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A Game-theoretic Model for Regulating Freeriding in Subsidy-Based Pervasive Spectrum Sharing Markets

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A GAME-THEORETIC MODEL FOR REGULATING FREERIDING IN SUBSIDY-BASED PERVASIVE SPECTRUM SHARING MARKETS

by

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ABSTRACT

Cellular spectrum is a limited natural resource becoming scarcer at a worrisome rate. To satisfy users' expectation from wireless data services, researchers and practitioners recognized the necessity of more utilization and pervasive sharing of the spectrum. Though scarce, spectrum is underutilized in some areas or within certain operating hours due to the lack of appropriate regulatory policies, static allocation and emerging business challenges. Thus, finding ways to improve the utilization of this resource to make sharing more pervasive is of great importance. There already exists a number of solutions to increase spectrum utilization via increased sharing. Dynamic Spectrum Access (DSA) enables a cellular operator to participate in spectrum sharing in many ways, such as geological database and cognitive radios, but these systems perform spectrum sharing at the secondary level (i.e., the bands are shared if and only if the primary/licensed user is idle) and it is questionable if they will be sufficient to meet the future expectations of the spectral efficiency. Along with the secondary sharing, spectrum sharing among primary users is emerging as a new domain of future mode of pervasive sharing. We call this type of spectrum sharing among primary users as "pervasive spectrum sharing (PSS)". However, such spectrum sharing among primary users requires strong incentives to share and ensuring a freeriding-free cellular market.

Freeriding in pervasively shared spectrum markets (be it via government subsidies/regulations or self-motivated coalitions among cellular operators) is a real techno-economic challenge to be addressed. In a PSS market, operators will share their resources with primary users of other operators and may sometimes have to block their own primary users in order to attain sharing goals. Small operators with lower quality service may freeride on large operators' infrastructure in such pervasively shared markets. Even worse, since small operators' users may perceive higher-than-expected service quality for a lower fee, this can cause customer loss to the large operators and motivate small operators to continue freeriding with additional earnings from the stolen customers. Thus,

freeriding can drive a shared spectrum market to an unhealthy and unstable equilibrium. In this work, we model the freeriding by small operators in shared spectrum markets via a game-theoretic framework. We focus on a performance-based government incentivize scheme and aim to minimize the freeriding issue emerging in such PSS markets. We present insights from the model and discuss policy and regulatory challenges.

I would like to dedicate this work to my parents. Without your support, it would not have been possible. You were always with me to fulfill the dream while fighting against all the odds.

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CHAPTER 1: INTRODUCTION

Day by day, the number of wireless mobile devices is increasing at an astonishing rate. According to [3], the number of cellular devices will hit to 24 billion by 2020. However, allocated spectrum for this kind of devices is finite, i.g., spectrum allocation in the US, Fig. 1.1 and the wireless spectrum is becoming scarce. The radio spectrum is a precious natural resource for our societies. The wireless/cellular service providers license spectrum bands after a competitive auction process arranged by a government authorized organization in different parts of the world such as the Federal Commission on Communications (FCC) in the U.S. Following such competitive process to gain the rights to operate their network on a particular band, the service providers do not want to share their spectrum bands with others unless they are financially benefited. Although we need more spectrum availability as societies, such economic and policy divisions limit our ability to utilize the spectrum.

Several schemes have already been taken to increase spectrum utilization, e.g., dynamic spectrum allocation to secondary providers, data roaming, and data offloading to neighboring networks. These techniques aim to share the spectrum on a *secondary* basis, i.e., only when the primary owner of the band is idle. Though secondary spectrum allocation has increased spectrum utilization, they focus more on providers' incentive than the actual end-users'. Thus, the main notion of spectrum sharing has been to utilize whatever is left from the primary user. Recent efforts are to take this sharing to *primary* level where sharing takes place even though the primary user may be busy. Such "aggressive" sharing seems necessary for optimizing "micro-opportunities" arising in radio propagation and maximizing overall users' wireless experience. For instance, one might go into dark spots even though his operator's coverage has the best overall quality. Such users in dark spots of their own provider could be served by another provider as a primary user, but this requires a new and extensive form of spectrum sharing among providers and users [4].

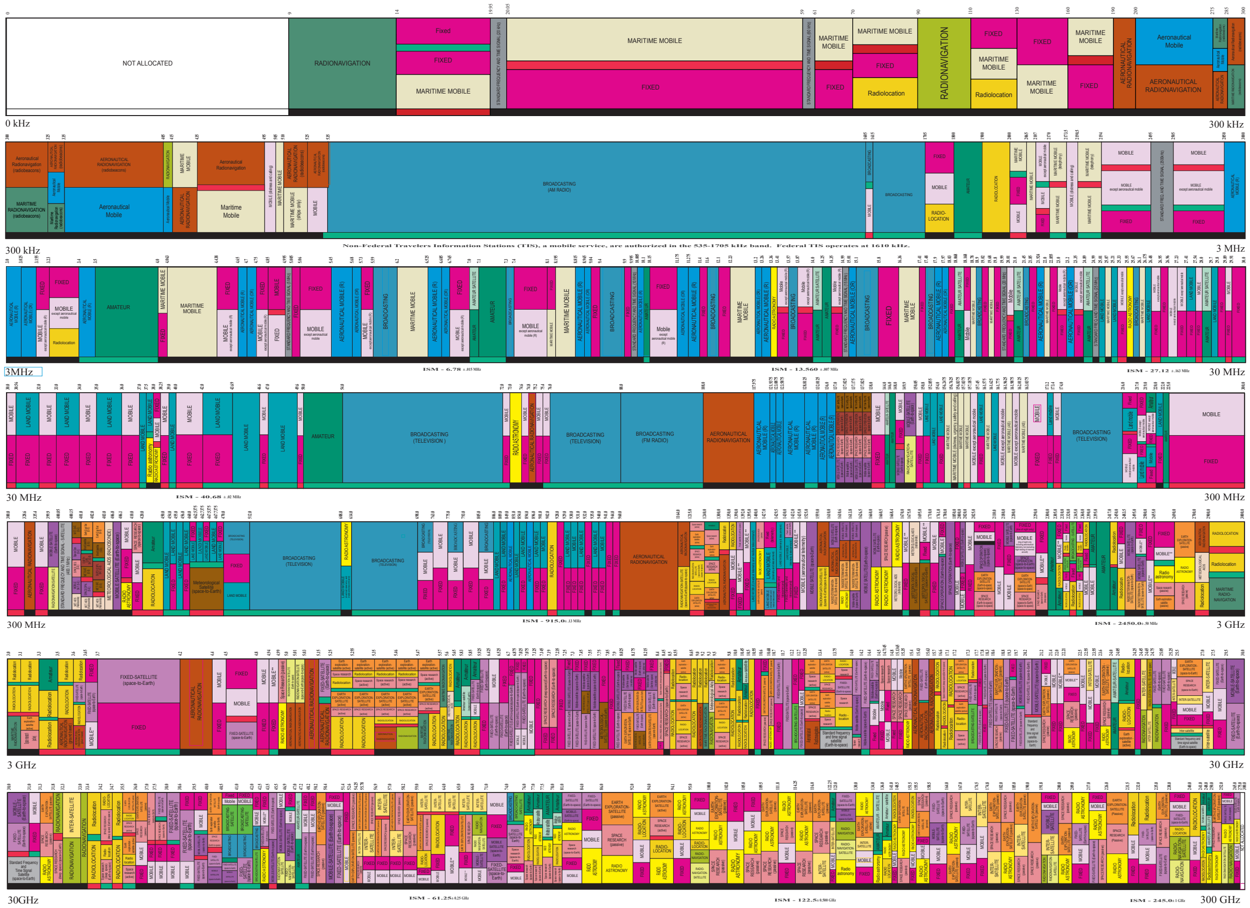

Figure 1.1: Spectrum Allocation in the US [1]

Aligned with these trends, the vision from U.S. National Broadband Plan (NBP) [5] and the direction wireless community is taking point to regimes where sharing is the norm and pervasive [4]. Such notion of pervasive sharing introduces the problem of freeriding. If strong providers are incentivized to share their spectrum bands with the users of weak providers, there may arise a tendency in the weak providers to freeride. In this thesis, we tackle this issue and explore regimes where freeriding can be minimized in spectrum markets shared at primary level.

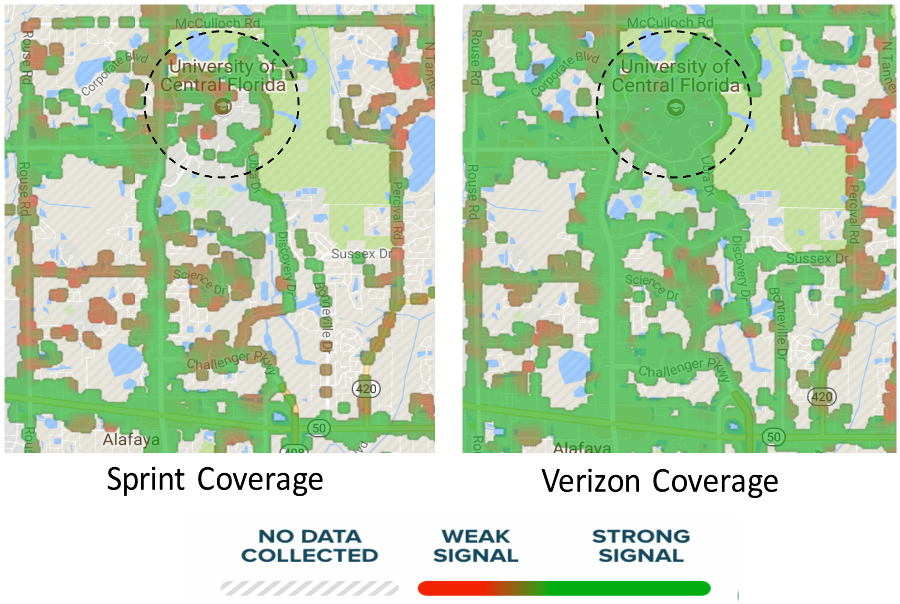

Figure 1.2: Overlapping Coverage Map [2]

A key challenge in primary spectrum sharing is the regulation of such markets. The only economic framework for primary level sharing (or cooperation) among wireless providers is roaming, [6]. A nice feature of roaming is that it does not require any governmental regulation. But, it works well only between providers with significantly non-overlapping coverage. To reap the benefits of spectrum sharing at smaller scales and utilize micro-opportunities for improving user experience, we need incentivized mechanisms for providers that may be competing with each other on overlapping regions of coverage. Fig. 1.2 shows a real-world example of signal strength difference between two providers within a small area. From this figure, we observe that provider Sprint and Verizon has overlapping coverage in University of Central Florida (UCF) area. However, in some places of UCF, a Sprint customer may find himself out of coverage or get weak coverage. At the same time, a Verizon customer is expected to enjoy seamless connectivity because of Verizon's better coverage. If we consider all overlapping areas within the US, we can find a large number of places where such differences are obvious. If there existed some primary level spectrum sharing, customers of all providers may have the opportunity to enjoy the much-improved signal coverage. One way to attain such cooperation among competing providers is to inject some governmental or semi-governmental regulation, although other motivations to improve the “larger good“ (e.g., public safety) do exist.

Recently, the idea of government subsidizing of spectrum bands picked up attention [7, 8]. In one particular spectrum subsidization scheme [7, 9], the government plays a vital role to provide subsidy to the service providers. As providers get subsidy on their allocated spectrum, they get more freedom to improve their infrastructure and can provide better service without charging more to end-users. In this way, spectrum subsidy aims to maximize users’ welfare. Providers get subsidy on the basis of Proof-of-Sharing, where a provider serves clients of other providers whose network coverage is not available or have weaker signal in a certain time and region.

We focus on this subsidy-based spectrum markets to develop a model to minimize freeriding. Despite its benefits in increasing primary level sharing, the subsidy-based spectrum management opens a tunnel for weaker providers to earn more money by relying on strong providers, and hence may negatively affect the healthy competition in the existing market. When a weak provider participates in the subsidy-based market, it can advertise relatively smaller subscription fees compared to the strong providers and can “steal” the customers who would go to the strong providers in a fair market. Since strong providers are motivated to serve the users of the weak providers in return of more subsidy from the government, the weak providers may exploit this situation by offering less-than-fair subscription fees and tend to freeride on the strong providers’ network infrastructure. Further, since strong providers have larger maintenance costs, they will not be able to drop their subscription fees under a certain level and may not prevent their potential customers to be stolen by the weak providers. We address this freeriding issue in spectrum markets heavily shared at the primary level and offer guidelines to providers as well as the government so as to minimize freeriding.

We develop a game-theoretic framework that can predict the strategies which should be taken by a provider in various circumstances of the subsidy-based spectrum sharing (SBSS) markets. A provider can include SBSS market parameters and run the model which will suggest appropriate strategies for that moment. If the initial number of base stations of two providers are 70 and 30 respectively, customers price averseness in the market is 0.9 and marginal signal improvement is 0.8 (all detailed in Chapter 5), our game simulation makes the large provider serve other providers' customers using a pure strategy when its willingness to share is at most 69% whatever the subsidy amount was given. If its willingness to share is more than 69%, it plays mixed strategy to ensure no revenue loss from freeriding. Now, if the strong provider's willingness to share is 0.8 and if the ratio of the small provider's fee to the fair market fee is below or equal to 41%, the large provider can operate sharing its infrastructure at any level, i.e., willingness to share being 1. Again, when the fees ratio is greater than 41%, the game converges to a mixed strategy to ensure market healthiness. Based on the subsidy amount given to the large provider, the mixed strategy varies. If the large provider is given too much subsidy, the game can end up in a freeriding regime. This gives us an idea of how much subsidy can be given to the large provider to avoid a freeriding equilibrium. If there exists a chance of higher signal improvement for both providers, a bit of subsidy can motivate a provider for more sharing (described in Chapter 5). Finally, due to customers' high price averseness, we observe that a spectrum sharing provider is more interested to play mixed strategy.

