# Site-Selective Excitation And Polarized Absorption Spectra Of Nd3+ In Sr-5(Po4)(3)F And Ca-5(Po4)(3)F 

John B. Gruber

Clyde A. Morrison
Michael D. Seltzer
Andrew O. Wright
Melvin P. Nadler

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## Authors

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John B. Gruber, Clyde A. Morrison, Michael D. Seltzer, Andrew O. Wright, Melvin P. Nadler, Toomas H. Allik, J. Andrew Hutchinson, and Bruce H. T. Chai

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John B. Gruber<br>Department of Physics, San Jose State University, San Jose, California 95192-0106

Clyde A. Morrison
Army Research Laboratory, Adelphi, Maryland 20783-1145
Michael D. Seltzer, Andrew O. Wright, and Melvin P. Nadler
Naval Air Warfare Center Weapons Division, China Lake, California 93555-6001
Toomas H. Allik
Science Applications International Corporation, 1710 Goodridge Drive, McLean, Virginia 22102
J. Andrew Hutchinson

Night Vision and Electronics Sensors Directorate, U.S. Army, Fort Belvoir, Virginia 22060-5806
Bruce H. T. Chai
Center for Research on Electro-optics and Lasers, University of Central Florida, Orlando, Florida 32836
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Polarized absorption and fluorescence spectra were analyzed to establish individual energy (Stark) levels of $\mathrm{Nd}^{3+}$ ions in host crystals of $\mathrm{Sr}_{5}\left(\mathrm{PO}_{4}\right)_{3} \mathrm{~F}$ (SFAP) and $\mathrm{Ca}_{5}\left(\mathrm{PO}_{4}\right)_{3} \mathrm{~F}$ (FAP). Site-selective excitation and fluorescence facilitated differentiation between $\mathrm{Nd}^{3+}$ ions in emitting sites associated with $1.06 \mu \mathrm{~m}$ stimulated emission, and nonemitting $\mathrm{Nd}^{3+}$ ions in other sites. Measurements were made on samples containing different concentrations of $\mathrm{Nd}^{3+}$ at 4 K and higher temperatures. Substitution of $\mathrm{Nd}^{3+}$ for $\mathrm{Sr}^{2+}$ or $\mathrm{Ca}^{2+}$ was accompanied by passive charge compensation during crystal growth. Crystal-field splitting calculations were performed according to site for Stark levels of $\mathrm{Nd}^{3+}$ ions identified spectroscopically. We obtained a final set of crystal-field parameters $B_{n m}$ for $\mathrm{Nd}^{3+}$ ions in fluorescing sites with a rms. deviation of $7 \mathrm{~cm}^{-1}$ ( 52 levels in Nd:SFAP) and $8 \mathrm{~cm}^{-1}$ ( 59 levels in Nd:FAP). For one of the nonemitting sites in Nd:FAP we obtained a final set of $B_{n m}$ parameters which gave a rms deviation of $6 \mathrm{~cm}^{-1}$ between 46 experimental and calculated levels. © 1996 American Institute of Physics. [S0021-8979(96)01403-2]

## INTRODUCTION

Fluorapatite crystals $\mathrm{Ca}_{5}\left(\mathrm{PO}_{4}\right)_{3} \mathrm{~F}$ (FAP), $\mathrm{Sr}_{5}\left(\mathrm{PO}_{4}\right)_{3} \mathrm{~F}$ (SFAP), and $\mathrm{Sr}_{5}\left(\mathrm{VO}_{4}\right)_{3} \mathrm{~F}$ (SVAP), containing trivalent rareearth ions, have been grown recently with sufficiently favorable optical properties to warrant further study of their spectroscopy and evaluation of their laser performance. ${ }^{1-8}$ The potential of these crystals as optical materials has been demonstrated by the efficient generation of stimulated emission at $1.04 \mu \mathrm{~m}$ in $\mathrm{Yb}: S V A P,{ }^{3,4,9}$ and by the successful use of Er:FAP as a saturable absorber for passive $Q$ switching of an $\mathrm{Er}^{3+}$-doped phosphate glass laser operating at $1.53 \mu \mathrm{~m} .{ }^{10-12}$ Interpretation of the Nd:FAP spectra has been of interest to a number of groups whose early studies (nearly 30 years ago) centered around the luminescence features of the crystal and its potential as a laser. ${ }^{13-18}$ More recently, spectroscopy and laser performance studies show that Nd:SVAP has promise as a new laser source. ${ }^{8}$

The complex optical spectra attributed to rare-earth ions occupying numerous crystal-field environments is a general characteristic of the fluorapatite crystals grown with rareearth ions as dopants. ${ }^{6,7,11}$ Heterovalent substitution by rareearth ions for Ca and Sr in two different positions in the hexagonal crystal lattice, $\mathrm{M}(\mathrm{I})$ and $\mathrm{M}(\mathrm{II})$, requires some form
of charge compensation. ${ }^{19-23}$ Possible charge-compensation mechanisms for $\mathrm{R}^{3+}$ ions in either or both host cation positions have been proposed by several groups. ${ }^{16,23,24}$ The possible substitution of two types of lattice cations and the location of charge-compensating ions and defects, relative to the $\mathrm{R}^{3+}$ ions, accounts for variation in the crystal-field environment experienced by individual $\mathrm{R}^{3+}$ ions. The particular environment, or site, occupied by a $\mathrm{R}^{3+}$ ion has considerable influence over the optical behavior of that ion. For instance, some of the $\mathrm{Nd}^{3+}$ ions in these hosts do not emit radiation upon excitation, and therefore are not likely to contribute to laser action. ${ }^{5,9,24}$ Furthermore, some appear to serve as quenching centers. ${ }^{24}$ Indeed, this characteristic is most evident for FAP crystals in which $\mathrm{Nd}^{3+}$ ions appear to be distributed among both emitting and nonemitting sites. In SFAP, $\mathrm{Nd}^{3+}$ ions appear to occupy emitting sites predominantly.

Our purpose is to identify the details of the crystal-field splitting of the energy levels of $\mathrm{Nd}^{3+}$ ions that occupy both emitting and nonemitting sites in Nd:FAP and Nd:SFAP using site-selective excitation and polarized absorption as optical probes. This approach allows us to dissect complex absorption spectra and identify features which arise from $\mathrm{Nd}^{3+}$ ions in specific sites. We find that the spectra of Nd:SFAP have fewer absorption and emission lines than comparable
spectra observed for Nd:FAP. This suggests that the availability of optical centers for appreciable occupation by $\mathrm{Nd}^{3+}$ ions in SFAP is limited relative to FAP.

In the course of our ongoing investigation of $\mathrm{R}^{3+}$-doped FAP and SFAP crystals, we have observed polarized absorption and fluorescence spectra for non-Kramers ions such as $\mathrm{Eu}^{3+}$ and $\mathrm{Pr}^{3+}$ consistent with selection rules associated with $C_{s}$ symmetry. ${ }^{6,7}$ In both FAP and SFAP, such polarization was observed for the principally occupied site, and therefore attributed to be $\mathrm{M}(\mathrm{II})$. Because $\mathrm{Nd}^{3+}$ is a Kramers ion, we have less direct evidence to suggest that $\mathrm{Nd}^{3+}$ prefers the M(II) location as well. However, resemblances between the spectra of Nd:SFAP and some of the features observed in the spectra of Nd:FAP, and similar comparisons made between other R:FAP and R:SFAP crystals, clearly indicate that the emitting $\mathrm{Nd}^{3+}$ ions in both FAP and SFAP occupy M(II) locations with $C_{s}$ symmetry. ${ }^{24}$ This hypothesis is consistent with the suggestion by other groups ${ }^{15,16,18,23}$ that the laseractive $\mathrm{R}^{3+}$ ions in FAP and SFAP occupy M(II) locations with $C_{s}$ symmetry.

In addition to spectral features attributed to the principally occupied sites in Nd:FAP we observe relatively weaker absorption lines which also polarize. Analysis of these spectra is consistent with selection rules operating for $\mathrm{Nd}^{3+}$ ions in sites having $C_{3}$ symmetry. Since no $\mathrm{Nd}^{3+}$ ion emission is observed for the site, we base this interpretation on the polarized absorption spectra reported in Table I. This site is one of several nonfluorescing sites described by Maksimova and Sobol ${ }^{15,16}$ who suggest that some $\mathrm{Nd}^{3+}$ ions may substitute into $\mathrm{M}(\mathrm{I})$ divalent cation sites. These sites have $C_{3}$ symmetry and represent $40 \%$ of all cation sites in the undoped lattice. They speculate that charge compensation for this site is sufficiently remote so that $C_{3}$ symmetry is preserved for $\mathrm{Nd}^{3+}$ ions in the site. ${ }^{25}$

Crystal-field splitting calculations were performed for Stark levels of $\mathrm{Nd}^{3+}$ ions in spectroscopically different sites. Using crystal-field parameters $B_{\mathrm{nm}}$ reported for Nd:SVAP as a starting set, ${ }^{5}$ we obtained a final set for $\mathrm{Nd}^{3+}$ ions in fluorescing sites with a rms deviation of $7 \mathrm{~cm}^{-1}$ ( 52 levels in $\mathrm{Nd}: \mathrm{SFAP}$ ) and $8 \mathrm{~cm}^{-1}$ (59 levels in Nd:FAP). Peale et al. ${ }^{5}$ obtained a rms of $6 \mathrm{~cm}^{-1}$ for 28 levels of $\mathrm{Nd}^{3+}$ in the emitting site in Nd:SVAP. Using $B_{n m}$ parameters obtained from a lattice-sum calculation for $\mathrm{Nd}^{3+}$ ions in sites of $C_{3}$ symmetry, we obtained a final set of $B_{n m}$ parameters which gave a rms of $6 \mathrm{~cm}^{-1}$ between 46 experimental and calculated levels for one of the nonemitting sites.

## EXPERIMENTAL DETAILS

Boules of Nd:SFAP and Nd:FAP containing nominally 1 at. wt \% neodymium, were grown by the standard Czochralski method at the University of Central Florida, CREOL. The hexagonal crystals were cut parallel and perpendicular to the crystalline optical axis (c axis). Crystals containing lesser quantities of $\mathrm{Nd}^{3+}$ ions were also examined spectroscopically. The actual amount present was determined using inductively coupled plasma atomic emission spectrometry on chemically digested portions of samples used in our studies.

As a check, one of the crystals (Nd:SFAP) was analyzed independently by Galbraith Laboratories (Knoxville, TN). The results were in agreement with our determinations. The absorption spectra reported in Table I were obtained from samples Nd:SFAP ( 0.29 at. wt \% Nd) and Nd:FAP (0.61 at. wt $\% \mathrm{Nd})$. The Nd densities in these crystals are $3 \times 10^{19}$ and $6 \times 10^{19}$ ions $/ \mathrm{cm}^{3}$, respectively.

