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Accurate Abinitio Calculation On The Low-Energy Elastic-Scattering Of Electrons From Helium

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The multiconfiguration Hartree-Fock method developed by Saha [Phys. Rev. A **39**, 5048 (1989)] to study scattering of electrons from atoms has been applied to the low-energy elastic scattering of electrons from helium atoms. The short-range electron correlation and the long-range dynamical polarization of the target by the scattering electron, which are very important in these calculations, have been taken into account in an accurate *ab initio* manner through the configuration-interaction procedure. Detailed results for phase shifts, elastic differential, integral-elastic, and momentum-transfer cross sections for electrons elastically scattered from helium are reported for the low and intermediate energies ranging from 0.58 to 50 eV. The present results are compared with accurate experimental measurements and theoretical calculations. It is found that the present multiconfiguration self-consistent-field method produces high-quality results that show excellent agreement with experimental measurements and compare well with other accurate theoretical calculations.

I. INTRODUCTION

In recent years the study of elastic scattering of electrons by atoms has been of great interest to both experimentalists and theoreticians because of much detailed information produced by recent measurements. It is found that normalization of the measured cross sections is a problem to the experimentalists in obtaining absolute differential and total-cross-section data. Most available data are the results of relative measurements, which is due to the difficulty of calibration in scattering experiments. If the electron scattering cross section for a particular process is known accurately, this could be used as a standard to normalize the data obtained from other cross-section measurements. Because inert gases have a simple structure, both experimentally and theoretically they have been the subject of extensive investigations. Among all the inert gases, helium is the most simple atom and can be easily handled experimentally. For this reason, electron-helium scattering has been the subject of much investigation as an ideal candidate to serve as a standard.

Elastic scattering is the dominant process arising from electron-helium scattering, which is why the determination of accurate elastic differential cross sections for electron-helium scattering has been given considerable importance.

In the past few years electron-helium total scattering cross sections have been measured by many experimentalists like Nickel *et al.*,[1] Kauppila *et al.*,[2] Charlton *et al.*,[3] Blaauw *et al.*,[4] Kennerly and Bonham,[5] and Stein *et al.*[6] The differential cross sections have been measured by Andrick and Bitsch,[7] Newell *et al.*,[8] and McConkey and Preston.[9] In addition, Register *et al.*[10] reported absolute differential, integral, and momentum-transfer cross sections for electrons elastically scattered from helium atoms. Scattering phase shifts have been derived from experimental measurements of the differential cross section of the scattered electrons by Andrick and Bitsch,[7] Williams[11] and Newell *et al.*[8] Recently, Buckman and Lohmann[12] measured absolute total cross sections for electrons scattered from helium over the energy range 0.1–20 eV using a time-of-flight spectrometer. On the other hand, there have been a large number of theoretical calculations on the electron scattering from the helium atom. McEachran and Stauffer[13] performed calculations on the low-energy elastic scattering of electrons from helium atoms in the exchange-adiabatic approximation. They examined the effects of the polarization potential in their investigation. They have not included exchange-polarization terms which may have a significant contribution, especially at low energies. They reported results for phase shifts, differential, total-elastic, and momentum-transfer cross sections. Fon *et al.*[14] carried out R-matrix calculations for the elastic scattering of electrons from helium atoms in which the ground-state wave function is coupled with a ^1P pseudostate to include the full static dipole polarizability. They calculated phase shifts, differential, integrated, and momentum-transfer cross sections for a wide range of energies. Nesbet[15] reported results for the elastic scattering of electrons from helium atoms using the matrix variational method. O'Malley *et al.*[16] made an R-matrix calculation for this atom. Scott and Taylor[17] applied many-body theory to calculate elastic scattering of electrons from the helium atom at energies ranging from 30 to 200 eV. Polarized orbital calculations have been made by LaBahn and Callaway,[18] Callaway *et al.*,[19] and Duxler *et al.*[20]

Saha *et al.*[21] and Saha[22] used the multiconfiguration Hartree-Fock (MCHF) approximation to calculate the photoionization cross section of the atoms. In their work, electron correlation and dynamical core polarization effects which are very important in this calculation were taken into account more accurately in an *ab initio* manner than by any other method. The results reported there were in good agreement with experimental measurements. Very recently Saha extended[23] the multiconfiguration self-consistent-field (MCSCF) method to consider the scattering of electrons from atoms and applied it to the low-energy elastic scattering of electrons

from neon atoms.[24] The results obtained were in excellent agreement with experimental measurements. It is the aim of this paper to apply the same *ab initio* method to the elastic scattering of electrons from helium atoms at low- and intermediate-impact energies to provide more detailed useful and accurate information. In low-energy scattering, the projectile polarizes the target. This polarization is different for projectiles carrying different kinetic energy. In our approach, distortion of the target orbitals due to the presence of the projectile are considered *ab initio* through the configuration-interaction procedure. The polarization effects, which are energy dependent, are taken into account through the bound configurations which represent the multipole polarization and by varying the bound and the scattering orbitals simultaneously for each kinetic energy of the projectile. The phase shifts of the scattered electron are calculated by this MCSCF method for a range of low and intermediate energies. These phase shifts are used to calculate elastic differential, total, and momentum-transfer cross sections.

In Sec. II, we present our MCSCF method of applications and Sec. III discusses computational procedures. Section IV is devoted to a discussion of the present results, and comparison with experimental and other theoretical results. Finally conclusions are given in Sec. V.

II. THEORY

A. MCHF wave function for a scattering state

The wave function for a scattering state with label γ, energy E, and term LS, in the multiconfiguration Hartree-Fock approximation can be expressed as

$$\Psi(\gamma LS;N+1)=\sum_{j}^{m_t} a_j\Phi(\gamma_j L_t S_t;N)\phi_{kl} + \sum_{i}^{m} c_i\Phi(\gamma_i LS;N+1)\ , \tag{1}$$

where the first term represents a correlated target wave function coupled with a scattering electron. The N-electron target that is an eigenstate of L_t and S_t is described in terms of N-electron bound configuration states $\Phi(\gamma_j L_t S_t;N)$, with configuration γ_j and term $L_t S_t$, and mixing coefficients a_j and the total energy E_t. This target is coupled to a scattering wave function ϕ_{kl} with orbital angular momentum l, to yield an antisymmetric configuration state for the $(N+1)$-electron system, with final term value LS and configuration $\gamma_j kl$. The second term represents $(N+1)$-electron, bound-state configurations, which are eigenstates with the same L and S and which are included to allow for polarization and electron correlation effects.

