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Failure Of The Integer Quantum Hall-Effect Without Dissipation

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Recent integer quantum Hall effect experiments on silicon samples have shown deviations from the quantized Hall resistance despite a vanishing longitudinal resistance. Here we argue that single short-range elastic scatterers at the edges can lead to observable deviations in the Hall conductivity without backscattering, at the large currents typical experimentally. We do so within an approach to steady-state mesoscopic transport which is not restricted to the linear-response regime (i.e., to small currents).

I. INTRODUCTION

In the integer quantum Hall effect (IQHE), discovered by von Klitzing, Dorda, and Pepper in 1980, the Hall resistivity ρ_H attains the values h/je^2, where h is Plancks's constant, e is the electron charge, and j is an integer.[1] When the Hall resistivity is quantized, the current flows without dissipation through the system.

An elegant explanation of the IQHE at low currents is given by the Landauer-Büttiker formalism, which explains both quantization and lack of dissipation in terms of edge channels that carry current without backscattering.[2,3] In this approach, it is assumed that each edge is separately in a local equilibrium described by a local chemical potential.[4] Ordinarily the local chemical potentials are presumed to differ infinitesimally [so that the Hall voltage V_H satisfies $eV_H/(\hbar\omega_c) \ll 1$, where $\omega_c = eB/(m^*c)$ is the cyclotron frequency]. This corresponds to an infinitesimal total current. However, in a typical experimental situation, the Hall voltage is much larger than the cyclotron energy. For example, in precision measurements at the National Institute of Standards and Technologies, the *lowest* Hall voltage ever measured is about sixteen times larger than $\hbar\omega_c/e$.[5] The Landauer-Büttiker approach, which is fundamentally a linear response theory,[6] does not yield[7] quantization when $eV_H > \hbar\omega_c$, even though it is observed experimentally. We have recently presented an approach to steady-state mesoscopic transport, based on the maximum entropy principle approach to nonequilibrium statistical mechanics, which is valid at arbitrary currents and explains the IQHE at arbitrary currents in an ideal system.[7] In the current paper we use this approach to show how an unusual instance of the *failure* of quantization can arise in certain IQHE systems.

The accuracy of the IQHE quantization, presently determined experimentally to be about one part per billion (ppb) in GaAs heterojunctions,[8] has led to the use of the IQHE to maintain the resistance standard. Recently, however, deviations in the Hall resistivity of up to 0.1 parts per million (ppm) from the quantized value have been observed[9] *despite the absence of dissipation* within experimental resolution. These deviations, observed in silicon metal-oxide semiconductor field-effect transistors (Si MOSFET's), are site dependent and depend on thermal history.[10] It is easy to think of processes such as inelastic scattering at the terminals or in the device, or backscattering, which destroy the quantization, but these then simultaneously lead to dissipation. The deviations observed in Si MOSFET's are therefore peculiar and worthy of further investigation for the following reasons. First, as stated above, they occur despite the absence of any measurable dissipation. Second, the deviations appear as fluctuations in the Hall resistance (not, for instance, as a constant offset in the Hall resistance from that of the GaAs reference device) as the gate voltage is varied at fixed current and magnetic field. Third, the fluctuations are reproducible at low temperatures but depend on thermal history. This suggests that defects such as ionized impurities in the inversion layer or interface charges,[11] which can diffuse at elevated temperatures, are likely to be responsible for the deviations. Such impurities typically act as short-range elastic scatterers of the electrons. In the GaAs heterostructures used in high-precision experiments, impurities give rise to smooth scattering potentials which only negligibly disturb the quantization. But in Si MOSFET's, impurities near the electron gas can provide short-range scatterers which, as we will argue, can cause observable deviations from quantization. Such deviations can arise from the mixing of discrete degenerate states in different Landau levels on one edge by a short-range elastic scatterer. This does not violate the result by Prange[12] that a single short-range scatterer does not cause any deviations from perfect quantization of the Hall resistivity. His result requires that all states perturbed by the scatterer must be occupied, and this may not be the case if the scatterer is close to an edge. In this work we will argue that in fact under certain circumstances short-range scatterers can indeed give deviations from quantization in a manner which appears to be consistent with the observations by Yoshihiro *et al.*[9] This could occur if (1) the scatterer is close to one edge so that only some of the perturbed states are occupied, (2) the scatterer mixes degenerate *discrete* states in different Landau levels,[13] and (3) the

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currents are large so that $eV_H \gtrsim \hbar\omega_c$ (but small compared to currents at which the breakdown of the IQHE begins[14]). Since this occurs without backscattering, the current flows without dissipation in the Hall bar.

