggestions were made to "take out the 1" and "change the 8 to a 9" with no discussion as to why they should do this. Kelsie's answer of 85 was addressed in a similar fashion. After she explained her answer, Kaitlyn said she "forgot to carry the one." Kelsie's response was that I had said "we can't regroup." Again, the thinking went back to the traditional algorithm without an understanding of its meaning. However, from this point on, the girls were more willing to challenge others' explanations and ask questions when they did not understand an explanation.

At this first instance of questioning an answer, I made a big deal of how impressed I was by Kelsie's thinking and willingness to question an answer that did not make sense to her. From that one moment of validation, the doors were opened to a new dimension of mathematical thinking for the girls. Each time we tackled a new problem,

the girls worked together and talked about and through their solution processes. The girls would partner together in order to solve a problem. I observed Crystal and Kelsie solving a 3-digit plus 3-digit addition problem during one session. They both set up base-ten blocks to represent each number of the problem. They then pushed all the blocks together and began to count the total amount. During this problem they interchanged terminology for the blocks. They called the hundreds, boxes, the tens were called tens, and the ones were called pieces. They then negotiated a starting point for counting. They agreed to start with the boxes and wrote down the total number. They then counted the tens and wrote down that total. Finally they counted the pieces and wrote down the total. At this point there was some discussion about how to find the final total. Crystal wanted to add the numbers on the paper and Kelsie wanted to count all the blocks together. Each girl did it her own way and both finished with a correct answer.

I also saw and heard evidence of increased understanding of place value and computation through their conversations and the apparent comfort level that they displayed with one another. There also seemed to be no confusion if one girl used boxes, rolls, and pieces while another girl used hundreds, tens, and ones to describe a solution process. When Kelly would mistakenly count tens as hundreds, Mary usually stopped her and made a comment such as, "Don't you mean 80?" indicating that she should have counted by tens rather than hundreds. The girls also began to recognize that if a problem consisted of 5 boxes, 8 rolls, and 7 pieces, the total number of pieces was 587 without doing any counting. Mary also began to recognize the total number of pieces in an addition problem without actual counting. If she had a total of 6 boxes, 12 rolls and 15

pieces, she automatically began combining the 10 rolls from the 12 rolls into a hundred and the ten pieces from the 15 pieces into a ten and mentally adding the total amount.

I also observed an increase in mathematical vocabulary within their explanations. When the girls explained their solutions or when they worked together, they used the terms hundreds, tens, and ones or boxes, rolls, and pieces. They remembered more often to include units when talking about their solution processes. They also easily switched from talking about boxes, rolls, and pieces, to talking about hundreds, tens, and ones. In one conversation, Kelsie mixed unit types referencing one box and then she continued counting by tens and ones. I more consistently used the terms boxes, rolls, and pieces. As the sessions progressed and I intermixed the use of the terms, the girls also began to use both sets of terms interchangeably and apparently comfortably. The following conversation regarding a problem in Cuffel's Candy Shop is one example of this vocabulary use.

> **Problem: Ms. Cuffel has 2 boxes, 5 rolls, and 6 pieces of strawberry candy in her candy shop. The candy makers brought in an additional 3 boxes, 8 rolls, and 5 pieces of strawberry candy. How many pieces of strawberry candy does Ms. Cuffel have in the candy shop now?**

> Instructor: Annie, please explain how you solved your problem.

> Annie: Two boxes and three boxes is five boxes, that is 500 pieces. And five tens and eight tens is 12, no 13 tens. That is (counts quietly by tens) 130 pieces. Then six ones and five ones is 11 pieces.

> Instructor: What do you need to do now?

> Annie: I need to get the total amount.

> Instructor: How will you get it?

> Annie: I have to add 500 and 130 and 11.

> Instructor: How will you add it?

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Annie: (She takes time to think and proceeds slowly.) 130 and 11 is 141. And 500 more is (long pause) 641 pieces.

Annie used candy factory vocabulary to talk about the hundreds, base-ten vocabulary to talk about tens and used both to describe the ones. The girls often mixed the different types of vocabulary. The problems were always presented in the context of the candy factory and used that vocabulary. Either set of vocabulary was acceptable for explanations.

A New Level of Participation

The apparent ease with which the girls began to discuss their mathematical thinking led me to believe that their understanding was increasing. The increased confidence that I observed as they explained answers and questioned others' reasoning carried over into the regular classroom causing increased participation during whole group instruction. This increased participation was particularly evident when I began using the candy factory within my regular classroom. Most of the girls repeatedly raised their hands in order to respond to a problem situation. Kaitlyn, Kelly, Mary, and Kelsie easily gave responses before the whole class using boxes, rolls, and pieces interchangeably with hundreds, tens, and ones. One example of this is Kaitlyn's response for a way to package 437. Her hand was eagerly raised and she was bouncing in her chair. When I called on her, she confidently explained her solution. Her solution was 3 boxes, 13 rolls, and 7 pieces and she explained that 3 boxes was 300 pieces, 13 rolls was 130 pieces, and 7 pieces was 7 pieces. She then added the pieces as follows: 130 pieces plus 7 pieces was 137 pieces. Then 137 pieces plus 300 pieces was 437 pieces. While this solution process was missing some elements of a completely effective explanation

(she did not explain how she knew that 3 boxes was 300 pieces), she had the most important elements.

Even though the girls did not always have the right answers, they were volunteering to answer and to explain their thinking. However, that level of participation was still shadowed by the mathematical ability level of each girl. Kaitlyn and Mary were the most enthusiastic in their regular classroom participation, while Kelsie and Annie participated more hesitantly. Even with the hesitation to participate, all the girls were still willing to explain their solution processes if I called on them.

Summary

Supported by the research of many (Bowers, Cobb, & McClain, 1999; Fuson & Burghardt, 2003; Fuson et al., 1997; Jones et al., 2007; Lopez & Allal, 2007; Yang & Cobb, 1995), I have seen within my own research group how social and sociomathematical norms are important for developing mathematical understanding. Discussions also provided opportunities to use mathematical vocabulary. Discussion of mathematical thinking brings to light not only children's knowledge and understanding, but misconceptions as well. It was often our conversations that caused me to change the course of instruction in order to address misunderstandings and a lack of depth of understanding throughout the study.

How Many Ways Can I Count?

Beyond the development of social norms, one of my goals was to increase student flexibility in both number representations and computational methods. In spite of the discussions about solution methods, the girls' methods were often not different enough to encourage this flexibility. While the use of the candy factory (Bowers et al., 1999;

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Cawley, Parmar, Lucas-Fusco, Kilian, & Foley, 2007; Hiebert & Wearne, 1992) has been shown to increase both flexibility in counting and computation, this was less apparent within my study. Cuffel's Candy Shop represented our candy factory, and the students used pieces of candy and packaged them as pieces (ones), rolls of ten pieces (tens), and boxes of ten rolls (hundreds). Inventory sheets (see figure 2) were used to keep track of the various ways that the candy could be packaged.

Cuffel's Candy Shop Inventory Sheet			
Boxes	Rolls	Pieces	Total Pieces
1	5	6	156
	15	6	156
1	3	26	156

Figure 2: Sample inventory sheet for Cuffel's Candy Shop

Initially the girls worked together to actually package the candy in order to have a physical connection to the packaging process. In later sessions of this activity, the girls used the individual pieces, but instead of packaging rolls of ten pieces or boxes of ten rolls, they used empty rolls and boxes to represent how the candy was packaged. After three weeks, we transitioned to using base-ten blocks in place of the candy packaging, but continued to utilize the context of Cuffel's Candy Shop.