A set of radial functions, say, $P_i(r)$, $i=1$ to m, defines the above scattering wave function. In the MCSCF method for the scattering states, all the radial functions describing the target are assumed to be fixed with the mixing coefficients a_j, and the other bound-state radial functions are determined variationally along with the radial function for the scattering electron. In this approximation, all the radial functions are the solutions of the second-order coupled integro-differential equations of the form[24]

$$\left[\frac{d^2}{dr^2}+\frac{2Z}{r}-\frac{l(l+1)}{r^2}\right]P_i(r) = \frac{2}{r}[Y_i(r)P_i(r)+X_i(r)+I_i(r)]+\sum_{i'}\varepsilon_{ii'}P_{i'}(r)\ , \tag{2}$$

where the off-diagonal energy parameters $\varepsilon_{ii'}$ are related to Lagrange multipliers that ensure orthogonality assumptions. The radial function for the scattering electron differs only in the boundary conditions it satisfies and the definition of the diagonal energy parameter. Boundary conditions satisfied by the bound radial functions are

$$P_i(r)\underset{r\to 0}{\to} r^{l+1},\quad P_i(r)\underset{r\to\infty}{\to} 0\ . \tag{3}$$

In this case the diagonal energy parameter ε_{ii} is an eigenvalue of the integro-differential equation and needs to be determined. The radial functions for the scattering orbital satisfy the conditions

$$P_i(r)\underset{r\to 0}{\to} r^{l+1}\ , \qquad P_i(r)\underset{r\to\infty}{\to} k^{-1/2}\sin\left[kr-\frac{l\pi}{2}+\delta_l\right]\ , \tag{4}$$

where δ_l is the phase shift, and $\varepsilon_{ii}=-k^2$. k^2 is the kinetic energy of the scattering electron in rydbergs. In the MCSCF method the bound and the scattering radial functions are determined by solving the above set of second-order coupled integro-differential equations under the proper boundary conditions. The scattering radial functions are normalized by fitting the computed values at two adjacent points to the regular and the irregular Bessel functions as soon as the region where the direct and the exchange potentials are found to be negligible is reached, which may be considerably smaller values of r than the asymptotic form given by the boundary condition of Eq. (4).

The coefficients c_i which need to be determined in the present approximation are the solutions of the system of equations derived from the condition that $\langle\Psi|H-E|\Psi\rangle$ be stationary with respect to variations in the coefficients where H is the Hamiltonian for the $(N+1)$-electron system and $E=E_t+k^2/2$ (in a.u.). The coefficients c_i are the solutions of the system of equations

$$\sum_{i'}^{m}\langle\Phi_i|H-E|\Phi_{i'}\rangle c_{i'}+\sum_{j}^{m_t}\langle\Phi_i|H-E|\Phi_j\rangle a_j=0\ , \qquad i=1,\ldots,m\ . \tag{5}$$

where

$$\Phi_j\equiv\Phi(\gamma_j L_t S_t;N)\phi_{kl},\quad j=1,\ldots,m_t$$

and

$$\Phi_i \equiv \Phi(\gamma_i LS; N+1), \quad i=1,\ldots,m \ .$$

The phase shifts δ_l of various partial waves for a range of low and intermediate energies are computed in the present work by the MCSCF method.

B. MCSCF theory of elastic scattering

This paper is concerned mainly with the low- and intermediate-energy elastic scattering of electrons from helium atoms. The elastic differential cross section $\sigma(\theta)$ in atomic units a_0^2/sr is given by[25]

$$\sigma(\theta)=\frac{d\sigma}{d\Omega}=|f(\theta)|^2 \ ,$$

where the scattering amplitude $f(\theta)$ is

$$f(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)[\exp(2i\delta_l)-1]P_l(\cos\theta) \ ,$$

where $P_l(\cos\theta)$ is the lth Legendre polynomial, δ_l is the real phase shift, and k is the electron momentum in atomic units.

The total elastic cross section in units of a_0^2 is

$$\sigma_T=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l \ ,$$

and the momentum-transfer cross section is (in units of a_0^2)

$$\sigma_M=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_l-\delta_{l+1}) \ .$$

The MCSCF method which was used earlier by Saha *et al.*[21] and Saha[22] for photoionization calculation is extended[23] further to take into account the dynamic target polarization and the electron correlation effects to very efficiently compute phase shifts for different angular momentum of the scattering electron.

As the polarization of the $1s^2\,{}^1S$ He target by the scattering electron and the electron correlation effects are very important in the calculation of the phase shifts, the most straightforward method to include these effects is to extend the configuration-interaction procedure through the multiconfiguration Hartree-Fock approximation. The phase shifts obtained in this way are used to calculate elastic differential, total-elastic, and momentum-transfer cross sections. Our results are compared with experimental measurements of these quantities and with other theoretical calculations.

III. COMPUTATIONAL PROCEDURE

In this paper calculations were performed using the MCHF program which has been extended[23] further to study the elastic scattering of electrons from atoms. First of all, the $1s$ wave function of the target helium atom is obtained by the Hartree-Fock (HF) calculation of the $1s^2\,{}^1S$ ground state. Finally, the correlated ground-state wave function of the target helium atom is calculated[26] by the MCHF wave-function expansion over the 35 configuration states coupled to form a ${}^1S$ term. These configurations are generated by the single and double replacements of the $1s$ orbital of the helium atom by the excited orbitals $2s$, $2p$, $3s$, $3p$, $3d$, $4s$, $4p$, $4d$, $4f$, $5s$, $5p$, and $5d$. The eigenenergy of the ${}^1S$ state of the helium atom was found to be $-2.903\,297\,1$ a.u. These wave functions are used as an input in the calculation of the scattering wave functions and the phase shifts for various partial waves.

As mentioned earlier, polarization of the $1s^2$ target atom by the scattering electron is very important in the low-energy e^--He scattering calculations. This polarization of the target atom has been taken into account very accurately through the configuration-interaction procedure. Polarization is nothing but the distortion of the $1s$ orbital of the helium atom due to the presence of the electric field of the scattering electron. It is found that only dipole polarization of the target is important in this case of the e^--He scattering problem. Quadrupole and multipole polarizations are not found to be important in this case. The dipole polarization effects have been taken into account by the bound configurations generated by the replacement $1s \rightarrow np$. All the configurations generated in this way, which give considerable contributions, are retained in the expansion of the scattering wave functions. The bound radial functions np are varied simultaneously along with the scattering radial function kl for each kinetic energy of the projectile. This procedure includes the dynamical polarization of the target more accurately in an *ab initio* manner.

A large number of configurations, which represent electron correlation and polarization effects, were used to calculate scattering wave functions for the various partial waves over a range of impact energies. For a particular partial wave, the same set of configurations is used for various kinetic energies of the scattering electron. As the dipole polarization of the target is energy dependent, both the bound and the scattering electron orbitals are varied simultaneously at each kinetic energy of the scattering electron. Partial waves up to $l=0$–6 are calculated directly by the MCHF method.[24] The effective range formula[27]

$$\tan\delta_l=\frac{\pi\alpha k^2}{(2l+3)(2l+1)(2l-1)} \qquad (6)$$

is used to calculate higher partial-wave contributions. α is the static dipole polarizability of the target atom. In the case of helium, experimental[28] dipole polarizability $\alpha=1.384\,138\ (a_0^3)$ is used in the present calculation. The effective range formula (6) provides reasonably accurate higher partial-wave phase shifts, since the contributions to the cross sections in the energy range considered are, in general, quite small.

The extended programs were vectorized and optimized according to the architecture of the supercomputer CYBER 205. All calculations in this paper were performed on the CYBER 205 supercomputer.

