Advances in Natural Science *ISSN 1715-7862* *Vol.2, No.1, 2009*
Canadian Research & Development Center of Sciences and Cultures *30/06/2009*
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Reliability Analysis of a Repairable C (2, 3; G) System with Repair Priority and one is " as good as new " [1]

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Abstract: In this paper, we discuss a repairable linear C (2, 3; G) system. One repairman carries out the maintenance of the system. It is assumed that the working time and the repair time of each component in the system are both exponentially distributed and only one component after repair is as good as new. Each component is classified as either a key component or an ordinary one according to its priority role to the system's repair. We apply the geometric process, supplementary variable technique and generalized Markov process to study a repairable linear C (2, 3; G) system. We obtain Laplace transforms of some reliability indices such as availability and reliability.
Key words: repairable system; generalized Markov process; key component; geometric process

1. INTRODUCTION

The linear or ring C (2, 3; G) system is one simple system in engineering, so it is interested. Kontoleon (Kontoleon, 1980) first studied linear k out of n system in 1980, Chiang and Nin (Chiang & Niu, 1981) further studied k out of n system in 1981. The reliability of the model is interesting for people, and it becomes a active discussion in reliability theory and application. Fang Kui (FANG & LUO, 1998)

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[1] **Supported by the Plan Projects of He Bei Education Office (No.2007323)**
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*Received 18 February 2009; accepted 10 June 2009

studied a special case: ring C (2, 3; G) system with dissimilar components, the all components can be as good as new，and the working time and the repair time of each component in the system are both exponentially distributed, then apply the Markov process to derive some reliability indices. Duan Qiu-shi (DUAN & ZHANG, 1998) studied ring C (2, 3; G) repairable system with dissimilar reparative case. Then Guan Ting-lu (GUAN, 2006) studied linear C (2, 3; G) repairable system with same components and all can not be as good as new, also apply the Markov process to derive some reliability indices. However, some components can not be as good as new after repair, and successive working times of the deteriorating components after repair will become shorter and shorter while the consecutive repair times after failure will become longer and longer.

Based on the paper (FANG & LUO, 1998), we discuss a repairable linear C (2, 3; G) system. It is assumed that only one component after repair is as good as new. Each component is classified as either a key component or an ordinary one according to its priority role to the system's repair. We derive Laplace transforms of some reliability indices such as availability and reliability.

2. DEFINITION AND ASSUMPTION

Definition 1 (GUAN, 2006)：A stochastic process $\{X_n, n = 1,2,\cdots\}$ is a sequence of independent non-negative random variables. If the distribution function of $\{X_n, n = 1,2,\cdots\}$ is $F(a^{n-1}t), n = 1,2,\cdots$, and if a is a positive constant, then $\{X_n, n = 1,2,\cdots\}$ is called a geometric process.

Definition 2 (GUAN, 2006)：A repairable system is in failure state, the system begins working over again after a fault component is repaired, then the component is called key component, otherwise it is called common component.

Assumption 1：Linear C (2, 3; G) repairable system consist of three dissimilar components and one repairman, component 1and 2 can not be as good as new, let X_k^i, Y_k^i be respectively the working time and the repair time of the two components in the k th cycle (Assume that the time interval form working to failure, then repair completely of component i is called one circle of component i), the distributions of X_k^i , Y_k^i are given by

$$F_i(t) = F_i(a^{k-1}t) = 1 - \exp\{-a^{k-1}\lambda_i t\}$$

$$G_i(t) = G_i(b^{k-1}t) = 1 - \exp\{-b^{k-1}\mu_i t\}$$

Where $t \geq 0, a > 1, 0 < b < 1, \lambda_i > 0, \mu_i > 0 \quad i = 1,2. \qquad k = 1,2\cdots$

Assumption 2：Component 3 is as good as new after repaired, ε_3, η_3 be respectively the working time and the repair time of component 3. The distributions of ε_3, η_3 are given by

$$F_3(t) = 1 - e^{-\lambda_3 t}, G_3(t) = 1 - e^{-\mu_3 t}$$

Where $t \geq 0, \lambda_3 > 0, \mu_3 > 0$

Assumption 3：The key component has priority in repair.

Assumption 4：The three components of the system are independent, and all components are new at the beginning.

Assumption 5：After system fault, the not fault components do not be fault.

