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DESIGN AND OPTIMIZATION OF NANOSTRUCTURED OPTICAL FILTERS

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Optics in the College of Optics and Photonics at the University of Central Florida Orlando, Florida

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ABSTRACT

Optical filters encompass a vast array of devices and structures for a wide variety of applications. Generally speaking, an optical filter is some structure that applies a designed amplitude and phase transform to an incident signal. Different classes of filters have vastly divergent characteristics, and one of the challenges in the optical design process is identifying the ideal filter for a given application and optimizing it to obtain a specific response. In particular, it is highly advantageous to obtain a filter that can be seamlessly integrated into an overall device package without requiring exotic fabrication steps, extremely sensitive alignments, or complicated conversions between optical and electrical signals.

This dissertation explores three classes of nano-scale optical filters in an effort to obtain different types of dispersive response functions. First, dispersive waveguides are designed using a sub-wavelength periodic structure to transmit a single TE propagating mode with very high second order dispersion. Next, an innovative approach for decoupling waveguide trajectories from Bragg gratings is outlined and used to obtain a uniform second-order dispersion response while minimizing fabrication limitations. Finally, high Q-factor microcavities are coupled into axisymmetric pillar structures that offer extremely high group delay over very narrow transmission bandwidths.

While these three novel filters are quite diverse in their operation and target applications, they offer extremely compact structures given the magnitude of the dispersion or group delay they introduce to an incident signal. They are also designed and structured as to be formed on an optical wafer scale using standard integrated circuit fabrication techniques. A number of frequency-domain numerical simulation methods are developed to fully characterize and model each of the different filters. The complete filter response, which includes the dispersion and delay characteristics and optical coupling, is used to evaluate each filter design concept. However, due to the complex nature of the structure geometries and electromagnetic interactions, an iterative optimization approach is required to improve the structure designs and obtain a suitable response. To this end, a Particle Swarm Optimization algorithm is developed and applied to the simulated filter responses to generate optimal filter designs. To my beloved wife Megan for her patience and encouragement through this long endeavor.

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LIST OF ABBREVIATIONS

Coupled-Resonator Optical Waveguide
Continuous Wave
Distributed Bragg Reflector
Electromagnetic
Finite Difference-Frequency Domain
Finite Difference-Time Domain
Finite Impulse Response
Free Spectral Range
Full-Width Half Max
Genetic Algorithm
Group Delay Ripple
Infinite Impulse Response
Method of Lines
Minimum Phase Filter
Numerical Aperture
Nano Dispersion-Amplified Waveguide
Photonic Crystal
Perfect Electric Conductors
Plasma Enhanced Chemical Vapor Deposition
Polarization Maintaining Fiber
Perfectly Matched Layers
Particle Swarm Optimization
Quasi-Phase Matching
Rigorous Coupled Wave Analysis
Spontaneous Emission
Second Harmonic Generation
Spatially Polarizing Autocloned Structure
Transverse Electric
Transverse Magnetic
Wavelength Division Multiplexing

CHAPTER 1 INTRODUCTION

1.1 Basics of Optical Filtering

Optical structures of all shapes and sizes rely on their ability to bend and control frequencies of light. The term "optical filter" is an exceedingly broad description that essentially includes any structure that purposely distinguishes between different frequency components of an incident signal and treats them in different ways.

The transformation applied to an input signal can be in terms of amplitude, phase, or both. The former is accomplished principally through some form of resonance or interference, while the latter involves dispersive and phase delay effects. While a number of structures, such as all-pass filters, offer minimal amplitude distortion, some degree of frequency-dependent phase is always applied by the filter to the incident signal.

Ignoring nonlinearities, optical filters consist of linear time-invariant systems, which may be characterized in the time domain in terms of an impulse response function, h(t) [1]. Given an input signal, x(t), the output, y(t), is defined as:

$$y(t) = x(t) * h(t) = \int x(\tau)h(t-\tau)d\tau$$
 (1.1)

In the frequency domain, this relationship becomes

$$Y(\omega) = X(\omega)H(\omega)$$
(1.2)

where

$$H(\omega) = |H(\omega)|e^{i\phi(\omega)}$$
(1.3)

The complex-valued transfer function, $H(\omega)$, is the primary focus of filter design problems. Its magnitude provides the amplitude distortion of the incident signal, while the frequency-dependent phase, $\phi(\omega)$, describes the phase accumulated for a given frequency component upon transmission through the filter. Some of the more interesting effects resulting from filters rely upon the spectral dependence of this phase function.

1.2 Frequency-Dependent Phase and Dispersion

1.2.1 Group Delay

The principles governing the induced spectrally-dependent phase, or dispersion, are well known, though it is illustrative to highlight a few basic details. The phase may be written in terms of a propagation constant, β , of an optical mode inside the filter

$$\phi(\omega) = -\beta(\omega)z = -\frac{\omega}{c}n_{eff}(\omega)z$$
(1.4)

where n_{eff} is the effective index of the guiding structure and *z* describes the propagation distance. Assuming that the amplitude distortion may be ignored and we are able to treat the filter as a guiding structure along which a signal travels¹, propagation of a specific frequency harmonic a distance *z* through a filter takes the form

$$a(z,\omega) = a(0,\omega)e^{-iz\beta(\omega)}$$
(1.5)

Since we generally consider optical pulses with a reasonably small frequency bandwidth propagating through media whose optical response has finite derivatives with respect to frequency, it is convenient to expand Eqn. 1.4 in a Taylor Series about a center frequency, ω_0 :

¹ While this treatment does not specifically apply to filters based on mechanisms other than guiding structures (such as cavities or gratings), the mathematical treatment may still be used to a certain degree, though the identified variables may take on a different physical meaning.

$$\beta(\omega) = \beta(\omega_0) + \left[\frac{1}{v_g}\right]_{\omega_0} \Delta\omega + \frac{1}{2} D_{\omega}|_{\omega_0} (\Delta\omega)^2 + O(\Delta\omega)^3$$
(1.6)

 v_g is known as the group velocity, and its inverse is the group delay per unit of propagation distance, which describes the phase accumulated by a propagating optical signal. It is given by

$$\frac{1}{v_g} = \frac{d}{d\omega} \beta(\omega) = \frac{n_{eff}(\omega)}{c} + \frac{\omega}{c} \frac{dn_{eff}}{d\omega}$$
(1.7)

This represents a delay experienced by optical signals propagating through the structure. The actual group delay for a signal propagating a distance, *z*, is given by

$$\tau_g = \frac{z}{v_g} = -\frac{d\phi(\omega)}{d\omega}$$
(1.8)

It is also reasonable to define a group index, n_g , which provides a descriptive "figure of merit" for evaluations of degree of delay a given structure provides. The group index is defined according to

$$n_g = \frac{c}{v_g} = c \frac{\tau_g}{z} = n_{eff}(\omega) + \omega \frac{dn_{eff}}{d\omega} = n_{eff}(\lambda) - \lambda \frac{dn_{eff}}{d\lambda}$$
(1.9)

and offers a value in terms of "delay per unit length." From this we see that group delay only becomes highly significant near sharp resonant peaks in the effective index curve. Thus, obtaining a large group delay requires designing a device either to operate near material resonance peaks or to rely on a geometry that creates a resonance condition or operates near a modal cutoff condition.

1.2.2 Quadratic Dispersion

 D_{ω} in Eqn. 1.6 describes the spreading, or dispersion, of the optical signal as a function of frequency. It is often convenient to work in terms of wavelength instead of frequency, so we use an equivalent definition for dispersion, *D*:

$$D = \frac{2\pi c}{\lambda^2} D_{\omega} = \frac{2\pi c}{\lambda^2} \frac{\partial^2 \beta}{\partial \omega^2} = \frac{\lambda}{c} \frac{\partial^2 n_{eff}}{\partial \lambda^2}$$
(1.10)

Returning to Eqn. 1.5, we know that an optical signal described in the frequency domain is related to the time domain description via a Fourier Transform pair:

$$a(0,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(0,t) e^{-i\omega t} d\omega$$
(1.11)

$$A(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} a(z,\omega) e^{i\omega t} d\omega \approx \frac{1}{2\pi} e^{i(\omega_0 t - \beta_0 z)} \int_{-\infty}^{\infty} a(0,\Delta\omega) e^{i\Delta\omega \left(t - \frac{z}{v_g} - \frac{1}{2}D_{\omega} z \Delta\omega\right)} d\omega$$
(1.12)

Eqn. 1.12 is analytically solvable for a few different types of input signals. The most common one is a Gaussian pulse of waist, τ_0 . After propagating a finite distance, z, the new beam waist has a dependence on the dispersion of the medium and is given by

$$\tau(z) = \tau_0 \left(1 + \frac{z^2}{z_0^2} \right)^{\frac{1}{2}}$$
(1.13)

where

$$z_0 = \frac{\pi c \tau_0^2}{\lambda^2 D} \tag{1.14}$$

The dispersive term in the exponent of Eqn. 1.12 induces frequency chirp whereby the spectral components of an optical signal are no longer centered on top of each other (Figure 1-1). This can work to one's advantage as a pulse incident on the dispersive structure already chirped

with a sign opposite of the frequency derivative of the propagation constant for the structure will experience compression instead of broadening since the frequency components are pushed back on top of each other by the structure's dispersion.



Figure 1-1: Relative Phase Delays of Frequency Components Causes Broadening of Signals.

1.2.3 Higher Order Dispersion

An additional consideration when dealing with pulse dispersion is the case where higher order dispersion is present. In some cases it is quite significant, though in most cases involving simple pulse compression and expansion it shows up as a slight ripple in an otherwise linear group delay curve. This can be represented as an additional frequency-dependent phase term incorporated into Eqn. 1.12. The phase can generally take the form

$$\Phi_{GDR} = \frac{\omega_p \tau_a}{2\pi} \cos\left(\frac{2\pi}{\omega_p} \Delta \omega + \phi_0\right)$$
(1.15)

where ω_p gives the frequency of the ripple and τ_a gives its amplitude. ϕ_0 defines the phase between a ripple peak and the spectral center of the signal. Following the derivation of [2] we note that this phase results in a term of the form

$$e^{iz\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\theta}$$
(1.16)

according to the Jacobi-Anger expansion. $J_n(z)$ is a Bessel function of the first kind. We can then evaluate the new pulse according to

$$A'(z,t) = \sum_{n=-\infty}^{\infty} A\left(z,t + \frac{2\pi n}{\omega_p}\right) i^n J_n\left(\frac{\omega_p \tau_a}{2\pi}\right) e^{in\phi_0}$$
(1.17)

Thus, group delay ripple (GDR) results in a continuum of overlapping signals with different amplitudes and temporal centers. The phase differences across the overlapping pulses result in a noticeable distortion of the final signal. Figure 1-2 demonstrates the effects of GDR on a pulse broadened from 1ps to 1ns. The frequency of the ripple is taken to be 1THz, and the results are plotted for different values of the amplitude. As can be seen, GDR causes oscillations in the output signal, though the magnitude has to be a noticeable percentage of the overall group delay before it significantly affects the output quality.



Figure 1-2: Pulse Broadened from 1 ps to 1 ns with Different Magnitudes of Group Delay Ripple.

1.2.4 Summary and Description of Dispersion

The frequency-dependent response of optical structures results in three different orders of dispersion that apply a phase transform to an incident signal in dispersive waveguides and delay line filters. The first is simply termed "delay" or "group delay" and describes a time delay applied to an optical signal. This is the principal consideration for nearly monochromatic signals or structures that are not dominated by quadratic dispersion. This is particularly applicable in a variety of situations, such as time synchronization in communications systems or optical buffering of data until it may be processed [3].

The second type of dispersion, known as "quadratic dispersion" or simply "dispersion," describes a difference in group delay for different frequency components of an optical signal. As discussed above, a chromatic pulse impinging upon a dispersive guiding structure receives a different delay for each component frequency and is thus broadened as it propagates.

Telecommunications, imaging systems, military uses for high-power lasers, and numerous other optical applications relying on ultra-short pulses require means for compensating or controlling dispersive effects on such optical signals. In particular, these methods make use of dispersive guiding structures to broaden pulses to better control individual frequency components. Additionally, in high-speed communication systems, dispersion-induced pulse broadening is already present, and dispersive structures are needed to compensate for this and to restore the original signals. Other applications [4] make use of dispersive delay for time synchronization or data buffering [3] or to enhance nonlinear interaction by slowing the group velocity [5].

To increase the rate at which a pulse is spread or compressed, the dispersion must be increased. For example, consider a 1 ps Gaussian pulse at 1.55 microns. Expanding the pulse to 2 ps within a 100 mm length of a dispersive waveguide requires a structure with dispersion on the order of 6800 ps nm⁻¹ km⁻¹. A typical optical fiber has a dispersion of around 10-20 ps nm⁻¹ km⁻¹ near this wavelength. To expand the same pulse up to 1 ns within the same length of waveguide requires a dispersion of nearly 4,000,000 ps nm⁻¹ km⁻¹.

The third category of dispersion involves the higher order terms in Eqn. 1.6. Whereas quadratic dispersion results from a convolution of the frequency components of a pulse with a linearly-varying frequency-dependent group velocity, the higher order terms result in a further convolution of the frequency-domain phase-shifted pulse with a set of Bessel functions. In general, the higher order dispersion results in noise in the output signal and is usually undesirable. For a relatively narrow signal bandwidth, these terms may have minimal effect, though for most optical filters the higher order terms are minimized as much as possible [6].

1.3 Classes of Optical Filters

Optical filters can largely be divided into two basic classes based on their filtering mechanisms [7]. Finite impulse response (FIR) filters are essentially single-pass devices. They do not rely on feedback mechanisms or optical reflections. Thus, signal delay is limited and strictly determined by the length of the structure. Dispersive waveguides and Mach-Zender based filters fit into this category [4, 8].

The alternative class, termed infinite impulse response (IIR) filters, relies on multiple reflections and feedback mechanisms. Filters in this class include Bragg gratings, resonators, and many all-pass filters. Their resonant nature allows them to produce extremely large group delays for frequencies near resonance.

The class of IIR filters may further be divided into two categories based on the relation (if any) between the amplitude response and phase response of the filter. Specifically, if $ln|H(\omega)|$ and $\phi(\omega)$ form a Hilbert Transform pair, either the phase or amplitude response is sufficient to fully characterize the filter. Given one, the other is completely determined [7]. These types of structures, termed minimum phase filters (MPF), include Fabry-Perot systems and some Bragg grating structures [4]. Non-MPF filters do not have the relationship between the phase and amplitude response and thus offer an additional degree of freedom in terms of the filter design process. The non-MPF category of IIR filters include chirped Bragg gratings and all-pass filters.

To improve the response of the filter it is common practice to couple multiple filters to each other [9]. This can provide a narrower line width for resonant transmission lines from certain filters, though coupling and fabrication errors can diminish the spectral response rather than improve it.

1.4 Dispersive Guiding Structures

1.4.1 Types of Dispersion in Waveguides

In bulk media, dispersion is caused by the variation of material refractive index with respect to wavelength. In waveguide structures dispersion is caused by two additional effects: the difference in propagation constants for different modes, and the dependence of a single mode's propagation constant on frequency. So long as the structure is single-moded, the former plays no part and may be ignored. Material dispersion is generally quite small in comparison with the dispersive values and is only dependent on the material used, so this type of dispersion cannot be adjusted through modifications to the waveguide shape. Hence this discussion will be confined to considerations of waveguide dispersion and an exploration of the ways different frequencies interact with the waveguide shape to allow the tuning of its phase response into a desired optical filter.

1.4.2 Highly Dispersive Guiding Structures

A waveguide with no longitudinal structural variation should introduce minimal amplitude distortion of the incident signal and must rely solely on a phase response as its filtering mechanism. Additionally, since the structure does not rely on resonance effects, the magnitude of the introduced dispersion must be as large as possible to provide a reasonable filter response within a relatively short waveguide length. Thus we specifically look for structures that contain a very high group delay or considerable quadratic dispersion depending on the application. We are primarily concerned with guiding structures of a wafer-size scale that may be formed using standard lithographic and nanofabrication processes and that produce dispersive delay on the order of 1 ps/nm. While bulk materials may yield substantial dispersion near

resonances, they also produce a rather significant degree of absorptive loss. The focus of this research will be on large dispersion magnitudes obtained through the actual structure of the waveguides. We also restrict our consideration to low intensity incident light and neglect nonlinear effects in these types of guiding structures.

1.4.3 Design of Dispersive Waveguides

A number of different approaches have been taken to varying the cross-sectional geometry of guiding structures to produce substantial group delay and dispersion. Although pairs or combinations of waveguides are not generally well known for their dispersion, primarily because most basic descriptions assume identical waveguides, they can prove quite effective under certain circumstances. When the structures differ substantially, dissimilar modes may actually form coupled modes with dispersive magnitudes dependent on the difference in group delays for the two individual guides. Their operation has been explored and discussed elsewhere [10, 11]. Plasmonic [12, 13] and photonic crystal waveguides [14-16] make use of combinations of material and geometrical properties to achieve resonance effects suitable for a strong frequency dependence.

An additional approach presented here adjusts the geometrical cross section of waveguides in the manner of a subwavelength grating or an effective index material pushing the modes close to cut-off to obtain highly dispersive behavior [17]. Neglecting scattering losses, the resulting filter function for a length, L, of the resulting waveguide will have a filter response of the form

$$H(\omega) = e^{-i\beta(\omega)L}$$
(1.18)

where $\beta(\omega)$ is the propagation constant described earlier. The design problem centers on optimizing β for the desired response.

To adequately obtain a specific magnitude of dispersion and group delay, one must carefully tune the geometric shape of the structure. This is particularly true when it operates very near mode cut-off conditions, as the structure bandwidth becomes quite limited and a design wavelength can be shifted into a scattering mode if the geometry changes too much. The design process requires a means to analyze the cross-sectional geometry, determine the effective refractive indices for a range of frequencies, and, from this information, derive an estimate of group delay and dispersion.

1.5 Bragg Grating Filters

Bragg structures and chirped Bragg gratings, especially as used in optical fibers, are some of the more recognizable optical filters [18-20], particularly in the realm of pulse compression and stretching. They involve the coupling of light from one propagating mode into another mode (either forwards or backwards propagating) [21, 22] in a manner highly dependent on frequency, or more specifically, on the ratio of the grating frequency to the propagating wave frequency. One of the biggest drawbacks with these structures is the inherent fabrication limitations which inevitably result in the introduction of non-ideal dispersion in the form of GDR to the system. Various studies have considered the magnitude of the impact of the GDR on the quality of the system as a whole [2, 23, 24], while others have considered a variety of means to compensate for this problem [25, 26]. Additional difficulties with Bragg structures include the significant length of the structures needed for any substantial degree of dispersion and the coupling loss for the typical Bragg reflector arrangement.

1.5.1 Bragg Grating Theory

There are a several ways to approach the operational theory of Bragg reflectors. We generally consider a shallow grating fabricated on top of a waveguide (often an optical fiber) as demonstrated in Figure 1-3 in a form known as a distributed Bragg reflector (DBR). For diffraction gratings, the well known Bragg condition is given by

$$\vec{k}_m = \vec{k}_{inc} - m\vec{K} \tag{1.19}$$

where k_m and k_{inc} give the propagation vectors for the mth diffracted order and the incident field, respectively, and *K* gives the grating vector. For sufficiently small grating periods the only propagating orders are the transmitted and reflected modes. The Bragg condition simplifies to

$$2n\Lambda = \lambda_{B} \tag{1.20}$$

which describes the grating period, Λ , needed to diffract the maximum possible energy into the reflected mode given an average waveguide index, *n*, and free space wavelength, λ_B .



Figure 1-3: Reflective Bragg Grating.

We generally utilize a grating that provides a very small index contrast, which puts it in the weak coupling regime. We can turn to coupled mode theory to further describe the behavior of the structure. If A gives the field strength of the forward propagating mode and B gives the strength for the reflected mode, the coupled-mode equations can be written as

$$\frac{dA}{dz} = -i\kappa B e^{i2\phi z}$$

$$\frac{dB}{dz} = i\kappa A e^{-i2\phi z}$$
(1.21)

where κ is the coupling constant and ϕ is given by

$$\phi = 2\pi \left(\frac{n}{\lambda_0} - \frac{1}{2\Lambda}\right) = 2\pi n \left(\frac{1}{\lambda_0} - \frac{1}{\lambda_B}\right)$$
(1.22)

which obviously goes to zero when the Bragg condition is met. The coupling constant for the Bragg grating structures is given by [4, 27]

$$\kappa = \frac{\pi \eta \Delta n}{2n\Lambda} = \frac{\pi \eta \Delta n}{\lambda_B}$$
(1.23)

where Δn is the index modulation depth and η is the confinement factor of the mode inside the waveguide.

By straightforward solution of Eqns. 1.21 it may be shown [27] that the solutions take the form

$$A(z) = A_0 \frac{\cos[\rho(z-L)] - i\frac{\phi}{\rho}\sin[\rho(z-L)]}{\cos(\rho L) + i\frac{\phi}{\rho}\sin(\rho L)}e^{i\phi z}$$

$$B(z) = A_0 \frac{i\frac{\kappa}{\rho}\sin[\rho(z-L)]}{\cos(\rho L) + i\frac{\phi}{\rho}\sin(\rho L)}e^{-i\phi z}$$
(1.24)

where the boundary conditions $A(0) = A_0$ and B(L) = 0 have been assumed (field incident from the left only). *L* is taken to be the length of the grating and $\rho = \sqrt{\phi^2 - \kappa^2}$. Thus the complex filter responses for the transmitted and reflected beams are given, respectively:

$$\frac{A(L)}{A_0} = \frac{1}{\cos(\rho L) + i\frac{\phi}{\rho}\sin(\rho L)}e^{i\phi L}$$

$$\frac{B(0)}{A_0} = \frac{-i\frac{\kappa}{\rho}\sin(\rho L)}{\cos(\rho L) + i\frac{\phi}{\rho}\sin(\rho L)}$$
(1.25)

Notice that if the incident light frequency falls within a stop band defined by $|\phi| < \kappa$, ρ becomes imaginary and the field strength of the forward propagating wave decays exponentially with distance along the waveguide. Exactly at the Bragg wavelength, the reflectance becomes $R = tanh^2(\kappa L)$. If the magnitude of the coupling constant is small (usually due to the introduction of a very small refractive index modulation) and the grating length is sufficiently long, very specific bands of frequencies may be selectively reflected. Thus, Bragg structures introduce a definite amplitude transformation to the optical signal in contrast with the dispersive waveguide structures where the filter response consists primarily of an induced phase variation.

1.5.2 Principles of Chirped Bragg Gratings

We now note that if the grating period is gradually changed or chirped along its length, different frequency components may be given a relative phase delay proportional to the distance between corresponding Bragg grating periods. This situation means that both κ and ϕ are functions of z. The exact solution of Eqns. 1.21 is nontrivial and is not usually solved directly. A better solution is obtained by considering each section of uniform-period grating individually and identifying a 2x2 transmission-line matrix for it. The full filter response is given by the product of each of these matrices [28]. Since the matrices are seldom trivial, the product is usually calculated numerically and may be used to evaluate the filter response of arbitrary nonuniform gratings [18]. However, an approximate response may be discussed from a purely analytical perspective.

To obtain uniform second-order dispersion from a chirped Bragg structure we would apply a uniform linear chirp to the grating period. The time delay between two spectral components reflected from different positions along the grating separated by a distance, L, is given by

$$\Delta t = n \frac{2L}{c} \tag{1.26}$$

where c is the speed of light and n is the effective index of the guiding structure. The spectral width between these components can be similarly determined by making use of Eqn. 1.20:

$$\Delta \lambda = \Delta \frac{\lambda_0}{n} = 2\Delta \Lambda \tag{1.27}$$

This gives a value for the dispersive delay of the structure as

$$LD = \frac{\Delta t}{\Delta \lambda} = \frac{nL}{c\Delta \Lambda}$$
(1.28)

1.5.3 Group Velocity in Bragg Gratings

Another effect common to resonant structures, such as Bragg gratings, involves the change of the group velocity of a mode. It was mentioned earlier that the phase response of a uniform Bragg grating is given by the Hilbert transform of its amplitude response. Thus, large changes in the filter's amplitude response will result in a correspondingly large change in its phase response. Such a variation in the phase corresponds directly to a large group delay, and a resonant structure operating near the band edge of a filter sees group velocity drop quickly to zero (see Figure 1-4). This effect can also be described in terms of the multiple reflections the

mode experiences before it is able to leak out of the structure. From either perspective, the interaction between the mode and the device geometry is significantly increased. This effect has been specifically employed to enhance the gain available to a given resonant cavity in the formation of one-dimensional band-edge lasers [29]. Similar effects have been obtained through careful tuning of two- and three-dimensional geometries [30, 31].



The large group delay introduced near the edges of the band can also be used to great advantage in certain delay line applications. Although there is only a very small bandwidth to which the large delay values apply, this approach offers a means by which the Bragg structures may be used in a transmission system, obviating the need for a potentially lossy coupling scheme to separate out the incident and reflected signals [19]. The group delay in the pass band is given by [32]

$$\tau_g = \frac{nL}{c} \frac{1}{\sqrt{1 - \left(\frac{\kappa}{\phi}\right)^2}}$$
(1.29)

which rapidly diverges as the wavelength approaches the stop band.
1.5.4 Design of Bragg Structures

Analysis of Eqn. 1.28 reveals a number of advantages and disadvantages of these structures. First, these structures can have an extremely large bandwidth given an appropriately large chirp. The dispersive delay can be increased by simply increasing the length of the structure, while actual dispersion is dependent strictly on the effective index of the structure and the total chirp. A device in glass may have a bandwidth of 15 nm and dispersion on the order of 10^6 ps nm⁻¹ km⁻¹ quite easily by using a total grating chirp of 5 nm.

Unfortunately, to make effective use of such magnitudes of dispersion, that chirp must be stretched out over tens of centimeters. At this point fabrication considerations come into play, and one must define some unique manner in which to uniformly increase the grating period by fractions of a nanometer without introducing significant GDR. Various approaches have been attempted to mitigate these difficulties [6].

A variety of other filtering applications can also be obtained through a nonlinear functional variation of the grating period along the grating length. This can be utilized to tune the filtering characteristics of the structure by introducing pass- and stop-bands to various frequency components in addition to the induced group delay. While the response may be evaluated for an arbitrary grating period function, determining the necessary function to obtain a desired filter response becomes a much more challenging problem.

1.6 Nano-Scale Resonant Structures

1.6.1 Resonant Optical Filters

Optical cavity resonators act much like their analogs in the acoustic world where tuning forks resonate at specific frequencies based on their size. Much of the terminology may also be borrowed from the area of electronics where resonators are composed of inductor-capacitor circuits.

The properties of microcavity resonators have been explored in great detail and are exploited for myriad applications. Whereas waveguides are based principally on the concept of transmitting signals, resonators are devices designed to store optical energy and build up high field intensities [33]. In recent years they have been most useful in quantum electrodynamics experiments and have provided excellent sources and filters in optical communications [34].

While many types of cavities have been fabricated, they commonly fit into one of three basic geometries. Fabry-Perot cavities are based on the concept of highly reflective coatings (often in the form of DBR layers) at either end of a guiding structure, typically forming a sort of pillar structure [35]. In contrast, whispering gallery cavities make use of a circular or elliptical path for the propagating components of the resonant mode. These often come in the form of microspheres or disk or ring resonators [36]. The final standard resonator geometry consists of a defect inside a photonic crystal structure [37, 38], where a standing wave takes on a more two- or three-dimensional nature in comparison to the Fabry Perot (where a standing wave is in the axial direction) and the whispering gallery (where a standing wave is oriented azimuthally) structures.

1.6.2 Modes of Optical Resonators

1.6.2.1 Wave Equation for Cavity Resonators

The derivation of the resonant modes inside cavity resonators begins with Maxwell's equations:

$$\nabla \times \vec{E} = -i\mu_0 \omega \vec{H}$$

$$\nabla \times \vec{H} = i\varepsilon_0 \varepsilon_r \omega \vec{E}$$
(1.30)

The standard derivation of the wave equation involves taking the curl of Faraday's Law and substituting Ampere's Law into it, arriving at

$$\nabla \times \left(\nabla \times \vec{E} \right) = \mu_0 \varepsilon_0 \omega^2 \varepsilon_r \vec{E} = k_0^2 \varepsilon_r \vec{E}$$
(1.31)

If the permittivity in the region can be treated as piecewise constant, we may obtain a similar expression for the magnetic field in each of the separate regions:

$$\nabla \times \left(\nabla \times \vec{H} \right) = \mu_0 \varepsilon_0 \omega^2 \varepsilon_r \vec{H} = k_0^2 \varepsilon_r \vec{H}$$
(1.32)

In rectangular coordinates, the left-hand side of both equations will simply further using the vector identity:

$$\nabla \times \left(\nabla \times \vec{E} \right) = \nabla \left(\nabla \bullet \vec{E} \right) - \nabla^2 \vec{E}$$
(1.33)

This can be further simplified by incorporating Gauss' Law:

$$\nabla \bullet \left(\varepsilon_r \vec{E}\right) = \varepsilon_r \left(\nabla \bullet \vec{E}\right) + \left(\nabla \varepsilon_r\right) \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{E} = -\frac{\nabla \varepsilon_r}{\varepsilon_r} \bullet \vec{E}$$
(1.34)

Since we are again assuming piecewise constant permittivity, the divergence of the electric and magnetic fields are both identically zero inside each region of uniform permittivity.

