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Mathematical Modeling in Law and Political Science: Learning from Public Health

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Mathematical Modeling in Law and Political Science: Learning from Public Health

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ABSTRACT: This paper provides an overview of mathematical modeling in public health policy and recommends the teaching of mathematical models in other fields, like law and undergraduate political science studies. First, I describe various facets of public health in terms of their scope and goals. The complex nature of public health lends way to a description of mathematical modeling and the role it can serve. Various mathematical solution concepts are also provided, including the SIR model, reproductive number, and game theory. Finally, I explain why knowledge of simple models is beneficial for students in pre-professional programs in law and political science. While these models are a cornerstone for public policy and health, their analogies are not as common within undergraduate studies in law and political science. This research aims to support extensions of these models.

KEYWORDS: mathematical modeling, public health, decision making

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ESSAY

Why don't undergraduate students of law or political science study mathematical modeling? By contrast, mathematical modeling has historically been applied to public health. Unlike law or political science, the goals of public health are to prevent and manage diseases and promote healthy behaviors among populations, and public health policies vary from relatively minor suggestions to sweeping regulations (Frenk, 1993). But like law and political science, the problems that public health addresses are often complex and involve multiple variables. Systems design in public health addresses these problems by building models and theoretical frameworks, activities with useful applications to legal and political problems.

Effective models, however, may present significant challenges when applied to real-world problems. For example, large-scale campaigns and the techniques to encourage hand washing can be difficult to implement, even among healthcare professionals. Bischoff, Renolds, Sessler, Edmond, and Wenzel (2000), for example, found that routine hospital handwashing compliance was unacceptably low at a leading teaching hospital. Furthermore, implementation of a simple education/feedback intervention failed to improve handwashing compliance. Despite this lack of compliance, making alcohol-based waterless antiseptic dispensers available did lead to significantly higher handwashing rates (Bischoff et al., 2000). A similar trend has been found among emergency medical service providers prior to arrival at the hospital (Bucher, Donovan, Ohman-Strickland, McCoy, 2014).

Ultimately, an informed systems framework can design more sophisticated models, better simulate real problems, and contribute to their solutions. Such sophisticated models have become increasingly quantitative and require an interdisciplinary approach to design, communicate, and execute (Basu & Andrews, 2013). Due to the complex nature of diseases, a systems approach allows a dynamic evaluation of more than the sum of individual factors (Aron, 2007). Yet the broadness of systems and model thinking can also be its Achilles' heel. As our understanding of quantitative biology becomes more refined, eclectic modeling for public health and for such fields as law and undergraduate political science studies becomes vital. My research aims to review these developments by advocating for an increased interdisciplinary approach between mathematical modeling and other disciplines. Examples of this blending will be explored, including mathematical modeling in epidemiology and interdisciplinary approaches within academia. My paper concludes by making an appeal to establish a bridge between these two fields: public health and law/political science.

While mathematical modeling may offer fresh perspectives for fields like law, its limitations cannot be discounted. Mathematical models are simple, while society and human action are not. Any attempt to distill the effectiveness of social programs, legal processes, or political movements to numerical values can easily become a reduction to the absurd. These broad strokes may offer new ways of thinking about old problems, but are just one of many tools for measuring effectiveness and analytical insight.

In the broadest sense, mathematical modeling is a description of a systems model that uses mathematical concepts and language (Leischow & Milstein, 2006). Increasingly, mathematical modeling has been applied to virtually every field, from physics to political science. Mathematical models can be used to build dynamic systems, statistical models, differential equations, or game theory models (Leischow & Milstein, 2006). Each one of these models provides a new set of tools and a fresh perspective when viewing complex or intractable problems. For example, game theory has transformed evolutionary biology (Axelrod & Hamilton, 1981). This application of mathematics to complex behaviors typically involves the relationship between variables. Common classifications of mathematical models include linear vs. nonlinear, static vs. dynamic, explicit vs. implicit, discrete vs. continuous, deterministic vs. stochastic, and deductive, inductive, or floating models. For example, within psychology, a linear relationship may follow a statistical model that represents a direct relationship, like age, obesity, and motor skills (Vameghi, Shams, and Dehkordi, 2013).

