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Access Games: A Game Theoretic Framework For Fair Bandwidth Sharing In Distributed Systems

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ACCESS GAMES: A GAME THEORETIC FRAMEWORK FOR FAIR BANDWIDTH SHARING IN DISTRIBUTED SYSTEMS

by

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ABSTRACT

In this dissertation, the central objective is to achieve fairness in bandwidth sharing amongst selfish users in a distributed system. Because of the inherent contention-based nature of the distributed medium access and the selfishness of the users, the distributed medium access is modeled as a non-cooperative game; designated as the Access Game.

A *p*-CSMA type medium access scenario is proposed for all the users. Therefore, in the Access Game, each user has two actions to choose from: “transmit” and “wait”. The outcome of the Access Game and payoffs to each user depends on the actions taken by all the users. Further, the utility function of each user is constructed as a function of both Quality of Service (QoS) and Battery Power (BP). Various scenarios involving the relative importance of QoS and BP are considered.

It is observed that, in general the Nash Equilibrium of the Access Game does not result into fairness. Therefore, Constrained Nash Equilibrium is proposed as a solution. The advantage of Constrained Nash Equilibrium is that it can be predicated on the fairness conditions and the solution will be guaranteed to result in fair sharing of bandwidth.

However, Constrained Nash Equilibrium is that it is not self-enforcing. Therefore, two mechanisms are proposed to design the Access Game in such a way that in each case the Nash Equilibrium of the Access Game satisfies fairness and maximizes throughput. Hence, with any of these mechanisms the solution of the Access Game becomes self-enforcing.

Dedicated to My Parents

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LIST OF ACRONYMS/ABBREVIATIONS

LAN	Local Area Network
WLAN	Wireless Local Area Network
MAC	Medium Access Control
CSMA	Carrier Sense Multiple Access
CSMA/CA	Carrier Sense Multiple Access/Collision Avoidance
CSMA/CD	Carrier Sense Multiple Access/Collision Detection
GPS	Generalized Processor Sharing
WFQ	Weighted Fair Queuing
WPFQ	Weighted Packet Fair Queuing
BP	Battery Power
QoS	Quality of Service
NE	Nash Equilibrium
CNE	Constrained Nash Equilibrium

CHAPTER ONE: INTRODUCTION

Fair sharing of bandwidth in computer networks is a well-researched issue; with the underlying philosophy being that users should receive bandwidth proportional to their weightages; with higher weightage implying a higher demand for bandwidth. This principle can be more or less efficiently implemented in centralized networks [1,2] where a central entity can switch to a user with higher requirement more frequently e.g. bit-level fair scheduling of Weighted Fair Queuing (WFQ) or packet-level fair scheduling of Weighted Packet Fair Queuing (WPFQ).

However, due to its very structure, a central switching entity can not be incorporated into a distributed system [5-10]. Therefore, fair sharing of bandwidth in a distributed system is a much more complicated than it is in a centralized system. This issue has been extensively researched; yet remains unresolved. Designing a suitable Medium Access Control (MAC) protocol holds the key to an efficient solution. The present work proposes a novel type of MAC protocol that results in fair sharing of bandwidth amongst users. In this work, considerable autonomy is granted to the users so that they can make intelligent decisions in their own selfish interest. This, combined with the contention-based nature of medium access in a distributed system, enables the formulation of distributed medium access as a Non-Cooperative Game.

A detailed description of the background work and the motivation behind the present work is provided in the next Chapter. In the rest of this Chapter, a general overview of our work is presented.

Consider the simple illustration of the Internet topology as given in figure 1: various Local Area Networks (LANs) connected through a Wide Area Network (WAN).

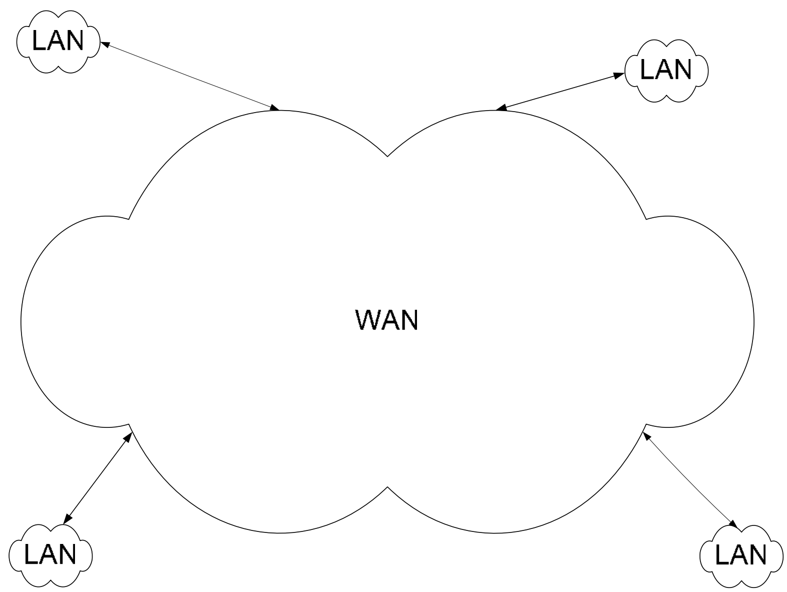

Figure 1: Internet

For the purpose of this dissertation, a LAN consists of a group of users sharing the local physical medium. The local physical medium is an electromagnetic frequency spectrum over which computers transmit data as signals. Because multiple computers use the same frequency spectrum for data transmission, a computer can successfully transmit its data only when that computer has the sole access to the medium. If any other computer transmits signals over the same frequency spectrum at the same time, interference occurs and none of the computers succeeds in transmitting their respective data successfully. In computer networking parlance, this is known as collision.

The MAC protocols are designed to avoid or reduce the possibility of collision in medium access. Collision can be altogether eliminated if a central entity (switch/scheduler) decides which user will transmit when. Centralized MAC protocols viz. HIPERLAN incorporate this idea. However, centralized MAC protocols suffer from the problems of scalability and robustness. Therefore distributed MAC protocols are more widely used for LANs. Figure 2 presents an illustration of a LAN using distributed MAC protocol.

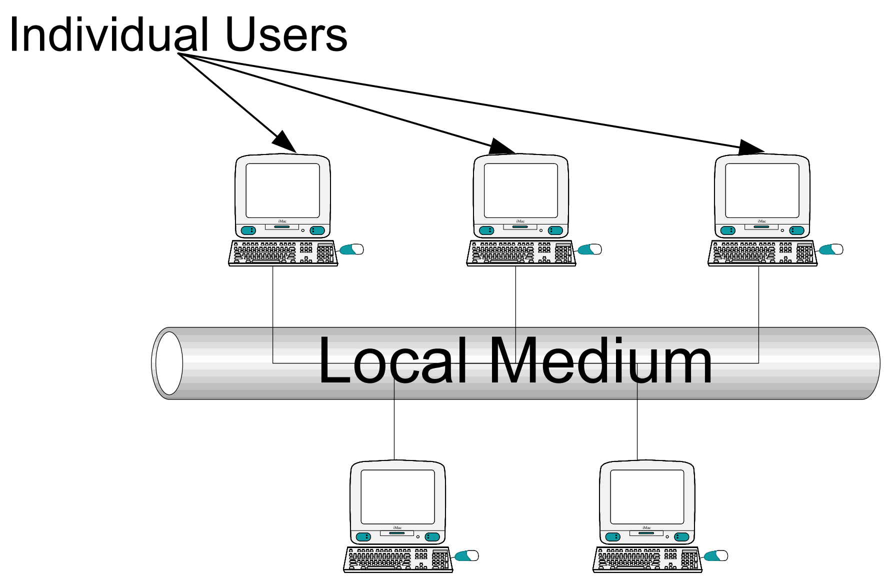

Figure 2: Distributed LAN

It can be noted from figure 2 that there is no central entity or switch arbitrating the medium access of the users. Moreover, there is no mechanism to coordinate transmission activities of the users. Therefore the probability of collision always exists. This probability of collision depends on how often the users have packets to transmit and how quickly they transmit a packet once they have it. From the previous discussion it is apparent that if a user succeeds in accessing the medium more frequently, it would be able to transmit more packets. In other words, successful medium access is directly related to issue of bandwidth usage. This linkage makes MAC protocols integral to the fair sharing of bandwidth.

We address the issue of weighted fairness in a distributed Local Area Network (LAN) setting with a finite but fixed number of users with different weightages. The objective is to achieve fair sharing of bandwidth where users will receive bandwidth proportional to their weightages. For the rest of this dissertation, we would use “fairness” to imply weighted fairness.

The two principal impediments in achieving fair sharing of bandwidth in distributed systems can be expressed as follows:

1. Lack of information: in a distributed system, users usually do not know about the number of other users.
2. Lack of coordination: in a distributed system, users cannot possibly coordinate their activity and determine who is going to transmit when.

Of these two problems, lack of coordination is more fundamental in nature. Even if all the users have complete information about the other users in the network, there is no possible way the medium access of users can be coordinated to avoid collision. A user with a packet to transmit does not know-unless it is a deterministic system with perfectly synchronized clocks-if the other user(s) are also trying to access the medium at the same time. Therefore, the medium access scenario in distributed systems is inherently contention-based in nature and the probability of collision always exists.

In order to alleviate the problem of lack of information, we propose a simple technique that enables users to have knowledge about other users [3]. However, the problem of lack of coordination remains and this inherently contention-based medium access scenario is modeled as a Non-Cooperative Game [14]; designated as the Access Game. Our findings are as follows:

1. Nash Equilibrium (NE) of the Access Game is usually inefficient.

2. Constrained Nash Equilibrium Solutions (CNE) of the Access Game results in fairness.

3. CNE is beset with stability problems and the Access Game can be suitably designed so that the NE results in fairness.

The rest of this dissertation is organized as follows: Chapter 2 provides a brief description of the related work and the motivation behind the present work, Chapter Three discusses some game-theoretic concepts, and models the Access Game, Chapter Five provides a simple analysis of an incomplete information game, Chapters 6-9 present the main results of this dissertation, Chapter Ten provides some stabilization mechanisms, and Chapter 11 concludes this dissertation.

CHAPTER TWO: BACKGROUND WORK AND MOTIVATION

In this Section, we provide a brief overview of the background work. First, we discuss a few of the well-known distributed MAC protocols. Secondly, work conducted in the area of fair sharing of bandwidth in distributed system is presented. Next, we explain the effect of MAC protocols vis-à-vis Quality of Service. Finally, we review the recent application of Non-Cooperative Game theory in distributed medium access.

2.1 Distributed MAC Protocols

Simple ALOHA [3] was one of the earliest distributed MAC protocols. ALOHA was attractive because of its simplicity. As soon as a user has a packet to transmit, it transmits without sensing the medium. However, a low maximum throughput of $1/2e$ could be achieved with simple ALOHA; assuming Poisson arrival of transmission packets. Improvements were suggested in the form of slotted ALOHA. In slotted ALOHA, transmission times are slotted and users can transmit only at the beginning of the slots. However, in this protocol also users do not sense the medium before transmitting. This protocol results in an improvement in throughput: $1/e$ assuming Poisson arrival of transmission packets.

Carrier Sense Multiple Access (CSMA) family of MAC protocols add an important functionality to the MAC protocol. As the name suggests, this family of MAC protocols senses the medium to determine whether somebody else is transmitting or not. Under the original classifications of this protocol, there are basically three types of CSMA protocols based on what the users do when the medium is sensed busy.

1. 1-persistent: Keep on sensing the medium and as soon as the medium is sensed idle, transmit

2. Non-persistent: Stop sensing the medium for a period of time.

3. p-persistent: keep on sensing the medium and when the medium becomes idle, transmit with probability p and do not transmit with probability $1-p$.

The CSMA family of MAC protocols is widely implemented in both business and academia. Ethernet, by far the most popular choice for media access in Local Area Networks (LANs), employs Carrier Sense Multiple Access/Collision Detection (CSMA/CD) algorithm. On the other hand, Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) algorithm is being widely used for Wireless Local Area Networks (WLANs) [10-12]. A comprehensive description of these protocols can be found in [4, Chapter 6].

2.2 Fairness and MAC Protocols

Fairness in bandwidth distribution is a well-researched topic. In centralized networks, the concept of Generalized Processor Sharing (GPS) forms the basis for achieving fairness. Informally, GPS guarantees a user resource allocation proportional to that user's relative weightage [38]. We would follow this simple, yet powerful definition of fairness for the present work also. GPS can not be implemented in practice because it relies on bit-by-bit switching whereas in computer networks, communication entity of interest is a packet. In [29], a practical packet-based implementation of GPS is presented. This algorithm is usually known as the Weighted Fair Queuing (WFQ) algorithm. In WFQ, each arriving packet is given virtual "start" and "finish" times based on the actual arrival time of the packet and the length of the packet. The packet with the smallest "finish" time is selected for transmission. A similar technique can be found in [39]. There, the authors combine the WFQ algorithm-designated as Packet GPS (PGPS)-with a Leaky Bucket Admission Control algorithm for a single server GPS and show that it is possible for the network to fulfill a wide range of performance guarantees using these algorithms. In [40], authors propose an improved GPS approximation algorithm, called Worst-case Fair Weighted Fair Queuing (WF2Q). Using WF2Q, only packets with a virtual "start" time that has been passed are considered for transmission. This scheme approximates GPS more accurately but increases the complexity of implementation.

The objective of bandwidth fair sharing in a distributed system is to resolve this contention in such a way that users get bandwidth proportional to their weightages. For distributed systems, most of the work concentrates on Carrier Sense Multiple Access (CSMA) family of MAC protocols. In [35], a Distributed Fair Scheduling (DFS) scheme based on a virtual clock mechanism has been proposed for Wireless Local Area Network (WLAN). As in WFQ, "start" and "finish" time of an arriving packet are computed and the packet with the smallest "finish" tag is transmitted. A distributed algorithm using the back-off interval mechanism of IEEE802.11 MAC [11, 12] is used to determine the packet with the smallest "finish" tag. However, in general "virtual clock" mechanism can not be implemented with ease in distributed systems. Another approach in differentiated bandwidth sharing in distributed systems is the "priority-based" access schemes. One of the earliest works incorporating priority in CSMA can be found in [41]. There, Tobagi presented a prioritized CSMA or P-CSMA[1]. In this scheme, access right is granted exclusively to the messages of the current highest priority

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[1] Different from *p-CSMA*

class. In [33], priority-based access schemes using the Carrier Sense Multiple Access (CSMA) protocol are analyzed for 1-persistent and non-persistent modes. *p*-persistence CSMA is not considered due to the difficulty in analysis. Specifically, the authors analyze three schemes: a) all packets transmitted in 1-persistent mode, b) higher priority packets transmitted in 1-persistent mode and lower priority packets transmitted in non-persistent mode and c) all packets transmitted in non-persistent mode. Assuming Poisson packet arrival and general packet length distributions mean packet delays are computed using approximate techniques. Another priority based scheme for CSMA is presented in [34]. Some other related work for distributed systems can be found in [31] and [32]. The problem with the "priority-based" access schemes is the lack of an explicit relation between and priority and fairness. The medium access strategy proposed by us is philosophically similar to the priority-based approach but completely different in modeling and analysis.

From the Quality of Service (QoS) point of view, the moot question is how users with different QoS requirements should transmit their packets so that a user with higher bandwidth requirement will have higher chance of success. A possible solution is to assign weightages to the users proportional (not necessarily directly) to their bandwidth requirements. However, as discussed above users will receive bandwidth directly proportional to their weightage only if the MAC protocol is suitably designed. Therefore, it can be seen that weighted fairness, QoS and MAC protocol design are interrelated and it is necessary to have suitable MAC protocols to satisfy the QoS demand of various users.