3. SYSTEM ANALYSIS

Now, let $\{N(t), t \geq 0\}$ be the system state at time t. According to the model assumptions, we have

$$N(t)=\begin{cases} 0, & \text{if at time t, three components work; the system works,} \\ 1, & \text{if at time t, the component 1 fails; the system works,} \\ 2, & \text{if at time t, the component 2 fails; the system is shut down,} \\ 3, & \text{if at time t, the compoment 3 fails; the system works,} \\ 12, & \text{if at time t, the compoments 1,2 fail; the system is shut down,} \\ 13, & \text{if at time t, the compoments 1,3 fail; the system is shut down,} \\ 31, & \text{if at time t, the compoments 3,1 fail; the system is shut down,} \\ 32, & \text{if at time t, the compoments 3,2 fail; the system is shut down.} \end{cases}$$

Obviously, the state space is $E=\{0,1,2,3,12,13,31,32\}$, the set of working states is $W=\{0,1,3\}$, and the set of failure states is $F=\{2,12,13,31,32\}$. Although the stochastic process $\{N(t), t \geq 0\}$ is not a Markov process, we can obtain a vector Markov process by introducing a supplementary variable $I_1(t), I_2(t)$. Let the supplementary variable $I_1(t)$ be the cycle of component 1 at time t; $I_2(t)$ be the cycle of component 2 at time t, then $\{N(t), I_1(t), I_2(t), t \geq 0\}$ forms a generalize vector Markov process.

Denote the state probability of the system by

$$P_{ik_1k_2}(t)=P\{N(t)=i, I_1(t)=k_1, I_2(t)=k_2\}, \quad i \in E. \text{ Where } k_1, k_2 = 1,2\cdots$$ 。

4. THEOREM AND RESULT

Theorem 1: the instantaneous availability of the system at time t is $A(t)$, and the Laplace transform of $A(t)$ is given by

$$A^*(s)=\sum_{k_1,k_2=1}^{+\infty}\left[P^*_{0k_1k_2}(s)+P^*_{1k_1k_2}(s)+P^*_{3k_1k_2}(s)\right]$$

Proof:

$$P_{0k_1k_2}(t+\Delta t)=P_{0k_1k_2}\left(1-a^{k_1-1}\lambda_1\Delta t\right)\left(1-a^{k_2-1}\lambda_2\Delta t\right)\left(1-\lambda_3\Delta t\right)+P_{1(k_1-1)k_2}b^{k_1-2}\mu_1\Delta t$$
$$+P_{2k_1(k_2-1)}b^{k_2-2}\mu_2\Delta t+P_{3k_1k_2}\mu_3\Delta t+o(\Delta t) \qquad k_1,k_2 \geq 2$$

Predigest formula:

$$\frac{\left[P_{0k_1k_2}(t+\Delta t)-P_{0k_1k_2}(t)\right]}{\Delta t}=\left(-a^{k_1-1}\lambda_1-a^{k_2-1}\lambda_2-\lambda_3\right)P_{0k_1k_2}(t)+b^{k_1-2}\mu_1P_{1(k_1-1)k_2}(t)$$
$$+b^{k_2-2}\mu_2P_{2k_1(k_2-1)}(t)+\mu_3P_{3k_1k_2}(t)$$

If $\Delta t\to 0$ and $k_1,k_2\geq 2$, we can obtain differential equation:

$$\left(\frac{\partial}{\partial t}+a^{k_1-1}\lambda_1+a^{k_2-1}\lambda_2+\lambda_3\right)P_{0k_1k_2}(t)=b^{k_1-2}\mu_1P_{1(k_1-1)k_2}(t)+b^{k_2-2}\mu_2P_{2k_1(k_2-1)}(t)+\mu_3P_{3k_1k_2}(t)$$

So we can obtain also:

$$\left(\frac{\partial}{\partial t}+a^{k_2-1}\lambda_2+\lambda_3+b^{k_1-1}\mu_1\right)P_{1k_1k_2}(t)=a^{k_1-1}\lambda_1P_{0k_1k_2}(t)+b^{k_2-2}\mu_2P_{12k_1(k_2-1)}(t)+\mu_3P_{31k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t}+b^{k_2-1}\mu_2\right)P_{2k_1k_2}(t)=a^{k_2-1}\lambda_2P_{0k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t}+b^{k_2-1}\mu_2\right)P_{12k_1k_2}(t)=a^{k_2-1}\lambda_2P_{1k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t}+b^{k_1-1}\mu_1\right)P_{13k_1k_2}(t)=\lambda_3P_{1k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t}+\mu_3\right)P_{31k_1k_2}(t)=a^{k_1-1}\lambda_1P_{3k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t}+b^{k_2-1}\mu_2\right)P_{32k_1k_2}(t)=a^{k_2-1}\lambda_2P_{3k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t}+a^{k_1-1}\lambda_1+a^{k_2-1}\lambda_2+\mu_3\right)P_{3k_1k_2}(t)=\lambda_3P_{0k_1k_2}(t)+b^{k_1-2}\mu_1P_{13(k_1-1)k_2}(t)+b^{k_2-2}\mu_2P_{32k_1(k_2-1)}(t)$$