1.1 Motivations

Existing cellular roaming or DSA is inadequate to provide the benefits of spectrum sharing where a cellular user goes temporarily out of coverage or experiences weak coverage. We have already seen an example, Fig 1.2, where current spectrum allocations cannot provide the service to the customers of different primary providers to get the advantages of spectrum sharing. Hence, we require a spectrum shared market where the providers are motivated to share and the customers enjoy maximum services from the sharing. We need to design a system that allows providers to trust the overall market norms that their customers will not be affected both in terms of numbers and services. Since different providers have various types of investments on the infrastructure, there should be an attractive way of incentivizing them for sharing the spectrum. Without any incentive, no provider will be motivated to share their valuable spectrum. Roaming-like pay-per-use framework may work for such pervasive spectrum sharing markets. However, users are going to pay the roaming charges at the end. If they have to do this for their daily cellular use, they may not find interest in this framework for the long run. So, we need a framework where cellular users will get the advantage of pervasive spectrum sharing without any additional fees and providers are motivated to share spectrum resources. We choose government subsidized spectrum shared market where providers are incentivized based on their sharing. As the subsidy money is coming from the taxpayers, there may arise some questions among taxpayers, e.g., why their tax should be given to the providers? The answer lies in the case where the government will give subsidy only to the providers who have actually shared their spectrum resources with other providers' customers. If a provider doesn't participate in sharing then he won't get any subsidy. If no such regulation exists, providers may charge higher pay-per-use fees and customers are obliged to pay for such services. If the government takes care of giving the incentive of the providers, customers do not need to worry about additional charges. However, this subsidy may create a situation where smaller providers may not find interest to invest in infrastructure improvement as they already know other strong providers will take care of their customers. Also, they may ask lower subscription fees to attract new customers. Which is legal and valid in any free market. No regulatory module can fix the lower bounds of the subscription fees. So, we need an effective framework to control such freeriders while maximizing spectrum sharing. We choose a game-theoretic framework to suggest to the providers' how much sharing should be done for any particular circumstance to avoid the adverse effect of freeriding. We also suggest how much subsidy should be given to the provider who is sharing to avoid freeriding. We aim to maintain market healthiness with maximizing user welfare while minimizing freeriding to motivate providers for spectrum sharing.

1.2 Contributions

Major contributions of this work to minimize freeriding in shared spectrum market at the primary level are as follows:

- We develop a game-theoretic approach to share the spectrum at primary level while minimizing freeriding.
- We formulated case-wise games, and calculated the Nash Equilibrium (NE) strategies and payoffs for each provider in a two-player scenario with one small and one large provider.
- Our game-theoretic model is able to determine the maximum sharing by the large provider without freeriding considering other conditions remain unchanged.
- Our game-theoretic model can prevent or reduce the small provider's freeriding opportunities by adjusting the subsidy given to the large provider.
- Also, to minimizing freeriding, our model sheds light on how to regulate providers' subscription fees (i.e., service prices) with respect to fair market fees.
- The game framework identifies operational regimes where the large provider earns at least its regular earnings (without spectrum sharing) irrespective of its willingness to share and the small provider's strategy to steal customers and freeride.
- Unless both the small and the large provider enters into the freeriding Pure Strategy Nash Equilibrium (PSNE) region (explained later in Section 5) due to the excessive subsidy which

was given to the large provider, we show the existence of operational regimes without freeriding. Before reaching this condition, the large provider can ensure at least its regular earning by playing a mixed strategy or a freeriding-free pure strategy. This condition ensures both providers maintain market healthiness in the shared spectrum market.

1.3 Organization of the Thesis

The rest of this thesis is organized as follows: In Chapter 2, we explore existing literature on spectrum sharing. Here, we cover related works on Dynamic Spectrum Access (DSA) for secondary spectrum sharing and auctioning procedures for such spectrum sharing, freeriding behaviors in networks, roaming models and associated regulatory policies to increase spectrum utilization. How government subsidy can contribute to increase sharing among primary providers without asking additional fees from customers was described in Chapter 3. It shows how a provider can behave maliciously, freeride on other providers infrastructure and trigger unfair customer switching. It also outlines a baseline model of an SBSS market and spectrum sharing among primary operators. Chapter 4 illustrates a game-theoretic model of two providers in an SBSS market. It also describes different scenarios of the game depending on small providers' subscription fees, large providers' willingness to share and associated equilibrium strategies for both providers to control unfair customer switching. Chapter 5 shows the results of this thesis work. Here, we illustrated the equilibrium strategies of both providers based on different subsidy amount. We have differentiated the game regions between freeriding and non-freeriding. We also show, how additional subsidy can restart freeriding game regions. In this chapter, customers price averseness effect on market was also examined in details. Finally, in Chapter 6, we have summarized our work and indicated future improvements on this work.

CHAPTER 2: LITERATURE REVIEW

Spectrum efficiency by any means has become a key concern due to its increasing scarcity. Several advanced methods have been introduced to increase spectrum utilization and the efficiency of spectrum sharing, such as auctions, dynamic pricing, dynamic spectrum access, and a recent form of spectrum sharing among primary operators.

2.1 Dynamic Spectrum Access (DSA) and Auction

Dynamic spectrum access (DSA) with truthful auction mechanism where secondary providers get spectrum in the form of certain fine-grained space-time unit was proposed in [10]. With this spectrum, secondary providers can introduce new service or improve its existing services to meet high demands from customers. Aiming at increasing primary providers' revenue from auctions, the authors discussed several auction procedures, e.g., Vickrey-Clarke-Groves (VCG) mechanism was used to maximize revenue and enforcing truthfulness. They also discussed polynomial time suboptimal auctions where monotone allocations and critical value payment was considered for enforcing truthfulness. The strength of this work was providing high revenue to the sellers which motivates a primary provider to share its spectrum and high spectrum utilization within the whole system.

Due to location-constrained interference, truthful auction becomes challenging. In some cases, it fails to address truthfulness and loss on spectrum utilization. In order to resolve this situation and reduce computation time, [11] described approximate truthfulness and proposed ETEX, a mechanism used to run seal-bid auction mechanism ensuring approximate truthfulness. The polynomial solution of ETEX outperforms some existing auction procedures in terms of user satisfaction, spectrum utilization.

Dynamic spectrum allocation and pricing based on sealed-bid knapsack auction was illustrated in [12]. Unlike to static allocation, service providers are offered a common pool of spectrum resource to bid. Providers bid for spectrum based on the necessity of growing users. Here, they developed a two-tier trading system where providers get spectrum from spectrum owners and provide spectrum services to users. The paradigm shift of static to dynamic spectrum allocation using sealed-bid knapsack auction from a common pool of spectrum resources with the help of a spectrum broker strengthened their work from the rest.

Withstanding interference by primary operators during secondary spectrum sharing was addressed in [13]. It analyzed the sharing in cooperative and coexistent manner. In the coexisting process, a secondary provider is allowed to transmit power without causing any harmful interference by making a query to a sensor network rather than direct communication with the primary network. This sensor network monitors primary's downstream communication and estimate upstream communications. Finally, the sensor network adjusts the interference tolerance level and shares this tolerance data with the secondary network when asked for.

Spectrum sharing with secondary users (SUs) under acceptable interference levels with primary user's receiver (PU-Rx) was introduced in [14]. According to it, a secondary link has multiple transmitters (SU-Tx) and single receiver (SU-Rx). SU-Tx broadcasts random beams to all available primary spectra, after receiving this beams a PU-Rx decides to send back information with interference level. Based on this return information, SU-Tx selects a set of beams with satisfying PUs interference and starts communication with SU-Rx over this beams. During the modeling, authors have considered three cases. Those are, interference level based on magnitude and phase, magnitude only, and a q-bit representation of its magnitude. For q-bit case, it finds the optimal quantizer thresholds and an optimal interference level for each quantization interval in a mean square error (MSE) sense. The strength of this work lies in the opportunistic beam-formation without excessive channel state information (CSI) feedback.

Spot pricing of the spectrum with the presence of nonelastic primary users, and elastic secondary users was discussed in [15]. The view of horizon reward problem and the stochastic dynamic solution of this show an efficient single pricing policy. Based on customer arrival, this model differentiates operating regions for different pricing policies. Their single-price deterministic optimal threshold pricing reaches near to globally optimal price. It uses unimodal profit function for threshold pricing. The weakness of this work is the single-pricing policy. The optimal price can changes over time depending on the channel occupancy which can make the spectrum access less predictable.

End-users' provider selection based on the sum of congestion and price announcement by primary and secondary providers was addressed in [16]. It assumes the congestion of primary provider is caused by its' subscribers and the congestion of secondary providers occurs with the presence of both subscribers.

2.2 Freeriding in Networks

Utility and extended point based freeriding control in peer to peer (P2P) system have been described in [17, 18]. Both research works consider the negative impact of freeriding in P2P file sharing, and they develop a model to study various patterns of file sharing behaviors among sharers and its impact on the community. They model a utility-based freeriding control scheme to provide an incentive to the users who share interesting and popular files. Here, they have considered the total number of files shared, the total size of the data, and the popularity of shared data, and they penalized users based on these. Their work differs from the other incentive-based freeriding control scheme in term of considering parameters to calculate incentive. They determine incentive points after taking care how much time a user spends in the network, his/her upload speed, how many simultaneous uploads are done by a user.

Minimizing freeriding in P2P networks using EigenTrust [19, 20] score has been introduced in, [21]. EigenTrust is a reputation-based metric for P2P networks. It assigns scores to each participating peer. Peers with higher EigenTrust score get additional benefits, e.g., more bandwidth, and more connection time while downloading or uploading data. Peers with a zero EigenTrust score are treated as freerider. If any peer is sharing popular content and a freerider wants to access that data, this model will restrict access to freerider. The strength of EigenTrust lies in determining different types of malicious freeriders where freeriders can act alone or collective manner. EigenTrust also determines malicious spies and camouflaged freeriders with high probability. It is effective against Sybil attack and virus-disseminators.

Freeriding control using passive monitoring neighbor nodes in the P2P network was proposed in [22]. A peer is a 'monitor' and at the same time 'controlled'. As a monitor, it observes neighbors' incoming and outgoing messages, and keeps statistical records of the number of data. The observing neighbor is called controlled peer. Monitoring peer is also being controlled by its neighbors as well. By monitoring each other's sharing amount and message passing record, a peer can decide the freeriding characteristic of its neighbors and take countermeasures to reduce freeriding effect in the network.

Payment-based freeriding control is another way to prevent freeriding in P2P systems. Two decentralized payment methods were delineated in [23], (1) sender uploads the required payments and each intermediate node earns a portion of it when the packet traverses through the node, and (2) each node buys a packet from the previous node and sells to the next node, finally, the receiver pays the total cost.

Agent-based dynamic freeriding was well investigated in [24, 25]. The contribution of one agent is decreasing with the increase of others' contributions. If equilibrium forces one agent to contribute more, then others reduce their contribution by the same amount in the aggregate.

2.3 Regulations for Spectrum Utilization

To meet the growing need of spectrum utilization, regulatory modules around the world (e.g., Federal Communications Commission (FCC) in the US, European Telecommunications Standards Institute (ETSI) in Europe) revise their regulatory policies regularly. Researchers on spectrum regulations are working actively to propose new sharing policies and help these regulatory modules to implement effective regulations for all participating network providers.

European Parliament and Council approved the first Radio Spectrum Policy Program (RSSP) in March 2012, [26, 27]. It aimed to implement two actions: (1) identifying beneficial sharing opportunities (BSOs) where net socio-economic benefits of multiple applications sharing a band surpass socio-economic benefits of a single application, (2) authorizing Licensed Shared spectrum Access (LSA) via spectrum sharing contracts with regulators handing out. A commission was formed to implement this two actions within European Union (EU) which acts as impartial technical advisers and registrars of the contract terms. The LSA users will use available unused cellular spectrum with temporary exclusive rights and role in a specific area.

International roaming is supported by almost all Mobile Network Operators (MNOs). To make data and voice roaming easier for cellular users, the Body of European Regulators for Electronic Communications (BEREC) has launched some regulation proposals, [28, 29, 30]. International mobile data roaming charge overhead on traveling cellular users within the European Union (EU) were analyzed in that proposal. It also analyzes the applicability of flat-rate pricing for roaming to reduce user burden and evaluate the three structural measurements proposed by European Commission (EC). They have considered the impact of competition, wholesale prices, retail prices, operators' investment for network infrastructure and finally, increase spectrum utilization.

The President's Council of Advisors on Science and Technology (PCAST) in the US, proposed a three-tier interference protection (Incumbent, Priority Access License (PAL), and General Authorized Access (GAA)), [31, 32], for utilizing spectrum allocations and obtaining incentive from spectrum market. This proposal allows commercial access of federal spectrum (3550 - 3700 MHz band) while protecting federal operations from interference. It also makes a way for the commercial users to upgrade their own technology to use their intended purpose. This federal spectrum sharing concept was termed as Spectrum Access System (SAS) which coordinates between multiple tiers of spectrum users. Federal ship-borne, radar operations, and fixed satellite services are included in the protected incumbent list. They are the highest tier and are protected from interference due to other users. Any non-federal incumbents are required to register their operation details with FCC or a SAS. PAL and GAA are authorized by SAS to use federal spectrum in specific locations. PAL users are protected from the interference created by GAA. A PAL user can have access up to 70MHz from 3550 - 3650 MHz bands. A GAA user can use the band which is not assigned to any PAL user between this range.