2.3 Non-Cooperative Game Theory and MAC

The drawbacks of both "virtual clock" and "priority-based" schemes have been indicated above. In this dissertation, we present a novel approach for contention resolution by modeling the contention for medium access as a Non-Cooperative Game, the Access Game. The Access Game model is predicated on an explicit relation between priority and fairness. Therefore, the solution of the Access Game satisfies the fairness definition as enunciated by GPS. Game Theory has been extensively used in other areas of computer communication [15-21]. However, application of Game Theory has been limited in distributed MAC designing. It has been introduced only recently in [13]. To the best of author's knowledge, the present work is the first attempt to formulate the fairness problem in a Game-Theoretic framework. There are two reasons why Game Theory is a suitable tool for analyzing distributed medium access. First, the contention-based nature of the medium access presents a natural application domain for Non-Cooperative Game Theory. Secondly, it is possible to conceive of "selfish" users in future choosing their individual access strategies to optimize their own selfish interests [13, 37, and 30]. The "virtual time"- based or "static priority"-based approaches described above are not suited for such situations. The Access Game model provides a theoretical formulation for achieving fair bandwidth sharing in the presence of "selfish" users. In addition to resolving the fairness problem, we also investigate in detail the interaction between the optimal "selfish" user strategies and the overall system performance.

CHAPTER THREE: MODELING THE ACCESS GAME

In this Chapter, we describe some relevant Game-Theoretic concepts and use these concepts to model the distributed medium access as a Non-Cooperative Game.

3.1 Game Theoretic Concepts

Formally, a finite simultaneous-move game G consists of a non-empty finite set I of players. A player, say i has a set of possible strategies/actions A_i. In order to play the game, all the players choose an action from the respective strategy sets simultaneously. At the end of the game, there is an outcome or result. Clearly, the outcome space of G can be given by $S = \times_i A_i$. Let $s \in S$ be a generic outcome of the game. Associated with the outcome s is a payoff to each of the players. Let us designate by $u_i = u_i(s)$ the payoff function for the i^{th} user. The payoff function of the game u is given by $u(s) = (u_1(s)...u_n(s))$

The concept of "mixed strategy" [26] can be described as follows. Instead of deciding for a particular action with certainty (i.e. pure strategy), a user i randomizes its decision and chooses a particular action from A_i with a probability. Consequently, the elements of the outcome set S also become probabilistic in nature. The payoff of the game is associated with the outcome of the game. It follows that in a mixed strategy game a non-negative probability is attached to the value of the payoff a user receives by playing the game. This entails the formulation of the utility function $\overline{u}_i$ which is essentially the expected payoff for player i from playing the game. We primarily consider two solution concepts for non-cooperative Game Theory: Nash Equilibrium and Constrained Nash Equilibrium. For both these solutions concepts, we assume that the users have complete information about all the other users. In the next Chapter, we consider the role of information in Game-Theoretic constructs in some detail.

Nash Equilibrium (NE) is arguably the most important solution concept for non-cooperative game theory. For each finite complete information game G, John Nash [26] proved the existence of equilibrium in mixed strategy. For this equilibrium, action of one user is completely independent of what other users are doing. Formally, the NE can be presented as follows. Let the generic mixed strategy of user i be denoted by π_i. Let π_{-i} denote the collective strategies of all the users other than i, i.e., $\pi_{-i} = (\pi_1,...\pi_{i-1},\pi_{i+1},...\pi_n)$. For an NE strategy $(\pi_1^*,..\pi_n^*)$ of the game, the following holds

$$\overline{u}_i(\pi_i^*,\pi_{-i}^*) \geq \overline{u}_i(\pi_i,\pi_{-i}^*) \qquad \forall i$$

We now provide a simple example to compute NE using best response correspondence of the users. Consider two users: X and Y. $A_X = \{x_1, x_2\}$ and $A_Y = \{y_1, y_2\}$. The payoffs for the game can be represented in a matrix form as follows (Table1):

Table 1: Payoff Matrix

Y \ X	x_1	x_2
y_1	(5,3)	(4,6)
y_2	(2,5)	(10,7)

As an example, the entry (5,3) implies that if X plays x_1 and Y plays y_1, the payoff to X is 5 and the payoff to Y is 6.

If X chooses to play x_1 with probability p and Y chooses to play y_1 with probability q, the utility function for X can written as

$$U_X = p \times q \times 5 + p \times (1-q) \times 2 + (1-p) \times q \times 4 + (1-p) \times (1-q) \times 10$$

Player X's best response correspondence for a value of q, is the set of p that would maximize U_X. Let us rewrite U_X as

$$U_X = p[9q-8] + 10 - 6q$$

In this case, the "best" value of p will depend on q as follows:

if $[9q-8] > 0$, U_X is maximized by $p = 1$

if $[9q-8] < 0$, U_X is maximized by $p = 0$

if $[9q-8] = 0$, U_X is maximized by any value of p in $(0,1)$

Similarly, $U_y = q[3p - 6p + 6 - 2p - 7 + 7p] + 2p + 7 - 7p = q[2p-1] + 7 - 5p$

Therefore,

if $[2p-1] > 0$, U_Y is maximized by $q = 1$

if $[2p-1] < 0$, U_Y is maximized by $q = 0$

if $[2p-1] = 0$, U_Y is maximized by any value of q in $(0,1)$.

NE is obtained when both U_X and U_Y are maximized simultaneously. From the above discussion, it is clear that it will happen when for a value of (p,q), both U_X and U_Y are maximized. If p is plotted as a function of q and q is plotted as a function of p, the set of NE will be the intersections of these two plots.

The three NE of this game are ($p=1$, $q=1$), ($p=0$, $q=0$), and ($p=1/2$, $q=8/9$).

In the formulation proposed by Nash, users take actions independent of each other. Rosen [28] considered the important case where strategies of users are constrained, i.e., some relations exist among the strategies. An example of such a scenario can be found in [42]. For such a case, Rosen proved the existence of equilibrium for concave utility functions. We refer to this equilibrium concept as the Constrained Nash Equilibrium, or CNE. As users do not communicate with each other about their actions, the solution for CNE still remains non-cooperative.

We use the game-theoretic concepts outlined above to model the distributed medium access scenario as the Access Game. Before presenting the model for Access Game, we briefly describe the proposed MAC protocol.

3.2 MAC Protocol

Consider a MAC protocol using CSMA. When a user has a packet to send, it senses the medium. If the medium is sensed busy, it can take several actions. Based on these actions, CSMA protocols can be classified as follows [4]:

1. 1-persistent: keep on sensing the medium and transmit the packet when the channel becomes idle.

2. Non-persistent: do not sense the medium for some time (i.e. backoff).

3. *p* -persistent: keep on sensing the medium as long as the medium is busy and when the medium becomes idle transmit with probability *p* [and wait with probability (1-p)].

We provide the following diagram (figure 3) for a schematic representation of *p*-CSMA.

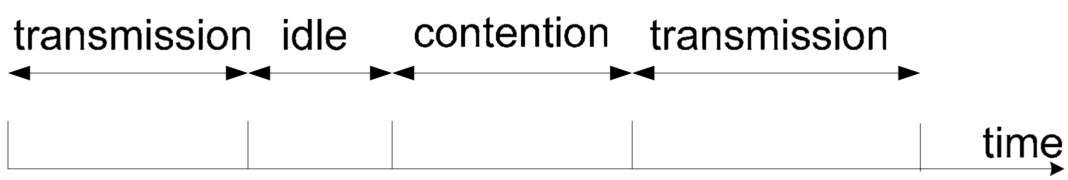

Figure 3: Successive States of the System in *p*-CSMA

It can be observed from figure 3 that at the end of a transmission period, there is a brief idle period after which users contend with each other to access the medium. This idle period essentially signals the end of the previous transmission period. Contention is eventually resolved in one user's favor and that user transmits next. Our proposed MAC strategy is similar to the *p*-CSMA. The differences are as follows. Different users have different values of "*p*". The values of these transmission probabilities depend on the number of users present in the system. Therefore, the transmission probabilities of users change over time.

3.3 Modeling

In order to model the distributed access scenario as a non-cooperative game, it is assumed that users are "selfish" [13, 15-21, 30] in nature i.e. users are solely interested in maximizing their own utility functions.

3.3.1 Players, Actions, and Payoffs

Players of the Access Game are the users trying to access the medium. We consider n users with user i having a weightage of w_i. All the users use p-CSMA type MAC protocol. At the beginning of each contention period, each player has two actions to choose from: "transmit" and "wait". User i transmits with probability p_i. If there are n users, then strictly speaking outcome space S consists of 2^n elements. However, note that from a user's point of view, the above outcomes can be interpreted in three distinct cases: "transmit and success", "transmit and failure" and "wait". Moreover, if a user "i" decides to wait, it should receive the same payoff irrespective of whether the game's outcome is "success" or "failure". We use subscripts "1","2" and "3" for "transmit and success", "wait" and "transmit and failure" respectively.

The payoffs to the users from playing the Access Game are as follows:

1. If only user i transmits, then the outcome of the game is "success". User i receives a payoff of $c_{1,i}$ and user $j[\neq i]$ receives a payoff of $c_{2,j}$.

2. If no user transmits, the game's outcome is "waste" and the user i receives a payoff of $c_{2,i}$.

3. If more than one user transmits, collision occurs and the outcome of the game is "failure". If user i had transmitted, then user i receives a payoff of $c_{3,i}$. If user i had not transmitted, it receives a payoff of $c_{2,i}$.

Note that the probability that user i transmits and succeeds is given by $p_i \times \prod_{j \neq i}^{n} (1 - p_j)$.

Probability for "transmit and failure" is $1 - p_i \times \prod_{j \neq i}^{n} (1 - p_j)$ and probability of "wait" is $1 - p_i$.

Therefore, the utility function u_i of user i can be written as

$$\overline{u}_i = p_i \times \prod_{j \neq i}^{n} (1 - p_j) \times c_{1,i} + p_i [1 - \prod_{j \neq i}^{n} (1 - p_j)] \times c_{3,i} + (1 - p_i) \times c_{2,i} \tag{3.1}$$

The NE of the Access Game will be a strategy profile $p^* = (p_1^* ... p_i^* ... p_n^*)$ satisfying the following

$$\overline{u}_i(p_i^*, p_{-i}^*) \geq \overline{u}_i(p_i, p_{-i}^*) \qquad \forall i \tag{3.2}$$

Similarly, for the existence of CNE the following condition should hold:

$$\frac{\partial^2 u_i}{\partial p_i^2} \leq 0 \qquad \forall i \tag{3.3}$$

We now discuss the nature of the payoffs in the Access Game.

3.3.2 Payoff Structure

Payoffs of an outcome underline the physical results of that outcome. When a user transmits and succeeds in accessing the medium, it can transmit a packet successfully over the network. Clearly, this is beneficial for QoS. However in order to transmit the packet, the user spends some Battery Power (BP) also. Therefore, the payoff for "success" has a positive QoS component and a negative BP component. On the other hand, if the user waits, then no or minimal BP is expended; however, the QoS component is adversely affected. Therefore, the payoff for not transmitting has a positive BP component and a negative QoS component. If the outcome is "failure", both the components are adversely affected. Logically the payoff for "failure" should have negative components of both BP and QoS and should be less than the payoffs for both "success" and "waste".

The objective of this dissertation is to achieve fair sharing of bandwidth in distributed networks. Consequently, QoS is considered relatively more important than BP

Following the above discussion, it can be said that the following relation usually holds between the payoffs for different outcomes:

$$c_{1,i} > c_{2,i} > c_{3,i} \tag{3.4}$$

In order to quantitatively represent the relative importance of QoS and BP in the Access Game, we define the payoff ratio r_i given as

$$r_i = \frac{(c_{2,i} - c_{3,i})}{(c_{1,i} - c_{3,i})} \tag{3.5}$$

Generally, $r_i < 1$ will hold.

3.4 Assumptions

We conclude this Chapter by outlining the assumptions made for the analysis in the subsequent Chapters.

A1. The Access Game is a complete information game.

A2. Users always have packets to transmit. This assumption has been made for simplicity and it is quite straightforward to relax this assumption. This is shown in Chapter Nine.

A3. Packets are of equal length. This assumption is also made for simplicity.

A4. The system is stable.

A5. The number of users playing the game is n and this number does not change. As $n = 1$ presents a trivial case, $n > 1$ is assumed.

In this Chapter, we provided a simple non-cooperative Game Theoretic model for distributed medium access. The Access Game is a simultaneous move, single-shot and complete information game. However, the assumptions of complete information can not be usually realized in realistic scenarios. We tackle this problem through an approximation mechanism to be presented subsequently in Chapter Nine. Detailed analysis of the complete information Access Game is presented in Chapters 5-8. In Chapter Nine, we present a simple approximation scheme to relax most of the assumptions made for the analysis in Chapters 5-8. In the next Chapter, we present a simple analysis for the incomplete information Access Game.

CHAPTER FOUR: INCOMPLETE INFORMATION GAMES

In Section 3.4, we outlined the assumptions made for our analysis in this dissertation. One of the assumptions was that the Access Game is a complete information game. In other words, all the users have complete information about all the users. However, realistically this is not true in a distributed medium access scenario. In this Chapter, our objective is to analyze the incomplete information Access Game. We show that an optimal access strategy exists for the incomplete information Access Game. We also show that users can make better decision in complete information Access Game. Consequently, in Chapter Nine, we propose a simple scheme to approximate an incomplete information game as a complete information game.

4.1 Incomplete Information Equilibrium

From the point of view of an individual user, say player i, all the other users can be combined together in one single player, $-i$. “Transmit” and “wait” for Player $-i$ are defined as follows: player $-i$ transmits if one or more of the constituent players transmit. Player $-i$ waits if none of the constituent players transmit. Using this argument, we describe the following two-player game G with the payoff matrix in Table 1.

Figure 4: Medium Access in Incomplete Information Game

Table 2 : Bi-matrix representation of players' payoff functions

Player$_i$\Player$_{-i}$	Transmit	Wait
Transmit	(u_i(f),u_{i-}(f))	(u_i(s),u_{i-}(w))
Wait	(u_i(w),u_{i-}(s))	(u_i(w),u_{i-}(w))

Formally, G is a game with two players i and $-i$. Strategy spaces of both i and $-i$ are {transmit, wait} and payoff functions are as given in Table 1. The game G is a game of incomplete information [27] because i does not know about the payoff functions of $-i$ and vice versa.

We consider C classes of traffic. Payoff functions of $-i$ will depend on the number of other users (besides player i) of different classes in the system. This information is *private* to $-i$ and can be expressed as a C-tuple $\chi_{-i} = (n_1..n_c..n_C)$ where n_c is the number of users in class c (other than player i) in the system. Player $-i$ does not know about the payoff function of the player i because it does not know the class of i. This is private information of i and is expressed as $\chi_i = (j)$ where j is the class of player i.

We use Theorem I in [27] to prove that the incomplete information Access Game has an optimal strategy. Before proving Lemma 4.1, we construct a complete information game G^* as considered by player i, in such a way that it is game-theoretically equivalent to G (as considered by player i). Our construction is along the line proposed in [27]. Following conditions need to be maintained in the construction

Condition 1. Payoff functions remain identical in G and G^*.

Condition 2. χ_i and χ_{-i} information and their range remain identical in G and G^*.

Before proving Lemma 4.1, we first prove the following lemma first.