Despite its usefulness, basic mathematical modeling is noticeably lacking in pre-professional programs in law and in fields like political science. This omission could be due to the compartmentalized nature of academia, since modeling is not a stand-alone field leading to a degree in its own right (Silecchia, 1996). Regardless, today's students will become our future healthcare providers and political leaders. All pre-professional programs are expected to shape ethical, socially-conscious students. In addition, these programs aim to foster high levels

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of critical thinking and analytical skills. These qualities are especially important when making decisions, and learning basic quantitative modeling may promote and nurture these critical thinking and analytical skills.

By way of example, Goldhaber-Fieber and Garnett (2006) survey links between decision science, mathematical modeling, and disease prevention. Specifically, they outline how decisions are made and strategies used, including the intuitive opinions of experts, quantitative frameworks, and explicit models to synthesize data. They then apply these broad categories to cervical cancer prevention and control. Goldhaber-Fieber and Garnett go on to discuss the complex questions explored by policy makers, including the rate of coverage between different groups, gender differences in vaccination, and the possibility of combined screenings and cost-effectiveness. In addition, they use longitudinal cohort models and applied deterministic and stochastic models to create schematic representations of the natural history of HPV and cervical cancer. Two examples of their mathematical approach to HPV and cancer can be seen in Figure 1.

The work of Goldhaber-Fieber and Garnett showcases quality representations of mathematical modeling in policy making. As public health becomes more integrated with evidence-based models, the integration of traditionally unrelated disciplines becomes increasingly useful. Another application of mathematical modeling, in biology, includes the field of epidemiology. Important to this application are the concepts of *endemicity*, *age at infection*, and *herd immunity*. In brief, *endemicity* refers to the persistence of an infection within a population, while *herd immunity* refers to the critical portion of a population that must be immune for a disease to be contained or removed from a community. All of these concepts can be expressed mathematically and are the bases of various models.

In particular, the SIR Model is a simple and useful model in epidemiology. In summary, the SIR model describes population densities of being composed of those susceptible (X), infected (Y), and removed from the population by either becoming immune or death (Z). Within a closed population, $X + Y + Z = N$. These basic parameters can then be used to model macro-level problems, including the mathematical movement of individuals between these various states as seen in Figure 2. In addition, these relationships can be plotted over time, as seen in Figure 3.

These basic equations have significant implications for public health and for other fields. For example, the SIR model has been used to predict outbreaks of measles, mumps, and rubella, and has also recently been applied to Ebola outbreaks. By predicting possible outbreaks, these simple models may help bring the disease of interest under control. In addition, these models can be applied to economics to describe optimal dosages through the forces of herd immunity. Herd immunity is a type of protection from infectious diseases which takes advantage of indirect forces (Aron, 2007). For example, once a majority of a population becomes immune and resistant to a disease, the infected individuals will become the minority and will thus diminish. This concept can be modeled with a threshold term: the basic reproduction number, or R_o. The basic reproduction number is the number of new cases caused by one case on average over the course of an infectious period (Aron, 2007). If $R_o < 1$, then the infection will die out in the long run. If $R_o > 1$, then the infection will spread in a population. In general, the higher the value of R_O, the more difficult control of an epidemic becomes. While Ebola has a low reproductive number (1.5-2.5; McCandless, 2014), smallpox has a higher reproductive number (5-7; McCandless, 2014), measles has among the highest (12-18; McCandless, 2014).