Our argument for such observable deviations requires that the density of current-carrying edge states be discrete and determined by a length scale $\lesssim 10^{-1}$ m. Showing that this occurs in devices takes some care; it is not enough simply to note that typical devices are of this size (or smaller). Typically in quantum Hall calculations the system is treated as a closed system of length L_0 with some boundary conditions (perhaps periodic) imposed. This yields a density of states which scales with the length L_0 of the device. The following simple argument shows that, for a closed system, the density of current-carrying edge states also scales with L_0. Each bulk Landau level gives rise to a channel of edge states which we may assume reside in a strip of length L_0 and width of order ℓ_B. [Here $\ell_B = (\hbar c/eB)^{1/2}$ is the magnetic length.] Each state fills an area $2\pi\ell_B^2$, so in the strip there are $L_0 l_B/(2\pi l_B^2)$ edge states per channel. The energies of the edge states within this strip vary an amount $\sim \hbar\omega_c$, so the density of edge states per channel is of order $L_0\ell_B/(2\pi\ell_B^2\hbar\omega_c)$. Thus for a closed system the density of current-carrying states scales with the length of the device. In a real experiment in an open mesoscopic system, however, there are terminals and leads attached to the actual device, and it is not clear if the "length" of the system should include these. That is, it is not entirely obvious whether or not the density of edge states is determined by the length L_0 of the device, or by some larger, effective length L which takes into account the existence of (and boundary conditions at) terminals and leads. In fact, in mesoscopic physics transport is usually regarded as a scattering process, with carriers injected at reservoirs, scattered in the device, and removed at other terminals. In this viewpoint, the number of carriers injected into the device from a reservoir depends only on the number of particles in the reservoir moving toward the device, so that the density of states of carriers is determined by the reservoirs. If the reservoirs are very large, the density of states of the injected carriers becomes effectively continuous. This density could be expressed in terms of a very large effective length L. This is assumed to be the case in most calculations in mesoscopic physics. The rather subtle point we will make here is that the density of injected current-carrying edge states is in fact discrete on a length L which, although larger than L_0, is small enough that it could render the effects of the discreteness of the density of edge states observable in high-precision measurements, at least for the Si MOSFET's used to study the IQHE.

II. COMPUTING THE HALL RESISTANCE

Let us begin by reinvestigating the conditions under which the Landauer-Büttiker formalism[2,3] leads to perfect quantization of the zero-temperature Hall resistivity. For clarity we will consider a two-terminal system with ideal terminals. Consider a system of non-interacting spinless electrons in a system of effective length L along the y direction. (We will later estimate this effective length.) The system is confined in the x direction by a potential $V(x)$, and a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ is applied. In the absence of a scatterer, which would break translational invariance in the y direction, the single-electron eigenstates in the Landau gauge are $\psi_{k',n}(x)e^{ik'y}$, with energies $\epsilon(k',n)$, where n denotes the bulk Landau level. These states are localized in the x direction within a strip of width $\sim \ell_B$ centered on a position $x_{k'}$ determined by k' and the details of the confining potential. We can require that the wave functions be eigenstates of the magnetic translation operator in the y direction, so that $k'L = 2\pi m_{k'} + \theta$, where $m_{k'} = 0, \pm1, \pm2, \ldots$. This corresponds to choosing periodic boundary conditions, which can be thought of as bending the system into a cylinder. Then $\theta/2\pi$ can be interpreted as an extra magnetic flux (measured in units of the flux quantum) through the cylinder's bore. This particular choice of boundary condition is convenient but otherwise of no consequence. What is important for our purposes is that the allowed values of k' are then discrete on a scale L^{-1}. Other choices of boundary conditions would select different sets of discrete k', but with a spacing still of order L^{-1}. The function $\epsilon(k',n)$ can formally be extended to a continuous function of k' by continuously varying θ.

Next consider a strong, short-range scatterer of strength $V_0 \sim \hbar\omega_c$ and range $\sim \ell_B$ positioned on one edge of the system. States within a lateral separation of the order of ℓ_B of the scatterer will then be phase shifted. That is, a state which originally had quantum numbers (k',n) will in the presence of the scatterer have energy $\epsilon(k,n)$, the same energy function evaluated at the shifted wave number $k = k' - \delta(k,n)/L$; this expression defines the phase shift $\delta(k,n)$. The current $i(k,n)$ carried by each state can be obtained using the Hellman-Feynman theorem,

$$i(k,n) = \frac{e}{\hbar}\frac{d\epsilon(k,n)}{d\theta} = \frac{e}{\hbar}\frac{d\epsilon(k,n)/dk}{L + d\delta/dk}. \qquad (1)$$

In the absence of backscattering, the total current in the system is obtained by summing $i(k,n)$ over all occupied states,

$$\begin{aligned} I &= \sum_{\substack{\text{occupied}\\ \text{states}}} i(k,n) \\ &= \frac{L}{2\pi}\sum_n \int i(k,n)\left[1 + \frac{1}{L}\frac{d\delta}{dk} + \mathcal{O}\left(\frac{1}{L^2}\frac{d^2\delta}{dk^2}\right)\right] dk \\ &= \frac{e}{h}\sum_n \int \frac{d\epsilon(k,n)}{dk}\left[1 + \mathcal{O}\left(\frac{1}{L^2}\frac{d^2\delta}{dk^2}\right)\right] dk, \end{aligned} \qquad (2)$$

where the integral extends over occupied states. In the usual Landauer-Büttiker formalism as applied to the IQHE, it assumed that, after injection by reservoirs and equilibration in the leads, the electrons on the two edges occupy states up to local chemical potentials (μ_L and $\mu_R = \mu + \Delta\mu$, say). The occupied single-particle states of maximum and minimum k have guiding centers positioned at the two edges, so Eq. (2) gives $I = j\frac{e}{h}\Delta\mu$ to order $L^{-2}d^2\delta/dk^2$, where j is the number of occupied Landau levels. If the measured Hall voltage is the work per unit charge needed to transfer charge from one edge to the other, then $V_H = \Delta\mu/e$, and one obtains the IQHE conductivity $\sigma_H = je^2/h$. More generally, in nonideal systems electrons have a probability of being reflected or backscattered. In this case Eq. (2) is modified by a factor in the integrand representing the probability of transmission, and the Hall conductivity is of course not quantized. Thus in the Landauer-Büttiker approach, the Hall quantization at zero temperature in the presence of impurities is explained by two facts: the lack of backscattering, and a cancellation between the change in current per state due to scattering and a corresponding change in the density of states. The lack of backscattering can be understood if states carrying current in opposite directions lie on widely separated edges.[2,3]