Absorption spectra measured between 2650 and 300 nm were obtained using a Cary model 2390 spectrophotometer equipped with a continuous-flow liquid-helium cryostat that allowed us to observe spectra at any temperature between 4 K and room temperature. Calibration of the instrument was achieved by measurement of standard deuterium emission lines in different orders. Spectral bandwidths were much less than the bandwidths of the majority of absorption peaks observed. The precision in measuring the spectra reported in Table I was generally to within 0.1 nm . Supplemental absorption spectra were recorded using a Perkin-Elmer Lambda 9 spectrophotometer covering the same wavelength range. The instrument was equipped with a liquid-helium conduction dewar. Absorption spectra measured between 4080 and 1920 nm were also obtained at room temperature with a Nicolet 60sx Fourier transform infrared (FTIR) spectrometer.

Using the Cary spectrophotometer, we obtained axial absorption spectra (with the light beam collinear with the $c$ axis of the crystal), and polarized transverse absorption spectra [with the $\mathbf{E}$ of the light beam perpendicular $(\sigma)$ and parallel $(\pi)$ to the $c$ axis of the crystal]. We observed no spectroscopic evidence that would suggest a phase change in either Nd:SFAP or Nd:FAP at temperatures below room temperature. ${ }^{26}$ Orientation of the optical axis was determined by placing the crystal between crossed polarizers and observing the characteristic "Maltese cross" pattern. We observed no temperature-dependent (hot band) absorption spectra in any of our samples. We conclude that the energy separation between the ground-state Stark level and the first excited Stark level in the ${ }^{4} I_{9 / 2}$ manifold is large relative to the thermal energy associated with the temperatures at which our measurements were made.

Site-selective excitation and fluorescence spectra were obtained at 4 and 80 K using a Quantel Nd:YAG laserpumped dye laser having an output bandwidth of approximately $0.1 \mathrm{~cm}^{-1}$. A 0.85 m double monochromator equipped with a R928 photomultiplier tube (PMT) was used for fluorescence detection and signals were processed using a boxcar averager and gated integrator. Data were collected and stored using a digital oscilloscope. A 0.22 m monochromator and liquid-nitrogen-cooled germanium detector were used to detect fluorescence between 1.0 and $1.5 \mu \mathrm{~m}$. An Oxford 1204D continuous-flow liquid-helium cryostat allowed us to obtain spectra from samples down to 4 K .

## THE OBSERVED SPECTRA

At 4 K numerous absorption bands are observed in the spectra of both crystals. Many bands are relatively broad and have satellite structure. We attribute the spectra to $\mathrm{Nd}^{3+}$ ions

TABLE I. Absorption spectra of $\mathrm{Nd}^{3+}$ in $\mathrm{Sr}_{5}\left(\mathrm{PO}_{4}\right)_{3} \mathrm{~F}$ and $\mathrm{Ca}_{5}\left(\mathrm{PO}_{4}\right)_{3} \mathrm{~F}^{\mathrm{a}}$

| ${ }^{2 s+1} L_{J}{ }^{\text {b }}$ | $\mathrm{Nd}: \mathrm{SFAP}^{\text {c }}$ |  |  | Nd:FAP ${ }^{\text {d }}$ |  |  | Nd:FAP ${ }^{\text {e }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\lambda(\AA){ }^{\text {f }}$ | $\alpha^{\text {g }}$ | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{h}}$ | $\lambda(\AA){ }^{\text {f }}$ | $\alpha^{\text {g }}$ | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{h}}$ | $\lambda(\AA){ }^{\text {f }}$ | $\alpha^{\text {g }}$ | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{h}}$ | $P^{\text {i }}$ |
| ${ }^{4} I_{13 / 2}$ | 26278 | 0.15 | 3804 | 26190 | 0.1 | 3817 | 25170 | 0.02 | 3972 | $\sigma, \pi$ |
|  |  |  |  |  |  |  | 25040 | 0.04 | 3992 | $\sigma, \pi$ |
|  |  |  |  |  |  |  | 24620 | 0.04 | 4061 | $\sigma$ |
|  |  |  |  |  |  |  | 24560 | 0.05 | 4070 | $\sigma$ |
|  |  |  |  |  |  |  | 24370 | 0.06 | 4102 | $\sigma, \pi$ |
|  |  |  |  | 23650 | 0.6 | 4227 |  |  |  |  |
|  | 23 390(b) | 0.10 | 4275 | 23520 | 0.4 | 4251 |  |  |  |  |
|  |  |  |  | 23220 | 0.4 | 4306 |  |  |  |  |
|  | 23104 | 0.80 | 4327 |  |  |  | 22980 |  | 4350 | $\sigma, \pi$ |
|  |  |  |  | 22910 | 0.6 | 4363 | 22930 |  | 4360 | $\sigma, \pi$ |
|  | 22880 | 0.54 | 4370 | 22780 |  | 4389 | 22860 |  | 4372 | $\sigma$ |
|  | 22542 | 0.66 | 4435 |  |  |  | 22590 |  | 4425 | $\sigma, \pi$ |
|  | 22476 | 0.64 | 4448 |  |  |  |  |  |  |  |
|  | 22397 | 1.01 | 4464 |  |  |  |  |  |  |  |
|  | 21976 | 0.48 | 4549 | 22200 |  | 4503 |  |  |  |  |
| ${ }^{4} I_{15 / 2}$ | 17510 | 0.45 | 5710 | 17470 |  | 5723 | 17375 |  | 5754 | $\sigma, \pi$ |
|  |  |  |  |  |  |  | $16950$ |  | $5898$ | $\sigma, \pi$ |
|  |  |  |  |  |  |  | 16610 |  | 6018 | $\sigma$ |
|  |  |  |  | 16130 |  | 6198 |  |  |  |  |
|  | 16010 | 0.15 | 6244 |  |  |  | 16034 |  | 6235 | $\sigma, \pi$ |
|  | 15848 | 0.27 | 6308 | 15850 |  | 6307 |  |  |  |  |
|  |  |  |  | 15740 |  | 6352 | $15740$ |  | 6352 | $\sigma$ |
|  | 15662 | 0.20 | 6383 |  |  |  | $15670$ |  | 6380 | $\sigma, \pi$ |
|  | 15549 | 0.23 | 6430 | 15523 |  | 6440 |  |  |  |  |
|  | 15500 | 0.23 | 6450 | 15500 |  | 6450 |  |  |  |  |
|  | 15370 | 0.21 | 6505 | 15210 |  | 6573 | 15280 |  | 6543 | $\sigma, \pi$ |
|  | 15136 | 0.19 | 6605 |  |  |  | 15130 |  | 6608 | $\sigma$ |
|  | 14880 | 0.19 | 6718 | 14887 |  | 6715 |  |  |  |  |
| ${ }^{4} F_{3 / 2}$ |  |  |  | 8870(b) | 0.04 | 11270 |  |  |  |  |
|  |  |  |  | 8839.8 | 0.73 | 11309 |  |  |  |  |
|  | 8813.0 | 0.12 | 11344 | 8815(b) | 0.46 | 11341 |  |  |  |  |
|  | 8806.0 | 1.23 | 11353 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | 8765.0 | 0.06 | 11406 | $\sigma, \pi$ |
|  |  |  |  |  |  |  | 8737(b) | $0.02$ | $11442$ | $\sigma, \pi$ |
|  |  |  |  |  |  |  | 8704(b) | $0.03$ | $11486$ | $\sigma, \pi$ |
|  |  |  |  |  |  |  | 8664.0 |  | 11539 | $\sigma$ |
|  |  |  |  | 8566.1 | 0.81 | 11671 |  |  |  |  |
|  | $8537.4$ | $1.33$ | 11710 |  |  |  |  |  |  |  |
|  | $8522.2$ | $0.48$ | 11731 |  |  |  |  |  |  |  |
| ${ }^{4} F_{5 / 2}$ | 8160.1 | 0.04 | 12251 | 8120(b) | 0.38 | 12312 |  |  |  |  |
|  | 8088.3 | 0.06 | 12360 | 8100.0 | 1.25 | 12342 |  |  |  |  |
|  | 8080.2 | 0.06 | 12373 |  |  |  | 8085.6 | 0.20 | 12364 | $\sigma, \pi$ |
|  | 8046.4 | 2.30 | 12425 |  |  |  | 8074(b) | 0.13 | 12382 | $\sigma, \pi$ |
|  | 8030.0 | 0.07 | 12450 |  |  |  | 8010.4 | 0.21 | 12480 | $\sigma, \pi$ |
|  |  |  |  | 8003.2 | 2.03 | 12492 |  |  |  |  |
|  |  |  |  | 7977.0 | 1.52 | 12533 | 7968.2 | 0.11 | 12546 | $\sigma$ |
| ${ }^{2} H_{9 / 2}$ |  |  |  | 7941.8 | 2.08 | 12588 |  |  |  |  |
|  | 7933.8 | 1.10 | 12601 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | 7920.7 | 0.10 | 12622 | $\sigma, \pi$ |
|  | 7905.2 | 0.43 | 12646 |  |  |  | 7906.1 | 0.10 | 12645 | $\sigma$ |
|  | $7881.1$ | 0.79 | 12685 | 7879.0 | 0.50 | 12688 |  |  |  |  |
|  | 7852(b) | 0.03 | 12730 |  |  |  | 7840(b) | 0.15 | 12752 | $\sigma, \pi$ |
|  | 7823.5 | 0.02 | 12779 | 7835.0 | 1.13 | 12760 |  |  |  |  |
|  | 7797(b) | 0.02 | 12823 | 7798.0 | 0.40 | 12820 |  |  |  |  |
|  |  |  |  |  |  |  | 7774.1 | 0.05 | 12860 | $\sigma$ |
|  | 7759.9 | 0.31 | 12883 |  |  |  | 7758.0 | 0.08 | 12886 | $\sigma, \pi$ |
|  | 7721.6 | 0.21 | 12948 |  |  |  |  |  |  |  |
|  | 7706.3 | 0.12 | 12973 |  |  |  |  |  |  |  |
|  |  |  |  | 7506(b) | 0.50 | 13020 |  |  |  |  |
|  | 7572(b) | 0.03 | 13202 |  |  |  |  |  |  |  |
| ${ }^{4} F_{7 / 2}$ |  |  |  |  |  |  | $7524.0$ | $0.15$ | $13287$ | $\sigma, \pi$ |
|  |  |  |  | 7504.4 | 2.64 | 13326 | $7505(\mathrm{sh})$ | $0.75$ | $13320$ | $\sigma, \pi$ |

TABLE I. (Continued).

${ }^{\text {a }}$ Spectra obtained at 4 K ; total Nd concentration ( 0.29 at. wt \% in SFAP); total Nd concentration ( 0.56 at. wt \% in FAP).
${ }^{\text {b }}$ Multiplet manifolds of $\mathrm{Nd}^{3+}\left(4 f^{3}\right)$ split by the crystal field.
${ }^{\text {c }}$ Total spectra of $\mathrm{Nd}^{3+}$ ions in SFAP.
${ }^{\mathrm{d}}$ Spectra of $\mathrm{Nd}^{3+}$ ions in fluorescing sites in FAP.
${ }^{e}$ Spectra of $\mathrm{Nd}^{3+}$ ions in nonfluorescing sites in FAP.
${ }^{\mathrm{f}}$ Wavelength in $\AA, b$ denotes broad.
${ }^{\mathrm{g}}$ Intensity of axial spectra; $\alpha$ is in units of absorbance/cm.
${ }^{\mathrm{h}}$ Energy in units of vacuum wave numbers.
${ }^{i}$ Polarization of the transverse spectra.