Mathematical modeling is especially helpful when exploring unique connections through a collaborative lens. For example, the spread of crime could be analyzed through the SIR model. In this case, $x(t) = X(t)/N$ is the susceptible fraction of the population. This parameter could include groups (e.g. hit and run cases) or populations (e.g. citizens of Ohio). Similarly, $y(t) = Y(t)/N$ could indicate the "infected" fraction of the population or those who are incarcerated. Finally, $z(t) = Z(t)/N$ would imply those "recovered" or post-conviction. These equations would carry the same limitations as the biological counterpart, including the disregard to any changes in the number of susceptible members. We would also assume that each incarcerated member would receive a similar punishment. Each variable would also change with time. These changes could then be compared to existing statistics to measure their outcomes compared to an ideal situation where a predictable number of members moves from susceptible to incarcerated to the general population. Furthermore, the effectiveness of certain programs could then be compared to experimental data. These methods have transformed public health and hold unique insights for legal studies as well.

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In addition to the SIR and reproductive models, game theory is another useful tool in the realm of health and public policy (Westhoff, Cohen, Cooper, Corvin., & McDermott, 2012). Game theory is a mathematical approach to understanding strategic decision-making. This understanding derives from the choices available to a player, such as the choice to compete or cooperate. The payoff is then determined by the responsive choices made by each player. These payoffs can be used to explain the benefits and costs of decision making (Westhoff et al., 2012).

The Prisoner's Dilemma is the paradigm example of game theory applied to law. The name of this model was originally coined by Professor Albert Tucker of Princeton University (Guerra Pujol & Martinez, 2014). The game describes two suspects who are being held separately by police, who inform them of the possible payoffs of various outcomes. For example, if prisoner A confesses while prisoner B does not, prisoner A will be released. A payoff table of the scenario is reproduced in Table 1.

The important aspects of this scenario are the inability for collaboration and the independent payoff rates. Both of these factors create a non-zero sum game where a "win-win" outcome is theoretically possible (in this case, if both suspects cooperate or remain loyal to each other). These parameters establish the rules of the game, and allow for us to model complex legal disputes and other real-world scenarios.

Moreover, this classic example is powerful for law students because it is realistic. This game seems to capture the legal system in action as these strategies are commonly used by law enforcement. When there is insufficient evidence to bring a case to trial, the ability to negotiate for information is critical for law enforcement professionals (Guerra-Pujol & Martinez, 2014). This deeper understanding of decision making thus sheds more precise descriptions of real-world problems and provides greater predictive power.

Broadly speaking, Meyer & Land (2003) describe threshold knowledge as concepts within higher education that transform the perception of a given subject. Their theory explains the significant relevance that certain concepts hold for various fields. Although a robust understanding of mathematical modeling might be impractical for many undergraduate students of law and political science, understanding the basic threshold concepts could illuminate other more complex topics in these fields in novel and innovative ways.

Attempts to increase mathematical literacy among biology and pre-health professions have been positive and warrant further study and funding. For example, Wallace & Rheinlander (2011) have been teaching a mathematics course targeting biology and pre-health students. The goal of the course is to introduce these students to calculus and mathematical modeling with systems of ordinary equations. The course was evaluated using formal mathematical assessments as well as inventories to measure confidence and the willingness to approach real-world problems through mathematics (Wallace & Rheinlander, 2011). Students who engaged in this type of course were able to read research literature on mathematical biology and carry out research of their own. The significance of this research is an increase in mathematical literacy among science students, and Wallace & Rheinlander's course plan could be extended to other fields like law and political science.

Moreover, being able to recognize patterns across disciplines is a critical skill for any profession. This is especially true in the field of public health, which requires a cross-discipline approach. This interdisciplinary approach is also nurtured in various conferences and workshops. These academic retreats provide networking opportunities between public health, biology, and math students at the undergraduate and graduate level. The staff at these events typically include professors and doctoral students from various institutions, who can also collaborate on research and discuss their findings. An example of such a conference is the Annual Outreach Conference to Increase Diversity in Mathematical Modeling and Public Health (Kutzko, 2015). This conference hosts 80 students and is a two-day event. It typically includes presentations from researchers and graduate students in infectious disease epidemiology (Kutzko, 2015). These enrichments are fundamental for working with diverse specialists in improving decision making processes and should be extended to law and political science.