The authors of [33] describe a two-stage pricing policy based on PCAST's 3-tiers interference protection policy. At the first stage, it takes static pricing policy for the specific level of commercial usage. The later stage was done by an optimal dynamic policy for controlling new admissions. This combination works efficiently for spectrum sharing without requiring additional spectrum, makes stable revenues for networks and provides the ability to adopt any change in the network.

Regulating Mobile Network Operators (MNOs) cognitive radio spectrum sharing, a simple rule framework consisting of six themes was proposed in [34]. The themes are: (1) The nature of the opportunity rule, (2) rules for conducting cellular business, (3) boundary rules to identify boundaries of the business, (4) priority ranking rules to differentiate critical to general decisions. (5) timing rules to identify, synchronize and pace things, and (6) exits rules to make the decision for exit or selecting things which should be stopped or given up. These simple rules help dominating and challenger MNOs to reform their business policy, such as a dominator can enhance small cell deployments to provider good QoS and traffic offloading, acting as a cognitive platform provider for other providers and a challenger can focus on specializing in governmental, enterprise customers, special mobile devices, Internet of Things (IoT) based operations to increase spectrum utilization.

Mobile service, technology, network value provisions were illustrated in [35]. To meet the best effort service delivery, operators should have the infrastructure for next-generation communication services such as real-time video. Certain regulatory and operations restrictions should be taken into consideration while allocating additional spectrum to the mobile network operators (MNO). In technology provision, LTE Time Division Duplex (TDD) is well suited for DL/UL asymmetric traffic with small applications though LTE Frequency Division Duplex (FDD) is used as the mainstream cellular networks. For value provisioning, a mobile business model should have the value proposition, revenue model, and architectural value provisioning.

2.4 Spectrum Sharing at Primary Level

Co-primary spectrum sharing in a decentralized way using Gibbs sampling based learning techniques was proposed in [36] with the goal of long-term spectrum sharing among small cell base stations from a common pool of spectrum resources. This Gibbs sampling based learning algorithm provides tenfold throughput gain compared to greedy or equal spectrum sharing algorithms among providers.

With the goal of maximizing social welfare in multi-operator spectrum sharing, [37] proposed a non-orthogonal spectrum allocation which is essentially a combinatorial optimization problem. They adopt a many-to-one form of a matching game to find a stable matching solution which correspondence to local optima. They also use generic Markov Chain Monte Carlo (MCMC) method to reach global optima which ensures the maximum social welfare. From their simulation, they also showed the significance of effective spectrum resource allocation over power allocation for social welfare system.

Spectrum sharing with the inter-operator device-to-device (D2D) communications capability was discussed in [38]. In this work, the authors create a common pool of spectrum resource from all operators' underutilized spectrums and play a non-cooperative game between providers to share their spectrum. They use Jacobi-play strategy instead of the best response for ensuring game equilibrium.

Authors of [39] constructed a protocol of asking and receiving spectrum resources in the form of favors. Any provider with the high load of users can ask favors from low loaded providers. Each of them keeps track of such favors. One favor gainer is expected to return the favor when others needed help. The authors proposed a repeated game to keep track for granting future favors. This work requires no monetary transaction for spectrum sharing and it doesn't reveal any operator specific information, which is the strength of this work.

Bidirectional spectrum sharing in the forms of multiplexing secondary users' communication with both primary and secondary users are described in [40]. The transmission times are divided into two time slots without interrupting spectrum sharing primary users' (PU) communication. In one time slot, a secondary user can act as a relay for primary users' transmission if the destination primary users are out of reach for the sender. On another time slot, an SU can communicate with another SU device as well as with a PU using two different links. Such simultaneous bidirectional communication protocol allows an SU to utilize spectrum more efficiently compared to the existing one-directional scheme without interfering PUs link.

The viability of spectrum sharing in millimeter-wave (mmWave) cellular networks was discussed in [41, 42]. They use cell association, coordination, beam forming, and bandwidth to analyze the effectiveness of such spectrum sharing as an alternate of traditional spectrum sharing. Its' mathematical framework integrates beam formation with the base stations focusing on maximizing throughput with guaranteed load balancing. Inter-operator coordination or without any coordination, this work discussed five key points. It shows (1) the feasibility of inter-operator spectrum sharing with light on-demand intra- and inter-operator coordination at higher mmWave frequencies (e.g. 73 GHz), (2) issues with directional communication (such as multiuser interference), (3) how a large number of antenna elements can be helpful for coordinating and simplifying spectrum sharing implementation, (4) how fair load balancing can be done in intra-operator coordination while neglecting inter-operator coordination in large antenna regimes, and finally, (5) how to protect critical information from the adverse effects of spectrum sharing by implementing critical control messages.

Two cooperative game models to address the user throughput, MNO markets, coalition cost and mobile data pricing was addressed in [43]. It finds a cooperative game between MNOs with sharing unique RAN for gaining spectrum aggregation and cost reduction. In this model, MNOs with large customer base are responsible for a larger fraction of the network cost while MNOs with larger spectrum resource should share the lower cost. Market share-based cost divisions' stability is not always guaranteed. Due to this instability, cost division based on spectrum contribution of each MNO makes this work as a better candidate for cost division policy.

CHAPTER 3: SBSS: SUBSIDY-BASED SPECTRUM SHARING

Earlier research on spectrum sharing has helped to increase spectrum utilization. Still, there remains space for more utilization if the sharing can be done among primary providers. This can increase end-user welfare as well. Though DSA increases the network availability, it considers less of the economic benefits for the users. Due to the roaming agreements between the home provider (i.e., the provider the user is subscribed to) and the foreign providers (i.e., providers the user is not subscribed to), users have to pay more than regular fees, [6]. In the SBSS markets, a *explicit* subsidy/incentive from the government or *implicit* subsidy/incentive from mutually beneficial relationships of providers can contribute to increasing user welfare. The external subsidizing organization can be a governmental, semi-governmental (e.g., similar to ISOs in power grid [44]), or an organization of providers who want to motivate providers to share. The explicit subsidy can come in a form of monetary benefit, low tax, or low license renewal fee whilst the implicit incentive can come in terms of favors, i.e., peering between different providers. In all these cases of SBSS markets, the end users will not need to pay more subscription fees for service when they are in suburban areas or out of their provider's coverage. Further, they will receive a better quality of experience due to micro-opportunities arising in urban settings. Thus, putting the end user's received quality of experience as the top priority is one of the main motivations for our work on SBSS markets. However, this subsidy scheme is still in academic research phase.

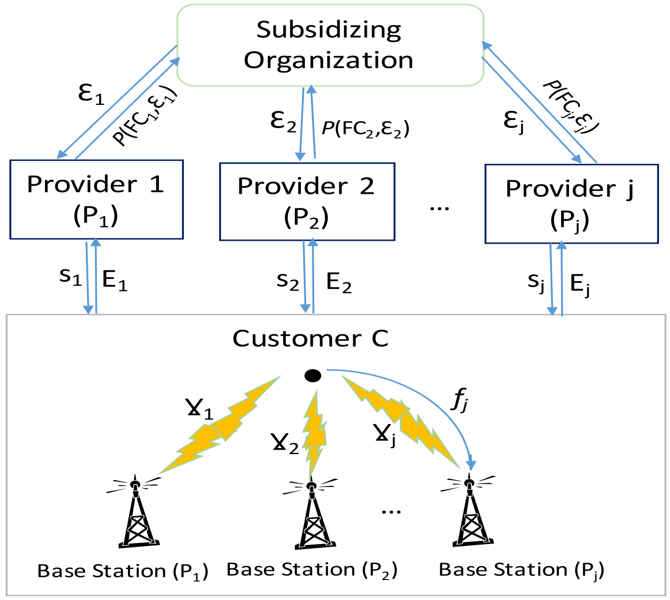

Figure 3.1: An SBSS Market

An SBSS market operating within a region is visualized in Fig. 3.1. Here, the subsidizing organization (SO), customers and the providers have specific roles to play. The customers act similar to the existing spectrum market. They choose providers based on the utility of the offered services. SO subsidizes the providers to share their spectrum with foreign customers (i.e., ones subscribed to another provider) and penalizes based on proof-of-sharing. The providers try to maximize revenue earnings by serving as many foreign calls as they can without hurting their service quality. However, such explicit SBSS can result in a situation that provides an opportunity to the weaker provider to freeride on the stronger providers' infrastructure and *unfairly* attract customers by offering lower-than-fair subscription fees. We will next explain how such unfair customer switching can emerge due to freeriding.

SBSS opens a tunnel for weaker providers to earn more money by relying on strong providers, and hence may negatively affect healthy competition in the existing market. When a weak provider participates in the SBSS market, it can advertise relatively smaller subscription fees compared to the strong providers and can unfairly attract customers who would normally go to the strong providers in a fair market. Since strong providers are motivated to serve the users of the weak providers because of the extra subsidy, the weak providers may exploit this situation and tend to freeride on the strong providers' network infrastructure. Further, since strong providers have larger maintenance costs, they will not be able to drop their subscription fees below a certain level, and may not retain their potential customers who eventually switch to other providers offering lower fees.

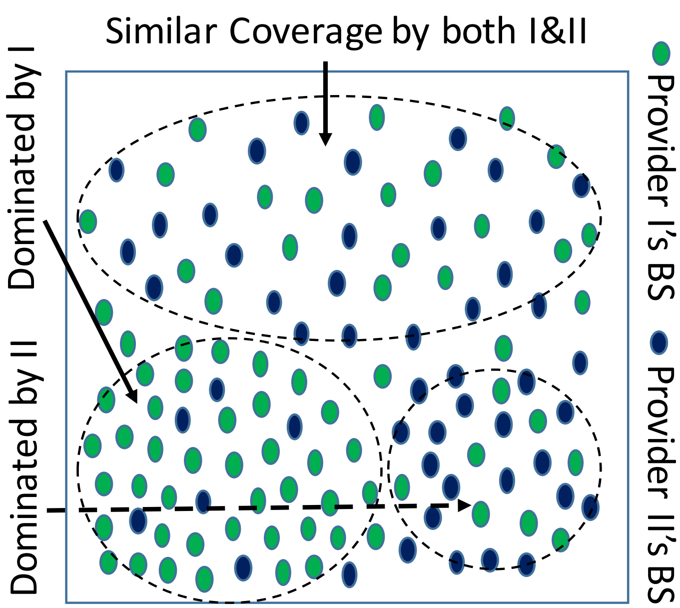

Figure 3.2: Provider's Signal Strength by Coverage Area

In [7], the authors considered the benefits of an SBSS market. However, they didn't analyze the potential risks of the equilibrium in such markets. Let's consider an explicit SBSS cellular market with Providers I and II. Fig. 3.2 shows the regions where these providers' strong coverage exists. Here, each small circle denotes a base station (BS). The dashed circles encapsulate each of the providers' dominated regions. Some regions are equally dominated by both providers while others dominated by one of them. Provider I is, overall, stronger as it has more BS infrastructure than II. In a normal market, Provider I's subscription fees would be higher due to its higher infrastructure costs. Due to the explicit subsidy in the SBSS market, all providers are motivated to serve each others' customers when it is needed.

Let's consider a region where Provider I has better network coverage. If a customer wants to get a *strong* connection, he should subscribe to Provider I in a normal market. In fact, if there was no explicit subsidy, Provider I would retain these customers. However, under the SBSS market, the weaker one, Provider II, can offer cheaper subscription fees due to its smaller infrastructure costs compared to Provider I. Further, Provider I is incentivized to serve Provider II's customers. Thus, if Provider I shares its BSes with Provider II's customers too much, the overall quality of the network service will appear to be similar for both Provider I's and II's subscribers. Then, for the customers, the only difference between two providers will be the subscription fees. This fee difference along with similar service quality will cause *unfair customer switching (UCS)* to weak provider, Provider II. To make it fair, Provider II should increase its fees to fair market levels (i.e., similar to I's fees), but it has no motivation to do so in the SBSS market. Due to this potential revenue loss from freeriding weak providers, no strong provider will agree to join the SBSS market. We address this freeriding and UCS issues in the explicit SBSS markets. We aim to minimize such freeriding to maintain SBSS market healthiness while increasing user welfare via subsidized sharing of the scarce spectrum.

3.2 A Baseline Model for SBSS Market

We assume a set of network providers, denoted by $J = \{1, 2, .., j\}$, competing in the same region. Customers are subscribed to a particular provider in return of a subscription fee, i.e., f_j, for the customers of provider j. For a customer subscribed to provider j, we call provider j as the "home provider" and any other provider, $k \neq j$, $k \in J$, as a "foreign provider".

Our subsidy-based market may result in a customer being served by a foreign provider in addition to its subscribed home provider, depending on the received signal quality from these providers as well as the providers' willingness to share their infrastructure. A customer subscribed to another provider will be treated as a "foreign customer". But, since providers are subsidized via explicit subsidy to share their resources, these foreign customers may get the chance to use a provider's resources even if they are not subscribed to that provider. This will be particularly helpful when a customer is at a spot where its home provider has a weaker signal.

A customer i will choose j as "home provider" based on the overall signal strength/quality, ψ_j, and the subscription fee, f_j, of provider j. Based on these selections (to be detailed next), the number of customers subscribing to provider j can be expressed as:

$$N_j = N(f_j, \psi_j) \tag{3.1}$$

where the demand function $N(\cdot)$ is a decreasing convex function with respect to (w.r.t.) f_j and an increasing concave function w.r.t. ψ_j.