As we began to use the inventory sheets to find multiple ways to represent numbers, I explained to the girls that the inventory sheets were like the Name Collection boxes that we used in our core mathematics curriculum. In Name Collection Boxes, the students created different ways to name a number (see figure 3). However, most of the

ways that students choose to name a number have little, if anything, to do with place value.

$$56$$

$$60 = 4$$

fifty-six   50

QQNP = 56¢

♥♥♥♥♥♥♥♥♥♥♥♥
♥♥♥♥♥♥♥♥♥♥♥♥
♥♥♥♥♥♥♥♥♥♥♥♥
♥♥♥♥♥♥♥♥♥♥♥
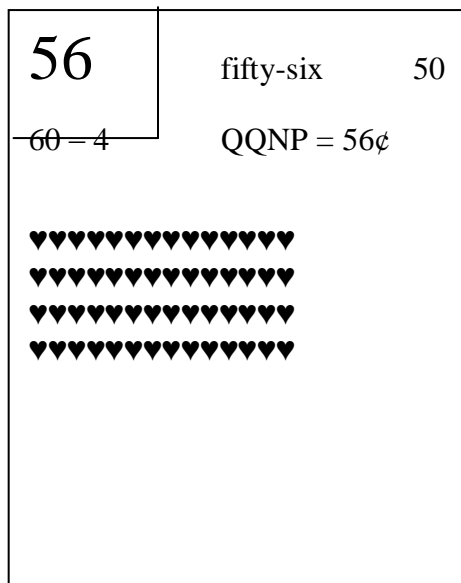
Figure 3: Sample Name Collection Box

Once the girls had been practicing with alternative representations of numbers, they began to see a pattern emerge and could follow the pattern if it was in front of them. Without the pattern, they had greater difficulty creating equivalent representations and did not make connections between the different representations and how they might be used for adding and subtracting multidigit numbers. This difficulty continued to persist. While the girls could give the initial representation of boxes, rolls and pieces of a number [291 is 2 boxes (200), 9 rolls (90), and 1 piece (1)], they still had difficulty finding different representations and determining if the alternate representations equaled the same total.

Figure 4 is a sample of Kelly's work where I had scaffolded the pattern for her near the beginning of the study. You can see the pattern of increasing the number of rolls

and decreasing the number of pieces.

Figure 4: Kelly's patterned candy factory inventory sheet

Figure 5 shows Kelly's work that was completed independently later in the study
and indicates a lack of reliance on a predictable packaging pattern. The changes in
packaging are more random, and as she explained her solutions, it became apparent that
her thinking was random as well. She would simply choose a number of boxes to use and
then choose a number of rolls and then get stuck trying to figure out how many pieces she
needed to make up the total number of pieces. While some of her packaging solutions
appear to be rather sophisticated (2 boxes, 19 rolls, and 66 pieces), some of her packaging
solutions are incorrect either in the total number of pieces for a quantity of rolls or in the
total packaging being equal to 456. It is my belief that she had not yet internalized the
pattern and still struggled with applying place value understanding in a given situation.

Boxes (100s)	Rolls (10s)	Pieces (1s)	Total Pieces
4 + 400	5 + 50	6 6	= 456
3 + 300	10 + 110	56 56	= 456
2 + 200	20 + 220	55	= 456
2 + 200	2 + 20	56 56	= 456
1 + 100	3 + 300	56 56	= 456
0 + 400	40 +	5 6	= 456
3 + 300	14 140	16	= 456
2 + 200	19 + 140	60	= 456

Figure 5: Kelly's independently created candy factory inventory sheet

In addition to counting quantities of candy, we also used candy factory problem solving situations that involved multidigit addition. There is some belief that the use of strictly two-digit numbers before three- or four-digit numbers is of limited value for promoting place value understanding or in developing flexibility in computation (Fuson & Burghardt, 2003). Even though there is evidence that two-digit numbers are of limited value for increasing place value understanding, I started problem solving with two-digit numbers to allow the girls time to adjust to this method of addition. However, we quickly transitioned to using three-digit numbers.

When it came to adding multidigit numbers, the girls were comfortable using base-ten blocks to find a total. At some point in their counting, each of the girls pushed all of each kind of block together and then counted them up, sometimes keeping track on paper of the total amount of hundreds, tens, and ones and then adding those numbers. Their first encounter adding base-ten blocks showed very similar methods of counting. In Table 3, each girl's counting method is explained. With one exception, Kelly, each girl pushed all the blocks together and then counted. Also note that most of the girls all counted the blocks twice to get their answer.

Table 3: Counting method used to add groups of base-ten blocks

Student	Method of Counting
Kelsie	Pushed each type of block (ones, tens, hundreds) together and wrote down the total for each type. Then went back to the blocks and counted them all to get a total
Kelly	Counted each type of block and then pushed each type together and recounted all the blocks.
Kaitlyn	Pushed each type of block together, counted each type, and wrote down the total for each type. Then used mental math to add the numbers she had written down.
Annie	Pushed each type of block together, counted the number of hundreds, and said the total. Then counted up the tens, said the total, and continued counting the ones. Then went back and counted all again.
Mary	Pushed each type of block together. Counted the hundreds. Then counted the tens and ones together and mentally added that number to the total number of hundreds.
Crystal	Pushed each type of block together. Counted each type and wrote down $100 + 70 + 13$. Then went back and recounted the blocks for the total.

Kelsie, Kelly, Annie and Crystal counted by the various units (ones, tens, and hundreds), but were not secure in their ability to find a total without recounting. Crystal was able to create a number sentence to represent the total of each unit, but was unable to add from the number sentence. She still chose to go back to the base-ten blocks to find the total. Kaitlyn and Mary were more confident of their ability to add all the blocks, moving from one unit to the next. Kaitlyn kept track on paper and did her mental addition from the numbers on the paper, while Mary maintained a continuous count from one unit to the next. Discussion of the various methods used did not raise any questions, nor did any of the girls feel that there were any problems with the methods they used. This was still early in the study, and we had not yet reached a level of discussion that encouraged questioning of methods.

A week later, the girls were given a two-digit plus two-digit addition problem. They were told that they could use any method to add except traditional regrouping. This caused a great deal of consternation. None of the girls was sure how to start. After some discussion about the options available for adding, they were able to choose a method. This is the same session where we had a breakthrough with social norms. Questions and challenges came up during the discussion of solutions. In spite of their apparent comfort using base-ten blocks for addition, this was not a first choice option for most of the girls. Table 4 shows the computation methods that each girl used and the resulting answers.

Table 4: Two-digit plus two-digit computation methods

Student	Computation Methods for adding 38 + 57
Annie	Added the ones and wrote down 15, then added the tens and wrote down 8, for a total of 815.
Kelsie	Used base-ten blocks. Combined the tens and the ones and then added them together for a total of 95.
Mary	Counted out 8 tens and 15 ones from the base-ten blocks and then added them together for a total of 95.
Kaitlyn	Used base-ten blocks. Counted the ones to get 15 ones. Recognized this as 1 ten and 5 ones. Added the 1 ten to the 8 tens to get 9 tens for a total of 95.
Kelly	Used base-ten blocks. Counted tens as hundreds and wrote down 8. Counted ones and wrote down 15 for a total of 815.
Crystal	Counted ones and wrote down 5. Counted tens as ones and wrote down 8 for a total of 85.

While the girls were not permitted to use the traditional regrouping algorithm, Kaitlyn evidenced a beginning understanding of the concept of regrouping with the explanation of her solution process.

Instructor: Kaitlyn, please explain how you got your answer.

Kaitlyn: Well, first I added 3 and 5 and got 8.

Instructor: Added 3 and 5 what?