FIG. 1. Emission spectrum at 4 K from the ${ }^{4} F_{3 / 2}$ to the ${ }^{4} I_{9 / 2}$ ground-state multiplet manifold; excitation of ${ }^{4} F_{9 / 2}(685 \mathrm{~nm}) \mathrm{Nd}: F A P$.
residing in different charge-compensated sites. The number of absorption peaks associated with any given multiplet manifold of $\mathrm{Nd}^{3+}\left(4 f^{3}\right)$ exceeds the expected number of $J+1 / 2$ transitions from the ground state of the ion in a single site. ${ }^{27,28}$ Most of the transitions observed in the spectra taken on crystals of Nd:SFAP (containing $0.29 \mathrm{wt} \% \mathrm{Nd}$ ) and Nd:FAP (containing $0.61 \mathrm{wt} \% \mathrm{Nd}$ ) appear in both $\sigma$ and $\pi$ polarizations in the transverse spectra. In Nd:FAP however, within each multiplet manifold we observe additional moderate to weak absorption peaks primarily in the $\sigma$ spectrum, along with weak spectra appearing in both polarizations.

Of the two fluorapatite crystals studied, the absorption spectrum of Nd:SFAP is easier to interpret because many fewer satellite peaks are observed within the wavelength range that spans a given multiplet. In fact Table I shows that for each manifold in Nd:SFAP there are usually $J+1 / 2$ relatively strong peaks accompanied by weaker and usually broader satellite features, suggesting that $\mathrm{Nd}^{3+}$ occupation of a single site may dominate the observed spectra. This feature


FIG. 2. Emission spectrum at 4 K from the ${ }^{4} F_{3 / 2}$ to the ${ }^{4} I_{11 / 2}$ multiplet manifold; excitation of ${ }^{4} G_{5 / 2},{ }^{2} G_{7 / 2}(588 \mathrm{~nm}) \mathrm{Nd}$ :FAP.


FIG. 3. Polarized absorption spectrum of the ${ }^{4} F_{9 / 2}$ multiplet manifold of $\mathrm{Nd}^{3+}$ in FAP at 4 K .
is attributed to the growth conditions of Nd:SFAP, the doping level of $\mathrm{Nd}^{3+}$ ions, ${ }^{8}$ and the intrinsic lattice properties of Nd:SFAP.

Site-selective excitation and fluorescence methods were used to identify transitions differentiating $\mathrm{Nd}^{3+}$ ions in the different sites in Nd:FAP and to confirm that there is a predominant site for $\mathrm{Nd}^{3+}$ ions in Nd:SFAP. The fluorescence spectra appearing in Figs. 1 and 2, representing transitions from the ${ }^{4} F_{3 / 2}$ manifold to the ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$ manifolds of $\mathrm{Nd}^{3+}$ in Nd:FAP, are primarily due to $\mathrm{Nd}^{3+}$ ions in only one of several sites. In Fig. 1 selectivity was achieved by narrow band ( $0.1 \mathrm{~cm}^{-1}$ ) excitation of the strongest peak ( 685.4 nm ) in the ${ }^{4} F_{9 / 2}$ manifold (see Fig. 3 and Table I) followed by nonradiative relaxation to the ${ }^{4} F_{3 / 2}$ manifold. In Fig. 2 we selectively excited one of the strong peaks in the ${ }^{4} G_{5 / 2},{ }^{2} G_{7 / 2}$ grouping at 588 nm . This excitation was followed by nonradiative relaxation to the ${ }^{4} F_{3 / 2}$ manifold. Similar experiments involving the same multiplets were carried out on Nd:SFAP where the results gave the expected number of transitions for


FIG. 4. Emission spectrum at 4 K from the ${ }^{4} F_{3 / 2}$ to the ${ }^{4} I_{9 / 2}$ ground-state multiplet manifold; excitation of ${ }^{4} F_{9 / 2}(683 \mathrm{~nm})$ Nd:SFAP.


FIG. 5. Emission spectrum at 4 K from the ${ }^{4} F_{3 / 2}$ to the ${ }^{4} I_{11 / 2}$ multiplet manifold; excitation of ${ }^{4} G_{5 / 2},{ }^{2} G_{7 / 2}(586 \mathrm{~nm}) \mathrm{Nd}$ :SFAP.
$\mathrm{Nd}^{3+}$ in a single site. Figure 4 and 5 present the emission spectra for the corresponding transitions in Nd:SFAP. From Figs. 1 and 2 as well as from Figs. 4 and 5 we can see clearly the large crystal-field splitting of the emitting $\mathrm{Nd}^{3+}$ ions. In both Nd:FAP and Nd:SFAP we observe fluorescence only from the ${ }^{4} F_{3 / 2}$ manifold. This observation is likely the result of a large crystal-field splitting that greatly mixes states above ${ }^{4} F_{3 / 2}$ and the presence of high-energy phonons that in coupling with electronic states can lead to strong nonradiative relaxation processes. ${ }^{9,24}$ The crystal-field splittings of the ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$ multiplets obtained from fluorescence measurements are given in Table II for both fluorapatites.

Figure 6 compares the 4 K excitation spectrum obtained by detecting fluorescence at $1063 \mathrm{~nm}\left({ }^{4} F_{3 / 2} \rightarrow{ }^{4} I_{11 / 2}\right.$, Nd:FAP) with the 4 K axial transmittance spectrum obtained between 500 and $540 \mathrm{~nm}\left({ }^{4} G_{9 / 2},{ }^{4} G_{7 / 2}\right)$. The site-selective excitation spectrum consists of five sharp peaks found between 505 and 517 nm (the number expected for ${ }^{4} \mathrm{G}_{9 / 2}$ ) and four strong peaks found between 520 and 530 nm (the number expected for ${ }^{4} G_{7 / 2}$ ). Conspicuously absent from the excitation spectrum are peaks associated with $\mathrm{Nd}^{3+}$ ions in other sites that are observed in the transmittance spectrum in Fig. 6. When excitation was carried out at wavelengths corresponding to some of these weak peaks in the transmittance spectrum, no fluorescence was observed at, or in the vicinity of 1063 nm .

In Nd:SFAP the absorption spectrum between 500 and 540 nm obtained at 4 K shows a total of nine strong peaks and several very weak broad bands indicating the dominance of a single site, with the ${ }^{4} G_{9 / 2},{ }^{4} G_{7 / 2}$ multiplets clearly resolved. Further evidence supporting a single fluorescing site in Nd:SFAP can be found by examining the 4 K absorption spectrum of the ${ }^{2} P_{1 / 2}$ multiplet. Over $90 \%$ of the integrated absorbance is found in a single peak at 431.5 nm with three other peaks at $430.3,430.9$, and 432.5 nm sharing less than $10 \%$ in total.

In Fig. 7 we compare the 4 K excitation spectrum produced by detecting fluorescence at $1063 \mathrm{~nm}\left({ }^{4} F_{3 / 2} \rightarrow{ }^{4} I_{11 / 2}\right)$


FIG. 6. (a) Transmittance spectrum of the ${ }^{4} G_{9 / 2},{ }^{4} G_{7 / 2}$ multiplet manifolds at 4 K ; (b) site-selective excitation of the ${ }^{4} G_{9 / 2},{ }^{4} G_{7 / 2}$ multiplets at 4 K Nd:FAP.
in Nd:FAP with the 4 K axial transmittance spectrum between 595 and $565 \mathrm{~nm}\left({ }^{2} G_{7 / 2},{ }^{4} G_{5 / 2}\right)$. The site-selective excitation spectrum is sharp, with three peaks (one possibly with a shoulder) found between 570 and 578 nm (the ${ }^{2} G_{7 / 2}$


FIG. 7. (a) Transmittance spectrum of the ${ }^{2} G_{7 / 2},{ }^{4} G_{5 / 2}$ multiplet manifolds at 4 K ; (b) site-selective excitation of the ${ }^{2} G_{7 / 2},{ }^{4} G_{5 / 2}$ multiplets at 4 K Nd:FAP.

TABLE II. Emission from ${ }^{4} F_{3 / 2}$ to ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$ at 4 K .

| $\lambda(\AA)^{\text {a }}$ | Nd:SFAP |  |  | Nd:FAP |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{b}}$ | $I^{\text {c }}$ | $\Delta E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{d}}$ | $\lambda(\AA)^{\mathrm{a}}$ | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{b}}$ | $I^{\text {c }}$ | $\Delta E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{d}}$ |
| 8797.0 | 11364 | vw | $\cdots$ | 8823.2 | 11331 | vw | $\cdots$ |
| 8812.0 | 11345 | vss | 0 | 8840.4 | 11309 | vss | 0 |
| 9217.1 | 10847 | m | 498 | 9163.2 | 10910 | vs | 399 |
|  |  |  |  | 9186.2 | 10883 | w |  |
| 9282.1 | 10771 | m | 574 | 9259.3 | 10797 | m | 512 |
| 9337.0 | 10707 | m | 638 | 9306.7 | 10742 | w | 567 |
| 9444 | 10585 | m | 760 | 9429 | 10602 | w | 707 |
| 10583 | 9447 | vs | 1899 | 10628 | 9407 | vss | 1902 |
| $11100(\mathrm{sh})$ | 9007 | w | 2340 | 11099 | 9007 | w | 2302 |
| 11148 | 8968 | w | 2378 | 11140 | 8974 | vw | 2335 |
|  |  |  |  | 11174 | 8947 | vw |  |
| 11212 | 8917 | m | 2429 | 11229 | 8903 | w | 2406 |
| 11264 | 8875 | m | 2470 | 11258 | 8880 | w | 2429 |
| 11355 | 8804 | w | 2541 | 11367 | 8795 | w | 2514 |

${ }^{\text {a }}$ Wavelength in $\AA$.
${ }^{\text {b }}$ Energy of transition in units of vacuum wave numbers.
${ }^{\mathrm{c}}$ Relative intensity within a manifold: vw (very weak); w (weak); m (moderate); s (strong); vs (very strong); vss (strongest).
${ }^{\mathrm{d}}$ Splitting of the multiplet manifold in $\mathrm{cm}^{-1}$; energy of the Stark level.
multiplet), and three peaks observed between 580 and 590 nm (the ${ }^{4} G_{5 / 2}$ multiplet). Clearly the simplicity and spectral resolution of the exciting laser, relative to the transmittance obtained from the Cary spectrophotometer, point to the advantage of using narrow-band laser excitation to differentiate between ions in different sites. The corresponding absorption spectrum at 4 K for Nd:SFAP (Table I) includes six very strong lines, a weak/broad band ( 570.6 nm ) and very weak shoulders around the base of most of the strong lines that were too difficult to measure systematically. The simplicity of the spectrum supports the notion that $\mathrm{Nd}^{3+}$ ions are found predominantly in a single site in Nd:SFAP.