The initial conditions are:

$P_{011}(0)=1;\quad P_{ik_1k_2}(0)=0,\quad i\neq 0;\quad P_{0k_1k_2}(0)=0,\quad k_{1,}k_2$ are not 1 at the same time.

The initial conditions are expressions about $P_{ik_1k_2}(t), i\in E$ when any $k_{1,}k_2$ are 1. So we set that is 0, when some $k_i=1\ (i=1,2)$ and the exponent of a or b less than 0.

Then taking the Laplace transform on the both sides of the above differential equation, and set $A_k=s+a^{k_1-1}\lambda_1+a^{k_2-1}\lambda_2+\lambda_3$

$B_k=s+a^{k_2-1}\lambda_2+\lambda_3+b^{k_1-1}\mu_1\qquad C_k=s+a^{k_1-1}\lambda_1+a^{k_2-1}\lambda_2+\mu_3$

We have:

$$P^*_{0k_1k_2}(s)=\frac{b^{k_1-2}}{A_k}\mu_1P^*_{1(k_1-1)k_2}(s)+\frac{b^{k_2-2}\mu_2a^{k_2-2}\lambda_2}{A_k\left(s+b^{k_2-2}\mu_2\right)}P^*_{0k_1(k_2-1)}(s)+\frac{\mu_3}{A_k}P^*_{3k_1k_2}(s)$$

$$P^*_{1k_1k_2}(s)=\frac{a^{k_1-1}\lambda_1}{B_k}P^*_{0k_1k_2}(s)+\frac{b^{k_2-2}\mu_2a^{k_2-2}\lambda_2}{B_k\left(s+b^{k_2-2}\mu_2\right)}P^*_{1k_1(k_2-1)}(s)+\frac{\mu_3\left(a^{k_1-1}\lambda_1\right)}{B_k\left(s+\mu_3\right)}P^*_{3k_1k_2}(s)$$

$$P^*_{3k_1k_2}(s)=\frac{\lambda_3}{C_k}P^*_{0k_1k_2}(s)+\frac{b^{k_1-2}\mu_1\lambda_3}{C_k\left(s+b^{k_1-2}\mu_1\right)}P^*_{1(k_1-1)k_2}(s)+\frac{b^{k_2-2}\mu_2a^{k_2-2}\lambda_2}{C_k\left(s+b^{k_2-2}\mu_2\right)}P^*_{3k_1(k_2-1)}(s)$$

$$P_{011}^*(s) = \frac{C_k}{A_k C_k - \mu_3 \lambda_3} \qquad P_{311}^*(s) = \frac{\lambda_3}{A_k C_k - \mu_3 \lambda_3}$$

$$P_{111}^*(s) = \frac{\lambda_1 C_k}{B_k (A_k C_k - \mu_3 \lambda_3)} + \frac{\mu_3 \lambda_1 \lambda_3}{B_k (s + \mu_3)(A_k C_k - \mu_3 \lambda_3)}$$

According to the model assumptions, the instantaneous availability of the system at time t is

$$A(t) = P\{N(t) = 0\} + P\{N(t) = 1\} + P\{N(t) = 3\}$$

And taking the Laplace transform on the both sides of the about t, we have

$$A^*(s) = \sum_{k_1, k_2 = 1}^{+\infty} \left[P_{0k_1k_2}^*(s) + P_{1k_1k_2}^*(s) + P_{3k_1k_2}^*(s) \right].$$

Theorem 2：The instantaneous rate of occurrence of failure of the system at time t is $W_f(t)$, and the Laplace transform of $W_f(t)$ is given by

$$W_f^*(s) = \sum_{k_1, k_2 = 1}^{+\infty} [\left(a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \lambda_3\right) P_{0k_1k_2}^*(s) + \left(a^{k_2 - 1}\lambda_2 + \lambda_3 + b^{k_1 - 1}\mu_1\right) P_{1k_1k_2}^*(s)$$
$$+ \left(a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \mu_3\right) P_{3k_1k_2}^*(s)].$$

Proof：According to reference (DUAN & ZHANG, 1999) about formula of $W_f(t)$, we have

$$W_f(t) = \left(a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \lambda_3\right) P\{N(t) = 0\} + \left(a^{k_2 - 1}\lambda_2 + \lambda_3 + b^{k_1 - 1}\mu_1\right) P\{N(t) = 1\}$$
$$+ \left(a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \mu_3\right) P\{N(t) = 3\}$$

And taking the Laplace transform, we can obtain $W_f^*(s)$.