Unfortunately, in non-Cartesian coordinate systems the Laplacian of the vector field, $\nabla^2 \vec{E}$, has no independent definition, and Eqns. 1.31 and 1.32 cannot be simplified in this manner, which would normally pose a problem for cylindrical geometry. However, this may be overcome by expressing the wave equations in terms of the longitudinal components of the fields [33]. Thus, combining Eqn. 1.33 with Eqns. 1.31 and 1.32, we obtain

$$\left(\nabla^2 + k_0^2 \varepsilon_r\right) \begin{bmatrix} E_z \\ H_z \end{bmatrix} = 0$$
(1.35)

where the Laplacian operator is expressed as

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$
(1.36)

The transverse components are obtained by direct application of Maxwell's Equations (Eqns. 1.30) to this result and solving in terms of the longitudinal components. If the field vectors are written as $\vec{E} = \vec{E}_T + E_z \hat{z}$ and $\vec{H} = \vec{H}_T + H_z \hat{z}$, the transverse components may be expressed as [39]

$$\vec{E}_{T} = \frac{1}{k_{0}^{2} \varepsilon_{r} - \beta^{2}} \left[\nabla_{T} \frac{\partial E_{z}}{\partial z} + i\omega \mu_{0} \nabla_{T} \times (H_{z} \hat{z}) \right]$$
(1.37)

$$\vec{H}_{T} = \frac{1}{k_{0}^{2}\varepsilon_{r} - \beta^{2}} \left[\nabla_{T} \frac{\partial H_{z}}{\partial z} - i\omega\varepsilon_{0}\varepsilon_{r} \nabla_{T} \times (E_{z}\hat{z}) \right]$$
(1.38)

where ∇_T is the transverse gradient operator, and we have assumed $\frac{\partial^2}{\partial z^2} = -\beta^2$. This relies on our ability to represent an arbitrary field distribution as a superposition of plane waves through a Fourier Transform.

1.6.2.2 Cylindrical Symmetry

The cavity geometries considered in this research are principally rotationally symmetric. In such cases, the azimuthal dependence of all solutions simplifies greatly. The boundary conditions require that a given solution and all its derivatives must be continuous at $\phi = 2\pi$. Thus, all resonant solutions for a rotationally symmetric cavity must have the form

$$E(r,\phi,z) = E(r,z)e^{-in\phi}$$
(1.39)

Therefore, the azimuthal derivatives used to determine the various field components may be expressed as

$$\frac{\partial}{\partial \phi} = -in \tag{1.40}$$

Note that the sign of the azimuthal mode number may be switched without affecting the solution of the wave equation. Although the opposite sign would result in an independent solution set, the radial dependence is identical, and it may safely be ignored without loss of generality.

1.6.2.3 Solutions for Dielectric Cylinder

As a specific example, consider the geometry depicted in Figure 1-5. A dielectric cylinder of permittivity ε_r with radius *a* and length L is embedded in a semi-infinite region of permittivity ε_c . The cylinder may be positioned so that the bottom end sits on a substrate region with permittivity ε_s .

The simplicity of the geometry suggests that a separable solution of the form $E_z(r,z) = E_z^{R}(r)E_z^{Z}(z)$ should be appropriate. Thus, Eqn. 1.35 becomes

$$\begin{bmatrix} \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{n^2}{r^2} - \beta^2 + k_0^2 \varepsilon \end{bmatrix} E_z^R(r) = 0$$

$$\begin{bmatrix} \frac{\partial^2}{\partial z^2} + \beta^2 \end{bmatrix} E_z^Z(z) = 0$$
(1.41)

and similarly for the H_z field. To obtain realistic behavior of the fields at very small and very large radial values, the appropriate solutions to the radial equation take the form

$$E_{z}^{R}(r) = \begin{cases} J_{n}\left(r\sqrt{k_{0}^{2}\varepsilon_{r}-\beta^{2}}\right) & r < a \\ K_{n}\left(r\sqrt{-k_{0}^{2}\varepsilon_{c}+\beta^{2}}\right) & r > a \end{cases}$$
(1.42)

The longitudinal solutions are given simply by

$$E_z^{\ Z}(z) = e^{\pm i\beta z} \tag{1.43}$$



An equivalent solution set may be expressed for the magnetic field [33].

1.6.2.4 Radial Solution

The resonant frequencies and hence wavenumbers, k_0 , of the cavity are one of the key qualities we look for when writing particular solutions to the equations above. To obtain the exact expression for the frequencies requires implementing boundary conditions and matching transverse field components at the edge of the cavity. Gauss' Law requires the electric and magnetic field components tangential to the radial surface of the dielectric cylinder to be continuous. Hence, the solutions for E_z , E_{ϕ} , H_z , and H_{ϕ} for radii less than *a* must be continuous with the solutions for radii larger than *a*.

To simplify the expressions somewhat, we make the following change of variables:

$$h = \sqrt{k_0^2 \varepsilon_r - \beta^2}$$

$$q = \sqrt{-k_0^2 \varepsilon_c + \beta^2}$$
(1.44)

Thus, from Eqn. 1.42 we have for E_z

$$AJ_n(ah) = CK_n(aq) \tag{1.45}$$

and for H_z

$$BJ_n(ah) = DK_n(aq)$$
(1.46)

where *A* and *B* are the amplitudes of the longitudinal electric and magnetic fields inside the cylinder and *C* and *D* are the amplitudes outside the cylinder.

To obtain the azimuthal field components, the solutions of Eqns. 1.42 and 1.43 are substituted into Eqns. 1.37 and 1.38. The field-matching expression for the azimuthal electric field component becomes

$$\frac{1}{h^2} \left[\frac{n\beta}{r} A J_n(ah) + i\omega\mu_0 h B J_n'(ah) \right] = -\frac{1}{q^2} \left[\frac{n\beta}{r} C K_n(aq) + i\omega\mu_0 q D K_n'(aq) \right]$$
(1.47)

Similarly, continuity of the azimuthal magnetic field component requires

$$\frac{1}{h^2} \left[\frac{n\beta}{r} BJ_n(ah) - i\omega\varepsilon_0 \varepsilon_r hAJ_n'(ah) \right] = -\frac{1}{q^2} \left[\frac{n\beta}{r} DK_n(aq) - i\omega\varepsilon_0 \varepsilon_c qCK_n'(aq) \right]$$
(1.48)

The general solution to all four continuity expressions results in the following [33]:

$$\left[\frac{J_n'(ah)}{ahJ_n(ah)} + \frac{K_n'(aq)}{aqK_n(aq)}\right] \left[\frac{\varepsilon_r J_n'(ah)}{ahJ_n(ah)} + \frac{\varepsilon_c K_n'(aq)}{aqK_n(aq)}\right] = n^2 \left[\left(\frac{1}{ah}\right)^2 + \left(\frac{1}{aq}\right)^2\right] \left(\frac{\beta}{k_0}\right)^2$$
(1.49)

While the general case is a rather convoluted expression, the simpler transverse cases based on n=0 simplify things considerably. It can easily be shown that the resonant frequencies for TE_{0mp} modes, which assume zero longitudinal electric field, come from setting the leftmost term in square brackets to zero, while the frequencies for the TM_{0mp} modes, derived from the assumption that $H_z = 0$, come from setting the rightmost term from the left hand side of Eqn. 1.49 to zero. By substituting in Eqns. 1.44 and solving numerically, one obtains values for k_0 in terms of β . For a complete numerical value, one must turn to the longitudinal solution to determine β . In general, there will be multiple numerical solutions to Eqn. 1.49. These values correspond to the different radial mode numbers, *m*.

1.6.2.5 Longitudinal Solution

The discussion in the previous section provides the standard solution for propagating fields in cylindrical waveguides. However, the resonant property of a cylindrical cavity involves standing waves rather than traveling waves. This may be represented by taking a forward and backward propagating mode of equal amplitude and summing the two to obtain a sinusoidal field dependence [39]:

$$E_z^{\ 2}(z) = \sin(\beta z + \gamma) \tag{1.50}$$

The standard approach at this point is to assume perfect electrical conductors (PECs) at the top and bottom surfaces of the cylinder, in which case, $\gamma = 0$ and $\beta L = p\pi$, where *p* takes on integer values and refers to the longitudinal mode number of the given resonance. A purely dielectric resonator does not include the requirement that the electric fields go to zero at the ends of the cylinder and the mode will extend outwards into the surrounding region resulting in a larger value for β .

1.6.3 Loss Mechanisms and Quality Factors

The results of the preceding discussion point to the presence of a set of discrete resonant modes in a given cavity. Further, those modes take the form of standing waves. For such solutions, the total energy contained in the cavity will be a time-independent constant value. Specifically, the energy will transition from electrical to magnetic and back again in an optical analog to the ideal circuit resonator consisting of an inductor and a capacitor. This theoretical model leads to two considerations. First, it is reasonable to presume that any non-ideal resonator present in any realistic environment must be able to contain optical signals at frequencies other than the specified resonances. This is particularly true in the case of "loaded resonators" which are coupled to other structures by some means. Light of various frequencies may be directly injected into the resonant cavity. In loaded resonators the resonant modes may exist at some frequency other than the specified resonance. The amplitude of the modes will decrease with the difference between the injected frequency and the ideal resonant frequency [40].

The second issue of note is that structures that permanently contain constant amounts of optical energy are of no practical value. A more useful structure is one into which we can couple a packet of energy, store it for some length of time, and then extract it again at some later time period. This requires that some mechanism exist by which the contained energy in the cavity may decrease over time. The primary reasons for this energy loss involve some combination of internal dissipative absorptive losses into the cavity materials and coupling between the internal resonant cavity modes and external scattering and traveling waves.

The resonant characteristics of optical cavities are termed in like manner to their circuit analogs. In addition to the resonant frequencies, the other property of note is the cavity quality factor (Q-factor) of the corresponding resonance. This unitless number is given as

$$Q = -\omega_r \frac{U}{dU/dt}$$
(1.51)

where ω_r is the frequency of a given resonance and U is the stored energy. Note that this is a differential equation for stored energy whose solution takes the form

$$U(t) = U(0)e^{-\frac{\omega_r}{Q}t}$$
 (1.52)

This suggests a suitable means to incorporate the Q-factor directly into the resonant frequency. If the frequency is assumed to be complex with the form $\tilde{\omega} = \omega_r + i\alpha$, the expression for the contained energy in the cavity results in the following [41]:

$$U(t) \propto \int_{V} |E(\vec{r},t)|^{2} + \int_{V} |H(\vec{r},t)|^{2} = \left[\int_{V} |E(\vec{r})e^{i\tilde{\omega}t}|^{2} + \int_{V} |H(\vec{r})e^{i\tilde{\omega}t}|^{2} \right]$$

= $\left[\int_{V} |E(\vec{r})|^{2} + \int_{V} |H(\vec{r})|^{2} \right] e^{-2\alpha t}$ (1.53)

Thus, we may express the Q-factor in terms of the ratio between the real and imaginary parts of the resonant frequency:

$$Q = \frac{\omega_r}{2\alpha} = \frac{Re[\widetilde{\omega}]}{2\,Im[\widetilde{\omega}]} \tag{1.54}$$

From circuit theory, the transfer function of a resonant circuit element is given by a Lorentzian function in terms of the input frequency:

$$H(i\omega) = \frac{1}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$
(1.55)

From this relation, one can define a frequency bandwidth, $\Delta \omega$, corresponding to the full-width half-max (FWHM) of the transfer function. This is related to the Q-factor in the following way:

$$Q = \frac{\omega_r}{\Delta \omega}$$
(1.56)

One additional aspect that is not immediately evident is that the Q-factor often increases for higher-order modes. This makes sense if one visualizes the higher-order resonator as a collection of smaller resonators containing lower-order modes. This could be accomplished by positioning PECs at each of the nodes in the original resonator. Thus, only the outermost resonator has imperfect lossy sidewalls, and the total power dissipation from the structure is significantly less than the sum of the energy losses from a collection of individual low-order resonators with imperfect sidewalls. On the other hand, the total energy contained inside the structure is exactly equal to the sum of the energy in each of the smaller resonators. Therefore, the ratio of the contained energy to power dissipation, and hence the quality factor, is significantly increased for higher-order resonances [40].

1.6.4 Mode Volume

An additional characteristic of interest for resonant cavities is the mode volume. While the contained cavity volume is immediately intuitive, the mode volume is an integral of the volume of space weighted by the field intensity. This value provides a better indication of the overall size of the contained mode and can indicate its degree of confinement. Large mode volumes are useful in cavity amplifiers, as the field is spread over a large gain region. On the other hand, cavity quantum electrodynamics experiments rely on very small mode volumes to obtain an enhancement to the spontaneous emission rate [34, 42, 43].

The effective mode volume is given by the volume integral of the field intensity divided by the peak intensity [42]:

$$V_m = \frac{2\pi \int_{-\infty 0}^{\infty} |E(r,z)|^2 r dr dz}{max \left(|E(r,z)|^2 \right)}$$
(1.57)

1.6.5 Resonant Enhancements

1.6.5.1 Spontaneous Emission and Purcell Factors

Some of the major applications for resonant microcavities involve quantum electrodynamics experiments and luminescent sources. Both cases typically make use of a quantum dot or dipole located inside a cavity and coupled to its resonant modes [34]. When the coupling is very strong, the atomic source interacts coherently with the resonant mode resulting in an entangled state and a splitting of the transmission peak [44].

The weak coupling system was originally described by Purcell [45], who predicted that if the dissipation time for a photon emitted by an atom or quantum dot inside a cavity were shorter than its radiative lifetime then reabsorption would be minimal and the spontaneous emission (SE) rate would be enhanced [42]. In a resonant cavity with a large Q-factor, the mode density is dramatically increased, which leads to the enhanced SE rate [46]. The enhancement is quantified by the Purcell factor

$$f = \frac{3}{4\pi^2} \left(\frac{\lambda}{n}\right)^3 \frac{Q}{V_m}$$
(1.58)

where λ/n gives the resonant wavelength inside the cavity.

1.6.5.2 Group Velocity and Slow Light

While the slow group velocity is typically observed in resonant devices incorporating a highly reflective structure at either end, which is usually effected by means of DBR layers, a similar result may be obtained in cavities with uniform geometry in the axial direction [47]. If a cylindrical cavity is coated in the radial direction with highly reflective layers (typically either DBR layers, metallic films, or a photonic crystal structure), a pair of resonant modes (such as the

TE and TM modes) will repel each other and can result in anomalous dispersion and even zero group velocity under the right conditions.

Under such conditions the Q-factor of the cavity is dramatically enhanced if the cavity dimensions are carefully balanced with the reflections at the end caps of the cavity. Without conditions at the ends restricting the field at the boundaries, a pair of modes exists for each resonance order (standing sine wave versus cosine wave). By expressing the transfer functions for the pair of modes in terms of cavity length and reflections at both ends, one may obtain its eigenvalues, ζ . The Q-factor for the pair of resonances is given in terms of the cavity length and group velocity as follows [48]:

$$Q = \frac{\omega_r L}{\left| v_g \left| \left(1 - \left| \varsigma \right|^2 \right) \right|}$$
(1.59)

At points where the transfer function eigenvalues are closest to ± 1 (their imaginary part drops to zero), the Q-factor peaks strongly. The distance between the peaks is closely related to the end cap reflections and cavity geometry [49].

1.6.5.3 Nonlinear Effects

In resonant optical filters electromagnetic energy can become highly concentrated in very small areas. This leads directly to nonlinear effects, which can either pose significant design challenges or provide an interesting and exploitable filter characteristic.

Third-order nonlinearities principally appear in the form of the Kerr effect in which the refractive index of a material varies as a function of the incident field intensity:

$$\Delta n = n_2 I \tag{1.60}$$

 n_2 is the material-dependent Kerr constant and *I* is the incident field intensity. In transmission lines, this effect results in self-phase modulation which can take the appearance of standard dispersion [50, 51]. For optical cavities expected to operate under specified field strengths, this effect may be calculated and applied as a perturbation to the cavity permittivity to fully analyze a given design. However, the actual shape of a resonant mode strongly influences the strength of the Kerr effect, so it is difficult to include in the first-order design phase.

Another nonlinear effect that may be explicitly incorporated into the filter design is second harmonic generation (SHG). This effect results in a polarization proportional to the square of the incident field:

$$\vec{P}(2\omega) = \overline{\vec{\chi}}^{(2)} : \vec{E}(\omega)\vec{E}(\omega)$$
(1.61)

This means that an incident pump beam can produce an output signal at twice its initial frequency. This has been experimentally demonstrated in a variety of structures, including photonic crystals [52], which may be designed to incorporate a sign reversal of the nonlinear term to provide quasi-phase matching (QPM) [53, 54].

In the case of optical microcavities, we have the possibility of dual-resonance cavities. By designing the structures to be resonant at both fundamental and harmonic modes we obtain significantly enhanced SHG resulting from a direct corollary of the Purcell effect outlined above [55, 56].

Several issues are cause for consideration when attempting to design such a cavity. First there are issues with phase and mode matching. SHG can result in a second-order pump polarization either exactly in phase or out of phase with the resonant harmonic signal. The former is preferred, while the latter should be suppressed by appropriate cavity design [57, 58].

Further, the modes must overlap sufficiently to create a phase-matched interaction. The relevant overlap integral may be expressed as [59]

$$S = \frac{\int E^2(\omega)E(2\omega)}{\left(\int E^2(\omega)\right)\sqrt{\int E^2(2\omega)}}$$
(1.62)

As an additional consideration, the polarizations of the selected resonant modes must match the corresponding values of the nonlinearity tensor. The tensor is commonly written in the form

$$d_{eff} = \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \\ d_{15} & d_{25} & d_{35} \\ d_{16} & d_{26} & d_{36} \end{bmatrix}$$
(1.63)

where the d_{iJ} terms of the tensor are given such that *i* gives the resulting second-order polarization (x, y, and z respectively) and *J* indicates the combination of the two fundamental field components to be multiplied (xx, yy, zz, yz, xz, and xy respectively). Most highly nonlinear materials only have a few nonzero tensor terms, thus the polarizations of the fundamental and harmonic modes for cylindrical cavity resonators must be chosen carefully.

For example, consider a resonator using a TE_{011} pump mode. If TE_{0mp} harmonic modes are expected, the nonlinear material must have large d_{11} and d_{22} tensor values (d_{33} may also be used if the crystal is oriented properly). Unfortunately, most materials do not exhibit such properties (though SiO₂ could be used if one is content with very low conversion). On the other hand, GaAs has a very large value for the d_{14} tensor element. This indicates an x-polarized harmonic given a product of an E_y and E_z fundamental. A TE_{011} pump mode has a polarization given by

$$\vec{E}(r,\phi) = (-\hat{x}\sin\phi + \hat{y}\cos\phi)E_{\phi}(r)$$
(1.64)

This allows us to obtain a second-order polarization of the form

$$\vec{P}(r,\phi) = -\hat{z}d_{14}E_{\phi}^{2}(r)\sin\phi\cos\phi = \hat{z}d_{14}E_{\phi}^{2}(r)\sin2\phi$$
(1.65)

Notice that this corresponds to the z-component of a second-azimuthal-order mode, and this then should be the choice of the resonant harmonic mode to be optimized.

1.6.6 Design of Resonant Cavities

Resonant optical cavities offer a unique and fitting approach to a wide variety of optical applications. The ability to store optical energy for some length of time, cause it to interact with some material geometry, and then release it, forms the basis for various optical filtering methodologies. Taking a broad spectrum of input frequencies and stripping out a narrow band centered on a specific resonance is essential to photoluminescent structures of various types.

However, as this section has highlighted, the operation and spectral characteristics of resonant cavities vary dramatically with shifts in the geometrical configuration of the structures. To make effective use of such cavities for the many possible applications mentioned previously, two design tools are crucial. First, a method to quickly obtain the optical characteristics of the cavity is vital. While a basic analytic approach has been outlined in this chapter, a rigorous model applicable to arbitrary cavity geometries is necessary.

The second tool of import to the cavity design process is a means to optimize the geometrical configurations and tweak the structures in various ways to obtain specific resonance

conditions. Simple cylindrical cavities can largely be designed by an analytical method, and even in more complicated structures a rough idea of the necessary cavity size can be estimated. However, the tolerances for some of these parameters are extremely limited if one is to obtain a high Q-factor resonance at a specific frequency, and they may be too difficult for back-of-theenvelope methods. Also, when realistic geometries, such as tilted cavity sidewalls and variations in layer thicknesses, are taken into account, the required parameter combinations shift beyond the range of prediction available to simplistic design methods.

Finally, even when cavities are designed for specific resonance conditions, a cavity-based optical filter requires a complete response function which involves coupling characteristics. In one-dimensional resonant filters or in those based on a waveguide coupling light into and out of resonant cavities, the response transfer function has sharp peaks around the resonant frequencies and relatively flat response away from them [60, 61]. In three-dimensional cavities, light that does not couple into the cavity is not necessarily re-coupled into a reflected mode in the manner of waveguide-based resonators. A large fraction of the energy away from resonance can couple into cavity scattering modes and leak out of the device. Thus, a complete analysis of cavity resonant filters requires a complete model for light coupled into and out of the structure as a function of frequency.

1.7 Sign Conventions

This research makes repeated use of Maxwell's Equations and various design and analysis methods derived from them. Solutions to Maxwell's Equations are typically expressed as traveling waves of the form

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) e^{\pm i(\omega t - \vec{k} \cdot \vec{r})}$$
(1.66)

Unfortunately, the sign in the exponent is not consistent across different fields. The "-" sign is more common among physicists, while engineers migrate towards the "+" sign (in addition to the use of "j" in place of "i"). While the different conventions have equivalent meanings, one must apply them consistently in any derivations, particularly in the formulation of numerical modeling approaches. This research makes use of the "+i" sign convention throughout.

1.8 Optical Filter Design Summary

Optical filters and resonators include a vast spectrum of devices and applications. To fully discuss optical filters in detail is well beyond the scope of this research. However, this chapter has provided a theoretical analysis of three different types of filters. Dispersive waveguide structures fall within the category of finite impulse response structures and rely on length scaling to introduce a time delay to the optical signal. Their filter response primarily introduces a frequency-dependent phase variation with little in the way of amplitude modulation (at least for frequencies above cut-off).

Infinite impulse response filters include the two presented examples of a Bragg grating structure and a resonant microcavity. These types of structures rely on resonance and multiple reflections and can produce magnitudes of delay that are independent of the device size. In addition, these two filters introduce an amplitude variation to the output signal and separate specific frequency components for either transmission or reflection. Large delay magnitudes usually occur either at or very close to the resonances of these structures (as would be predicted from the Hilbert transform of the amplitude response).

However, in some ways these two types of structures are opposites of each other. Bragg structures rely on resonant properties to reflect narrow bands of frequencies and largely transmit the rest. Microcavities are used primarily to select and transmit narrow resonant frequencies while reflecting (or simply rejecting) off-resonant components.

A basic theoretical framework for the description of each of these filters has been presented in this chapter. Succeeding chapters will deal with the tools needed to adequately model the realistic response of each type of filter. Additionally, we explore a proposed method for taking the actual filter response and feeding it back into its geometrical description. This allows us to optimize the filter design for a desired spectral response.

CHAPTER 2 RESEARCH OVERVIEW

2.1 Optical Filter Design

Optical filters find uses in a vast array of applications from telecommunications to signal processing and detecting. One of the most significant challenges in obtaining a filter for a given application is the ability to quickly decipher the necessary structure geometry that will provide the given filter response.

In the optical filter design process, one must first determine the types of filtering capabilities required. A filter that simply seeks to introduce a phase delay over a given frequency band will use a very different device than one that needs to selectively remove very narrow frequency components from a broadband signal. Beyond that, an appropriate method must be devised both to model the response of a designed optical filter and to adjust its geometry to obtain a response function that more closely matches that required by the given application.

Three classes of optical filters have been identified and discussed in detail. Innovative design approaches will be presented for filters in each category, and the necessary numerical modeling and optimization tools required to develop each filter will be discussed.

2.2 Design and Optimization Tools

In order to adequately analyze and design any such structures, a number of numerical techniques is required. The first category of design tools consists of methods that evaluate Maxwell's Equations for a given geometry in an attempt to determine the interaction of electric and magnetic fields with the structure. These tools allow one to analyze a given structure and

determine its overall electromagnetic properties and its response to given boundary conditions (expressed as optical sources). The other type of design tool used in this research is a numerical optimization method. Optical filters typically have a wide variety of geometrical variables, each affecting the filter response in a different manner. The goal of numerical optimization is to find a combination of geometrical parameters that allows the filter to perform to some predefined criteria.

2.2.1 Numerical Analysis of Optical Filters

CHAPTER 3 presents a number of frequency-domain numerical techniques. These methods assume monochromatic conditions and evaluate Maxwell's Equations across a given filter geometry for a single optical frequency. To find the overall spectral response, one must recalculate the response at a large number of closely spaced frequencies across the band of interest.

The frequency-domain techniques presented herein are threefold. First, an eigenfrequency solver involves expressing Maxwell's Equations as an eigenvalue problem with the optical frequency given by the eigenvalues of the system. This allows one to obtain the resonant frequencies and corresponding Q-factors of a given filter geometry. Since resonance conditions are inherently assumed, this method is primarily used to analyze the spectral characteristics of resonant cavities.

The second numerical method similarly expresses Maxwell's Equations as an eigensystem, but in contrast with the first approach assumes that the field may be expressed as waves propagating along the z-axis. In this manner, the eigenvalues of the system are the wave

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propagation constants for a given optical frequency. This method lends itself well to the analysis of dispersive waveguides.

The final technique, known as the method of lines (MOL) is largely an extension of the second method to geometries that vary in the propagation direction. Specifically, propagation constants can be obtained in each discrete layer in the axial direction, and then the sets are connected using electromagnetic boundary conditions to obtain a single transfer matrix describing the response of the entire structure. This approach allows one to obtain both reflectance and transmittance for an arbitrary incident electric field and can provide both phase and amplitude response. It can be used for most types of optical filters and will be specifically applied to the analysis of Bragg grating structures and optical cavity filters in this research.

2.2.2 Optimization of Optical Filters

Evaluation of the phase and amplitude response for a given geometry is essential to the overall analysis of optical filters, but the design phase typically requires incremental improvements to designs to produce a specific filter response. For some simple filters and response functions, requisite geometries may often be derived from first principles. However, complex response functions and elaborate filter geometries may require an iterative numerical optimization method. CHAPTER 4 outlines the particle swarm optimization (PSO) tool, one of the more recently developed probabilistic search algorithms. This method provides a reasonably fast and efficient way to adjust filter geometries to obtain desired filter responses.

2.3 Dispersive Waveguide Filter Design

The first major category of optical filters discussed above involves waveguides with large phase response. In these structures the geometry is designed to increase group delay and dispersion. CHAPTER 5 explores a specific example of a dispersive waveguide filtering mechanism (illustrated in Figure 2-1). This geometry operates very close to cutoff for TE-type propagating modes, and this combined with the periodic nature of the structure results in very large dispersive magnitudes. The actual capabilities and response of the structure (termed a "Nano Dispersion Amplified Waveguide," or "Nano-DAWG") is discussed in detail. Coupling to the structure and alternative approaches are also explored.



Figure 2-1: Nano Dispersion Amplified Waveguide Structure.

2.4 Bragg Grating Filter Design

The second major area of optical filters explored in this research makes use of Bragg gratings to form the filter response. CHAPTER 6 explores an innovative approach to the design and fabrication of Bragg grating waveguide structures. The grating vector is decoupled from the waveguide trajectory, introducing an additional degree of flexibility in the design of filter response without succumbing to fabrication constraints. Figure 2-2 illustrates the basic approach

whereby a radial grating is fabricated on an optical substrate and a waveguide with an arbitrary trajectory is placed directly on top. The combination of the two produces some filter response. The decoupling approach, in addition to providing a significant degree of flexibility in terms of possible responses, allows one to re-use the same grating for a variety of different waveguides and corresponding filter responses. A special case design based on a linear group delay curve is discussed in detail and the parameters and approach necessary for a more general case is outlined.



Figure 2-2: Spiral Waveguide, Radial Grating, and the Resulting Combination.

2.5 Microcavity Filter Design

The final class of optical filters of interest to this research departs from the waveguide nature of the first two filter classes and moves into the realm of three-dimensional filter design using resonant microcavities as the foundational element. CHAPTER 7 describes the design and optimization of axially symmetric resonant microcavities and a number of approaches to optical filtering using such structures. One approach uses exotic cavity geometries to confine the resonant mode and obtain large Q-factors to allow a single cavity to filter out narrow transmission bands around design wavelengths. A more interesting case makes use of multiplexed cavities with somewhat lower Q-factors (as illustrated in Figure 2-3). By coupling a large number of these structures together, the Q-factor is increased, although additional spectral features begin to appear due to resonance and oscillations between cavities. Additionally, by making use of higher Q cavities using DBR layers for confinement significantly higher group delay magnitudes are obtained, albeit over a much narrower bandwidth. These features and possible uses for these structures are discussed in detail, and some specific design examples are demonstrated and analyzed.



Figure 2-3: Multiplexing of GaAs/AlAs Cavity Filter.

2.6 Research Summary

The fundamental goal of this research is to provide the design tools and approaches necessary for the formulation of various types of nanostructured optical filters. The applicable numerical tools for design, analysis, and optimization are derived and discussed in detail. In addition, three classes of optical filters are explored and innovative designs in each category are presented and characterized. Coupling and fabrication challenges are noted and accounted for in

the analysis, and calculations and predictions for the overall filter responses for the different designs are presented.

CHAPTER 3 FREQUENCY DOMAIN MODELING OF OPTICAL FILTERS

3.1 Eigenstates of Optical Filters

The preceding chapters outlined a few varieties of optical filters, each of which has a unique response function in the frequency domain. Delay lines are highly dispersive devices whose geometrical configurations are used to introduce a large shift in the propagation constant as a function of wavelength. The variation of the propagation constant of the fundamental guided mode provides a means to calculate the structure's dispersion. Optical resonators involve a cavity around which electromagnetic fields circulate. The resonant modes of the cavity are caused by standing waves of the field around the circumference of the cavity. The geometry of the cavities and the requirement that the accumulated phase around the perimeter be an integer multiple of 2π tends to result in a very narrow spectral width of the resonant modes for microcavity resonators. This gives them a very significant quality factor which makes them advantageous choices for a number of optical filtering applications. The difficulty is in designing and predicting the behavior of both of these types of optical filters.

Frequency domain simulation methods are based fundamentally on solving Maxwell's Equations either at or for a single frequency value. This is in direct contrast to the time domain approach presented in the next chapter. These models are based on steady-state behavior of systems. In order to obtain broadband results, frequency domain simulations must be run multiple times to ascertain the response at each individual spectral component. Thus, there are certain limitations to these models. Dispersion, nonlinear effects, and other aspects of some

optical filters are not directly attainable from these models, though they may be estimated based on filter behavior calculated for various relevant wavelengths.