The wealth of knowledge that simple models offer pre-professional students is staggering. Moreover, models provide an opportunity to exercise critical thinking skills in collaborative environments. Models they are used by professionals in the real world to solve important problems. Mathematical modeling and systems design have begun to foster interdisciplinary collaboration, but more work is yet to be done. While modeling conferences

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and integrative coursework are promising, these paths are still in their early stages at the pre-professional level of pre-law and political science.

To sum up, mathematical modeling continues to dominate the field of public health. The industries of virology, bioinformatics, and healthcare administration make modeling and systems design central to their work (Motta & Pappalardo, 2013). As mentioned, mathematical modeling is already being introduced at the undergraduate level to pre-medical and biology students. It follows that undergraduate pre-law and political science students will also benefit from modeling.

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APPENDIX A

Figure 1: Two schematic representations of the natural history of HPV and cervical cancer used in mathematical models (Goldhaber-Feiber & Garnett, 2006). These models describe the symptomatic periods through immunity, including cervical intraepithelial neoplasia (CIN), low-grade squamous intraepithelial lesions (LSIL), and high-grade squamous intraepithelial lesions (HSIL).

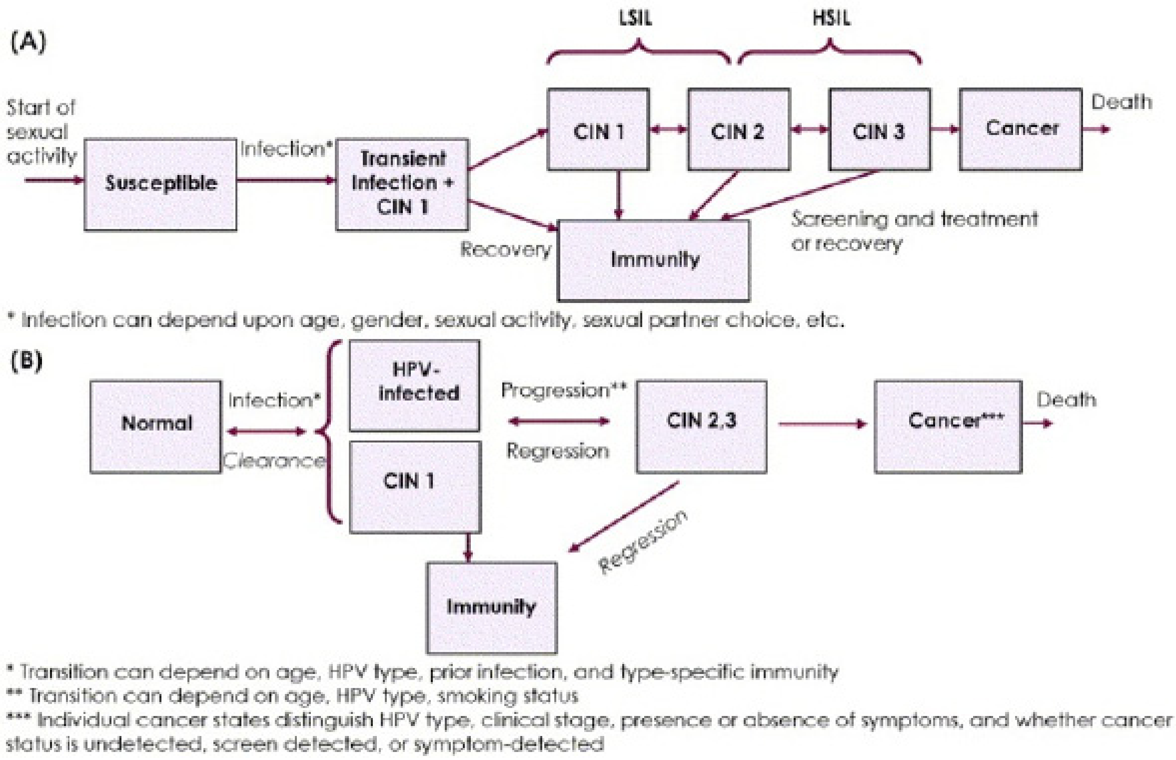

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Figure 2: SIR Model with transfer groups (Aron, 2007).