As mentioned above, the Landauer-Büttiker approach only works at small currents. If the Hall voltage exceeds the Landau-level spacing $\hbar\omega_c$, so that only states carrying current in one direction are occupied in some Landau level, this approach does not give a quantized Hall resistance even in an ideal system.[7] We have recently developed an approach to steady-state nonlinear mesoscopic transport which overcomes this difficulty and gives the correct quantization at large currents and voltages, in ideal systems and in the case of smooth impurities.[7] Here we will summarize this method. Our approach is based on the formulation of nonequilibrium statistical mechanics using the maximum entropy principle,[15] and on recent work by Hershfield.[16] In this approach, the probability of occupying a microstate is determined by maximizing the (information) entropy, subject to certain macroscopic constraints: on energy, particle number, and, in the case of steady-state transport, net current. In the case of mesoscopic transport (and neglecting electron-electron interactions), this can be done exactly. In the case of a two-terminal system, the resulting occupancies of single-particle states (k,n) are[7]

$$f(k,n) = \frac{1}{1 + \exp\left[\beta(\epsilon(k,n) - \mu - \xi i(k,n))\right]}. \qquad (3)$$

Here β is the inverse temperature $1/(k_BT)$, μ is the chemical potential, and ξ is a Lagrangian multiplier which enforces the constraint on the net current through the system. In the present paper we consider only the case of no backscattering, but $i(k,n)$, the current carried by a single-particle state (k,n), can be modified as in Eq. (1) by the presence of elastic scatterers.

At zero temperature states are occupied up to an energy $\mu+\xi i(k,n)$. If only one Landau level is occupied, this corresponds to a local Fermi energy at each edge, similar to the Landauer-Büttiker approach [see Fig. 1(a)]. But if more than one level is occupied, states on one edge are occupied up to different energies in each level [Fig. 1(b)]. In this case the distributions differ from the local equilibrium distributions of the Landauer-Büttiker approach; that is, the occupancies on the edges are *not* described by local chemical potentials μ_L, μ_R. Consequently we cannot simply define the Hall voltage in terms of a chemical potential difference $\mu_R - \mu_L$ between the two edges. Instead, we compute the Hall voltage (as described in Ref. 7) in terms of the work required to move charge from edge to edge (or, generally, terminal to terminal). This gives a set of linear equations for the voltage in each terminal.

The case of a parabolic confining potential (and no

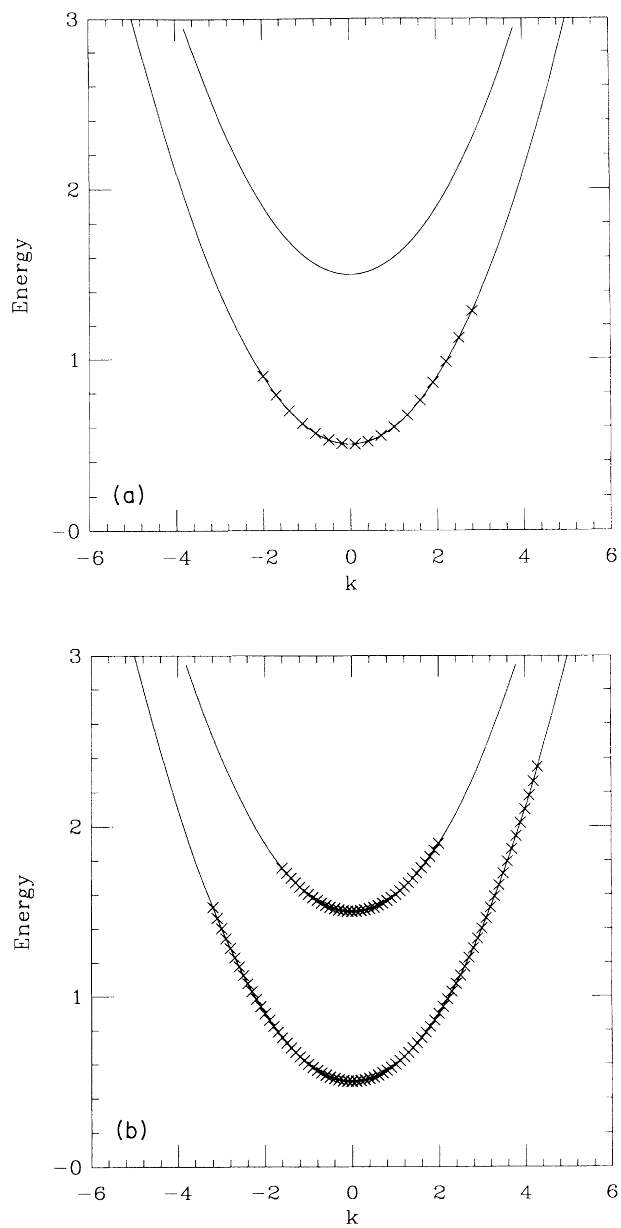

FIG. 1. Schematic occupancies of the two lowest Landau levels in a parabolic confinement. At zero temperature, states are occupied up to an energy $\mu + \xi i(k,n)$ in Landau level n. The wave number k is proportional to the transverse positions of the states' guiding centers. Occupied states are indicated by $\times$. (a) With states in only one Landau level occupied, states at each edge are occupied up to a single energy which can be viewed as a local Fermi energy. (b) With states in more than one Landau level occupied, states in different Landau levels on the same edge are occupied up to different energies.