TABLE I. Comparison of phase shifts with other theories and experiment.

k (a_0^{-1})	η_0					η_1				η_2			
	Present	MS (Ref. 13)	FBH (Ref. 14)	N (Ref. 15)	Expt. (Ref. 7)	Present	MS (Ref. 13)	FBH (Ref. 14)	N (Ref. 15)	Expt. (Ref. 7)	Present	MS (Ref. 13)	FBH (Ref. 14)	N (Ref. 15)	Expt. (Ref. 7)
0.3834	2.6051	2.6372			2.616	0.0489	0.0540			0.052	0.0059	0.0060			
0.6000	2.3488	2.3662	2.3463	2.3486		0.1211	0.1344	0.116	0.1209		0.0143	0.0156	0.0142	0.0149	
0.6062	2.3406	2.3588			2.323	0.1234	0.1371			0.135	0.0146	0.0159			
0.7000	2.2320	2.2518	2.2309	2.2349		0.1592	0.1775	0.1524	0.1588		0.0197	0.0217	0.0193	0.0203	
0.8000	2.1228	2.1449	2.1234	2.1261		0.1973	0.2202	0.1883	0.1960		0.0258	0.0288	0.0253	0.0265	
0.9000	2.0213	2.0456	2.0244	2.0253		0.2334	0.2600	0.2221	0.2305		0.0328	0.0369	0.0320	0.0335	
0.9391	1.9829	2.0086	1.9880	1.9891	1.985	0.2468	0.2745	0.2346	0.2433	0.259	0.0357	0.0403	0.0348	0.0365	
1.0	1.9275	1.9537	1.9339	1.9359		0.2661	0.2953	0.2528	0.2626		0.0406	0.0457	0.0396	0.0414	
1.1	1.8411	1.8689	1.8516	1.8568	1.814	0.2950	0.3254	0.2796	0.2932		0.0492	0.0550	0.0482	0.0501	
1.1817	1.7746	1.8044	1.7907	1.8034		0.3164	0.3461	0.2990	0.3158	0.325	0.0566	0.0628			

IV. RESULTS AND DISCUSSION

Detailed calculations are performed to determine the effects of polarization on the low-energy elastic scattering of electrons from helium atoms. In the present paper, the phase shifts, elastic differential, elastic integral, and momentum-transfer cross sections are calculated for the following process:

$$e^- + \mathrm{He}(^1S) \rightarrow e^- + \mathrm{He}(^1S)$$

at impact energies ranging from 0.58 to 50 eV.

A. Phase shifts

In Table I, the present phase shifts for $l=0$, 1, and 2, calculated in the MCSCF approximation for few impact energies, are compared with the experimental phase shifts of Andrick and Bitsch[7] and the theoretical results of McEachran and Stauffer[13] (MS), Fon *et al.*[14] (FBH), and Nesbet[15] (N).

The present s- and p-wave phase shifts are in excellent agreement with the experimental measurements of Andrick and Bitsch.[7] Andrick and Bitsch[7] derived s- and p-wave phase shifts from their experimental angular distributions of electrons elastically scattered from ground-state He. Their results are available for energies $k=0.3834$, 0.6062, 0.9391, and 1.1817 (a_0^{-1}). Comparison of the present s-, p-, and d-wave phase shifts with the theoretical calculations of Nesbet[15] and Fon *et al.*[14] indicates that the present results are in excellent agreement with these calculations. Excellent agreement is also observed between the calculations of Fon *et al.*[14] and of Nesbet[15] and the experimental measurements of Andrick and Bitsch.[7] Nesbet[15] used the matrix variational method to calculate phase shifts for low-energy electron-helium scattering. Fon *et al.*[14] performed an R-matrix calculation for the elastic scattering of electrons from helium atoms in which the He ground-state wave function is coupled with a ^1P pseudostate to include the full static dipole polarizability. There is good agreement between the present calculation and theoretical calculation made by McEachran and Stauffer.[13] The results obtained by McEachran and Stauffer[13] are slightly higher than the present results, the other theoretical results, and also the experimental measurements of Andrick and Bitsch.[7] McEachran and Stauffer[13] applied adiabatic exchange approximation to calculate phase shifts. They examined the effects of polarization and the exchange on the elastic scattering of electrons from the helium atom for a range of low energies. They did not include the exchange-polarization terms in their calculation. This might have a significant effect on the calculation of phase shifts.

The present phase shifts for $l=0-2$ obtained in the MCSCF approximation are compared with the experimental measurements of Williams[11] and the theoretical results of Nesbet[15] and of Fon *et al.*[14] in Table II for impact energies ranging from 0.58 to 20 eV. Williams[11] has

TABLE II. Comparison of phase shifts with the experimental measurements of Williams and other theories.

E (eV)	η_0 Present	Expt. (Ref. 11)	FBH (Ref. 14)	N (Ref. 15)	η_1 Present	Expt. (Ref. 11)	FBH (Ref. 14)	N (Ref. 15)	η_2 Present	Expt. (Ref. 11)	FBH (Ref. 14)	N (Ref. 15)
0.58	2.8676	2.868		2.8672	0.0133	0.015		0.0140	0.0018	0.0014		0.0018
1.23	2.7375	2.745		2.7386	0.0299	0.035		0.0308	0.0037	0.0031		0.0037
1.51	2.6930	2.702		2.6939	0.0370	0.043		0.0380	0.0045	0.0038		0.0046
2.176	2.6030	2.613		2.6029	0.0542	0.060		0.0552	0.0064	0.0056		0.0066
2.75	2.5374	2.534		2.5378	0.0689	0.077		0.0698	0.0081	0.0068		0.0084
3.40	2.4725	2.464		2.4734	0.0852	0.093		0.0860	0.0100	0.0087		0.0104
3.90	2.4277	2.417		2.4282	0.0975	0.105		0.0980	0.0114	0.0107		0.0119
4.44	2.3832	2.370		2.3834	0.1104	0.116		0.1105	0.0129	0.0118		0.0135
4.91	2.3472	2.332	2.3454	2.3477	0.1213	0.127	0.1163	0.1212	0.0143	0.0132	0.0143	0.0149
5.02	2.3392	2.322	2.3375	2.3398	0.1238	0.129	0.1187	0.1236	0.0148	0.0136	0.0146	0.0153
5.51	2.3047	2.283	2.3034	2.3062	0.1349	0.140	0.1291	0.1344	0.0161	0.0152	0.0160	0.0168
6.10	2.2659	2.242	2.2651	2.2687	0.1476	0.152	0.1412	0.1471	0.0180	0.0172	0.0176	0.0186
6.66	2.2317	2.207	2.2313	2.2353	0.1592	0.161	0.1522	0.1586	0.0196	0.0188	0.0192	0.0203
7.01	2.2119	2.181	2.2111	2.2153	0.1661	0.168	0.1589	0.1656	0.0207	0.0198	0.0203	0.0213
8.00	2.1575	2.134	2.1580	2.1617	0.1849	0.186	0.1766	0.1839	0.0236	0.0228	0.0232	0.0243
8.71	2.1219	2.098	2.1233	2.1260	0.1975	0.196	0.1884	0.1960	0.0258	0.0251	0.0253	0.0265
10.0	2.0629	2.036	2.0656	2.0670	0.2185	0.216	0.2080	0.2161	0.0296	0.0302	0.0290	0.0304
11.0	2.0213	2.003	2.0252	2.0261	0.2332	0.229	0.2219	0.2302	0.0327	0.0334	0.0319	0.0335
12.0	1.9829	1.968	1.9880	1.9891	0.2468	0.242	0.2346	0.2433	0.0357	0.0372	0.0348	0.0365
13.0	1.9472	1.936	1.9536	1.9552	0.2591	0.254	0.2463	0.2555	0.0387	0.0406	0.0378	0.0395
14.0	1.9138	1.908	1.9215	1.9238	0.2706	0.267	0.2569	0.2671	0.0417	0.0444	0.0408	0.0426
15.0	1.8825	1.881	1.8917	1.8948	0.2812	0.278	0.2667	0.2781	0.0447	0.0471	0.0438	0.0456
16.0	1.8531	1.857	1.8638	1.8682	0.2910	0.289	0.2756	0.2886	0.0476	0.0501	0.0468	0.0487
17.0	1.8255	1.833	1.8378	1.8443	0.3000	0.298	0.2840	0.2984	0.0507	0.0527	0.0498	0.0517
18.0	1.7993	1.815	1.8135	1.8228	0.3089	0.305	0.2917	0.3074	0.0534	0.0556	0.0526	0.0547
19.0	1.7746	1.800	1.7907	1.8034	0.3164	0.311	0.2990	0.3158	0.0566	0.0580	0.0555	0.0578
20.0	1.7512	1.784			0.3241	0.315			0.0597	0.0602		