Lemma 4.1: G^* exists and it is unique.

Proof: We prove the above by using Theorem 3 in [27]. In G, player i computes subjective probability $R_i(\chi_{-i} / \chi_i)$ and player $-i$ computes subjective probability $R_i(\chi_i / \chi_{-i})$. The game is consistent [27] if these subjective probabilities can be derived from some basic probability. We prove that the game G is consistent. We do so by proving that if $P(\eta_1..\eta_C)$ describes the system wide probability distribution of users in different classes, then $P(\eta_1..\eta_C)$ can be used to compute $R_i(\chi_{-i} / \chi_i)$ and $R_i(\chi_i / \chi_{-i})$. Clearly,

$$R_i(\chi_{-i} = (n_1..n_j..n_C) / \chi_i = j) = [\sum_{n_j > 0, \sum_{u=1}^{C} n_u \leq N} P(n_1..n_j..n_C)] \times P(n_1..n_j + 1..n_C)$$

where, N is the maximum number of users allowed in the system.

Similar results can be derived for $R_i(\chi_i / \chi_{-i})$. Therefore, we can see that we can compute the subjective probabilities using the basic probability distribution $P(\eta_1..\eta_C)$. Hence, G is consistent.

Next, we prove that G is indecomposable [27]. In order for G to be decomposable, player i should be able to predict each of $n_1..n_j..n_C$ within a range with probability 1. However, player i has no information regarding these attributes and $P(\eta_1..\eta_j..\eta_C)$ can not predict a range with probability 1. Hence G is not decomposable[2] i.e. G is indecomposable. Hence, by Theorem III in [27], the general probability distribution R^* of G^* exists and is unique. Therefore, G^* also exists and is unique. Moreover, $R^* \equiv P$. This proves Lemma 4.1.

∎

We now use Lemma 4.1 to prove the existence of optimal strategy.

Lemma 4.2: An optimal strategy exists for each individual player.

Proof: Using Theorem I in [27] we know that Nash equilibrium of G^* is the Bayesian equilibrium of G. We have proven in Lemma 4 the existence and uniqueness of G^*. As G^* is a finite game, there exists at least one equilibrium point in mixed strategies in G^* (Theorem I in [26]). From the definition of Nash equilibrium, this equilibrium is also an optimal strategy for player i. This completes the proof.

∎

4.2 Expected Utility and Optimal Strategy

In Section 4.1, we proved the existence of an optimal strategy for player i. In this Section, we provide a mechanism to compute these strategies. In order to do this, we first need to compute an expression for expected utility to player i in terms of payoff functions.

---

[2] A more rigorous proof has been omitted

As mentioned before, there are C classes of users. Probability of a player of class j having a packet to transmit at the beginning of the game is denoted as $p_{p,j}$ and the probability that the user transmits the packet is denoted as $p_{t,j}$. $p_{w,j}$ is the probability that the user does not transmit the packet. Naturally,

$$p_{t,j} + p_{w,j} = 1 \quad \forall j \tag{4.1}$$

Hence, the probability $p_{W,j}$ that a user of class j waits is given by

$$p_{W,j} = (1 - p_{p,j}) + p_{p,j}\, p_{w,j} = 1 - p_{p,j}\, p_{t,j} \tag{4.2}$$

Let the number of users including player i, presently playing the game be n. Let $p_\eta(n)$ be the probability distribution of the number of users playing the game and we assume that all the users know this distribution. $\sum_{n=0}^{N} p_\eta(n) = 1$, where N is the maximum number of users allowed in the system.

$n-1$ players other than player i are distributed amongst C classes. Let us say $n-1$ users can be distributed amongst C classes in X number of ways.

$$\mathrm{X} = \binom{n-1+C-1}{C-1} = \binom{n-2+C}{C-1} \tag{4.3}$$

Let us denote by $S_\chi = (\xi_1 \ldots \xi_C)$ {such that $\sum_{k=1}^{C} n_k = n-1$} be the state of the system representing number of players in different classes.

We assume that the probability distribution $p_\chi(S_\chi)$ is also known by the users. Hence, we have $\sum_{\chi}^{X} p_\chi(S_\chi) = 1$.

Let us denote by $p_{w,-i}$ and $p_{t,-i}$ the probability that player $-i$ waits and transmits respectively.

Clearly,

$$p_{t,-i} + p_{w,-i} = 1. \tag{4.4}$$

With the above assumptions, $p_{w,-i}$ can now be computed as follows:

$$p_{w,-i} = \sum_{n=1}^{N} p_\eta(n) \sum_{S_\chi} p_\chi(S_\chi) \prod_{j=1}^{C} (p_{,W,j})^{n_j}] \tag{4.5}$$

Note that (4.5) is an invariant for all the users.

Player i will use (4.5) to make an estimate of the action to be taken by its opponents in a collective way. This estimate may or may not be equal to the actual value. However, if player i plays the game sufficiently large number of times, the above will be the average probability of "wait" for player $-i$. player i uses $p_{w,-i}$ as the "wait" probability. From (4.4) and (4.5) we have

$p_{t,-i} = 1 - p_{w,-i} = 1 - \sum_{n=1}^{N} p_\eta(n) \sum_{S_\chi} p_\chi(S_\chi) \prod_{j=1}^{C} (p_{,W,j})^{n_j}]$ as the "transmit" probability for player i_ in the 2×2 game described above. Expected utility for player i is given by

$$U_i = p_{t,i} p_{w,-i} u_s + p_{t,i} p_{t,-i} u_f + (1 - p_{t,i}) u_w .$$

For optimal strategy,

$$\frac{\partial U_i}{\partial p_{t,i}} = 0 \qquad\qquad \forall i$$

Before presenting the optimal transmission probability, a general form for (4.5) is presented for a special case.

We now derive a generalized simplification for (4.5).

Proposition 4.3: Assuming $p_x(S_x) = constant = p$ *, we have* $p_{w,i\_} |_n = p \sum_{i=1}^{C} \frac{a_i^{n+C-2}}{\prod_{j=1, j\neq i}^{C} (a_i - a_j)}$

Proof: For, $C = 2$, we have

$$p_{w,-i} |_n = p \sum_{i=1}^{2} \frac{a_i^{n+2-2}}{\prod_{j=1, j\neq i}^{2} (a_i - a_j)} = p_{w,-i} |_{\substack{n \\ C=2}} = p \sum_{i=1}^{3} \frac{a_i^{n+C-2}}{\prod_{j=1, j\neq i}^{3} (a_i - a_j)}$$

Let us say the following holds for that for $C = c - 1$ classes of traffic,

$$p_{w,-i} |_{\substack{n \\ C=c-1}} = p \sum_{i=1}^{C} \frac{a_i^{n+C-2}}{\prod_{j=1, j\neq i}^{C} (a_i - a_j)}$$

For, $C = c$ classes of traffic, we have

$$p_{w,-i}\,|_{\substack{n\\C=c}} = p[\sum_{n_1=0}^{n-1} a_1^{n_1}[\sum_{\sum_{j=2}^{c} n_j = n-1-n_1} \prod_{j=2}^{c} a_j^{n_j}]]$$

$$= p\sum_{n_1=0}^{n-1}[a_1^{n_1}\sum_{j=2}^{c}\frac{a_j^{(n-1-n_1)+c-2}}{\prod_{k=2,\,j\neq k}^{c}(a_j-a_k)}] = p\sum_{j=2}^{c}\frac{a_j^{c-2}(a_1^n-a_j^n)}{(a_1-a_j)\prod_{j=2,\,j\neq k}^{c}(a_j-a_k)} = -a_1^n p\sum_{j=2}^{c}\frac{a_j^{c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_j-a_k)} - p\sum_{j=2}^{c}\frac{a_j^{n+c-2}}{(-1)\prod_{k=1,\,j\neq k}^{c}(a_j-a_k)}$$

$$= -a_1^n p[\sum_{j=1}^{c}\frac{a_j^{c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_j-a_k)} - \frac{a_1^{c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_1-a_k)}] + p\sum_{j=2}^{c}\frac{a_j^{n+c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_j-a_k)}$$

$$= -a_1^n p[p_{w,-i}\,|_{\substack{n=0\\C=c}} - \frac{a_1^{c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_1-a_k)}] + p\sum_{j=2}^{c}\frac{a_j^{n+c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_j-a_k)}$$

Note that the scenario with $n=0$ players is not defined; hence the corresponding probabilities are zero. $\therefore\ p_{w,-i}\,|_{\substack{n=0\\C=c}} = 0$.

Therefore, for equally likely cases:

$$p_{w,-i}\,|_{\substack{n\\C=c}} = p\sum_{j=1}^{c}\frac{a_j^{n+c-2}}{\prod_{k=1,\,j\neq k}^{c}(a_j-a_k)} \tag{4.6}$$

$\blacksquare$

4.3 Optimal Transmission Probabilities

In this Section, we compute optimal probabilities for two important cases. We consider two important Cases. In the first Case, all the users belong to the same class and therefore identical. In the second Case, users of two classes are considered.

4.3.1 Case 1: Single Class of Users

As all the users are indistinguishable, equilibrium will symmetric i.e. all the users will have identical optimal strategy. Let p_w and p_t be the probability for all the players to wait and transmit respectively. Expected utility functions for i can formulated as follows [with $p = p_t$]:

$$U_i = pp_{w,-i}u_s + pp_{t,-i}u_f + (1-p)u_w \tag{4.7}$$

where u denotes payoff function (as in Table 1) and s, w, f are subscripts for success (when only player i transmits), wait (when player i waits) and failure (when both the players transmit) respectively.

The values of $p_{w,-i}$ and $p_{t,-i}$ can be computed as follows [$p_\chi(S,n) = 1 \forall \chi, n$]:

$$p_{w,i\_} = \sum_{j=1}^{N} p_\eta(n=j)[p_p p_w + (1-p_p)]^{j-1} \tag{4.8}$$

and $p_{t,-i} = 1 - p_{w,-i}$

We now compute p_η . If we assume that the users join the system according to a Poisson process with parameter λ and departure time from the system is exponentially distributed with parameter μ , we have:

$$p_\eta(n=j) = \frac{\rho^j(1-\rho)}{1-\rho^{N+1}} \qquad \text{where,} \qquad \rho = \frac{\lambda}{\mu}$$

Hence, we have the following:

$$\therefore p_{w,-i} = \frac{\rho(1-\rho)(1-k^N)}{(1-\rho^{N+1})(1-k)} \tag{4.9}$$

where,

$$k = \rho(p_p p_w + (1-p_p)) = \rho(1-p_p p) \tag{4.10}$$

$$\Rightarrow p = \frac{(\rho - k)}{p_p \rho} \tag{4.11}$$

Using (4.9) and (4.10) in (4.7), we have

$$U_i = [p\{\frac{\rho(1-\rho)(1-k^N)}{(1-\rho^{N+1})(1-k)}u_s + (1 - \frac{\rho(1-\rho)(1-k^N)}{(1-\rho^{N+1})(1-k)})u_f\} + (1-p)u_w]$$

$$or, U_i = pF_1 + (1-p)F_2$$
$$where, F_1 = \frac{\rho(1-\rho)(1-k^N)}{(1-\rho^{N+1})(1-k)}(u_s - u_f) + u_f$$
$$F_2 = u_w$$

Optimization with respect to p requires the following:

$$F_1 + pF_1' = F_2 \tag{4.12}$$

We have:

$$F_1 = \frac{\rho(1-\rho)}{(1-\rho^{N+1})}\frac{(1-k^N)}{(1-k)}d_1 + u_f$$

$$where, d_1 = u_s - u_f$$

$$\therefore F_1' = \frac{p_p d_1 \rho^2 (1-\rho)}{(1-\rho^{N+1})} \frac{[Nk^{N-1}(1-k)-(1-k^N)]}{(1-k)^2} \tag{4.13}$$

Using (4.13) in (4.12), we have:

$$\frac{\rho(1-\rho)}{(1-\rho^{N+1})}\frac{(1-k^N)}{(1-k)}d_1 + \frac{(\rho-k)d_1\rho(1-\rho)}{(1-\rho^{N+1})}\times[\frac{Nk^{N-1}(1-k)-(1-k^N)}{(1-k)^2}] = u_w - u_f = d_2$$

$$\therefore \frac{(1-k^N)}{(1-k)} + \frac{(\rho-k)[Nk^{N-1}(1-k)-(1-k^N)]}{(1-k)^2} = \frac{d_2(1-\rho^{N+1})}{d_1\rho(1-\rho)} \tag{4.14}$$

In a general case, p^* [optimal solution of p from (4.14)] can be found out through numerical methods. We now consider the limiting case with $N \to \infty$. The above equation reduces to

$$\frac{(u_w - u_f)}{(u_s - u_f)} = \frac{\rho(1-\rho)^2}{(1-k)^2}$$

$$or, \frac{(1-k)}{(1-\rho)} = \sqrt{\frac{\rho(u_s - u_f)}{(u_w - u_f)}}$$

$$\Rightarrow p^* = \frac{(1-\rho)}{p_p \rho}[\sqrt{\frac{\rho(u_s - u_f)}{(u_w - u_f)}} - 1] \tag{4.15}$$

Let us now consider normalized payoff functions with $u_s = 1, u_f = 0$ and $u_f \le u_w \le u_s$. Furthermore, let $u_w = r$. The above equation is simplified as follows:

$$p^* = \frac{(1-\rho)}{p_p \rho}[\sqrt{\frac{\rho}{r}} - 1] \tag{4.16}$$

It can be easily verified that this value of p^* maximizes the utility function.

We show that for a given value of r the optimal transmission probability, p^* in (4.16) maximizes (4.7). From (4.10), we have

$$\frac{\partial k}{\partial p} = -\rho p_p < 0 \tag{4.17}$$

$$\therefore F_1' = \rho(1-\rho)d_1(-(1-k)^{-2})(-\frac{\partial k}{\partial p}) < 0$$

Now,

$$F_1'' = -\rho(1-\rho)d_1(\rho p_p)(-2(1-k)^{-3})(-\frac{\partial k}{\partial p})$$

$$or, F_1'' = -\rho(1-\rho)d_1(\rho p_p)(1-k)^{-2}[(-2(1-k))\rho p]$$

$$\therefore F_1'' = -2(1-k)\rho p_p F_1' \tag{4.18}$$

We have, from (4.7)

$$U_1 = pF_1 + (1-p)F_2$$

$$\therefore U_1'' = 2F_1' + pF_1'' = 2F_1'(1 - p\rho p_p(1-k)) < 0$$

Therefore, from (4.17) and (4.18),

$U_1'' = 2F_1'(1 - p\rho p_p(1-k)) < 0$

This shows that the value of p obtained maximizes the utility function.

$p^* = 0$ for $\rho = 1$ and p^* is not defined when $\rho = 0$. Bounds on r can be found out from $0 \le p^* \le 1$. Using the above equation:

$$[\frac{(1-\rho)}{p_p\rho + 1 - \rho}]^2 \le \frac{r}{\rho} \le 1 \tag{4.19}$$

From the above relationship, we find that for $p_p \to 0$,

$$\Rightarrow \lim_{p_p \to 0} \frac{r}{\rho} = \lim_{p_p \to 0} [\frac{(1-\rho)}{p_p\rho + 1 - \rho}]^2 = 1$$

Substituting this value in (4.16), we have

$$\lim_{p_p \to 0} p^* = \lim_{p_p \to 0} \frac{1-\rho}{p_p\rho} (\frac{p_p\rho}{p_p\rho + 1 - \rho}) = 1 \tag{4.20}$$

This is an intuitive result also in the sense that if the probability of others having a packet is slim, go ahead and transmit your packet.