That said, frequency domain models are typically quite fast. Further, they are absolutely essential if one wishes to observe very narrow spectral linewidths in the optical response. Time domain models must be run for exorbitant amounts of time to be able to even approximate the sharp frequency-dependent changes in the response function.

While there are myriad frequency domain models, those relevant to this research are all based, in part, on formulating Maxwell's Equations into an eigenvalue problem, where the eigenvalues will represent either a propagation constant or resonant frequency depending on the formulation.

3.2 Modeling Fields on a Grid

3.2.1 Finite Difference Approximations

Critical to formulating the equations for any model and to representing the fields and material parameters is the determination of the best numerical representation. The principle models used in this research are based on the finite difference representation for derivatives and a rectangular grid. In this way, continuous functions are fit to a grid where the values are specified at discrete points. I make use of a rectangular grid for ease of implementation, though others have demonstrated improved accuracy and efficiency with more elaborate gridding schemes such as variable grid spacing [62] and conformal surface gridding [63].

The standard approach for the finite difference expressions involves a simple Taylor's series expansion allowing for second order accurate derivatives [64]. The first derivatives are expressed according to

$$f'(x) = \frac{f\left(x + \frac{\Delta x}{2}\right) - f\left(x - \frac{\Delta x}{2}\right)}{\Delta x} + O(\Delta x)^2$$
(3.1)

This provides the first derivative midway between two grid points separated by a distance Δx given the field value at both points. A second derivative may be obtained by applying the first derivative operator twice.

This equation is suitable for most situations, though additional terms may be included to provide derivatives of higher-order accuracy. Additionally, others [65, 66] have demonstrated alternative formulations of the derivative operators that provide a significant improvement in the order of accuracy under certain conditions.

3.2.2 Field Representation

The numerical methods used throughout this research are explicitly based only on the curl equations rather than the full set of Maxwell's Equations. In general, the divergence equations are a direct consequence of a charge-free medium. However, when the field components are snapped to a finite grid extra care must be taken to ensure the divergence conditions are satisfied, particularly for two- and three-dimensional problems. If the field components are oriented incorrectly or if they are all co-located on the grid, the divergence conditions will not necessarily be satisfied.

The most common finite difference gridding algorithm was introduced by Yee [67] and is illustrated in Figure 3-1. The H-fields are interlaced around the E-fields so that they rotate around each other. This method offers a number of convenient attributes [68]:

1) The location of the field components lends itself immediately to a straightforward implementation of the curl expressions with second-order accurate derivatives.

- 2) The tangential components are naturally maintained across the interface of dissimilar materials without the need to apply additional requirements to strictly enforce the boundary conditions.
- 3) The divergence conditions are automatically satisfied.
- 4) The nonphysical numerical dispersion resulting from the finite grid is significantly less with the Yee grid than with co-located grids [69].





Axisymmetric Simulations.

To provide sufficiently accurate results, the grid spacing must be significantly smaller than the wavelength of interest. On the other hand, if the grid spacing is too small, the computation time and computer memory required to complete the simulation become unreasonable. A reasonable compromise between speed and accuracy occurs when the grid spacing is roughly between $\frac{\lambda_{min}}{10n_{max}}$ and $\frac{\lambda_{min}}{40n_{max}}$, where λ_{min} is the smallest wavelength

of interest and n_{max} is the largest refractive index present in the simulation region.

3.3 Maxwell's Equations Formulation

As with most numerical methods, we begin the derivation with Maxwell's Equations:

$$\nabla \times \vec{E} = -i\mu_0 \omega \vec{H}$$

$$\nabla \times \vec{H} = i\varepsilon_0 \varepsilon_* \omega \vec{E}$$
(3.2)

The standard derivation of the wave equation involves taking the curl of Faraday's Law and substituting Ampere's Law into it, arriving at

$$\nabla \times \left(\nabla \times \vec{E} \right) = \mu_0 \varepsilon_0 \omega^2 \varepsilon_r \vec{E} = k_0^2 \varepsilon_r \vec{E}$$
(3.3)

In rectangular coordinates, the left hand side will simplify further using the vector identity

$$\nabla \times \left(\nabla \times \vec{E} \right) = \nabla \left(\nabla \bullet \vec{E} \right) - \nabla^2 \vec{E}$$
(3.4)

This can be further simplified by incorporating Gauss' Law:

$$\nabla \bullet \left(\varepsilon_r \vec{E} \right) = \varepsilon_r \left(\nabla \bullet \vec{E} \right) + \left(\nabla \varepsilon_r \right) \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{E} = -\frac{\nabla \varepsilon_r}{\varepsilon_r} \bullet \vec{E}$$

(3.5)

Eqn. 3.3 then simplifies to

$$\nabla^2 \vec{E} + \nabla \left(\frac{\nabla \varepsilon_r}{\varepsilon_r} \bullet \vec{E} \right) + k_0^2 \varepsilon_r \vec{E} = 0$$
(3.6)

In non-Cartesian coordinate systems, the vector Laplacian, $\nabla^2 \vec{E}$, is not defined apart from the curl operator (Eqn. 3.4), and Eqn. 3.3 cannot be simplified in this manner.

Note that the curl operators may be written in matrix form according to

$$\nabla \times \vec{A} = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$
(3.7)

so long as we are operating in rectangular coordinates. The case is similar with cylindrical coordinates, though the matrix is slightly more complicated:

$$\nabla \times \vec{A} = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{1}{r} \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial r} \\ -\frac{1}{r} \frac{\partial}{\partial \phi} & \frac{1}{r} + \frac{\partial}{\partial r} & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_{\phi} \\ A_z \end{pmatrix}$$
(3.8)

It will also be convenient to represent the divergence operator in cylindrical coordinates in matrix form:

$$\nabla \bullet \vec{A} = \left(\frac{1}{r} + \frac{\partial}{\partial r} \quad \frac{1}{r} \frac{\partial}{\partial \phi} \quad \frac{\partial}{\partial z}\right) \begin{pmatrix} A_r \\ A_{\phi} \\ A_z \end{pmatrix}$$
(3.9)

3.4 Perfectly Matched Layer Boundary Conditions

The boundary conditions require extra care to eliminate non-physical reflections from the edges of the simulated area. The most common formulation for absorbing boundaries is known as the perfectly matched layers (PML) and incorporates what is known as the stretched coordinate method [70, 71]. In this method, each successive layer at the boundary has a somewhat higher electrical conductivity than the one before it, while at the same time a simulated magnetic conductivity is applied to the layer to ensure impedance matching and to minimize reflections.

In some cases, Dirichlet boundaries may be used without adversely affecting the simulation results. This is only the case if the field is sufficiently small at all boundaries so that reflections are negligible. This does allow some flexibility in the formulation of the expressions, but also requires extra caution, as the hard boundaries can cause the appearance of non-physical guided modes that would not exist apart from the boundary conditions.

Additionally, the estimation of loss terms of both guided modes and resonant frequencies requires a mechanism for said loss to occur. PML boundaries allow energy from the simulated modes to leak out of the region, resulting in complex eigenvalues whose imaginary part provides a mechanism to estimate the loss.

3.4.1 Material Tensor Expressions

Permeability and permittivity both become tensors representing uniaxial materials inside PML regions [70]. For minimal reflections, they can be shown [68] to take the form of an invertible diagonal matrix

$$\overline{\overline{s}} = \begin{pmatrix} \frac{s_3 s_2}{s_1} & 0 & 0\\ 0 & \frac{s_3 s_1}{s_2} & 0\\ 0 & 0 & \frac{s_1 s_2}{s_3} \end{pmatrix} \qquad s_1(u_1) = \kappa_1(u_1) + \frac{\sigma_1(u_1)}{i\omega\varepsilon} \\ s_2(u_2) = \kappa_2(u_2) + \frac{\sigma_2(u_2)}{i\omega\varepsilon} \\ s_3(u_3) = \kappa_3(u_3) + \frac{\sigma_3(u_3)}{i\omega\varepsilon} \end{cases}$$
(3.10)

where u_1 , u_2 , and u_3 are the coordinates for the three dimensional region (*x*,*y*,*z* for Cartesian coordinates or *r*, ϕ ,*z* for cylindrical coordinates). The κ and σ terms scale the real and imaginary parts of the permittivity (and permeability) as a function of depth inside the PML.

There are different methods for expressing the κ and σ terms [68], though I make use of a polynomial grading of the form

$$\kappa(u) = 1 + (\kappa_{max} - 1) \left(\frac{u}{d}\right)^{m}$$

$$\sigma(u) = \sigma_{max} \left(\frac{u}{d}\right)^{m}$$
(3.11)

where *m* is the order of the grading and *d* is the width of the PML region. A significant amount of experimentation [72] has been performed on various combinations of parameters, though $m \approx$ 3, $d \approx 10$ grid points, and $\sigma_{max} \approx \frac{3.5n}{\Delta}$ (where *n* is refractive index and Δ is the grid spacing) appear to provide reasonably good performance. Optimal values of κ_{max} vary according to the problem geometry, though it is usually between 1 and 5.

3.4.2 Cylindrical Coordinates

There are additional considerations in cylindrical coordinates. The ϕ component might not appear to need any PML considerations due to a lack of any boundary in that direction. However, the radial and azimuthal components are not independent, so a radially-dependent azimuthal PML term is still required [73, 74]. Eqn. 3.10 is still suitable for the *r* and *z* components (s_1 and s_3), but the azimthal term is given by

$$s_{\phi}(r) = \frac{r_0 + \int_{r_0}^r s_r(r')dr'}{r}$$
(3.12)

where r_0 is the radius at the edge of the PML region.

3.4.3 Modification of Maxwell's Equations

Since the PML tensor scales both the permittivity and permeability, both of Mawell's curl Equations are affected:

$$\nabla \times \vec{E} = -i\bar{s}\mu_0 \omega \vec{H}$$

$$\nabla \times \vec{H} = i\bar{s}\varepsilon_0 \varepsilon_r \omega \vec{E}$$
(3.13)

The resulting expressions are no longer as easy to simplify into a succinct form as they were previously. However, the PML tensor is an invertible diagonal matrix, and the matrix forms for the curl expressions allow us to solve the pair with a limited degree of manipulation.

3.5 Eigenfrequency Calculations

When attempting to model and design micro-cavities, one needs a method for obtaining the resonant modes of the structures. The structures will typically resonate at a variety of different frequencies, each having its own field distribution and Q-factor. There are a number of different methods for obtaining these resonant modes [75-77], each method offering a different set of advantages and disadvantages. For sufficiently small problems, or geometries where symmetry may reduce the necessary simulation points, methods based on linear algebra [78-80] provide a fast and efficient algorithm for obtaining the modes.

Maxwell's Equations express the field components in the cavity. The resonance condition and cavity geometry are used to combine and simplify the equations into a pair of differential equations in terms of the electric field. These are expressed in terms of an eigenvalue problem, where the calculated eigenvalues are interpreted as the resonant frequencies of the cavity.

When we calculate eigenfrequencies, we are looking for two pieces of information: the resonant frequencies of the cavity and the Q-factors of each resonance. PML boundaries are essential to the estimation of Q-factors, as mentioned above, so Eqns. 3.13 are central to the formulation of the model. Further, the polar coordinate system adds a degree of complexity to the operators, and the formulation of the curl expressions in Eqn. 3.8 is used.
3.5.1 **Problem Symmetry**

We assume a rotationally symmetric cavity geometry, which implies an angular symmetry for the fields as well. For a resonant mode inside such a structure, the boundary conditions require

$$\vec{E}(r,z,\phi) = \vec{E}_n(r,z)e^{-in\phi}$$

$$\vec{H}(r,z,\phi) = \vec{H}_n(r,z)e^{-in\phi}$$
(3.14)

where n is an integer describing the azimuthal variation of the mode. The mode equations are separately derived for each n value and are solved independently. The excited field in the region should be the superposition of all the resonant modes from each n value. This indicates that the angular derivative can be expressed analytically

$$\frac{\partial}{\partial \phi} \vec{E} = -in\vec{E}$$
(3.15)

and similarly for the *H* field.

With n given, the problem can be reduced to two dimensions by incorporating the divergence condition (assuming a charge-free region). Since Gauss' Law is defined in terms of electric displacement rather than electric field, we make a change of variables at this point:

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \tag{3.16}$$

The divergence condition then gives

$$\nabla \bullet \vec{D} = \left(\frac{1}{r} + \frac{\partial}{\partial r} - \frac{-in}{r} - \frac{\partial}{\partial z}\right) \begin{pmatrix} D_r \\ D_{\phi} \\ D_z \end{pmatrix} = 0$$
(3.17)

The azimuthal component may now be eliminated by a convenient transform matrix:

$$\begin{pmatrix} D_r \\ D_{\phi} \\ D_z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{i}{n} - \frac{ir}{n} \frac{\partial}{\partial r} & \frac{-ir}{n} \frac{\partial}{\partial z} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D_r \\ D_z \end{pmatrix} = 0$$
 (3.18)

This allows us to write an eigenvalue problem in terms of D_r and D_z . However, the terms involving *n* in the denominator suggest that an alternative approach must be applied when working with zero-order modes. Eqn. 3.17 indicates that the azimuthal term is independent of the radial and z-directed field components in this case. The same is true with the magnetic field, which suggests that the zero-order modes may be represented as a function of either E_{ϕ} or H_{ϕ} exclusively. The electric displacement vector components are co-located with the corresponding electric field components.

3.5.2 Derivative Operators and Boundary Conditions

Since several terms incorporate an inverse of the radial coordinate (which becomes singular on the z-axis) proper positioning of field components is critical. The grid is structured so that only field components that go to zero on the z-axis are positioned on integer radial grid locations [68]. Components that are nonzero on-axis are located on the half-grid in the radial direction, as illustrated in Figure 3-2.



Figure 3-2: Field Component Locations for Axisymmetric Simulations

Eqn. 3.1 provides the formulation for the derivative operators, but the positioning of the field components requires slight differences in the exact construction of the corresponding derivatives. For example:

$$H_{\phi}\Big|_{m} = H_{\phi}\Big|_{m\Delta r} \propto \frac{\partial D_{z}}{\partial r}\Big|_{m\Delta r} = \frac{1}{\Delta r} \left(D_{z} \Big|_{m} - D_{z} \Big|_{m-1} \right)$$

$$E_{\phi}\Big|_{m} = E_{\phi}\Big|_{m\Delta r} \propto \frac{\partial H_{z}}{\partial r}\Big|_{m\Delta r} = \frac{1}{\Delta r} \left(H_{z} \Big|_{m+1} - H_{z} \Big|_{m} \right)$$
(3.19)

Thus, we define four different derivative operators:

$$\left(\widetilde{D}_{r,E}A\right)_{r,z} = \frac{A_{r,z} - A_{r-1,z}}{\Delta r}$$
(3.20)

$$\left(\widetilde{D}_{r,H}A\right)_{r,z} = \frac{A_{r+1,z} - A_{r,z}}{\Delta r}$$
(3.21)

$$\left(\widetilde{D}_{z,E}A\right)_{r,z} = \frac{A_{r,z+1} - A_{r,z}}{\Delta z}$$
(3.22)

$$\left(\widetilde{D}_{z,H}A\right)_{r,z} = \frac{A_{r,z} - A_{r,z-1}}{\Delta z}$$
(3.23)

The operators are identified so that the *E*-labeled derivative operators are used in the curl of the electric field in Eqn. 3.13, while the *H*-labeled operators are incorporated in the curl of the magnetic field. Eqns. 3.20 and 3.23 operate on integer grids and result in values for field components on the half-integer grids. Eqns. 3.21 and 3.22 operate on the half integer grids and result in values located on the integer grids. This allows each of the derivative operators to be implemented as matrices with nonzero elements strictly along the main diagonal and a single additional diagonal. The field components can then be represented as a vector, allowing the system of equations to be solved via linear algebraic methods.

3.5.3 Transverse Modes

As discussed earlier, the zero-order modes are handled in a slightly different manner than the higher-order ones. The divergence condition (Eqn. 3.17) indicates that the azimuthal component of the electric field is linearly independent from the radial and axial components. This indicates that the zero-order modes can be divided into two categories: transverse electric (E_{ϕ}) and transverse magnetic (H_{ϕ}) . These will be designated as TE_{0mp} and TM_{0mp} modes respectively.

3.5.3.1 TE_{0mp} Modes

Since the azimuthal derivatives are all zero, the cross-product operator from Eqn. 3.8 simplifies to the following:

$$\nabla \times \vec{A} = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} & 0 \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial r} \\ 0 & \frac{1}{r} + \frac{\partial}{\partial r} & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_{\phi} \\ A_z \end{pmatrix}$$
(3.24)

Since we are only concerned with the E_{ϕ} H_r , and H_z field components, Maxwell's Equations in operator form become

$$\begin{pmatrix} -\frac{\partial}{\partial z} \\ \frac{1}{r} + \frac{\partial}{\partial r} \end{pmatrix} E_{\phi} = \begin{pmatrix} -\widetilde{D}_{z,E} \\ \frac{1}{r}\widetilde{D}_{r,E}r \end{pmatrix} E_{\phi} = -i\overline{s}\mu_{0}\omega \begin{pmatrix} H_{r} \\ H_{z} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial}{\partial z} & -\frac{\partial}{\partial r} \end{pmatrix} \begin{pmatrix} H_{r} \\ H_{z} \end{pmatrix} = (\widetilde{D}_{z,H} & -\widetilde{D}_{r,H} \begin{pmatrix} H_{r} \\ H_{z} \end{pmatrix} = i\overline{s}\varepsilon_{0}\varepsilon_{r}\omega E_{\phi}$$
(3.25)

Since $\overline{\overline{s}}$ is invertible, the first equation may be solved for the magnetic field

$$\begin{pmatrix} H_r \\ H_z \end{pmatrix} = i\overline{\overline{s}}^{-1} (\mu_0 \omega)^{-1} \begin{pmatrix} -\widetilde{D}_{z,E} \\ \frac{1}{r} \widetilde{D}_{r,E} r \end{pmatrix} E_{\phi}$$
(3.26)

which is then substituted into the second equation:

$$(\widetilde{D}_{z,H} - \widetilde{D}_{r,H})\overline{s}^{-1} (\mu_0 \omega)^{-1} \begin{pmatrix} -\widetilde{D}_{z,E} \\ \frac{1}{r} \widetilde{D}_{r,E} r \end{pmatrix} E_{\phi} = \overline{s} \varepsilon_0 \varepsilon_r \omega E_{\phi}$$
(3.27)

After some simplification and substitution of the appropriate PML matrix elements, the operator matrix equation becomes

$$\frac{1}{\varepsilon_r s_{22}} \left(\widetilde{D}_{z,H} \left(\frac{1}{s_{11}} \widetilde{D}_{z,E} \right) + \widetilde{D}_{r,H} \left(\frac{1}{r s_{33}} \widetilde{D}_{r,E} (r) \right) \right) E_{\phi} = -k_0^2 E_{\phi}$$
(3.28)

We may simplify this expression a bit further if we expand the PML terms. The terms are expressed according to

$$s_{11} = \frac{s_{\phi}(r)s_{z}(z)}{s_{r}(r)}$$

$$s_{22} = \frac{s_{r}(r)s_{z}(z)}{s_{\phi}(r)}$$

$$s_{33} = \frac{s_{r}(r)s_{\phi}(r)}{s_{z}(z)}$$
(3.29)

Some of the PML terms may be pulled out of the partial derivatives resulting in

$$\left[\frac{1}{\varepsilon_r s_z}\widetilde{D}_{z,H}\left(\frac{1}{s_z}\widetilde{D}_{z,E}\right) + \frac{s_{\phi}}{\varepsilon_r s_r}\widetilde{D}_{r,H}\left(\frac{1}{r s_r s_{\phi}}\widetilde{D}_{r,E}(r)\right)\right]E_{\phi} = -k_0^2 E_{\phi}$$
(3.30)

The operators on the left hand side may all be combined into a single matrix, resulting in a standard eigenvalue problem. The eigenvalues of the system give the complex wavenumbers of each of the resonant TE_{0mp} modes, while the eigenvectors give the azimuthal electric field for the mode.

3.5.3.2 TM_{0mp} Modes

For the TM modes, the first concern is the boundary conditions. As with the TE case, the azimuthal field component is zero on-axis, which suggests that the coordinate system indicated in Figure 3-2 would be better served if the H and E field components were all swapped.

Following the same method as the derivation of the TE modes, we assume that the only field components present are H_{ϕ} E_r , and E_z , which instead results in

$$\begin{pmatrix} \widetilde{D}_{z,H} & -\widetilde{D}_{r,H} \begin{pmatrix} E_r \\ E_z \end{pmatrix} = -i\overline{s}\mu_0 \omega H_{\phi}$$

$$\begin{pmatrix} -\widetilde{D}_{z,E} \\ \frac{1}{r}\widetilde{D}_{r,E}r \end{pmatrix} H_{\phi} = i\overline{s}\varepsilon_0\varepsilon_r \omega \begin{pmatrix} E_r \\ E_z \end{pmatrix}$$

$$(3.31)$$

Solving for the electric field and substituting gives

$$\left(\widetilde{D}_{z,H} - \widetilde{D}_{r,H}\right)\left(\overline{\overline{s}}\varepsilon_r\right)^{-1} \left(\frac{-\widetilde{D}_{z,E}}{\frac{1}{r}\widetilde{D}_{r,E}}r\right) H_{\phi} = \overline{\overline{s}}k_0^2 H_{\phi}$$
(3.32)

Expanding and simplifying further produces the following:

$$\frac{1}{s_{22}} \left(\widetilde{D}_{z,H} \left(\frac{1}{s_{11}} \widetilde{\mathcal{E}}_r \widetilde{D}_{z,E} \right) + \widetilde{D}_{r,H} \left(\frac{1}{rs_{33}} \widetilde{\mathcal{E}}_r \widetilde{D}_{r,E} r \right) \right) H_{\phi} = -k_0^2 H_{\phi}$$
(3.33)

Once again, canceling appropriate PML terms results in the final eigenvalue expression:

$$\left[\frac{1}{s_z}\widetilde{D}_{z,H}\left(\frac{1}{s_z\varepsilon_r}\widetilde{D}_{z,E}\right) + \frac{s_\phi}{s_r}\widetilde{D}_{r,H}\left(\frac{1}{rs_rs_\phi\varepsilon_r}\widetilde{D}_{r,E}r\right)\right]H_\phi = -k_0^2H_\phi$$
(3.34)

This equation is solved in the same manner as the TE equation to obtain the TM_{0mp} resonant modes.

3.5.4 Higher Order Modes

The derivation of the eigenvalue expression for the higher order modes closely follows that of the TE modes, though with a greater degree of complexity. One major difference is that the equations are derived in terms of electric displacement defined by Eqn. 3.16 as opposed to electric field. As before, Maxwell's Equations from Eqn. 3.13 are expressed in matrix form:

$$\begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{1}{r} \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial r} \\ -\frac{1}{r} \frac{\partial}{\partial \phi} & \frac{1}{r} + \frac{\partial}{\partial r} & 0 \end{pmatrix} \frac{1}{\varepsilon_0 \varepsilon_r} \begin{pmatrix} D_r \\ D_{\phi} \\ D_z \end{pmatrix} = -i\mu_0 \omega \begin{pmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & 0 \\ 0 & 0 & s_{33} \end{pmatrix} \begin{pmatrix} H_r \\ H_{\phi} \\ H_z \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{1}{r} \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial r} \\ -\frac{1}{r} \frac{\partial}{\partial \phi} & \frac{1}{r} + \frac{\partial}{\partial r} & 0 \end{pmatrix} \begin{pmatrix} H_r \\ H_{\phi} \\ H_z \end{pmatrix} = i\omega \begin{pmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & 0 \\ 0 & 0 & s_{33} \end{pmatrix} \begin{pmatrix} D_r \\ D_{\phi} \\ D_z \end{pmatrix}$$
(3.35)

After replacing the derivatives with the appropriate operators and simplifying, the equations become

$$\frac{-1}{i\varepsilon_{0}\mu_{0}\omega} \begin{pmatrix} s_{11}^{-1} & 0 & 0 \\ 0 & s_{22}^{-1} & 0 \\ 0 & 0 & s_{33}^{-1} \end{pmatrix} \begin{pmatrix} 0 & -\widetilde{D}_{z,E} & \frac{-in}{r} \\ \widetilde{D}_{z,E} & 0 & -\widetilde{D}_{r,E} \\ \frac{in}{r} & \frac{1}{r}\widetilde{D}_{r,E}(r) & 0 \end{pmatrix} \frac{1}{\varepsilon_{r}} \begin{pmatrix} D_{r} \\ D_{\phi} \\ D_{z} \end{pmatrix} = \begin{pmatrix} H_{r} \\ H_{\phi} \\ H_{z} \end{pmatrix}$$

$$\begin{pmatrix} s_{11}^{-1} & 0 & 0 \\ 0 & s_{22}^{-1} & 0 \\ 0 & 0 & s_{33}^{-1} \end{pmatrix} \begin{pmatrix} 0 & -\widetilde{D}_{z,H} & \frac{-in}{r} \\ \widetilde{D}_{z,H} & 0 & -\widetilde{D}_{r,H} \\ \frac{in}{r} & \frac{1}{r}\widetilde{D}_{r,H}(r) & 0 \end{pmatrix} \begin{pmatrix} H_{r} \\ H_{\phi} \\ H_{z} \end{pmatrix} = i\omega \begin{pmatrix} D_{r} \\ D_{\phi} \\ D_{z} \end{pmatrix}$$
(3.36)

The two equations may now be combined and simplified further, resulting in

$$\begin{pmatrix}
0 & -s_{11}^{-1}\widetilde{D}_{z,H} & \frac{-in}{rs_{11}} \\
s_{22}^{-1}\widetilde{D}_{z,H} & 0 & -s_{22}^{-1}\widetilde{D}_{r,H} \\
\frac{in}{rs_{33}} & \frac{1}{rs_{33}}\widetilde{D}_{r,H}(r) & 0
\end{pmatrix} \times \\
\begin{pmatrix}
0 & -\frac{1}{s_{11}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right) & \frac{-in}{s_{11}r\varepsilon_{r}} \\
\frac{1}{s_{22}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right) & 0 & -\frac{1}{s_{22}}\widetilde{D}_{r,E}\left(\frac{1}{\varepsilon_{r}}\right) \\
\frac{in}{s_{33}r\varepsilon_{r}} & \frac{1}{s_{33}r}\widetilde{D}_{r,E}\left(\frac{r}{\varepsilon_{r}}\right) & 0
\end{pmatrix} (3.37)$$

Combining matrices once again and canceling PML terms yields the following:

$$\begin{pmatrix} \frac{1}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) - \frac{n^{2}}{s_{\phi}^{2}r^{2}\varepsilon_{r}} \\ in\frac{s_{\phi}}{s_{r}}\widetilde{D}_{r,H}\left(\frac{1}{s_{r}s_{\phi}r\varepsilon_{r}}\right) \\ -\frac{1}{rs_{r}s_{\phi}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) \end{pmatrix} D_{r} + \begin{pmatrix} -\frac{1}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{r,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) \\ \frac{in}{s_{z}r}\widetilde{D}_{z,H}\left(\frac{1}{\varepsilon_{r}}\varepsilon_{z}\right) \\ \frac{1}{rs_{r}s_{\phi}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{r,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) - \frac{n^{2}}{s_{\phi}^{2}r^{2}\varepsilon_{r}} \end{pmatrix} D_{z} \\ + \begin{pmatrix} \frac{in}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) + \frac{s_{\phi}}{s_{r}}\widetilde{D}_{r,H}\left(\frac{1}{s_{r}s_{\phi}r}\widetilde{D}_{r,E}\left(\frac{r}{\varepsilon_{r}}\right)\right) \\ \frac{in}{s_{z}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right) \end{pmatrix} D_{z} = -k_{0}^{2} \begin{pmatrix} D_{r} \\ D_{\phi} \\ D_{z} \end{pmatrix}$$

$$(3.38)$$

This matrix is then combined with Eqn. 3.18 to eliminate the azimuthal term:

$$\begin{pmatrix} \frac{1}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) - \frac{n^{2}}{s_{\phi}^{2}r^{2}\varepsilon_{r}} + \frac{1}{s_{\phi}^{2}r^{2}}\widetilde{D}_{r,E}\left(\frac{r}{\varepsilon_{r}}\widetilde{D}_{r,H}(r)\right) \\ - \frac{1}{rs_{r}s_{\phi}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) + \frac{1}{rs_{\phi}^{2}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\widetilde{D}_{r,H}(r)\right) \end{pmatrix} D_{r} + \\ \begin{pmatrix} \frac{1}{s_{\phi}^{2}}r^{2}}\widetilde{D}_{r,E}\left(\frac{r^{2}}{\varepsilon_{r}}\widetilde{D}_{z,H}\right) - \frac{1}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{r,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) \\ \frac{1}{s_{\phi}^{2}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\widetilde{D}_{z,H}\right) + \frac{1}{rs_{r}s_{\phi}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{r,E}\left(\frac{1}{\varepsilon_{r}}\right)\right) - \frac{n^{2}}{s_{\phi}^{2}}r^{2}\varepsilon_{r} \end{pmatrix} D_{z} = -k_{0}^{2}\binom{D_{r}}{D_{z}}$$

$$(3.39)$$

Eqn. 3.39 is the full eigenvalue expression for the complete set of higher order modes. Once the electric displacement vectors for each resonant mode are calculated, Eqns. 3.16 and 3.18 may be used to obtain the full 3-D vector form of the electric field for each of these modes.

3.5.5 Quality Factors of Eigenfrequencies

Properly implemented PML boundary conditions will incorporate a loss mechanism into the eigenvalues computed from the systems of equations, which provides an estimate of the energy leakage out of the geometry. Thus, the resonant frequencies calculated in the solver will, in general, be complex-valued. The imaginary part gives the loss and is directly related to the Qfactor of the cavity [81] according to

$$Q = \frac{Re[\omega]}{2 Im[\omega]}$$
(3.40)

3.5.6 Model Benchmarks

To verify the predicted resonant frequencies and Q-factors from the model, a variety of test cases were used to benchmark the model against published values. [73] gives values for a cylindrical dielectric resonator surrounded by air. The cylinder radius is 5.25mm, its length is

4.62mm, and its permittivity is 38. The simulated resonances compared to those reported is demonstrated in Table 3-1. [75] reports values for a few higher order modes for a similar resonator except with a length of 4.6mm. These comparisons are presented in Table 3-2.