3.2.1 Customer's Provider Selection

During a fixed time period, we assume each customer, i, on average makes γ calls. Among them, $\beta_{j,i} = \sigma(X_j)$ are home calls where $\sigma(\cdot)$ is an increasing concave function w.r.t. the number of base stations, X_j, of its home provider j. The rest of the calls are treated and served as foreign call, $\alpha_{j,i}$. Here,

$$\alpha_{j,i} = \gamma - \beta_{j,i} \tag{3.2}$$

We assume each customer i, on average, makes γ calls during a fixed time period. Among them, he makes $\beta_{j,i} = \sigma(X_j)$ home calls where $\sigma(\cdot)$ is an increasing concave function w.r.t. the number of base stations, X_j, of his home provider j. He also makes $\alpha_{j,i} = \gamma - \beta_{j,i}$ foreign calls.

We model a customer's selection of a home provider via its utility of the service quality. Let a customer's utility function, $u(\cdot)$, be an increasing concave function of the signal intensity/quality of service available to that customer. Then, customer i's overall utility from subscribing to provider j can be expressed as:

$$U(i,j) = \beta_{j,i} u(\psi_j) - f_j \tag{3.3}$$

Based on (3.3), customer i will select provider j as his home provider with the following probability:

$$P(i,j) = \frac{U(i,j)}{\sum_{j=1}^{J} U(i,j)} \tag{3.4}$$

We get this probabilistic provider selection from Contest Theory, [45, 46], where a customer will most likely choose a provider which offers best utility services. We haven't considered other selection criteria, such as: a customers' knowledge on all offered services, brand-name of a provider, etc., we expect a customer will consider only signal quality and fees to choose a "home provider". Thus, a provider would attract more customers, not all of them.

3.2.2 Provider's Revenue Maximization

With the target of maximizing revenue, provider j receives subsidy in addition to its regular subscription fees. Further, it can generate revenue by freeriding on other providers' networks. From (3.4), we can say, for a market with n customers, that provider j will attract $N_j(= nP(i,j))$ customers with a subscription revenue of $R_j(= f_jN_j)$, in a normal market. However, if provider j joins in the SBSS market, it will get the subsidy ϵ_j. From this subsidy, it can spend s_j to improve its infrastructure, e.g., by increasing the number of base stations, X_j. It also has licensed bandwidth, b_j, to run cellular operations. Available bandwidth and the number of base stations determine the signal strength of provider j, which can be expressed as:

$$\psi_j = Q(X_j, b_j) \tag{3.5}$$

where $Q(\cdot)$ is an increasing concave function w.r.t. both X_j and b_j.

Let's assume that provider j serves FC_j foreign calls, which is the proof of sharing their BSes with other providers' customers. The providers are penalized on their subsidy based on the number of foreign calls they served. The higher FC_j, the lower the penalty. We denote this penalty function for provider j as:

$$p_j = p(FC_j) \tag{3.6}$$

where $p(\cdot)$ is a decreasing convex function w.r.t. FC_j.

Based on the provider selection problem (3.4), a customer i of provider j is more likely to choose another provider k if $U(i,k) \geq U(i,j)$. Also, any change of subscription fees can change provider selection criteria along with signal strength, which is the main driving factor behind $U(\dot{})$. In an SBSS market, the service utility received from providers j and k can be similar due to the subsidy for sharing, i.e., $U(i,j) \approx U(i,k)$. In such a case, if $f_j < f_k$, customer i of provider k will more likely subscribe to provider j. Let's assume, by advertising lower-than-fair subscription fees, $f_j < f_k$, provider j can unfairly attract $N_k P_{cs,j}$ subscribed customers of k, where $P_{cs,j}$ is the probability of unfair customer switching (UCS) from provider k to provider j. Now, the revenue maximization problem for provider j becomes

$$\max_{\{s_j\},\{b_j\},f_j} R_j + (1 - p_j)\epsilon_j - s_j + \sum_{k \neq j}^{J} f_j(N_k P_{cs,j} - N_j P_{cs,k}) \tag{3.7}$$

$$s_j \leq \epsilon_j \tag{3.8}$$

$$s_j, b_j, f_j \geq 0, \forall j \tag{3.9}$$

$$f_j \leq f_k, \forall j \tag{3.10}$$

The first term of (3.7) denotes the earnings of provider j from its subscribed customers. The second term describes the leftover subsidy money of provider j, which depends on given subsidy, ϵ_j, and the penalty function, $p(FC_j)$. If provider j does not serve a satisfactory number of foreign calls, it has to return some or all of ϵ_j. The third term is the money which provider j invests from subsidy money to improve its signal quality. Provider j has to carefully invest this subsidy money within its regions of operation to make maximum profit, i.e., to both increase its competitiveness for more subscribed customers and opportunities to serve more foreign calls. The fourth term delineates the earnings from net switching customers.

CHAPTER 4: GAME-THEORETIC MODEL OF A TWO-PROVIDER SBSS MARKET

Let's consider two providers, Provider I and Provider II, under an SBSS cellular market. Here, the maximum number of foreign calls I and II can get from the market is $\sum_{i=1}^{N_2} \alpha_{2,i}$ and $\sum_{i=1}^{N_1} \alpha_{1,i}$ respectively. Both providers can serve up to this many foreign calls and claim more subsidy. Also, they can take a strategic decision to limit serving foreign calls. Let's assume that Providers I and II are willing to serve FC_1 and FC_2 number of incoming foreign calls respectively, where

$$0 \leq FC_1 \leq \sum_{i=1}^{N_2} \alpha_{2,i} \tag{4.1}$$

$$0 \leq FC_2 \leq \sum_{i=1}^{N_1} \alpha_{1,i} \tag{4.2}$$

Again, we consider both providers serve foreign calls with the probabilities ω_1 and ω_2 respectively. Then the total number of foreign calls served by I and II will be $FC_1 = \omega_1 \sum_{i=1}^{N_2} \alpha_{2,i}$ and $FC_2 = \omega_2 \sum_{i=1}^{N_1} \alpha_{1,i}$ respectively. The customers of I and II can choose both providers with probabilities θ_1 and θ_2 when making a call under the incentivized market. Now, the overall signal quality customers of I and II will get, respectively, are:

$$\psi_1^* = \theta_1 \psi_1 + (1 - \theta_1)\psi_2 \tag{4.3}$$

$$\psi_2^* = \theta_2 \psi_2 + (1 - \theta_2)\psi_1 \tag{4.4}$$

We assume that Provider I has stronger coverage, i.e., $\psi_1 > \psi_2$. So, Provider I offers better utility than II, i.e., $U(i, 1) > U(i, 2)$.

If we consider the existing market and assume each provider maintains its existing business policy, then the profit maximization problem for both providers should be:

$$E_1 = f_1 n P(i,1) + (1 - p_1)\epsilon_1 - s_1 \tag{4.5}$$

$$E_2 = f_2 n P(i,2) + (1 - p_2)\epsilon_2 - s_2 \tag{4.6}$$

However, the SBSS cellular market makes a tunnel for the weaker provider to make extra earnings by freeriding on the stronger provider. Under the subsidized market, each provider's offered cellular utility, $U(\cdot)$, could become almost equal, and there may remain only a small perceived difference between cellular providers' service quality. Assuming no changes of previously subscribed customers' fees and considering the possibility of UCS, the revenue maximization problem under the subsidized market becomes

$$E_1 = f_1 n P(i,1) + (1 - p_1)\epsilon_1 - s_1 + f_1(N_2 P_{cs,1} - N_1 P_{cs,2}) \tag{4.7}$$

$$E_2 = f_2 n P(i,2) + (1 - p_2)\epsilon_2 - s_2 + f_2(N_1 P_{cs,2} - N_2 P_{cs,1}) \tag{4.8}$$

From (4.7) and (4.8), if $f_1 \leq f_2$, most of the potential customers will go for Provider I's service in those areas where I has better coverage than II. This is because Provider II offers a similar utility of service, i.e., $U(i,1) \approx U(i,2)$, with higher subscription fees under the SBSS market. So, no freeriding takes place in this case, which is in alignment with the existing market equilibrium. However, if Provider II aims to initiate UCS by lowering its fees, i.e., $f_1 > f_2$, then the market equilibrium changes. Now, Provider II is able to offer similar utility, i.e., $U(i,1) \approx U(i,2)$, with lower subscription fees. This will attract the subscribed customers of I to II. Provider I will lose its customers to II, even though it has a stronger signal. Here, Provider I will get subsidy money for serving these foreign calls. If no subsidy agreement exists, Provider I could consider these earnings as the revenue from subscribed customers. However, depending on the Subsidizing Organization's offerings, this subsidy money can be less than the subscription revenue lost due to II's freeriding. As a result, UCS can demotivate Provider I from the SBSS market. We model the number of unfairly switching customers from each provider as a function, $F(.)$, which is increasing concave w.r.t. the provider's own subscription fees and decreasing convex w.r.t. the other provider's fees. That is, for Providers I and II, we express this as:

$$N_1^l = F(f_1, f_2) \tag{4.9}$$

$$N_2^l = F(f_2, f_1) \tag{4.10}$$

4.1 Two-Provider Game Formation

To model the freeriding problem, we envision a simple 2x2 non-cooperative strategic form game. The two players are a large provider and a small provider. They are competing in an area where the large provider is able to offer much better service than the small provider. Now consider customers not subscribed to the large provider, who venture into this area and wish to make calls. The large provider then has a decision to make: it can either cover such calls or else not cover them. Covering them has the benefit of earning subsidy profit from the Subsidizing Organization, but runs the risk of the freeriding problem described below.

At the same time, the small provider decides whether to "undercut" the large provider, by offering low-fee service to these customers. If the large provider is "covering" these customers' calls, the small provider can freeride, i.e., it enrolls the customers by offering the low-fee service, collects the fees, but doesn‘t have to provide any better service because the large firm is doing so by sharing its base stations with these customers.

		Player II (Small Provider)	
		Low Fees	Fair Fees
Player I (Large Provider)	Cover Calls	(a, b)	(c, d)
	Doesn’t Cover Calls	(e, f)	(g, h)

Figure 4.1: 2x2 Game Model for Freeriding

The situation outlined above is modeled by the simple 2x2 non-cooperative strategic form game in Fig. 4.1. The two strategies for the large provider (or “Player I” or “The Row Player”) are listed on the left, while those for the small provider (“Player II” or “The Column Player”) are across the top. The four possible outcomes are represented by the four cells. Of interest here is the freeriding outcome in the top left (if the large provider “covers calls” and the small provider charges “low fees”). Also note that there is another outcome which the Subsidizing Organization would prefer over the freeriding outcome, namely for the large provider to cover foreign calls, but for the small provider to charge fair market fees and so no freeriding occurs. This is the top right cell of the bimatrix.

The two entries in each cell represent the payoffs for Player I (large provider) and Player II (small provider), respectively, if the players play the corresponding strategies. For example, if the large provider does not cover calls while the small provider charges low fees, the payoff is e for the large provider and f for the small provider.

We now derive expressions for the quantities $a, b, ..., h$. We begin by simplifying (3.7) above to

$$E_1 = N_1 f_1 + (1 - p_1)\epsilon_1 + f_1(N_2^l - N_1^l) \tag{4.11}$$

$$E_2 = N_2 f_2 + (1 - p_2)\epsilon_2 + f_2(N_1^l - N_2^l) \tag{4.12}$$

respectively. For simplicity, we have dropped spending amounts, s_1, s_2, from the payoffs and assumed only Provider I is losing customers to Provider II. As a result, we have $N_1^l > 0$ and $N_2^l = 0$. Also, Provider II cannot earn subsidy money as it's not sharing ($0 \leq p_1 < 1$, $p_2 = 1$) the spectrum.

Table 4.1: Each Provider's Payoff

Symbol	*Value*
a	$N_1 f_1 + (1 - p_1)\epsilon_1 - N_1^l f_1$
b	$N_2 f_2 + N_1^l f_2$
c	$N_1 f_1 + (1 - p_1)\epsilon_1$
d	$N_2 f_2^*$
e	$N_1 f_1$
f	$N_2 f_2$
g	$N_1 f_1$
h	$N_2 f_2^*$

We have denoted the fair market fee or reservation price as f_2^* for Provider II. Alternatively, f_2 is the undercutting low fee Provider II could charge. Using (4.11) and (4.12), we present expressions for $a, b, ..., h$ in Table 4.1.

Provider I and provider II get payoff a and b respectively when provider I is covering the calls and provider II is charging lower fees. This is the case when freeriding occurs and provider I loses revenue due to UCS by provider II. In the same time, I is earning subsidy as it is sharing spectrum and the UCS customers from the provider I are contributing to provider II's revenue. c and d are the payoffs of provider I and provider II respectively when provider I covers the calls and provider II charges fair market fees. This is the desired case when no UCS takes place. As the provider I is sharing spectrum, it continues earning subsidy here. When provider I doesn't cover any calls and provider II charges low fees, the payoffs for the provider I and II becomes e and f respectively. Payoffs g and h refers to the existing revenue earning scenario for provider I and provider II when no sharing takes place and provider II charges fair market fees.

Observing above defined payoffs, we can say $e = g$, $g < c$ and $a < c$ which remain unchanged always. In case of payoffs e and g, provider I doesn't cover any calls. So there remains no opportunity of UCS. Which ensures the equal revenue ($e = g$) earning of provider I in both cases. The provider I earns subsidy by covering calls along with regular subscription revenues when the provider II charges fair fees which results in the payoff c. It earns g when it's not covering any calls and provider II is charging fair market fees which mean no subsidy earning for provider I. From this two conditions, we can say $g < c$. Again, based on our assumption, we know provider I loses customers to provider II due to UCS when provider II charges low fees. From here, we can say $a < c$. Only changing relationship is found between a and e. If $(1 - p_1)\epsilon_1 - N_1^l f_1$ from a results in any positive value then we get $a > e$. This can happen if the government gives too much subsidy to the provider I or the penalty (p_1) of provider I becomes too small compared to the revenue loss due to UCS or both. In this case, provider I can overcome revenue loss occurred by UCS customers. If none of these happen then payoff a becomes smaller than payoff e. Another situation may arise when we observe $a = e$. This can happen if revenue loss due to UCS becomes equal to subsidy earning from spectrum sharing.