Kaitlyn: Oh, yeah, I added 3 tens and 5 tens and got 8 tens.

Instructor: Okay. What did you do next?

Kaitlyn: Well, then I uh, um, added 8 (pause) um, ones and 7 ones and got 15 ones.

Instructor: Then what did you do?

Kaitlyn: Okay. So I um, had 15 ones and 8 tens. (Pause). And then I thought that um, 15 ones is um 1 ten and um 5 ones. So I uh I could um make a ten and put it with um the other tens and uh that made um 9 tens. And that's 95.

Instructor: How did you know that you could make a ten?

Kaitlyn: (Pauses to think). Well, I had uh 15 ones. And um 10 of the ones is a ten.

Instructor: Thank you for your explanation.

There were no comments or questions about Kaitlyn's explanation. Crystal's explanation

showed that she did not have the same understanding as Kaitlyn about regrouping.

Instructor: Crystal, would you please explain how you solved the problem?

Crystal: Well, first I added um 8 ones and 7 ones and got 15 ones. And I wrote down a 5.

Instructor: Then what did you do?

Crystal: Well, then I added 3 ones and 5 ones and got 8 ones.

Instructor: What do you mean? Are the 3 and the 5 ones?

Crystal: Um, well, I thought I could count um them as ones.

Instructor: What answer did you get?

Crystal: 85.

Instructor: Kaitlyn got 95. Which answer do you think is correct?

Crystal: I don't know.

At this point, I wanted Annie to explain her solution process. She had seemed to

have a great deal of difficulty when told that she could not use regrouping in the form of

the traditional algorithm. She seemed very unsure of herself, but did not attempt to use

base-ten blocks.

Instructor: Annie, would you please explain your answer.

Annie: Well, I didn't really get it.

Instructor: Tell me what you tried.

Annie: Well, you said I couldn't regroup. So, I um added the ones and got 15. I didn't know what to do so I wrote down 15. Then I added the tens and got 8 and wrote down 8.

Instructor: So what is your answer?

Annie: 815.

Kelsie: That doesn't make sense.

Instructor: Why doesn't that make sense?

Kelsie: It can't be 815. The one can't be there.

Instructor: What do you mean?

Kelsie: It doesn't work.

Instructor: Anyone else?

Mary: You can't just put down 15 ones.

Instructor: Why not?

Kaitlyn: You need to change it.

Instructor: How do you need to change it?

Mary: Leave out the one.

Kelsie: Change the 1 to a 9 so you get 95.

Though Mary, Kaitlyn, and Kelsie understood that there was a problem, they had difficulty specifically stating their objections. Kelly did not participate in the discussion at this point since she had gotten the same answer as Annie. I then asked the girls how we could tell that the answer 815 did not make sense to the problem 38 + 57. I also asked

Kelly how she got 815 as the answer. She responded that she had added 3 tens and 5 tens and got 800. She then added the ones and then added 15 to 800 for a total of 815. When I restated the first part of her explanation, she looked confused, but was uncertain of the problem. It took several more minutes of discussion before she realized that 3 tens and 5 tens is 8 tens for a total of 80.

When given the option to solve a multidigit addition problem in any way they desired, every one of the girls chose to use the traditional regrouping algorithm. By now, all the girls could accurately regroup multidigit addition problems most of the time, but their explanations were primarily procedural in nature. For example, the explanation for $86 + 35 = 121$ would proceed as follows: $6 + 5 = 11$, put down the 1 and carry the 1. $8 + 3 = 11$, plus 1 more is 12, put down the 12. The answer is 121. This indicated to me that while the girls could add with regrouping, most of them still did not have a conceptual understanding of what they were doing. In another example, each girl was given a multidigit addition problem that I thought was commensurate with her current level of proficiency with place value and multidigit addition. In this instance they were asked to deliberately use the regrouping algorithm and then write an explanation of how they solved the problem. As the explanations in Figures 6, 7, and 8 seem to suggest, their understanding of regrouping was still at a procedural level. In each written explanation, regrouping is "putting the number on top." Crystal does use the word "regroup," and Mary indicates that she puts "it in the tens (hundreds) place," but there is no indication in the written responses that they understand what they are regrouping.

$$\begin{array}{r} \overset{1}{1}27 \\ +\ 184 \\ \hline 211 \end{array}$$

frist I did 7+4 and got 11 so I had
to regroup so I put it on top of the
2 then I Jcst added 1+1=2 and I
got all together 211
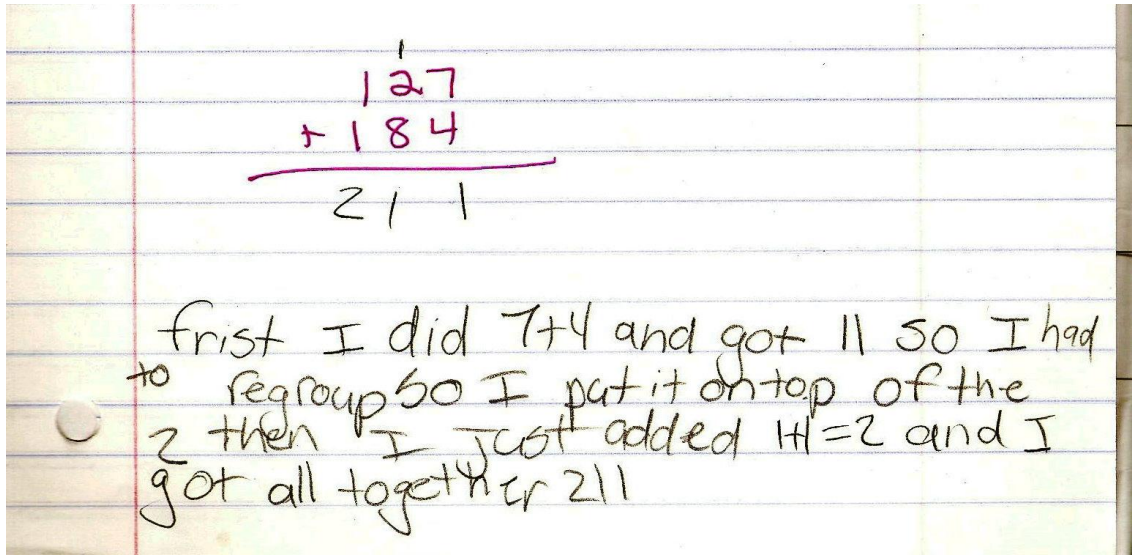
Figure 6: Crystal's explanation of regrouping

Crystal explained that she added numbers and regrouped. However, even though she added the tens correctly, she forgot to regroup to the hundreds column and produced an incorrect answer. Her verbal explanation also gave no indication that she conceptually understood the procedure.

$$\begin{array}{r} \overset{1}{1}23 \\ +\ \ 86 \\ \hline 209 \end{array}$$

6+3= 9 and
8+2= 10 but I
put left the 0
and put the 1
on top of the
next number.

Figure 7: Kelsie's explanation of regrouping

Kelsie had been able to add with regrouping since the beginning of the school year. While she was proficient in computation, her conceptual understandings of mathematics were very weak. In this example, even her procedural explanation is

missing information. Her written explanation also gave no indication that she understood the procedure at a conceptual level.