Returning to Fig. 7, we tried excitation at wavelengths corresponding to absorption by $\mathrm{Nd}^{3+}$ ions in other sites in Nd:FAP, but failed to generate fluorescence. This suggests that perhaps while $\mathrm{Nd}^{3+}$ ions in numerous sites may absorb energy, only those in one of the sites fluoresce. Those that reside in the remaining so-called "dead" sites lose their energy nonradiatively; energy transfer to a different fluorescing center is not evident.

Laser excitation of Nd:FAP was also carried out while total fluorescence from the ${ }^{4} F_{3 / 2}$ manifold was measured nondispersively. This was accomplished by placing a highpass optical filter in front of the germanium detector to allow detection of wavelengths greater than 800 nm . This permitted the simultaneous detection of all fluorescence associated with transitions between the ${ }^{4} F_{3 / 2}$ manifold, and the ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$ manifold for $\mathrm{Nd}^{3+}$ ions in all sites. These experiments indicate that the fluorescence originated primarily from a single site, identified as the fluorescing site.

## DISCUSSION AND CALCULATIONS

The spectra reported in Table I were obtained from $\mathrm{Nd}^{3+}$-doped fluorapatite crystals which have a hexagonal structure that belongs to the $P 6_{3} / m$ (\# 176) space group
with two molecules per unit cell. ${ }^{22}$ The divalent metal ions ( $\mathrm{Sr}, \mathrm{Ca}$ ) occupy two sites, $\mathrm{M}(\mathrm{I})$ in the $4 f$ site with $C_{3}$ symmetry and $\mathrm{M}(\mathrm{II})$ in the $6 h$ site with $C_{s}$ symmetry. The ratio of metal ions in these two sites is 60 to 40 . Surrounding the $\mathrm{M}(\mathrm{I})$ site are six nearest-neighbor oxygen ions that form a distorted triangular prism. The $M(I I)$ sites sit at the corners of equilateral triangles with the $\mathrm{F}^{-}$ion in the center. Substitution of trivalent neodymium for a divalent cation was achieved by passive charge compensation during crystal growth. ${ }^{1}$

Recently, Morrison ${ }^{27}$ completed a point charge analysis of symmetry-preserving charge compensation and vacancies in the fluorapatites doped with $\mathrm{Nd}^{3+}$ ions. Our approach in the present study has been to calculate the crystal-field splitting of the energy levels of Nd:SFAP and Nd:FAP by considering the symmetry of the site ${ }^{27,28}$ and the subsequent data sets as the primary guide to the phenomenological set of crystal-field parameters $B_{n m}$ obtained from the analysis. To interpret the splitting of the levels in the fluorescing sites, we used as a starting set of $B_{n m}$ the values reported by Peale et al. ${ }^{5}$ for Nd:SVAP. Our calculations were based on $C_{s}$ symmetry, the same symmetry used in Ref. 5.

The total Hamiltonian for the $\mathrm{Nd}^{3+}$ ion includes terms representing the free-ion and the crystal electric-field interactions. The Racah and spin-orbit parameters entering into the free-ion Hamiltonian were derived from an analysis of the aqueous solution spectra. ${ }^{30}$ The appropriate values as well as the details of the computation are given elsewhere. ${ }^{30-34}$ The crystal-field Hamiltonian is of the form

$$
\begin{equation*}
H_{\mathrm{CEF}}=\sum_{n} \sum_{\text {even }}^{n} B_{n=-n}^{*} \sum_{i=1}^{N} C_{n m}(i), \tag{1}
\end{equation*}
$$

where the $B_{n m}$ are the crystal-field parameters and the expressions $C_{n m}$ are given as

TABLE III. Crystal-field splitting: $\mathrm{Nd}^{3+}$ ions in $C_{s}$ sites

| ${ }^{2 S+1} L_{J}{ }^{\text {a }}$ | No. ${ }^{\text {b }}$ | $\begin{gathered} E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{c}} \\ (\text { expt. }) \end{gathered}$ | $\begin{gathered} E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{d}} \\ (\text { calc. }) \end{gathered}$ | Free-ion percent mixture |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{4} I_{9 / 2}$ | 1 | 0 | 6.0 | $97.7{ }^{4} I_{9 / 2}+2.12{ }^{4} I_{11 / 2}+0.12{ }^{4} I_{13 / 2}$ |
| 545 | 1 | 0 | -0.4 | $98.1{ }^{4} I_{9 / 2}+1.72{ }^{4} I_{11 / 2}+0.10{ }^{4} I_{13 / 2}$ |
| (482) | 2 | 498 | 486 | $98.9{ }^{4} I_{9 / 2}+0.76{ }^{4} I_{11 / 2}+0.19{ }^{4} G_{5 / 2}$ |
|  | 2 | 399 | 387 | $98.7{ }^{4} I_{9 / 2}+1.00{ }^{4} I_{11 / 2}+0.15{ }^{4} G_{5 / 2}$ |
|  | 3 | 574 | 566 | $99.2{ }^{4} I_{9 / 2}+0.55{ }^{4} I_{11 / 2}+0.11{ }^{4} G_{5 / 2}$ |
|  | 3 | 512 | 512 | $99.5{ }^{4} I_{9 / 2}+0.32{ }^{4} I_{11 / 2}+0.10{ }^{4} G_{5 / 2}$ |
|  | 4 | 638 | 647 | $98.2{ }^{4} I_{9 / 2}+1.64{ }^{4} I_{11 / 2}+0.08{ }^{4} I_{13 / 2}$ |
|  | 4 | 567 | 575 | $99.0{ }^{4} I_{9 / 2}+0.84{ }^{4} I_{11 / 2}+0.10{ }^{4} G_{5 / 2}$ |
|  | 5 | 760 | 764 | $99.0{ }^{4} I_{9 / 2}+0.73{ }^{4} I_{11 / 2}+0.13{ }^{4} G_{5 / 2}$ |
|  | 5 | 707 | 712 | $98.8{ }^{4} I_{9 / 2}+0.96{ }^{4} I_{11 / 2}+0.11{ }^{4} G_{5 / 2}$ |
| ${ }^{4} I_{11 / 2}$ | 6 | 1899 | 1907 | $95.8{ }^{4} I_{11 / 2}+2.17{ }^{4} I_{13 / 2}+1.88{ }^{4} I_{9 / 2}$ |
| 2361 | 6 | 1902 | 1911 | $97.0{ }^{4} I_{11 / 2}+1.84{ }^{4} I_{13 / 2}+1.04{ }^{4} I_{9 / 2}$ |
| (2332) | 7 | 2340 | 2339 | $97.4{ }^{4} I_{11 / 2}+1.62{ }^{4} I_{9 / 2}+0.87{ }^{4} I_{13 / 2}$ |
|  | 7 | 2302 | 2307 | $98.1{ }^{4} I_{11 / 2}+1.15{ }^{4} I_{9 / 2}+0.64{ }^{4} I_{13 / 2}$ |
|  | 8 | 2378 | 2368 | $98.3{ }^{4} I_{11 / 2}+0.77{ }^{4} I_{9 / 2}+0.68{ }^{4} I_{13 / 2}$ |
|  | 8 | 2335 | 2327 | $98.2{ }^{4} I_{11 / 2}+0.89{ }^{4} I_{9 / 2}+0.71{ }^{4} I_{13 / 2}$ |
|  | 9 | 2429 | 2418 | $98.9{ }^{4} I_{11 / 2}+0.54{ }^{4} I_{13 / 2}+0.43{ }^{4} I_{9 / 2}$ |
|  | 9 | 2406 | 2393 | $99.0{ }^{4} I_{11 / 2}+0.58{ }^{4} I_{9 / 2}+0.32{ }^{4} I_{13 / 2}$ |
|  | 10 | 2470 | 2471 | $98.6{ }^{4} I_{11 / 2}+0.71{ }^{4} I_{9 / 2}+0.50{ }^{4} I_{13 / 2}$ |
|  | 10 | 2429 | 2431 | $98.5{ }^{4} I_{11 / 2}+0.80{ }^{4} I_{9 / 2}+0.56{ }^{4} I_{13 / 2}$ |
|  | 11 | 2541 | 2546 | $98.5{ }^{4} I_{11 / 2}+0.80{ }^{4} I_{13 / 2}+0.47{ }^{4} I_{9 / 2}$ |
|  | 11 | 2514 | 2521 | $98.4{ }^{4} I_{11 / 2}+0.93{ }^{4} I_{13 / 2}+0.45{ }^{4} I_{9 / 2}$ |
| ${ }^{4} I_{13 / 2}$ | 12 | 3804 | 3815 | $96.6{ }^{4} I_{13 / 2}+1.69{ }^{4} I_{15 / 2}+1.54{ }^{4} I_{11 / 2}$ |
| 4323 | 12 | 3817 | 3799 | $97.6{ }^{4} I_{13 / 2}+1.38{ }^{4} I_{15 / 2}+0.95{ }^{4} I_{11 / 2}$ |
| (4271) | 13 | 4275 | 4275 | $96.4{ }^{4} I_{13 / 2}+2.25{ }^{4} I_{11 / 2}+1.20{ }^{4} I_{15 / 2}$ |
|  | 13 | 4227 | 4220 | $97.3{ }^{4} I_{13 / 2}+1.80{ }^{4} I_{11 / 2}+0.78{ }^{4} I_{15 / 2}$ |
|  | 14 | 4327 | 4328 | $99.1{ }^{4} I_{13 / 2}+0.45{ }^{4} I_{15 / 2}+0.19{ }^{4} I_{11 / 2}$ |
|  | 14 | 4251 | 4258 | $98.9{ }^{4} I_{13 / 2}+0.59{ }^{4} I_{15 / 2}+0.40{ }^{4} I_{11 / 2}$ |
|  | 15 | 4370 | 4370 | $99.1{ }^{4} I_{13 / 2}+0.55{ }^{4} I_{15 / 2}+0.27{ }^{4} I_{11 / 2}$ |
|  | 15 | 4306 | 4321 | $99.3{ }^{4} I_{13 / 2}+0.34{ }^{4} I_{15 / 2}+0.25{ }^{4} I_{11 / 2}$ |
|  | 16 | ... | 4419 | $98.5{ }^{4} I_{13 / 2}+0.75{ }^{4} I_{11 / 2}+0.63{ }^{4} I_{15 / 2}$ |
|  | 16 | 4363 | 4357 | $98.4{ }^{4} I_{13 / 2}+0.89{ }^{4} I_{11 / 2}+0.57{ }^{4} I_{15 / 2}$ |
|  | 17 | 4464 | 4455 | $99.0{ }^{4} I_{13 / 2}+0.47{ }^{4} I_{15 / 2}+0.39{ }^{4} I_{11 / 2}$ |
|  | 17 | 4389 | 4395 | $99.0{ }^{4} I_{13 / 2}+0.50{ }^{4} I_{15 / 2}+0.36{ }^{4} I_{11 / 2}$ |
|  | 18 | 4549 | 4549 | $98.7{ }^{4} I_{13 / 2}+0.75{ }^{4} I_{15 / 2}+0.28{ }^{4} I_{11 / 2}$ |
|  | 18 | 4503 | 4505 | $98.7{ }^{4} I_{13 / 2}+0.84{ }^{4} I_{15 / 2}+0.32{ }^{4} I_{11 / 2}$ |
| ${ }^{4} I_{15 / 2}$ | 19 | 5710 | 5701 | $98.2{ }^{4} I_{15 / 2}+1.66{ }^{4} I_{13 / 2}+0.09{ }^{4} I_{11 / 2}$ |
| 6362 | 19 | 5723 | 5733 | $98.9{ }^{4} I_{15 / 2}+0.99{ }^{4} I_{13 / 2}+0.06{ }^{4} I_{11 / 2}$ |
| (6361) | 20 | 6244 | 6250 | $98.4{ }^{4} I_{15 / 2}+1.38{ }^{4} I_{13 / 2}+0.08{ }^{4} F_{9 / 2}$ |
|  | 20 | 6198 | 6203 | $98.6{ }^{4} I_{15 / 2}+1.15{ }^{4} I_{13 / 2}+0.06{ }^{4} F_{9 / 2}$ |
|  | 21 | 6308 | 6312 | $99.3{ }^{4} I_{15 / 2}+0.46{ }^{4} I_{13 / 2}+0.07{ }^{4} F_{9 / 2}$ |
|  | 21 | 6307 | 6298 | $99.2{ }^{4} I_{15 / 2}+0.66{ }^{4} I_{13 / 2}+0.05{ }^{4} F_{7 / 2}$ |
|  | 22 | 6383 | 6387 | $99.2{ }^{4} I_{15 / 2}+0.57{ }^{4} I_{13 / 2}+0.07{ }^{4} I_{11 / 2}$ |
|  | 22 | 6352 | 6357 | $99.4{ }^{4} I_{15 / 2}+0.48{ }^{4} I_{13 / 2}+0.04{ }^{4} I_{11 / 2}$ |
|  | 23 | 6450 | 6453 | $99.0{ }^{4} I_{15 / 2}+0.79{ }^{4} I_{13 / 2}+0.05{ }^{4} F_{9 / 2}$ |
|  | 23 | 6440 | 6433 | $99.6{ }^{4} I_{15 / 2}+0.32{ }^{4} I_{13 / 2}+0.04{ }^{4} I_{11 / 2}$ |
|  | 24 | 6505 | 6510 | $99.5{ }^{4} I_{15 / 2}+0.36{ }^{4} I_{13 / 2}+0.03{ }^{4} I_{11 / 2}$ |
|  | 24 | 6450 | 6457 | $99.0{ }^{4} I_{15 / 2}+0.74{ }^{4} I_{13 / 2}+0.07{ }^{4} F_{9 / 2}$ |
|  | 25 | 6605 | 6602 | $99.6{ }^{4} I_{15 / 2}+0.32{ }^{4} I_{13 / 2}+0.03{ }^{4} F_{7 / 2}$ |
|  | 25 | 6573 | 6569 | $99.6{ }^{4} I_{15 / 2}+0.28{ }^{4} I_{13 / 2}+0.04{ }^{4} F_{7 / 2}$ |
|  | 26 | 6718 | 6710 | $99.5{ }^{4} I_{15 / 2}+0.26{ }^{4} I_{13 / 2}+0.07{ }^{4} F_{9 / 2}$ |
|  | 26 | 6715 | 6708 | $99.5{ }^{4} I_{15 / 2}+0.28{ }^{4} I_{13 / 2}+0.05{ }^{4} F_{7 / 2}$ |
| ${ }^{4} F_{3 / 2}$ | 27 | 11353 | 11358 | $93.7{ }^{4} F_{3 / 2}+4.61{ }^{4} F_{5 / 2}+0.56{ }^{4} F_{7 / 2}$ |
| 11666 | 27 | 11309 | 11322 | $94.1{ }^{4} F_{3 / 2}+4.42{ }^{4} F_{5 / 2}+0.50{ }^{4} G_{5 / 2}$ |
| (11 614) | 28 | 11710 | 11706 | $91.8{ }^{4} F_{3 / 2}+5.00{ }^{4} F_{5 / 2}+1.40{ }^{5} G_{5 / 2}$ |
|  | 28 | 11671 | 11658 | $92.1{ }^{4} F_{3 / 2}+5.22{ }^{4} F_{5 / 2}+1.25{ }^{4} G_{5 / 2}$ |
| ${ }^{4} F_{5 / 2}$ | 29 | 12425 | 12431 | $82.1{ }^{4} F_{5 / 2}+7.48{ }^{2} H_{9 / 2}+5.37{ }^{4} F_{7 / 2}$ |
| 12697 | 29 | 12342 | 12345 | $78.1{ }^{4} F_{5 / 2}+12.9{ }^{2} H_{9 / 2}+4.55{ }^{4} F_{7 / 2}$ |
| (12 593) | 30 | 12601 | 12601 | $52.7{ }^{4} F_{5 / 2}+41.7{ }^{2} H_{9 / 2}+2.53{ }^{4} F_{3 / 2}$ |
|  | 30 | 12492 | 12475 | $76.7{ }^{2} H_{9 / 2}+20.7{ }^{4} F_{5 / 2}+1.16{ }^{4} F_{3 / 2}$ |