scatterers) can be solved analytically[7] and gives exact quantization at $T = 0$ for any value of current (ignoring current-induced breakdown[14]). At finite temperatures, the corrections to the Hall resistance are of order $\exp(-\hbar\omega_c/k_BT)$. For other confining potentials we find numerically that this approach gives extremely accurate quantization in systems with no impurities. This is illustrated by the solid line in Fig. 2. When smooth impurities are added to the system, or when all the states disturbed by a short-range scatterer are occupied, the Hall conductivity is quantized with corrections of order L^{-2}, as suggested by our discussion of the ordinary Landauer-Büttiker approach.

In the present paper we investigate the effect of short-range scatterers on the Hall quantization. We will argue that the terms formally of order L^{-2} [such as the $L^{-2}d^2\delta/dk^2$ term in Eq. (2)] can, in fact, sometimes give rise to observable deviations in the conductivity. This can occur in the presence of short-range scatterers positioned at an edge, as a consequence of elastic intra-edge, inter-Landau-level scattering. Again we emphasize that this deviation can occur for systems despite the absence of any backscattering which would give dissipation (and hence a nonzero longitudinal resistivity in a four-terminal measurement). In the following section we will present the results of calculations of the Hall conductivity in the

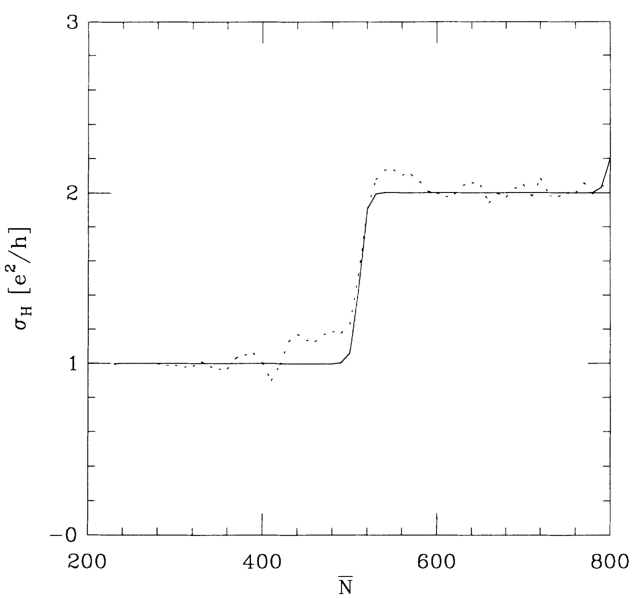

FIG. 2. Hall conductivity vs particle number. The system is a strip of length $L = 80\pi\ell_B$ confined by a parabolic potential $V(x) = 0.045m^*\omega_c^2x^2$, with a temperature $k_BT = 0.005\hbar\omega_c$ and a current $I = 0.2e\omega_c$. This corresponds to a Hall voltage $V_H = 1.257\hbar\omega_c/e$. The first ~ 500 electrons go into the lowest Landau level; additional electrons go into higher Landau levels. Full line: the Hall conductivity with no scatterers. At the first plateau, the numerically-obtained conductivity is quantized to within 20 ppb, and at the second plateau, to within 0.02%. Dashed line: the Hall conductivity with a strong, short-range repulsive Gaussian scatterer of strength $\hbar\omega_c$ and range ℓ_B added at $(5.5, 125)\ell_B$. The deviations at the center of the plateaus are in this case of the order of a few percent. For larger systems they scale as ℓ_B/L.

presence of short-range scatterers. We will focus on the simplest case of a two-terminal system, but similar results are obtained in the multiterminal case. The calculations proceed as follows. The single-particle eigenstates of a system of effective length L (and with periodic boundary conditions) are calculated in the presence of a static potential. This potential is the sum of the confining potential $V(x)$ plus Gaussian potentials of strength $V_0 \sim \hbar\omega_c$ and range $\sim \ell_B$ describing short-range elastic scatterers at an edge. The occupancy of a state is given by Eq. (3), where the parameter ξ and the chemical potential μ are determined by requiring that $\sum_{nk} f(k,n)i(k,n) = I$ and $\sum_{nk} f(nk) = N$. Here I is the net current through the device and N is the number of electrons present. I can be varied from low currents to the much higher values typical experimentally. The Hall voltage is then obtained by inverting a set of linear equations, as mentioned above and explained in Ref. 7.

III. CONSEQUENCES OF INTRA-EDGE ELASTIC SCATTERERS

Let us now examine the circumstances under which short-range elastic scatterers can cause the conductivity to deviate from its quantized value. We will proceed in three steps, considering first the case of a single Landau level and low currents, second the case of multiple levels still at low currents, and third the case of multiple levels at high (but experimentally typical) currents. While all of these in principle could give deviations in quantization, we will see that only the last can give rise to deviations of observable magnitude.