From the payoffs of provider II, we find that the unchanged conditions under any circumstances are $f < h$, $h = d$ and $f < b$. As the provider I doesn't cover calls so lowering fees will generate less revenue for provider II which holds the inequality of $f < h$. Again as per our assumption, provider II doesn't share its spectrum, so it earns equal revenue ($h = d$) in cases when Provider II charges fair fees and Provider I covers calls or doesn't. When provider two charges low fees, it will get additional revenues if provider I covers calls compared to provider I doesn't cover. Covering calls of customers of provider II creates the opportunity of additional earning for provider II by triggering UCS which holds the condition $f < b$. The only changing relationship is found between payoffs b and d. When the numbers of UCS customers are large and the fees charged by II are closer to fair market fees then we can observe the scenario where payoff b becomes larger than d. Again, if the numbers of UCS customers are small or the asking fees are too small compared to the fair market fees then we get the opposite relation ($d > b$). Also, $d = b$ can be found. This happens when the loss occurred by lowering fees becomes equal to the revenue earning from UCS customers.

From these, we see that the following relationships necessarily hold: $e = g$, $g < c$, $a < c$, $f < h$, $h = d$, and $f < b$. These relationships are used in all four equilibrium cases detailed later.

We now analyze this game using standard non-cooperative game theory. A mixed strategy for Player I is a two component vector $(p, 1-p)$ in which p ($\geq$0) represents the probability that he will cover all calls and 1-p is the probability he doesn't. Similarly, a mixed strategy for Player II is given by $(q, 1-q)$, in which q is the probability that she charges low fees. Given mixed strategies for each player, it is easy to calculate the expected payoff for each player: $pqa + p(1-q)c + (1-p)qe + (1-p)(1-q)g$ for Player I and $pqb + p(1-q)d + (1-p)qf + (1-p)(1-p)h$ for Player II.

A Nash equilibrium (NE) is a pair of mixed strategies, one for each player, in which both players are maximizing their expected payoff given what the other is doing. It is the most used solution concept in noncooperative game theory, and we use it here to analyze the game above.

Considering uncertain relationships between a, e and b, d, we get four different cases for the game. They are:

Case I ($a < e$ ***AND*** $d < b$***).*** This is the interesting case, with no dominating strategies. The unique NE is for Player I to play $(p*, 1{-}p*)$ and Player II to play $(q*, 1{-}q*)$, where $p* = (h - f)/(b - f)$ and $q* = (c - g)/(c - a)$.

Now observe what happens if the Subsidizing Organization raises the 'reward' to the large provider for covering calls. This raises both a and c by the same amount, say x. For small x, the effect is to raise $q*$, i.e., to make the small provider more likely to undersell. If $x = e{-}a$, there is a continuum of NEs, all with $q* = 1$, i.e., all with small provider underselling. Finally, if $x > e - a$, the unique NE outcome is the freeriding outcome of $p* = q* = 1$. Hence, we see how the Subsidizing Organization raising the subsidy triggers the freeriding problem.

So, how can the Subsidizing Organization encourage its desired outcome? It merely needs to raise the payoffs (to small provider) for not underselling. In terms of our bimatrix, this would raise the quantities d and h. If done in conjunction with the subsidy idea above, it could force the NE to be desired the outcome, i.e., $p* = 1$ and $q* = 0$. In practice, this can be done by either erecting a price floor for phone service (so that charging too small a fee is a crime), or perhaps by allowing providers to keep more of their fees via lowering particular penalties on the subsidy benefits by the Subsidizing Organization.

Case II ($a \geq e$ ***AND*** $d < b$***).*** In this case, covering calls is a dominating strategy for the large provider (i.e. it is best for the large provider to cover calls, no matter what the small provider does). Since $d < b$, the small provider's best response to this is to charge low fees. Hence the unique NE is the freeriding outcome.

Case III ((**$d > b$ **AND any comparative relation between** a **AND** e**) OR (**$a < e$ **AND** $d = b$**)). In this case, charging high fees is a dominating strategy for the small provider. Since $c > g$, the large provider's best response to this is to cover calls. Hence, the unique NE is the desired freeriding-free outcome.

Case IV (**$a \geq e$ **AND** $d = b$**). When a game forms like this, we get a continuum of NEs. For all NEs, Player I covers calls. Player II can play any mixed strategy.

CHAPTER 5: PERFORMANCE EVALUATION OF A TWO-PROVIDER SBSS MARKET

In Chapter 4, we observe different NEs of a two-provider game based on different game conditions. In this Chapter, we will perform quantitative analysis of these NEs using simulation. To analyze the providers' problem in an SBSS market, we consider a two-provider game within a single region where one is a large provider and another is a small provider. In the simulation, we will find customer utility, demand, and call quality first. After that, we will analyze customers' price averseness and UCS functions which are used to determine game NEs. After running the game simulation, we observe the game equilibriums and find the strategies associated with each NE. Then, we find the effect of government subsidy on determining NE. We also observe the freeriding effect of a large subsidy. Marginal Signal Improvement (MSI) has a significant contribution on determining NE. We analyze its effect on equilibrium. Small provider's UCS strategy can contribute to shifting the game NE from one state to another. Similarly, large provider's willingness to share also plays a big role in determining NE. We have simulated the effects of both (small and large) providers' such strategies and finally illustrated the observed NE.

Table 5.1: Initial Parameters

Parameter	*Value*
n	1000
ω_1, ω_2	0.8, 0
D	100
MSI	0.8
X_1, X_2	70, 30
n_1, n_2	$\frac{nX_1}{X_1+X_2}, \frac{nX_2}{X_1+X_2}$
f_1^*, f_2^*	$\frac{n_1}{X_1^{MSI}}, \frac{n_2}{X_2^{MSI}}$
V_1, V_2	$n_1 f_1^*, n_2 f_2^*$
T_{mv}	$V_1 + V_2$
ϵ_1, ϵ_2	$0.05T_{mv}$, 0

5.1 Experimental Setup

We assume Provider I is the larger provider. Table 5.1 describes the initial parameters where n and D denote the number of customers within the experimental region and the total number of calls each customer makes respectively. We assume Providers I and II have $X_1(= 70)$ and $X_2(= 30)$ base stations and their willingness to share are initialized to $\omega_1 = 0.8$ and $\omega_2 = 0$. A 35×35 square grid is considered as the experimental region. Base stations of each provider were set up randomly in this square region. We calculate the number of customers, n_1, n_2, subscribed to each provider based on the number of base stations each provider has. We select a customers' position in the square region randomly.

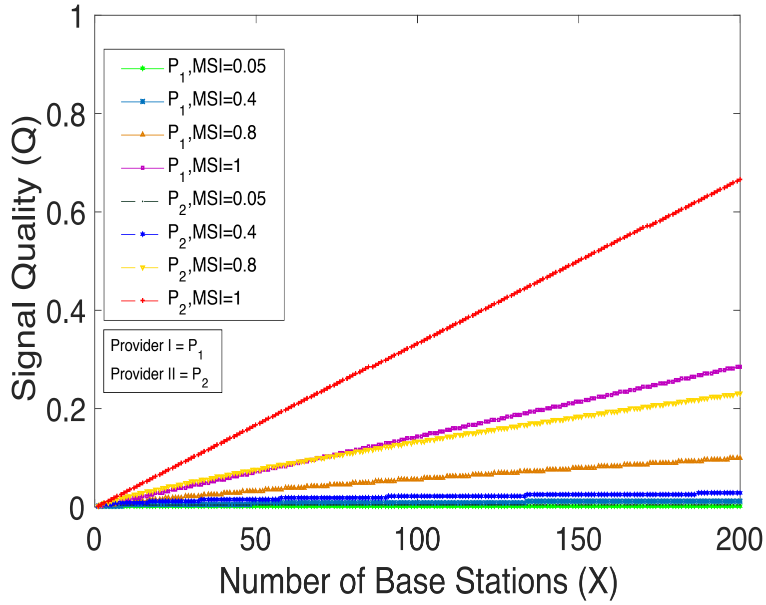

Figure 5.1: Signal Quality w.r.t. MSI

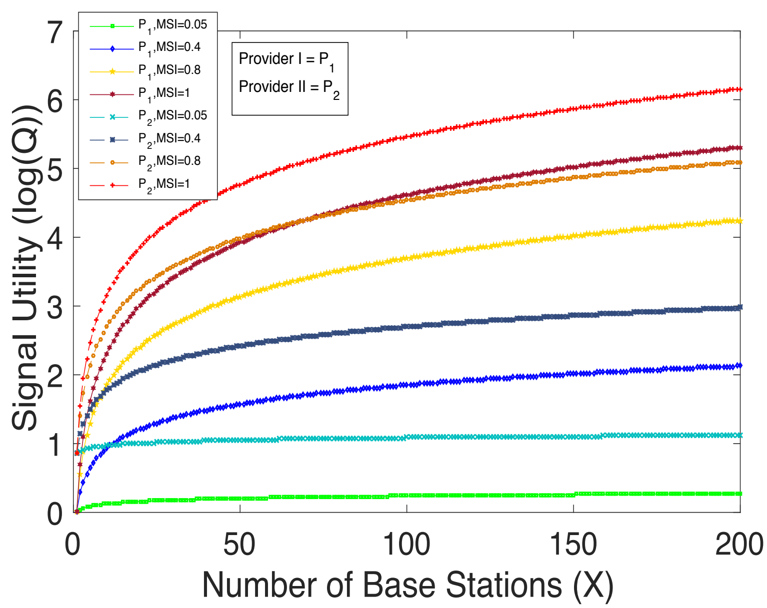

Figure 5.2: Utility w.r.t. MSI

5.1.1 Customer Utility and Demand

We model a customers' perceived signal utility from Providers I and II as $u_1 = log(Q_1)$ and $u_2 = log(Q_2)$, respectively. We model the signal quality as an increasing concave function of X and decreasing convex function of the number of customers. In particular, we use $Q_1 = X_1^{MSI}/n_1$ and $Q_2 = X_2^{MSI}/n_2$ to represent the offered signal quality to the customers of Providers I and II, where the Marginal Signal Improvement, MSI, is a constant expressing the benefit of having more base stations on the signal strength. To assure concavity, MSI must be in (0, 1). It describes, within a fixed size region, how the marginal improvement on the offered signal quality will diminish w.r.t. the number of base stations. As shown in Fig. 5.1 and Fig. 5.2, the small provider's offered signal quality and utility will improve more than a large providers' if we increase the number of base stations by the same amount. In our experiments, we chose a default value of $MSI = 0.8$ for both providers as this value attains a solid concave signal quality and logarithmic utility.

MSI also helps to determine fair market fees for both providers. Using the marginal utilities, u'_1 and u'_2, we define the fair market fees as: $f_1^* = u_1^{'} = 1/Q_1 = n_1/X_1^{MSI}$ and $f_2^* = u_2^{'} = 1/Q_2 = n_2/X_2^{MSI}$. For $MSI < 1$, we get $f_1^* > f_2^*$ which is expected in a normal market where market leaders charge more. Using fair market fees, we determine both providers' market values, V_1, V_2, and the total market value, T_{mv}, Table 5.1.

5.1.2 Call Quality

To measure the signal strength, ψ, of each provider, we have considered the distance from a caller to the base station. We have defined $\psi_1 = 1/d_1^2$ and $\psi_2 = 1/d_2^2$ where d_1 and d_2 represent the distance from a customer to the nearest base station of Provider I and Provider II, respectively. The lower the distance is, the better the signal. As we are considering a simplified game model where only Provider I is willing to share ($\omega_1 > 0$, $\omega_2 = 0$), so all calls of its customers are considered as home calls. However, for Provider II, this depends on the signal strength of both providers. If the nearest base station is one of Provider I's, then the decision to serve as a foreign call is made based on a random number ranged between 0 to 1. If the generated number is less than or equal to ω_1 then it is served by Provider I. Otherwise, Provider II serves as a home call. Considering the randomness of the locations of base stations, the positions of customers and determining of foreign vs. home call, we have run our simulation experiment 7 times and took the average of them when calculating the call qualities ψ_1 and ψ_2.