Figure 8: Mary's explanation of regrouping
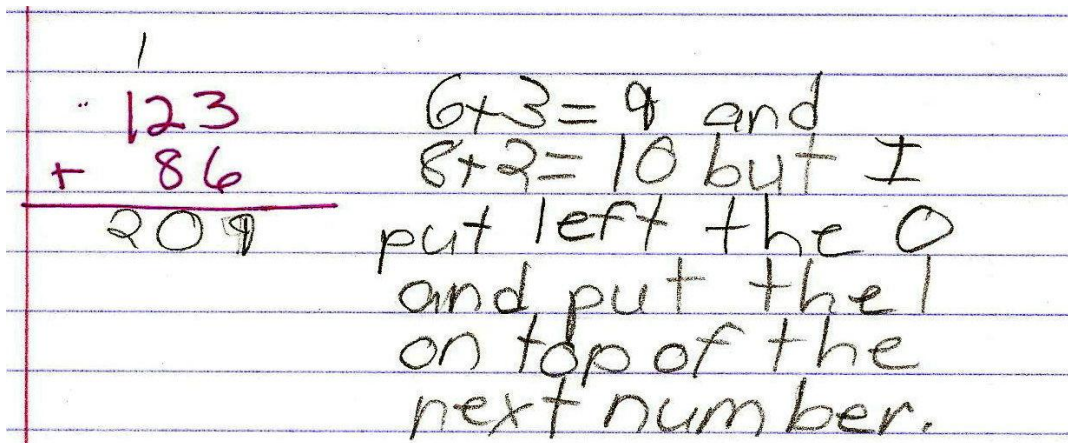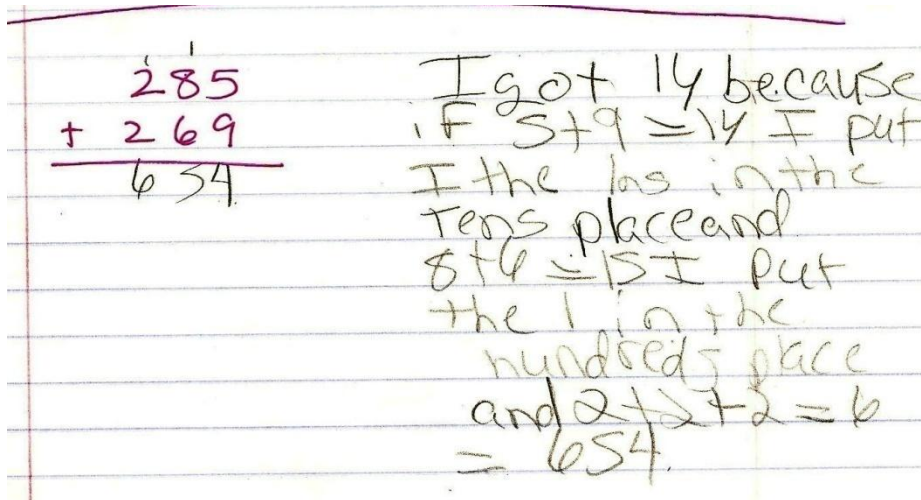
While Mary has the procedure correct, she made two computational errors. She says that 8 + 6 equals 15. When questioned during her verbal explanation, she corrected herself to say that 8 + 6 + 1 equaled 15. I also asked her to check the addition of her hundreds, which she did, and corrected the answer to 554. Although she mentioned regrouping to the next place value position, she had not indicated that she knew what she was regrouping.

My data seem to suggest that the girls' flexibility in representing numbers was increasing, but was not always accurate. Further, the girls seemed to lack of flexibility for multidigit addition. The girls either resorted totraditional regrouping when presented with a problem or used base-ten blocks and their counting procedures. At the very end of the study, the girls were moving away from using the actual blocks and attempting to use mental computation. As a result frequent mistakes in addition were made as they either left parts of numbers out when adding or combined numbers inappropriately.

Instability with the Application of Understanding

As we neared the end of the study, I felt comfortable that while the girls did not exhibit a lot of flexibility with computation, they were gaining a solid understanding of place value and its application to flexibly representing numbers since they were able to find multiple ways to represent quantities of candy with the inventory sheets. However, my inclinations seemed to be misplaced. As I began to use the candy factory with my entire class, I anticipated that the girls would take the lead in helping the other students to develop their own understanding of place value. This is not what happened.

The first time I introduced the candy factory to the entire class and began to teach about multiple representations of numbers, every girl expressed some level of confusion and did not seem to remember what we had previously done. By the end of that first lesson, they had remembered the principle ideas, but were still having difficulty accurately representing numbers in different ways. The same error patterns and randomness that was mentioned in the previous section were still taking place, though with less frequency for some of the girls. The work samples that follow were all completed independently following whole class instruction.

Boxes (100s)	Rolls (10s)	Pieces (1s)	Total Pieces
6 + 600	7 700 +	5 50 =	675
6 + 600	6 100 +	7 5 =	675
6 + 600	2 200 7	6 60 =	675
6 + 600	100 +	5 50 =	675
6 600	3 300 I	4 40 =	675
6 + 600	4 400 +	5 50 =	675
66 + 600	5 500 +	7 70 =	675
500 + 5	7 70 +	5 50 =	675

Figure 9: Crystal's representations of 675

In Figure 9, you can see that Crystal seemed to have forgotten how many pieces of candy are in a roll. She consistently counted the number of rolls as hundreds except for one time. She also apparently did not accurately add the total number of pieces, simply assuming that her representations were 675.

Boxes (100s)	Rolls (10s)	Pieces (1s)	Total Pieces
0 +	0 +	675 =	675
6 + 600	7 + 70	5 = 5	675
6 600 +	0 0 +	75 75 =	675
6 600 +	6 60 +	15 15 =	675
6 600 +	5 50 +	25 25 =	675
5 500 +	10 100 +	75 75 =	675
6 + 600	4 40 +	88 =	675
6 600 +	1 10 +	65 =	675

Figure 10: Kaitlyn's representations of 675

Kaitlyn's work is shown in Figure 10. Her work was somewhat random, but representations were accurate with one exception. Both her quantity of pieces and her total number of pieces were incorrect in the second to last representation, 600 + 40 + 8.

Boxes (100s)	Rolls (10s)	Pieces (1s)	Total Pieces
6	7	5	675
0	600 +	75 =	675
4 400	200	75	675
1 + 100	50 300 +	75 =	675
2 + 200	40 400 +	75 =	675
1 + 100	50 500 +	75 =	675
3 + 300	30 300 +	75 =	675
(circle)	30 300	375	675

Figure 11: Kelly's representations of 675

Kelly's work is shown in Figure 11. Her work, while somewhat random, does show a level of sophistication that is not apparent with the other girls. She consistently used much larger quantities of rolls in representing her numbers.

While the girls initially seemed to have forgotten what they had learned in our small group sessions, most of them seemed to quickly regain their apparently lost understanding. Initially, even Crystal was misrepresenting the rolls in her number representations; when she was specifically reminded of what constituted a roll, she returned to greater accuracy. These whole class lessons continued for a week before the

end of my study. On completion of the study, I interviewed each of the girls and asked them to solve a multidigit addition problem using the candy factory inventory sheets and then explain how they solved the problem.

The final interviews provided further insight into the instability of my girls understanding. The girls often made mistakes in their explanations; as I had them solve a Cuffel's Candy Shop addition problem. For example, Mary said that 9 boxes of candy were 100 pieces, even though she had written down 9 boxes was 900 pieces of candy. In another example, Kaitlyn was regrouping and adding boxes, rolls, and pieces at the same time, causing her to write down an incorrect answer for one of the candy quantities. I also saw the same type of computational mistakes that had been occurring during the study. For example, Kelsie solved $900 + 150 + 13$ with an answer of 953. She dropped the 100 from 150 and the 10 from 13. Mary made the same mistake.