performed a phase-shift analysis of his experimental angular distribution of electrons elastically scattered from the ground state of He and has obtained *s*-, *p*-, and *d*-wave phase shifts. Comparing the present *s*- and *d*-wave phase shifts with the experimental results of Williams[11] and the theoretical results of Nesbet[15] and Fon *et al.*,[14] it is found that the present phase shifts are in excellent agreement with the theoretical results of Nesbet[15] and of Fon *et al.*[14] (where available) and the experimental results of Williams[11] throughout the range of energies considered. There is excellent agreement between the present *p*-wave phase shifts and those of Nesbet[15] and the experimental measurements of Williams.[11] The results obtained by Fon *et al.*[14] in the *R*-matrix calculation for the *p*-wave phase shifts are also very close to the present results and the experimental results of Williams.[11] Their results are slightly lower than the present results; the difference increases with the increase of energy.

B. Differential cross sections

Comparison of differential cross sections with the experimental measurements and other theoretical calculations is an excellent test of any theory. In this paper the differential cross sections computed from the present calculated phase shifts obtained in the MCSCF approximation for $k^2=5$, 12, 20, 30, and 50 eV are compared in Tables III–VII from 0° to 180° with the experimental measurements and the theoretical calculations. As mentioned earlier, the phase shifts for $l=0-6$ have been calculated directly by the MCSCF method. The contributions of phase shifts for $l=7-500$ have been calculated by using the effective-range formula (6). It is found that the differential cross sections converge by $l=60$. As the convergence is very slow in the forward direction, we use the following relation[29] for the elastic differential cross section in the forward direction:

TABLE III. Comparison of differential cross sections (10^{-16} cm^2/sr) at $E=5$ eV, with experiments and other theories.

θ (deg)	Present	RTS (Ref. 10)	AB (Ref. 7)	FBH (Ref. 14)	MS (Ref. 13)	N (Ref. 15)
0	0.2450			0.2408	0.244	0.245
5	0.2419			0.2391	0.240	0.242
10	0.2403			0.2388	0.236	0.240
15	0.2403		0.250		0.233	0.239
20	0.2417	0.2465	0.257	0.2419	0.232	0.241
25	0.2447		0.264		0.232	0.244
30	0.2493	0.2503	0.267	0.2510	0.235	0.248
35	0.2554		0.268		0.239	0.255
40	0.2631	0.2720	0.277	0.2662	0.245	0.263
45	0.2724		0.285		0.254	0.272
50	0.2833	0.2993	0.294	0.2871	0.264	0.283
55	0.2958		0.311		0.276	0.296
60	0.3097	0.3164	0.321	0.3136	0.290	0.310
65	0.3250		0.340		0.306	0.325
70	0.3416	0.3407	0.355	0.3449	0.323	0.342
75	0.3592		0.374		0.342	0.360
80	0.3777	0.3810	0.387	0.3801	0.362	0.378
85	0.3971		0.403		0.383	0.397
90	0.4170	0.4157	0.425	0.4181	0.405	0.417
95	0.4375		0.446		0.428	0.438
100	0.4582	0.4624	0.470	0.4575	0.452	0.458
105	0.4790		0.498		0.475	0.479
110	0.4996	0.5061	0.512	0.4970	0.499	0.499
115	0.5199		0.536		0.522	0.520
120	0.5397	0.5442	0.581	0.5320	0.545	0.539
125	0.5588		0.587		0.566	0.558
130	0.5770	0.5905	0.607	0.5701	0.587	0.576
135	0.5942		0.635		0.607	0.593
140	0.6101	0.6267	0.657	0.6009	0.625	0.609
145	0.6248		0.665		0.641	0.623
150	0.6380	0.6567		0.6262	0.656	0.635
155	0.6495				0.668	0.646
160	0.6593			0.6449	0.679	0.655
165	0.6671				0.687	0.662
170	0.6729			0.6564	0.693	0.668
175	0.6763				0.697	0.671
180	0.6774			0.6601	0.698	0.672

$$\left[\frac{d\sigma}{d\theta}\right]_{\theta=0^\circ} = \left|\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l\right|^2$$
$$= \left[\frac{d\sigma}{d\Omega}\right]_{\theta=0^\circ}^{l<l_0} + \left[\frac{\pi\alpha l_0}{4l_0^2-1}\right]\sum_{l=0}^{l_0-1}(2l+1)\sin 2\delta_l + \left[\frac{\pi\alpha l_0 k}{4l_0^2-1}\right]^2,$$

where $l_0=7$ and the effective-range formula, Eq. (6), has been used for $l \geq 7$ to derive this formula.

In Table III, we compare present differential cross sections at 5 eV with the experimental results of Register *et al.*[10] (RTS) and of Andrick and Bitsch[7] (AB) and the theoretical results of Fon *et al.*,[14] McEachran and Stauffer,[13] and of Nesbet.[15] Register *et al.*[10] obtained absolute differential cross sections from the measurements of angular distributions in a crossed-beam geometry using a collimated, differentially pumped atomic-beam source which requires no effective-path-length correction. The agreement with the experimental measurements of Register *et al.*[10] is excellent for all angles at this energy. The experimental differential cross sections of Andrick and Bitsch[7] are very close but slightly higher than the present results and the results obtained by Register *et al.*[10] The theoretical results obtained by Fon *et al.*[14] and of Nesbet[15] agree very well with the present and the experimental results of Register *et al.*[10] and of Andrick and Bitsch.[7] The results obtained by McEachran and Stauffer[13] agree well with the present and the other theoretical and the experimental results. Their results are slightly lower for $\theta=0^\circ-110^\circ$ and are little higher for

TABLE IV. Comparison of differential cross sections (10^{-16} cm^2/sr) at $E=12$ eV, with experiments and other theories.