TABLE III. (Continued.)

| ${ }^{2 S+1} L_{J}{ }^{\text {a }}$ | No. ${ }^{\text {b }}$ | $\begin{gathered} E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{c}} \\ (\text { expt. }) \end{gathered}$ | $\begin{gathered} E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{d}} \\ (\text { calc. }) \end{gathered}$ | Free-ion percent mixture |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{2} H_{9 / 2}$ | 31 | 12664 | 12685 | $61.1{ }^{2} H_{9 / 2}+35.2{ }^{4} F_{5 / 2}+1.67{ }^{4} F_{3 / 2}$ |
| 12912 | 31 | 12533 | 12532 | $56.1{ }^{4} F_{5 / 2}+38.6{ }^{2} H_{9 / 2}+2.75{ }^{4} F_{3 / 2}$ |
| (12 375) | 32 | ... | 12736 | $56.4{ }^{4} F_{5 / 2}+40.1{ }^{2} H_{9 / 2}+1.08{ }^{4} F_{3 / 2}$ |
|  | 32 | 12588 | 12589 | $50.4{ }^{2} H_{9 / 2}+46.4{ }^{4} F_{5 / 2}+1.21{ }^{4} F_{3 / 2}$ |
|  | 33 | ... | 12820 | $76.1{ }^{2} H_{9 / 2}+20.3{ }^{4} F_{5 / 2}+1.48{ }^{4} F_{3 / 2}$ |
|  | 33 | 12688 | 12676 | $63.4{ }^{2} H_{9 / 2}+32.9{ }^{4} F_{5 / 2}+1.90{ }^{4} F_{3 / 2}$ |
|  | 34 | 12948 | 12951 | $88.9{ }^{2} H_{9 / 2}+9.07{ }^{4} F_{5 / 2}+1.13{ }^{4} F_{7 / 2}$ |
|  | 34 | 12760 | 12772 | $85.1{ }^{2} H_{9 / 2}+13.0{ }^{4} F_{5 / 2}+1.08{ }^{4} F_{7 / 2}$ |
|  | 35 | 12973 | 12984 | $81.1{ }^{2} H_{9 / 2}+16.7{ }^{4} F_{5 / 2}+1.63{ }^{4} F_{7 / 2}$ |
|  | 35 | 12820 | 12829 | $70.3{ }^{2} H_{9 / 2}+27.5{ }^{4} F_{5 / 2}+1.51{ }^{4} F_{7 / 2}$ |
|  | 36 | ... | 13215 | $96.5{ }^{2} H_{9 / 2}+2.04{ }^{4} F_{5 / 2}+0.63{ }^{4} F_{9 / 2}$ |
|  | 36 | 13020 | 13024 | $96.9{ }^{2} H_{9 / 2}+1.81{ }^{4} F_{5 / 2}+0.54{ }^{4} F_{9 / 2}$ |
| ${ }^{4} S_{3 / 2}$ | 37 | 13393 | 13396 | $89.8{ }^{4} F_{7 / 2}+3.82{ }^{4} F_{5 / 2}+3.52{ }^{4} F_{9 / 2}$ |
| 13554 | 37 | 13322 | 13327 | $92.3{ }^{4} F_{7 / 2}+2.87{ }^{4} F_{9 / 2}+2.54{ }^{4} F_{5 / 2}$ |
| (13 498) | 38 | 13510 | 13513 | $93.5{ }^{4} S_{3 / 2}+5.07{ }^{4} F_{7 / 2}+0.49{ }^{4} G_{5 / 2}$ |
|  | 38 | 13460 | 13461 | $80.4{ }^{4} S_{3 / 2}+17.3{ }^{4} F_{7 / 2}+0.83{ }^{4} F_{5 / 2}$ |
| ${ }^{4} F_{7 / 2}$ | 39 | 13565 | 13561 | $61.0{ }^{4} F_{7 / 2}+32.8{ }^{4} S_{3 / 2}+2.76{ }^{4} F_{5 / 2}$ |
| 13676 | 39 | 13481 | 13490 | $73.3{ }^{4} F_{7 / 2}+20.1{ }^{4} S_{3 / 2}+3.29{ }^{4} F_{5 / 2}$ |
| (13 597) | 40 | 13572 | 13574 | $70.4{ }^{4} S_{3 / 2}+26.2{ }^{4} F_{7 / 2}+1.55{ }^{4} F_{5 / 2}$ |
|  | 40 | 13517 | 13515 | $97.1{ }^{4} S_{3 / 2}+2.12{ }^{4} F_{7 / 2}+0.18{ }^{4} F_{9 / 2}$ |
|  | 41 | 13692 | 13692 | $93.4{ }^{4} F_{7 / 2}+2.76{ }^{4} F_{9 / 2}+1.21{ }^{4} F_{5 / 2}$ |
|  | 41 | 13613 | 13611 | $94.4{ }^{4} F_{7 / 2}+2.24{ }^{4} F_{9 / 2}+1.33{ }^{4} F_{5 / 2}$ |
|  | 42 | ... | 13998 | $95.1{ }^{4} F_{7 / 2}+1.58{ }^{2} H_{9 / 2}+1.10{ }^{2} G_{7 / 2}$ |
|  | 42 | 13920 | 13910 | $95.7{ }^{4} F_{7 / 2}+1.21{ }^{4} F_{9 / 2}+1.20{ }^{2} H_{9 / 2}$ |
| ${ }^{4} F_{9 / 2}$ | 43 | 14641 | 14627 | $95.9{ }^{4} F_{9 / 2}+1.96{ }^{4} F_{7 / 2}+0.89{ }^{4} F_{5 / 2}$ |
| 14901 | 43 | 14586 | 14584 | $96.9{ }^{4} F_{9 / 2}+1.42{ }^{4} F_{7 / 2}+0.67{ }^{4} F_{5 / 2}$ |
| (14 845) | 44 | 14808 | 14813 | $94.3{ }^{4} F_{9 / 2}+4.42{ }^{4} F_{7 / 2}+0.54{ }^{2} H_{11 / 2}$ |
|  | 44 | 14745 | 14742 | $95.3{ }^{4} F_{9 / 2}+3.66{ }^{4} F_{7 / 2}+0.47{ }^{2} H_{11 / 2}$ |
|  | 45 | 14939 | 14946 | $94.3{ }^{4} F_{9 / 2}+3.20{ }^{4} F_{7 / 2}+1.90{ }^{2} H_{11 / 2}$ |
|  | 45 | 14890 | 14893 | $94.9{ }^{4} F_{9 / 2}+2.94{ }^{4} F_{7 / 2}+1.58{ }^{2} H_{11 / 2}$ |
|  | 46 | ... | 15083 | $95.5{ }^{4} F_{9 / 2}+2.62{ }^{2} H_{11 / 2}+1.09{ }^{2} G_{7 / 2}$ |
|  | 46 | 15020 | 15020 | $95.8{ }^{4} F_{9 / 2}+2.33{ }^{2} H_{11 / 2}+0.96{ }^{2} G_{7 / 2}$ |
|  | 47 | 15113 | 15117 | $94.5{ }^{4} F_{9 / 2}+4.59{ }^{2} H_{11 / 2}+0.38{ }^{2} H_{9 / 2}$ |
|  | 47 | 15052 | 15054 | $95.1{ }^{4} F_{9 / 2}+4.06{ }^{2} H_{11 / 2}+0.36{ }^{2} H_{9 / 2}$ |
| ${ }^{2} H_{11 / 2}$ | 48 | ... | 16086 | $98.4{ }^{2} H_{11 / 2}+0.94{ }^{2} G_{7 / 2}+0.20{ }^{4} F_{9 / 2}$ |
|  | 48 | 16067 | 16069 | $98.7{ }^{2} H_{11 / 2}+0.60{ }^{2} G_{7 / 2}+0.21{ }^{4} F_{9 / 2}$ |
| 16152 | 49 | 16124 | 16119 | $98.5{ }^{2} H_{11 / 2}+0.50{ }^{4} F_{9 / 2}+0.50{ }^{2} G_{7 / 2}$ |
| (16 128) | 49 | 16086 | 16093 | $98.7{ }^{2} H_{11 / 2}+0.52{ }^{4} F_{9 / 2}+0.36{ }^{2} G_{7 / 2}$ |
|  | 50 | 16148 | 16145 | $97.7{ }^{2} H_{11 / 2}+1.17{ }^{4} F_{9 / 2}+0.51{ }^{2} G_{7 / 2}$ |
|  | 50 | 16119 | 16123 | $98.2{ }^{2} H_{11 / 2}+1.02{ }^{4} F_{9 / 2}+0.33{ }^{2} G_{7 / 2}$ |
|  | 51 | 16185 | 16189 | $96.8{ }^{2} H_{11 / 2}+1.46{ }^{2} G_{7 / 2}+1.25{ }^{4} F_{9 / 2}$ |
|  | 51 | 16165 | 16169 | $96.9{ }^{2} H_{11 / 2}+1.41{ }^{4} F_{9 / 2}+1.14{ }^{2} G_{7 / 2}$ |
|  | 52 | 16232 | 16243 | $98.1{ }^{2} H_{11 / 2}+1.05{ }^{4} F_{9 / 2}+0.57{ }^{2} G_{7 / 2}$ |
|  | 52 | 16227 | 16210 | $98.6{ }^{2} H_{11 / 2}+0.68{ }^{4} F_{9 / 2}+0.42{ }^{2} G_{7 / 2}$ |
|  | 53 | 16303 | 16300 | $93.4{ }^{2} H_{11 / 2}+5.03{ }^{4} F_{9 / 2}+0.73{ }^{2} H_{9 / 2}$ |
|  | 53 | ... | 16261 | $94.5{ }^{2} H_{11 / 2}+4.37{ }^{4} F_{9 / 2}+0.58{ }^{2} H_{9 / 2}$ |
| ${ }^{4} G_{5 / 2}$ | 54 | 17044 | 17041 | $56.4{ }^{4} G_{5 / 2}+41.0{ }^{2} G_{7 / 2}+0.93{ }^{4} F_{3 / 2}$ |
| 17164 | 54 | 16995 | 16987 | $91.9{ }^{4} G_{5 / 2}+5.03{ }^{2} G_{7 / 2}+1.93{ }^{4} F_{3 / 2}$ |
| (17 041) | 55 | ... | 17137 | $67.5{ }^{4} G_{5 / 2}+29.3{ }^{2} G_{7 / 2}+1.51{ }^{4} F_{3 / 2}$ |
|  | 55 | 17090 | 17087 | $78.1{ }^{4} G_{5 / 2}+20.1{ }^{2} G_{7 / 2}+0.56{ }^{4} F_{3 / 2}$ |
|  | 56 | 17243 | 17239 | $75.1{ }^{2} G_{7 / 2}+22.6{ }^{4} G_{5 / 2}+0.81{ }^{2} H_{11 / 2}$ |
|  | 56 | 17219 | 17234 | $60.2{ }^{4} G_{5 / 2}+37.1{ }^{2} G_{7 / 2}+1.11{ }^{4} F_{5 / 2}$ |