One exception Annie, gave the most coherent explanation of her solution process. Annie did not always demonstrate this type of understanding. She also expressed that she was less confident participating in the whole class because more people make her nervous. Since the interview was one-on-one, she may have been more comfortable giving her explanation, yet some elements of her computation would indicate otherwise. She was able to determine without counting that 6 rolls of candy was 60 pieces, and she knew in the same way that 15 rolls was 150 pieces of candy, yet she had to count on her fingers by tens to find the total of 9 rolls of candy. She had also made the connection that having 4 boxes, 6 rolls, and 5 pieces is 465 without the need to do any counting. However, when she was computing the total number of pieces of candy for the problem, she ran into some difficulty. Her explanation for adding $900 + 150 + 13$ follows.

Annie: There is 3 here so that goes here (indicated the ones place). The 1 and 5 is 6 and it goes here (indicated the tens place). Then 9 and 1 and 1 is 11 (answer 1,163).

It was at the point of adding the hundreds that she added in an extra one. She added the one ten in fifteen twice, as a ten and as a hundred. It took several minutes to straighten out what she had done incorrectly. Until the very end of her explanation (adding the hundreds), there was no confusion to her explanation, and she proceeded through the process without any hesitations.

As evidenced by the presented examples, the girls' understanding was not firmly grounded. They would demonstrate proficiency one time and then have confusion the next. Some performed better when in a group, such as Mary and Kaitlyn, and some performed better when they were one-on-one with the teacher, such as Annie. This would seem to give credence to the research that indicates that place value develops over time and that I should not expect my students to *get it* and be done (Hiebert & Wearne, 1992).

Summary

This data from this research brought several ideas to light. Social norms can develop without specific instruction, but it is even more important to make both social and sociomathematical norms an integral part of instruction due to the social nature of mathematics learning (Brown, Collins & Duguid, 1989; Cobb & Yackel, 1996; Greeno, 1991; Lerman, 1996; Schoenfeld, 1987; Sfard, 1994; Yang & Cobb, 2007). Flexibility in representing numbers and flexibility with computation do not seem to develop easily. There seem to be aspects of classroom culture that affect this development as well as the proficiency of the students involved and their prior exposure to traditional algorithms.

Finally, application of understanding may take time to fully develop. As was evidenced with my students, understanding seemed to be dependent on situations and the students' level of comfort with the situation. This indicated a need to continually present students with multiple opportunities to experience and utilize place value and computation. In the next chapter, I examine the implications of the data to my future teaching practices.

CHAPTER FIVE: CONCLUSION

Introduction

The purpose of this study was to examine my practices of teaching place value using strategies to develop conceptual understanding. The questions that I chose to address were:

1. How does my use of conceptually-based strategies for teaching place value affect my students' ability to flexibly use strategies to add and subtract multidigit numbers?

2. How does my practice of making explicit connections between place value, number representations, and computation affect my students' ability to flexibly use strategies to add and subtract multidigit numbers?

In order to investigate my use of conceptually-relevant instructional strategies, I planned a course of instruction for my third-grade students that included practices that had been shown to increase conceptual understanding of place value such as the candy factory (Andreasen, 2006; Bowers, Cobb & McClain, 1999; Fuson et al., 1997; Roy, 2008). Throughout the study, I continually adjusted the course of instruction as I reflected upon student responses and their apparent understanding or lack thereof. The rest of this chapter addresses my reflections and how they will affect my future choices for mathematical instruction of place value and computation.

The Importance of Classroom Community

Much research has been done on classroom community, social and sociomathematical norms and their effects on the conceptual understanding of mathematics (Bowers, Cobb, & McClain, 1999; Dixon, Andreasen, & Stephan, in press; Fuson & Burghardt, 2003; Fuson et al., 1997; Jones et al., 2007; Lopez & Allal, 2007; Yang & Cobb, 1995). Though I did not set out to directly address these norms, it became apparent that the norms were developing. Mathematical conversations increased and were more varied as the study progressed. During the interviews, several girls suggested that they became more comfortable sharing within the regular classroom. They attributed that comfort level with the belief that they were now more proficient in mathematics. This information has caused me to rethink my initial approach to teaching mathematics from the beginning of the school year.

I believe that in order for there to be effective classroom communication about mathematics, there needs to be a level of trust that enables all students to participate. While Annie was comfortable sharing within the small group, she admitted that she felt nervous about sharing within the whole class. She is not a particularly shy child, but she is easily intimidated by the comments of others. Mary and Kaitlyn both became more participatory in whole class, but seemed to become very nervous during their final interviews. I wondered if the one-on-one with the tape recorder was somehow intimidating to them. These experiences may be an indication that the culture of the particular setting influenced student behavior. Mary and Kaitlyn may have felt a stronger need to be correct and therefore stumbled through their explanations when they were one-on-one with me. This makes it important to establish the norms necessary for all children

to feel comfortable sharing in the classroom. Some of these norms could include: all children are entitled to share answers; mistakes are acceptable and important for learning; and all answers are initially acceptable, but may be improved or later discarded. These norms should help students to develop a level of comfort that would enable them to share their thinking and listen to the thinking of other students.

Though I did not set out to explicitly address social and sociomathematical norms during this research, I found them to be an integral part of the learning process. The more the girls discussed their thinking, the more they were able to correct their thinking. Often their mistakes were simply a matter of forgetting what they were talking about, but as Annie demonstrated in her exit interview, she was capable of thinking through the process of solving a problem and explained her reasoning with little assistance from me. At the beginning of the study, she exhibited one of the lowest levels of mathematics proficiency. Though she still had a considerable amount of ground to cover, she understood more at the end of the study than she did at the beginning.

At the beginning of the next school year, I will set out to establish both social and sociomathematical norms within my classroom. Some of these norms would include an expectation for explaining answers, respect for all students, what constitutes an effective solution process, and what constitutes a different solution. These norms as well as others that could be developed should help to increase student flexibility with numbers and computation processes. I will start the process and then assist the children in negotiating the further development of those norms. As a teacher leader, I will also share these ideas with other teachers in order to further encourage the growth of conceptual understanding in mathematics. If our schools are to see an increase in mathematical proficiency, it will

require that all teachers join in developing these types of practices within our classrooms (Lopez & Allal, 2007). Social and sociomathematical norms can create a framework in which students can comfortably share their thinking. When students share their thinking, this may increase flexible use mathematical strategies and improve conceptual understanding.

<u>Let Me Count the Ways</u>

One of the goals of this study was to increase my students' ability to flexibly represent numbers and to have flexibility in solving multidigit computation. This flexibility did not seem to develop through the course of my study. I spent much time reflecting on this apparent lack of development looking for reasons and ways to adjust instruction to encourage flexibility.

One conclusion that I reached and that I believe has merit is that all the girls were close to the same proficiency level in mathematics as indicated by the district benchmark assessments. Since there was little disparity in their ability levels, they had fewer options to draw on. The girls did not look for additional methods of computation, and when I suggested other methods, they preferred to continue to use what was already familiar. They had all had some experience with base-ten blocks and were therefore comfortable using them. The candy pieces also became familiar because of their connection to the base-ten blocks. Each was essentially the same type of representation. The girls almost never used pictorial representations of the blocks and had difficulty with mental computation. Writing numbers horizontally also caused many addition mistakes, though this method was consistently used on the inventory sheets. This aspect of flexibility may be improved with exposure to students who are more proficient in mathematics. Being

part of a larger group with more diverse thinkers could encourage the development of computational flexibility (Fuson et al., 1997).

Next year, I will address this issue by routinely mixing the types of groups in which children work. There is a place for ability grouping, but it should not dominate my instructional practices. Creating diverse groups with children of all abilities should enable every student to grow. Working in both small and whole class groups will also diversify mathematical discussions. This diversity should lead to greater flexibility since the students will be exposed to a wider variety of solution processes and the opportunity to investigate the methods that other students use.