Table 5.2: Intermediate Variables

Variable Name	*Symbol*
Number of foreign calls served	FC_1, FC_2
Total foreign calls to Provider I	FC_t
Penalty	p_1, p_2
Signal strength	ψ_1, ψ_2
Probability of unfair customer switching	$P_{cs,1}$, $P_{cs,2}$
Number of home calls	$\beta_{1,i}$, $\beta_{2,i}$
Number of foreign calls	$\alpha_{1,i}$, $\alpha_{2,i}$

5.1.3 Price Averseness (C) and Unfair Customer Switching

When simulating the equilibrium strategies of the game, we need to calculate a few intermediate variables as listed in Table 5.2. From the count of home and foreign calls made by each customer i, we get the values for $\beta_{1,i}$, $\beta_{2,i}$, $\alpha_{1,i}$ and $\alpha_{2,i}$. The probability of UCS, $P_{cs,2}$, is one of the key factors of freeriding. If the perceived signal quality by Provider II's customers, ψ_2^*, is greater than or equal to the perceived signal quality by Provider I's customers, ψ_1^*, and the fair market fee of Provider I, f_1^*, is higher than any low fees of Provider II, f_2, then we model the probability of the UCS initiated by Provider II as: $P_{cs,2} = 1 - e^{-Cf_1^*/f_2}$. In this model, we introduce C as customers' averseness to price, Fig. 5.3. It ranges from 0 to 1. When C is high, customers averseness with price is also high. In a normal market, we believe customers are very averse with the price. Hence we use a high default value for C: $C = 0.9$

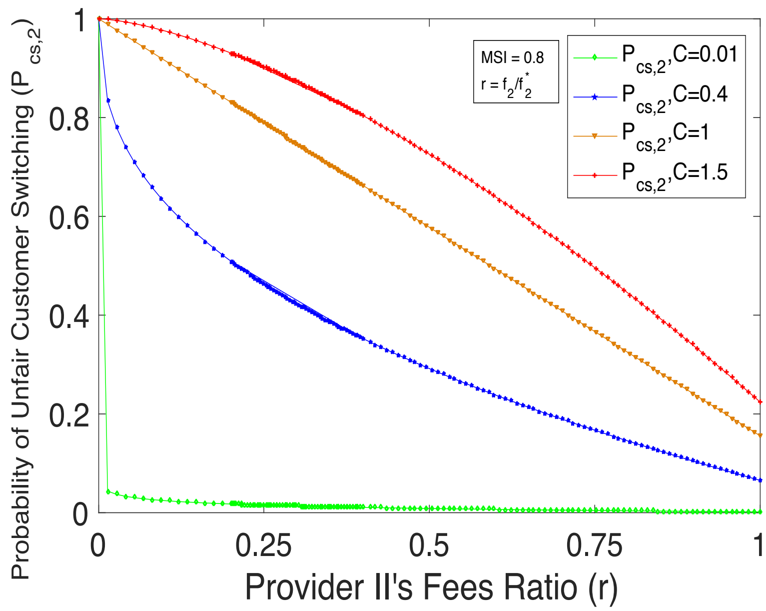

Figure 5.3: Effect of Price Averseness on $P_{cs,2}$

Fig. 5.4 and 5.5 describe the equilibrium strategies for both providers on a set of C. If customers are not too price averse, Provider I's higher sharing region increases. When it is too low (C = 0.01), Provider I plays higher sharing pure strategy always. However, in a real market, customers are price averse. As a result, with the increase of C, Provider I is more interested to play mixed strategy. Due to UCS, Provider I's lost customers $n_1 P_{cs,2}$, can range between 0 to n_1 depending on $P_{cs,2}$. When $f_1^*/f_2 \approx \infty$, we get $P_{cs,2} \approx 1$. And if $f_1^*/f_2 \approx 0$, we have $P_{cs,2} \approx 0$.

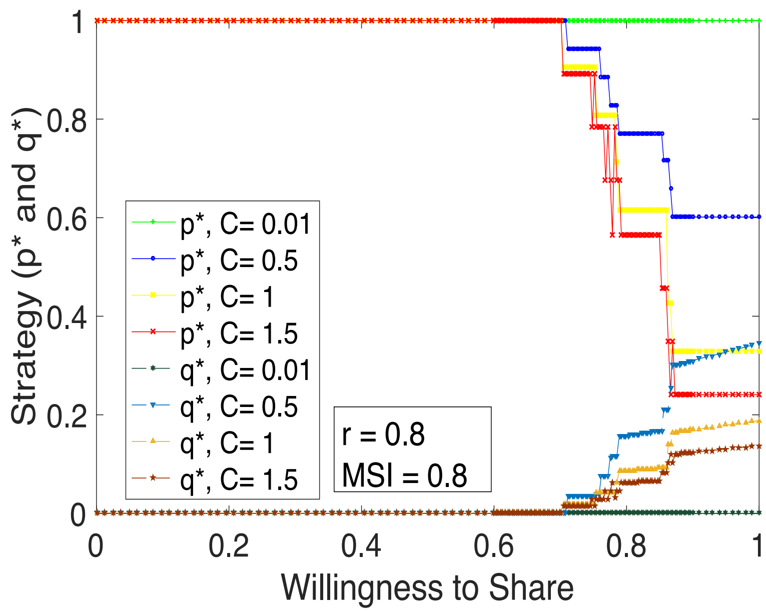

Figure 5.4: Fixed Fees Equilibrium Strategies with a set of C

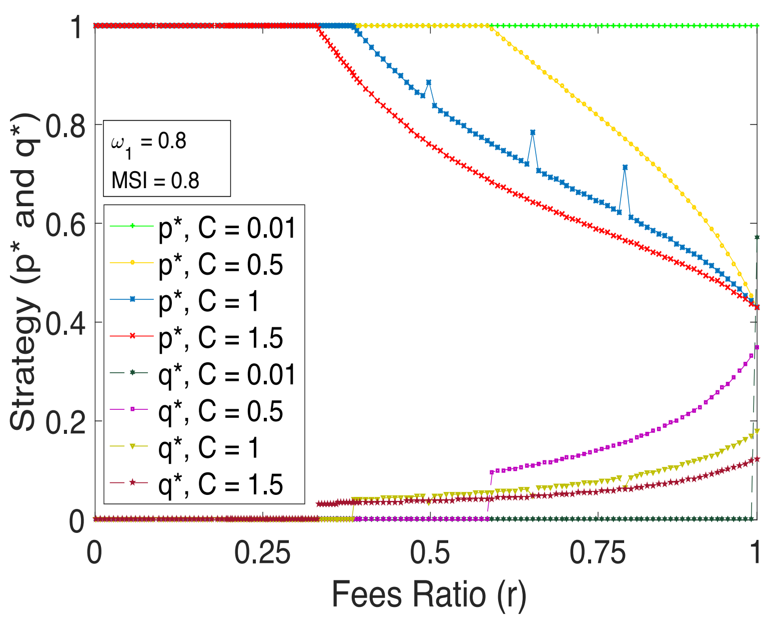

Figure 5.5: Fixed Willingness to Share Equilibrium Strategies with a set of C

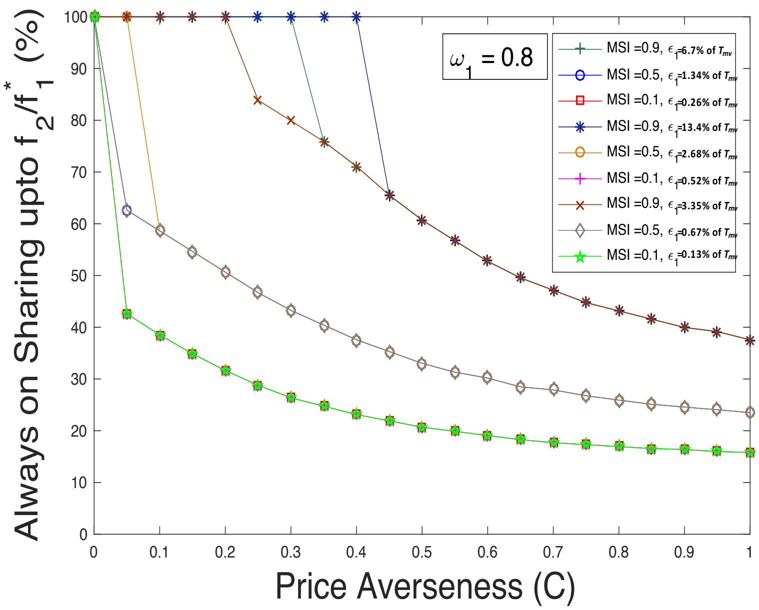

Figure 5.6: Always on Sharing w.r.t. Price Averseness

Fig. 5.6 gives us the idea of Provider I's always on sharing (covering all foreign calls) pure strategy region related to Provider I's and Provider II's fees ratio. It tells us the fees ratio (f_2/f_1^*) up-to which Provider I will play higher sharing pure strategy whatever Provider II does. Above this fees ratio, Provider I starts playing mixed strategy. We observe that, with the increase of C and MSI, Provider I is more willing to share its spectrum resource. If MSI is higher, a spectrum sharing provider can tolerate higher price averseness and continue high sharing. Also, If we increase the amount of subsidy, the region of always on sharing up to agreed willingness to share increases. For this case, we have considered ω_1 as 80%. Our assumption of this high value of ω_1 comes from the experiment (described in Subsubsection 5.2.4) where we found that for any given subsidy, our model can ensure Provider I to play pure strategy up to 69% of willingness to share approximately.

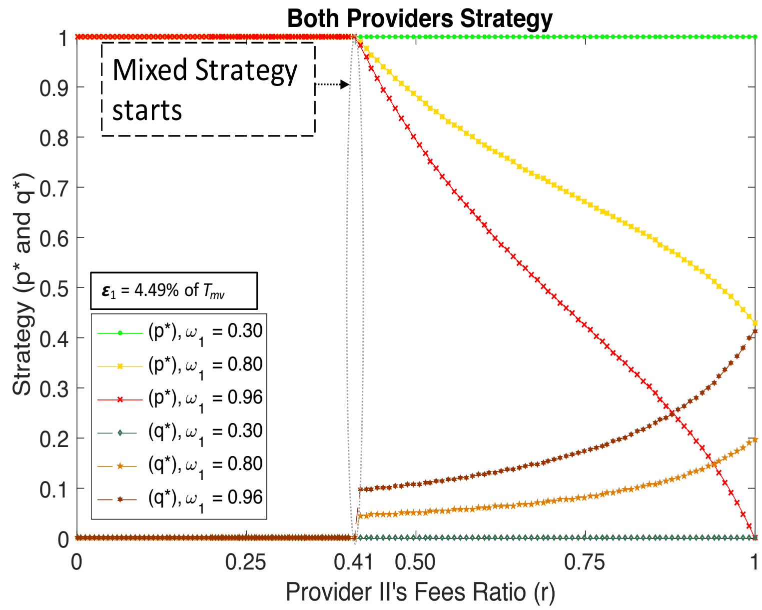

Figure 5.7: Equilibrium Strategies w.r.t. Fees Ratio

5.2 Results at Equilibrium

Both providers change their strategic decisions with the change of game scenario which generates a new NE. The subsidy amount also has a significant contribution to changing NEs of a game. The excessive subsidy can drive a game from a non-freeriding to a freeriding regime.

5.2.1 Strategies of Large and Small Providers at Equilibrium

From the experimental results, Fig. 5.7 describes the scenario where we have plotted the strategy taken by Provider I, $p*$, and the strategy taken by Provider II, $q*$, against the ratio, r. We have taken 30, 50 and 70 different data points between the ranges $0 \leq r \leq 0.199$, $0.2 \leq r \leq 0.4$ and $0.41 \leq r \leq 1$ respectively for the evaluation. Here, we draw the graph with three different willingness to share ($\omega_1 = 0.3, 0.8, 0.96$) of Provider I. That player is found to play a less sharing strategy (lowering $p*$) with the increase of ω_1 if we continue to increase the fee of Provider II towards the fair market fee, i.e., $r -- > 1$. This is because increasing the fee nearer to fair market fee causes less revenue loss for Provider II and switched customers add more revenue to Provider II. As a result, Provider II is willing to play undersell more (higher $q*$). This revenue loss scenario forces Provider I to play less sharing most of the time. However, we have found a transition of $p*$ from pure strategy, $p* = 1$, to a mixed strategy between fees ratio, $r = 0.41$ to $r = 0.42$. This r $(= 0.41)$ is the terminal sharing percentage up-to which Provider I can play pure strategy, $p* = 1$, without facing any loss. From the test cases, if $\omega_1 = 0.3$, our game ensures no freeriding. When $\omega_1 = 0.8$ or 0.96, freeriding takes place and Provider I changes its strategy. Instead of playing pure strategy it starts playing mixed strategy to overcome the loss due to UCS. Thus, it is possible to ensure market healthiness and provide subsidy to spectrum sharing provider.

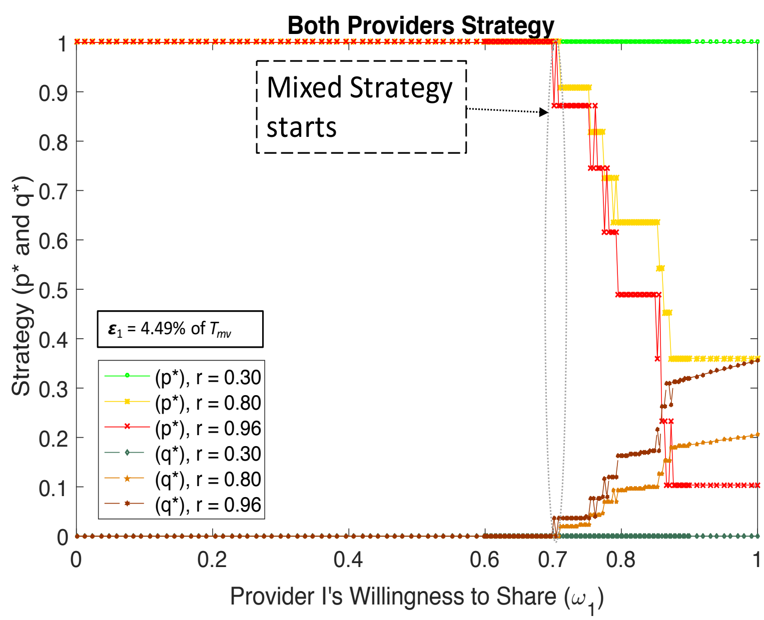

Figure 5.8: Equilibrium Strategies w.r.t. Willingness to Share (ω_1)

Fig. 5.8 describes the situation where we have plotted the strategies $p*$ and $q*$ against the willingness to share, ω_1, of Provider I. We have considered 50, 90 and 10 different data points between the ranges $0 \leq \omega_1 \leq 0.599$, $0.6 \leq \omega_1 \leq 0.9$ and $0.91 \leq \omega_1 \leq 1$ respectively for this simulation. Here, we have plotted the graph for three different values of $r = 0.3, 0.8, 0.96$. Similar to the previous graph, we find that Provider I is more willing to play less sharing with the increase of fees of Provider II to the fair market fee. The closer to the fair market fee of Provider II with the same sharing percentage by Provider I, the more earning by Provider II. As a result, we found that Provider II will charge low fees which forces Provider I to share less most of the time. We observe both players play the freeriding-free pure strategy, $p* = 1$, $q* = 0$, when $r = 0.3$. For the case of $r = 0.8$ or 0.96, both players play a mixed strategy above a certain willingness to share. We have seen a transition phase of $p*$ (from 1 to a proper fraction) when the sharing is 0.69. As we calculate $p*$ with the fraction of $(h - f)$ to $(b - f)$, it is expected to reduce $p*$ to that level where the fraction is located. We also observe that both providers maintain the same strategy for certain period of willingness to share. During this period, Provider II cannot start UCS anymore. We calculate $p*$ based on the payoff of Provider II. As low fees, f_2, and fair fee, f_2^*, of Provider II remains unchanged while changing willingness to share, the fraction of $(h-f)$ to $(b-f)$ gives the same result as no more switched customers have subscribed to Provider II. This also encourages Provider I to follow the same strategy for certain period.