First let us consider the case of very small net currents. For clarity we will in this case use the Landauer-Büttiker formalism, which gives correct results for the resistivity at small currents. A single short-range scatterer causes the phase shift $\delta(k,n)$ to vary quickly for single-particle states in its vicinity. As pointed out by Prange, when all of the perturbed states are occupied, the Hall resistance remains quantized exactly.[12] But if only some are occupied, the resistance can deviate from its quantized value. This appears in Eq. (2) as the terms of order $L^{-2}d^2\delta/dk^2$. While formally of order L^{-2}, these deviations can become unusually large when the Fermi edge (the highest-energy occupied single-particle state) is in the immediate vicinity of the scatterer where $\delta(k,n)$ varies abruptly. This is illustrated in Fig. 3, which shows the differential conductivity, $e\Delta I/\Delta\epsilon$, as a function of the Fermi energy (i.e., the energy of the highest-energy occupied single-particle state) in the presence of a repulsive Gaussian scatterer of strength $V_0 = \hbar\omega_c$ and range ℓ_B. This plot is for the simplest case in which only states in the lowest Landau level are occupied. The differential conductivity is the ratio of the increase in current to that in Fermi energy when one more eigenstate is occupied. Like the total conductivity, it is quantized with corrections of order $L^{-2}d^2\delta/d^2k$. It is clear from Fig. 3 that the deviation is largest when the Fermi energy sweeps through the region of maximum curvature of δ (see the inset). The deviations from quantization in this case are nonetheless unobservably small in a macroscopic system.

Next let us see the effect of having more than one Lan-

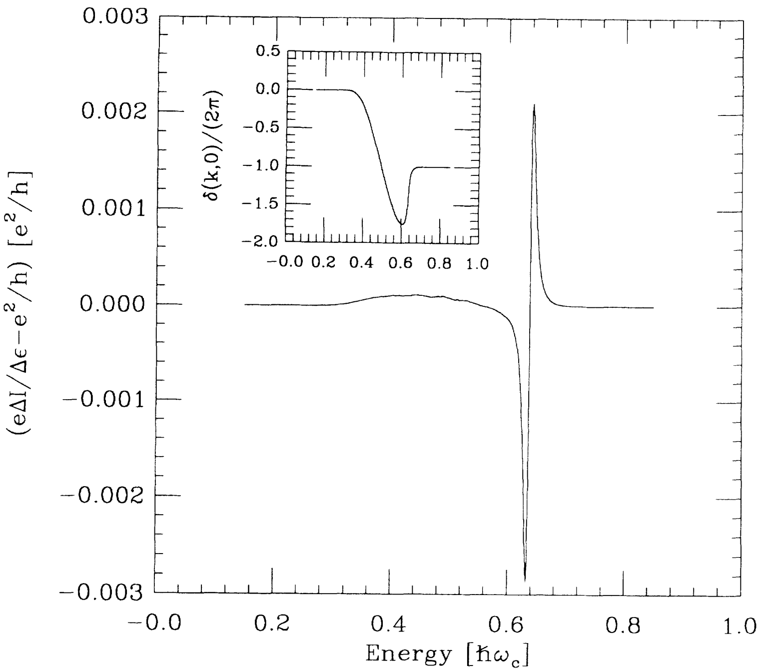

FIG. 3. The differential conductivity for states in the $n = 0$ Landau level as a function of the Fermi energy. The system is a strip of length $L = 80\pi\ell_B$ in the presence of an electric field $E\hat{\mathbf{x}} = 0.05\hbar\omega_c/(e\ell_B)\hat{\mathbf{x}}$ and a repulsive Gaussian scatterer of strength $\hbar\omega_c$ and range ℓ_B. The field is weak enough that states in adjacent Landau levels which are degenerate in the absence of the scatterer are well separated in the x direction. At points where the curvature of the phase shift is large, the differential conductance deviates from e^2/h. *Inset:* The phase shift $\delta(k,0)$ for states in the lowest Landau level.

dau level participating in transport, to begin with in the low-current limit. In this case it turns out that elastic scattering between nearly degenerate states in different levels at the edge can cause δ to vary so abruptly that terms of order $L^{-2}d^2\delta/dk^2$ are dramatically increased.[17] Consider a pair of states in two adjacent Landau levels which, in the absence of scatterers, are nearly degenerate. These states are separated by a lateral distance of order ℓ_B. Call these states $(k_0,0)$ and $(k_1,1)$. Typically the pairs of discrete states closest to these $[(k_0\pm 2\pi/L,0)$ and $(k_1 \pm 2\pi/L,1)]$ are much less close to degeneracy. (This is the case whenever the two Landau levels, as usual, are not parallel, i.e., $d\epsilon(k,n')/dk$ differs sufficiently for $n' = 0,1$.) Now put in a scatterer of strength V_0 in the vicinity of these states. As a consequence of the scatterer these states will mix, and so the phase shifts $\delta(k,n')$ will fluctuate abruptly in a range of k about these states of order $\Delta k \sim 1/L$. An example of this is shown in Fig. 4(a). This fluctuation reaches its maximum (call it $\delta\phi$) at the originally nearly-degenerate states, and so the curvature $d^2\delta/dk^2$ reaches a maximum in this vicinity of order $\delta\phi/(\Delta k)^2 \sim L^2\delta\phi$. That is, the phase shift will exhibit isolated rapid fluctuations in the vicinity of certain pairs of states which in the absence of the scatterer were nearly degenerate. As the Fermi level sweeps through these states, there will be a fluctuation in the differential Hall conductivity of order $L^{-2}d^2\delta/dk^2 \sim \delta\phi$. Consequently deviations $\Delta\sigma$ in the Hall conductivity, formally of order $L^{-2}d^2\delta/dk^2$, become of order