Norms and flexible representation of numbers and flexibility with computational practices increase conceptual understanding. However, in order to further deepen conceptual understanding and allow its transfer to novel situations, it is important to make connections between place value and computational practices.

Building Bridges

Another goal of this study was to make explicit connections between representations. This was an area which I feel I did not adequately address. Hiebert and Wearne (1992) suggested that making connections between representations is a critical element of conceptual understanding of place value. Brain research supports the idea that making connections between ideas promotes more extensive learning as well as better retention of the learning (Jensen, 2000).

While I did make some explicit connections during the study such as connecting numerical symbols to base-ten blocks, and specifically demonstrating how regrouping works with base-ten blocks, I believe that some were not done effectively. I intended to

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be explicit about making connections during my study and planned it as part of my instructional sequence, but I found that I was often caught up in a particular issue of misconception or misunderstanding during a lesson so that I unintentionally bypassed making connections. Perhaps, if I had been more effective at making connections, my students' application of their understanding would not have been so questionable. Often I expect students to automatically see the connections, but this is not always the case. My students need support and guidance in order to make effective connections. These connections can then bridge ideas and levels of representation in order to bring about a fuller understanding of the mathematics topic (Hiebert & Carpenter, 1992).

In order to improve this aspect of my instruction, I believe that I need a fuller understanding of how to make explicit connections for my students. I really did not expect to have such difficulty in this area. However, what I understand intuitively, or have had ingrained in my understanding for a long period of time, needs to be drawn out and made explicit. For example, I automatically adjusted my method of computation to a given situation. I intuitively understand when estimating is most effective, when to use mental math, and when I need a calculator. My students do not have these understandings, and it is up to me to find ways to build the bridge that will cover the gap of understanding between one representation and another and between one concept and another. This will probably be my hardest task since it will require me to deeply examine my own mathematical understanding in order to break it down for my students. I also need to recognize that what I see as a connection may not always work for every student, and I need to be open to other interpretations. For example, I automatically see "counting up" which is an addition strategy as the simplest method for solving a subtraction

problem. I discovered in this study that students don't always understand that connection between addition and subtraction. There is almost always more than one path to understanding.

A Road Full of Detours

Finally, I would like to address how my instructional sequence was modified throughout this study. Weeks one through three progressed as noted in chapter three, but from this point on, lessons were frequently modified. Some of the girls had occasional difficulty counting the base-ten blocks and sometimes confused counting tens with counting hundreds, so practice counting base-ten blocks was continually incorporated into the other lessons that included the addition of multidigit numbers. This continued to be a problem that surfaced throughout the study. Rather than progressing linearly through the instructional sequence, I often returned to the practice of counting blocks and creating various representations of numbers through the candy factory. Since the girls were using the traditional algorithm, but not understanding the concept of regrouping, I explicitly taught the concept of regrouping. I used base-ten blocks to show the exchange of one unit for another, such as exchanging 10 ones for one ten and then adding the ten to the tens column. I wanted to extend this into regrouping for subtraction, but the girls had such significant misconceptions about subtraction that I decided to remain with addition. The girls would take the smaller number from the larger number in a multidigit subtraction problem regardless of its meaning as part of the whole number or the entire problem. For example, $342 - 196$ would result in an answer of 254. They compartmentalized the subtraction to each unit rather than look at each number holistically.

As teachers we plan what we perceive to be the best course of instruction. If we truly have our students' learning at the heart of our instruction, that instruction needs to be adjusted when it does not seem to work. These are the reasons that I made the choices that I did regarding the order of instruction. I will continue to keep these ideas at the heart of my instructional planning in the hope that I will make good choices for my students.

An Additional Thought

One observation that I made during the course of my study was that the beginning mathematical proficiency level of the students seemed to be correlated to the rate at which the students gained understanding. Both Mary and Kaitlyn started this research project at a higher level of mathematical understanding as assessed by the district benchmark assessment. They acquired understanding of the concepts more quickly than the other girls. Additionally, Annie and Kelly, who had the lowest level of mathematical understanding as assessed by the district benchmark assessment, had the most difficulty acquiring conceptual understanding. Yet, in the final interview, Annie gave one of the best explanations of her solution process. Students' mathematical proficiency would need to be taken into consideration when planning instruction and the need for extra support within the classroom.

Implications for the Future

One aspect to this study that may affect the development of conceptual understanding is previous knowledge of and exposure to traditional computational algorithms (Hiebert & Carpenter, 1992). When given the ability to choose a computational method, the girls' first choice was the traditional regrouping algorithm.

While this may be the most efficient method for most paper and pencil computation in most cases, it is not the best choice in all situations. This was the method with which they were most familiar and comfortable regardless of the accurate use of the algorithm. The knowledge of the traditional algorithm that was learned in second grade seemed to interfere with their ability to develop flexibility in the use of computational strategies. It might be appropriate to address this issue with second grade teachers in order to promote a deeper understanding of place value and computation.

I also found that my instructional sequence seemed to follow a traditional progression of content. I moved from basic place value concepts, to number representation to computation much as a traditional textbook would even though I used conceptually based strategies such as the candy factory. In order to move my instruction to a new level, I need to become more comfortable with allowing my students productively struggle to gain insight and understanding. A stronger level of mathematical inquiry may promote stronger conceptual understanding of place value.

Finally, some of my conclusions could have been more strongly represented if I had had my students write their thinking and solution processes more often. While I used multiple data sources such as my observations, student work, and recorded conversations, I believe that my evidence would have been stronger if I had had additional support.

<u>Summary</u>

Through this research, I learned that teaching place value for understanding is an ongoing process that takes time and repeated exposures. Students bring unique understanding to the classroom, and this understanding needs to be brought into the open in order to fully understand each student's perspective before proceeding with instruction.

I discovered the importance of this during my research. We were working on the strategy of counting up with a number line in order to solve a subtraction problem. Annie was reflecting confusion and frustration and was unable to use the strategy. When I asked her how she solved basic subtraction problems such as $15 - 8$, she demonstrated the strategy of counting down. When I asked her if she ever solved that type of problem by counting up, she told me she hadn't and really was not sure what I was talking about. I had made an assumption that all of my students knew how to use the strategy of counting up. It turned out that Annie was not the only one.

Finally, as noted by Hiebert and Wearne (1992), it is not necessary to completely separate place value from multidigit addition and subtraction. Their belief that conceptual understanding of place value develops over time and develops along a continuum allows for an interrelated connection between the two areas of understanding. Multidigit addition and subtraction can inform place value understanding and vice versa. In light of this research and the results of my study, I believe that integrating these two areas would be a wise course of action. Incorporating both areas into instruction in a carefully guided and scaffolded manner may allow for more connections to be made sooner and to further promote conceptual understanding of both place value and computation.

This study has caused me to spend significant amounts of time examining my instructional practices. This is, after all, one of the goals of action research (Holly, Arhar, & Kasten, 2005). The research I have read and the data I have collected have caused me to reflect deeply on how I must change my instructional practices so that I may assist my students in achieving the deepest possible mathematical understanding. Additionally, I

see opportunities for future action research such as which computational practices work best for my students and how to effectively develop classroom norms with third-grade students. As I put into practice what I have learned this year, I will continue to analyze my instructional practices and modify them in order to best serve my students.