Mode	Simulated freq.	Reported freq.	Simulated Q	Reported Q
TE ₀₁₁	4.8406GHz	4.8713GHz	38.6	51
TE ₀₂₁	9.1108GHz	9.1199GHz	43.2	47.4
 TM_{011}	7.5229GHz	7.5083GHz	74.9	86

The model was also benchmarked against more complicated geometries [82] with similar degrees of accuracy. To ensure the simulation accuracy is good over a range of geometries, the periodic enhancement of Q-factors as a function of cavity length described and simulated through a time domain calculation in [49] was also confirmed by the eigenmode simulation, as demonstrated in Figure 3-3. The resonant frequencies matched very closely with the published values and indicate excellent agreement, particularly considering the vast differences in simulation methodology and data gridding. The strong agreement between published data and simulation results for a range of geometries and resonances provides greater confidence in the model results.

Table 3-2: Comparison of Various Mode Resonances.								
Mode	Simulated freq.	Reported freq.	Simulated Q	Reported Q				
TE ₀₁₁	4.855GHz	4.829GHz	39.3	45.8				
TM_{011}	7.577GHz	7.524GHz	74.5	76.8				
HEM_{111}	6.290GHz	6.333GHz	42.8	30.7				
HEM ₂₁₁	7.703GHz	7.752GHz	316.0	327.1				



Figure 3-3: Q-factor Dependence on Cavity Length (a) Published and (b) Simulated.

3.6 Eigenmode Calculations

As discussed previously, delay lines offer another type of optical filter in which the total delay scales linearly with length. In this case it is desirable to obtain a guiding cross-section that offers an extremely high dispersion around the desired operating frequency. To design such a structure, one needs a reliable method for calculating the frequency response of an arbitrarily-shaped waveguide cross-section.

While material and modal dispersions will have some overall effect, we are primarily concerned with designing structures containing extremely large waveguide dispersion, which is related to the frequency dependence of the effective index of the waveguide. As such, we simply need to calculate the effective index over a range of frequencies to obtain the dispersion of the delay line.

While the previous section outlined a method for obtaining the standing wave resonant frequencies of a structure, a slight variation of that formulation allows us instead to obtain the

propagation constants corresponding to traveling waves as a given frequency. In this formulation we make the assumption that light of a specific frequency is being guided through the delay line perpendicular to its cross-section with some propagation constant. By a straightforward derivation from Maxwell's Equations, we arrive at a linear algebraic formulation wherein the propagation constants are given by the eigenvalues. We may solve this expression over a band of closely spaced frequencies to obtain the spectral response of the delay line. This approach is known as an eigenmode formulation, whereas the approach for the resonators is termed the eigenfrequency formulation.

3.6.1 Mode Expression

For these waveguides, we assume that propagation is along the z-axis, so the electric field takes the form

$$\vec{E}(x,y,z) = \vec{E}(x,y)e^{-i\beta z}$$
(3.41)

This allows us to eliminate the z-derivatives. Eqn. 3.6 then simplifies to

$$\nabla_T^2 \vec{E} + \nabla \left(\frac{\nabla \varepsilon_r}{\varepsilon_r} \bullet \vec{E} \right) + \left(k_0^2 \varepsilon_r - \beta^2 \right) \vec{E} = 0$$
(3.42)

This equation is also an eigenmode expression where the eigenvalues are the propagation constants of the corresponding eigenmodes. A TE mode may be obtained by assuming there is no electric field component in the direction of propagation (E_z =0). Maxwell's Equations may then be solved in terms of H_z :

$$\varepsilon_r \nabla \bullet \left(\frac{1}{\varepsilon_r} \nabla H_z\right) + \varepsilon_r k_0^2 H_z = \beta^2 H_z$$
(3.43)

Similarly, a TM mode may be obtained by assuming $H_z=0$ and solving in terms of E_z :

$$\nabla^2 E_z + \varepsilon_r k_0^2 E_z = \beta^2 E_z \tag{3.44}$$

These equations are all available in Comsol's Femlab software, which, due to its efficiency and ease of use, provided most of the propagation constants for the dispersive waveguide structures discussed in this research. The values were benchmarked against a custom solver script implemented according to the finite difference methods described in [79].

3.6.2 Dispersion Calculations

The results of eigenmode simulations are often given in terms of an "effective refractive index" for the structure expressed according to

$$\beta = \frac{2\pi}{\lambda} n_{eff} \tag{3.45}$$

where β is the calculated propagation constant and λ is the simulated wavelength. Once the propagation constants are determined at fine increments over a band of wavelengths, the dispersion may also be obtained. Dispersion is expressed according to the following equation:

$$D = \frac{2\pi c}{\lambda^2} \frac{\partial^2 \beta}{\partial \omega^2} = \frac{\lambda}{c} \frac{\partial^2 n_{eff}}{\partial \lambda^2}$$
(3.46)

We can then apply the finite difference expressions to the derivative term to obtain the magnitude of dispersion for the guiding structure over a given frequency band.

3.6.3 Bent Waveguide Modes

Another mode-solver problem arises when we allow the waveguide trajectory to bend along a curved path. An angular propagation direction will rely on a rather different mode shape than will propagation down a straight-line waveguide. For gentle curvatures the difference in the propagation constants is not substantial, but radial loss terms can become significant for long bent waveguides. It is worthwhile to implement a complex-valued mode solver to ascertain the loss terms as a function of waveguide curvature in order to determine the limitations for a curved waveguide filter.

3.6.3.1 Model Formulation

We again start with Maxwell's Equations (Eqns. 3.13) and assume that the propagating eigenmode takes the form

$$\vec{E}(r,\phi,z) = \vec{E}(r,z)e^{-i\beta R\phi}$$
(3.47)

where *R* is the radius of curvature of the waveguide and β is the complex-valued propagation constant as discussed previously. Thus, based on the derivations in section 3.5.4, Maxwell's Equations in matrix form may be written as follows:

$$\frac{-1}{i\varepsilon_{0}\mu_{0}\omega} \begin{pmatrix} s_{11}^{-1} & 0 & 0 \\ 0 & s_{22}^{-1} & 0 \\ 0 & 0 & s_{33}^{-1} \end{pmatrix} \begin{pmatrix} 0 & -\widetilde{D}_{z,E} & \frac{-i\beta R}{r} \\ \widetilde{D}_{z,E} & 0 & -\widetilde{D}_{r,E} \\ \frac{i\beta R}{r} & \frac{1}{r} \widetilde{D}_{r,E}(r) & 0 \end{pmatrix} \begin{pmatrix} E_{r} \\ E_{\phi} \\ E_{z} \end{pmatrix} = \begin{pmatrix} H_{r} \\ H_{\phi} \\ H_{z} \end{pmatrix}$$

$$\begin{pmatrix} s_{11}^{-1} & 0 & 0 \\ 0 & s_{22}^{-1} & 0 \\ 0 & 0 & s_{33}^{-1} \end{pmatrix} \begin{pmatrix} 0 & -\widetilde{D}_{z,H} & \frac{-i\beta R}{r} \\ \widetilde{D}_{z,H} & 0 & -\widetilde{D}_{r,H} \\ \frac{i\beta R}{r} & \frac{1}{r} \widetilde{D}_{r,H}(r) & 0 \end{pmatrix} \begin{pmatrix} H_{r} \\ H_{\phi} \\ H_{z} \end{pmatrix} = i\omega\varepsilon_{r} \begin{pmatrix} E_{r} \\ E_{\phi} \\ E_{z} \end{pmatrix}$$
(3.48)

Note that these equations are identical to those in section 3.5.4 with the slight adjustment to the azimuthal derivative: $n \rightarrow \beta R$. Thus, we may write the general expression for the modes in bent waveguides in the following manner:

$$\begin{pmatrix}
k_{0}^{2}\varepsilon_{r} + \frac{1}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{z,E}\right) + \frac{1}{s_{\phi}^{2}r^{2}}\widetilde{D}_{r,E}\left(\frac{r}{\varepsilon_{r}}\widetilde{D}_{r,H}(r\varepsilon_{r})\right) \\
- \frac{1}{rs_{r}s_{\phi}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{z,E}\right) + \frac{1}{rs_{\phi}^{2}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\widetilde{D}_{r,H}(r\varepsilon_{r})\right)
\end{pmatrix}E_{r} + \\
\begin{pmatrix}
\frac{1}{s_{\phi}^{2}r^{2}}\widetilde{D}_{r,E}\left(\frac{r^{2}}{\varepsilon_{r}}\widetilde{D}_{z,H}(\varepsilon_{r})\right) - \frac{1}{s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{r,E}\right) \\
k_{0}^{2}\varepsilon_{r} + \frac{1}{s_{\phi}^{2}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\widetilde{D}_{z,H}(\varepsilon_{r})\right) + \frac{1}{rs_{r}s_{\phi}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{r,E}\right)
\end{pmatrix}E_{z} = \frac{\beta^{2}R^{2}}{s_{\phi}^{2}r^{2}}\left(\frac{E_{r}}{E_{z}}\right)$$
(3.49)

This can be simplified a bit further to obtain a convenient eigenvalue expression:

$$\begin{pmatrix}
\frac{s_{\phi}^{2}r^{2}k_{0}^{2}\varepsilon_{r}}{R^{2}} + \frac{s_{\phi}^{2}r^{2}}{R^{2}s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{z,E}\right) + \frac{1}{R^{2}}\widetilde{D}_{r,E}\left(\frac{r}{\varepsilon_{r}}\widetilde{D}_{r,H}\left(r\varepsilon_{r}\right)\right) \\
-\frac{rs_{\phi}}{R^{2}s_{r}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{z,E}\right) + \frac{r}{R^{2}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\widetilde{D}_{r,H}\left(r\varepsilon_{r}\right)\right) \\
\begin{pmatrix}
\frac{1}{R^{2}}\widetilde{D}_{r,E}\left(\frac{r^{2}}{\varepsilon_{r}}\widetilde{D}_{z,H}(\varepsilon_{r})\right) - \frac{s_{\phi}^{2}r^{2}}{R^{2}s_{z}}\widetilde{D}_{z,H}\left(\frac{1}{s_{z}}\widetilde{D}_{r,E}\right) \\
\frac{s_{\phi}^{2}r^{2}k_{0}^{2}\varepsilon_{r}}{R^{2}} + \frac{r^{2}}{R^{2}}\widetilde{D}_{z,E}\left(\frac{1}{\varepsilon_{r}}\widetilde{D}_{z,H}(\varepsilon_{r})\right) + \frac{rs_{\phi}}{R^{2}s_{r}}\widetilde{D}_{r,H}\left(\frac{rs_{\phi}}{s_{r}}\widetilde{D}_{r,E}\right)
\end{pmatrix} E_{z} = \beta^{2}\binom{E_{r}}{E_{z}}$$
(3.50)

While this is accurate for arbitrary hybrid modes, we typically assume that the ridge waveguides contain either horizontally or vertically polarized modes. We can consider each of these cases individually.

3.6.3.2 Bent Waveguide TE Modes

In this formulation we define the TE modes as those for which the electric field is perpendicular to the waveguide curvature. Thus, $E_r = 0$ by assumption, and the eigenvalue problem simplifies to the following:

$$\frac{r^2}{R^2} \left(s_{\phi}^2 k_0^2 \varepsilon_r + \widetilde{D}_{z,E} \left(\frac{1}{\varepsilon_r} \widetilde{D}_{z,H} (\varepsilon_r) \right) + \frac{s_{\phi}}{rs_r} \widetilde{D}_{r,H} \left(\frac{rs_{\phi}}{s_r} \widetilde{D}_{r,E} \right) \right) E_z = \beta^2 E_z$$
(3.51)

In similar manner to the eigenfrequency calculations outlined above, this equation may be expressed in matrix form and solved numerically using an eigenvalue solver.

3.6.3.3 Bent Waveguide TM Modes

We define the TM modes as those with the electric field in the same plane as the waveguide curvature. Thus, $E_z = 0$ and we write the eigenvalue problem in terms of E_r . The equation simplifies to the following:

$$\frac{r^2}{R^2} \left(s_{\phi}^2 k_0^2 \varepsilon_r + \frac{s_{\phi}^2}{s_z} \widetilde{D}_{z,H} \left(\frac{1}{s_z} \widetilde{D}_{z,E} \right) + \frac{1}{r^2} \widetilde{D}_{r,E} \left(\frac{r}{\varepsilon_r} \widetilde{D}_{r,H} \left(r \varepsilon_r \right) \right) \right) E_r = \beta^2 E_r$$
(3.52)

3.6.3.4 Loss Terms

The eigenvalue solver returns complex eigenvalues that we interpret as the propagation constant:

$$\beta = k_0 \left(n_{eff} - i\kappa \right) = k_0 n_{eff} - i\frac{\alpha}{2}$$
(3.53)

The amplitude of the modes then decays according to

$$\vec{E}(\phi) = \vec{E}_0 e^{-ik_0 n_{eff} R \phi} e^{-\frac{\alpha}{2} R \phi}$$
(3.54)

The electric field intensity loss (in dB) may then be written as

$$10\log\left(\left|\frac{\vec{E}(\phi)}{\vec{E}_0}\right|^2\right) = -10\alpha R\phi\log(e)$$
(3.55)

The propagation distance along the waveguide is given by $L = R\phi$, so loss is expressed as

Loss (dB per unit length) =
$$10\log(e)\alpha = 4.343\alpha = -8.686 \operatorname{Im}(\beta)$$
 (3.56)

3.7 Method of Lines Calculations

While the above methods allow one to predict certain characteristics of an optical filter's response, it is often necessary to simulate the entire complex response function for a given incident source condition. There are a variety of frequency domain propagation methods available, though this work will focus on the MOL approach [83-86]. This method operates under similar assumptions as the eigenmode simulation. Specifically, propagation is assumed to be along the z-axis. However, instead of simply evaluating a scalar propagation constant for a single region, the MOL approach involves calculating a transmission matrix (based on the eigenvalue formulations expressed above) for each individual region and then building up matrices that describe the transmission and reflection for an entire structure.

3.7.1 Method of Lines Formulation

This method is based on the assumption that permittivity and permeability are piecewiseconstant in the propagation direction. Thus, the structure may be divided into individual layers, each with its own permittivity distribution in the r, ϕ plane (see Figure 3-4). In each layer the electric field takes the form

$$\vec{E}(r,\phi,z) = e^{-i\overline{G}z}\vec{A}(r,\phi) + e^{i\overline{G}z}\vec{B}(r,\phi)$$
(3.57)

Note that the A term refers to forward-propagating waves (in the $+i\omega$ sign convention used here) and the B term deals with backward-propagating waves.



Figure 3-4: Layering of Permittivity Regions.

3.7.2 Derivation of Transfer Matrix

From Maxwell's Equations (per Eqn. 3.13) we can write the following:

$$-i\overline{\overline{s}}\mu_{0}\omega\overline{H} = \nabla \times \overline{E} = \begin{pmatrix} -\frac{\partial}{\partial z}E_{\phi} - \frac{in}{r}E_{z} \\ \frac{\partial}{\partial z}E_{r} - \frac{\partial}{\partial r}E_{z} \\ \frac{in}{r}E_{r} + \frac{1}{r}\frac{\partial}{\partial r}(rE_{\phi}) \end{pmatrix}$$
(3.58)

and

$$i\overline{\overline{s}}\varepsilon_{0}\varepsilon_{r}\omega\overline{E} = \nabla \times \overline{H} = \begin{pmatrix} -\frac{\partial}{\partial z}H_{\phi} - \frac{in}{r}H_{z} \\ \frac{\partial}{\partial z}H_{r} - \frac{\partial}{\partial r}H_{z} \\ \frac{in}{r}H_{r} + \frac{1}{r}\frac{\partial}{\partial r}(rH_{\phi}) \end{pmatrix}$$
(3.59)

If we consider only zero-order azimuthal modes, the equations simplify to

_

$$-i\overline{s}\mu_{0}\omega\vec{H} = \nabla \times \vec{E} = \begin{pmatrix} -\frac{\partial}{\partial z}E_{\phi} \\ \frac{\partial}{\partial z}E_{r} - \frac{\partial}{\partial r}E_{z} \\ \frac{1}{r}\frac{\partial}{\partial r}(rE_{\phi}) \end{pmatrix}$$
(3.60)

and

$$i\overline{\overline{s}}\varepsilon_{0}\varepsilon_{r}\omega\vec{E} = \nabla \times \vec{H} = \begin{pmatrix} -\frac{\partial}{\partial z}H_{\phi} \\ \frac{\partial}{\partial z}H_{r} - \frac{\partial}{\partial r}H_{z} \\ \frac{1}{r}\frac{\partial}{\partial r}(rH_{\phi}) \end{pmatrix}$$
(3.61)

The TE_0 modes can be written in terms of the azimuthal electric field component. In this case Maxwell's Equations combine to produce the following:

$$s_{22}\varepsilon_r k_0^2 E_{\phi} = -\frac{1}{s_{11}} \frac{\partial^2 E_{\phi}}{\partial z^2} - \frac{\partial}{\partial r} \left(\frac{1}{s_{33}r} \frac{\partial}{\partial r} \left(r E_{\phi} \right) \right)$$
(3.62)

The longitudinal derivative term may be eliminated using Eqn. 3.57:

$$\overline{\overline{G}}^{2}E_{\phi} = \varepsilon_{r}k_{0}^{2}E_{\phi} + s_{11}\frac{\partial}{\partial r}\left(\frac{1}{rs_{33}}\frac{\partial}{\partial r}\left(rE_{\phi}\right)\right)$$
(3.63)

Similarly, the TM_0 modes are expressed in terms of the azimuthal magnetic field component. In such a case, the following expression may be obtained:

$$s_{22}k_0^2 H_{\phi} = -\frac{1}{s_{11}\varepsilon_r} \frac{\partial^2 H_{\phi}}{\partial z^2} - \frac{\partial}{\partial r} \left(\frac{1}{s_{33}\varepsilon_r r} \frac{\partial}{\partial r} \left(rH_{\phi} \right) \right)$$
(3.64)

Eliminating the longitudinal derivative and simplifying produces the following:

$$\overline{\overline{G}}^{2}H_{\phi} = \varepsilon_{r}k_{0}^{2}H_{\phi} + \varepsilon_{r}s_{11}\frac{\partial}{\partial r}\left(\frac{1}{s_{33}\varepsilon_{r}r}\frac{\partial}{\partial r}\left(rH_{\phi}\right)\right)$$
(3.65)

In terms of the derivative operators defined above, these equations may be written in the following manner:

TE₀ case:

$$\overline{\overline{G}}^{2}E_{\phi} = \left[\varepsilon_{r}k_{0}^{2} + s_{11}\widetilde{D}_{r,H}\left(\frac{1}{rs_{33}}\widetilde{D}_{r,E}(r)\right)\right]E_{\phi}$$
(3.66)

TM₀ case:

$$\overline{\overline{G}}^{2}H_{\phi} = \left[\varepsilon_{r}k_{0}^{2} + \varepsilon_{r}s_{11}\widetilde{D}_{r,H}\left(\frac{1}{rs_{33}\varepsilon_{r}}\widetilde{D}_{r,E}(r)\right)\right]H_{\phi}$$
(3.67)

3.7.3 Evaluation of Matrix Functions

As an aside, one may note that we actually have the formulation for the square of the propagation matrix. Additionally, the formulation of the proposed solution for the electric field is based on an exponential function of the matrix. Powers and exponential functions of such matrices may be obtained in a straightforward manner by method of diagonalization. Specifically, the matrix may be expressed as

$$\overline{\overline{G}}^{2} = \overline{\overline{T}}\lambda^{2}\overline{\overline{T}}^{-1}$$
(3.68)

where λ^2 and T are the eigenvalues and eigenvectors of $\overline{\overline{G}}^2$. Then we may write the following:

$$\overline{\overline{G}} = \overline{\overline{T}}\lambda\overline{\overline{T}}^{-1}$$
(3.69)

and

$$e^{\pm i\overline{G}z} = \overline{\overline{T}}e^{\pm i\lambda z}\overline{\overline{T}}^{-1}$$
(3.70)

3.7.4 Layer Transitions

At an interface between layers we know that the tangential field components are continuous. Specifically, for E_{ϕ} and H_r (or H_{ϕ} and E_r) at the boundary between the mth and $(m+1)^{th}$ layers we obtain

$$A_{\phi,m} + B_{\phi,m} = A_{\phi,m+1} + B_{\phi,m+1}$$

$$\overline{\overline{G}}_{m}A_{\phi,m} - \overline{\overline{G}}_{m}B_{\phi,m} = \overline{\overline{G}}_{m+1}A_{\phi,m+1} - \overline{\overline{G}}_{m+1}B_{\phi,m+1}$$
(3.71)

If we assume that there is no field incident from the right ($B_{m+1} = 0$), we can obtain reflection and transmission coefficients:

$$B_{\phi,m} = R_{m,m+1}A_{\phi,m} = \left(I + \overline{\overline{G}}_{m}^{-1}\overline{\overline{G}}_{m+1}\right)^{-1} \left(I - \overline{\overline{G}}_{m}^{-1}\overline{\overline{G}}_{m+1}\right)A_{\phi,m}$$

$$A_{\phi,m+1} = T_{m,m+1}A_{\phi,m} = 2\left(I + \overline{\overline{G}}_{m}^{-1}\overline{\overline{G}}_{m+1}\right)^{-1}A_{\phi,m} = \left(I + R_{m,m+1}\right)A_{\phi,m}$$
(3.72)

Similarly, by setting $A_m = 0$ we can derive transmission and reflection coefficients for light incident on the opposite side of the interface. The full set of coefficients are given as

$$R_{m,m+1} = \left(I + \overline{\overline{G}}_{m}^{-1} \overline{\overline{G}}_{m+1}\right)^{-1} \left(I - \overline{\overline{G}}_{m}^{-1} \overline{\overline{G}}_{m+1}\right)$$

$$T_{m,m+1} = I + R_{m,m+1}$$

$$R_{m+1,m} = -R_{m,m+1}$$

$$T_{m+1,m} = I - R_{m,m+1}$$
(3.73)

Of greater interest are the reflection and transmission coefficients for multiple layers. There is a variety of methods to accurately account for the transitions. The simplest approach, known as the transmission matrix formulation [87], simply involves matching the forward- and backward-propagating amplitudes at each boundary to formulate a transfer matrix. However, the exponential term for the backward-propagating fields can become numerically unstable, leading to erroneous simulations. A better approach is to directly sum the reflected and transmitted components in an infinite series to account for multiple reflections and propagations across a given layer. This ultimately leads to the following expressions [88, 89]:

$$R_{AB}^{m} = R_{m,m+1} + T_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} R_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \left(I - R_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} R_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \right)^{-1} T_{m,m+1}$$

$$T_{AB}^{m} = T_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \left(I - R_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} R_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \right)^{-1} T_{m,m+1}$$

$$R_{BA}^{m} = R_{BA}^{m+1} + T_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} R_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \left(I - R_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} R_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \right)^{-1} T_{BA}^{m+1}$$

$$T_{BA}^{m} = T_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \left(I - R_{AB}^{m+1} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} R_{m+1,m} e^{i\overline{\overline{G}}_{m+1}\Delta_{m+1}} \right)^{-1} T_{BA}^{m+1}$$
(3.74)

Here, R_{AB}^{m} and T_{AB}^{m} represent the reflection and transmission coefficients for a wave incident through the mth layer from the left, and R_{BA}^{m} and T_{BA}^{m} are the coefficients for a wave incident from the right; $R_{m,m+1}$, $T_{m,m+1}$, $R_{m+1,m}$ and $T_{m+1,m}$ are the reflection and transmission coefficients for the single interface between the m and (m+1) layers expressed in Eqns. 3.73; and Δ_{m+1} gives the thickness of the m+1 layer.

Thus, the coefficient matrices may be built up based on the matrices defined for succeeding layers. The algorithm iterates backward through the entire structure consisting of M layers to obtain a single overall set of filter response matrices for a given frequency. To begin, one introduces a "ghost" M+1 layer with zero reflection and unity transmission coefficients. Additionally, a similar m=0 layer may be introduced with like coefficients to provide the appropriate transitions from a source plane to the first layer interface.

3.7.5 Doubling Algorithm

In some cases we wish to model the transmission and reflection through structures of a periodic nature. In such cases, the reflection and transmission coefficients outlined above may be calculated for a single period and then appropriately transformed through a doubling

algorithm to describe the overall structure. The more general case of this algorithm allows us to stitch together potentially disparate adjacent regions whose coefficients have been calculated previously. If C denotes the leftmost region and D denotes the rightmost region, the reflection and transmission coefficients for the resulting combination are given by the following:

$$R_{AB}^{CD} = R_{AB}^{C} + T_{BA}^{C} R_{AB}^{D} \left(I - R_{BA}^{C} R_{AB}^{D} \right)^{-1} T_{BA}^{C}$$

$$T_{AB}^{CD} = T_{AB}^{D} \left(I - R_{BA}^{C} R_{AB}^{D} \right)^{-1} T_{AB}^{C}$$

$$R_{BA}^{CD} = R_{BA}^{D} + T_{AB}^{D} R_{BA}^{C} \left(I - R_{AB}^{D} R_{BA}^{C} \right)^{-1} T_{BA}^{D}$$

$$T_{BA}^{CD} = T_{BA}^{C} \left(I - R_{AB}^{D} R_{BA}^{C} \right)^{-1} T_{BA}^{D}$$
(3.75)

Obviously, for periodic structures regions C and D are identical. Thus, once the coefficients for the pair are calculated, the doubling algorithm may be applied a second time to represent 4 periods, a third time to provide the coefficients for 8 periods, and so on. This allows us to quickly and efficiently build up the transition coefficients for a large periodic structure.

3.8 Frequency Domain Models Summary

This chapter has outlined three basic frequency domain modeling tools. The eigenfrequency calculations provide a fast and efficient way to evaluate resonant cavity filters. While they do not offer a means to predict the response function for an arbitrary incident source condition, they do produce the various resonant characteristics of interest. Specifically, the predicted complex eigenfrequencies give the essential information to calculate the spectral positioning and the bandwidth of the relevant resonances. The eigenmode simulations are used to determine the dispersive characteristics of delay line filters. The eigenvalues represent the propagation constants of the modes of interest, and the variation of the propagation constant with frequency allows us to predict the filter's dispersion. Finally, the MOL simulation allows us to

simulate the results of an arbitrary source condition incident on a given optical filter. It yields both reflectance and transmittance, and gives us a means to visualize how these change with incident frequency and source conditions.

CHAPTER 4 OPTIMIZATION OF ELECTRO MAGNETIC STRUCTURES

4.1 **Principles of Optimization**

For various electromagnetic design problems it is usually necessary to make use of an optimization of the geometry of the physical elements of the optical system or device. Depending on the complexity of the design, this may be trivial or exceedingly difficult.

Any optimization problem relies on two components. The first is a collection of numerical values or variables offering a complete description of the component to be optimized. The number of independent variables in this collection defines the dimensionality of the possible set of solutions. This set, termed parameter space, contains the entire set of possible designs or solutions to the optimization problem expressed in vector form. The second aspect of the optimization problem is a function that maps a vector in parameter space describing a possible solution to a value expressing how closely the behavior of the solution comes to the absolute optimum. This mapping function is typically termed the fitness function or cost function. (Often the former term lends itself more naturally to maximization problems and the latter to minimizations, though they are usually used interchangeably.) A simple optimization example could consist of an N-dimensional parameter space with a De Jong cost function:

$$f(X) = \sum_{i=1}^{N} x_i^2$$
 (4.1)

One of the difficulties in optimization is that the cost function rarely has as simple or obvious a solution as the example above. De Jong's function has a single minimum at $x_i=0$. However, most fitness functions have a large number of minima throughout parameter space. Only one value provides the global minimum, though simple optimization algorithms relying on gradients and steepest descent will often get caught in one of the myriad local minima. A good example of this is Rastrigin's function

$$f(X) = 10N + \sum_{i=1}^{N} \left(x_i^2 - 10 \cos(2\pi x_i) \right)$$
(4.2)

which is illustrated in two dimensions in Figure 4-1.



Figure 4-1: Fitness Map of Rastrigin's Function.

A problem with a large number of minima (or maxima) requires a more brute-force search algorithm that can exhaustively go through all of parameter space and evaluate every local minimum before settling in the global best. However, as the dimensionality of parameter space grows, exhaustive, or deterministic, search algorithms become highly inefficient and we must rely on probabilistic methods that combine aspects of a global exhaustive search with fast localized optimization methods.

4.2 Methods of Probabilistic Searches

One of the primary methods of probabilistic optimization was the Genetic Algorithm (GA) [90, 91]. It developed into a very robust search algorithm applicable to a wide variety of problems and attracted significant attention and interest, becoming quite mature over the years. Eventually, a number of variations of GAs were applied to optical and electro magnetic design problems [92-96].

The central concept of evolutionary optimization algorithms is a group, or population, of potential solutions in parameter space. The quality, or fitness, of the solutions is evaluated using whatever criteria is appropriate for the given optimization problem. The solutions are then shifted in a quasi-random manner with a bias toward points in parameter space previously identified to be more optimal. In GAs, the better solutions are selected to pass on their information to future generations and directly exchange information through a process known as crossover. The parameters of two solutions from the current generation are combined together to form two "children" solutions for the next generation, allowing the solutions to quickly move around parameter space while exchanging information about optimal points. An additional operation known as mutation takes a single solution and shifts it in parameter space by some relatively small perturbation, allowing the more optimal individuals of the population to perform a localized search and zero in on a globally optimal solution.

The success of the GA approach spurred additional research into using behavior from nature to solve mathematical and physical problems. In 1995, the swarming or flocking nature of insects and birds was first applied to mathematical search problems in what was termed Particle Swarm Optimization [97, 98]. Instead of using a genetic approach where genes compete

to pass on their information to future generations, the solutions behave as social insects moving randomly through space and adjusting their velocities towards other solutions that identify more optimal positions.