TABLE III. (Continued.)

| ${ }^{2 S+1} L_{J}{ }^{\mathrm{a}}$ | No. ${ }^{\mathrm{b}}$ | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{c}}$ <br> $($ (expt.) | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{d}}$ <br> $(\mathrm{calc}$.) | Free-ion percent mixture |
| :--- | :---: | :---: | :---: | :---: |
| ${ }^{2} G_{7 / 2}$ | 57 | 17293 | 17296 | $90.7{ }^{2} G_{7 / 2}+6.87{ }^{4} G_{5 / 2}+1.24{ }^{4} F_{5 / 2}$ |
| 17217 | 57 | 17361 | 17358 | $74.2{ }^{2} G_{7 / 2}+24.1{ }^{4} G_{5 / 2}+1.16{ }^{4} F_{5 / 2}$ |
| $(17323)$ | 58 | 17396 | 17392 | $63.7{ }^{2} G_{7 / 2}+33.4{ }^{4} G_{5 / 2}+0.97{ }^{2} H_{11 / 2}$ |
|  | 58 | 17412 | 17404 | $92.9{ }^{2} G_{7 / 2}+5.35{ }^{4} G_{5 / 2}+0.69{ }^{2} H_{11 / 2}$ |
|  | 59 | 17459 | 17462 | $64.4{ }^{2} G_{7 / 2}+32.4{ }^{4} G_{5 / 2}+0.98{ }^{4} F_{9 / 2}$ |
|  | 59 | 17462 | 17468 | $72.7{ }^{2} G_{7 / 2}+26.1{ }^{4} G_{5 / 2}+0.30{ }^{2} H_{11 / 2}$ |
|  | 60 | 17540 | 17540 | $73.7{ }^{4} G_{5 / 2}+24.4{ }^{2} G_{7 / 2}+0.58{ }^{4} F_{7 / 2}$ |
|  | 60 | 17525 | 17527 | $89.2{ }^{2} G_{7 / 2}+7.64{ }^{4} G_{5 / 2}+1.02{ }^{4} F_{7 / 2}$ |

${ }^{\text {a }}$ Multiplet manifold of $\mathrm{Nd}^{3+}\left(4 f^{3}\right)$; the centroid is given in $\mathrm{cm}^{-1}$; the number without parentheses is the energy for Nd:SFAP; the number with parentheses is the energy for Nd:FAP.
${ }^{\mathrm{b}}$ First number in the pair represents the splitting for Nd:SFAP; the second number represents the splitting for Nd :FAP in the fluorescing site.
${ }^{\text {c }}$ Experimental data from Tables I and II.
${ }^{\mathrm{d}}$ Calculated splitting based on $B_{n m}$ parameters given in Table IV.

$$
\begin{equation*}
C_{n m}(i)=\left(\frac{4 \pi}{(2 n+1)}\right)^{1 / 2} Y_{n m}\left(\theta_{i}, \phi_{i}\right) \tag{2}
\end{equation*}
$$

with

$$
\begin{equation*}
C_{n-m}=(-1)^{m} C_{n m}^{*}, \tag{3}
\end{equation*}
$$

and the $Y_{n m}$ are the usual spherical harmonics. In $C_{s}$ symmetry, the values of $m$ are restricted by $n+m=0, \pm 2, \pm 4$, with $|m| \leqslant n$ and $n=2,4$, and 6 . The crystal-field splitting is predominately determined by the $n$ even $B_{n m}$ parameters. For $C_{s}$ symmetry this number is restricted to 15 , but with a simple rotation about the principal axis ( $c$ axis), $B_{22}$ can be made real and positive and the number of parameters is then reduced to 14 .

In Table III we present the results for the lowest 13 mul tiplet manifolds of $\mathrm{Nd}^{3+}$ ions in sites that fluoresce in Nd :SFAP and Nd:FAP. Because of the size of the splittings, the free-ion percent mixture is also included. Multiplet manifolds ${ }^{4} F_{5 / 2},{ }^{2} H_{9 / 2}$, and ${ }^{4} F_{7 / 2},{ }^{4} S_{3 / 2}$ are highly mixed making

TABLE IV. Final set of $B_{n m}$ parameters for $\mathrm{Nd}^{3+}$ ions in fluorescing sites.

| $B_{n m}$ <br> parameter | SFAP $^{\mathrm{a}}$ <br> $\left(\mathrm{cm}^{-1}\right)$ | FAP $^{\mathrm{b}}$ <br> $\left(\mathrm{cm}^{-1}\right)$ | SVAP $^{\mathrm{c}}$ <br> $\left(\mathrm{cm}^{-1}\right)$ |
| :---: | :---: | :---: | :---: |
| $B_{20}$ | 2647 | 2528 | 2452 |
| $B_{22}$ | 533 | 520 | 410 |
| $B_{40}$ | 2075 | 1896 | 2219 |
| $\operatorname{Re} B_{42}$ | -138 | -278 | 320 |
| $I_{m} B_{42}$ | 883 | 439 | 57.6 |
| $\operatorname{Re} B_{44}$ | 136 | 53.4 | -148 |
| $I_{m} B_{44}$ | 223 | 444 | -186 |
| $B_{60}$ | 592 | 483 | 435 |
| $\operatorname{Re} B_{62}$ | 173 | 289 | -81.3 |
| $I_{m} B_{62}$ | 514 | 427 | 253 |
| $\operatorname{Re} B_{64}$ | -13.6 | -170 | -255 |
| $I_{m} B_{64}$ | -514 | -473 | 122 |
| $\operatorname{Re} B_{66}$ | -362 | -407 | 266 |
| $I_{m} B_{66}$ | -16.9 | -131 | 395 |

${ }^{\text {a }}$ rms derivation: $7 \mathrm{~cm}^{-1}$, 52 levels (this work).
${ }^{\mathrm{b}} \mathrm{rms}$ derivation: $8 \mathrm{~cm}^{-1}, 59$ levels (this work).
${ }^{\mathrm{c}}$ rms derivation: $6 \mathrm{~cm}^{-1}, 28$ levels (Ref. 5).
their ${ }^{2 S+1} L_{J}$ manifold assignments problematic for individual Stark levels. We made repeated attempts to improve our analysis by using $B_{n m}$ parameters obtained from latticesum calculations by Morrison ${ }^{27}$ based on different chargecompensated models for $\mathrm{Nd}^{3+}$ ions in $\mathrm{M}(\mathrm{II})$ sites. These efforts did not lead to any success. The final set of phenomenological $B_{n m}$ parameters is compared in Table IV with the set reported for Nd:SVAP (our initial starting set). The similarity in sign and magnitude among the dominant terms, including parameters $B_{20}, B_{22}$, and $B_{40}$, suggest a common symmetry site is involved in all three crystals. Variation among smaller parameters is due to our inclusion of numerous excited multiplet manifolds that are so highly mixed (see Table III) that relatively small changes in the crystal-field environment cause large changes in the $J$ mixing among these manifolds. Peale et al. ${ }^{5}$ restricted their analysis to the splitting of the ${ }^{4} I_{J}$ and ${ }^{4} F_{3 / 2}$ manifolds alone.