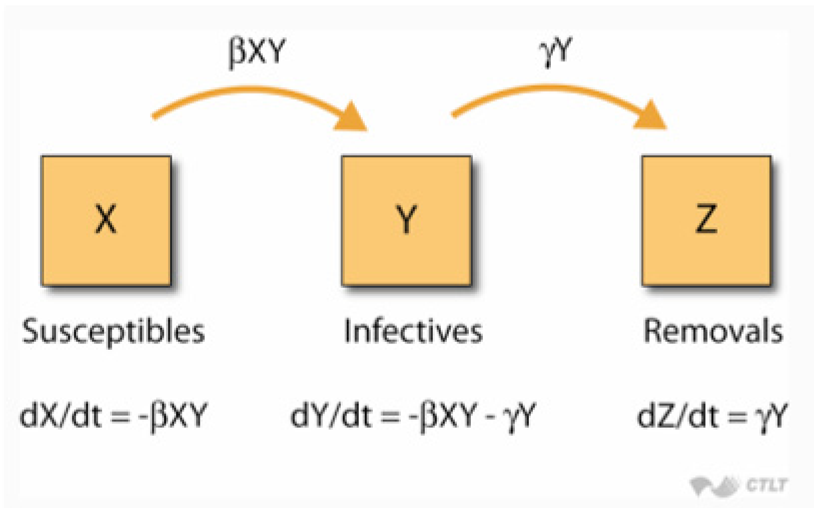
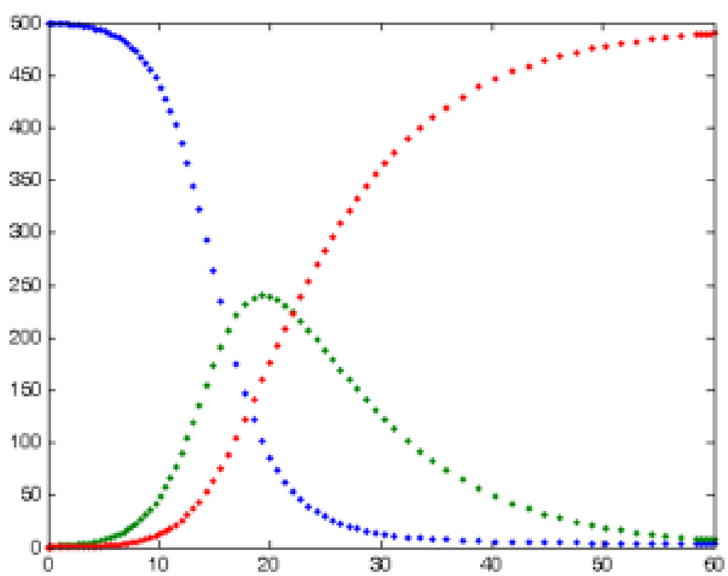


Figure 3: SIR Graph (blue represents X, green represents Y, and Blue represents Z; SIR Model, 2006).



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APPENDIX B

Table 1: Payoff schedule for various outcomes in the classic Prisoner's Dilemma.

	Prisoner B Remains Loyal	**Prisoner B Betrays**
Prisoner A Remains Loyal	Each serves 6 months	Prisoner A: 3 years Prisoner B: Released
Prisoner A Betrays	Prisoner A: Released Prisoner B: 3 years	Each serves 2 years

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