Value of

$$p^* |_{p_p=1} = \frac{(1-\rho)}{\rho} [\sqrt{\frac{\rho}{r}} - 1] \tag{4.21}$$

Throughput and r : An important question is how should r be chosen? We choose r such that the average throughput is maximized.

Probability of a slot being successfully used with n users in the system is given by

$$\theta_n = n \times p_p \times p \times p_W^{n-1} \tag{4.22}$$

The expected value for throughput is given by

$$\therefore \overline{\theta} = (1-\rho) \times \sum_{i=1}^{\infty} \Pr(n) \times \theta_n = (1-\rho)^2 p_p \rho \times \frac{p}{(1-p_W \rho)^2} \tag{4.23}$$

Note that in (4.23), p is given by (4.16).

For optimization, $\frac{\partial \overline{\theta}}{\partial r} = 0$. Differentiating (4.23) with respect to r, we get

$$[1-\rho - p\rho p_p] \times \frac{\partial p}{\partial r} = 0 \tag{4.24}$$

From (4.16), we have $\frac{\partial p}{\partial r} = \frac{(1-\rho)}{p_p \rho}[-\sqrt{\frac{\rho}{r}} \times \frac{1}{2r}] \neq 0$

Hence, for throughput optimization $[1-\rho - p\rho p_p] = 0$

$$\therefore p = \frac{1-\rho}{\rho p_p} \tag{4.25}$$

From (4.25) and (4.16), we have

$$\frac{(1-\rho)}{p_p\rho}[\sqrt{\frac{\rho}{r}}-1]=\frac{1-\rho}{\rho p_p}$$

$$r^*=\frac{\rho}{4} \tag{4.26}$$

(4.26) gives the value of r maximizing throughput.

The optimal value of throughput is given by

$$\bar{\theta}^*=(1-\rho)^2 p_p\rho\times\frac{p}{(1-p_W\rho)^2}=\frac{(1-\rho)^3}{\rho p_p 4(1-\rho)^2}$$

$$\therefore \bar{\theta}^*=\frac{1-\rho}{4} \tag{4.27}$$

Equation (4.27) gives the optimal value for throughput for incomplete information games. Let us compare this throughput value with an equivalent complete information system.

In this system, users arrive according to a Poisson process with mean arrival rate λ and all exit time of a user is exponentially distribute with mean $\frac{1}{\mu}$. The optimal throughput for the system in an incomplete scenario is given by [30]

$$\bar{\theta}^*=\frac{(1-\rho)}{4}$$

A simple comparison with the corresponding complete information game shows the following difference in average throughput for various values of ρ. The corresponding average throughput for the complete information case can be given as

$$\Theta^* = (1-\rho)\sum_{n=1}^{\infty} \rho^n (1-\frac{1}{n})^{n-1}$$

Let us designate by δ the difference in optimal throughputs.

$$\therefore \delta = \Theta^* - \overline{\theta}^* = (1-\rho)[\sum_{n=1}^{\infty} \rho^n (1-\frac{1}{n})^{n-1} - \frac{1}{4}]$$

The following figure (Figure 5) provides an illustration of the relation between δ and ρ.

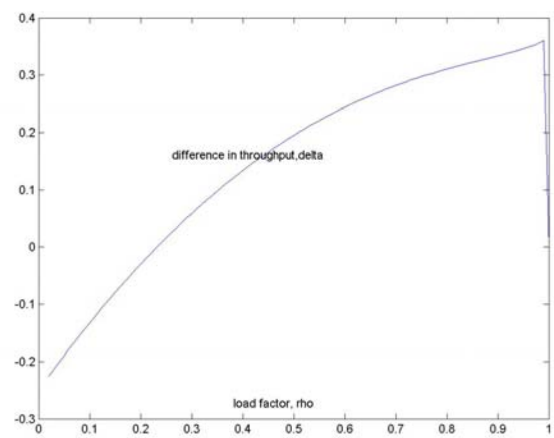

Figure 5: Comparison between Incomplete and Complete Information Game

It is easy to verify that $\delta > 0$ for all values of $\rho \in [0.25,1]$ i.e. load factors lying between (but not including) 0.25 and 1. In the next figure, δ has been plotted against ρ. The value of δ becomes positive shortly after $\rho = 0.2$. The point is that $\delta > 0$ for most of the values of ρ.

4.3.2 Case 2: Two Classes of Users

We now calculate optimal strategies for two classes of traffic. In the previous scenario, all the users were indistinguishable with the same transmission probability.

$$\mathrm{X} = \binom{n-1+2-1}{2-1} = n \tag{4.28}$$

Assuming equally likelihood of every combination we have:

$$p_{\chi}(S,n) = \frac{1}{n} \forall \chi \tag{4.29}$$

$$\therefore p_{w,-i}|_n = [\sum_{n=1}^{N} p_{\eta}(n) \prod_{\sum_{j=1}^{2} n_j = n-1} k_i^{n_i}] \frac{1}{n} \tag{4.30}$$

where,

$$k_i = (p_{p,i} p_{w,i} + (1 - p_{p,i})) = (1 - p_{p,i} p_i)$$

Assuming, $p_{p,1} = p_{p,2}$

$$p_{w,-i}|_n = \sum_{n=1}^{N} p_{\eta}(n) \frac{\sum_{j=0}^{n-1} [(p_p p_{w,1} + (1-p_p))^j (p_p p_{w,2} + (1-p_p))^{n-1-j}]}{n}$$

$$\therefore p_{w,-i}|_n = \sum_{n=1}^{N} p_{\eta}(n) \frac{b^n - a^n}{n(b-a)}$$

where,

$$b = p_p p_{w,2} + (1 - p_p); a = p_p p_{w,1} + (1 - p_p)\,; b - a = p_p(p_1 - p_2) \tag{4.31}$$

For, $p_\eta(n = j) = \dfrac{\rho^j (1-\rho)}{1-\rho^{N+1}}$ we have

$$p_{w,-i} = \sum_{n=1}^{N} \frac{\rho^n (1-\rho)}{n(1-\rho^{N+1})} \frac{(b^n - a^n)}{(b-a)}] \tag{4.32}$$

Neglecting b^i, a^i for $i > 2$ and considering $N \to \infty$ we have:

$$\lim_{N\to\infty} p_{w,-i} \approx (1-\rho)\rho[1 + \rho \frac{b+a}{2}]$$
$$= (1-\rho)\rho[1 + \rho \frac{1 - p_p p_2 + 1 - p_p p_1}{2}]$$

$$\therefore \lim_{N\to\infty} p_{w,-i} = (1-\rho)\rho[1 + \rho - p_p \frac{p_2 + p_1}{2}] \tag{4.33}$$

We have the following expression for calculating utility for each player:

$$U_i = pp_{w,-i}u_s + pp_{t,-i}u_f + (1-p)u_w$$

Hence,

$$U_1 = p_1 p_{w,-i} u_{s,1} + p_1 p_{t,-i} u_{f,1} + (1-p_1)u_{w,1}$$

$$or, U_1 = p_1 p_{w,-i}(u_{s,1} - u_{f,1}) + p_1(u_{f,1} - u_{w,1}) + u_{w,1}$$

$$or, \frac{\partial U_1}{\partial p_1} = (u_{f,1} - u_{w,1}) + (u_{s,1} - u_{f,1})[p_{w,-i} + p_1 \frac{\partial p_{w,-i}}{\partial p_1}]$$

For maximization, $\frac{\partial U_1}{\partial p_1} = 0$

Hence, for maximization $r_1 = p_{w,-i\_} + p_1 \frac{\partial p_{w,-i}}{\partial p_1}$

We have, $\frac{\partial p_{w,-i}}{\partial p_1} \approx (1-\rho)\rho[-\frac{p_p}{2}]$.

Therefore, we get

$$(1-\rho)\rho[1+\rho - p_p \frac{2p_1 + p_2}{2}] = r_1$$

$$\therefore \ p_1 + \frac{p_2}{2} = \frac{(1+\rho)}{p_p} - \frac{r_1}{p_p(1-\rho)\rho} \tag{4.34}$$

Similarly,

$$p_2 + \frac{p_1}{2} = \frac{(1+\rho)}{p_p} - \frac{r_2}{p_p(1-\rho)\rho} \tag{4.35}$$

Solving (4.34) & (4.25), we get:

$$\begin{aligned} p_1^* &= \frac{2}{3}[\frac{(1+\rho)}{p_p} + \frac{r_2 - 2r_1}{p_p(1-\rho)\rho}] \\ p_2^* &= \frac{2}{3}[\frac{(1+\rho)}{p_p} + \frac{r_1 - 2r_2}{p_p(1-\rho)\rho}] \end{aligned} \tag{4.36}$$

In a scenario involving more than one class, it is desirable to have

$$p_2^* = wp_1^* \tag{4.37}$$

where, $w > 1$.

Relationships between r_1 and r_2 can be derived from (4.36) and (4.37).

We have conducted our analysis of the incomplete information Access Game in this Chapter. In Section 4.1, we proved the existence of optimal access strategies. In Section 4.2, we analyzed two simple cases of the incomplete information Access Game. Our subsequent analysis of the complete information Access Game will demonstrate that the complete information Access Game is simpler to analyze and gives more efficient Access Strategies.

CHAPTER FIVE: AN IMPORTANT SPECIAL CASE OF COMPLETE INFORMATION ACCESS GAME

In Chapter Four, simple scenarios with incomplete information were considered. In Chapters Six-Eight, a general analysis is presented for the complete information scenario. Before presenting the general analysis, we consider the important special case of the wired networks in this Chapter. In order to analyze this scenario, we first recall the utility function as expressed in (3.1),

$$\bar{u}_i = p_i \times \prod_{j \neq i}^{n} (1 - p_j) \times c_{1,i} + p_i [1 - \prod_{j \neq i}^{n} (1 - p_j)] \times c_{3,i} + (1 - p_i) \times c_{2,i}$$

For the subsequent analysis, we drop the “bar” in $\bar{u}_i$.

In the case of the wired networks, Battery Power (BP) is not considered important. Therefore, we have $c_{2,i} = c_{3,i} = 0$.

Consequently, $u_i = p_i \prod_{j \neq i}^{n} (1 - p_j)$

5.1 Nash Equilibrium

We now prove that the Nash Equilibriums of the Access Game for the wired networks is inefficient.

Theorem 5.1: In the Nash Equilibrium for the wired networks, there is at least one user i *such that* $p_i = 1$.

Proof: The utility function of user i is given by

$$u_i = p_i \prod_{j \neq i}^{n} (1 - p_j) \tag{5.1}$$

Of interest is the expression $\prod_{j \neq i}^{n} (1 - p_j)$. Let us call it δ_i

$$\delta_i = \prod_{j \neq i}^{n} (1 - p_j) \tag{5.2}$$

Clearly,

$$\delta_i \geq 0 \qquad \forall i$$

Therefore the best response of user i to $\delta(p_i)$ is as follows

$$\delta_i > 0 \;\Rightarrow\; p_i = 1 \tag{5.3a}$$

$$\delta_i = 0 \;\Rightarrow\; p_i = \{x : x \in [0,1]\} \tag{5.3b}$$

We now prove the Theorem by contradiction.

Let us assume that

$$p_i \neq 1 \qquad \forall i \tag{5.4a}$$

It immediately follows that $\delta_i > 0 \;\; \forall i$ and by (5.3a)

$$p_i = 1 \qquad \forall i \tag{5.4b}$$

It is easy to see that (5.4a) and (5.4b) are clear contradictions. Therefore our assumption that $p_i \neq 1 \;\; \forall i$, is not correct.

■

We now discuss the above Theorem with the help of $n = 2$ case.

Following (5.3a) and (5.3b), we have plotted the best response in p_1 as a function of p_2 in fig 6a. As a way of explanation, let us start at $p_2 = 0$. The value of p_1 is “1” at $p_2 = 0$. The value of p_1 remains “1” as long as $p_2 < 1$. At $p_2 = 1$, the best response in p_1 is any value in the interval $[0,1]$ i.e. there are infinite number of best responses in p_1 for $p_2 = 0$. This is represented by the straight line joining (1,1) and (0,1).

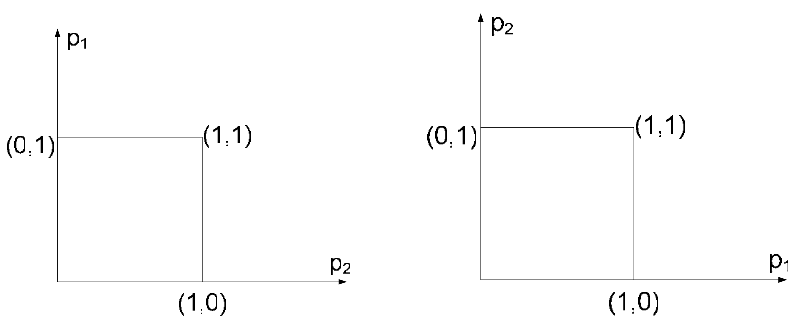

Figure 6a and 6b: Nash Equilibrium

In fig 6b, we have plotted the best response in p_2 as a function of p_2. It can be clearly seen that the best response plots completely overlap. As the NE are given by the intersections of the best response correspondences, there are an infinite number of NE for the Access Game.

Corollary 5.2: There are infinite numbers of NE for the Access” game

Proof: Using Theorem 5.1 and following the logic for $n = 2$

■

Let us introduce the following definition.

Definition 1: A solution of the Access Game is acceptable if and only if the probability of success is non-zero for all the users.

Using Definition 1, we have the following result,

Corollary 5.3: Nash Equilibriums for the Access Game are not acceptable.

Proof: There are two possible cases. Case1: only one user i is transmitting with $p_i = 1$. $\Pr\{success\} = 0$ for all the other users. Case2: More than one user are transmitting with probability "1". Hence, $\Pr\{success\} = 0$ for all the users.

$\blacksquare$

From the above results, we see that although an infinite number of NE exist, they are all inefficient. Therefore, rational users would be willing to adhere to a set of constraints if these constraints benefit them.

5.2 Constrained Nash Equilibrium

In the previous Section, we observed that the NE is inefficient in nature. Therefore, the selfish users will be willing to follow a set of constraints if this set of constraints results in fairness. In order to remove any ambiguity, we will consider a set of explicit fairness constraints as the constraint set for the Access Game. First, we investigate the feasibility of solution with constraints.

Before proceeding further, we reproduce Theorem 1 of [28] for completeness and ready reference.

Theorem 5.4: An equilibrium exists for every concave n-person game.

For Theorem 5.4 to hold, the utility functions of all the users should be concave in nature with respect to their respective strategies. The striking characteristic of this Theorem is that the strategy spaces of different users need not be orthogonal (independent). For such strategies, Theorem 4 ensures an equilibrium if the utility functions are concave.

We have $\frac{\partial^2 u_i}{\partial p_i^2} = 0$. Therefore, u_i is concave in p_i and we can apply Theorem 4 to our problem.

In order to derive our following results, we now present the concept of fairness as a precise set of constraints. In the present context of distributed medium access, the fairness is defined as follows [24].

Definition 2: Fairness is achieved if the probability that user i will access the medium successfully is proportional to its weightage $w_i\,(>0)$.