There is no specific selection operator in PSO whereby lesser solutions are eliminated from the "gene pool." However, since the solutions (termed particles) are continuously moving, they have a low probability of returning to non-optimal positions in space. Instead, each particle is accelerated slightly towards the best point in parameter space it had previously located, providing a means of localized search akin to the mutation operator of the GA. Additionally, the particles communicate information to each other about the current global optimum, and each experiences an acceleration towards this point [99].

PSO has been applied to a wide variety of problems and has been found to perform extraordinarily well, in many cases locating an optimal solution faster and more efficiently than GAs and other similar search methods [99, 100]. With the interest surrounding PSO, it was quickly applied to electrical and electro magnetic design problems [101, 102], and very recently to problems in the optical regions of the spectrum [103, 104].

4.3 Particle Swarm Mechanics

4.3.1 Initialization

The PSO algorithm consists of a population of M individual particles moving around in N-dimensional parameter space. In each iteration or generation of the population, all particles are updated according to their current velocity vectors. The m^{th} particle of the n^{th} generation is defined according to

$$X_m^{\ n} = \{x_1, x_2, x_3, \dots, x_N\}$$
(4.3)

and its velocity vector is defined as

$$\mathbf{V}_{m}^{n} = \{ v_{1}, v_{2}, v_{3}, \dots v_{N} \}$$
(4.4)

The initial particle positions and velocities are generated based on a uniform random distribution across parameter space as indicated in Figure 4-2.



Figure 4-2: Initial Particle Distribution in Parameter Space.

4.3.2 Solution Evaluation

Prior to updating the particles further, one must evaluate each proposed solution based on the given design criteria. A cost or fitness function accepts the particle position vector as input and returns a scalar quantity describing the performance of the given solution. The cost function is the only point of the PSO algorithm where the particles receive any meaning and are treated in anything other than an abstract manner. Each design problem will require a different cost function incorporating various evaluation criteria. Also, it is convenient to incorporate penalties of various sorts into the cost function to push particles away from solutions that might mathematically appear optimal but offer unrealistic or undesirable behavior.

In designing the PSO algorithm, one must choose whether to maximize or minimize the cost function. The PSO used here minimizes the fitness function of the population. If certain aspects of the design are expressed in terms of maximizing characteristics of the optical performance (Q-factors of resonators for example), we will simply take the inverse to obtain a fitness value that may be minimized.

Prior to updating the velocities and positions, the PSO algorithm evaluates the fitness of each particle:

$$f_m^{\ n} = F\left(X_m^{\ n}\right) \tag{4.5}$$

At this point, one must determine how the current generation matches up against previous ones. PSO keeps track of both the best point located globally and the best points located by the individual particles:

$$\left(f_{m}^{n} < f_{m}^{pbest}\right) \implies \begin{array}{c}X_{m}^{pbest} = X_{m}^{n}\\f_{m}^{pbest} = f_{m}^{n}\end{array}$$
(4.6)

The current global optimum, X_{gbest} , is simply the best of the X_m^{pbest} solutions.

4.3.3 Particle Motion

Prior to allowing the particles to move, the velocity vectors are perturbed with small accelerations toward optimal points in parameter space. One component accelerates the particles toward the current global optimum, X_{gbest} , offering the communication between particles needed for good global searching of parameter space. An additional acceleration component pushes

each particle towards the best position it has located on its own, X_m^{pbest} . Thus, the velocity update equation takes the form

$$V_m^{n+1} = w V_m^n + c_1 r_1 \left(X_m^{pbest} - X_m^n \right) + c_2 r_2 \left(X_{gbest} - X_m^n \right)$$
(4.7)

where w, c_1 , and c_2 are constants describing the relative contributions of each velocity component, and r_1 and r_2 are random numbers between 0 and 1, which provide a degree of randomness to the search algorithm. The exact values of the parameters w, c_1 , and c_2 for efficient optimization vary somewhat from problem to problem. However, while varying the parameter values may increase the speed of the optimization, it rarely effects the ability of the PSO to ultimately converge [99].

The first of the coefficients, *w*, describes the particle's inertial weight and defines what fraction of the initial velocity is maintained from one time step to the next. It effectively provides a balance between localized and global exploration. Larger values result in fast moving particles that explore more globally, while smaller values cause the particle to focus more on localized searches and information about local minima provided by other particles. Better convergence may be obtained by initially setting this parameter to a larger value (near unity) and decreasing it over time to allow the swarm to move globally early on, while gradually shifting toward local refinement late in the simulation [105].

The other two coefficients, c_1 and c_2 , are the cognitive and social coefficients respectively. They provide the accelerations toward the global and nearest local minima. Exact values are not usually critical, though fine tuning can increase the speed of convergence [106]. In the absence of fine tuning, a standard choice of values is $c_1 = c_2 = 2$ [107]. An additional constraint found to aid convergence is a vector that defines maximum velocity. Since there is no immediate constraint on velocity in Eqn. 4.7, large accelerations can induce the particle to jump past minima and miss them altogether. Angeline [100] found that imposing a maximum velocity threshold, V_{max} , in each dimension significantly aided convergence. The magnitude of each component of the velocity vector is compared to the corresponding component of V_{max} and is set to that value if that component exceeds the maximum. Large values of V_{max} result in particles moving across parameter space too quickly and missing solutions, while small values impede the global search mechanism. Suitable values for bounded problems are around one quarter to one half of the allowed range in each dimension.

Once the velocity vector is appropriately updated, it is added to the current particle position:

$$X_m^{n+1} = X_m^n + V_m^{n+1} \Delta t$$
 (4.8)

In general, we may simply define the time step, Δt , to be unity and incorporate any desired variation to this into the velocity update parameters in Eqn. 4.7 [101]. Others have shown that the time step may be replaced by a velocity constriction factor that scales the velocity to prevent too large or too small of a step through parameter space. However, similar behavior may be obtained through the inertial weight and the maximum velocity thresholds outlined above [99, 100].

The process of the velocity and position update is illustrated in Figure 4-3. The dark blue arrows indicate the initial velocity vectors, while the dashed arrows denote acceleration components towards the particle's previous best location (light blue) and the current globally optimal solution (orange). The black arrows indicate the resulting velocity vector that is used to

update the particle position. This also becomes the initial velocity for the next generation (second dark blue arrow).



4.4 Boundary Conditions

An additional consideration of importance in developing the PSO algorithm involves the range of physically realizable geometries available for the cavity or other design problem. Standard PSO implementation offers no constraints to bound parameter space, yet this can lead to completely unrealistic solutions such as negative layer thicknesses or cavity radii too small to be fabricated. Such behavior may lead the search algorithm toward undesirable solutions and can produce unexpected and meaningless results from the fitness function. For the types of problems considered in this research and in other related electromagnetic design problems it is appropriate to constrain the particles to specified limits [101]. The bounds may typically be

expressed a priori based on known fabrication capabilities and from rough estimations of cavity shapes suitable for the design target.

The concept of bounding search space for electromagnetic problems is not unique to this research. Robinson [101] expressed three conceptual approaches to the processing of particles that cross over the boundary. Absorbing boundaries stopped incongruous particles at the boundary and zeroed out the normal component of said particles' velocity. Reflecting boundaries resulted in the particles bouncing completely off the edges of parameter space and reversed the sign of the normal component of the particles' velocity. Invisible boundaries allowed the particles to wander at will but prevented erroneous fitness calculations from occurring on particles that strayed outside the bounds. Instead, the fitness was set to an extremely poor value to prevent it from enticing additional particles to cross the edges of defined parameter space. Figure 4-4 illustrates the mechanics of the (a) absorbing, (b) reflecting, and (c) invisible boundary conditions. Robinson found modest differences in the rate of convergence between these types of boundaries depending on the problem considered.



In this research, I developed a more generalized boundary system that allows one to specify how repulsive the parameter space walls are to the particles. In keeping with the concept of particles moving through parameter space, they are forced to bounce off the parameter bounds in the manner of Robinson's reflecting boundaries. However, I generalized the expression to define an elasticity of the boundary. In like manner to the mechanical analog, a fully elastic wall (elasticity parameter of 1) was perfectly reflecting while an inelastic wall (elasticity parameter of 0) forced the particle to stop at the boundary and lose all momentum normal to the boundary. Elasticity values between zero and one indicated reflections off the boundaries with loss of momentum.

There are also occasions where optimal solutions are expected to occur near or outside of the specified extents of parameter space. In such cases, negative values of the elasticity parameter can be used to allow the particles to pass through the boundaries to varying degrees. When it is set to a value of -1, the effective result is to allow the particle to completely ignore the boundaries altogether. Smaller negative values result in a deceleration of the particle as it crosses the boundary. This allows limited searching outside the edges of parameter space with acceleration components tending to pull the particles back inside.

Figure 4-5 illustrates a particle that is violating the defined boundaries as a result of the position update defined in Eqn. 4.8. The updated particle's distance outside the boundaries may be expressed in vectoral form by

$$D = X^n - B \tag{4.9}$$

where B gives the vector form of the boundaries. The sign of the components of the distance vector, D, are given by

$$d_{k} \begin{cases} > 0 \qquad x_{k}^{n} > u_{k} \\ < 0 \qquad x_{k}^{n} < l_{k} \\ 0 \qquad l_{k} \le x_{k}^{n} \le u_{k} \end{cases}$$
(4.10)
where u_k are the components of the upper bounds and l_k are the components of the lower bounds.



Figure 4-5: Particle Attempting to Leave Bounded Parameter Space.

After defining the distance vector for a straying particle, its position and velocity vector components corresponding with nonzero values of the distance vector are updated according to

$$x_{k}^{n} = x_{k}^{n} - (1 + \rho)d_{k}$$
(4.11)

$$v_k^{\ n} = -\rho v_k^{\ n} \tag{4.12}$$

where ρ is the elasticity parameter for the boundaries. Physically speaking, the position is reset to the boundary and then pushed back inside (or outside if the sign of ρ is negative) by a distance proportional to the elasticity. The final position of the particle for both the (a) positive and the (b) negative case of the elasticity coefficient is illustrated in Figure 4-6.



Figure 4-6: Particle Swarm Boundary Transforms.

4.5 Direct Comparison of Particle Swarm and Genetic Algorithm Approaches

As a test of the optimization speed of PSO in comparison to that of the GA approach for the optical filters under consideration in this research, I formulated a test case for both algorithms. A micro-cavity filter geometry was optimized to have a high Q-factor resonance for the TE_{011} mode at a specified wavelength (as will be discussed in detail in later chapters). The two algorithms were implemented and supplied with the same fitness function. The resulting curves for optimal fitness as a function of iteration number are illuminating.

It was immediately evident that the rate of convergence can vary greatly depending on how close the random initial populations of solutions are to optimal values. Additionally, restricting the parameter space as much as possible significantly speeds the rate of convergence. With these factors in mind, both algorithms were supplied with identical boundaries and initial populations. The few parameters specific to PSO were set to default values as outlined earlier in this chapter. The GA used in this comparison was supplied by MathWork's MATLAB software. Default settings for most parameters were also used for this case, with one exception. The intermediate value approach was used for the crossover option, which produces "children" from a weighted average of two parent solutions. This offers a better approach for problems composed of a small number of continuous (real-valued) parameters as opposed to long strings of binary or integer-valued data (in which case a direct exchange of values between parents is the preferred approach [91]).

Figure 4-7 shows the results of the comparison optimizations with (a) the optimal and average fitness as a function of generation for the GA and (b) the fitness of the best particle in the PSO. It is clear that the GA is able to quickly move the population into the region of an

optimal solution much faster than the PSO. However, it gets caught in a local minimum (in this case, a higher order mode) and is never able to fully jump out. In fact, after about 200 generations the entire population appears to consist of solutions in the close neighborhood of this non-optimal solution. In contrast, the PSO steps downward toward an optimal solution at a more gradual rate. However, since the particles are continuously moving throughout parameter space, they are far less likely to get permanently stuck in a non-optimal local minimum.



Figure 4-7: (a) Genetic Algorithm Versus (b) Particle Swarm Convergence Rate.

4.6 Particle Swarm Optimization Summary

PSO provides a unique and efficient approach to a variety of probabilistic optimization problems. It provides an extremely general framework and a means to efficiently obtain a global minimum, making it particularly suitable for a variety of electromagnetic design problems. It is quite efficient and demonstratedly more robust than the more conventional GA approach for the types of problems of interest in this research.

Various authors [61, 63, 66, 68] have thoroughly explored choices for algorithm parameters of a number of different optimization problems and have provided very reasonable

values for efficient optimization. Thus, all that remains is to determine a well-defined fitness function that can take a vector string of numbers, interpret it as a geometrical description of an optical device, and evaluate its optical performance, returning a scalar numerical value expressing that performance. This is typically straightforward, as we might express the geometrical aberrations of an optical system defined by the vector [94] or the difference from a desired diffraction pattern resulting from a given grating filter [104], should that be the design problem in question. More pertinent to this research, the vector may be easily used to express the geometry of a cavity resonator, dispersive waveguide, or Bragg structure, while the cost function evaluates these in terms of resonant frequency, loss, and spectral output.

CHAPTER 5 DISPERSIVE WAVEGUIDE FILTERS

Dispersive waveguides and delay lines have filter responses that are primarily phasebased over the range of operation of the device². In the absence of scattering and leakage losses, the response function takes the form

$$H(\omega) = e^{-i\beta(\omega)L}$$
(5.1)

where the propagation constant may be expressed as

$$\beta(\omega) = \beta(\omega_0) + \left[\frac{1}{v_g}\right]_{\omega_0} \Delta\omega + \frac{1}{2} D_{\omega}\Big|_{\omega_0} (\Delta\omega)^2 + O(\Delta\omega)^3$$
(5.2)

These expressions were discussed in more detail in Chapter 1.2. The design of these structures involves the task of tuning the group velocity, v_g , and the second order dispersion, D_{ω} , to obtain a specific filter response. The specific focus of this discussion is to design a waveguide with a very large dispersive magnitude, which allows these types of structures to provide potential solutions to challenges in the area of pulse broadening and compression.

5.1 Dispersion in Standard Waveguides

Ridge waveguides operating near cutoff tend to have a rather significant amount of dispersion. Figure 5-1 shows the dispersion for a 400nm x 200nm silicon ridge waveguide on a silicon dioxide substrate as determined by the effective index method. The structure must be this small in order to maintain a single-mode profile. The dispersive magnitude is relatively high,

 $^{^2}$ Scattering and leakage losses are certainly critical components of the filter response function but are difficult to control and rarely provide meaningful frequency-dependent effects in these types of structures. We usually attempt to minimize these effects rather than try to tune them for specific filtering applications.

and it can be increased by a factor of 5 or more by shrinking the ridge down to 320 nm x 160 nm since it pushes the operation very close to modal cutoff. Similar structures (termed nano-wires) have been tuned for a dispersion peak as high as 80,000 ps nm⁻¹ km⁻¹ albeit with a very difficult and sensitive fabrication process [108, 109].

However, such structures have some intrinsic drawbacks. With such a minute waveguide size, coupling light into the structure becomes highly problematic, though not impossible. Additionally, the high index contrast results in a highly confined mode in a very small area. Confining a pulse of any significant intensity to such a tiny area virtually guarantees that nonlinear effects will play a significant role. Depending on the application, this may be an advantage, but for pulse compression and stretching applications, nonlinearities tend to hurt much more than they help.



Figure 5-1: TE Mode Dispersion in Ridge Waveguide.

5.2 Nanostructured Waveguides

While coupling and nonlinear issues can be accounted for, it would be advantageous to have a much more reasonably sized waveguide while simultaneously maintaining (or increasing)

the overall device dispersion. SiON has a significantly lower index than Si and could provide one alternative in the area of nano-wires. However, this research presents another option. Instead of restricting ourselves to simple ridge structures, we make use of a grating-based structure in the shape of a ridge [17]. The high dispersion of a ridge-like waveguide can be taken advantage of, but the grating nature lowers the effective index of the ridge and forces the mode to spread over a much larger area. If the period is too large, the structure will act as a diffraction grating and spread the light in multiple orders. However, for an appropriate choice of structure size and period, only the fundamental diffraction order will propagate and the structure forms a single-mode waveguide. Figure 5-2 illustrates the geometry of such a structure with a 275 nm period, 150 nm fins, and 1.25 micron depth. 6 periods are used in this structure.



Figure 5-2: Nano Dispersion Amplified Waveguide Structure.

5.3 Analysis of Dispersion Amplified Waveguides

Comsol's FEMLab software implements a finite element solver which is used to evaluate the waveguide propagation constants as a function of wavelength for the two different types of modes, TE and TM. The TE-like modes are an expression of the wave equation in terms of H_z and are based on the assumption that there is no component of the E-field along the z-axis. The TM-like modes are solutions of the formulation of the wave equation in terms of E_z with the assumption that the H-field is entirely transverse to the z-axis. The formulation for the TE-like case is

$$-\nabla \cdot \left(\frac{\nabla H_z}{n^2}\right) - \mu_r k_0^2 H_z = -\frac{\beta^2}{n^2} H_z$$
(5.3)

and for the TM-like case is

$$-\nabla \cdot \left(\frac{\nabla E_z}{\mu_r}\right) - n^2 k_0^2 E_z = -\frac{\beta^2}{\mu_r} E_z$$
(5.4)

Note that this definition for the TE and TM modes is not specifically identical to the more traditional definition in the waveguide community where the TE mode is defined in terms of E-fields parallel to the grating vector for this structure (in the horizontal direction in all the waveguide profile diagrams) and the TM mode is based on H-fields parallel to the grating vector for this structure (with the E-fields primarily lying in the vertical direction in the waveguide profile diagrams). In contrast, this definition is more reminiscent of the radial types of TE and TM modes defined for fibers. For the remainder of this chapter, "TE", "TM", "TE-like", and "TM-like" will refer to the FEMLab definition of the modes, while "true TE" and "true TM" will refer to the traditional definitions used in the waveguide community as defined above.

The mode equations were expressed as eigensystems and were solved for the eigenmode propagation constant, β . The results were additionally verified using an independent solver based on standard finite difference methods [79]. The finite difference solver was based on a

fixed rectangular grid and thus had a significantly lower accuracy than the FEMLab results, but the results agreed to within a few percent and added confidence in the FEMLab simulations. Additionally, the simulations were tested for convergence based on grid sampling to further ensure the accuracy of the simulations. Dispersion is calculated using the finite difference approximation to the second derivative of the propagation constants. Although this assumes negligible material dispersion, we know that material dispersion provides nowhere near the dispersive magnitude we require and should be negligible in comparison to the desired large waveguide dispersion.

The mode profile for the propagating TE mode in the designed nano dispersion amplified waveguide (Nano-DAWG) operating at 1.55 microns is shown in Figure 5-4. The plot coloration indicates the H_z field component, while the arrows illustrate the E-field. Figure 5-4 provides a graph of the energy density of the propagating TE and TM modes. It is interesting to note that the majority of the energy for the TE case is actually in the substrate instead of the fins, while the converse is true of the TM mode.



Figure 5-3: TE Mode Profile in Nano-DAWG Structure.

In some ways this seems counterintuitive, but in reality it is consistent with what appears to be occurring. In the TE case the short period of the fins results in an effective medium above the substrate layer that has an effective refractive index too low to support a mode by itself, but it is sufficient to allow a mode to propagate just beneath it in a similar manner to modes formed in a silicon layer just beneath a small silicon ridge. In the TE case, the energy is obviously much more tightly confined, but it is still spread over a significantly larger area than the single moded ridge waveguide mentioned earlier. Additionally, one should note that shorter wavelengths have propagation constants further from cutoff, and thus the energy shifts out of the substrate and back into the fins, resulting in a situation more reminiscent of the energy distribution for the presented TM case. Interestingly, this offers additional possibilities because the structure could potentially be used in an amplifier configuration where the pump beam is used to excite a signal in a neighboring medium.





After solving for the propagation constants (and hence the effective refractive indices) at a range of incident wavelengths, the software evaluates dispersion through a difference approximation to the second derivative. The TE mode has remarkably high calculated dispersion, while the TM case, although having reasonably high dispersion, is not single moded. The dispersion for the principle modes in both cases is shown in Figure 5-5.



Of additional interest is the extreme birefringence this structure exhibits. The refractive index for the principle TE mode at a wavelength of 1.55 microns is 1.4663, while the index for the TM mode at that wavelength is 2.7078. The equation for the birefringent beat length,

$$L_B = \frac{\lambda}{B} = \frac{\lambda}{|n_{TM} - n_{TE}|}$$
(5.5)

gives a beat length magnitude of 1.25 microns. Typical polarization-maintaining fibers (PMFs) have beat lengths on the order of 2-3 mm.

The silicon-air interface introduces an index contrast that is quite large, and hence the mode is highly confined. Using SiON (index of 2.0) for the fins instead of Si (index of 3.47) allows the energy to spread out across the interfaces to a much greater degree and further increases the dispersion. Additionally, the lower guiding index means that the fins must be significantly larger to maintain a single-moded guiding structure, which makes fabrication somewhat easier. The TE case is single-moded for the SiON Nano-DAWG with a period of 825 nm. The duty cycle is maintained between the two cases (which gives 450 nm width fins), as is

the grating height (1.25 microns). The energy density for the TE and TM cases are presented in Figure 5-6.



Figure 5-6: Second Nano-DAWG Structure. (a) TE Mode Energy Density; (b) TM Mode Energy Density.

Once again, dispersion for the TE and TM modes is calculated from the propagation constants. Figure 5-7 shows a plot of dispersion for each case. Interestingly, the dispersion is double that of the first Nano-DAWG design. Also, the calculated beat length for this design at a wavelength of 1.55 microns is 6.556 microns.

As can be seen, the Nano-DAWG dispersion is nearly a full order of magnitude greater than that of the ridge waveguide structures, albeit about an order of magnitude less than that available to chirped Bragg structures. Also there is a noticeable higher-order dispersion component present, which limits the bandwidth a great deal more than for the grating structures. However, the bandwidth, structure length, and dispersive magnitude are not so closely interlinked for these structures. Increasing the length of these devices to obtain a greater dispersive delay is simply a matter of fabricating a longer device, whereas in the grating systems such an increase requires a corresponding decrease in grating chirp per unit length to maintain the magnitude of second-order dispersion.



5.4 Theoretical Analysis

There are several interesting characteristics in these structures. First, the grating provides an additional dispersion effect. The effective index for the true TE mode in a grating structure includes a higher order dependency on the ratio of wavelength to grating period [110]

$$n_{eff}^{2} = \varepsilon_{P0} \left(1 + \frac{\pi^{2}}{3} \left(\frac{\Lambda}{\lambda} \right)^{2} f^{2} \left(1 - f \right)^{2} \frac{\left(n_{g}^{2} - n_{c}^{2} \right) \varepsilon_{T0}}{n_{c}^{2}} \frac{\varepsilon_{P0}}{n_{g}^{2} n_{c}^{2}} \right) + \mathcal{O} \left(\left(\frac{\Lambda}{\lambda} \right)^{4} \right)$$
(5.6)

where

$$\varepsilon_{T0} = fn_g^2 + (1 - f)n_c^2$$

$$\varepsilon_{P0} = \left(\frac{f}{n_g^2} + \frac{(1 - f)}{n_c^2}\right)^{-1}$$
(5.7)

f is the grating fill factor, Λ is the grating period, n_g is the refractive index of the grating, and n_c is the refractive index of the surrounding region. While this expression indicates the presence of a higher-order grating dispersion, it is not entirely accurate in this case due to the high contrast

materials and the extremely small size of the grating period. As such, to fully analyze these structures, many more terms from the permittivity expansion need to be incorporated. We turned to a rigorous coupled wave analysis (RCWA) [111] to calculate an effective index for the Nano-DAWG structures. The effective index and corresponding dispersion for an infinite grating using the design parameters for the first structure are shown in Figure 5-8.



Figure 5-8: Effective Index and Dispersion for True TE Mode in an Infinite Grating.

Although the dispersion is of a similar magnitude to the Nano-DAWG dispersion, its slope is in the opposite direction. However, as noted earlier and illustrated in Figure 5-4(a) and Figure 5-6(a), much of the mode energy is not actually contained in the ridge. In addition, the slope of the dispersion in Figure 5-1 is of the same sign as what we see from the model of the Nano-DAWGs. This suggests that the structures are actually acting much more like ridge waveguides very close to cutoff, which we would expect to have high dispersion. For comparison, a model was made of a ridge waveguide of the same as in the fin structure, but the grating region was replaced by a solid rectangular guiding region with an index of 1.567, which is the effective index of the grating at a wavelength of 1.55 according to the RCWA. The

dispersion of this structure gives the limit for the Nano-DAWG structure where the grating effect goes to zero. The dispersion is the dotted line in Figure 5-9.

The shape of the curve was very similar to the curves for the Nano-DAWGs, though the magnitude was noticeably smaller. To analyze this further, the constant refractive index of the ridge structure was replaced by the index values predicted from the RCWA at each wavelength analyzed. The propagation constants were then found and analyzed at each wavelength to determine dispersion. The modal analysis predicted that there were no propagating modes for wavelengths above $1.55 \,\mu\text{m}$. Below that cutoff point, this representation provided an upper limit for the Nano-DAWG dispersion where the grating effect reaches the highest magnitude. Dispersion for the propagating mode is plotted as the dashed line in Figure 5-9.

The dispersion for the infinite grating limit is much higher than what was predicted for the Nano-DAWGs. However, the infinite grating structure is non-guiding at frequencies where the Nano-DAWG actually supports a mode, which suggests that the RCWA is not an entirely accurate indicator of the effective index in the guiding structure. This is a sensible conclusion since the RCWA is predicated on an infinite grating structure whereas the actual Nano-DAWG contains only six grating periods. On the other hand, the infinite grating limit and constant index limit do provide an upper and lower bound to the expected dispersion for the actual structure. As the figure demonstrates, the modeled Nano-DAWG dispersion falls comfortably between the two limits, as we would expect.



Figure 5-9: Dispersion for TE Mode in a Similarly Dimensioned Ridge Waveguide Operating Near Cutoff.

Essentially then, there are two effects present in the Nano-DAWG structures. The dominant one is the waveguide effect for a ridge-like waveguiding structure with an extremely low index contrast. The grating nature allows us to fabricate a ridge with a substantially lower index contrast than could be obtained with standard materials. The second effect is the grating dispersion. This induces a decrease in the effective index as a function of wavelength, which amplifies the tendency of the waveguide to approach cutoff at higher wavelengths and causes the slope and magnitude of the dispersion to be increased even further.

These combined effects offer broader insight into the nature of the dispersion plots in Figure 5-5(a) and Figure 5-7(a). The first Nano-DAWG design actually has an extrema in the dispersion curve. It seems that below 1.54 μ m the structure is far enough from cutoff that the waveguide effect is no longer dominant and the grating effect plays a larger role, giving the curve a negative slope. Also, the second design actually has a significantly larger dispersion despite lower-index materials and a significantly larger structure. However, this makes sense

when we note that the configuration approximates a ridge waveguide near cutoff, since the energy is spread over a much larger area than in the first design. Hence, in light of a low index contrast ridge-like waveguide approximation, the dispersion curves do make a great deal of sense.

5.5 Alternative Configurations

One significant concern regarding these structures is the ability to accurately fabricate such small grating lines with a large aspect ratio. There are a few alternative configurations that could alleviate these fabrication issues. If we rotate the fin sections by 90 degrees relative to the substrate we can fabricate them by means of deposition of layers of alternating materials with different refractive indices. Depending on the nature of the materials, it would be possible to utilize different etch responses and induce undercutting in alternating layers to increase the index contrast between them (Figure 5-10).



Figure 5-10: Energy Density and Electric Field for Alternative Fin Waveguide Layout.

The illustrated horizontally-layered example Nano-DAWG consists of a 4-period structure. The longer layers have a width of 1 micron, height of 0.1um, and refractive index of

2.5. The smaller layers have a width of 0.8um, height of 0.1um, and refractive index of 1.6. This structure also supports only a single propagating TE mode. However, the previous structures attained high dispersive magnitudes when the mode was pushed downward into the substrate region, a characteristic unavailable to the horizontally-layered structures. Indeed, the calculated dispersion for the horizontal structure is on the order of 800 ps nm⁻¹ km⁻¹ – nearly an order of magnitude smaller than the original design. It is possible that further optimization of the geometry could yield a higher dispersive magnitude, particularly if it operates closer to cut-off conditions. However, since the mode cannot be pushed out into an adjacent substrate region so easily (without losing the grating effect altogether) the structure is unlikely to attain dispersive magnitudes as large as the previous one.

There is another fabrication approach that offers significant merit. When plasmaenhanced chemical vapor deposition (PECVD) is used to grow an oxide layer in a narrow trench, the physical dynamics of the process results in a layer that bulges slightly towards the center of the trench and tapers off toward the edges. This effect has been exploited to produce structures termed "trench-bulge waveguides" [112, 113]. Because of the very wide shallow nature of the waveguides, they contain only a single propagating TE mode. All others are cut off. They also exhibit fairly significant leakage into the resulting slab waveguide on top of the substrate, though fabrication processes can alleviate that somewhat.

The design of these structures can become much more involved because the available parameters consist solely of the refractive indices of the layers, the deposition times, and the dimensions of the trench. The exact layer thicknesses have to be calculated from these parameters based on PECVD deposition models [114, 115]. While these structures were not

actively explored in the course of this research, it is possible to estimate a possible geometry and simulate the propagating mode. The energy density and electric field for an approximation to the guide illustrated in **Error! Reference source not found.** and a version with a large number of layers are both presented in Figure 5-11. When the layer thicknesses are designed for operation close to cutoff, the calculated dispersion is approximately 10⁶ ps nm⁻¹ km⁻¹. This simulation does not account for leakage losses, which would certainly pose a significant issue for any applications for these structures, but the magnitude of the dispersion suggests that this may be a valuable avenue to pursue to obtain viable Nano-DAWG structures.



Figure 5-11: Energy Density and Electric Field in (a) Dual-Layer and (b) Multi-Layered Trench Bulge Waveguides.