θ (deg)	Present	RTS (Ref. 10)	AB (Ref. 7)	FBH (Ref. 14)	MS (Ref. 13)	N (Ref. 15)
0	0.5570			0.5171	0.608	0.552
5	0.5143			0.4826	0.573	0.509
10	0.4764			0.4466	0.529	0.470
15	0.4420	0.4603	0.460		0.488	0.435
20	0.4110	0.4271	0.415	0.3859	0.451	0.403
25	0.3833	0.3885	0.393		0.416	0.375
30	0.3588	0.3641	0.360	0.3389	0.386	0.351
35	0.3373	0.3364	0.339		0.357	0.330
40	0.3189	0.3206	0.319	0.3043	0.334	0.312
45	0.3035		0.305		0.313	0.298
50	0.2911	0.2918	0.287	0.2813	0.297	0.286
55	0.2816		0.279		0.284	0.278
60	0.2750	0.2768	0.273	0.2689	0.275	0.272
65	0.2711		0.274		0.269	0.268
70	0.2698	0.2708	0.270	0.2660	0.266	0.268
75	0.2706		0.272		0.267	0.269
80	0.2736	0.2717	0.275	0.2713	0.269	0.272
85	0.2783		0.271		0.275	0.277
90	0.2847	0.2848	0.281	0.2831	0.282	0.283
95	0.2924		0.294		0.291	0.291
100	0.3013	0.2986	0.297	0.2997	0.301	0.300
105	0.3111		0.313		0.313	0.310
110	0.3216	0.3206	0.322	0.3194	0.325	0.320
115	0.3324		0.327		0.338	0.331
120	0.3435	0.3438	0.347	0.3403	0.351	0.342
125	0.3545		0.350		0.364	0.353
130	0.3653	0.3681	0.366	0.3609	0.377	0.364
135	0.3757		0.387		0.389	0.374
140	0.3857	0.3915	0.388	0.3797	0.400	0.384
145	0.3951		0.405		0.411	0.393
150	0.4037	0.4138		0.3956	0.420	0.401
155	0.4116				0.428	0.408
160	0.4184			0.4077	0.435	0.414
165	0.4240				0.441	0.419
170	0.4282			0.4152	0.445	0.422
175	0.4308				0.447	0.424
180	0.4316			0.4176	0.448	0.425

$\theta=115°-180°$ than the present results.

At 12 eV, the present differential cross sections are compared with the experimental measurements of Register *et al.*[10] and of Andrick and Bitsch[7] and the theoretical results of Fon *et al.*,[14] McEachran and Stauffer,[13] and of Nesbet,[15] in Table IV. Comparison with the experimental measurements of Register *et al.*[10] and of Andrick and Bitsch[7] shows that the present results are in excellent agreement with these experimental measurements. The two experimental results are very close to each other. The present differential cross sections agree remarkably well with the theoretical results of Nesbet.[15] The results obtained by Fon *et al.*[14] agree very well with the present results. Except at a few angles, their results are slightly lower than the present results. The results obtained by McEachran and Stauffer[13] are a little higher than the present results for $\theta=0°-55°$. For $\theta=65°-100°$, their results are little lower than the present results. For the remaining angles their results are again a little higher. Both experimental and the theoretical results show a minimum at $\theta=70°$. It is also found that the magnitude of the minimum obtained by different experimental measurements and the different theoretical calculations at this energy are in excellent agreement.

TABLE V. Comparison of differential cross sections (10^{-16} cm^2/sr) at $E=20$ eV, with experiments and other theories.

θ (deg)	Present	RTS (Ref. 10)	FBH (Ref. 14)	MS (Ref. 13)
0	0.7839		0.7132	0.833
5	0.7043		0.6470	0.771
10	0.6338		0.5800	0.696
15	0.5701	0.5854		0.625
20	0.5129	0.5175	0.4681	0.561
25	0.4618	0.4549		0.503
30	0.4163	0.4092	0.3808	0.451
35	0.3762	0.3706		0.404
40	0.3409	0.3319	0.3142	0.363
45	0.3103			0.327
50	0.2839	0.2730	0.2652	0.297
55	0.2617			0.271
60	0.2431	0.2340	0.2311	0.250
65	0.2281			0.232
70	0.2161	0.2061	0.2091	0.218
75	0.2069			0.208
80	0.2002	0.1928	0.1969	0.201
85	0.1957			0.196
90	0.1930	0.1872	0.1921	0.193
95	0.1918			0.192
100	0.1920	0.1846	0.1927	0.193
105	0.1932			0.195
110	0.1953	0.1872	0.1970	0.198
115	0.1979			0.201
120	0.2010	0.1942	0.2032	0.205
125	0.2044			0.210
130	0.2080	0.2012	0.2103	0.214
135	0.2117			0.219
140	0.2155	0.2061	0.2171	0.223
145	0.2194			0.227
150	0.2231	0.2154	0.2229	0.231
155	0.2267			0.234
160	0.2301		0.2273	0.237
165	0.2329			0.239
170	0.2351		0.2300	0.241
175	0.2364			0.242
180	0.2368		0.2309	0.243

In Table V, we compare the present differential cross sections at 20 eV with the experimental results of Register *et al.*[10] and the theoretical results of Fon *et al.*[14] and of McEachran and Stauffer.[13] The present results are in excellent agreement with the experimental measurements of Register *et al.*[10] The theoretical results obtained by Fon *et al.*[14] agree very well with the present results. Their results are little lower than the present results for $\theta=0°-90°$ and for $\theta=150°-180°$. For $\theta=100°-140°$ their results are slightly higher than the present results. The present results are in very good agreement with the theoretical results obtained by McEachran and Stauffer.[13] Except for a few angles their results are slightly higher than the present results. The difference gradually decreases as the angle increases, goes to a minimum, and then increases. At this energy the present results go to a minimum at $\theta=95°$. This agrees very well with the minimum obtained by the experimental measurements and the other theoretical calculations.

At 30 eV, the present differential cross sections are compared with the experimental results of Register *et al.*[10] and the theoretical results of Fon *et al.*,[14] McEachran and Stauffer,[13] and of Scott and Taylor,[17] in Table VI. Scott and Taylor[17] (ST) used many-body theory to the electron-helium atom scattering and reported results for elastic differential cross sections obtained from their calculated phase shifts. The present differential cross sections at this energy agree well with the experimental results of Register *et al.*[10] For $\theta=20°-150°$ the present results are slightly higher than the experimental results. There is good agreement between the theoretical results obtained by Fon *et al.*[14] and the present results. Their results for $\theta=30°-100°$ are slightly lower than the present results but they are closer to the experimental results of Register *et al.*[10] For $\theta=110°-180°$, their results are a little higher. The results obtained by McEachran and Stauffer[13] also agree well with the present results but their results are slightly higher than the present results throughout the angles considered. The present results agree very well with the theoretical results obtained by Scott and Taylor.[17] For $\theta=25°-110°$, their results are lower than the present results but closer to the experimental results of Register *et al.*[10] For $\theta=120°-140°$, their results are slightly higher. The minimum at this energy occurs at $\theta=130°$, whereas the experimental minimum is at $\theta=110°$. The minima in the differential cross sections, obtained by Fon *et al.*,[14] McEachran and Stauffer,[13] and Scott and Taylor,[17] at this energy occur, respectively, at $\theta=110°$, 120°, and 100°.