We also carried out crystal-field splitting calculations for $\mathrm{Nd}^{3+}$ ions occupying $C_{3}$ sites in Nd:FAP. In this case we found that one of Morrison's lattice-sum calculations provided us with a reasonable starting set of $B_{n m}$ parameters. ${ }^{27,35,36}$ This model involves the substitution of $\mathrm{Nd}^{3+}$ ions into $\mathrm{M}(\mathrm{I})$ sites, assuming that the charge compensation is sufficiently remote so that $C_{3}$ symmetry is preserved. Maksimova and Sobol ${ }^{15,16}$ also describe a chargecompensation model of this type. Since we observe no fluorescence for this site on which to base our assignments, we chose the symmetry of the ground-state Stark level to be $\Gamma_{4,5}$ as predicted using the Morrison parameters. ${ }^{27}$ This choice is also consistent with the observed polarized absorption spectra using $C_{3}$ symmetry selection rules. ${ }^{28}$ The results of these calculations are given in Table V for multiplet manifolds above ${ }^{4} I_{11 / 2}$ where the experimental energy levels can be drawn from Table I since no emission to the ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$ manifolds was observed. Again, the most difficult Stark levels to identify are those with a high degree of $J$ mixing. The ${ }^{4} F_{5 / 2},{ }^{2} H_{9 / 2}$ manifolds and the ${ }^{4} F_{7 / 2},{ }^{4} S_{3 / 2}$ manifolds represent the most difficult manifolds to analyze.

In Table VI we see that relatively small changes in the

TABLE V. Crystal-field splitting: $\mathrm{Nd}^{3+}$ ions in $C_{3}$ sites in Nd:FAP.

| ${ }^{2 S+1} L_{J}{ }^{\text {a }}$ | Level | $\begin{gathered} E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{b}} \\ (\mathrm{expt} .) \end{gathered}$ | $\begin{gathered} \Gamma_{n}^{\mathrm{c}} \\ \text { (expt.) } \end{gathered}$ | $\begin{gathered} E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{d}} \\ (\text { calc. }) \end{gathered}$ | $\begin{gathered} \Gamma_{n}{ }^{\mathrm{d}} \\ \text { (calc.) } \end{gathered}$ | Free-ion percent mixture |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & { }^{4} I_{9 / 2} \\ & (427) \end{aligned}$ | 1 | 0 |  | 0.2 | 4,5 | $98.3{ }^{4} I_{9 / 2}+1.51{ }^{4} I_{11 / 2}+0.12{ }^{4} I_{13 / 2}$ |
|  | 2 | $\cdots$ |  | 270 | 6 | $97.5{ }^{4} I_{9 / 2}+2.07{ }^{4} I_{11 / 2}+0.32{ }^{4} I_{13 / 2}$ |
|  | 3 | $\ldots$ |  | 356 | 4,5 | $97.4{ }^{4} I_{9 / 2}+2.40{ }^{4} I_{11 / 2}+0.06{ }^{4} G_{5 / 2}$ |
|  | 4 | $\cdots$ |  | 547 | 4,5 | $98.7{ }^{4} I_{9 / 2}+0.90{ }^{4} I_{11 / 2}+0.16{ }^{4} I_{13 / 2}$ |
|  | 5 | $\cdots$ |  | 675 | 6 | $97.3{ }^{4} I_{9 / 2}+2.45{ }^{4} I_{11 / 2}+0.09{ }^{4} I_{13 / 2}$ |
| $\begin{aligned} & { }^{4} I_{11 / 2} \\ & (2261) \end{aligned}$ | 6 | ... |  | 2043 | 6 | $96.4{ }^{4} I_{11 / 2}+2.20{ }^{4} I_{9 / 2}+1.29{ }^{4} I_{13 / 2}$ |
|  | 7 | $\ldots$ |  | 2133 | 4,5 | $96.8{ }^{4} I_{11 / 2}+2.54{ }^{4} I_{9 / 2}+0.29{ }^{4} I_{13 / 2}$ |
|  | 8 | $\cdots$ |  | 2147 | 4,5 | $95.2{ }^{4} I_{11 / 2}+2.38{ }^{4} I_{9 / 2}+2.25{ }^{4} I_{13 / 2}$ |
|  | 9 | $\cdots$ |  | 2348 | 4,5 | $96.8{ }^{4} I_{11 / 2}+1.72{ }^{4} I_{9 / 2}+1.31{ }^{4} I_{13 / 2}$ |
|  | 10 | $\cdots$ |  | 2387 | 4,5 | $98.9{ }^{4} I_{11 / 2}+0.40{ }^{4} I_{9 / 2}+0.33{ }^{4} I_{13 / 2}$ |
|  | 11 | $\cdots$ |  | 2395 | 6 | $96.6{ }^{4} I_{11 / 2}+2.32{ }^{4} I_{9 / 2}+0.91{ }^{4} I_{13 / 2}$ |
| $\begin{aligned} & { }^{4} I_{13 / 2} \\ & (4249) \end{aligned}$ | 12 | 3992 | 4,5 | 3989 | 4,5 | $96.9{ }^{4} I_{13 / 2}+2.01{ }^{4} I_{11 / 2}+1.02{ }^{4} I_{15 / 2}$ |
|  | 13 | 4061 | 6 | 4064 | 6 | $96.2{ }^{4} I_{13 / 2}+1.95{ }^{4} I_{15 / 2}+1.49{ }^{4} I_{11 / 2}$ |
|  | 14 | 4102 | 4,5 | 4099 | 4,5 | $96.4{ }^{4} I_{13 / 2}+3.02{ }^{4} I_{15 / 2}+0.42{ }^{4} I_{11 / 2}$ |
|  | 15 | 4350 | 4,5 | 4350 | 4,5 | $97.9{ }^{4} I_{13 / 2}+1.02{ }^{4} I_{11 / 2}+0.74{ }^{4} I_{15 / 2}$ |
|  | 16 | 4360 | 4,5 | 4365 | 4,5 | $96.3{ }^{4} I_{13 / 2}+1.85{ }^{4} I_{15 / 2}+1.61{ }^{4} I_{15 / 2}$ |
|  | 17 | 4372 | 6 | 4375 | 6 | $97.9{ }^{4} I_{13 / 2}+1.07{ }^{4} I_{15 / 2}+0.67{ }^{4} I_{11 / 2}$ |
|  | 18 | 4425 | 4,5 | 4417 | 4,5 | $98.1{ }^{4} I_{13 / 2}+1.00{ }^{4} I_{11 / 2}+0.74{ }^{4} I_{15 / 2}$ |
| $\begin{aligned} & { }^{4} I_{15 / 2} \\ & (6211) \end{aligned}$ | 19 | 5754 | 4,5 | 5752 | 4,5 | $96.9{ }^{4} I_{15 / 2}+2.91{ }^{4} I_{13 / 2}+0.05{ }^{4} F_{9 / 2}$ |
|  | 20 | 5898 | 4,5 | 5899 | 4,5 | $98.5{ }^{4} I_{15 / 2}+1.18{ }^{4} I_{13 / 2}+0.22{ }^{4} I_{11 / 2}$ |
|  | 21 | 6018 | 6 | 6014 | 6 | $98.9{ }^{4} I_{15 / 2}+0.91{ }^{4} I_{13 / 2}+0.05{ }^{4} F_{9 / 2}$ |
|  | 22 | 6235 | 4,5 | 6236 | 4,5 | $98.5{ }^{4} I_{15 / 2}+1.17{ }^{4} I_{13 / 2}+0.11{ }^{4} I_{11 / 2}$ |
|  | 23 | 6352 | 6 | 6358 | 6 | $99.2{ }^{4} I_{15 / 2}+0.42{ }^{4} I_{13 / 2}+0.11^{4} F_{9 / 2}$ |
|  | 24 | 6380 | 4,5 | 6376 | 4,5 | $98.8{ }^{4} I_{15 / 2}+0.74{ }^{4} I_{13 / 2}+0.14{ }^{4} F_{9 / 2}$ |
|  | 25 | 6543 | 4,5 | 6547 | 4,5 | $98.5{ }^{4} I_{15 / 2}+1.11{ }^{4} I_{13 / 2}+0.10{ }^{4} F_{9 / 2}$ |
|  | 26 | 6608 | 6 | 6605 | 6 | $98.2{ }^{4} I_{15 / 2}+1.64{ }^{4} I_{13 / 2}+0.07{ }^{4} I_{11 / 2}$ |
| $\begin{aligned} & { }^{4} F_{3 / 2} \\ & (11520) \end{aligned}$ | 27 | 11406 | 4,5 | 11399 | 4,5 | $97.5{ }^{4} F_{3 / 2}+0.88{ }^{4} F_{3 / 2}+0.57{ }^{2} H_{9 / 2}$ |
|  | 28 | 11539 | 6 | 11543 | 6 | $93.9{ }^{4} F_{3 / 2}+3.88{ }^{4} F_{5 / 2}+1.17{ }^{4} F_{7 / 2}$ |
| $\begin{aligned} & { }^{4} F_{5 / 2} \\ & (12536) \end{aligned}$ | 29 | 12364 | 4,5 | 12361 | 4,5 | $85.8{ }^{4} F_{5 / 2}+11.7{ }^{2} H_{9 / 2}+1.19{ }^{4} F_{7 / 2}$ |
|  | 30 | 12480 | 4,5 | 12483 | 4,5 | $65.2{ }^{4} F_{5 / 2}+28.5{ }^{2} H_{9 / 2}+4.92{ }^{4} F_{7 / 2}$ |
|  | 31 | 12546 | 6 | 12546 | 6 | $86.3{ }^{2} H_{9 / 2}+11.1{ }^{4} F_{5 / 2}+1.12{ }^{4} F_{3 / 2}$ |
| ${ }^{2} H_{9 / 2}$ | 32 | 12622 | 4,5 | 12626 | 4,5 | $70.5{ }^{2} H_{9 / 2}+28.0{ }^{4} F_{5 / 2}+0.74{ }^{4} F_{7 / 2}$ |
|  | 33 | 12645 | 6 | 12644 | 6 | $83.0{ }^{4} F_{5 / 2}+12.6{ }^{2} H_{9 / 2}+3.58{ }^{4} F_{3 / 2}$ |
| (12 726) | 34 | 12752 | 4,5 | 12759 | 4,5 | $87.4{ }^{2} H_{9 / 2}+10.5{ }^{4} F_{5 / 2}+1.42{ }^{4} F_{7 / 2}$ |
|  | 35 | 12860 | 6 | 12854 | 6 | $98.7{ }^{2} H_{9 / 2}+0.69{ }^{4} F_{5 / 2}+0.23{ }^{4} F_{9 / 2}$ |
|  | 36 | 12886 | 4,5 | 12891 | 4,5 | $97.8{ }^{2} H_{9 / 2}+1.34{ }^{4} F_{7 / 2}+0.33{ }^{4} F_{5 / 2}$ |
| $\begin{aligned} & { }^{4} F_{7 / 2} \\ & (13466) \end{aligned}$ | 37 | 13287 | 4,5 | 13280 | 4,5 | $90.5{ }^{4} F_{7 / 2}+3.21{ }^{4} S_{3 / 2}+2.83{ }^{4} F_{5 / 2}$ |
|  | 38 | 13320 | 4,5 | 13323 | 4,5 | $94.8{ }^{4} S_{3 / 2}+3.93{ }^{4} F_{7 / 2}+0.44{ }^{4} G_{5 / 2}$ |
|  | 39 | 13353 | 6 | 13351 | 6 | $94.8{ }^{4} S_{3 / 2}+4.15{ }^{4} F_{7 / 2}+0.18{ }^{4} G_{5 / 2}$ |
| ${ }^{4} S_{3 / 2}$ | 40 | 13499 | 6 | 13504 | 6 | $91.8{ }^{4} F_{7 / 2}+4.36{ }^{4} S_{3 / 2}+1.21{ }^{2} H_{9 / 2}$ |
|  | 41 | 13514 | 4,5 | 13512 | 4,5 | $95.2{ }^{4} F_{7 / 2}+2.44{ }^{4} F_{5 / 2}+1.03{ }^{4} F_{9 / 2}$ |
| (13 346) | 42 | ... | 4,5 | 13610 | 4,5 | $93.9{ }^{4} F_{7 / 2}+3.30{ }^{4} F_{9 / 2}+1.76{ }^{4} F_{5 / 2}$ |
| ${ }^{4} F_{9 / 2}$ | 43 | 14687 | 4,5 | 14687 | 4,5 | $97.3{ }^{4} F_{9 / 2}+0.81{ }^{2} H_{11 / 2}+0.57{ }^{4} F_{7 / 2}$ |
|  | 44 | 14755 | 4,5 | 14754 | 4,5 | $95.1{ }^{4} F_{9 / 2}+3.55{ }^{4} F_{7 / 2}+0.64{ }^{2} H_{11 / 2}$ |
| (14 797) | 45 | 14763 | 6 | 14765 | 6 | $98.0{ }^{4} F_{9 / 2}+0.79{ }^{4} F_{7 / 2}+0.37{ }^{4} F_{5 / 2}$ |
|  | 46 | 14837 | 4,5 | 14833 | 4,5 | $98.3{ }^{4} F_{9 / 2}+0.83{ }^{4} F_{7 / 2}+0.28{ }^{2} G_{7 / 2}$ |
|  | 47 | 15058 | 6 | 15055 | 6 | $99.2{ }^{4} F_{9 / 2}+0.23{ }^{2} G_{7 / 2}+0.21{ }^{2} H_{9 / 2}$ |
| $\begin{aligned} & { }^{2} H_{11 / 2} \\ & (16101) \end{aligned}$ | 48 | ... | 4,5 | 16051 | 4,5 | $98.3{ }^{2} H_{11 / 2}+1.09{ }^{2} G_{7 / 2}+0.14{ }^{4} F_{7 / 2}$ |
|  | 49 | 16067 | 6 | 16063 | 6 | $98.9{ }^{2} H_{11 / 2}+0.76{ }^{2} G_{7 / 2}+0.10{ }^{4} F_{7 / 2}$ |
|  | 50 | 16086 | 4,5 | 16092 | 4,5 | $97.7{ }^{2} H_{11 / 2}+1.68{ }^{2} G_{7 / 2}+0.34{ }^{4} F_{9 / 2}$ |
|  | 51 | 16119 | 6 | 16106 | 6 | $98.8{ }^{2} H_{11 / 2}+0.39{ }^{2} G_{7 / 2}+0.39{ }^{4} F_{9 / 2}$ |
|  | 52 | 16130 | 4,5 | 16134 | 4,5 | $98.6{ }^{2} H_{11 / 2}+0.66{ }^{4} F_{9 / 2}+0.31{ }^{2} H_{9 / 2}$ |
|  | 53 | 16165 | 4,5 | 16189 | 6 | $99.2{ }^{2} H_{11 / 2}+0.37{ }^{4} F_{9 / 2}+0.29{ }^{2} G_{7 / 2}$ |
| $\begin{aligned} & { }^{4} G_{5 / 2} \\ & (17291) \end{aligned}$ | 54 | $\begin{aligned} & 17071 \\ & 17192 \\ & 17345 \end{aligned}$ | 4,5 | 17070 | 4,5 | $83.1{ }^{4} G_{5 / 2}+15.3{ }^{2} G_{7 / 2}+0.38{ }^{4} S_{3 / 2}$ |
|  | 55 |  | 4,5 | 17193 | 4,5 | $75.7{ }^{4} G_{5 / 2}+23.3{ }^{2} G_{7 / 2}+0.42{ }^{2} H_{11 / 2}$ |
|  | 56 |  | 6 | 17349 | 6 | $67.8{ }^{2} G_{7 / 2}+30.5{ }^{4} G_{5 / 2}+0.84{ }^{2} H_{11 / 2}$ |