Hence, fairness can be quantitatively expressed as

$$\frac{p_1 \prod_{j \neq 1}^{n} (1 - p_j)}{w_1} = \ldots \frac{p_i \prod_{j \neq i}^{n} (1 - p_j)}{w_i} = \ldots \frac{p_n \prod_{j=1}^{n-1} (1 - p_j)}{w_n} \tag{5.5}$$

As we are interested in only non-trivial and acceptable solutions,

$$p_i \neq 0,1 \qquad \forall i \tag{5.6}$$

Combining (5.5) and (5.6), we have

$$\frac{p_1/(1-p_1)}{w_1} = \ldots \frac{p_i/(1-p_i)}{w_i} = \ldots \frac{p_n/(1-p_n)}{w_n} = \frac{1}{K} \tag{5.7}$$

where, K is a constant and

$$K > 0 \tag{5.8}$$

From (5.7) we have,

$$[\frac{1}{p_i} - 1] = \frac{w_j}{w_i}[\frac{1}{p_j} - 1] \quad \forall i, j \tag{5.9}$$

From the above discussions and Theorem 5.4, we have the following lemma.

Lemma 5.5: A CNE exists for the Access Game satisfying the fairness condition in (5.7).

We now define the concept of "social welfare". Loosely speaking, social welfare is achieved when no body can be made better off without making somebody worse off. In the present context, we can have the following definition:

Definition 3: Social Welfare is achieved if users get fair share of bandwidth.

With the above formulations and definitions, we have the following result

Theorem 5.6: *At CNE satisfying social welfare, the transmission probabilities satisfy the following:*

$$\sum_{i=1}^{n} p_i = 1$$

Proof: Using (5.1) and (5.6), we have

$$u_i = p_i \prod_{j \neq i}^{n} (1 - p_j) = \frac{p_i}{1 - p_i} g \tag{5.10}$$

where, $g = \prod_{j=1}^{n} (1 - p_j)$ and from (5.6)

$$g = \prod_{j=1}^{n} (1 - p_j) \neq 0 \tag{5.11}$$

For CNE [28], $\frac{\partial u_i}{\partial p_i} = 0$ and $\frac{p_1 /(1 - p_1)}{w_1} = .. \frac{p_i /(1 - p_i)}{w_i} = .. \frac{p_n /(1 - p_n)}{w_n} = \frac{1}{K}$

From $\frac{\partial u_i}{\partial p_i} = 0$, we have

$$g \frac{\partial (\frac{p_i}{1 - p_i})}{\partial p_i} + \frac{p_i}{1 - p_i} \frac{\partial g}{\partial p_i} = 0 \tag{5.12}$$

From (5.7), we have $\frac{p_i / 1 - p_i}{w_i} = \frac{p_j / 1 - p_j}{w_j} \quad \forall i, j$

It follows that $\frac{\partial p_j}{\partial p_i} = \frac{p_j^2}{p_i^2} \times \frac{w_i}{w_j} = \frac{p_j^2}{p_i^2} \times \frac{p_i}{1 - p_i} \times \frac{1 - p_j}{p_j}$

$$\therefore \frac{\partial p_j}{\partial p_i} = \frac{p_j}{1 - p_i} \times \frac{1 - p_j}{p_i} \tag{5.13}$$

We compute $\frac{\partial g}{\partial p_i}$ by using (5.13)

In order to demonstrate how the proof works, we first prove the case of two users. We have

$$g\frac{\partial[\frac{p_1}{1-p_1}]}{\partial p_1}+\frac{p_1}{1-p_1}\frac{\partial g}{\partial p_1}=0 \tag{5.14}$$

$$g=(1-p_1)(1-p_2)$$

$$\frac{\partial g}{\partial p_1}=-[(1-p_2)]+(1-p_1)[\frac{\partial}{\partial p_1}(1-p_2)]$$

The chain partial differentiation in the next step is the "trick" in this proof

$$\frac{\partial g}{\partial p_1}=-[(1-p_2)]+(1-p_1)[\{\frac{\partial}{\partial p_2}(1-p_2)\}\frac{\partial p_2}{\partial p_1}]$$

From (5.13) $\frac{\partial p_2}{\partial p_1}=\frac{p_2}{1-p_1}\times\frac{1-p_2}{p_1}$

$$\therefore\frac{\partial g}{\partial p_1}=-[(1-p_2)+(1-p_1)\frac{p_2(1-p_2)}{p_1(1-p_1)}]$$

$$or,\frac{\partial g}{\partial p_1}=-[(1-p_2)\frac{p_1}{p_1}+\frac{p_2(1-p_2)}{p_1}]=-\frac{(1-p_2)}{p_1}[p_2+p_1]$$

$$\therefore\frac{\partial g}{\partial p_1}=-\frac{g}{p_1(1-p_1)}[p_2+p_1]=-\frac{g}{p_1(1-p_1)}\sum_{i=1}^{2}p_i \tag{5.15}$$

Using (5.15) in (5.14) and the fact that $\frac{\partial(p_i/1-p_i)}{\partial p_i}=\frac{1}{(1-p_i)^2}$ we have

$$\frac{g}{(1-p_1)^2} - \frac{p_1}{1-p_1} \times \frac{g}{p_1(1-p_1)} \times (\sum_{i=1}^{2} p_i) = 0$$

From (5.11), $g \neq 0$

$$\therefore \sum_{i=1}^{2} p_i = 1$$

For a general case involving n users, following the exact same steps and the chain differentiation trick, we have

$$g \frac{\partial(\frac{p_i}{1-p_i})}{\partial p_i} + \frac{p_i}{1-p_i} \frac{\partial g}{\partial p_i} = 0$$

and $\dfrac{\partial g}{\partial p_i} = -\dfrac{g}{p_i(1-p_i)} \sum_{i=1}^{n} p_i$

Hence, we have

$$\sum_{i=1}^{n} p_i = 1 \tag{5.16}$$

∎

Theorem 5.6 leads to the following result.

Theorem 5.7: The CNE of the Access Game is unique.

Proof: From (5.7), we have

$$p_i = \frac{w_i}{K + w_i} \quad \forall i \tag{5.17}$$

Using (5.16) and (5.17), we have

$$\sum_{i=1}^{n} \frac{w_i}{K + w_i} = 1 \tag{5.18}$$

We show that the solution for K in (5.18) is unique. We prove the uniqueness of the solution in 3 parts.

a. All the solutions of K in (5.18) are real.

Proof: Let there be complex solution for K; $\alpha \pm \sqrt{-1}\beta$. We then have

$$\sum_{i=1}^{n} \frac{w_i}{\alpha + \sqrt{-1}\beta + w_i} = 1 \text{ and } \sum_{i=1}^{n} \frac{w_i}{\alpha - \sqrt{-1}\beta + w_i} = 1$$

$$\therefore \sum_{i=1}^{n} \left(\frac{w_i}{\alpha + \sqrt{-1}\beta + w_i} - \frac{w_i}{\alpha - \sqrt{-1}\beta + w_i}\right) = 0$$

$$\therefore -2\alpha\sqrt{-1}\beta \sum_{i=1}^{n} \frac{w_i}{(\alpha + w_i)^2 + \beta^2} = 0$$

As $\sum_{i=1}^{n} \frac{w_i}{(\alpha + w_i)^2 + \beta^2} \neq 0$

$$\beta = 0 \tag{5. 19}$$

b. There is at least one positive root for K.

Proof: (5.18) has n solutions in K. Let these roots be $r_1....r_n$.

By expanding (5.18), it can be clearly seen that some of the coefficients are negative. Hence, there is at least one positive root. Formally,

$$\exists k : r_k > 0 \qquad (5.20)$$

c. If the number of positive roots in (5.18) is more than one, they are equal.

Proof: Let there be more than one positive root; α_1, α_2. From (5.18), we have:

$$\sum_{i=1}^{n}\left(\frac{w_i}{\alpha_1 + w_i} - \frac{w_i}{\alpha_2 + w_i}\right) = 0$$

$$\therefore (\alpha_2 - \alpha_1)\sum_{i=1}^{n}\frac{w_i}{(\alpha_1 + w_i)(\alpha_2 + w_i)} = 0$$

$$or, \alpha_2 = \alpha_1 \qquad (5.21)$$

From (5.19)-(5.21), we have that the solution of K in (5.18) is unique. Hence, from (5.17) the CNE is unique.

∎

A discussion on computing K is presented in Chapter Eight (pp. 64-65).

5.3 Throughput

Finally, we show that the system throughput is optimized at the CNE.

Lemma 5.8: At the CNE satisfying fair share, $\frac{\partial u_i}{\partial p_j} = 0 \ \forall i, j$

Proof: For CNE, $\frac{\partial u_i}{\partial p_i} = 0 \quad \forall i$ and $\frac{p_i/(1-p_i)}{w_i} = \frac{p_j/(1-p_j)}{w_j} \quad \forall i, j$

We have $u_i = \frac{p_i}{1-p_i} g$

It follows that at CNE, $u_i = \frac{w_i}{w_j} \frac{p_j}{1-p_j} g = \frac{w_i}{w_j} u_j$

But at CNE, $\frac{\partial u_j}{\partial p_j} = 0 \qquad \forall j$

$$\therefore \frac{\partial u_i}{\partial p_j} = \frac{w_i}{w_j} \frac{\partial u_j}{\partial p_j} = 0 \qquad \forall i, j \tag{5.22}$$

Theorem 5. 9: CNE satisfying fairness maximizes the system throughput.

Proof: Throughput θ can be expressed as

$$\theta = \sum_{i=1}^{n} p_i \times \prod_{j \neq i}^{n} (1 - p_j)$$

From (5.6), $\theta = \sum_{i=1}^{n} \frac{p_i}{1-p_i} \times g$

From (5.10)

$$\theta = \sum_{i=1}^{n} u_i \tag{5.23}$$

For throughput maximization, $\frac{\partial \theta}{\partial p_j} = 0 \qquad \forall j$

From (5.23)

$$\frac{\partial \theta}{\partial p_j} = \sum_{i=1}^{n} \frac{\partial u_i}{\partial p_j} \tag{5.24}$$

Following Lemma 5.8, we have

$$\frac{\partial \theta}{\partial p_j}\Big|_{CNE} = 0 \qquad \forall j$$

$\blacksquare$

The results in this Chapter outline some important properties of the MAC strategy considered here. We observed that the CNE satisfying social welfare is unique and it optimizes the system throughput as well. The implications are as follows:

- The equilibrium is unique and results in fairness. Consequently, this is a desirable equilibrium and the users have no reason to deviate from these equilibrium transmission probabilities.

- Throughput is optimized. Consequently, independence to the user is a feasible idea from the system-performance view.

- The solution is Pareto-optimal. This requires some explanation. While formulating the problem, we saw that there were several contradictory objectives: contention amongst users and the conflict between fairness for users and the system throughput. As we have shown in this Chapter, all these contradictory objectives are fulfilled by the equilibrium transmission probabilities. Therefore, the transmission probabilities qualify as Pareto-Optimal.

CHAPTER SIX: CLASSIFICATIONS OF COMPLETE INFORMATION ACCESS GAME

In this Chapter we provide a classification of the Access Game based on the relative importance of the QoS and the BP components in the utility functions.

$$\Pr\{success\}_i = p_i \prod_{j \neq i}^{n} (1 - p_j) \tag{6.1}$$

Hence, fairness can be quantitatively expressed as

$$\frac{p_1 \prod_{j \neq 1}^{n} (1 - p_j)}{w_1} = \ldots \frac{p_i \prod_{j \neq i}^{n} (1 - p_j)}{w_i} = \ldots \frac{p_n \prod_{j=1}^{n-1} (1 - p_j)}{w_n} \tag{6.2}$$

It can be observed from (6.2) that if $p_i \in \{0,1\} \quad \forall i$, the fairness criteria are satisfied but the probability of success is zero for all or some of the users. Clearly, these solutions are of no interest.

For acceptable solutions

$$p_i \neq 0,1 \qquad \forall i \tag{6.3}$$

6.1 Nash Equilibrium

As mentioned before, for the sake of convenience, we drop the “bar” in $\overline{u}_i$. The payoff function of user i is denoted simply as u_i.

Rewriting (3.1),

$$u_i = p_i \times [\prod_{j \neq i}^{n} (1 - p_j) \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i})] + c_{2,i} \tag{6.4}$$

Let,

$$\delta_i = [\prod_{j \neq i}^{n} (1 - p_j)] \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i}) \tag{6.5}$$

For mixed strategy equilibrium

$$\delta_i = 0 \qquad \forall i \tag{6.6}$$

Consequently, for NE

$$[\prod_{j \neq i}^{n} (1 - p_j)] \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i}) = 0 \tag{6.7}$$

Rewriting (6.7)

$$\prod_{j \neq i}^{n} (1 - p_j) = \frac{(c_{2,i} - c_{3,i})}{(c_{1,i} - c_{3,i})} \tag{6.8}$$

Using (3.5), equation 6.8 can be rewritten as

$$\prod_{j \neq i}^{n} (1 - p_j) = r_i \tag{6.9}$$

From (6.9) it can be seen that for NE to exist,

$$0 \leq r_i \leq 1 \qquad \forall i \tag{6.10}$$

Therefore, for the existence of NE following relations are permissible amongst the payoff values of a user i

$c_{3,i} > c_{2,i} > c_{1,i}$

$c_{2,i} = c_{3,i} \neq c_{1,i}$

$c_{1,i} = c_{2,i} \neq c_{3,i}$

$c_{1,i} > c_{2,i} > c_{3,i}$

It should be noted that although $c_{3,i} > c_{2,i} > c_{1,i}$ presents a mathematical possibility, it does not reflect any physical reality: payoff for failure is more than payoffs for either success or waste. As per our previous discussion, this case is discarded. Therefore, we have the following distinct cases:

Case 1: $c_{2,i} = c_{3,i} \neq c_{1,i}$ (6.11a)

Case 2: $c_{1,i} = c_{2,i} \neq c_{3,i}$ (6.11b)

Case 3: $c_{1,i} > c_{2,i} > c_{3,i}$ (6.11c)

Comments: All the three cases of 6.11a-c represent interesting situations:

- Case 1 can represent two distinct scenarios depending on the relationship between $c_{2,i}$ and $c_{1,i}$. If $c_{1,i} > c_{2,i}$, then it can be said that QoS is much more important than BP constraints. If this is true for all the users, then we have the important special case of wired networks as analyzed in the previous Chapter. On the other hand, if $c_{2,i} > c_{1,i}$, then the scenario can be thought of as the case where BP is much more important than QoS. This second scenario does not hold any special significance for the purpose of this dissertation. Nevertheless, we combine these two scenarios and present a unified analysis in the next Chapter.

- Discarding the possibility of $c_{3,i} > c_{1,i}, c_{2,i}$, Case 2 can be rewritten as

$$\text{Case 2:}\; c_{1,i} = c_{2,i} > c_{3,i} \tag{6.11d}$$

  Case 2 as represented in (6.11d) can be interpreted as an interesting special scenario where both QoS and BP are equally important.

- Case 3 corresponds to (3.4) i.e. it represents the general case where QoS is more important than power constraints.

We have the following result for Cases 1 and 2.

Theorem 6.1: Nash Equilibriums for the Access Game with payoff structures as in (6.11a) and (6.11d) are not acceptable.

Proof: In order to prove the Theorem, let us say that Case 1 holds for a particular user i, then at the NE,

$$\prod_{j \neq i}^{n} (1 - p_j) = 0$$

Therefore,

$$\Pr\{success\}_i = p_i \prod_{j \neq i}^{n} (1 - p_j) = 0$$

Similarly, if Case 2 holds for some user i, then

$$\prod_{j \neq i}^{n} (1 - p_j) = 1$$

Therefore,

$$p_j = 0 \quad \forall j \neq i$$

$$\therefore \Pr\{success\}_j = 0 \qquad \forall j \neq i$$

As per Definition 2, the NE for Case 1 and Case 2 are not acceptable.