The present differential cross sections at 50 eV are compared with the experimental results of Register *et al.*[10] and of McConkey and Preston[9] (MP) and the

theoretical results of Fon *et al.*,[14] McEachran and Stauffer,[13] and Scott and Taylor,[17] in Table VII. The present differential cross sections agree well with the experimental measurements of Register *et al.*[10] and of McConkey and Preston.[9] The present results are slightly higher for $\theta=15°-130°$ and are slightly lower for $\theta=140°-150°$ than the experimental results of Register *et al.*[10] For $\theta=30°-90°$, the present results are in better agreement with the experimental results of McConkey and Preston.[9] The measurements of McConkey and Preston[9] are limited in the range of scattering angles $20°\leq\theta\leq90°$. The results obtained by Fon *et al.*[14] agree well with the present results. For $\theta=10°-90°$, their results are closer to the experimental results of Register *et al.*[10] than the present results. For $\theta=100°-180°$, the present results are in better agreement. Good agreement is observed between the results of McEachran and Stauffer[13] and the present results. Their results are slightly higher than the present results. The results obtained by Scott and Taylor[17] are lower than the present results for $\theta=0°-110°$. Their results are higher for $\theta=120°-140°$.

C. Integral-elastic cross section

We compare in Table VIII the present integral-elastic cross sections with the theoretical results of McEachran and Stauffer,[13] Fon *et al.*,[14] Nesbet,[15] Scott and Taylor,[17] de Heer and Jansen[30] (DJ) and the experimental results of Buckman and Lohmann[12] (BL), Nickel *et al.*[1] (NIRT), Charlton *et al.*[3] (CGHT), Kauppila *et al.*[2] (K), Newell *et al.*[8] (NBS), Register *et al.*,[10] Blaauw *et al.*[4] (BWBH), Kennerly and Bonham[5] (KB), Andrick and Bitsch,[7] and McConkey and Preston.[9] We presented our results for

TABLE VI. Comparison of differential cross sections (10^{-16} cm^2/sr) at $E=30$ eV, with experiments and other theories.

θ (deg)	Present	RTS (Ref. 10)	FBH (Ref. 14)	MS (Ref. 13)	ST (Ref. 17)
0	0.8990		0.8601	0.934	0.890
5	0.7865		0.7581	0.848	0.804
10	0.6897		0.6599	0.746	0.700
15	0.6045	0.6172		0.653	0.605
20	0.5299	0.5255	0.4997	0.571	0.521
25	0.4647	0.4478		0.498	0.445
30	0.4077	0.3862	0.3798	0.435	0.381
35	0.3581	0.3256		0.379	0.325
40	0.3150	0.2872	0.2921	0.332	0.277
45	0.2778			0.291	
50	0.2459	0.2219	0.2290	0.256	0.207
55	0.3190			0.227	
60	0.1964	0.1718	0.1847	0.202	0.161
65	0.1777			0.182	
70	0.1623	0.1412	0.1545	0.166	0.134
75	0.1499			0.153	
80	0.1400	0.1231	0.1347	0.142	0.119
85	0.1321			0.134	
90	0.1260	0.1114	0.1227	0.128	0.112
95	0.1213			0.123	
100	0.1178	0.1049	0.1165	0.120	0.110
105	0.1152			0.118	
110	0.1134	0.1006	0.1146	0.116	0.111
115	0.1123			0.116	
120	0.1116	0.1029	0.1156	0.115	0.113
125	0.1112			0.116	
130	0.1111	0.1055	0.1184	0.116	0.116
135	0.1112			0.117	
140	0.1115	0.1086	0.1221	0.117	0.119
145	0.1119			0.118	
150	0.1125	0.1089	0.1258	0.119	
155	0.1131			0.120	
160	0.1138		0.1289	0.120	
165	0.1145			0.121	
170	0.1150		0.1310	0.122	
175	0.1154			0.122	
180	0.1154		0.1316	0.122	

impact energies from 0.58 to 50 eV. The present results are in excellent agreement with the theoretical results of Nesbet.[17] The results obtained by McEachran and Stauffer[13] were slightly lower at 2.0 and 5.0 eV, whereas their results are a little higher for the other energies. The present results agree very well with the theoretical results obtained by Fon *et al.*[14] in the *R*-matrix method. The results obtained by Scott and Taylor[17] in the many-body theory for energies 30, 40, and 50 eV are lower than the present results. The semiempirical result obtained by de Heer and Jansen[30] at 5 eV is higher than the present results. At other energies shown in the table, their results are lower but close to the present results. At energies 1.0 and 1.51 eV the present cross sections agree very well with the experimental results of Kennerly and Bonham.[5] The experimental result obtained by Buckman and Lohmann[12] at 1.0 eV is also in very good agreement with the present result. The experimental results of Kauppila *et al.*[2] at energies 3.40, 4.44, 5.40, 6.40, 7.40, 8.40, 9.40, 10.40, 11.40, 12.40, 13.40, 14.40, 15.40, 16.40, 17.40, and 18.40 eV show excellent agreement with the present results and the theoretical calculations of Nesbet[15] and of Fon *et al.*[14] At 2.0 and 3.0 eV, there is excellent agreement between the present results and the experimental measurements of Buckman and Lohmann,[12] Kennerly and Bonham,[5] and Kauppila *et al.*[2] In comparison with the measurements at energies 5.0, 6.0, 8.0, 10.0, and 12.0 eV, the present results show excellent agreement with the experimental results of Buckman and Lohmann,[12] Kennerly and Bonham,[5] and Nickel *et al.*[1] The measurements of Register *et al.*[10] at 5.0 and 12.0 eV agree very well with the present calculations. The results of Charl-

TABLE VII. Comparison of differential cross sections (10^{-16} cm^2/sr) at $E=50$ eV, with experiments and other theories.