APPENDIX A

IRB APPROVAL

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901, 407-882-2012 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Notice of Expedited Initial Review and Approval

From : **UCF Institutional Review Board**
FWA00000351, Exp. 6/24/11, IRB00001138

To : **Terry Cuffel**

Date : **August 18, 2008**

IRB Number: **SBE-08-05764**

Study Title: **Conceptually-based teaching practices of place value and their effects on student computation practices**

Dear Researcher:

Your research protocol noted above was approved by **expedited** review by the UCF IRB Chair on 08/15/2008. **The expiration date is 08/14/2009.** Your study was determined to be minimal risk for human subjects and expeditable per federal regulations, 45 CFR 46.110. The category for which this study qualifies as expeditable research is as follows:

6. Collection of data from voice, video, digital, or image recordings made for research purposes.

7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

The IRB has approved a **consent procedure which requires participants to sign consent forms.** Use of the approved, stamped consent document(s) is required. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Subjects or their representatives must receive a copy of the consent form(s).

All data, which may include signed consent form documents, must be retained in a locked file cabinet for a minimum of three years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained on a password-protected computer if electronic information is used. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

To continue this research beyond the expiration date, a Continuing Review Form must be submitted 2 – 4 weeks prior to the expiration date. Advise the IRB if you receive a subpoena for the release of this information, or if a breach of confidentiality occurs. Also report any unanticipated problems or serious adverse events (within 5 working days). Do not make changes to the protocol methodology or consent form before obtaining IRB approval. Changes can be submitted for IRB review using the Addendum/Modification Request Form. An Addendum/Modification Request Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at http://iris.research.ucf.edu .

Failure to provide a continuing review report could lead to study suspension, a loss of funding and/or publication possibilities, or reporting of noncompliance to sponsors or funding agencies. The IRB maintains the authority under 45 CFR 46.110(e) to observe or have a third party observe the consent process and the research.

On behalf of Tracy Dietz, Ph.D., UCF IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 08/18/2008 02:01:46 PM EDT

IRB Coordinator

University of
Central Florida

Notice of Expedited Review and Approval
of Requested Addendum/Modification Changes

From: **UCF Institutional Review Board**
        FWA00000351, Exp. 10/8/11, IRB00001138

To: **Terry Cuffel**

Date: **February 10, 2009**

IRB Number: **SBE-08-05764**

Study Title: **Conceptually-based teaching practices of place value and their effects on student computation practices**

Dear Researcher:

Your requested addendum/modification changes to your study noted above which were submitted to the IRB on 02/08/2009 were approved by **expedited** review on 2/10/2009.

Per federal regulations, 45 CFR 46.110, the expeditable modifications were determined to be minor changes in previously approved research during the period for which approval was authorized.

Use of the approved, stamped consent document(s) is required. The new forms supersede all previous versions, which are now invalid for further use. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Subjects or their representatives must receive a copy of the consent form(s).

This addendum approval does NOT extend the IRB approval period or replace the Continuing Review form for renewal of the study.

On behalf of Tracy Dietz, Ph.D., IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 02/10/2009 12:54:53 PM EST

IRB Coordinator

Internal IRB Submission Reference Number: 004888
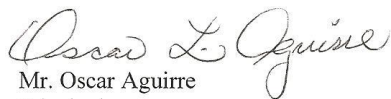
APPENDIX B

PRINCIPAL APPROVAL

Dear IRB Coordinator,

Ms. Terry Cuffel has notified me of the action research project she would like to conduct in the fall of the 2008-2009 school year with her third grade students. Ms. Cuffel will study the effects of conceptually based instructional strategies on students' ability to understand place value, flexibly represent numbers and flexibly utilize strategies for multi-digit addition and subtraction.

I have reviewed the parental consent form that will be sent home and the student assent form requesting the participation of her students. Student grades will not be affected if they choose to not participate in the study.

I authorize Ms. Cuffel to conduct her proposed research project with her students beginning September, 2008.

Sincerely,

Mr. Oscar Aguirre
Principal

APPENDIX C

SCHOOL DISTRICT APPROVAL

RESEARCH REQUEST FORM

Requester's Name _Terry Cuffel_   Date _7-21-08_

Address: Home _3222 Caulfield St, Apopka 32703_ Phone _407-774-2455_

Project Director or Advisor _Dr. Juli Dixon_   Phone _407-823-4140_

Address _UCF 4000 Central FL Blvd. Orlando, FL 32816_

Degree Sought: ☐ Associate   ☐ Bachelor's   ☒ Master's   ☐ Specialist
(check one)   ☐ Doctorate   ☐ None

Project Title _Conceptually-based teaching practices of place value and their effects on student computation practices_

ESTIMATED INVOLVEMENT

PERSONNEL/CENTERS	NUMBER	AMOUNT OF TIME (DAYS, HOURS, ETC.)	SPECIFY/DESCRIBE GRADES, SCHOOLS, SPECIAL NEEDS, ETC.
Students	22	2hr/wk for 16wks	3rd grade -
Teachers	1	same + planning	"
Administrators			
Schools/Centers	1	same as above	
Others (specify)			

Specify possible benefits to students/school system: _Improved understanding of mathematical concepts_

ASSURANCE

Using the proposed procedures and instrument, I hereby agree to conduct research in accordance with the policies of the ▮▮▮▮▮▮▮▮▮▮▮s. Deviations from the approved procedures shall be cleared through the Senior Director of Accountability, Research, and Assessment. Reports and materials shall be supplied as specified.

Requester's Signature _Terry Cuffel_

Approval Granted: ☒ Yes   ☐ No   RECEIVED AUG 08 2008 Date: _8-8-08_

Signature of the Senior Director for Accountability, Research, and Assessment _Jee Baker_

NOTE TO REQUESTER: When seeking approval at the school level, a copy of this form, signed by the Senior Director, Accountability, Research, and Assessment, should be shown to the school principal.

Reference School Board Policy GCS, p. 249

7PL202   FORM ID #GB0103/23-1/1FY   REV 1/04

APPENDIX D

PARENTAL CONSENT

February, 2009

Dear Parents,

I am writing to request reconsent for your child to participate in a study that I am conducting in our classroom this year. I have made slight changes to the research process and this requires that I obtain new consent for participation. I am currently working as a Lockheed Martin Scholar towards a Master's Degree in Mathematics and Science Education at the University of Central Florida.

I will be studying the effects of specific teaching strategies on how third graders learn place value and the addition and subtraction of multi-digit numbers. The purpose of this research is to see if the teaching strategies improve student understanding of these mathematics concepts. The research will begin in September and end by the end of the school year. Students will receive my usual mathematics instruction in addition to the specific lessons related to place value and adding and subtracting.

There are minimal anticipated risks with this study. The identity of your child will be kept confidential and I will be using pseudonyms in all written documentation and any discussions with my peers or advisors. I will be using audio recordings during small group work related to this study in order to have a clear understanding of student thinking during the problem solving process.

I will also periodically conduct student focus groups or interviews and will audio record the focus groups or interviews to gain further insight into student thinking processes. The audio tapes will only be listened to by myself or my advisors and will be destroyed when the study is completed. Students may choose to not answer any question during the interview. A pre- and post-test will be administered in order to assess student knowledge before and after the study. These tests will not be part of any class grade. Additionally, students will keep written journals to further explain their mathematical thinking and I will be reviewing these journals throughout the study.

No compensation will be provided, but I would be happy to share the results with you upon completion.

Participation in this study is voluntary and grades will not be affected in any way. Students who do not participate in the study will do the same work as the rest of the class, but their work and journal entries will not be used for the study nor will they be recorded during any small group or whole group activities. You and your child may withdraw consent at any time. If you have any questions about this research project, please call me at 407-884-2235 ext. 4422. You may also contact my faculty supervisor, Dr. Juli K. Dixon, at 407-823-4140. Questions or concerns about research participants' rights may be directed to the UCFIRB Office of Research, Orlando Tech Center, 12443 Research Parkway, Suite 302, Orlando, FL 32806. The hours of operation are 8:00 AM to 5:00 PM, Monday through Friday except on University of Central Florida official holidays. The phone number is 407-823-2901.