$$\Delta\sigma \sim \frac{e}{hV_H}\int \frac{d\epsilon}{dk}\delta\phi\, dk \tag{4}$$

integrated over the range $\Delta k \sim L^{-1}$ over which $\delta\phi$ occurs [see Eq. (2)]. The magnitude of $\delta\phi$ is determined by the splitting of the degeneracy caused by the scatterer, which is given by the off-diagonal matrix element between the nearly degenerate states. These matrix elements are of order $V_0\ell_B/L$. We expand the energies $\epsilon(k,n)$ to obtain the shift in energy caused by a phase shift $\delta\phi$ as $(\delta\phi/L)(d\epsilon/dk)$. By equating this with the energy splitting $V_0\ell_B/L$, we obtain the following estimate for $\delta\phi$:

$$\delta\phi \sim \frac{V_0\ell_B}{(d\epsilon/dk)}. \tag{5}$$

This inserted in Eq. (4) then yields a deviation $\Delta\sigma$ in the conductivity of order

$$\frac{\Delta\sigma}{e^2/h} \sim \frac{1}{eV_H}\frac{V_0\ell_B}{L}. \tag{6}$$

For a strong scatterer, $V_0 \sim \hbar\omega_c$, and for typical voltages $V_H \sim 10\hbar\omega_c/e$. Thus typical fluctuations under these circumstances are of order

$$\frac{\Delta\sigma}{e^2/h} \sim \frac{1}{10}\frac{\ell_B}{L}. \tag{7}$$

In fact, these deviations can be significant only if one of the two perturbed states is occupied. Due to the mixing of the two states, the current of the state evolving from the lower level is decreased while that of the state evolving from the upper is increased, and the pair's degeneracy is lifted by an amount $\Delta\epsilon \sim V_0\ell_B/L$. If both states are occupied, which is likely when the total current is small, the deviation is negligible, essentially because the decrease in current carried by the state in the lower Landau level is offset by the increase in current carried by the state in the upper one. This is illustrated in the main portion of Fig. 4(b). There large jumps in the total conductivity arise whenever one member of the pair is occupied, but this is immediately canceled as soon as the other member is filled. Hence deviations at infinitesimal currents are observable only if the experimental resolution is sufficient to resolve the splitting of the degeneracy.[18] The Hall voltage can be measured at sufficient resolution (for example, in the experiment by Yoshihiro *et al.*,[9] the resolution was 0.01 nV, while for a device of effective length $L \sim 10^{-3}$ m, in a strong field with $V_0 \sim 10$ meV and $\ell_B \sim 10^{-8}$ m, the energy split is of order 0.1 nV). But resolving the energy split also requires $k_B T$ to be smaller than the energy spacing $\Delta\epsilon$. This would require a temperature T of order 1 mK. Thus, while elastic scattering between levels at one edge can in principle give large fluctuations in resistance quantization, these are not experimentally observable at small currents.

Finally, let us consider the case of currents as large as those typically realized experimentally, in the presence of such a short-range scatterer on one edge, with multiple Landau levels involved in transport. In this case the deviations from quantization can be observably large. We just showed that if both states of a pair (nearly degenerate in the absence of a scatterer, split by its presence) are occupied, there is no significant deviation from quantization. But if only one state is occupied, deviations result. Distributions of precisely this sort arise in our maximum entropy approach to mesoscopic transport, as we explained earlier, when the currents become finite (and nonzero). At such currents, states in the lower Landau level are occupied to a higher energy than those in the upper level. In the presence of a large current, there will be many pairs of once-degenerate states lying in the range of energy between the maximum occupied states