θ (deg)	Present	RTS (Ref. 10)	FBH (Ref. 14)	MS (Ref. 13)	ST (Ref. 17)	MP (Ref. 9)
0	0.9574		0.9663	0.971	0.893	
5	0.7994		0.8195	0.851	0.770	
10	0.6687	0.6765	0.6763	0.715	0.641	
15	0.5591	0.5475		0.596	0.526	
20	0.4674	0.4364	0.4497	0.498	0.428	0.358
25	0.3912	0.3514		0.416	0.344	0.329
30	0.3279	0.2810	0.2999	0.348	0.278	0.285
35	0.2756	0.2334		0.291	0.223	0.257
40	0.2323	0.1931	0.2061	0.244	0.181	0.218
45	0.1965			0.206		0.187
50	0.1671	0.1316	0.1480	0.175	0.125	0.164
55	0.1431			0.149		0.140
60	0.1235	0.0984	0.1118	0.129	0.093	0.117
65	0.1076			0.112		0.103
70	0.0948	0.0771	0.0888	0.098	0.074	0.088
75	0.0845			0.087		0.084
80	0.0762	0.0623	0.0738	0.078	0.062	0.077
85	0.0695			0.071		0.067
90	0.0640	0.0541	0.0639	0.066	0.055	0.062
95	0.0595			0.061		
100	0.0558	0.0474	0.0573	0.057	0.051	
105	0.0528			0.054		
110	0.0503	0.0429	0.0529	0.052	0.050	
115	0.0482			0.050		
120	0.0464	0.0403	0.0500	0.048	0.051	
125	0.0450			0.047		
130	0.0439	0.0388	0.0481	0.046	0.051	
135	0.0431			0.045		
140	0.0424	0.0426	0.0470	0.044	0.052	
145	0.0419			0.043		
150	0.0416	0.0426	0.0463	0.043		
155	0.0415			0.043		
160	0.0414		0.0460	0.043		
165	0.0414			0.042		
170	0.0415		0.0458	0.042		
175	0.0415			0.042		
180	0.0415		0.0458	0.043		

TABLE VIII. Comparison of integral-elastic cross section (10^{-16} cm^2) with experiments and other theories.

E (eV)	Present	MS (Ref. 13)	FBH (Ref. 14)	N (Ref. 15)	ST (Ref. 17)	DJ (Ref. 30)	NBS (Ref. 8)	RTS (Ref. 10)	BWBH (Ref. 4)	KB (Ref. 5)	BL (Ref. 12)	K (Ref. 2)	AB (Ref. 7)	MP (Ref. 9)	NIRT (Ref. 1)	CGHT (Ref. 3)
0.58	6.0888															
1.0	6.1312			6.115						6.23	6.08					
1.23	6.1253															
1.51	6.0981			6.084						6.18						
2.0	6.0148	5.806		6.027						6.06	5.88	5.934	6.22			5.463
2.176	5.9883															
2.75	5.8736															
3.0	5.8186			5.816						5.78	5.70	5.785	6.18			5.551
3.40	5.7325			5.727								5.68				5.400
3.90	5.6221															
4.44	5.5043			5.513								5.53				
4.91	5.4040															
5.0	5.3848	5.312	5.365	5.379		5.629		5.482		5.25	5.26	5.388	5.699		5.346	5.226
5.02	5.3798															
5.40	5.2995		5.273	5.287								5.30				
5.51	5.2767															
6.0	5.1745		5.138	5.152						5.04	5.06	5.192	5.433		5.122	
6.10	5.1540															
6.40	5.0931		5.051	5.066								5.09				
6.66	5.0390															
7.0	4.9713		4.923	4.942						4.83		5.007	5.158			
7.01	4.9649															
7.40	4.8923		4.841	4.864								4.96				
8.0	4.7776		4.720	4.749						4.64	4.67	4.836	4.929		4.735	
8.4	4.7034		4.642	4.675								4.73				
8.71	4.6469															
9.0	4.5945		4.527	4.566						4.46		4.604	4.704			
9.4	4.5232		4.453	4.495								4.54				
10.0	4.4194	4.486	4.345	4.390			4.223			4.30	4.30	4.388	4.508		4.389	4.144
10.4	4.3515		4.275	4.322								4.25				
11.0	4.2529											4.223	4.326			
11.4	4.1888		4.108	4.155								4.26				
12.0	4.0950	4.188	4.010	4.059				4.127		3.96	3.96	4.091	4.144		4.065	
12.4	4.0340		3.949	3.996								3.94				
13.0	3.9440											3.906	3.948			
13.4	3.8868		3.799	3.848								3.92				
14.0	3.8021		3.713	3.763						3.69	3.67	3.786	3.850		3.777	
14.4	3.7479		3.657	3.709								3.64				
15.0	3.6678	3.781	3.576			3.596	3.503					3.610	3.696	3.428		3.431
15.4	3.6156		3.524	3.580								3.58				

TABLE VIII. (*Continued*).

E (eV)	Present	MS (Ref. 13)	FBH (Ref. 14)	N (Ref. 15)	ST (Ref. 17)	DJ (Ref. 30)	NBS (Ref. 8)	RTS (Ref. 10)	BWBH (Ref. 4)	KB (Ref. 5)	BL (Ref. 12)	K (Ref. 2)	AB (Ref. 7)	MP (Ref. 9)	NIRT (Ref. 1)	CGHT (Ref. 3)
16.0	3.5404		3.448	3.506					3.428	3.43	3.41	3.450	3.556		3.520	
16.4	3.4911		3.398	3.458								3.41				
17.0	3.4193								3.293			3.341	3.450			
17.4	3.3729		3.279	3.341								3.27				
18.0	3.3072	3.423	3.212	3.274				3.347	3.240	3.22	3.17	3.212	3.332		3.274	
18.4	3.2621		3.168	3.230								3.20				
19.0	3.1979	3.313							3.156			3.10	3.234			
20.0	3.0972	3.208	3.442			2.946		3.00±0.09	3.036	3.03	2.99	2.999		2.811	3.095	2.938
25.0	2.6435	2.753	2.952			2.307		2.51±0.10	2.641			2.585		2.193	2.680	
30.0	2.2898	2.390	2.535		2.13	2.237		2.11±0.13	2.372	2.36		2.34		2.125	2.391	2.305
35.0	2.0177					1.883		1.89±0.09	2.156	2.14				1.786		
40.0	1.7915	1.862	1.968		1.61	1.680		1.58±0.06	1.952	1.95				1.596	2.001	2.006
50.0	1.4391	1.506	1.573		1.27	1.385		1.26±0.05	1.728	1.68		1.733		1.316	1.715	1.680