Sincerely,

Ms. Terry Cuffel

_____ I have read the procedure described above.

_____ I give consent for my child _____ to participate in Ms. Cuffel's study on place value and addition and subtraction.

 I would/would not like to receive a copy of the procedure description.

_____   _____
                    Parent/Guardian   Date

APPENDIX E

STUDENT ASSENT

January, 2009

Dear Students,

I am doing a project to find out how you learn math. I want to help you become better math students. I am doing this project as part of my college classes.

Sometimes I will be recording what you say when you work in small groups. Sometimes I will ask questions of just you. I will also be saving your work from this project. Sometimes I may ask you questions and will record your answers. The only ones who will listen to these tapes will be my teacher and me. I will destroy the tapes when my project is done. When I write about my project, I will not use your real name. It is all right if you don't want to be part of my project. You can stop being part of my project at any time. You can choose to not be recorded if you don't want. Would you like to take part in my project?

_____ I want to take part in Ms. Cuffel's project.

_____   _____
Student's Signature   Date

_____
Student's Printed Name

REFERENCES

Andreasen, J. B., (2006). Classroom mathematical practices in a preservice elementary mathematics education course using an instructional sequence related to place value and operations. *Dissertation Abstracts International,* 67 (03). (UMI No. 3210341).

Baroody, A. J. (1999). Children's relational knowledge of addition and subtraction. *Cognition and Instruction, 17,* 137-175.

Bowers, J., Cobb, P., McClain, K. (1999). The evolution of mathematical practices: A case study. *Cognition and Instruction, 17,* 25-64.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18,* 32-42.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Cawley, J., Parmar, R., Lucas-Fusco, L., Kilian, J., & Foley, T. (2007). Place value and mathematics for students with mild disabilities: Data and suggested practices. *Learning Disabilities, 5,* 21-39.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29,* 573-604.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31,* 175-190.

Cobb, P., Yackel, E., Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23,* 2-33.

Dixon, J. K., Andreasen, J. B., Stephan, M. (in press). Establishing social and sociomathematical norms in an undergraduate mathematics content course for prospective teachers: The role of the instructor.

Fehr, H. (1955). A philosophy of arithmetic instruction. *Arithmetic Teacher, 2,* 27-32.

Fraenkel, J. R., & Wallen (2006). *How to design and evaluate research in education* (6th ed.). New York: McGraw-Hill.

Fuson, K. (1990). A forum for researchers: Issues in place-value and multidigit addition and subtraction learning and teaching. *Journal for Research in Mathematics Education, 21,* 273-280.

Fuson, K. C., Briars, D., J. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education, 21,* 180-206.

Fuson, K. C., Burghardt. B. H. (2003). Multidigit addition and subtraction methods invented in small groups and teacher support of problem solving and reflection. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills constructing adaptive expertise* (pp. 267-304). Mahwah, NJ: Lawrence Erlbaum Associates.

Fuson, K. C., Wearne, D., Heibert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., et al. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28,* 130-162.

Geist, E. A., King, M. (2008). Different, not better: Gender differences. *Journal of Instructional Psychology, 35,* 43-52.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22,* 170-218.

Hiebert, J. (Ed.). (1986). *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research of mathematics teaching and learning* (65-97). New York: MacMillan Publishing Co.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., et al. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education. 23,* 98-122.

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction, 14,* 251-283.

Holly, M. L., Arhar, J., Kasten, W. (2005). *Action research for teachers: Following the yellow brick road* (2nd ed.). Upper Saddle River, NJ: Pearson Education, Inc.

Janvier, C. (Ed.). (1987). *Problems of representation in the teaching learning of mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

Jensen, E. (2000). *Brain-based learning: the new science of teaching & learning* (Rev. ed.) San Diego, CA: The Brain Store Publishing.

Jones, G. A., Thornton, C. A., Putt, I. J., Hill, K. M., Mogill, T., Rich, B., et al. (1996). Multidigit number sense: A framework for instruction and assessment. *Journal for Research in Mathematics Education, 27,* 310-336.

Kamii, C., Lewis, B. A., & Livingston, S. J. (1994). Primary arithmetic: Children inventing their own procedures. *Arithmetic Teacher, 41,* 200-203.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27,* 29-63.

Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education, 27,* 133-150.

Lopez, L. M., & Allal, L. (2007). Sociomathematical norms and the regulation of problem solving in classroom microcultures. *International Journal of Education Research, 46,* 252-265.

Michener, E. R. (1978). Understanding understanding mathematics. *Cognitive Science, 2,* 361-383.

NCTM (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Nataraj, M. S., & Thomas, M. O. J.(2007). Developing the concept of place value. In J. Watson & K. Beswick (Eds.), 30th Annual Conference of the Mathematics Education Research Group of Australasia: Vol. 2. Mathematics: Essential research, essential practice (pp.523-532). Wahroonga, Australasia: MERGA, Inc.

Nickson, M. (1992). The culture of mathematics classroom: An unknown quantity. In D. A. Grouws (Ed.), *Handbook of research of mathematics teaching and learning* (101-114). New York: MacMillan Publishing Co.

Polya, G. (1957). *How to solve it* (2nd ed.). Garden City, NY: Doubleday Anchor Books.

Psychometric report for the ----------- county public schools (2008). The Princeton Review. New York.

Raphael, D., & Wahlstrom, M. (1989). The influence of instructional aids on mathematics achievement. *Journal for Research in Mathematics Education, 20,* 173-190.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3, pp. 41-95). Hillsdale, NJ: Lawrence Erlbaum.

Roy, G. J., (2008). *Prospective teachers' development of whole number concepts and operations during a classroom teaching experiment.* University of Central Florida.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-216). Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. D. A. Grouws (Ed.), *Handbook of research of mathematics teaching and learning* (334-370). New York: MacMillan Publishing Co.

Sfard, A. (1994) Reification as the birth of metaphor. *For the Learning of Mathematics, 14,* 44-55.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26,* 114-145.

Sowder, J., & Schappelle, B. (1994) Number sense-making. *Arithmetic Teacher, 41,* 342-345.

Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20,* 498-505.

Steffe, L. P., Cobb, P., & von Glaserfeld, E. (1988). *Construction of arithmetical meanings and strategies.* New York: Springer-Verlag.

Van Engen, H. (1949). An analysis of meaning in arithmetic. *Elementary School Journal, 49,* 321-329.

Varelas, M., Becker, J., (1997). Children's developing understanding of place value: Semiotic aspects. *Cognition and Instruction, 15,* 265-286.

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb & H. Bauersfeld (Eds.). *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 163-202). Hillsdale, NJ: Lawrence Erlbaum.

Wearne, D., Hiebert, J., Campbell, P., (1994). Place value addition and subtraction. *Arithmetic Teacher, 41,* 272-275.

Wertheimer, M. (1959). *Productive thinking.* New York: Harper & Row.

Wolfe, P. (2001). *Brain matters: Translating research into classroom practice.* Alexandria, VA: Association for Supervision and Curriculum Development.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458-471.

Yang, M., & Cobb, P. (1995). A cross-cultural investigation into the development of place-value concepts of children in Taiwan and the United States. *Educational Studies in Mathematics, 28,* 1-33.

Zohar, A., Gershikov, A. (2008). Gender and performance in mathematical tasks: Does the context make a difference? *International Journal of Science and Mathematics Education, 6,* 677-693.