5.6 Coupling Considerations

Perhaps the other most significant concern for these structures is the ability to couple light into and out of them. With the strange shape for the mode profile it is expected that significant losses will be present at the interface, so direct end-fire coupling is unlikely to provide a reasonable approach. The most likely candidate is an evanescent coupling scheme using a propagation constant-matched ridge waveguide structure. In this case, a 1um square ridge waveguide composed of a SiON layer with refractive index 1.7125 is sufficiently matched to allow a pair of coupled modes to exist between it and the original Nano-DAWG structure.

The symmetric and antisymmetric modes are shown in Figure 5-12. With a 3um center-to-center spacing, the effective refractive indices of the two modes are 1.46639 and 1.46604 respectively. The coupling length is given by

$$L = \frac{\pi}{\Delta\beta} = \frac{\lambda}{2(n_s - n_a)}$$
(5.8)

For this configuration, the coupling length is expected to be approximately 2.2mm.



Figure 5-12: (a) Symmetric and (b) Antisymmetric Modes for Nano-DAWG Coupling Approach.

Interestingly, this configuration may provide an additional approach of merit. Specifically, coupling between highly dissimilar waveguides of otherwise modest dispersion can result in extremely high dispersion [10, 11]. The symmetric an antisymmetric each exhibit significant dispersion but with opposite signs. This effect occurs when two waveguides with differing group velocities are coupled. At the wavelength where both waveguides have identical propagation constants the dispersion for the two coupled modes peaks. The values at a given frequency, ω , can be estimated from

$$D_{+/-} \approx D_0 \pm \left(\frac{1}{\delta\omega}\right)^2 \left(\left(\frac{\omega - \omega_0}{\delta\omega}\right)^2 + 1\right)^{-3/2}$$
(5.9)

where D_0 is the average dispersion of the individual waveguides and ω_0 is the frequency at which the individual propagation constants are equal. The characteristic bandwidth, $\delta\omega$, is given by

$$\delta\omega = \frac{2\kappa}{\left|\frac{1}{v_1} - \frac{1}{v_2}\right|}$$
(5.10)

where κ is the coupling constant and v_1 and v_2 are the group velocity of the individual guides. By decreasing the coupling constant and increasing the difference of the group delays between the two waveguides, one may obtain extremely large dispersive magnitudes.

5.7 Nano-DAWGS as Delay Lines

The Nano-DAWGs were designed and optimized for large second-order dispersion rather than a substantial group delay. However, it is worth exploring the delay-line applications of these structures as well. As discussed previously, FIR filters, including these devices, lack a structural enhancement to the delay and primarily achieve large phase delays simply by increasing the length of the structure. Delay peaks generally occur near band edges [4], which are not present in the Nano-DAWGs. Additionally, large group delay is often associated with small bandwidth as was illustrated in section 1.5.3. The substantial bandwidth (100nm) of the Nano-DAWGs further suggests that the overall group delay will be limited. Since delay scales with structure length, it is advantageous to express a group delay figure of merit independent of length. The ratio of the delay time to the structure length is (when multiplied by c) the structure's group index (Eqn. 1.9). The group index for the two Nano-DAWG designs is shown in Figure 5-13. As expected, these structures provide a relatively minor increase of about 30% over a simple optical fiber [27]. Thus, while the Nano-DAWGs offer remarkable dispersive properties, they are only marginally better the other much simpler waveguide geometries for delay line purposes.



Figure 5-13: Group Index for (a) First and (b) Second Nano-DAWG designs.

5.8 Dispersive Waveguide Summary

This chapter has presented the design and analysis of transmission-based dispersive optical filters. These filters have a uniform cross-section in the longitudinal direction and were designed to exhibit a great deal of second-order dispersion. The Nano-DAWG structures operate close to cutoff conditions and their filtering capability is enhanced by the grating nature of the structures.

These designs offer a number of challenges, particularly in terms of fabrication and coupling. In addition, since they operate near cutoff and fabrication is likely to induce roughness and discontinuities in the structure, one could reasonably expect a relatively significant degree of scattering loss. However, Nano-DAWGS are not without their benefits as well. While they have significantly lower dispersion than similarly sized Bragg grating structures, they involve a uniform structure that may be wrapped in a spiraling manner [116] to form a structure with significant group delay. Additionally, the structure dispersion and structure length are independent of each other, unlike the grating devices. This means that total dispersive delay is far less limited for these structures, though losses certainly provide an upper bound on it.

As far as the coupling and fabrication issues go, a number of approaches have been presented. Evanescent coupling may be obtained through a properly designed ridge waveguide, though the coupling constant is fairly small, which means that a fairly significant coupling length is required. However, the evanescent coupling may also provide a means to further increase the overall structure dispersion rather substantially.

Alternative configurations for Nano-DAWGS have been presented, including a horizontally-oriented structure and a trench-bulge waveguide formulation. Fabrication is much more straightforward for both alternatives since dielectric layers can be deposited with a great deal of uniformity, and alignment of separate patterns is not required for this case. Hence, the only fabrication step requiring careful consideration and optimization is the etching and possible undercutting of alternating layers, though the methods for these operations are fairly well documented.

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CHAPTER 6 SPIRAL BRAGG GRATING FILTERS

Bragg structures and chirped Bragg gratings are the result of a periodic (or slightly aperiodic) perturbation of the refractive index along the length of a given optical guiding structure [117]. The fabrication of permanent Bragg gratings in fibers was demonstrated by Hill in 1978 [118], and the concept of linearly adjusting the grating period to yield a constant dispersion structure was later noted by Ouellette [20]. Today these structures find myriad applications in telecommunications areas such as dispersion compensation [19], wavelength selection and multiplexing [119], and fiber lasers [120], as well as optical storage [121], sensing [122], and delay line [4] uses. The periodic corrugation of the refractive index profile couples an incident signal from one propagating mode into another mode (either forwards or backwards propagating) [21, 22] in a manner highly dependent on frequency, or more specifically, on the ratio of the grating frequency to the propagating wave frequency. One of the biggest drawbacks with these structures is the inherent fabrication limitations which inevitably result in the introduction of non-ideal dispersion in the form of group delay ripple (GDR) to the system. Various studies have considered the magnitude of the impact of the GDR on the quality of the system as a whole [2, 23, 24], while others have considered a variety of means to compensate for Additional difficulties with chirped Bragg structures include the this problem [25, 26]. significant length of the structures needed to obtain a sufficient degree of dispersion and the coupling loss for the typical Bragg reflector arrangement (where reflected signals have to be separated from the incident signal).

Much of the attention given to Bragg structures focuses on fiber-based filters due to their low losses and ready incorporation into other telecommunication devices. However, there is also a great deal of interest in wafer-based Bragg waveguide structures [123, 124]. Such structures allow one to utilize a wider range of fabrication techniques to more directly tailor the waveguide and grating shapes to a specific application and improve the filter response [125] and to access more exotic guiding and regeneration materials. Unfortunately, waveguides typically have somewhat higher losses than optical fibers and the high precision required from the grating fabrication can introduce significant challenges, but the ability to directly incorporate the Bragg structures into other wafer-based devices without resorting to fiber-wafer coupling can largely mitigate these issues for a variety of applications.

6.1 Bragg Filter Characteristics

Bragg grating structures provide one of the more common approaches to optical filtering. Unlike dispersive waveguides, Bragg structures introduce both amplitude and phase changes to the incident signal. Eqns. 1.24 give the complete complex filter response for a forward and backward propagating beam assuming a uniform constant-period grating. In such structures, the amplitude response is completely determined by the phase response by a Hilbert Transform and vice versa.

In the more general case where the grating period can vary, the response can no longer be expressed analytically, and the phase and amplitude responses become independent of each other. When the resulting grating can be approximated by a sequence of constant-period gratings (where $L >> \Lambda$ for each section) the overall response can be formulated in terms of the product of 2x2 transfer matrices whose components are given by a form of the solutions expressed in Eqns. 1.24 (with the caveat that the boundary condition that the amplitude of backward propagating waves goes to zero at z = L must be removed) [18]. However, for rapid changes in the grating period or index modulation, a more accurate response requires a different set of transfer matrices that consider a single grating period at a time [126]. In either case, the filter response may be calculated numerically. The goal at this point is to design and optimize the grating period to tune the filter to a desired response.

6.2 Spiral Bragg Structure Description

In a wafer-based Bragg structure, attempting to accurately vary a grating period from point to point along the waveguide in a quasi-arbitrary manner can be very difficult. However, it is possible to completely decouple the waveguide from the grating. In this way, the grating can be fabricated to an extremely high precision and reproduced using such techniques as nanoimprint. Afterwards, a waveguide can be aligned to the grating and fabricated in such a way that its trajectory produces the desired grating chirp. This offers a number of advantages, the biggest of which is the wide range of dispersion curves accessible from a single grating structure. Additionally, measurable errors in the grating structure can be compensated somewhat by applying an appropriate perturbation to the waveguide trajectory. The advantages become particularly evident when we consider the possibility of a radial grating combined with a spiral waveguide trajectory, which allows us to obtain arbitrarily long waveguides (and correspondingly large group delays) on a single substrate.

The grating function, $P(r,\theta)$, and the waveguide trajectory, $f(\theta)$ (both expressed in polar coordinates), can be defined completely independently of each other. The two are then combined to obtain an extremely flexible functional dependence of grating period on distance

traveled in a waveguide in the manner illustrated in Figure 6-1. Although a transmissive filter requires the waveguide to exit the center of the spiral by crossing over itself, the nearly perpendicular nature of the intersections and small waveguide width help minimize scattering losses at those points.



Figure 6-1: Spiral Waveguide, Radial Grating, and the Resulting Combination.

This decoupling also allows one to design a specific grating structure and use it repeatedly for a number of completely different filter response functions. In most cases we will make use of the simplest case for the grating function and assume that it has no azimuthal dependence

$$P(r,\theta) = \frac{\Lambda_0}{r_0} r \tag{6.1}$$

where r_0 is some initial radius at which the actual period takes on the initial value, Λ_0 .

If the desired filter response can be translated to a simple analytic functional form for the grating period, the exact waveguide trajectory can often be obtained analytically. However, the more general case involves an arbitrary numerically-expressed filter response. The waveguide

trajectory is then best designed through an iterative optimization approach (such as PSO). The trajectory is expressed in terms of the radius and has a functional dependence on the angular coordinate. For a realistic waveguide, the function must be continuous and smooth (continuous in the first derivative). The simplest expression would be a polynomial dependence expressed as

$$f(\theta) = r_0 \left[1 + a_1 (\theta - \theta_0) + a_2 (\theta - \theta_0)^2 + \dots \right]$$
(6.2)

where r_0 and a_i are optimized parameters.

 θ_0 is not an independent parameter but may be included to describe a final angle. This is specifically useful if the desired filter response would be better described in piecewise fashion with different trajectories connected together at specific radii and angles. In such cases, the separate waveguide sections must merge smoothly to ensure minimal scattering losses. This is sufficiently accomplished by continuity of the functions and their first derivatives. If we assume the sections f_n and f_{n+1} are to meet at angle θ_{n+1} , the appropriate constraints on the second waveguide are given by

$$r_{n+1,0} = r_{n,0} \Big[1 + a_{n,1} \big(\theta_{n+1,0} - \theta_{n,0} \big) + a_{n,2} \big(\theta_{n+1,0} - \theta_{n,0} \big)^2 + \dots \Big]$$

$$a_{n+1,1} = r_{n,0} \Big[a_{n,1} + 2a_{n,2} \big(\theta_{n+1,0} - \theta_{n,0} \big) + 3a_{n,3} \big(\theta_{n+1,0} - \theta_{n,0} \big)^2 + \dots \Big]$$
(6.3)

6.3 Linearly Chirped Bragg Structure

The standard and most obvious case of a Bragg filter is a simple linearly-chirped Bragg grating. The grating period is given by

$$\Lambda(s) = \Lambda_0 - \alpha s \tag{6.4}$$

where Λ_0 is the initial grating period, *s* is the distance along the waveguide, and α is a constant describing the grating chirp per unit length. For convenience, let us define another constant, γ , which we will label the chirp parameter, by

$$\gamma = \frac{\Lambda_0}{\alpha} = L \frac{\Lambda_0}{\Delta \Lambda}$$
(6.5)

where $\Delta \Lambda$ describes the total chirp over the length of the grating, and *L* is the length of the grating. With this definition, we rewrite Eqn. 6.4 in the following manner:

$$\Lambda(s) = \Lambda_0 \left(1 - \frac{s}{\beta} \right)$$
(6.6)

We can now equate Eqns. 6.6 and 6.1 to solve for the waveguide trajectory

$$s = \gamma - \frac{\gamma}{r_0} r \tag{6.7}$$

where *s* is the total arc length of the waveguide, which is given in calculus texts in terms of the radial function as

$$s = \int_0^\theta \sqrt{f^2(\theta) - \left(\frac{\partial f}{\partial \theta}\right)^2} d\theta$$
(6.8)

If we assume the total desired chirp to be very small in comparison to the length of the grating, the overall change in radius must be quite small in comparison to the radius of curvature of the waveguide, and we can expect

$$\left|\frac{\partial f}{\partial \theta}\right| \ll \left|f(\theta)\right| \tag{6.9}$$

Therefore,

$$f(\theta) \approx \frac{ds}{d\theta} = -\frac{\gamma}{r_0} \frac{df}{d\theta}$$
(6.10)

by equating the arc length with Eqn. 6.7 and letting the radius, r, take the functional dependence $f(\theta)$. The solution of this equation is a simple exponential dependence of the waveguide radius on angle

$$f(\theta) = r_0 e^{-\frac{r_0}{\gamma}\theta}$$
(6.11)

where θ is given in radians and is allowed to take on positive values. We can substitute Eqn. 6.11 back into Eqn. 6.1 to solve for the angle at which a given final period $\Lambda_0 - \Delta \Lambda$ is reached:

$$\theta_f = \frac{\gamma}{r_0} \ln \left(\frac{\Lambda_0}{\Lambda_0 - \Delta \Lambda} \right)$$
(6.12)

By numerical comparison to the exact result determined from Eqn. 6.8, the error based on the assumption expressed by Eqn. 6.9 is less than 1urad over a total angular extent of tens of radians for reasonable sets of design parameters. Thus the assumption may be considered appropriate.

As a final point, it is worth noting the physical significance of the chirp parameter, γ , which is directly related to the relative time delay experienced by two different incident wavelengths:

$$\frac{\Delta\tau}{\Delta\lambda} = \frac{\frac{2n\Delta L}{c}}{\frac{2n\Delta\Lambda}{c\Delta\Lambda}} = \frac{\Delta L}{c\Delta\Lambda} = \frac{1}{c\psi} = \frac{\gamma}{c\Lambda_0}$$
(6.13)

6.4 Coupling Strength Considerations

Section 1.5.1 showed that the complex filter responses for the transmitted and reflected beams are given by

$$\frac{A(L)}{A_0} = \frac{1}{\cos(\rho L) + i\frac{\phi}{\rho}\sin(\rho L)}e^{i\phi L}$$

$$\frac{B(0)}{A_0} = \frac{-i\frac{\kappa}{\rho}\sin(\rho L)}{\cos(\rho L) + i\frac{\phi}{\rho}\sin(\rho L)}$$
(6.14)

where $\rho = \sqrt{\phi^2 - \kappa^2}$, κ is the coupling constant expressed as

$$\kappa = \frac{\pi \eta \Delta n}{\lambda_B} \tag{6.15}$$

and the detuning parameter, ϕ , is given by

$$\phi = 2\pi n \left(\frac{1}{\lambda_0} - \frac{1}{\lambda_B} \right)$$
(6.16)

Again, at the Bragg wavelength the reflectance becomes $R = tanh^2(\kappa L)$, but towards the edges of the stop band, defined by $|\phi| < \kappa$, the reflectance starts to drop off. The sharpness of the band edges and the overall structure reflectance are directly related to the product κL , though the structure bandwidth is determined solely by the coupling constant, and ultimately by the grating index contrast. While larger bandwidth and reflectance are generally desirable, there are a couple drawbacks to a higher index contrast. First, the very nature of corrugations on top of a

waveguide results in some degree of scattering, particularly at frequencies that are not Braggmatched. For a typical waveguide, index contrasts on the order of 5% results in loss of about 3 dB/mm, while contrasts of 1% and less produce losses more than an order of magnitude smaller [127]. Secondly, higher contrast results in a larger overall impact on the optical signal from each grating period, and any fabrication errors due in surface roughness or period size will have a more pronounced influence on the optical signal.

Since the grating reflectance depends strongly on a combination of the grating length and the coupling strength, the two factors can be used to compensate for each other. For good band extinction, the product should be reasonably large. However, reflectance starts to drop off towards the edges of the band. At $\kappa L = 2$ the reflectance at the center of the band is about 93% but is only 90% of that at $|\phi| = 0.9\kappa$. When $\kappa L > 4$, reflectance is over 98% for 90% of the rejection band. More of the band becomes useable as the product increases, but $\kappa L = 4$ serves as a suitable working minimum for coupling strength and grating length.

This leads to a limitation on the maximum possible chirp (and bandwidth) of the spiral grating structure. The difficulty comes from the length necessary to provide good reflectance over the entire band. As the waveguide spirals inwards, the grating period, and hence the effective Bragg wavelength, decreases. We require that a given incident wavelength remain in the reflection band for a minimum distance *L*. Based on the analysis above, it is reasonable to require the wavelength in question, λ_0 , to remain within the 90% of full bandwidth for the structure, meaning that the coupling constant (which determines the bandwidth) plays a significant role.

Thus, over a waveguide length of $\frac{L}{2}$ the Bragg wavelength is allowed to decrease from λ_0 to the point where $|\phi| = 0.9\kappa$. From Eqn. 6.16 and the definition of the coupling constant, we find

$$\frac{1}{\lambda_B} = \frac{1}{\lambda_0} \left(1 + \frac{0.9\eta \Delta n}{2n} \right)$$
(6.17)

For small index contrasts, we may approximate the maximum chirp by

$$\Delta \Lambda \approx \frac{0.9\eta \Delta n}{2n} \Lambda_0$$
 (6.18)

If we make use of the assumption noted above that the minimum length is given by $\kappa L > 4$, we may express the maximum chirp per unit length:

$$\frac{\Delta\Lambda}{L} = \frac{0.45\eta\Delta n}{nL}\Lambda_0 = 0.177 \left(\frac{\eta\Delta n}{n}\right)^2$$
(6.19)

which clearly increases with the square of the grating index contrast. Based on the analysis in Eqn. 6.13, we can obtain a minimum relative delay:

$$\min\left(\frac{\Delta\tau}{\Delta\lambda}\right) = \frac{5.65}{c} \left(\frac{n}{\eta\Delta n}\right)^2$$
(6.20)

For reasonable index contrasts and confinement factors, this works out to a minimum relative delay of approximately 1.2 ps/nm.

6.5 Additional Design Constraints

It is important at this point to note that the spiral cannot simply be made arbitrarily small. The minimum radius of curvature is strongly limited by bend leakage. A method for estimating the leakage loss of these guides as a function of curvature is outlined in section 3.6.3. For dielectric waveguides in the spectral region of interest, the loss grows exponentially with curvature and is on the order of a few dB per mm for radii of curvature around 2-3mm [128]. Based on this, we take 5mm as a reasonable minimum for the radius of curvature to have minimal losses. This gives the minimum possible radius for the spiral structure. Further, if the device is to be used in transmission, the innermost spiral loop will have to curve 180 degrees and exit through the center of the spiral. This final bend has double the curvature of the highest curvature of the spiral, and thus, if the radius of curvature of any part of the structure is to be no less than 5 mm, the innermost radius for the spiral can be no less than 10mm.

Another important quantity to consider is the distance between successive loops of the spiral. To have minimal coupling between waveguide loops, a minimum reasonable separation distance between them is about 15-20um. From analysis of Eqn. 6.11, we find the radial distance between two rings to be

$$f(\theta - 2\pi) - f(\theta) = \left(e^{2\pi \frac{r_0}{\beta}} - 1\right) f(\theta)$$
(6.21)

This distance takes on a minimum value when $f(\theta)$ is at a minimum (i.e. the radius from Eqn. 6.1 at which the final grating period is obtained). For a linearly chirped grating, given a desired grating period, total chirp, initial waveguide radius, and minimum allowed radial distance between rings, Eqn. 6.21 in combination with Eqn. 6.5 can yield a maximum grating length. In most cases this is not a severe limitation. If the initial radius is 30mm, initial period is 500nm, and total chirp is 10nm stretched out over a length of 5m, the smallest radial separation between successive loops is still 22um while the structure's delay is 1.67ns/nm.

In more general terms, we can rewrite this as a maximum grating chirp parameter in terms of the initial radius and grating periods, the minimum radial separation between spiral loops, and the overall structure chirp:

$$\max(\gamma) = \frac{2\pi r_0}{\ln\left(\frac{\Lambda_0 \min(\Delta r)}{r_0(\Lambda_0 - \Delta \Lambda)} + 1\right)}$$
(6.22)

Since the minimum radial spacing is much smaller than the initial spiral radius, a Taylor series expansion simplifies this to

$$\max(\gamma) = \frac{2\pi (\Lambda_0 - \Delta \Lambda) r_0^2}{\Lambda_0 \min(\Delta r)}$$
(6.23)

Using Eqn. 6.13, we can also derive a maximum relative delay for a structure (assuming multiple spiral loops are required):

$$\max\left(\frac{\Delta\tau}{\Delta\lambda}\right) = \frac{2\pi(\Lambda_0 - \Delta\Lambda)r_0^2}{c\Lambda_0^2 \min(\Delta r)}$$
(6.24)

While a delay of 1.67ns/nm was noted for a larger structure above, if we instead assume a smaller spiral structure with an outer radius of 10mm and a minimum loop spacing of 25um, a period of 500nm with a chirp of 10nm yields a maximum relative delay of 164ps/nm.

6.6 Quantification of Fabrication Issues

An additional potential advantage of the decoupling of the grating function from the waveguide trajectory involves fabrication accuracy. It can be quite difficult to position grating lines with an accuracy significantly better than 1nm. Yet grating lines snapped to the nearest nanometer will not produce a desirable uniform phase response, and in the extreme case can result in a discrete reflection spectrum similar to that illustrated in Figure 6-2. The radial nature

of the grating and the curved waveguide trajectory allow one to overcome any sort of finite grid issues introduced by fabrication limitations [6].



In a straight-line grating, the pixel snapping would be uniform across the width of the grating, but because of the angular nature of the grating lines in the spiral method, the grating period varies significantly point by point across the width of a single segment (see Figure 6-3). However, while the edge will theoretically be written in a very jagged manner, the spacing between the points is on the order of 1.25nm (depending on the resolution used during e-beam writing). Further, the nature of the grating fabrication process will usually cause pixilations to wash out, resulting in a smooth line. This results in an averaging of the period length and spreads out any remaining errors over the length of the structure.


Figure 6-3: Pixilation of a Single Grating Period.

To model the grating structure, we calculated the exact snapped distance between grating lines for pairs of corresponding points in the waveguide. The calculations were repeated and averaged for several hundred points spanning the width of each section of waveguide. This provided a numerical approximation to the averaging the light mode actually undergoes. Ideally the successive periods would demonstrate a smooth linear curve according to Eqn. 6.6. Simulations indicate that there is still a degree of error in the periods due to e-beam pixel snapping, but the errors are reduced by several orders of magnitude from the straight-line case. In the case of a grating written with a 1.25nm resolution (the best resolution achievable on the e-beam system), three standard deviations of the error in the periods fall within 0.235nm, which is substantially better than one might expect. Also, this calculation was based on a very rudimentary estimation that overestates the error. A more accurate error determination should vield significantly better results. Sumetsky [24] provides an analytic and quantitative analysis of

how such errors in the grating periods will affect the group delay curve of the system. Figure 6-4 illustrates the calculated reflectance and group delay for the designed structure. Figure 6-5 shows the group delay ripple expected for structures written with 1.25nm and 5nm pixels.



Figure 6-4: Expected Reflectance and Group Delay for 1m Linearly Chirped Bragg Structure.



Figure 6-5: Expected Group Delay Ripple for Linearly Chirped Structure Written with 1.25nm and 5nm Pixels.

6.7 Alternative Group Delay Functions

6.7.1 Constant Period Delay Line

Another application of Bragg grating structures is in the area of delay lines. Although the relative delay across the reflection band is not particularly substantial, transmitted frequencies at the edge of the band experience dramatic group delay as predicted by [32]:

$$\tau_g = \frac{nL}{c} \frac{1}{\sqrt{1 - \left(\frac{\kappa}{\phi}\right)^2}}$$
(25)

These structures have a very small bandwidth, and the grating should maintain a fixed period to keep the specified frequency at the band edge. This would be accomplished via a waveguide with constant radius. Note however that the geometrical concept is amenable either to multiple waveguides acting in parallel or to a single waveguide that curves inwards a short distance to yield a different effective grating periods for subsequent waveguide sections.

For structures of this sort, it is advantageous to produce as much delay as possible over a short distance. Thus, we employ a reasonably large index contrast without going so large as to incur substantial scattering penalties. The central Bragg wavelength was taken to be 1.55um, and the grating layers had indices of 1.57 and 1.56. This gives an expected coupling constant of 0.013um⁻¹ and a reflection bandwidth of 6.3nm. A 2mm length structure is therefore sufficient to yield a very sharp bandedge at 1.5468um. If we take the expected waveguide loss to be 0.1dB/mm and the scattering loss due to the grating to be another 0.3dB/mm, and the bend loss to be 1dB/mm, the overall loss is less than 3dB. The structure offers a delay of 0.7ns over the 2mm grating length, which gives a group index of 105. On the other hand, if the index contrast is reduced by 50%, the coupling is cut in half and the structure must be doubled in length to

obtain the same sharp bandedge, albeit at a slightly longer wavelength. In this case the delay is twice as much, but the overall group index remains the same. In addition, since radiation losses due to the curvature of the waveguide are expected to dominate, the shorter device experiences significantly lower loss. Group delay for both structures is illustrated in Figure 6-6.



Figure 6-6: Group Delay from Constant Radius Waveguide with (a) 0.01 and (b) 0.005 Index Differential.

6.7.2 Long Period Gratings

An additional design consideration involves longer period gratings. At certain periods, longer period gratings have been used to couple light into forward-propagating cladding modes for band rejection and dispersion purposes [129], but they may also be designed to couple to reverse-propagating modes at a higher diffraction order. In particular, when the grating period is given by

$$\Lambda = 3\frac{\lambda_B}{2n} \tag{26}$$

the Bragg condition is still met for the backwards propagating mode.

The larger period eases fabrication tolerances somewhat, but does introduce the possibility of scattered diffraction orders (though these don't meet the Bragg condition for maximum diffraction efficiency). This also means that the coupling strength is reduced and the minimum waveguide length is increased, but for a dispersion-type application we are rarely working near the minimum grating length for reasonable reflectance. There is also significantly higher scattering loss as the propagating mode can couple to radiating diffraction orders. On the other hand, there are significant advantages to this approach, particularly when fabrication issues are considered. Small errors in the period sizes pose a much more significant problem when the grating periods themselves are small. Figure 6-7 illustrates this situation. A 1nm amplitude normally distributed random perturbation was applied to normal first-order gratings and long-period third-order gratings (both based on the same design as the dispersive Bragg structure outlined in the previous section). Clearly the GDR is substantially worse for the short-period structure, and sufficiently so as to have a larger impact on device performance than the additional loss terms introduced by the long-period structure.



Figure 6-7: Group Delay Ripple for (a) First-Order Bragg Grating and (b) Third-Order Bragg Grating with 1nm Normally Distributed Random Perturbations to Period Size.

6.7.3 Tailored Phase Delay for Dispersion Compensation

Earlier, this chapter indicated that a uniform linear delay slope requires an exponential dependence in the waveguide trajectory. However, it is illustrative to consider the filter response for a waveguide with a linear dependence on angle:

$$f(\theta) = r_0 - \alpha \theta \tag{6.27}$$

Based on the arc length formulas above, the path length as a function of angle becomes

$$s = \theta \left(r_0 - \frac{\alpha}{2} \theta \right) = \frac{r_0^2 - r^2}{2\alpha}$$
(6.28)

Thus, instead of obtaining a linear relationship between the grating period and grating length we instead have

$$\Lambda = \frac{\Lambda_0}{r_0} r = \Lambda_0 \sqrt{1 - \frac{2\alpha}{r_0^2} s}$$
(6.29)

which suggests that the resulting group delay curve will be of a quadratic nature. Indeed, Figure 6-8 demonstrates the difference between the resulting group delay and the linear curve produced by the exponential spiral.

It is worth noting that this deviation is only about 0.5% and would likely be swallowed up in the GDR noise from a realistic grating structure. However, structures with larger overall group delays and greater bandwidths could experience a much larger impact from this effect. On the other hand, with appropriate optimization, the dispersion introduced by a structure of this form could easily be used to compensate for other devices that introduce unwanted dispersion and phase distortions.



Figure 6-8: Departure from Linear Group Delay Curve for Waveguides with Linear Angular Dependence.

6.7.4 Spiral Bragg Structures as Amplitude Response Filters

Another potential application for these structures involves amplitude filtering of closely spaced spectral bands for wavelength division multiplexing (WDM) related devices. In this case we are not looking for a continuous group delay, but rather a band of closely spaced transmission (or reflection) bands. As is discussed in CHAPTER 7, coupled resonant cavities yield sharp closely spaced resonant transmission lines. While it is difficult to obtain the same behavior with Bragg waveguide structures without introducing extremely large waveguide curvatures, the concept is suggestive of a potential approach utilizing the decoupling method.

Instead of spiraling the waveguide inward in a continuous manner to obtain a smooth group delay curve, we designed the waveguide to have a constant radius with the exception of a periodic perturbation of the form

$$f(\theta) = r_0 + \Delta r \, e^{-8\sin^2(2\theta\theta)}$$
(6.30)

This results in a trajectory of the form shown in Figure 6-9. Note that the periodic ripple has been exaggerated significantly to demonstrate the overall shape. With an initial radius of 23.9mm, the actual total radial shift was approximately 130um for this structure.



Figure 6-9: Sinusoidal Waveguide Trajectory (Exaggerated for Effect).

As may be expected, the trajectory produces a periodic variation in the grating period as shown in Figure 6-10, and the amplitude response of the structure is shown in Figure 6-11. The reflection bands are spaced by approximately 0.2nm, though they are not particularly uniform in this case. However, a more complete optimization of the structure should be suitable to obtain a better overall response.