ton *et al.*[3] at 5.0 and 10.0 eV and of Newell *et al.*[8] at 10.0 eV are lower than the present results. At energies 7.0, 9.0, and 11.0 eV, the present results are in excellent agreement with the experimental result of Kauppila *et al.*[2] At 13.0 eV, the result obtained by Andrick and Bitsch[7] shows excellent agreement with the present result. The present result at 14.0 eV is in excellent agreement with the experimental result of Kauppila *et al.*[2] and Nickel *et al.*[1] The results obtained by Buckman and Lohmann,[12] Kennerly and Bonham,[5] and Andrick and Bitsch[7] are also very close to the present result. The results obtained by Andrick and Bitsch[7] and Kauppila *et al.*[2] at 15.0 eV show excellent agreement with the present result. Excellent agreement is also observed between the present cross section at 16.0 eV and the experimental results of Andrick and Bitsch[7] and Nickel *et al.*[1] The results of Blaauw *et al.*,[4] Buckman and Lohmann,[12] Kennerly and Bonham,[5] and Kauppila *et al.*[2] at this energy are very close and are lower than the present result. The measurement of Andrick and Bitsch[7] agrees very well with the present cross section at 17.0 eV. The experimental results of Andrick and Bitsch,[7] Nickel *et al.*,[1] and Register *et al.*,[10] at 18.0 eV are in excellent agreement with the present result. The results obtained by Blaauw *et al.*,[4] Buckman and Lohmann,[12] Kennerly and Bonham,[5] and Kauppila *et al.*[2] are very close to each other but are slightly lower than the present result. At 19.0 eV, the present result agrees very well with the experimental results of Blaauw *et al.*[4] and of Andrick and Bitsch.[7] The present results at 20.0 and 25.0 eV are remarkably in best agreement with the results of Nickel *et al.*,[1] Blaauw *et al.*,[4] and of Register *et al.*[10] The results of Buckman and Lohmann[12] and of Kennerly and Bonham[5] at this energy are also very close. The results obtained by McConkey and Preston[9] and Kauppila *et al.*[2] at 25.0 eV are lower than the present result. The present cross section at 30.0 eV is in excellent agreement with the experimental result of Charlton *et al.*,[3] Kauppila *et al.*,[2] Kennerly and Bonham,[5] Nickel *et al.*,[1] Register *et al.*[10] and Blaauw *et al.*[4] The present result at 35.0 eV, shows excellent agreement with the experimental results of Kennerly and Bonham,[5] Blaauw *et al.*,[4] and of Register *et al.*[10] Good agreement is observed between the present cross section at 40.0 eV and the experimental measurements of Kennerly and Bonham,[5] Blaauw *et al.*,[4] and of Register *et al.*[10] Comparison of the present cross section at 50.0 eV indicates that there is good agreement with the experimental result of Charlton *et al.*,[3] Kennerly and Bonham,[5] and of Register *et al.*[10]

D. Momentum-transfer cross section

In Table IX, the momentum-transfer cross sections are compared with the theoretical results of Nesbet,[15] McEachran and Stauffer,[13] and Fon *et al.*[14] and the experimental results of Newell *et al.*,[8] Register *et al.*,[10] Milloy and Crompton[31] (MC), Andrick and Bitsch,[7] and of Crompton *et al.*[32] (CER), at impact energies ranging from 0.58 to 50 eV. Comparing the present cross section with the theoretical results, it is found that the present results are in excellent agreement with the results ob-

tained by Nesbet.[15] The present results are also in very good agreement with the theoretical results of Fon *et al.*[14] Their results are slightly lower than the present results except for energies from 20 to 50 eV, for which their results are higher. The present results also agree very well with the theoretical results of McEachran and Stauffer.[13] Their results are little higher than the present results. The experimental results of Crompton *et al.*[32]

TABLE IX. Comparison of momentum-transfer cross sections (10^{-16} cm^2) with experiments and other theories.

E (eV)	Present	MS (Ref. 13)	FBH (Ref. 14)	N (Ref. 15)	NBS (Ref. 8)	RTS (Ref. 10)	MC (Ref. 31)	AB (Ref. 7)	CER (Ref. 32)
0.58	6.6500								
1.0	6.8573			6.880					6.85
1.23	6.9369								
1.51	6.9750			6.980					6.96
2.0	6.9609	6.832		7.005				7.2	6.99
2.176	6.9695								
2.75	6.8967								
3.0	6.8490			6.856					6.89
3.40	6.7643								
3.90	6.6412								
4.44	6.4940								
4.91	6.3574								
5.0	6.3306	6.342	6.278	6.320		6.451	6.31	6.64	6.26
5.02	6.3228								
5.40	6.2090								
5.51	6.1747								
6.0	6.0198		5.959	5.994			6.00		6.01
6.10	5.9881								
6.40	5.8931								
6.66	5.8088								
7.0	5.6999		5.633	5.670			5.68		
7.01	5.6928								
7.40	5.5719								
8.0	5.3832		5.315	5.354			5.35		
8.4	5.2590								
8.71	5.1643								
9.0	5.0754		5.007	5.049			5.03		
9.4	4.9572								
10.0	4.7824	4.864	4.714	4.755	4.601		4.72		
10.4	4.6682								
11.0	4.5027		4.438	4.476			4.44		
11.4	4.3957								
12.0	4.2400	4.323	4.180	4.213		4.270	4.15	4.28	
12.4	4.1395								
13.0	3.9930		3.938						
13.4	3.8995								
14.0	3.7632		3.713						
14.4	3.6759								
15.0	3.5491	3.625	3.504		3.498				
15.4	3.4683								
16.0	3.3501		3.309						
16.4	3.2743								
17.0	3.1647		3.130						
17.4	3.0951								
18.0	2.9940	3.060	2.963	2.996		3.041		3.013	
18.4	2.9276								
19.0	2.8329	2.897	2.809	2.86				2.86	
20.0	2.6847	2.745	3.028			2.576			
25.0	2.0789	2.132	2.376			1.947			
30.0	1.6384	1.697	1.896			1.506			
35.0	1.3292								
40.0	1.1253	1.145	1.281			0.977			
50.0	0.7957	0.824	0.913			0.702			

are in excellent agreement with the present results. Good agreement is observed between the measurement of Andrick and Bitsch[7] and the present result at 2.0 eV. The present result at 5.0 eV shows excellent agreement with the experimental result of Milloy and Crompton.[31] It also agrees well with the experimental result of Register *et al.*[10] The result obtained by Andrick and Bitsch[7] at this energy is higher than the present result. At 12 eV, the present result is in excellent agreement with the experimental results of Register *et al.*[10] and of Andrick and Bitsch.[7] The experimental result obtained by Milloy and Crompton[31] is slightly lower than the present result. There is good agreement between the present results and the experimental results of Newell *et al.*[8] at 10 and 15 eV. Excellent agreement is observed between the present result and the experimental results of Andrick and Bitsch[7] and of Register *et al.*[10] at 18.0 eV. The experimental result of Andrick and Bitsch[7] shows excellent agreement with the present result at 19.0 eV. There is very good agreement between the present results and the experimental results of Register *et al.*[10] at energies 20, 25, 30, 40, and 50 eV.

V. CONCLUSION

Accurate *ab initio* calculations of the elastic scattering of low-energy electrons by helium atoms have been performed with the multiconfiguration Hartree-Fock method. The dynamical polarization of the target and the electron correlation effects, which are very important in this case, have been taken into account more accurately in an *ab initio* manner through the configuration-interaction procedure. The beauty of the present method lies in the fact that both the bound and the scattering electron wave functions are varied simultaneously for each kinetic energy of the scattering electron to obtain the energy-dependent target polarization very accurately. It is found that only the dipole part of the polarization is important in this simple case of electron-helium scattering. We have carried out detailed calculations on phase shifts, elastic differential, integral-elastic, and momentum-transfer cross sections for a range of energies from 0.58 to 50 eV. The phase shifts calculated in the present MCSCF method are in excellent agreement with the experimental phase shifts derived by Williams[11] and Andrick and Bitsch[7] from their elastic differential cross-section measurements. Comparison of differential, integral-elastic, and momentum-transfer cross sections with experimental measurements and other theoretical calculations, which is an accurate test of any theory, shows that the present MCSCF calculations achieve excellent agreement with accurate experimental measurements and theoretical calculations on these quantities over the range of energies considered. Finally, we conclude from our present results that the MCSCF calculation is capable of producing high-quality data on the elastic scattering of electrons from atoms over the low- and intermediate-energy ranges.

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