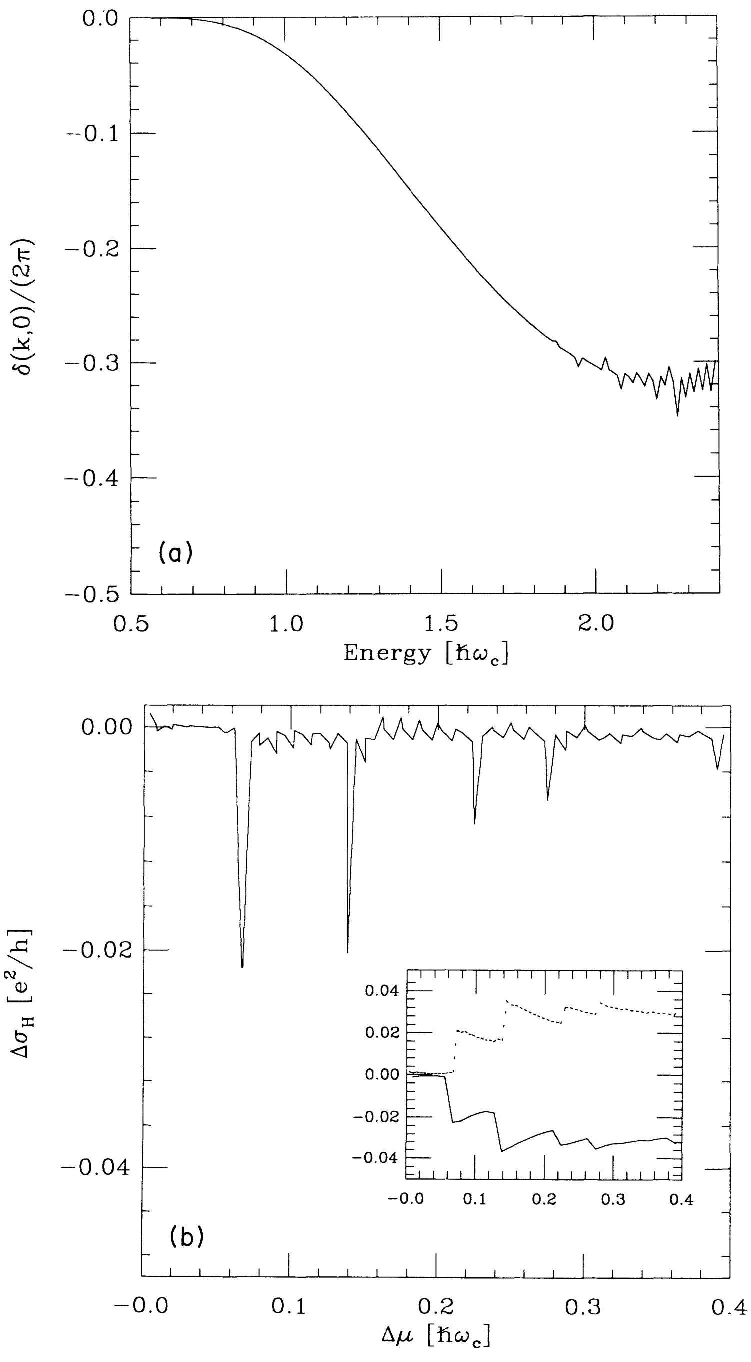

FIG. 4. (a) Phase shift vs energy for states in the $n = 0$ Landau level in the presence of a repulsive scatterer. This is the system whose total conductivity at a finite current is shown as the dashed line in Fig. 2. For energies greater than about $1.8\hbar\omega_c$, the phase shift exhibits rapid local fluctuations due to the strong mixing of degenerate states. (b) Total conductivity for the same system vs chemical potential difference $\Delta\mu$ at low currents, obtained by summing the conductivities from the two Landau levels. At points where the Fermi level sweeps past strongly mixed states, there are large deviations in the conductivities from their ideal values. The left movers in both Landau levels are occupied to an energy $\mu = 1.80\,\hbar\omega_c$, and the right movers are occupied to $\mu + \Delta\mu$. *Inset:* Conductivity per Landau level vs $\Delta\mu$ (solid line for n=0, dashed for n=1). The sum of the conductivities of both Landau levels is nearly constant, except when only one member of a strongly-mixed pair is occupied.

of the two levels. Then in the lower Landau level many such states (half of once-degenerate pairs) will be occupied while the corresponding states in the upper Landau level remain unoccupied. At each of these states, there is an abrupt variation $\sim \delta\phi$ in the phase shift δ. Unlike the situation with a small current, in this case the deviations are cumulative, due to the effect of occupying only one member of many such nearly degenerate pairs. This can lead to a large effect (compared to the ppb accuracy of measurements) which, as shown in Eq. (6), scales as ℓ_B/L over a large range of electron density or magnetic field. This is illustrated in Fig. 2(b).

IV. DISCUSSION

We have here argued that single short-range elastic scatterers on an edge can give rise to observable deviations in the Hall conductivity. Note that these deviations from quantization are not generic. When the disorder is smooth, the phase shifts are smooth. Even in the case of a sharp scatterer, if all the states affected by it in both Landau levels are occupied, the deviations are not observable. Only in the case of a short-range scatter on the edge—influencing states in a range of energy where one Landau level is occupied and not the next—can observable deviations occur. In this case the conductivity will fluctuate about its quantized value as the Fermi energies sweep across the scatterer. This holds true with several scatterers on one edge as long as the mean free path between scattering events is much greater than the magnetic length $\ell_B = \sqrt{\hbar c/eB}$. With a high density of scatterers, so that the mean free path is of order ℓ_B, the scattering phase shifts become smooth.

We emphasize that this deviation occurs at steady state in the absence of backscattering or inelastic scattering in the device, and at currents below those at which the current-induced breakdown occurs.[14] In fact, the mechanism for deviation that we present is significant only when there is no backscattering. With backscattering, transport properties become dominated by the details of the transmission and reflection matrices, which swamp the contribution of elastic forward scattering.

Let us briefly discuss the circumstances under which this deviation could be observable. First, short-range scatterers on the edge are needed. These are more likely to be present in Si MOSFET's, where ionized impurities in the inversion layer, interface charges, and interface roughness provide strong, short-range elastic scatterers, than in GaAs heterojunctions (see, however, Ref. 10). The magnitude of the deviation depends on the difference between the Fermi levels of different levels on on edge, $\epsilon_1 - \epsilon_0 = \xi[i(k_1, 1) - i(k_0, 0)]$. This grows with total current (since ξ then grows), and is significant only when the currents $i(k, n)$ in the two levels differ. This in turn depends on the slopes $d\epsilon_n/dk$, which differ most when the confinement is very stiff, or has curvature. In fact, these slopes are equal only in the case of a potential $V(x)$ which is linear in x. The deviation is ultimately traced to fluctuations in phase shift of order $\delta\phi \sim V_0\ell_B/(d\epsilon/dk)$, so the effect is stronger in weak magnetic fields. For the purposes of metrology, it is desirable to minimize the deviations due to short-range scatterers. In the absence of other considerations, it is best to sit on the $\nu = 1$ plateau to avoid interlevel scattering,[19] or else to keep the current small, with a Hall voltage much less than $\hbar\omega_c$ (provided a good signal-to-noise level can be obtained at lower currents).