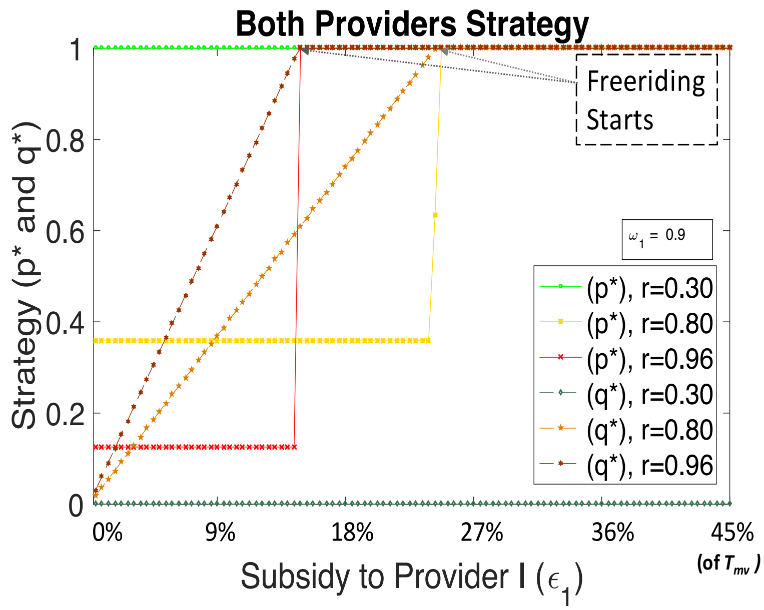

Figure 5.9: Equilibrium Strategies for Different Fees Ratio ($r = f_2/f_2^*$)

5.2.2 *Effect of Subsidy on Equilibrium*

We draw graphs which describe the scenario where equilibrium points shift with the changing of given subsidy to Provider I. Fig. 5.9 delineates the cases where equilibrium points are found against three different values of $r = 0.3, 0.8, 0.96$, and Fig. 5.10 describes the case where we graph w.r.t. three different willingnesses to share ($\omega_1 = 0.3, 0.8, 0.96$) for Provider I. We have considered 100 different subsidy amounts between 0% and $4.5\%\ of\ T_{mv}$ for this run. We have seen that with the increase of Provider II's fees closer to fair market fees, Provider I is less interested in sharing. However, if we continue increasing the subsidy, Provider I is finally able to overcome its loss due to unfair customer switching. Additional subsidy encourages it to play the most sharing strategy, $p* = 1$. Also, we observe the start of freeriding from here. We have seen, when $\omega_1 = 0.3$, Provider I and Provider II play $p* = 1$ and $q* = 0$ respectively. When $\omega_1 = 0.8$ and $\omega_1 = 0.96$, Provider

I and Provider II play mixed strategies up to $\epsilon_1 = 0.238T_{mv}$ and $\epsilon_1 = 0.139T_{mv}$ respectively. After reaching these ϵ_1's, freeriding takes place. Fig. 5.10 tells the same story from a different perspective. Higher willingness to share of Provider I with the same r $(= 0.9)$ encourages Provider II to undersell mostly, and this forces Provider I to a play less sharing strategy. In case of $\omega_1 = 0.8$, if we continue increasing the subsidy to Provider I, finally it is able to overcome the customer loss when $\epsilon_1 = 0.202T_{mv}$, which encourages Provider I to share more $(p* = 1)$. In this case, Provider II continues its mixed strategy onwards. We find the start of freeriding when the subsidy amount is $\epsilon_1 = 0.175T_{mv}$ where $\omega_1 = 0.96$.

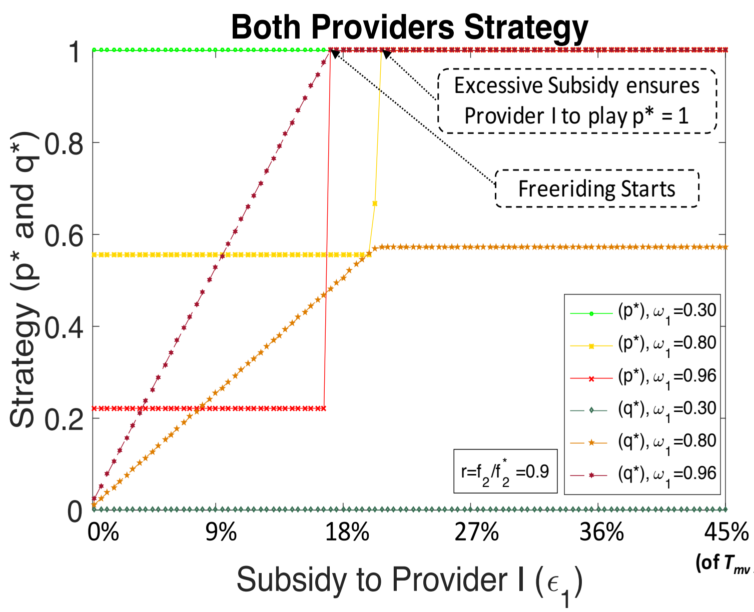

Figure 5.10: Equilibrium Strategies for Different Willingness to Share (ω_1) and Subsidy of Provider I

5.2.3 *Effect of Marginal Signal Improvement (MSI)*

Equilibrium strategies w.r.t. different MSI values have been shown in Fig. 5.11 and 5.12. If we increase same weight on MSI for both providers, Provider I's higher sharing region increases.

This happens due to two reasons, (1) the higher the MSI is, the closer each other's fees are, and (2) if the customers get equal improvement on signal strength from both providers, the large provider still remain ahead of the small provider. As a result, unfair customer switching probability remains lower, which encourages Provider I for higher sharing. We also observe, if MSI remains higher Provider I can switch to play higher sharing pure strategy from a mixed strategy, Fig. 5.12. Because, higher sharing makes more profit from the subsidy in spite of low customer loss. It happens when Provider II's charging fee stays near to fair market fee. In such competitive market, a little subsidy is well enough to drive a provider for higher sharing. In a case, where $MSI = 0.95$, the ratio of a low fee, f_2, to fair market fee, f_2^*, of Provider II, $r = 0.93$, subsidy $\epsilon_1 (= 0.045 T_{mv})$ is well enough to motivate Provider I to switch from a mixed strategy to the higher sharing pure strategy. Initially, the willingness to share is set to $\omega_1 = 0.8$ and $\omega_2 = 0$. For determining penalty on given subsidy to Provider I, we used a linear penalty, p_1, where $p_1 = 1 - FC_1/FC_t$.

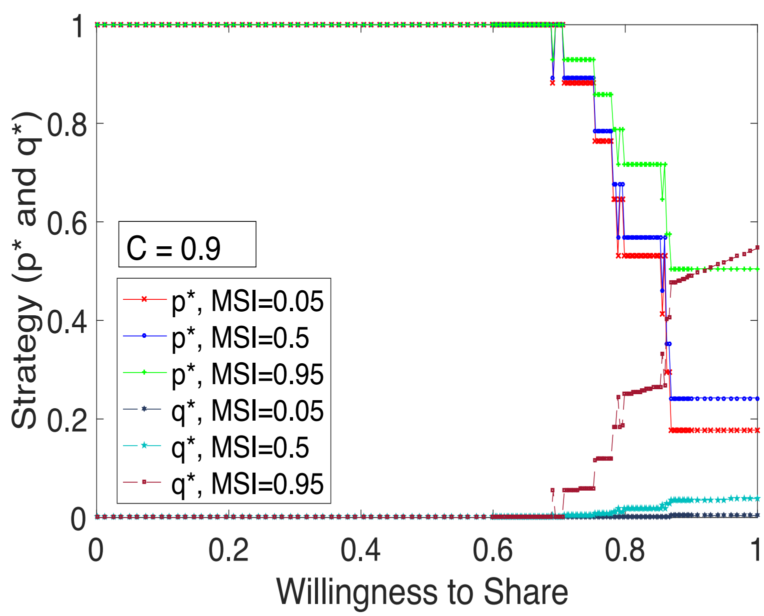

Figure 5.11: Equilibrium Strategies with a set of MSI w.r.t. ω_1

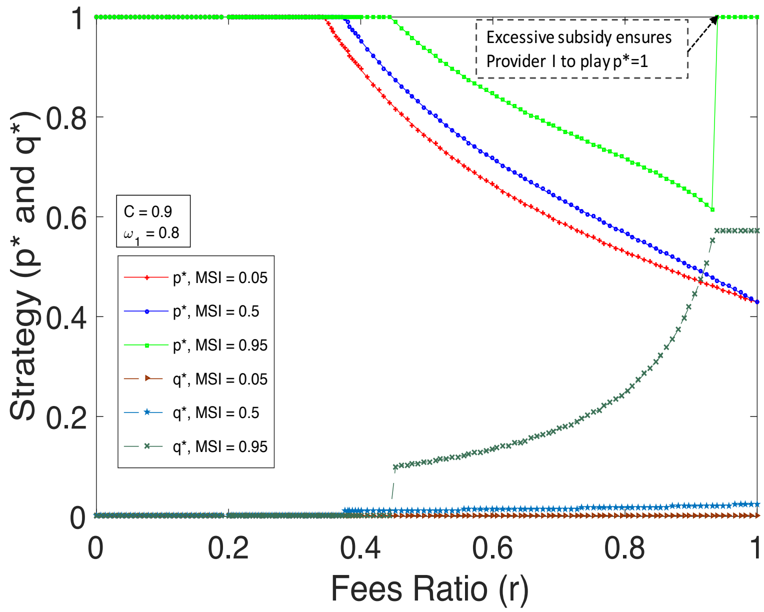

Figure 5.12: Equilibrium Strategies with a set of MSI w.r.t. Fees Ratio

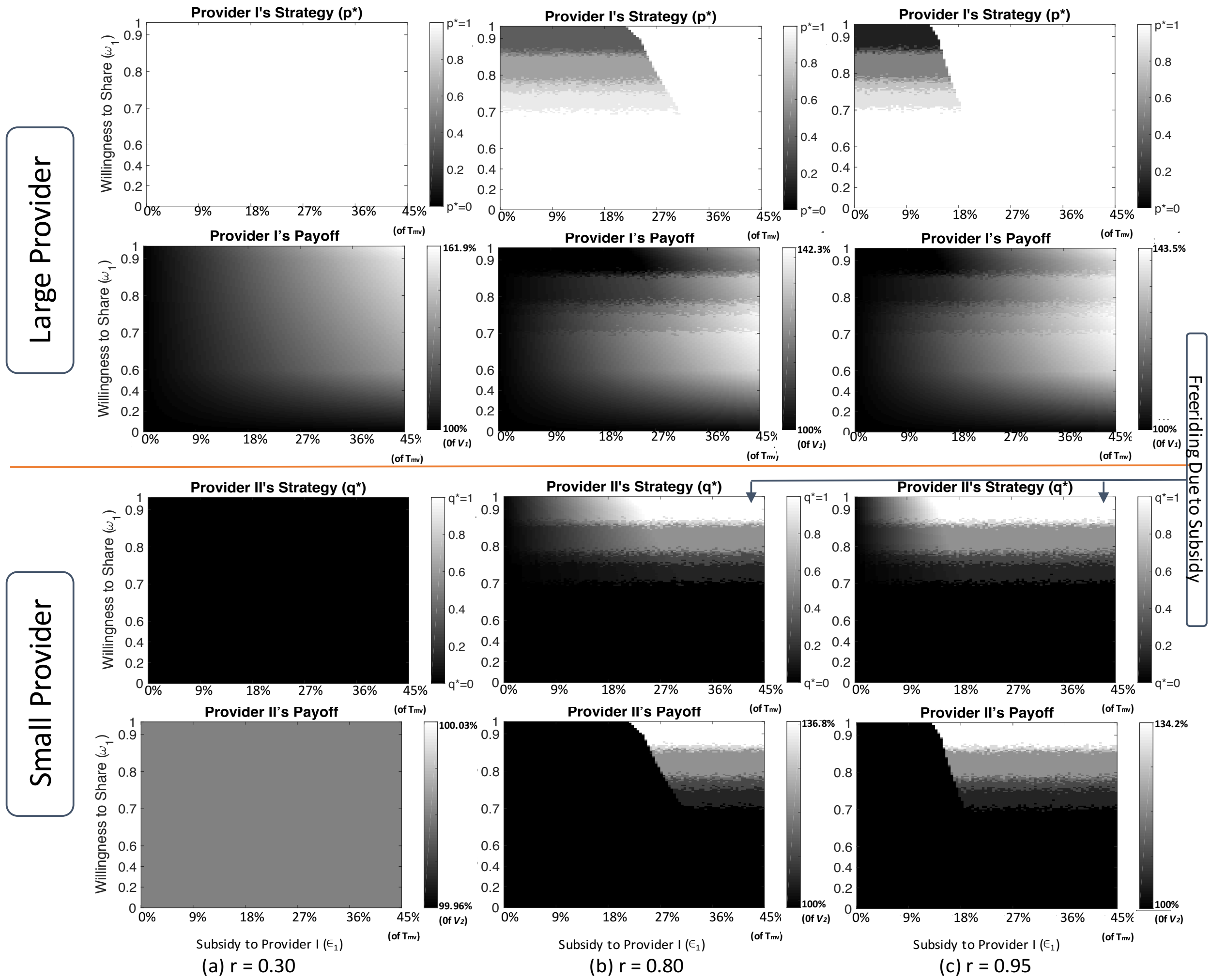

Figure 5.13: Equilibrium Strategies and Payoffs with different fees ratio ($r = f_2/f_2^*$)