$\blacksquare$

Theorem 6.1 proves that Cases 1 and 2 have only trivial solution in NE. However, both these cases represent interesting special scenarios. Therefore, the concept of CNE is used in Chapter Four to analyze these two cases. Before presenting the detailed analysis in Chapter Four, some preliminary analysis required for computing CNE is conducted in Chapter 6.3.

6.2 Preliminaries for CNE

The rationale behind computing CNE is that if NE is inefficient, then selfish users would be willing to adhere to some set of constraints; more so if the end-result is beneficial. Clearly, the fairness constraints of (6.2) and (6.3) are beneficial for the users and can be used for the computation of CNE.

In order to compute CNE, the condition in (2.3) needs to be checked first.

From (6.6),

$$\frac{\partial^2 u_i}{\partial p_i^2} = 0 \qquad \forall i \tag{6.12}$$

Hence, (2.3) is satisfied.

The fairness constraints of (6.2) and (6.3) are used as constraints. Combining (6.2) and (6.3), the fairness constraints can be expressed as:

$$[\frac{1}{p_i} - 1] = \frac{w_j}{w_i}[\frac{1}{p_j} - 1] \qquad \forall i, j \tag{6.13}$$

Comments: From (6.13), it can be observed that there are infinite number of ways fairness can be achieved. In order to see this note that (6.13) can be rewritten as

$$w_i y_i - w_j y_j = w_i - w_j \qquad \forall i, j$$

where, $1/p_j = y_j, 1/p_i = y_i$. Therefore, there are $n-1$ independent equations involving n variables. Consequently, the number of possible solutions is infinite. However, CNE gives the solutions that optimize the individual utility functions (i.e. interests) of users with fairness given by (6.13) as constraints. Therefore, the solution given by CNE is efficient in nature.

It should be noted that although (6.13) was derived using (6.3), (6.13) is satisfied by $p_i = p_j = 1$. As (6.13) is used for computing CNE, it may so happen that for some user i, $p_i^{CNE} = 1$. This is not a problem because these solutions are simply discarded.

From (6.13), following partial derivatives are computed for future use:

$$\frac{\partial p_j}{\partial p_i} = \frac{p_j^2}{p_i^2} \times \frac{w_i}{w_j} \tag{6.14}$$

It is convenient to express (6.14) in terms of the probabilities only. From (6.13),

$$\frac{w_i}{w_j} = \frac{[1/p_j - 1]}{[1/p_i - 1]} \tag{6.15}$$

However, (6.15) is not valid for $p_i = 1$. In order to incorporate $p_i = 1$, the following technique is used.

Multiplying both sides of (6.15) by $[1/p_i - 1]$ we have

$$[1/p_i - 1]\frac{w_i}{w_j} = [1/p_j - 1] \tag{6.16}$$

This is a valid expression.

In order to use (6.16) in (6.14), (6.14) is multiplied by $[1/p_i - 1]$. Therefore,

$$[1/p_i - 1] \times \frac{\partial p_j}{\partial p_i} = \frac{p_j^2}{p_i^2} \times \frac{1 - p_j}{p_j} = p_j \frac{1 - p_j}{p_i^2}$$

$$\therefore (1-p_i)\times\frac{\partial p_j}{\partial p_i} = p_j\,\frac{1-p_j}{p_i} \tag{6.17}$$

In this Chapter, we analyzed the Access Game in some detail and delineated three Cases of importance. We have also shown that two of these cases result in inefficient Nash Equilibriums. In Chapter Seven, we consider these two cases and compute their Constrained Nash Equilibrium. Results from Section 6.2 are extensively used for this purpose.

CHAPTER SEVEN: TWO SPECIAL CASES OF COMPLETE INFORMATION ACCESS GAME

In Chapter Six, we proved that Case 1 and Case 2 have inefficient and unacceptable Nash Equilibriums. In this Chapter, we analyze these two special cases for their Constrained Nash Equilibriums.

7.1 Case 1

As mentioned before, wired networks present a scenario for Case1. Our following analysis is true for wired networks as considered in Chapter Five. However, some of the results derived in Chapter Five will not be valid for the unified analysis present here. For the following analysis, it is assumed that (6.11a) holds for all the users. Therefore,

$$\begin{aligned} &c_{2,i} = c_{3,i} \neq c_{1,i} \\ &\therefore r_i = 0 \end{aligned} \qquad \forall i \tag{7.1}$$

Note that if $c_{1,i} > c_{2,i}\ \forall i$, (7.1) represents wired network. However the results in this Chapter will also hold for $c_{1,i} < c_{2,i}\ \forall i$. Another interesting point is that for (7.1), weightages are assigned based on only one component: either QoS or BP i.e. a user with higher QoS (or BP) requirement should be allocated higher weightage.

For (7.1), the following result holds:

Theorem 7.1: For (7.1), there is a unique acceptable CNE subject to (6.13).

Proof: The utility function for user i can be written as

$$u_{i,Case1} = p_i \times [\prod_{j \neq i}^{n} (1 - p_j) \times (c_{1,i} - c_{3,i})] + c_{2,i}$$

Rewriting,

$$u_{i,Case1} = p_i \times d_i^{Case1} [\prod_{j \neq i}^{n} (1 - p_j) + c_{2,i} \qquad (7.2)$$

where,

$$d_i^{Case1} = (c_{1,i} - c_{3,i}) \neq 0$$

For CNE [28],

$$\frac{\partial u_{i,Case2}}{\partial p_i} = 0 \qquad \forall i \qquad (7.3)$$

subject to (6.13)

From (7.3)

$$d_i \times [\frac{\partial (p_i \times \prod_{j \neq i}^{n} (1 - p_j))}{\partial p_i}] = 0$$

$$or, \prod_{j \neq i}^{n} (1 - p_j) + p_i \times \sum_{j \neq i}^{n} (-\frac{\partial p_j}{\partial p_i}) \times \prod_{k \neq i,j}^{n} (1 - p_k) = 0 \qquad (7.4)$$

In order to use (6.17) in (7.4), both sides of (7.4) are multiplied by $1-p_i$. Therefore,

$$(1-p_i)\prod_{j\neq i}^{n}(1-p_j)+p_i\times\sum_{j\neq i}^{n}(-(1-p_i)\frac{\partial p_j}{\partial p_i})\times\prod_{k\neq i,j}^{n}(1-p_k)=0 \tag{7.5}$$

Using (6.17) in (7.5):

$$\prod_{j=1}^{n}(1-p_j)-p_i\times\sum_{j\neq i}^{n}\frac{p_j(1-p_j)}{p_i}\times\prod_{k\neq i,j}^{n}(1-p_k)=0$$

$$or,\prod_{j=1}^{n}(1-p_j)-\prod_{j\neq i}^{n}(1-p_j)\times\sum_{j\neq i}^{n}p_j=0 \tag{7.6a}$$

Multiplying both sides of (7.6a) by $1-p_i$,

$$(1-p_i)\prod_{j=1}^{n}(1-p_j)-\prod_{j=1}^{n}(1-p_j)\times\sum_{j\neq i}^{n}p_j=0$$

$$or,\prod_{j=1}^{n}(1-p_j)\times(1-\sum_{j=1}^{n}p_j)=0 \tag{7.6b}$$

For (7.6b) at least one of the following should hold true:

$$\prod_{j=1}^{n}(1-p_j)=0$$

$$(1-\sum_{j=1}^{n}p_j)=0$$

However, $\prod_{j=1}^{n}(1-p_j)=0\Rightarrow\exists j:p_j=1$.

Therefore, this is not an acceptable solution

Hence, the acceptable CNE is given by

$$\sum_{j=1}^{n} p_j = 1 \tag{7.7}$$

$\blacksquare$

Computation of K: A closed form solution for K from (7.10) cannot be obtained for $n \geq 5$ (Abel's Impossibility Theorem). Expressions for K for some special cases are as follows:

1. Two users: $K = \sqrt{w_1 w_2}$ (7.8)

2. n identical users: $K = (n-1)w$ (7.9)

3. Two classes (1,2) of users and class i has n_i users. Each user of class i has weight w_i:

$$K = \frac{\sqrt{b^2 + 4ac} - b}{2a} \tag{7.10}$$

where,

$$a = (n_1 + n_2 - 1) \tag{7.11a}$$

$$b = w_2(n_1 - 1) + w_1(n_2 - 1) \tag{7.11b}$$

$$c = w_1 w_2 \tag{7.11c}$$

Numerical Results: The following numerical examples show that the value of K increases with the increase in the number of users.

1. Consider five classes of users. The weightage of class 1 is "1", weightage of class 2 is "2" etc. Each class has N number of users.

Table 3: Computation of K

N	1	2	3	4	5	6
K	11.5	26.4	41.4	56.4	71.36	86.4

2. Consider N classes. Each class has one user and the weightage of class i is "i"

Table 4: Computation of K

N	3	4	5	6	7	8
K	6.77	7.11	11.45	16.77	26.12	30.46

Both table 1 and table 2 show that the value of K increases with the increase in number of users. This implies that the value of p_i^{CNE} decreases with the increase in number of users. This trend points towards the necessity of Call Admission Control (CAC).

7.2 Case 2

For the analysis of Case 2, it is assumed that

$$c_{1,i} = c_{2,i} > c_{3,i} \qquad \therefore r_i = 1 \qquad \forall i \tag{7.12}$$

The following result holds for (7.12)

Theorem 7.2: For (7.12), the CNE subject to (6.13) is unique and it is given by $p_i = 1$ $\forall i$ *. Hence, it is not acceptable.*

Proof: For this case, the utility function can be written as follows:

$$u_{i,Case2} = p_i \times d_i^{Case2}[\prod_{j \neq i}^{n}(1 - p_j) \times -1] + c_{2,i} \tag{7.13}$$

where,

$$d_i^{Case2} = c_{1,i} - c_{2,i}$$

For CNE,

$$\frac{\partial u_{i,Case2}}{\partial p_i} = 0 \qquad \forall i \tag{7.14}$$

subject to (6.13)

From (7.14),

$$d_i \times [\frac{\partial(p_i \times \prod_{j \neq i}^{n}(1 - p_j))}{\partial p_i} - 1] = 0$$

$$or, \prod_{j \neq i}^{n} (1 - p_j) + p_i \times \sum_{j \neq i}^{n} (-\frac{\partial p_j}{\partial p_i}) \times \prod_{k \neq i, j}^{n} (1 - p_k) = 1$$

Using exactly the same techniques as in Section 7.1, we have

$$\prod_{j=1}^{n} (1 - p_j) \times (1 - \sum_{j=1}^{n} p_j) = (1 - p_i)^2$$

$$\therefore (1 - p_i)[\prod_{j \neq i}^{n} (1 - p_j) \times (1 - \sum_{j=1}^{n} p_j) - (1 - p_i)] = 0 \qquad (7.15)$$

For (7.15), at least one of the following should hold true:

$$1. \prod_{j \neq i}^{n} (1 - p_j) \times (1 - \sum_{j=1}^{n} p_j) - (1 - p_i) = 0 \qquad (7.16a)$$

$$2. 1 - p_i = 0 \qquad (7.16b)$$

We first consider (7.16a): $\prod_{j \neq i}^{n} (1 - p_j) \times (1 - \sum_{j=1}^{n} p_j) - (1 - p_i)$

Introducing, $\Delta_1 = \prod_{j \neq i}^{n} (1 - p_j)$ and $\Delta_2 = \sum_{j \neq i}^{n} p_j$

We have

$$\prod_{j \neq i}^{n} (1 - p_j) \times (1 - \sum_{j=1}^{n} p_j) - (1 - p_i) = \Delta_1 \times (1 - p_i - \Delta_2) - (1 - p_i) = (1 - p_i)(1 - \Delta_1) - \Delta_1 \Delta_2$$

Therefore, for (7.16a) to hold, Δ_2 must be zero. However, from (6.3) $p_j \neq 0$

$p_j \neq 0 \qquad \forall j \qquad \Rightarrow \qquad \Delta_1 \neq 1, \Delta_2 \neq 0$

Therefore, $\prod_{j \neq i}^{n} (1 - p_j) \times (1 - \sum_{j=1}^{n} p_j) - (1 - p_i) \neq 0$

Therefore, (7.16b) must hold true.

It follows that we have the following unique solution

$p_i = 1$

Hence, the unique CNE is given by

$p_i = 1 \qquad \forall i$

$\blacksquare$

From Theorem 7.2 it can be concluded that the Access Game model is not a suitable model for the scenario described by (7.12).

In this Chapter, we have computed Constrained Nash Equilibriums for Case 1 and Case 2. Case 1 is the generalization of the wired network scenario analyzed in Chapter Five. For Case 1, the CNE is acceptable and unique. However, for Case 2 the CNE is not acceptable. We propose that scenarios represented through Case 2 should be modeled differently.

CHAPTER EIGHT: GENERAL CASE OF COMPLETE INFORMATION ACCESS GAME

In the previous Chapter, two special cases were analyzed. In this Chapter we analyze the general case as expressed in 6.11c.

8.1 Nash Equilibrium

From Theorem 6.1, it can be said that if Case 1 or Case 2 holds for even one user, the Nash Equilibrium is not acceptable.

Therefore, for the analysis in this Chapter, the following holds

$$\begin{aligned} & c_{1,i} > c_{2,i} > c_{3,i} \\ & \therefore 0 < r_i < 1 \end{aligned} \qquad \forall i \tag{8.1}$$

Theorem 8.1: If an NE exists for the Access Game satisfying (8.1), it is unique.

Proof: Continuing from (6.9).

Taking natural logarithm on both sides in (6.9)

$$\ln[\prod_{j \neq i}^{n} (1 - p_j)] = \ln r_i \tag{8.2}$$

Introducing $x_j = \ln(1 - p_j) < 0$ and $k_i = \ln r_i < 0$, we have from (8.2)

$$\sum_{j \neq i}^{n} x_j = k_i \qquad \forall i \tag{8.3}$$

(8.3) can be expressed in a matrix form

$$A_{n\times n}\vec{x}=\vec{b} \tag{8.4}$$

where,

$A(i, j)=1$ if $i \neq j$

$A(i, j)=0$ if $i = j$

$\vec{x}=[x_1 ... x_n]^T$

$\vec{b}=[k_1 ... k_n]^T$

The unique solution for $\vec{x}$ in (8.4) is given by

$$\vec{x}=B_{n\times n}\vec{b} \tag{8.5}$$

where, $B = inv(A)$ and

$$B_{n\times n}(i, j)=\frac{1}{(n-1)} \qquad \text{if } i \neq j \tag{8.6a}$$

$$B_{n\times n}(i, j)=-\frac{n-2}{(n-1)} \qquad \text{if } i = j \tag{8.6b}$$

From (8.5) and (8.6),

$$x_i = \frac{\sum_{j\neq i}^{n} k_j}{(n-1)} - \frac{(n-2)\times k_i}{(n-1)}$$

$$\therefore x_i = \frac{\sum_{j=1}^{n} k_j - (n-1)k_i}{(n-1)}$$

But $x_j = \ln(1-p_j) < 0$ and $k_i = \ln r_i < 0$

Therefore,

$$\ln[1-p_i] = \frac{\sum_{j=1}^{n} \ln r_j - \ln r_i^{n-1}}{(n-1)} = \frac{\ln \frac{\prod_{j=1}^{n} r_j}{r_i^{n-1}}}{(n-1)}$$

$$\Rightarrow 1-p_i = \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} \tag{8.7}$$

Clearly, for an acceptable NE to exist (from (6.3))

$$0 < \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < 1 \qquad \forall i \tag{8.8}$$

As $r_i > 0 \ \forall i$, the existence condition (8.8) can be rewritten as

$$\frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < 1 \qquad \forall i \tag{8.9}$$

Rewriting (8.9), we have

$$\prod_{j=1}^{n} \rho_j < \frac{1}{r_{\min}} \tag{8.10}$$

where,

$r_{\min} = \min\{r_1 \ldots r_n)$ and $\rho_j = \dfrac{r_j}{r_{\min}}$

If (8.10) holds, the unique NE is given by

$$p_i^{NE} = 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} \qquad \forall i \tag{8.11}$$

$\blacksquare$

Note that the criterion of (6.4) is satisfied by (8.11). Therefore, it is possible to achieve fairness.