Lemma 1：Let absorb states of $\{N(t), I_1(t), I_2(t), t \geq 0\}$ is $\{\tilde{N}(t), \tilde{I}_1(t), \tilde{I}_2(t), t \geq 0\}$, then the $\{\tilde{N}(t), \tilde{I}_1(t), \tilde{I}_2(t), t \geq 0\}$ forms a stochastic process.

Theorem 3：By the definition, the Laplace transform of availability of the system at time t is given by

$$R^*(s) = \sum_{k_1 = 1}^{\infty} Q_{0k_11}^*(s)[1 + \frac{a^{k_1 - 1}\lambda_1}{s + a^{k_2 - 1}\lambda_2 + \lambda_3 + b^{k_1 - 1}\mu_1} + \frac{\lambda_3}{s + a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \mu_3}]$$

Proof: Set $Q_{ik_1k_2}(t) = P\{\tilde{N}(t) = i, \tilde{I}_1 = k_1, \tilde{I}_2 = k_2\}, i \in E.$

According to probability analysis, we can obtain

$$\left(\frac{\partial}{\partial t} + a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \lambda_3\right) Q_{0k_1k_2}(t) = b^{k_1 - 2}\mu_1 Q_{1(k_1 - 1)k_2}(t) + \mu_3 Q_{3k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t} + a^{k_2 - 1}\lambda_2 + \lambda_3 + b^{k_1 - 1}\mu_1\right) Q_{1k_1k_2}(t) = a^{k_1 - 1}\lambda_1 Q_{0k_1k_2}(t)$$

$$\left(\frac{\partial}{\partial t} + a^{k_1 - 1}\lambda_1 + a^{k_2 - 1}\lambda_2 + \mu_3\right) Q_{3k_1k_2}(t) = \lambda_3 Q_{0k_1k_2}(t)$$

And taking the Laplace transform on the both sides of the about t, we have

$$Q^*_{1k_1k_2}(s) = \frac{a^{k_1-1}\lambda_1}{s + a^{k_1-1}\lambda_1 + \lambda_3 + b^{k_1-1}\mu_1} Q^*_{0k_1k_2}(s)$$

$$Q^*_{3k_1k_2}(s) = \frac{\lambda_3}{s + a^{k_1-1}\lambda_1 + a^{k_2-1}\lambda_2 + \mu_3} Q^*_{0k_1k_2}(s)$$

$$Q^*_{0k_1k_2}(s) = \frac{A_k C_k b^{k_1-2}\mu_1 a^{k_1-2}\lambda_1}{(A_k C_k - \lambda_3\mu_3)B_k} Q^*_{0(k_1-1)k_2}(s)$$

if $k_2 = 1$,

$$Q^*_{011}(s) = \frac{1}{s + \lambda_1 + \lambda_2 + \lambda_3 - \dfrac{\lambda_3\mu_3}{s + \lambda_1 + \lambda_2 + \mu_3}}$$

If $k_2 \neq 1$, $Q^*_{01k_2}(s) = 0$

So we can obtain the Laplace transform of $R(t)$:

$$R^*(s) = \sum_{k_1=1}^{\infty} Q^*_{0k_11}(s)[1 + \frac{a^{k_1-1}\lambda_1}{s + a^{k_2-1}\lambda_2 + \lambda_3 + b^{k_1-1}\mu_1} + \frac{\lambda_3}{s + a^{k_1-1}\lambda_1 + a^{k_2-1}\lambda_2 + \mu_3}]$$

5. CONCLUDING REMARKS

The n-1/n(G) system is researched, when $n = 3$, and the working time and the repair time of each component in the system are both exponentially distributed and repair condition is different, we obtain Laplace transforms of some reliability indices such as availability and reliability. We consider from application, and supply some information for engineer, that make them get expediently some index of reliability, at the same time supply some help for further studying linear and ring C(k,n;G/F) systems.

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