The observability of the deviations depends on the discreteness in the density of current-carrying states on the edges, which is characterized by the effective length L. It is clear from the preceeding section that the details of the particular boundary conditions we used here played no role beyond establishing the discreteness. To obtain the deviations consistent with the experimental observations requires $L/\ell_B \lesssim 0.1$ ppm $= 10^{-7}$, or, in a 10 T field, $L \lesssim 8$ cm. Clearly if the discreteness is simply determined by the length L_0 of the device, this not very stringent condition is satisfied. But, as mentioned earlier, it is not obvious that $L \sim L_0$. We now turn to an estimate of L, and show that it is small enough to give rise to deviations of observable magnitude. A Si MOSFET consists essentially of an inversion layer below a SiO_2 spacer which in turn is below a gate. On either end of the inversion layer is a three-dimensional n^+-doped layer to which contacts are attached to provide the current source and sink. These might be thought to serve as the three-dimensional 'reservoirs' which are key to the ordinary understanding of mesoscopic transport. From this viewpoint, the reservoirs determine the density of states injected into the device; if the reservoirs are very large this density will be nearly continuous. This is a common assumption in the mesoscopic literature, but, we argue, is not the case here: the discreteness of the density of states should be observable in high-precision measurements. It is a simple matter to relate the density of states in the three-dimensional reservoirs to the effective length L characterizing the density of states in the two-dimensional device. We now do so, and by requiring that this L be small enough to obtain the experimentally observed deviations we will obtain a value for the size of the source and drain which compares well with the actual device parameters. The source and drain on the device used by Yoshihiro *et al.* were made by ion implantation and have a depth $D \sim 10^{-7}$ m, and an area a. The density of states $g(\epsilon)$ of Landau level n in the source is (per spin direction, ignoring the Zeeman energy)

$$g(\epsilon) = \frac{a}{2\pi\ell_B^2}\frac{D}{2\pi}\left[\frac{2\hbar^2}{m^*}\left(\epsilon - \left(n + \frac{1}{2}\right)\hbar\omega_c\right)\right]^{-1/2}. \quad (8)$$

We evaluate this expression at $\epsilon \approx 4.5$ meV, corresponding to an energy between the $n = 0$ and $n = 1$ Landau levels. The result is $g(4.5 \text{ meV}) \approx a \times 10^{18}$ eV^{-1} m^{-2}. If the energies at the edge of the inversion layer increase by $0.5\hbar\omega_c$ in a distance ℓ_B, then an L of 8 cm (required to get the observed deviations in a 10 T field) corresponds to a density of edge states of about 10^9 eV^{-1}. By equating these two expressions for the density of states, we obtain $a \approx 1000$ μm^2. This corresponds to an area of implanted ions of approximate dimensions 10×100 μm, which is not unreasonable. This simple argument shows that the density of current-carrying states in a Si

MOSFET device could indeed be expected to be discrete on the scale needed for the deviations to have the observed magnitude.

Finally, we propose the following experimental tests of the mechanism that we have proposed here. It may be difficult to directly manufacture in a controlled way a device which exhibits deviations of the type described here.[20] However, a device already found to exhibit them, such as the device studied by Yoshihiro *et al.*,[9] can be used to systematically study the magnitude of the deviations. If these are studied as a function of magnetic field for a given range of filling factor, so that the same interval along the Hall plateau is studied at different magnetic fields, a definite dependence on magnetic field should be detected for the mechanism proposed here. According to our arguments earlier, the deviations should scale as ℓ_B/L, where L is the effective length of the system. It then appears that there are two possibilities. (1) It may be the case that the density of current-carrying states is determined by three-dimensional reservoirs subjected to the external magnetic field, i.e., the source and the drain. By equating the density of states in these reservoirs [Eq. (8)] with the density of edge states $L/(2\pi\ell_B\hbar\omega_c)$ in a device of length L, we obtain $L \sim \hbar\omega_c/\ell_B \sim B^{3/2}$. The result is then that the deviations scale with the magnetic field as B^{-2}. (2) On the other hand, it may be the case that the effective length, i.e., the density of current carrying states, is independent of magnetic field. This would be the case if, for example, the density of states is determined by the actual device, and not by three-dimensional reservoirs which are also subjected to the magnetic field. Even if under ordinary circumstances L is determined by the reservoirs, then this could occur if they become so small that L becomes of the order of L_0. It follows that the magnitude of the deviations then scale as $B^{-1/2}$. As a by-product, such an experiment will then yield information which can help our understanding of what determines the density of current-carrying states.

The calculation presented here is a semiquantitative explanation of the deviations which may be expected from the mechanism that we propose: intra-edge, inter-Landau-level elastic scattering by short-range impurities. Even so, we stress the fact that we are not aware of any other explanation for the deviations observed by Yoshihiro *et al.*[9] A more quantitative analysis of the mechanism that we have proposed here requires better knowledge of the density of current-carrying states. Some information about this could be provided, for example, by the experiments proposed in the preceding paragraph.

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[20] It should, however, be noted that E. Palm raised the possibility of burning a nanometer sized hole on the edge of a GaAs device. Such a hole could presumably act as a scatterer with a range of the order of the magnetic length [E. Palm (unpublished)].