5.2.4 *Weak Provider's Effort to Trigger Unfair Customer Switching*

Equilibrium strategies against given subsidy to Provider I and for a given set of fees ratio has been described in Fig. 5.13. When Provider II's ratio of fee to fair market fee is 30%, we see both providers are interested to play freeriding-free pure strategies, i.e., $p* = 1$ and $q* = 0$. With the increase of this ratio, both change their pure strategies to a mixed strategy. When $\omega_1 = 0.69$, for any r and given subsidy ϵ_1, our model ensures a freeriding-free PSNE. However, any further increase of r, ϵ_1, ω_1, equilibrium leads us in a region where Provider I covers calls and Provider II lowers fees. This happens because UCS was triggered by Provider II and additional subsidy earning by Provider I. We also see different freeriding starting points for different ω_1, ϵ_1 and r. High subsidy continues freeriding output. However, this situation compromises the ideal market. We also observe that when $r = 0.8$, Provider II's maximum revenue earning is greater compared to the revenue earning when $r = 0.95$. It happens because of UCS, the more Provider II's fee is closer to its fair market fee, the lower UCS is. And, it ensures less customer loss by Provider I. If we observe the revenue earnings of Provider I, we see its earning is higher with a higher value of r. However, there exists a threshold value of r, bellow which UCS cannot guarantee Provider II to earn at least its regular revenue. As a result, Provider II doesn't play freeriding strategy ($q^* = 1$). Here, $r = 0.3$ is located within the range of that threshold to $r = 0$. We also observe that, in this case, Provider II's maximum revenue is higher compared to two more values of r.

Fig. 5.14 describes both providers equilibrium strategies and payoffs w.r.t. to ω_1. When $\omega_1 = 0.4$, both providers play the freeriding free PSNE. With the increase of sharing, both switch to a mixed strategy. However, when the fees ratio r is high, the game exhibits a freeriding equilibrium with the increase of subsidy. The more willingness to share, the more we get the freeriding equilibrium when all game conditions remain unchanged. From Fig. 5.14b and Fig. 5.14c, when $r \leq 0.41$, it ensures both providers to form freeriding-free PSNE. Above of this ratio, we observe both players adjust their pure strategy to mixed strategy for maximizing their profit. In this case, $r = 0.41$ is the threshold value of r, bellow which Provider II isn't interested to play freeriding strategy $q* = 1$ to make the profit from UCS. The higher ω_1, the higher freeriding PSNE. Higher sharing encourages Provider II to play customers' switching strategy, ($q* = 1$), and higher subsidy amount minimizes revenue loss of Provider I. These two conditions help to continue freeriding PSNE when sharing is too high. The brighter an area is, the more earning for a provider.

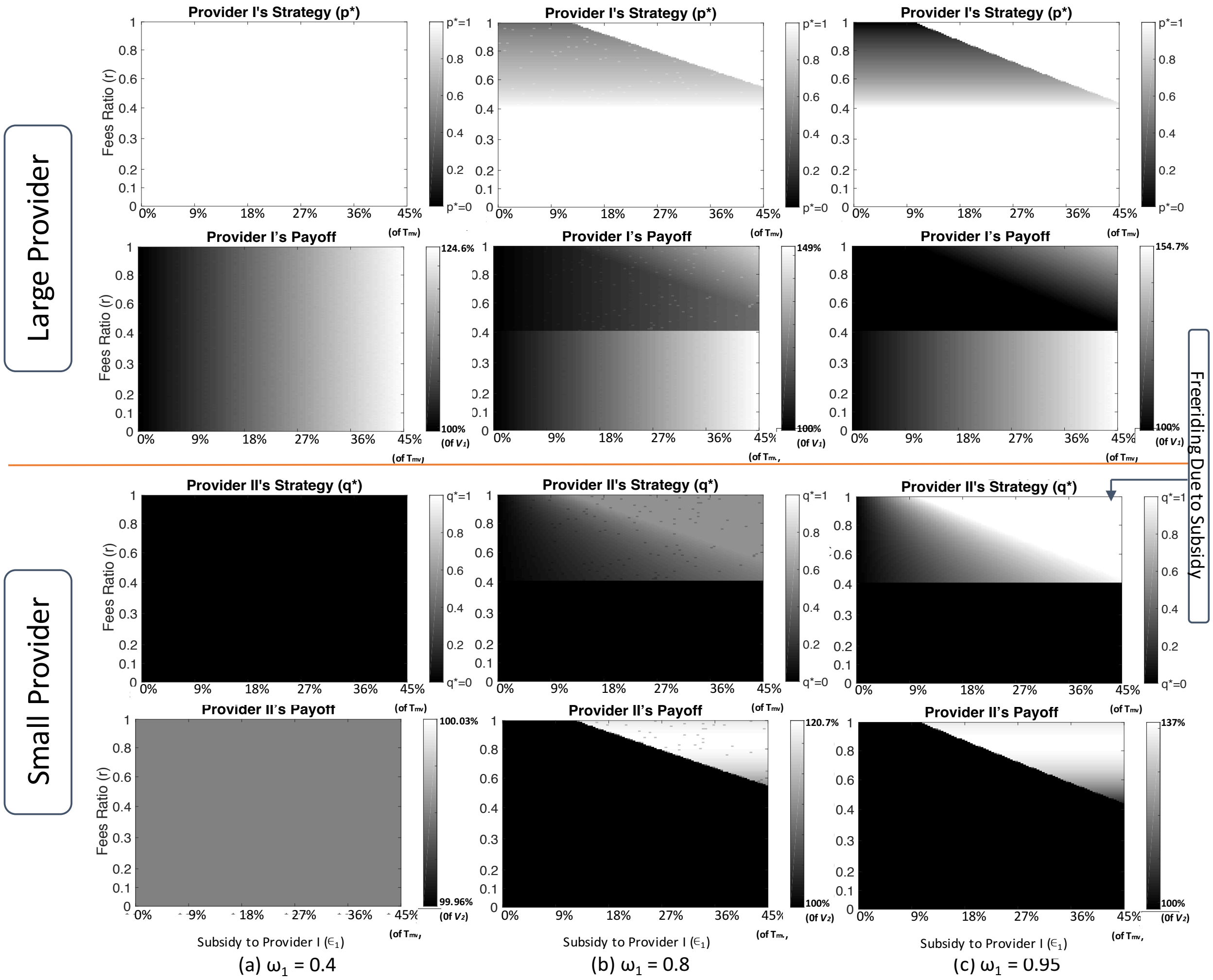

Figure 5.14: Equilibrium Strategies and Payoffs with different Willingness to Share (ω_1)

5.2.5 Large Provider's Willingness to Share

Higher sharing is expected in the SBSS market. However, higher sharing increases the chance of revenue loss for the provider who shares spectrum. Lowering fee is beneficial for the smaller provider which gets the benefits of sharing. If the fees are too low compared to fair market fee, lowering fees can be a bad move on its' own. There is a willingness to share, below which any sharing can be done with PSNE. After that percentage of sharing, players are interested in choosing a mixed strategy. From our experiment, we can say there exists a value of ω_1 (here it is 0.69), below which any sharing percentage ensures Provider I to play $p* = 1$ and Provider II to play $q* = 0$. Also, other NEs ensure at least the earnings from unshared market for both Providers. From the payoffs of all cases, we observed that Provider I earns at least its regular earning which it can earn while not taking part in SBSS market. In the freeriding PSNE, both providers earn more than the earnings of existing cellular market. From such observations, a spectrum sharing provider can make a strategic decision to limit service of foreign calls. In this way, the spectrum sharing provider's strategic decision guarantees it at least the earnings of an unshared spectrum market. Also, the freeriding PSNE gives the incentivizing organization an idea of how much subsidy should be given to large provider to stop freeriding.

CHAPTER 6: CONCLUSION

Freeriding is always harmful to any stable market. In computer communication networks, freeriders may arise in different forms. There already exist different countermeasures against them. However, in the cellular market, due to the static licensing of spectrum allocations, such freeriders haven't arisen previously. Now, cellular users are increasing rapidly which requires more spectrum utilization and more sharing to accommodate a huge number of cellular devices. Different approaches have already been taken to increase spectrum utilization, e.g., dynamic spectrum sharing among providers. Monetary transactions are mostly found in this types of sharing. However, it may cost additional money of the customers eventually. Government subsidy-based spectrum sharing may reduce this burden from the users. However, it also creates tunnels for the opportunistic freeriding providers which can interrupt existing market healthiness, trigger unfair customer switching and create an unstable cellular market.

In this thesis, we have discussed a subsidy-based spectrum sharing model, where the amount of customers' switching triggered by a smaller provider can be reduced and the earnings of large providers' remain at least as much as the earnings from the existing unshared spectrum market. Here, we have developed a game-theoretic framework which can ensure a large provider to serve up to a certain sharing percentage of total incoming foreign calls for all times. Above this willingness to share, our model helps both providers to maintain a mixed strategy to control freeriding. Also, there exists a fees ratio, below which a spectrum sharing provider can always share. We have considered customers' price averseness to determine customer switching probability. Also, we have shown the impact of subsidy in an SBSS cellular market.

We have considered a two-provider non-cooperative game for SBSS markets. We leave the more-than-two provider game as future work. Also, the exploration of the signal strength indicator,

MSI, is another worthy future work. Here, we have only used the same MSI for both providers. In a competitive market with probable higher signal improvement, a little subsidy can motivate a provider for higher sharing. Also, different MSI values can be used to run this experiment. It may substantially increase/ decrease the size of the desired/undesired equilibrium regions of our experiment. Also, we have not considered Mobile Virtual Network Operators (MVNOs) as part of the game-theoretic model. As such virtual operators without any infrastructure are being considered, looking at the effect of such operators on the SBSS markets will be an interesting direction to take.

APPENDIX : SYMBOLS AND NOTATIONS

$$n = Number\ of\ total\ customers$$

$$\sigma(\cdot) = Function\ to\ calculate\ number\ of\ home\ calls\ made\ by\ each\ customer$$

$$\beta_{j,i} = Number\ of\ home\ calls\ made\ by\ each\ customer\ i\ of\ provider\ j$$

$$\alpha_{j,i} = Number\ of\ outside\ calls\ made\ by\ each\ customer\ i\ of\ provider\ j$$

$$\gamma = Number\ of\ total\ calls\ made\ by\ each\ customer$$

$$f_j = The\ fees\ charged\ by\ provider\ j\ to\ its\ customers$$

$$\psi_j = The\ intensity/quality\ of\ provider\ j's\ signal$$

$$X_j = Provider\ j's\ number\ of\ base\ stations$$

$$b_j = Government\ subsidy\ to\ Provider\ j$$

$$Q(.) = An\ increasing\ concave\ function\ takes\ X_j\ and\ b_j\ to\ calculate\ \psi_j$$

$$u(.) = Customers'\ utility\ function,\ parameterized\ with\ received\ signal\ intensity\ \psi_j$$

$$U(i,\mathrm{j}) = Overall\ utility\ determining\ function\ of\ customer\ i\ by\ choosing\ provider\ j$$

$$P(i,j) = Probabilistic\ function\ to\ choose\ provider\ j\ by\ customer\ i$$

$$SO = Subsidizing\ Organization$$

$$\epsilon_j = Explicit\ subsidy\ amount\ that\ provider\ j\ receives\ from\ an\ SO$$

$$\epsilon = Total\ subsidy\ budget\ of\ an\ incentivizing\ organization$$

$$R_j = Provider\ j's\ revenue\ from\ subscribed\ customers\ only$$

$$E_j = Provider\ j's\ overall\ revenue$$

$$s_j = A\ portion\ of\ the\ amount\ \epsilon_j\ that\ provider\ j\ spends\ for\ infrastructure$$

$$FC_j = Number\ of\ outside\ calls\ that\ provider\ j\ serves$$

$$\omega_j = Provider\ j's\ willingness\ to\ share$$

$$N_j^l = Number\ of\ customers\ lost\ by\ provider\ j$$

$$N_j = Number\ of\ customers\ of\ provider\ j$$

$$p(\cdot) = Proportionate\ subsidy\ returning\ penalty\ function$$

$$P_{cs,j} = Probability\ of\ unfair\ customer\ switching\ initiated\ by\ provider\ j$$

$$UCS = Unfair\ Customer\ Switching$$

$$BS = Base\ Station$$

$$SBSS = Subsidy - Based\ Spectrum\ Sharing$$

$$PSS = Pervasive\ Spectrum\ Sharing$$

$$PSC = Public\ Safety\ Communication$$

$$RAN = Radio\ Access\ Network$$

$$MNO = Mobile\ Network\ Operator$$

$$MVNO = Mobile\ Virtual\ Network\ Operator$$

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