Figure 6-10: Sinusoidal Grating Periods.



Figure 6-11: Amplitude Response for Sinusoidal Waveguide Trajectory.

6.8 Spiral Bragg Filter Summary

This chapter presented a unique approach to Bragg grating filter design in which the functional forms of both the waveguide trajectories and grating structures are fully independent and quite arbitrary. The method provides for reuse of gratings and eased fabrication tolerances while offering a great deal of flexibility in terms of applications and optical delay curves.

Design constraints included a minimum radius of curvature to prevent significant radial leakage and a minimum grating chirp to prevent coupling between successive spiral loops. Coupling strength as a result of corrugation depth plays a significant role in the overall structure response by introducing scattering loss and determining the width of the rejection band. Additionally, by incorporating higher-order grating periods into the device design fabrication errors can be substantially mitigated. Based on the derivations presented herein, this approach is capable of providing relative delays between spectral components of anywhere between 1 ps/nm and 2 ns/nm depending on the chosen design constraints. While dispersion compensation is one

direct application of this approach, this range of relative delay values offers a unique approach for WDM-related applications and tunable sources.

The radial nature of the grating lines and the curved waveguide trajectory combine through very straightforward fabrication processes. The angular nature of the structure components average out fabrication errors and offer a substantially reduced GDR than can be obtained with linear gratings fabricated with the same processes. This chapter demonstrated the design process and the response model for a structure incorporated a 1m long waveguide onto a standard wafer. The structure had a flat reflectance band over a bandwidth of about 40nm and a dispersive delay of 250ps/nm. In addition, we provided a delay line approach utilizing high index contrast gratings to obtain an extremely large group delay with an effective group index of 105 with expected leakage well below 3 dB.

Alternative trajectory designs were also presented as examples, demonstrating the wide range of filtering applications attainable with this approach. Long period gratings offer reduced fabrication complexity. Other simple functional forms of the waveguide trajectory may be used to obtain a variety of nonlinear delay curves, which lend themselves well to correcting and offsetting frequency delay already present in an incident signal. On the other hand, an appropriately designed periodic perturbation to the waveguide trajectory can be used to obtain unique transmission bands for selecting specific frequency components. PSO or other optimization approaches are easily applied to the functional forms to obtain specific filter responses tailored to a wide range of applications.

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CHAPTER 7 AXISYMMETRIC RESONANT CAVITY FILTERS

Optical cavities offer another type of resonant structure that finds uses in many different applications. While more commonly used for optical sources and quantum electrodynamics experiments, they may also be used in filtering applications [60]. The standard approach for resonant filters involves a ring structure coupled to a waveguide. In such conditions, the response of the signal coupled into and transmitted through the resonator has a narrow bandwidth centered on the structure resonances. The non-resonant bands do not couple into the structure and remain in the waveguide. The filter's phase response also involves large group delays near the resonant frequencies.

In this chapter we explore the application of three-dimensional axisymmetric cavity structures to optical filtering and different ways to approach these types of structures. In general, a good resonance-based filter must have a large Q-factor. Certain geometries may be exploited to obtain this with a single cavity, though such structures are often better suited for optical sources and various other electrodynamics applications than to direct filtering of transmitted and reflected signals. On the other hand, chains of weakly coupled low-Q cavities can be built up into a guiding structure with significant filtering properties.

7.1 Single Cavity Optimization

Earlier in this work methods for analyzing and modeling microcavity resonators were outlined and discussed in detail. Of interest in this section is to apply those tools and the optimization approach discussed previously to design and optimize resonators for specific spectral outputs and resonance characteristics. In particular, we would like to provide an optimization program with a set of design criteria that specify that a designed cavity should resonate with the highest Q-factor possible at a given frequency, ω_0 , and with a specified mode number (e.g. TE₀₁₁). The program then returns the cavity geometry necessary to obtain those characteristics.

In general, the design and optimization of resonant cavities is nontrivial. Although certain aspects can be determined analytically – such as the dimensions required for a basic cylindrical structure to resonate at a specified frequency – an optimal resonator design may reasonably be expected to come only through a numerical optimization approach. Further, the process can be rather time consuming. Each newly proposed design suggested by the optimization algorithm must be modeled and tested for the desired resonant characteristics. In a PSO approach, an average swarm size of 30 particles must be re-evaluated at every iteration. A typical optimization that may take 200 iterations to converge then requires 6000 separate cavity models. Time domain simulations, which can take anywhere from a few minutes to several hours to provide a full diagnostic of a given cavity, become completely unusable in this scenario. On the other hand, the eigenmode analysis, discussed earlier in this work, has been shown to work well with PSO for the cavity design process [130].

7.1.1 Particle Swarm Optimization of Cavity Designs

One of the most significant difficulties in the design problem comes when we consider the range of geometric parameters available to the optimization algorithm. For example, consider the cavity geometries illustrated in Figure 7-1. The cavity radius and length would nearly always be optimized to obtain the desired resonant frequency or wavelength. But a dielectric cavity by itself has a rather low Q-factor, and one must additionally optimize the surrounding regions to obtain narrow resonances. In the pillar structure, one might optimize the individual layers to adjust for a tapering of the structure width introduced during fabrication [35, 42, 131]. In pillars, radial confinement is provided simply by the index contrast in a longitudinal guiding situation.

One alternative use relies on refractive index contrast in the vertical direction while introducing a highly reflective confinement medium in the radial direction. Cavities encased by metallic films are common in microwave applications, but the lossy nature of metals at optical frequencies tends to be far too great to be useful in these types of configurations. However, highly reflective radial confinement structures with no intrinsic absorptive loss are capable of supporting extremely high Q-factors under certain conditions [47, 49]. It is useful to optimize the geometry to obtain this effect, which may be further accentuated by optimizing layers separating the cavity from a bulk substrate region.



Figure 7-1: Depiction of Single-Cavity Geometries.

With such a large parameter space to consider, particularly when we have noted previously that the resonance characteristics can have very narrow peaks in parameter space (see Figure 3-3), an exhaustive or deterministic search algorithm would be hopelessly inefficient. Further, it is very reasonable to assume that parameter space will contain a number of locally-optimal solutions that perform far worse than some globally-optimal cavity design, which means that calculus-based search algorithms will inevitably fail. This makes the cavity resonator design an ideal problem for the PSO algorithm discussed earlier.

7.1.2 Mode Choice

It has been shown [132] that cylindrical waveguides surrounded by metallic cladding layers offer substantially reduced absorptive loss to the TE_{01} propagating mode. This results from the electric field being polarized tangential to the dielectric-metallic interface. In like manner, a TE_{01p} mode maintains a fixed relationship between electric polarization and the interface between the cavity and the confinement region. The difficulty of coupling to and from the cavity require that the light be focused to as small a spot as possible. Radially symmetric modes may be focused to a significantly smaller spot than can linearly polarized or other hybrid modes [133, 134]. Additionally, modes of this type are significantly smaller than the higherorder hybrid modes, which allow for smaller cavities and reduced mode volumes.

One of the principle reasons this mode is largely ignored is the difficulty in coupling to it. The HE₁₁₁ modes involve linearly polarized light, which is easy to obtain. Thus, mode matching to cavities based on that mode is fairly straightforward, allowing one to couple light in and out of such structures without tremendous difficulty. On the other hand, obtaining azimuthally (TE₀₁) and radially (TM₀₁) polarized propagating modes is not nearly so trivial. An approach for coupling to azimuthally and radially polarized resonant modes will be explored in subsequent sections.

7.1.3 Determination of Cavity Fitness

To provide a robust optimization approach for the construction of resonant filters, all important design criteria have to be appropriately represented and given suitable weight. An improperly defined fitness function may result in undesirable solutions that may behave well in some aspects and very poorly in others. In the case of cavity resonators, the most crucial requirement is that the cavity resonate at a specified wavelength, λ_0 , with as high a Q-factor as possible. It is also worthwhile to include a component to force this wavelength to correspond to a specific mode number.

Thus, the eigenmode solver outlined in an earlier chapter offers a suitable tool to obtain the metrics needed for the fitness function. The eigenmode solver will take the cavity geometry as an input and return the resonant wavelengths, Q-factors, and electric field distributions. Since the problem is assumed to be axisymmetric and the azimuthal mode number is specified as an input to the eigenmode solver, it need not factor into the fitness function. On the other hand, the field distribution can be used to determine the radial and axial mode numbers. These may all be combined into a single fitness mapping function of the following form:

$$F = \left[1 + \left(A|\lambda - \lambda_0|\right)^{a_1}\right] |Q|^{-a_2} \left[\left(1 + |M_r - M_{r0}|\right)\left(1 + |M_z - M_{z0}|\right)\right]^{a_3}$$
(7.1)

In this expression, λ_0 gives the design wavelength and M_{r0} and M_{z0} specify the desired radial and axial mode numbers respectively (typically both unity). λ , M_r , and M_z are the corresponding characteristics of the nearest resonant mode of given cavity geometry. Q is the calculated quality factor of the cavity. The three parameters in the exponent, a_1 , a_2 , and a_3 (all assumed to be positive-valued) provide weights for each of the cavity characteristics. The remaining coefficient, A, is simply a scale factor to remove units from the expression and determines what magnitude of difference between the desired and actual wavelengths is significant. For example, a value of 100 um^{-1} essentially means that deviations of the resonant wavelength from design are not significant until they are of the magnitude of 100 um^{-1}

This formulation of the cavity fitness function provides several features. The function is designed to be minimized, so values smaller than 1 necessitate high Q-factors. The different cavity characteristics are combined geometrically rather than algebraically, which prevents the function from relying solely on a single aspect while neglecting the others. Since the mode number is integer-valued, it makes sense that it should have no impact on the overall cavity fitness, so long as it matches the desired mode. As such, in this formulation, the portion dealing with mode numbers becomes unity when they are correct. Likewise, the resonant wavelength contribution also approaches unity as it nears the design wavelength. This leaves the Q-factor alone to produce the required small fitness value.

A very respectable resonant cavity design would have a Q-factor about 10,000. The design criteria used for optimization in this research assumed that a sufficiently optimized cavity would have a TE_{011} resonant mode at a wavelength near 1.5um with a Q-factor over 100,000. Based on trial and error, the exponent weights in Eqn. 7.1, a_1 and a_2 , for the Q-factor and resonant wavelength were set to 2. This allowed the fitness function to be sufficiently narrowed in the region of an optimal solution so as to force the particles down into it quickly while still being sufficiently wide that the particles did not speed by without locating it. The wavelength

scaling coefficient, A, was set to 100um^{-1} , which forced the resonance to be within about 50nm of the design wavelength. The exponent weight on the mode numbers, a_3 , was set to 8, which increased the fitness value by over two orders of magnitude for cavity resonant modes other than the specified design mode number. With these fitness function parameters, the design criteria were satisfied with a fitness value smaller than 10^{-10} . This was chosen as the convergence threshold for the PSO approach.

7.1.4 High Q-factor Cavity Design

Under very specific combinations of cavity dimensions a very strong enhancement in the Q-factor is observed if a lossless highly reflective region surrounds the cavity in the radial direction [47, 49]. This effect was discussed by Ibenescu and demonstrated numerically using PECs as a surrounding medium. The basic concept is demonstrated in Figure 7-2 and the resonances were calculated and plotted in Figure 3-3 as a benchmark for the eigenmode solver. Essentially, for this type of situation, the geometry causes a repulsion between polarization states resulting in a region with very small group velocity at some point in the dispersion relationship [49]. This occurs at very specific values of cavity radius and geometry depending on the reflectivities at either end of the cavity.



While PECs used for radial confinement allow the Q-factors to reach 10⁴-10⁵, replacing them with real metal films results in a dramatic reduction of the Q-factor. It should be noted that metal films work well in the microwave region of the spectrum (and have been used to form microcavities for decades) where the frequency-dependent permittivity approaches that of a PEC. Further, when media with sufficiently high gain are present, such structures have been shown to function as low-threshold lasers [135]. There is an alternative approach to the geometry that offers a similar effect. Yariv originally proposed the idea of a radial Bragg grating encasing optical fibers [136], and the concept has also been used to aid confinement in microcavities [137, 138]. This configuration has similar peaks of the Q-factor for certain combinations of geometrical parameters as in the PEC-encased cavity scenario [47].

Because of the narrow range of parameters offering enhanced Q-factors, PSO offers a convenient approach as a microcavity design tool [130, 139]. An example of an optimized DBR-encased cavity and its corresponding amplitude response is shown in Figure 7-3, and the resonant wavelengths and Q-factors for various combinations of cavity lengths and radii are shown in Figure 7-4. Notice that the resonant wavelength depends primarily on the radius, while the Q-factor is affected more strongly by the cavity length.



Figure 7-3: Cylindrical-Bragg Microcavity and Amplitude Response.



Figure 7-4: (a) Resonant Wavelength and (b) Q-factor for Combinations of Cavity Geometries.

To improve confinement further, it is worth exploring a means of reducing leakage into the substrate. A conventional approach is to place a set of DBR layers between the substrate and the cavity (as shown in Figure 7-5(a)) to reduce downward propagation. Indeed, in our simulations, this improved the peak Q-factor from around 30,000 to about 50,000. However, it has been noted previously [139, 140] that the small size of the cavities results in a great deal of diffraction at the ends of the cavity. Thus, the longitudinal leakage consists of a large range of wave vectors, which reduces the effectiveness of the DBR. The structure may be optimized somewhat to provide the highest reflection for the peak wave vector, but this offers only a small improvement. An alternative is to optimize each reflecting layer independently. This was seen to increase the Q-factor by nearly a factor of 2 to approximately 90,000.



Figure 7-5: Cavity with Longitudinal Confinement Provided by (a) DBR Layers and (b) Optimized Reflecting Layers.

7.2 Coupling to Axisymmetric Modes

While we may optimize single cavities for high Q-factors at specific resonance frequencies, we would like to apply these cavities to the field of optical filtering. As such, it is absolutely essential to be able to couple an incident signal into the structure and obtain a well-defined response for the signals transmitted through and reflected from the device. In the case of axisymmetric cavity modes, this involves converting a signal to either a radial or azimuthal polarization state and focusing it down to a sufficient size for mode matching with the cavity in the manner illustrated in Figure 7-6.



Figure 7-6: Coupling Methodology for Microcavity Resonators.

A suitable polarization-converting element was originally fabricated by Mohammed [132] and was developed into an appropriate structure for oxide wafers by Rumpf and Mehta [141]. The latter case, a spatially polarizing autocloned element (SPACE) based on a spatially-varied effective-index grating structure has been used experimentally to convert a linearly polarized incident beam into both azimuthal and radial polarizations. The structure and its output beam are illustrated in Figure 7-7.



Figure 7-7: SPACE Structure and Output Azimuthally Polarized Beam.

Once the incident beam is in the correct polarization state, it must also be focused down to the appropriate size to couple into the cavity filters. Notice that for the appropriate cavity radii for the resonances we have described, the spot size is below the geometrical diffraction limit of most lenses. Thus, we need a high-NA lens and we should numerically calculate the resulting spot size directly using the full diffraction integral since polarization state will strongly influence the resulting spot size and shape for such lenses [142, 143]. With modern fabrication methods it is possible to formulate an ultra-high NA lens on a small scale. An example of such a lens (courtesy of P. Srinivasan) is shown in Figure 7-8.



Figure 7-8: High NA Lenses (NA = 1.45).

To obtain the resulting field distribution for the spot at the focus of such a lens, we turn to vector diffraction theory [133]. In the case of large Fresnel numbers, we can ignore diffraction effects from the edge of the lens and decompose the incident signal into a series of plane waves. At the focus of the lens, the resulting field distribution is calculated according to the superposition of each plane wave

$$\vec{E}(\vec{r}_{2}) = \iint dk_{x} dk_{y} \frac{\vec{A}(k_{x}, k_{y})}{k_{z}} e^{i\vec{k} \cdot (\vec{r}_{2} - \vec{r}_{1})}$$
(7.2)

where *A* is the complex vector amplitude of the plane waves on transmission through the lens aperture. If we assume an azimuthally polarized beam (TE_{01}) incident on the lens given by

$$\vec{E}_{inc}(r,\phi) = \hat{\phi} E_0 r e^{-r^2/w_0^2}$$
(7.3)

the resulting field at the spot may be expressed according to

$$\vec{E}(r,\phi,z) = \hat{\phi}kfE_0 \int_0^{\theta_1} \sin\alpha \sqrt{\cos\alpha} e^{-(f^2/w_0^2)\sin^2\alpha} e^{ikz\cos\alpha} J_1(-kr\sin\alpha)d\alpha$$
(7.4)

where z is the axial distance between the observation plane and the lens focus, f is the focal length of the lens, and θ_1 is the maximum angular extent of the lens as seen from the focal point. This distribution is illustrated in Figure 7-9 for several values of the focusing lens numerical aperture. Similarly, if the polarization of the incident field is switched to the radial direction, the resulting field distribution becomes

$$\vec{E}(r,\phi,z) = kfE_0 \int_0^{\theta_1} \sin\alpha \sqrt{\cos\alpha} e^{-(f^2/w_0^2)\sin^2\alpha} e^{ikz\cos\alpha} \begin{bmatrix} (\cos\alpha)J_1(-kr\sin\alpha)\hat{r} \\ (\sin\alpha)J_0(-kr\sin\alpha)\hat{z} \end{bmatrix} d\alpha$$
(7.5)

Clearly the TM_{01} case has both longitudinal and radial polarization components. These are shown in Figure 7-10, while the resulting electric field intensity is demonstrated in Figure 7-11.



Figure 7-9: Azimuthally Polarized Field at High NA Lens Focus.



Figure 7-10: (a) E_r and (b) E_z Components of Radially Polarized Beam at High NA Lens Focus.



Figure 7-11: Electric Field Intensity of Radially Polarized Beam at High NA Lens Focus.

7.3 Coupled Resonator Filters

So far we have demonstrated a reasonable approach to designing and optimizing single cavities for narrow resonances and we have outlined a feasible method to couple an incident signal into such structures. The final step needed to construct a useful optical filter is to express the effects introduced by coupling multiple cavities to each other in a chain. In such a situation it is readily apparent that the exotic geometries discussed earlier in this chapter do not lend themselves well to a multiplexed device. However, the chaining of a large number of cavities together allows us to make use of much weaker resonators without introducing large loss terms to the response function.

7.3.1 Coupled Resonator Optical Waveguides

There are a number of ways to approach a structure composed of coupled resonators. A very simplistic first-order approximation may be obtained by noting that each successive resonator receives as input the filtered signal output from the previous resonator in the line. In other words, if we ignore backward propagation and oscillations between neighboring cavities, the overall amplitude transmission function is approximated by

$$T_N \approx T_0^N \tag{7.6}$$

If the impulse response of a single filter is given by a Lorentzian function centered at the cavity's resonant frequency, the output of the second filter will be the Lorentz filter function applied to the broadband signal twice, and so on. Thus, it is reasonable to assume that the Q-factor for an N-cavity filter is approximately equal to NQ_0 where Q_0 is the Q-factor of a single resonant cavity.

Now this approach obviously ignores coupling and oscillations, which are extremely significant factors and have particular implications for the resonant frequency. The simple case of a pair of resonators is comparable to a coupled waveguide structure in which the nominally identical propagating modes of two waveguides combine and split into a symmetric mode and an antisymmetric mode according to coupled-mode theory. The difference between the propagation constants of the two modes is directly proportional to the coupling constant of the waveguide pair [27]. In like manner, a pair of identical coupled resonators will resonate at two different modes centered on the original resonant frequency. The frequency separation between the two modes is directly proportional to the coupling constant between the two modes is directly proportional to the coupling separation between the two modes is directly proportional to the coupling separation between the two modes is directly proportional to the coupling separation between the two modes is directly proportional to the coupling separation between the two modes is directly proportional to the coupling separation between the two modes is directly proportional to the coupling constant frequency separation between the two modes is directly proportional to the coupling constant between the two modes is directly proportional to the coupling constant between the two modes is directly proportional to the coupling constant between the two cavities [144].

Yariv originally proposed that a line of coupled resonators could act as a waveguide with minimal loss over a transmission band, even if sharp bends occur in the propagation direction [145]. Assuming a quasi-infinite linear combination (parallel to the z-axis) of high-Q resonators, one may assume that the solutions for the eigenmodes satisfy the Bloch theorem. Thus,

$$\vec{E}_{K}(\vec{r},t) = E_{0}e^{i\omega_{K}t}\sum_{n}e^{-inKL}\vec{E}_{\Omega}(\vec{r}-nL\hat{z})$$
(7.7)

where $\vec{E}_{\Omega}(\vec{r})$ is the eigenmode of the individual resonators, Ω is the single-cavity resonant frequency, and *L* is the spacing between resonators. Under the assumption that the individual eigenmode is normalized to 1 and that the full filter mode, $\vec{E}_{\kappa}(\vec{r})$ satisfies Maxwell's Equations, Yariv shows that the following dispersion relation holds:

$$\omega_{K}^{2} = \Omega^{2} \frac{1 + \sum_{n \neq 0} \beta_{n} e^{-inKL}}{1 + \Delta \alpha + \sum_{n \neq 0} \alpha_{n} e^{-inKL}}$$
(7.8)

where $\Delta \alpha$, α_n , and β_n are coupling constants given by $(n \neq 0)$

$$\alpha_{n} = \int \varepsilon(\vec{r}) \vec{E}_{\Omega}(\vec{r}) \bullet \vec{E}_{\Omega}(\vec{r} - nL\hat{z}) d^{3}\vec{r}$$

$$\beta_{n} = \int \varepsilon'(\vec{r} - nL\hat{z}) \vec{E}_{\Omega}(\vec{r}) \bullet \vec{E}_{\Omega}(\vec{r} - nL\hat{z}) d^{3}\vec{r}$$

$$\Delta \alpha = \int [\varepsilon'(\vec{r}) - \varepsilon(\vec{r})] \vec{E}_{\Omega}(\vec{r}) \bullet \vec{E}_{\Omega}(\vec{r}) d^{3}\vec{r}$$
(7.9)

and where ε is the permittivity distribution for a single resonator and ε ' is the permittivity due to the additional resonators. In the weak coupling limit (and assuming identical cavities so that $\Delta \alpha$ = 0), the dispersion relation reduces to

$$\omega_{K} = \Omega[1 + \kappa_{1} \cos(KL)]$$
(7.10)

where $\kappa_1 = \beta_1 - \alpha_1$. It should be noted for completeness that the "+" in Eqn. 7.10 should in reality be replaced by "±" to account for positive and negative group velocities, though the latter is unlikely to occur physically [146]. From this relationship we can obtain a group index:

$$n_G = \frac{c}{\Omega \kappa_1 L} \tag{7.11}$$

Note that this indicates an increase in the initial group delay by a factor of approximately $\frac{1}{\kappa_1}$.

From Eqn. 7.10 we can easily derive the full frequency bandwidth of the structure:

$$\Delta \omega_{use} = 2\kappa_1 \Omega \tag{7.12}$$

The eigenmodes of the coupled resonator structure are spaced evenly in *K*-space with values ranging from 0 to $\frac{\pi}{L}$ [147]. Thus, if the wave vectors are spaced out by

$$\Delta K = \frac{\pi}{(N-1)L} \tag{7.13}$$

the frequency spacing of the modes is approximated by

$$\Delta \omega \approx \Omega \kappa_1 \frac{\pi}{(N-1)} \sin(KL)$$
(7.14)

This means that the eigenfrequencies should be closely spaced toward the edges of the band and spread out more toward the middle. However, when a relatively small number of cavities is present, the separation is less easily expressed, and the average spacing can offer greater meaning.

Based on the derived group index, we can calculate the time required to traverse a single resonator:

$$\tau = \frac{Ln_G}{c} = \frac{1}{\Omega\kappa_1}$$
(7.15)

Consequently, the leakage out of one resonator into the next, which corresponds to its effective Q-factor, can be expressed in the following manner [148]:

$$Q_{eff} = \Omega \tau = \frac{1}{\kappa_1}$$
(7.16)

The overall Q-factor for a resonance of the entire device is also easily obtainable. The transit time through the device is simply the single-cavity lifetime, τ , scaled by the number of cavities, *N*. Thus, the Q-factor for a device resonance line may be expressed as

$$Q_{dev} = \frac{N}{\kappa_1} \tag{7.17}$$

This derivation does not directly account for radial losses out of the overall structure or potential intrinsic absorptive losses which may be expressed in terms of an intrinsic Q-factor. A more detailed accounting for these losses may be found elsewhere [148], though it is worth noting that so long as the intrinsic cavity lifetime is substantially larger than the lifetime due to coupling (τ from Eqn. 7.15 above), which occurs when $Q_{int} >> Q_{eff}$, the effective Q-factor will dominate in the expression of overall response function. In other words, as long as signal energy leaks out of the device significantly slower than it is allowed to move between successive resonant cavities, the cavity coupling dominates the overall filter response.

7.3.2 Coupling of Low Q-factor Resonators

This analysis has assumed relatively large Q-factors for the coupled resonators. The standard coupled-resonator optical waveguides (CROW) approaches make use of either ring

resonators [146, 149] or photonic crystal (PC) [148, 150] structures. One limitation of both approaches is their confinement to a two-dimensional plane: the filters may be tiled in only a single direction. A more flexible approach would enable free-space optics to couple incident light into a two-dimensional array of three-dimensional optical filters.

The simplest approach to this problem is to fabricate an array of cylindrical coupledresonator filters. Unfortunately, simple cylindrical dielectric resonant cavities tend to have relatively low Q-factors. This introduces a number of variations to the previous analysis. First, and most obviously, the low Q-factor means that energy is not stored in individual resonators for as long, meaning that the overall structure delay is expected to be significantly reduced. Additionally, the large frequency bandwidth of the resonators means that the filter transmission band will be significantly larger than if high-Q resonators are used. However, as noted above, the Q-factor for device resonances can be expected to scale with the number of resonators used.

Another issue results from the low modal confinement. This leads directly to much higher coupling than one would expect from high-Q filters. From Eqns. 7.8 and 7.9 we expect to find multiple spectral orders in the transmission and reflection bands. On the other hand, as the coupling strength is reduced, one may reasonably expect fewer and narrower resonance lines. As an example, consider an isolated (in air) cylinder of SiON (refractive index 1.936) with radius 400nm and height 300nm. The resonant wavelength is approximately 1.4um and the Q-factor is approximately 10. The amplitude response for a stack of 16 uniformly spaced identical cylinders as the center-to-center spacing varies from 400nm to 600nm is demonstrated in Figure 7-12. Notice the widely separated bands that close in and narrow as the separation increases and the coupling constant decreases.



Figure 7-12: Amplitude Response of 16 Coupled Cylindrical Cavities Separated by (a) 100 nm, (b) 200 nm, and (c) 300 nm.

A final consideration for these 3-D structures involves the regions separating the resonant cavities. In the case of ring resonators and PC cavities there is no guiding provided in the separating regions. However, if we are making use of pillars composed of layers of different dielectric materials, it is likely that certain ranges of structure geometries will provide substantial confinement and guiding in the regions between cavities. Such structures will still have CROW aspects but will additionally act, in part, as Bragg grating structures. This means that, in addition to the resonance peaks, we can expect large band rejection regions corresponding to the various Bragg grating orders in the transmission spectrum. While this complicates a full analysis of the structures, it also suggests a means for eliminating some of the transmission lines introduced by strong coupling between resonators.

7.3.3 GaAs/AIAs Cavity Filter

As the basis for a coupled resonator optical filter, we begin with a cylindrical GaAs cavity (refractive index 3.5) supported by an AlAs post (refractive index 3.0) with a slightly smaller radius (as shown in Figure 7-13). The cavity was optimized for a resonance near 1.5um, and the resulting radius was 383nm and the cavity length was 523nm. The AlAs post radius was

200nm, which has a cutoff for propagating modes at approximately 1.5um. The resulting structure had a resonant wavelength of 1.48um with a Q-factor of 6. A radial cross section of the resonant azimuthal (TE_{0mn}) eigenmode at this wavelength is illustrated in Figure 7-14(a), while Figure 7-14(b) demonstrates the filter's amplitude response given an incident source based on the results of the high-NA coupling described above.



Figure 7-13: Cross-Section and 3-D Profile of GaAs/AlAs Filter Unit Cell.



Figure 7-14: (a) Resonant Eigenmode and (b) Amplitude Response of GaAs/AlAs Filter.

7.3.3.1 Multiplexed Filter Device

It is readily apparent that a significant portion of the incident energy is scattered radially out of the filter, and only light within a specific band passes through the structure to any significant degree. The more interesting case occurs when we begin to build a coupled network of these structures as is illustrated in Figure 7-15. In this case the center-to-center spacing between cavities was 923nm (pillar length was 400nm).



Figure 7-15: Multiplexing of GaAs/AlAs Cavity Filter.

As was outlined above, several different effects begin to occur. First, the subsequent transmission of a filtered signal from one cavity to the next results in a steady narrowing of the transmission peak. Additionally, the alternating regions of high and low effective index produces a Bragg grating effect, which means we should expect to see a strong reflection band in the amplitude response as the number of cavities grows. Finally, strong coupling between the cavities results in a splitting of the resonant modes. This produces multiple closely-spaced peaks in the structure response function. The amplitude response of a structure with (a) 4 and (b) 64 cavities is shown in Figure 7-16. All three of these features are clearly evident in the response curves.



Figure 7-16: Amplitude Response of a Device with (a) 4 and (b) 64 Cavities.

7.3.3.2 Bragg Reflections and Cavity Spacing Dependence

The Bragg reflections may be explained quantitatively if we look at the effective index of the propagating modes in the two different filter regions. As is shown in Figure 7-17, the support post operates fairly close to cutoff in the region of interest. This suggests a subtle connection to the Nano-DAWG filters explored previously and might imply that dispersive effects could play an important role in these structures. Additionally, note that in the wavelength band near the Bragg reflection peak (centered at approximately 1.37um), the effective index of the post is about 1.2, and that of the cavity region is roughly 3.0. This results in a Bragg wavelength of 4.14um, which has a third diffraction-order reflection peak at 1.38um.