TABLE V. (Continued.)

| ${ }^{2 S+1} L_{J}{ }^{\mathrm{a}}$ | Level | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{b}}$ <br> $($ expt. $)$ | $\Gamma_{n}{ }^{\mathrm{c}}$ <br> $($ expt. $)$ | $E\left(\mathrm{~cm}^{-1}\right)^{\mathrm{b}}$ <br> $(\mathrm{calc})$. | $\Gamma_{n}{ }^{\mathrm{d}}$ <br> $($ calc. $)$ | Free-ion percent mixture |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{2} G_{7 / 2}$ | 57 | 17525 | 4,5 | 17528 | 4,5 | $97.4^{2} G_{7 / 2}+1.02{ }^{2} H_{11 / 2}+0.97{ }^{4} G_{5 / 2}$ |
| $(17459)$ | 58 | 17600 | 4,5 | 17596 | 4,5 | $84.2{ }^{2} G_{7 / 2}+14.6{ }^{4} G_{5 / 2}+0.39{ }^{2} H_{11 / 2}$ |
|  | 59 | 17661 | 4,5 | 17653 | 4,5 | $74.3{ }^{2} G_{7 / 2}+23.5{ }^{4} G_{5 / 2}+1.11{ }^{2} H_{11 / 2}$ |
|  | 60 | $\cdots$ | 6 | 17681 | 6 | $68.3{ }^{4} G_{5 / 2}+30.3{ }^{2} G_{7 / 2}+0.44{ }^{2} H_{11 / 2}$ |

${ }^{\text {a }}$ Multiplet manifold of $\mathrm{Nd}^{3+}\left(4 f^{3}\right)$; the centroid in parentheses is given in $\mathrm{cm}^{-1}$.
${ }^{\mathrm{b}}$ Experimental levels taken from Table I.
${ }^{c}$ Assignments based on selection rules for $C_{3}$ symmetry assuming the ground-state level is $\Gamma_{4,5}$.
${ }^{\mathrm{d}}$ Calculated splitting and predicted symmetry labels $\left(\Gamma_{4,5}\right.$ or $\left.\Gamma_{6}\right)$ based on $B_{n m}$ parameters reported in Table VI.
$B_{n m}$ parameters are necessary to obtain a final rms value of 6 $\mathrm{cm}^{-1}$ between 46 calculated and experimental levels. Overall agreement suggests that the lattice-sum model is appropriate for describing those $\mathrm{Nd}^{3+}$ ions in one of the nonfluorescing sites in Nd:FAP. We were not able to obtain sufficient data sets to articulate other sites. The fact that we observed no conclusive hot band data in absorption is not contradicted by the predicted splitting of ${ }^{4} I_{9 / 2}$. For $\mathrm{Nd}^{3+}$ ions that occupy this site, transitions from a Stark level $270 \mathrm{~cm}^{-1}$ above the ground state would very likely be too weak and broad to observe even at room temperature.

In conclusion, site-selective spectroscopy has permitted differentiation between $\mathrm{Nd}^{3+}$ ions in multiple sites in the fluorapatite crystals $\mathrm{Nd}: \mathrm{SFAP}$ and $\mathrm{Nd}: \mathrm{FAP}$. Attempts to refine the point-charge lattice-sum model are ongoing in hopes of obtaining improved understanding of the complex nature of charge compensation associated with heterovalent substitution of $\mathrm{R}^{3+}$ ions in crystals with the fluorapatite structure. ${ }^{24,27}$

TABLE VI. Crystal-field parameters for $\mathrm{Nd}^{3+}$ ions in $C_{3}$ sites in Nd:FAP.

| $B_{n m}$ <br> parameter | Lattice sum $^{\mathrm{a}}$ <br> $\left(\mathrm{cm}^{-1}\right)$ | Final set $^{\mathrm{b}}$ <br> $\left(\mathrm{cm}^{-1}\right)$ |
| :---: | :---: | :---: |
| $B_{20}$ | 1247 | 1251 |
| $B_{40}$ | -2252 | -2518 |
| $B_{43}$ | 1016 | 605 |
| $B_{60}$ | -1176 | -1237 |
| $\operatorname{Re} B_{63}$ | 890 | 899 |
| $I_{m} B_{63}$ | -260 | -126 |
| $\operatorname{Re} B_{66}$ | -117 | -14.6 |
| $I_{m} B_{66}$ | 356 | 474 |
| $I_{2}(B)^{\mathrm{c}}$ | 1247 |  |
| $I_{4}(B)$ | 2671 |  |
| $I_{6}(B)$ | 1544 |  |

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[^0]:    ${ }^{\text {a }}$ Charge on oxygen taken as -1.8 , phosphorus +4.2 , in electron charge units; x-ray crystallography data taken from Ref. 22 for symmetry properties and bond distances; for details regarding calculations see Ref. 35.
    ${ }^{\mathrm{b}}$ Since fluorescence is not observed, fitting analysis involved 14 multiplet manifolds higher in energy than the ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$; original estimates for the centroids of ${ }^{4} I_{9 / 2}$ and ${ }^{4} I_{11 / 2}$ were taken from Table III.
    ${ }^{\mathrm{c}}$ Rotational invariants, $I_{n}(B)$, are in units of $\mathrm{cm}^{-1}$ (Ref. 31).