The natural question that arises is whether this NE satisfies fairness or not. The answer is that for a general set of payoff ratios, the NE will not satisfy fairness. However if there is some flexibility in designing the payoff ratios, the NE can satisfy the fairness criteria. We have the following result.

Consider a set of probability $p^* = \{p_1^* \dots p_n^*\}$ that satisfies fairness. In order to see that an infinite number of such probability sets exist, note that the fairness criteria can be rewritten as

$$[\frac{1}{p_i} - 1] = \frac{w_j}{w_i}[\frac{1}{p_j} - 1] \qquad \forall i, j \tag{8.12}$$

Equation (8.12) can be rewritten as

$$w_i y_i - w_j y_j = w_i - w_j \qquad \forall i, j$$

where, $1/p_j = y_j, 1/p_i = y_i$.

Therefore, there are $n-1$ independent equations involving n variables. Consequently, the number of possible solutions is infinite.

For such a set of probabilities, we have the following result

Lemma 8.2: If $r_i = \prod_{j \neq i}^{n} (1 - p_j^*)$, *then the Nash Equilibrium given by (8.11) satisfies fairness.*

Proof**:** Let,

$$r_i = \prod_{j \neq i}^{n} (1 - p_j^*) \tag{8.13}$$

First, we check for (8.9) to determine if the NE exists or not.

Using equation (8.13)

$$\frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} = \frac{[\prod_{j=1}^{n}\prod_{k\neq j}^{n}(1-p_k^*)]^{\frac{1}{(n-1)}}}{\prod_{k\neq i}^{n}(1-p_j^*)} = \frac{[\prod_{j=1}^{n}\prod_{k\neq j}^{n}(1-p_k^*)]^{\frac{1}{(n-1)}}}{\prod_{k\neq i}^{n}(1-p_j^*)} = \frac{[\prod_{j=1}^{n}\prod_{k\neq j}^{n}(1-p_k^*)]^{\frac{1}{(n-1)}}}{\prod_{k\neq i}^{n}(1-p_j^*)} = \frac{[\prod_{j=1}^{n}\frac{\prod_{k=1}^{n}(1-p_k^*)}{(1-p_j^*)}]^{\frac{1}{(n-1)}}}{\frac{\prod_{k=1}^{n}(1-p_k^*)}{(1-p_i^*)}}$$

$$= \frac{(1-p_i^*)\times[\prod_{j=1}^{n}\prod_{k=1}^{n}(1-p_k^*)]^{\frac{1}{(n-1)}}}{[\prod_{j=1}^{n}(1-p_j^*)^{\frac{1}{(n-1)}}]\times\prod_{k=1}^{n}(1-p_k^*)} = (1-p_i^*)\times\frac{\prod_{k=1}^{n}(1-p_k^*)]^{\frac{n}{(n-1)}}}{\prod_{k=1}^{n}(1-p_k^*)]^{\frac{n}{(n-1)}}} = (1-p_i^*) < 1$$

$$\therefore \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} = (1-p_i^*) < 1 \quad \forall i \tag{8.14}$$

Therefore, (8.9) is satisfied and NE exists. Moreover, using (8.14) in (8.11) we have

$$\therefore p_i^{NE} = p_i^*$$

■

Therefore, if the payoff ratios are designed accordingly, the unique NE satisfies (3.13) and maximizes throughput.

Corollary 8.3: $p^{NE} = p^*$ *if and only if* $r_i = \prod_{j\neq i}^{n}(1-p_j^*)$.

Proof: In Lemma8.2, we have proved the "if" part of the proof. In order to see the uniqueness or the "only if" part, note that for a given set of NE we have

$$1 - p_i^{NE} = \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i}$$

$$or, \ln[1 - p_i^{NE}] = \frac{1}{(n-1)} \sum_{j=1}^{n} \ln r_j - \ln r_i$$

$$\therefore [\sum_{j \neq i}^{n} \ln r_j - (n-2) \ln r_i] = (n-1) \ln[1 - p_i^{NE}] \tag{8.15}$$

If the objective is to design payoff ratios such that a particular Nash Equilibrium is achieved, then the Right Hand Side of (8.15) is known and (8.15) can be expressed as

$$C_{n \times n} \vec{r} = \vec{d} \tag{8.16}$$

where,

$C[i, j] = -(n-2)$ if $i = j$; $C[i, j] = 1$ if $i \neq j$

$\vec{r} = [r_1 ... r_n]^T$; $\vec{d} = (n-1)[k_1 ... k_n]^T$;

$k_i = \ln[1 - p_i^{NE}]$

Solution of $\vec{r}$ from (8.16) is unique and given by

$$r_i = \prod_{j \neq i}^{n} (1 - p_j^{NE}) \tag{8.17}$$

In other words, if the Access Game is to operate at a particular known NE, the payoff ratios must be as in (8.17). Therefore, if the NE is to operate at p^*, the payoff ratios must be

$$r_i = \prod_{j \neq i}^{n} (1 - p_j^*)$$

$\blacksquare$

It is easy to see that for $n = 2$, the existence condition of (8.9) or (8.10) is satisfied for any values of the payoff ratios. However, for higher values of n, the situation becomes problematic. Let us give a few numerical examples. For the first two examples $1/r_{\min} = 2.5$ and for the last example, $1/r_{\min} = 10$

Example1: Let there be four different users with payoffs of "0.4", "0.5", "0.6", "0.7". In this case, $\prod_{j=1}^{n} \rho_j = 3.1875 > \frac{1}{r_{\min}}$. Hence, (8.10) is not satisfied

Example2: Consider 11 users. The j^{th} user has a payoff ratio of $r_j = 0.4 + (i-1)*.01$. In this case,

$$\prod_{j=1}^{n} \rho_j = 3.5549 > \frac{1}{r_{\min}}$$

Example3: Consider 11 users. The j^{th} user has a payoff ratio of $r_j = 0.1 + (i-1)*.01$. In this case,

$$\prod_{j=1}^{n} \rho_j = 67.0443 > \frac{1}{r_{\min}}$$

These examples show that it is difficult to hold the existence condition if the number of users are large. Therefore, CNE is considered for the general case also.

8.2 Constrained Nash Equilibrium

For Case 3, the utility function is same as in (3.1). Therefore,

$$u_{i,Case3} = p_i \times \prod_{j \neq i}^{n} (1 - p_j) \times c_{1,i} + p_i [1 - \prod_{j \neq i}^{n} (1 - p_j)] \times c_{3,i} + (1 - p_i) \times c_{2,i} \tag{8.18}$$

For CNE,

$$\frac{\partial u_{i,Case3}}{\partial p_i} = 0 \qquad \forall i \tag{8.19}$$

subject to (6.13)

Following procedures similar to Chapter Seven, we have the following for CNE

$$\frac{g}{(1 - p_i)^2} (1 - \sum_{i=1}^{n} p_i) = \frac{c_{2,i} - c_{3,i}}{c_{1,i} - c_{3,i}} = r_i$$

$$\therefore g(1 - \sum_{i=1}^{n} p_i) = r_i (1 - p_i)^2 \qquad \forall i \tag{8.20}$$

where,

$$g = \prod_{j=1}^{n} (1 - p_j) \tag{8.21}$$

Therefore,

$$r_i (1 - p_i)^2 = r_j (1 - p_j)^2 \qquad \forall i, j$$

$$\therefore \frac{(1 - p_i)}{(1 - p_j)} = \sqrt{\frac{r_j}{r_i}} \tag{8.22}$$

As the probabilities in (8.22) satisfy fairness, equation (8.22) can be rewritten using (6.13)

$$\frac{p_i}{w_i / \sqrt{r_i}} = \frac{p_j}{w_j / \sqrt{r_j}} \tag{8.23}$$

As the CNE is guaranteed to exist, solution of (8.20) will give transmission probabilities that satisfy fairness. Moreover these probabilities will satisfy the relationship in (8.23). From (8.23) it can be concluded that equilibrium transmission probability is proportional to the weightage and inversely proportional to the square root of the payoff ratio. This means that as the importance of battery power relative to the importance of QoS increases, equilibrium transmission probability decreases to reduce the probability of collision where battery power is wasted for no benefit. On the other hand as QoS is more important, equilibrium transmission probability increase with increasing weightage.

We provide some simple numerical examples for the CNE of general case.

Example1: Consider two user with $w_1 = 1, w_2 = 2$.

Let, $r_1 = 0.3$ and $r_2 = 0.6$.

The transmission probabilities satisfying fairness are given by $p_1 = 0.3135$, $p_2 = 0.4434$

Example2: Consider the previous example with $r_1 = 0.4, r_2 = 0.5$. The transmission probabilities satisfying fairness are given by

$p_1 = 0.1841$, $p_2 = 0.3294$

Example3: Consider three users with $w_1 = 1, w_2 = 2, w_3 = 3$ and $r_1 = 0.2, r_2 = 0.3, r_2 = 0.4$.

The transmission probabilities satisfying fairness can be given by

$p_1 = 0.1428, p_2 = 0.2332, p_3 = 0.3029$

Theorem 8.4: The CNE computed from (8.15) is unique

Proof: From (8.20), we have

$$\prod_{j=1}^{n}(1-p_j)(1-\sum_{i=1}^{n}p_i) = r_i(1-p_i)^2 \qquad \forall i$$

From (8.23), we have

$$p_j = \frac{w_j / \sqrt{r_j}}{w_i / \sqrt{r_i}} p_i = c_{i,j} p_i \tag{8.24}$$

where, $c_{i,j} = \dfrac{w_j / \sqrt{r_j}}{w_i / \sqrt{r_i}}$ (8.25)

Now, for fairness (from 6.13)

$$\frac{p_i /(1-p_i)}{w_i} = \frac{p_j /(1-p_j)}{w_j}$$

$$or, \frac{p_j / w_j}{p_i / w_i} = \frac{(1-p_j)}{(1-p_i)}$$

From (8.23), we have $\dfrac{p_j / w_j}{p_i / w_i} = \dfrac{\sqrt{r_i}}{\sqrt{r_j}}$

Therefore,

$$\frac{\sqrt{r_i}}{\sqrt{r_j}} = \frac{(1-p_j)}{(1-p_i)}$$

$$\therefore (1-p_j) = \frac{\sqrt{r_i}}{\sqrt{r_j}}(1-p_i) = k_{i,j}(1-p_i) \tag{8.26}$$

where, $k_{i,j} = \dfrac{\sqrt{r_i}}{\sqrt{r_j}}$ (8.27)

Using (8.24) and (8.26) in (8.20), it can be seen that $\prod_{j=1}^{n}(1-p_j)(1-\sum_{i=1}^{n} p_i) = r_i(1-p_i)^2$ can be rewritten as follows:

$$(1-p_i)^n \prod_{j=1}^{n} k_{i,j} \times (1-p_i \sum_{i=1}^{n} c_{i,j}) = r_i (1-p_i)^2$$

$$or, (1-p_i)^{n-2} \times (1-p_i \sum_{i=1}^{n} c_{i,j}) = \frac{r_i}{\prod_{j=1}^{n} k_{i,j}}$$

$$or, (1-p_i)^{n-2} \times (1-K_1 p_i) = K_2 \tag{8.28}$$

where,

$$K_1 = \sum_{j=1}^{n} \frac{w_j / w_i}{\sqrt{r_j / r_i}}, \; K_2 = \frac{r_i}{\prod_{j=1}^{n} k_{i,j}} \tag{8.29}$$

Rewriting (8.28) we have

$$K_1 (1-p_i)^{n-1} - (K_1 - 1)(1-p_i)^{n-2} = K_2 \tag{8.30}$$

As the utility functions are concave, we know from [28] that a positive solution exists for p_i in (8.20) such that $0 < p_i \le 1$. It is easy to see from (8.20) that $p_i = 1$ in not a solution. Therefore, there is a solution of p_i in (8. 30) such that $0 < p_i < 1$. From here, we simply apply Descartes' Rule of Signs. Descartes' Rule of Signs states that the number of positive roots of a polynomial $p(x)$ with real coefficients does not exceed the number of sign changes of the nonzero coefficients of $p(x)$. More precisely, the number of sign changes minus the number of positive roots is a multiple of two. As $K_1, K_1 > 0$, there is exactly one positive real root in this case.

$\blacksquare$

8.3 Throughput

The results in the previous Chapter showed that fairness as defined in (6.2) and (6.3) can be achieved in a non-cooperative fashion by computing the CNE of the Access Game. In order to complete our description of the Access Game, we now analyze the throughput performance of the Access Game.

Throughput is defined as the collective probability of “success”. Therefore, it can be given as:

$$\theta = \sum_{i=1}^{n} \Pr\{success\}_i = \sum_{i=1}^{n} p_i \times \prod_{j \neq i}^{n} (1 - p_j) \qquad (8.31)$$

Theorem 8.5: Consider a set of acceptable transmission probabilities $p = (p_1 \ldots p_i \ldots p_n)$. *If these probabilities satisfy fairness, then throughput is maximized if and only if* $\sum_{i=1}^{n} p_i = 1$

Proof: For throughput optimization, the following should hold:

$$\frac{\partial \theta}{\partial p_i} = 0 \qquad \forall i \qquad (8.32)$$

If a set of acceptable transmission probabilities satisfy the fairness criteria in (6.13), (8.31) can be rewritten as

$$\theta = \sum_{i=1}^{n} p_i \times \prod_{j \neq i}^{n} (1 - p_j) = p_i \times \prod_{j \neq i}^{n} (1 - p_j) \times \sum_{j=1}^{n} \frac{w_j}{w_i} \qquad \forall i$$

Therefore, for throughput maximization [from (8.32)]

$$\left[\frac{\partial(p_i \times \prod_{j \neq i}^{n}(1-p_j))}{\partial p_i}\right] = 0 \qquad \forall i$$

Following the same procedure as in the proof of Theorems 6.1 and 7.1, we have

$$\prod_{j=1}^{n}(1-p_j) \times (1-\sum_{j=1}^{n} p_j) = 0$$

The only acceptable solution is

$$1-\sum_{j=1}^{n} p_j = 0$$

$$\therefore \sum_{j=1}^{n} p_j = 1$$

$\blacksquare$

Corollary 8. 6: The CNE for (8.20) subject to (6.13) does not optimize throughput.

Proof: For an acceptable solution, $\sum_{j=1}^{n} p_j = 1$ is a necessary and sufficient condition for throughput optimization. However, from (8.20), it can be seen that for any acceptable CNE in Case 3

$$\sum_{i=1}^{n} p_i^{CNE,Case3} \neq 1 .$$

∎

Corollary 8. 7: The CNE for (7.1) subject to (6.13) optimizes throughput

Proof: Theorem 8.5 and (7.7).