Figure 7-17: Effective Index of Propagating Mode in (a) Cavity and (b) Post Regions.

Thus, the effective index approach to the calculation of the reflectance spectrum is in agreement with the observed values. It is also suggestive of a means to tailor the response function; if the spacing between cavities is reduced, two effects will occur. First, the cavities will couple more strongly, resulting in a larger separation between eigenfrequencies, which should lead to a wider transmission bandwidth. Additionally, the Bragg reflection will shift to shorter wavelengths. For a slight increase in cavity spacing, the opposite will occur for both phenomena. Based on the effective index calculations, it is reasonable to expect that a 50nm shift in the spacing will yield a shift in the third diffraction-order Bragg reflection peak of roughly 40nm. Amplitude responses for cavity spacings of (a) 100nm, (b) 375nm, (c) 425nm, and (d) 475nm are shown in Figure 7-18.



Figure 7-18: Amplitude Response with Cavity Spacing of (a) 100nm, (b) 375nm, (c) 425nm, and (d) 475nm.

7.3.3.3 Resonance Shift and Cavity Size Dependence

A final interesting perturbation that may be applied to the cavity geometry to tune the overall response involves a variation in the cavity radius. This is a bit more complex than the cavity spacing situation, as it directly changes the resonant frequencies as well as the cavity effective index (which will shift the Bragg reflection peaks). Figure 7-19 shows the change in the filter response when the radius is decreased or increased by 26nm. It is very clear that the resonant wavelength shifts by about 0.7um in each case while the Bragg reflection band shifts

much less. What is particularly interesting is that in the case of a 357nm radius, the transmission peak moves inside the reflection band. The dramatic change in the amplitude response for a relatively small change in the cavity geometry indicates that these structures require very careful tuning to obtain a desired response function, though it is also highly suggestive of a possible application. Specifically, an optically active material in the cavity regions could receive a small electrically- (or temperature-) induced variation to its refractive index (which offers a similar effect as a change in cavity radius) and could easily switch the resonant transmission line on and off by pushing it into or out of the reflection band.



Figure 7-19: Amplitude Response for Cavity Radius of (a) 357nm and (b) 409nm.

7.3.3.4 Feasible Structure Response

While the structure response has been demonstrated and evaluated with 64 cavities present, such a device is extremely difficult to fabricate. Since the period is nearly 1um, the structure would consist of an 800nm diameter free-standing pillar nearly 60um tall. This would severely stretch the bounds of reasonable fabrication processes and is therefore unrealistic. A more appropriate design would consist of 16 periods. Such a structure would be slightly less
than 15um tall. While this is still difficult to create, it is not impossible (and a slightly shorter structure consisting of 12-14 periods would not have a significantly different response function). The amplitude response of this structure calculated via (a) MOL and (b) FDTD is shown in Figure 7-20. Note that the time domain requires a much lower spatial resolution to perform the calculations in a reasonable time frame and is thus not quite as accurate. However, the agreement between the two simulations offers some reassurance that the spectral results are correct. The time domain simulation is also capable of plotting the single-frequency CW field inside the structure at peak transmission, as shown in Figure 7-21.



Figure 7-20: Amplitude Response for 16-Cavity Filter Calculated via (a) Method of Lines and (b) Finite Difference-Time Domain.



Figure 7-21: Radial Cross-Section of Resonant Frequency Field Intensity in Filter.

The FWHM bandwidth of the transmission line is approximately 30nm, which corresponds to a device Q-factor of 50. While this is not particularly high in terms of spectral selection, the nature of the structure is suggestive of reasonable delay line functionality. The magnitude and phase of the transmitted and reflected modes are plotted in Figure 7-22. The transmitted signal has a reasonably flat phase across it, which could be suitable for calculating phase delay as a function of frequency.



A better approach is to apply a Hankel Transform to both the incident and transmitted signals. The transform pair takes the form

$$F_{\nu}(k) = \int_{0}^{\infty} f(r) J_{\nu}(kr) r dr$$

$$f(r) = \int_{0}^{\infty} F_{\nu}(k) J_{\nu}(kr) k dk$$

(7.18)

where F_v is the transformed signal and J_v is a Bessel function of the first kind with order v. This approach is the cylindrical analog to using the Fourier Transform to express a signal in terms of plane waves. Here we express the signal in terms of an infinite sum of Bessel functions, which is appropriate for cylindrical coordinates since the solutions to the wave equations naturally take this form. For azimuthal polarizations, it is appropriate to make use of first-order Bessel functions and refer to the propagating Bessel-wave with peak amplitude. The frequency-dependent variation of the phase on this wave provides an adequate means of estimating the structure's dispersion. Figure 7-23 shows the amplitude response and group delay for (a) the transmitted and (b) reflected signals. Note that the delay in the reflected signal grows quite large at the band edges, while the transmitted signal delay is roughly flat across the entire bandwidth as was previously predicted and observed in the case of ring-resonator and photonic crystal CROWs [149, 150]. From Eqn. 1.9 we may estimate the group index of the structure to be 8.1. By reducing the inter-cavity coupling and increasing the Q-factor of the individual resonators it is reasonable to expect that the delay could be increased significantly.



Figure 7-23: Amplitude Response and Group Delay for (a) Transmitted and (b) Reflected Signals.

7.3.4 Coupled High-Q Cavities Filter

For a variety of other applications it is desirable to design structures with significantly increased group delay over a fairly small transmission bandwidth. To obtain such structures without sacrificing the ease of fabrication and the ability to multiplex them in a two-dimensional

array, it is useful to return to micropost cavities composed of monolithic pillars of alternating dielectric layers (in the form of DBRs for confinement purposes) surrounding a small cavity layer [35, 131]. These structures do not require any exotic fabrication methods beyond standard deposition, lithography, and etching. Further, coupled cavity filters may be formed by adding thicker cavity layers after a specified number of DBR layer pairs as illustrated in Figure 7-24. The principle fabrication concern involves the aspect ratio of the resulting pillar. As additional cavities are stacked, the pillar height grows, which can pose a substantial challenge for accurate fabrication processes.



Figure 7-24: Coupled Cavity Dielectric Pillar Filter.

7.3.4.1 Single Cavity Response

Resonant cavity design largely follows the method outlined earlier in this chapter. Based on characterized fabrication processes, we chose to use dielectric films with refractive indices of 1.936 (SiON) and 1.4496 (SiO₂). Additionally, the large index contrast offered higher reflectance and confinement which provided higher Q-factors without requiring extra DBR layers. The pillar radius was set to 1um, and the optimal cavity layer thickness was found to be 0.53um. This results in a TE_{012} resonant mode at 1.55um. With this operating wavelength, one would normally expect the quarter-wave thicknesses for the two Bragg layers to be 200nm and 267nm. However, the narrow structure diameter decreases the effective index of the various layers substantially. To compensate and obtain the desired reflectance, the layer thicknesses were increased to 245nm and 328nm respectively. Figure 7-25 demonstrates the amplitude response of this cavity with (a) 4, (b) 6, and (c) 8 DBR layer pairs on either side of it. Clearly, as DBR layers are added, the bandwidth of the resonance line decreases, resulting in an increased cavity Q-factor. The calculated value of the Q-factor for each case is (a) 65, (b) 210, and (c) 530. Note that the response is based on coupling a realistic source condition into the resonator and back out again. Thus, the reduced reflectance ceiling is due to scattering losses as the incident signal couples to a propagating mode.



Figure 7-25: Single Cavity Amplitude Response with (a) 4 DBR Pairs; (b) 6 DBR Pairs; and (c) 8 DBR Pairs.

7.3.4.2 Coupled Resonator Structure

To form a coupled cavity system, the structure described in the previous section receives an additional cavity and set of DBR layers directly on top. Thus, the 6 DBR coupled structure has two cavities separated by 6 DBR layer pairs with additional sets of 6 pairs on either end for a total of 21 layers (the middle DBR section requires an extra low-index layer for symmetry). Note that the coupling constant in the case of the single cavity (which described coupling the incident signal to the structure) is now identical to that describing coupling between the cavity pairs. Thus, based on the analysis of section 7.3.1, we should expect the two cavity structures to have Q-factors approximately double that of the single cavity case, and the spacing between resonances to decrease as additional DBR layers are used since the resulting coupling constant is reduced. Figure 7-26 illustrates the amplitude response of the two cavity structures. (a) shows the 4 DBR layer pair case and has a resonance spacing of 46.3nm and an average Q-factor of 130; (b) has 6 DBR pairs, a spectral separation of 24.9nm and an average Q-factor of 450; in (c) the 8 DBR pairs case has a wavelength separation of 14.5nm and a Q-factor of 1000.



Pairs; and (c) 8 DBR Pairs.

Although these results have merit, a more applicable structure would have significantly more cavities present. Thus, we extend the structure pattern and demonstrate the response for 4- and 8-cavity devices in Figure 7-27. Notice that the structure bandwidth matches closely with that of Figure 7-26(b). In this case the average Q-factors were 1080 and 2000 respectively, and the average resonance spacings were 13.8nm and 7.0nm. This matches well with the theoretical

analysis outlined above. The Q-factor scales linearly with the number of cavities present, while the line spacing is inversely proportional to the same.



Figure 7-27: (a) 4 Coupled Cavities and (b) 8 Coupled Cavities Amplitude Response with 6 DBR Layer Pairs.

7.3.4.3 Stronger Coupling for Flat Transmission Bands

While the results thus far have matched well with theoretical predictions and offer combfunction transmission filters suitable for a variety of applications, we are specifically targeting a device with a relatively broad transmission band and extremely high group delay. Unfortunately, by making the inter-cavity coupling constant the same as the end-coupling constant we have prevented the transmission peaks from overlapping. In other words, as additional DBR layers were added to separate the cavities, thereby reducing the inter-cavity coupling and narrowing the free spectral range (FSR), the single-cavity Q-factor correspondingly increased and the transmission lines narrowed. A better approach would allow one to tune the coupling constant independent of the intrinsic cavity characteristics. To this end, we began by inserting a low-index (1.4496) layer midway between each of the resonance cavities and attempted to optimize this layer thickness to obtain a relatively flat transmission band. Unfortunately, this had the undesired effect of creating an additional set of resonance cavities. The overlapping set of coupled resonances resulted in a rather convoluted filter response. A better approach would involve a separation layer that only supported leaky modes, thus reducing the chance for a secondary Fabry-Perot effect to occur.

The new design had a pillar radius of 0.65um and a cavity layer thickness of 0.525um. The low-index DBR layer had a refractive index of 1.65 and a thickness of 288nm while the high-index (1.936) layer was 245nm thick. The separation layer had a refractive index of 1.4496 and a thickness of 0.3um. With 10 DBR layer pairs present, the single-cavity structure had a resonance at 1.521um with a Q-factor of 470. The amplitude response of the structure with (a) 1, (b) 2, (c) 4, and (d) 8 coupled cavities is shown in Figure 7-28. Note that the transmission band is relatively flat without significant ripples due to the additional cavities present. As cavities are added, the band edges become much sharper in like manner to Bragg reflection gratings.





Constant.

Since the bandwidth for these structures is constant, obtaining a Q-factor for the structure is not immediately intuitive and has limited meaning besides. However, the characteristic of interest is the structure's group delay, which is shown in Figure 7-29. Notice that the delay peaks strongly at the band edge and is reasonably flat, though somewhat reduced, across the middle. Based on the layer thicknesses and the calculated delay, Eqn. 1.9 gives a group index of 16 for the structure near the middle of the transmission band and 24 for frequencies at the edge of the band. However, it is reasonable to expect that a minor increase in the spacer layer thickness should substantially increase the delay and the structure's group index. Note though that this degree of delay compares quite favorably to that obtained by other CROW structures. For example, a CROW consisting of 12 ring resonators offered a total delay of 110ps for a group index of 23 but with a bandwidth of less than 0.2nm [149].

7.3.4.4 Increased Mode Confinement for Higher Group Delay

The delay for the demonstrated pillar cavities is substantial, but one would reasonably expect that it could be increased further if the Q-factors of the individual cavities could be substantially increased. Section 7.1.4 outlined a means for increasing modal confinement and demonstrated a marked improvement in the Q-factors for such structures. Specifically, encasing the pillar structure radially in a highly-reflective layer dramatically increases its Q-factor.



Figure 7-29: Group Delay for Reduced Coupling Constant Structures.

To exploit this effect, I used the same basic pillar structure surrounded radially by a PEC layer. Similar results should be attainable from realistic reflecting structures such as highly reflective metallic films (especially in the microwave region of the spectrum) and the radial Bragg gratings discussed earlier. I returned to the 1.936/1.4496 index layers design and re-optimized the cavity radius and thickness and the overall thickness of the Bragg layers to obtain a high-Q structure resonant near 1.5um. The resulting structure (illustrated in Figure 7-30) had a diameter of 1.6um, a cavity thickness of 450nm, and DBR layer thicknesses of 271nm and 362nm. With 8 DBR pairs on either side of the cavity, the single-cavity pillar resonated at

1.469um with a Q-factor of 11,000. Even with only 6 DBR pairs present, the structure still had a Q-factor of 1500. The single cavity amplitude response for both cases is shown in Figure 7-31.



Figure 7-30: High Confinement Couple Cavities.



Figure 7-31: Single Cavity Amplitude Response with (a) 6 and (b) 8 DBR Layer Pairs.

As was noted in the previous section, the transmission band can be flattened out somewhat if κ_{int} , the intrinsic cavity coupling constant (which defines the coupling into the overall filter), is adjusted independently of κ_{eff} , the inter-cavity coupling constant. Figure 7-32 demonstrates the transmittance for 4 coupled cavities separated by 12 DBR layer pairs (L) as the DBRs at each end of the filter, L₀, is adjusted from 5 to 7 layer pairs. The amplitude response over the transmission band is flattest when the intrinsic coupling constant is sufficiently larger than the cavity to cavity coupling constant, that is, when there are twice as many DBR layer pairs between cavities as at either end of the structure. If the intrinsic coupling strength is increased too much, it begins to dominate the overall filter response and becomes a larger factor in the structure bandwidth (which also results in a significant reduction in group delay). Conversely, when the intrinsic coupling is reduced sufficiently below the inter-cavity coupling strength, the transmission lines narrow and the amplitude response becomes bumpy.³



Figure 7-32: Amplitude Response as a Function of Coupling Constants.

³ This issue is less significant when a large number of cavities can be coupled. However, these pillar structures are somewhat limited in their overall length due to fabrication difficulties, and thus, the number of cavities that may be present is severely restricted.

While it appears likely that a finer tuning of the coupling constants could further smooth the transmission curve (as was explored in the preceding section), setting the cavity spacing to be twice the number of DBR pairs surrounding a single cavity provides reasonable results. Under this assumption, the phase response for coupled cavities separated by 10, 12, 14, and 16 DBR layer pairs was calculated. The results are summarized in Table 7-1 and illustrated in Figure 7-33.

Table 7-1. Summary of TEC-Encased Coupled Cavities Thase Response.						
	Cavity Spacing	Single Cavity	Approx. 8 Cavity	Group Index	Approx.	
		Length (um)	Delay (ps)		Bandwidth (nm)	
	10 DBR pairs	6.512	8	46	2.5	
	12 DBR pairs	7.779	20	96	0.93	
	14 DBR pairs	9.046	50	210	0.35	
	16 DBR pairs	10.312	140	510	0.13	

Table 7-1: Summary of PEC-Encased Coupled Cavities Phase Response.





Figure 7-33: Phase Response of Coupled Cavities Spaced by (a) 10 (b) 12 (c) 14 and (d) 16 DBR Layer Pairs.

7.3.4.5 Tolerance to Pillar Sidewall Angles

One of the most significant difficulties in the fabrication of pillar structures is the introduction of nonzero sidewall slopes. When we are considering such narrow resonance lines, perturbations in the sidewalls can produce significant variations to the output spectrum. Indeed, section 7.3.3.3 demonstrated that a shift of 50nm in the radius changes the transmission spectrum dramatically.

To quantify the effect for high-Q cavities, I made use of the 8-DBR design presented in section 7.3.4.2. The cavity-to-cavity spacing for that structure was 4.7um, which meant that a 0.5° sidewall angle shifted the effective radius of successive cavities by 41nm. In addition, there are two other elements affecting the structure response. First, as noted in section 7.3.4.1, the reflection band for a given set of DBR layers is dependent on the pillar diameter. Thus, a substantial variation in diameter will adversely affect the ability of the DBRs to confine the intended resonance lines. Likewise, the propagation direction has a strong influence on the filter

response. Specifically, when light is incident on the narrow end of the pillar, the first cavity will reflect much of the longer wavelength band which would otherwise be transmitted through the resonance lines of successive cavities. The opposite would be true for light incident on the large end of the pillar. The former case is represented by positive sidewall angles and the latter by negative angles.

Figure 7-34 demonstrates the 2- and 4-cavity filter response due to a $\pm 0.5^{\circ}$ sidewall angle. The variations in cavity radii caused the resonances to spread substantially, especially for the outer resonance lines of the 4-cavity structure. The central resonance lines shifted outward by approximately 1.5nm, while the outer lines moved roughly 9nm. However, the resonance lines for the positive and negative angles did occur at the same wavelengths. The principle effect of the sidewall angle sign was to determine whether the lower or higher wavelength resonances would be better transmitted.



Figure 7-34: Amplitude Response Changes Due to Sloped Sidewalls for (a) 2 Cavity and (b) 4 Cavity Structures.

7.3.5 Three-Dimensional Filters

One of the motivations behind coupled cavity pillar filters is the ability to tile them across a wafer, forming a truly three-dimensional filter. In addition, it is useful to be able to individually tune sections of the array to obtain different transmission bands. This has been obtained in the past via PC structures [151]. However, this work suggests another alternative. The coupled cavity structures (particularly the high-Q structures demonstrated in section 7.3.4.4) offer filter functions with high group delay and extremely narrow bandwidth. By simply adjusting the pillar diameter, one may obtain a shift in the transmission band. Calculations for the PEC-encased pillar with cavities separated by 12 DBR layer pairs suggest that a variation of 20nm in the pillar diameter (a shift of 1.25%) moves the transmission band by approximately 8.5nm, nearly 10 times the structure bandwidth, without noticeably affecting the phase response. Thus, one could reasonably create a three-dimensional filter with very high group delay and a spatially dependent transmission spectrum.



Figure 7-35: Amplitude Response as a Function of Coupling Constants.

7.4 Microcavity Filters Summary

Much like Bragg gratings, microcavities rely on resonance effects, multiple reflections, and low group velocity to obtain a strong filter response. However, whereas Bragg grating structures are designed around a reflection band, these filters have a narrow transmission line at the resonance wavelength. Additionally, microcavity filters are three-dimensional free-space structures which may easily be placed in a large array for large-scale filtering, while Bragg gratings and dispersive waveguides are confined to the surface of a substrate in a single waveguide.

However, to use such structures as optical filters, one must be able to couple light into and out of them. Since the cavities focused on in this research involved the azimuthally and radially polarized modes in ultra-small regions, we needed a way to convert incident light to these polarization states and focus it to a small enough spot. The optics necessary for both aspects of the coupling method have previously been demonstrated theoretically and experimentally. An autocloned polarization converting element, which takes a linearly polarized beam and converts it to either the TM_{01} or TE_{01} propagating modes, and adequate high-NA lenses have both been fabricated in our facilities. A theoretical analysis of the focusing of the two different polarizations suggests that a lens with an NA of 2.0 focusing through a region with refractive index 3.5 should be more than sufficient for GaAs/AlAs cavities. Slightly smaller NA lenses will also work, though coupling to the resonant modes is somewhat reduced.

This discussion focused on the design and optimization of microcavities in two different configurations. First, we explored an optimization approach to enhance the Q-factors of single microcavities. Through this approach, the geometry was designed and adjusted to contain the optical energy at the resonance wavelength. To better achieve this, the cavity geometry and a set of reflecting layers between the cavity and substrate were optimized using the Particle Swarm approach. The optimized reflecting layers offered nearly a 3-fold improvement over a bare substrate and almost doubled the Q-factor achieved with a simple DBR.

While isolated high-Q cavities are ideal for certain applications, coupled cavities offer a much greater range of filtering capabilities, particularly since relatively low-Q cavities may be chained together in a long sequence to obtain an improved filter response. The filtering capabilities of a chain of GaAs/AlAs cavities was evaluated and discussed at length. The amplitude response is controlled by three different factors: 1) cut-off of the propagating modes; 2) Bragg reflections due to the periodicity of the structure; and 3) weakly coupled resonant microcavities resulting in a narrower transmission line than a single isolated structure. Each of these three features combines to produce the overall response function, and each one depends heavily on the overall filter geometry. In addition to the amplitude response, the phase response was obtained. There is a relatively flat band across the width of the transmission line with a significant group delay. Similarly, the reflection band due to the Bragg effect has a relatively flat delay across its center but has large delay peaks at the edges.

Although the coupled low-Q devices have a relatively wide transmission line and moderate phase delay, the ability to tune the transmission lines into and out of reflection bands is suggestive of a potential switching application. This is especially true in the case of a twodimensional array of independently controlled structures of very similar design. Spatially separated transmission bands could be independently switched on and off with relative ease. On the other hand, high-Q coupled filters offered extremely large group delay magnitudes, albeit over very narrow transmission bandwidths. By separating cavities by DBR layers and further confining the modes through highly reflective radial coatings we were able to obtain a delay-per-unit-length as much as 300 times that offered by the other filter structures explored in this research. While the fabrication tolerances of these structures were shown to be particularly tight, we demonstrated that slight variations in pillar diameters may be exploited over a two-dimensional filter array to achieve a spatially-dependent filter response.

CHAPTER 8 CONCLUSION

8.1 Background

This work developed a body of knowledge essential for the design and optimization of nanostructured optical filters. The basics and overall theory concerning filters in general and the classes specifically investigated in this research were outlined at length in CHAPTER 1. Three types of filters were explored at length and innovations to each category are presented here. We designed and optimized each filter for different target applications and provided a full analysis of the resulting structures and their filter functions.

8.2 Optical Filter Design and Modeling

This research made use of a variety of numerical methods to obtain suitable optical filter designs. The principle modeling methods (described in CHAPTER 3) were frequency-based and incorporated Maxwell's curl equations into a suitable form for a variety of different types of filters. They directly related given filter geometries to actual filter responses for given incidence conditions and offered a means to characterize different optical structures. PSO provided the optimization approach required to adjust the filter geometries iteratively and converge on optimal structure designs. This algorithm was outlined in detail in CHAPTER 4.

8.2.1 Numerical Modeling Tools

Eigenvalue simulations were based on the assumption that the field vectors in Maxwell's Equations could be expressed as traveling waves. The problem geometry was converted into matrix form in such a way that the problem's eigenvalues represented the propagation constants

for the fields. This approach was particularly convenient for long structures with constant crosssections, such as the Nano-DAWGs. While it did not immediately account for coupling and amplitude response for the given structure, it did provide a means for quickly characterizing the group delay and dispersion of the device.

The eigenfrequency simulation was well suited for the design and optimization of optical cavities. It provided a quick and efficient way to obtain the resonance characteristics of a given structure geometry. However, the approach lacked the ability to express the filter response for a given source condition. To obtain a complete structure response, the equations were reformulated in a manner similar to the eigenvalue problems, but were not solved directly for the propagation constants. Instead, this MOL approach expressed Maxwell's Equations in matrix form such that its product with an incident source vector represented either the reflected or transmitted field. This meant that both the phase and amplitude of the reflected and transmitted signals could be calculated for arbitrary source conditions. Such an approach was vital for fully characterizing the coupled cavity filters, and a variation of this method was used to calculate the response of the spiral Bragg structures.

8.2.2 Numerical Optimization Approach

PSO was used in conjunction with the modeling tools to complete the optical filter design. The algorithm was quite simple while also extremely general and was well suited to a wide variety of problems, so long as a well-defined fitness function could be expressed. While the overall optimization speed was not always quite as fast as other approaches such as GAs, the PSO was quite robust and consistently converged to good solutions that the GA was occasionally unable to locate.

8.3 Summary of Results

8.3.1 Nano-DAWGs

CHAPTER 5 described and characterized dispersive waveguide structures termed Nano-DAWGs. These devices consisted of an exotic waveguide cross-section optimized to contain a single TE mode propagating near mode cutoff. The periodicity of the structure coupled with the mode cutoff condition resulted in extremely high second-order dispersion ranging from 5000 to 15,000 ps nm⁻¹ km⁻¹. The actual group delay of the structures was relatively low, particularly when expressed in terms of delay per unit length, though that was not the central motivation for the design.

In addition to characterizing the structures, a method for evanescently coupling light into the waveguides was described. Further, a number of variations to the structures were identified and described, offering specific avenues of exploration for this class of optical filters.

8.3.2 Spiral Bragg Structures

An innovative approach to the fabrication of Bragg grating structures was described in CHAPTER 6. Instead of attempting to directly apply a specific grating period to a given waveguide, the two were described independently of each other. In the examples presented, the grating was given a radial geometry, and a waveguide trajectory was optimized to obtain specific filter responses when used in conjunction with the grating. The linearly chirped structure was 1m long with a bandwidth of 40nm and relative group delay of about 5ns. This corresponds to a constant dispersion of 250,000 ps nm⁻¹ km⁻¹.

The approach offered a few advantages. First, fabricating the structures independently allowed the gratings to more closely approximate the specific design required despite potential

fabrication errors. The magnitude of GDR introduced through e-beam writing of the gratings was quantified to demonstrate the efficacy of this approach. The second advantage of this approach is the extreme flexibility offered in terms of potential filter responses and applications. Specific examples demonstrated in the chapter described both phase responses for dispersion compensation and amplitude responses for WDM filtering applications.

Additional applications included tunable sources and WDM-related structures where very precise relative time delays between frequency components are critical. This research demonstrated that the spiral structures are capable of providing relative delays over a range of 1ps/nm to 2ns/nm.

8.3.3 Coupled Cavity Filters

Microcavity filters were described at length in CHAPTER 7. These resonant structures operate principally as band-pass filters in contrast with the Bragg structures that reflect the spectral band of interest. The initial discussion involved ways to increase confinement and narrow the resonance of a single cavity to produce a high-Q cavity with a specific designed resonance wavelength. PSO was instrumental in optimizing the cavity geometry, and Q-factors of nearly 10⁵ were observed.

Coupling to the cavity structures was obtained through polarization converting elements and high NA lenses previously fabricated by the research group. The coupling characteristics were described analytically and quantified so that additional simulations could make use of realistic incident source conditions.

The discussion also explored the coupling of successive microcavities to obtain ultra-high group delays. The single cavity work was applied to the coupled cavity pillar structure to

increase the intrinsic Q-factors and realize higher delay magnitudes. The structures were fully characterized with realistic incident signals to give a meaningful amplitude and phase response. Although the total group delay was about 140ps for the best structures, the required structure length was only 82um, meaning that the delay per unit was over 300 times larger than that of the other filters presented, albeit over a transmission bandwidth of 0.13nm. Also, by slight adjustment of the structure diameter the transmission band can be shifted significantly without affecting the group delay profile.

8.3.4 Filter Comparisons

This research has explored three different classes of very different optical filters suitable for a variety of different applications. There is a degree of overlap between each of the structures, though they all are focused in slightly different directions and tailed for different applications. Each has a variety of quite divergent characteristics, some of which are summarized in Table 8-1.

	Nano-DAWGs	Spiral Bragg Structures	Coupled Microcavities
Principle Characteristic	High second order	Constant dispersion over	High group delay over
	dispersion over wide	wide reflected band	narrow transmission
	transmission band		band
Group Index	1.9	1.5-105	510
Bandwidth	100nm	40nm (or larger)	0.13nm
Structure Length	100mm+	1m+	82um

Table 8-1: Comparison of Optical Filter Designs.

In addition to the listed distinctives, Nano-DAWGs provide a fairly flexible approach for dispersive waveguides. Coupling is challenging but somewhat straightforward, and one need not be concerned with separating incident and reflected signals.

Conversely, spiral Bragg structures can offer a similar degree of dispersion over a comparable bandwidth. However, the signal is reflected, which requires one to separate it from the source, which introduces an immediate 3dB loss to the system. On the other hand, the approach is extremely general and coupling does not introduce further difficulties.

Finally, the coupled microcavities offer extremely high group delays and a very narrow transmission band. The structures are quite compact and can be arrayed across a wafer to provide a spatial variation to the complete optical filter device. Coupling and fabrication sensitivities are the greatest challenges with these structures, although further work may be able to obviate these issues somewhat.

8.4 Recommendations for Future Work

This work has described, explored, and characterized three different classes of optical filters. However, the work in each area has indicated additional areas of development, several of which have been specifically outlined in the text.

While the Nano-DAWGS offer a unique approach to dispersive waveguide design and has been further explored elsewhere [152], other geometries, such as dissimilar coupled waveguide and trench-bulge structures, offer other approaches that make use of the same ideas. In particular, the trench-bulge concept makes use of the low confinement and periodic nature of the structure in a manner that is much more convenient to fabricate. On the other hand, the evanescent coupling approach offers a means not only to shift optical energy into the Nano-DAWG structure but also to further increase the overall device dispersion.

The decoupling approach to Bragg grating structures is an innovative concept offering a great deal of promise. Specific areas of future work include fabricating and optically testing the

structures and further optimizing the waveguide trajectories for alternative applications. In particular, such structures have the potential to operate in much the same manner as the coupled cavity designs, only with reduced fabrication difficulties. By using a non-constant radial grating function in conjunction with appropriate waveguides, one should be able to obtain resonant regions in the waveguide separated by Bragg reflectors. Further, identical waveguides fabricated at a variety of radial coordinates should offer an array of slightly offset transmission bands with extremely high group delay.

The coupled resonant microcavity structures provide a means of obtaining large group delays over very narrow transmission bands. The fabrication tolerances of these devices are quite narrow, and a particular area for additional work would be to develop a less sensitive geometry that is more easily fabricated. One option would involve rectangular waveguides fabricated along the length of a wafer rather than the described pillar geometry. Correspondingly, the designs should be expanded to other polarization states both for other applications and for ease of coupling to and from the resonant modes. Finally, specific realistic geometries should be defined for increasing the mode confinement and intrinsic cavity Q-factor to replace the PECs used in the high group delay designs.

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