∎

Corollary 8.8: There is a unique set of transmission probabilities that optimize throughput with fairness conditions as the constraints.

Theorem 8.5 and Corollaries 8.6 and 8.7 illustrate that throughput is a QoS related issue. For Case 1 where QoS is more important than BP, throughput is maximized. However, in the general case where both QoS and BP are important, a more general view of the system level performance including the expected battery life of the users should be used. We are presently working on this aspect.

CHAPTER NINE: INCOMPLETE INFORMATION AND APPROXIMATION

As discussed at the onset of the analytical analysis presented in this dissertation, the analysis presented in Chapters Five, Six, Seven, and Eight assume that all the users have complete information about all the other users. This scenario however does not reflect the reality. Therefore, in this Chapter we propose a mechanism so that realistic scenarios-that are incomplete information in nature-can be approximated as a complete information scenario.

The key to our approximation process is as follows: in most of the distributed systems, there is usually a registration authority (henceforth designated by R) that performs several accounting functions. We use this authority to gather and disseminate information.

9.1 Information Gathering and Dissemination

Let us call the approximation scheme as *appxm*. There are two conceptual steps in *appxm*. Information is sent to R by users (information gathering) and the information accumulated from all the users is broadcast over the network by R (information dissemination). There is a wide body of work dealing with information gathering and dissemination. Our objective is to provide a simple, case-specific mechanism to achieve approximation. The particulars of the mechanism are as follows:

1. Users are divided into C classes. Associated with a class c are two parameters: weightage w_c and the probability of having a packet at the beginning of a transmission slot $p_{p,c} \neq 0,1$. Users know these parameters a priori. The parameter $p_{p,c}$ makes the scheme approximate because users do

not know exactly whether the other users have packets to transmit or not. Instead, users estimate whether other users have packets to transmit or not.

2. R maintains a table containing the number of different classes of users in the system.

3. When a user enters the system, it is assigned a class and the number of users in the corresponding class is increased by one. Similarly, when a user leaves the system, it informs R about its decision to leave. The number of users in the corresponding class of users is decreased by one. Separate control channels are to be used for the registration and leaving process.

4. When the number of users changes, R broadcasts the number of users of each class present in the system.

The above steps are depicted in the following figure.

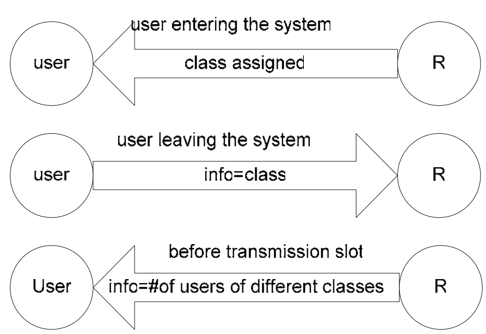

Figure 7: Approximation Scheme *appxm*

9.2 Analysis

In order to show the equivalence between the analysis conducted in Chapter Eight and the analysis to be conducted in this Chapter; we use, instead of using class-specific notations, notations used in Chapter Eight.

When *appxm* is used, a user knows about the number of other users present in the system. In other words, a user also knows about the values of w_i s and $p_{p,i}$ s.

Utility function of user i can be given as (from (2.1))

$$u_i = p_i \times [\prod_{j \neq i}^{n} (1 - \hat{p}_j) \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i})] + c_{2,i} \tag{9.1}$$

where $1 - \hat{p}_j$ is the "estimated" probability that user j does not transmit a packet. Note that a user j does not transmit if

1. It does not have a packet to transmit OR
2. It has a packet to transmit but decides not to transmit

Therefore, $1 - \hat{p}_j$ can be computed as follows

$$1 - \hat{p}_j = p_{p,j} \times (1 - p_j) + (1 - p_{p,j}) = 1 - p_{p,j} \times p_j$$

$$\therefore \hat{p}_j = p_{p,j} \times p_j \neq 0,1 \tag{9.2}$$

Rewriting the utility function in (9.1), we have

$$\therefore u_i = \frac{1}{p_{p,i}} \hat{p}_i \times [\{\prod_{j \neq i}^{n} (1 - \hat{p}_j) \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i})\} + c_{2,i} p_{p,i}] \tag{9.3}$$

As $p_{p,i}$ is a non-zero positive constant, the value of the utility function is dependent on the numerator in (9.3).

Theorem 9.1: If NE for the utility functions in (9.3) exists, it is unique.

Proof: NE is given by

$$[\prod_{j \neq i}^{n} (1 - \hat{p}_j) \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i})] = 0 \qquad \forall i \tag{9.4}$$

Therefore, following the exact procedures as in Theorem 1, we have

$$1 - \hat{p}_i = \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} \qquad \forall i \tag{9.5}$$

$$or, p_i = \frac{1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i}}{p_{p,i}} \qquad \forall i$$

Therefore p_i^{NE} exists if the following condition holds:

$$0 < \frac{1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i}}{p_{p,i}} < 1 \qquad \forall i \tag{9.6}$$

(9.6) can be rewritten as follows:

$$0 < 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < p_{p,i} \qquad \forall i \tag{9.7}$$

If (9.7) is satisfied, then the NE for utility functions in (9.3) can be given as

$$p_i^{NE} = \frac{1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i}}{p_{p,i}} \qquad \forall i \tag{9.8}$$

$\blacksquare$

Corollary 9.2: Let $\vec{p}_{appxm}^{NE} = [p_{1,appxm}^{NE} \ldots p_{n,appxm}^{NE}]^T$, *where* $p_{i,appxm}^{NE}$ *is given by (9.8). Let* $\vec{p}_{exact}^{NE} = [p_{1,exact}^{NE} \ldots p_{n,exact}^{NE}]^T$ *where* $p_{i,exact}^{NE}$ *is given by (8.11). If* p_{appxm}^{NE} *exists,* p_{exact}^{NE} *also exists. Moreover, the following relation holds:*

$$\vec{p}_{appxm}^{NE} = T_{n \times n} \vec{p}_{exact}^{NE} \text{ where } T[i,j] = \frac{1}{p_{p,i}} \text{ if } i = j \text{ and } T[i,j] = 0 \text{ if } i \neq j .$$

Proof: The NE p_{appxm}^{NE} for the approximate case exists if (9.7) holds. Note that (9.7) can be rewritten as follows

$$0 < 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < p_{p,i} < 1$$

$$or, 0 < 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < 1$$

Subtracting “1”, we have

$$-1 < -\frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < 0$$

$$\therefore 0 < \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} < 1$$

Therefore, if condition of (9.7) holds, the condition of (8.9) also holds. Hence if p_{appxm}^{NE} exists, p_{exact}^{NE} also exists.

From (9.8) we have

$$p_{appxm}^{NE}[i] = {}^{1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i}} \Big/ {}_{p_{p,i}}$$

From (8.11), we have

$$p_{exact}^{NE}[i] = 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i}$$

Therefore, we have from (9.8) and (8.11) $\vec{p}_{appxm}^{NE} = T_{n\times n}\,\vec{p}_{exact}^{NE}$

$\blacksquare$

9.3 Fairness

Note that for the approximate scenario, the probability of success can be written as

$$\Pr(success)_i = \frac{\hat{p}_i}{1-\hat{p}_i}\hat{g}$$

where, $\hat{g} = \prod_{i=1}^{n}(1-\hat{p}_i)$

Consequently, the fairness constraints can be represented as

$$\frac{\Pr(success)_1}{w_1} = \ldots \frac{\Pr(success)_n}{w_n}$$

$$\therefore \frac{\hat{p}_1/(1-\hat{p}_1)}{w_1} = \ldots = \frac{\hat{p}_n/(1-\hat{p}_n)}{w_n} \tag{9.9}$$

Corollary 9.3: If the NE of (8.11) satisfies fairness, so will NE of (9.8).

Therefore, if the payoff ratios can be designed suitably then fairness can be achieved by NE. However, if there is no flexibility is designing the fairness ratios NE will not result in fairness in a general case. Therefore, we need to compute CNE for the general case. From (9.1), it can be said that the utility function is maximized if

$\hat{p}_i \times [\prod_{j \neq i}^{n} (1 - \hat{p}_j) \times (c_{1,i} - c_{3,i}) - (c_{2,i} - c_{3,i})] + c_{2,i} p_{p,i}$ is maximized.

Therefore, the variable of interest is $\hat{p}_i = p_i p_{p,i}$

For CNE, we have

$$\frac{\partial u_i}{\partial p_i} = 0 \qquad \forall i \tag{9.10}$$

subject to (9.9)

As $\hat{p}_i = p_i p_{p,i}$, $\frac{\partial u_i}{\partial p_i} = 0$ can be written as $p_{p,i} \frac{\partial u_i}{\partial \hat{p}_i} = 0$

Therefore, we have for CNE

$$\frac{\partial u_i}{\partial \hat{p}_i} = 0 \qquad \forall i \tag{9.11}$$

subject to (9.9)

Lemma 9.4: CNE for the approximate case is unique.

Proof: Following the exactly same steps as in Theorem 8.4, we see that solution in terms of $\hat{p}_i$ is unique and noting that $\hat{p}_i = p_i p_{p,i}$.

■

In this Chapter it was shown that the results for the complete information Access Game can be easily extended to the incomplete information Access Game. Although these results are approximate in nature, with good historical data, reasonable estimates can be made for $p_{p,i}$. Consequently, we can design a realistic medium access protocol as proposed in this paper.

CHAPTER TEN: STABILIZATION

In the previous Chapters (Chapter Five-Chapter Eight), the CNE was proposed as a solution to the problem of fairness in the Access Game. The CNE is computed by satisfying the fairness conditions. Therefore, the CNE satisfies our primary objective of fair bandwidth sharing. However, the CNE is not self-enforcing in nature. It is based on a mutual agreement amongst the users and each user should optimize its utility function with the fairness conditions as the constraints. In case one or more than one users deviate from the mutual agreement for selfish reasons, there is no enforcing mechanism in the system to force the deviating users back to the point of mutual agreement i.e. the CNE. This is a drawback of the CNE as a solution concept. In this Chapter, we provide two mechanisms to provide an effective solution to this problem. For the rest of this Chapter, the scenario of interest is the general scenario considered in Chapter Eight.

Before providing the mechanisms, we briefly discuss the solution strategy. We have shown in Theorem 8.1 that the NE of the Access Game, if it exists, is unique. As the NE is unique, it is self-enforcing in nature. Even if some users deviate from the NE strategy, they will revert back to the equilibrium strategy in their own interest. This self-enforcing nature of the NE makes it an attractive and natural solution for the stability problem. However, as noted before the NE does not in general result in fair sharing of bandwidth. Therefore, for the NE to be considered as a viable solution, it needs to satisfy the fairness conditions. In order to achieve this goal, our proposed mechanisms design the system parameters in such a way that the NE actually satisfies the fairness conditions and maximizes the throughput. From Corollary 8.8, we know that there is a unique set of transmission probabilities that satisfies fairness and maximizes throughput. Moreover, this unique set of transmission probabilities p_i's are computed from (5.17) and (5.18). For convenience, we reproduce the expression these probabilities.

$$p_i = \frac{w_i}{K + w_i} \quad \forall i \tag{10.1}$$

where, K is computed from $\sum_{i=1}^{n} \frac{w_i}{K + w_i} = 1$.

In the remainder of this Chapter, we propose two schemes so that the NE of the Access Game corresponds to the set of transmission probabilities as described in (10.1). Before presenting the results, we provide a short discussion on these schemes.

The first approach involves choosing the weightages of the users suitably. The probabilities in (10.1) are dependent solely on the weightages of the users whereas the NE is dependent solely on the payoff ratios. As payoff ratios are intrinsic to the use users, there is little scope for modifying the payoff ratios. Weightages on the other hand, can be assigned to the users based on their resource requirement. We assign the weightages so that the NE corresponds to (10.1). The second approach is based on punishing the deviating users. Under this scheme, if a user i transmits with a probability higher than the p_i as in (10.1), the system takes a punitive action against the user. The idea is to force the user to transmit with p_i as in (10.1).

We now provide our analysis based on these two approaches.

10.1 Computing Weightages

In this Section, we ask the following question: given a set of payoff ratios $(r_1 ... r_n)$, how should the weightages $(w_1 ... w_n)$ be chosen so that the NE of the Access Game satisfies fairness and maximize the throughput.

Our objective is to design the NE of the general Access Game in such a way that the NE corresponds to the transmission probabilities described in (10.1). For convenience, we reproduce the formula for NE in the general scenario from (8.11)

$$p_i^{NE} = 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} \qquad \forall i \tag{10.2}$$

From (10.2), it is apparent that the Nash Equilibrium is dependent on the payoff ratios. Therefore, it can be argued that by choosing the payoff ratios properly the Nash Equilibrium can have the desirable properties. However, the payoff ratios are strictly user specified and can not modified easily. On the other hand, the transmission probabilities in (10.1) depend solely on the weightages and these weightages can be chosen accordingly so that the transmission probabilities as computed from the NE satisfy fairness and maximize throughput subject to satisfying fairness.

Mathematically, this translates to the following:

$$p_i^{NE} = 1 - \frac{[\prod_{j=1}^{n} r_j]^{\frac{1}{(n-1)}}}{r_i} = \frac{w_i}{K + w_i} \qquad \forall i \text{ where, } K \text{ is computed from } \sum_{i=1}^{n} \frac{w_i}{K + w_i} = 1$$

We have the following result.

Proposition 10.1 If $w_i = \left. \left(r_i - \frac{n-1}{\sum_{j=1}^{n} 1/r_j} \right) \middle/ \left(r_1 - \frac{n-1}{\sum_{j=1}^{n} 1/r_j} \right) \right.$ $\forall i \neq 1$ exists and $w_1 = 1$, then NE as given (8.11) satisfies fairness and maximizes the throughput.

Proof: For the Nash Equilibrium, we have from (6.9),

$$r_i = \prod_{j \neq i}^{n} (1 - p_j^{NE})$$

$$\therefore r_i (1 - p_i^{NE}) = r_j (1 - p_j^{NE}) \quad \forall i, j$$

Let

$$r_i (1 - p_i^{NE}) = \lambda \tag{10.3}$$

Now we have,

$$\sum_{j=1}^{n} p_j^{NE} = 1$$

$$or, \sum_{j=1}^{n} 1 - p_j^{NE} = n - 1$$

Using (10.3),

$$\lambda = \frac{n-1}{\sum_{j=1}^{n} 1/r_j} \tag{10.4}$$

Using (10.3) and (10.4) together,

$$\therefore r_i (1 - p_i^{NE}) = \lambda$$

$$or, (1 - p_i^{NE}) = \frac{\lambda}{r_i}$$

$$\therefore\ p_i^{NE} = 1 - \frac{\lambda}{r_i} = 1 - \frac{1}{r_i} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j} \qquad (10.5)$$

On the other hand, we have

$$p_i^{NE} = \frac{w_i}{K + w_i} \qquad (10.6)$$

$$\Rightarrow w_i = \frac{Kp_i^{NE}}{1 - p_i^{NE}}$$

Without the loss of any generality, we can consider $w_1 = 1$. Therefore,

$$p_1^{NE} = \frac{1}{K+1} \Rightarrow K = \frac{1 - p_1^{NE}}{p_1^{NE}}.$$

Using (10.6),

$$K = \frac{\frac{1}{r_1} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}}{1 - \frac{1}{r_1} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}} \qquad (10.7)$$

Using (10.5), (10.6), and (10.7) we have:

$$\therefore\ w_i = \frac{\frac{1}{r_1} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}}{1 - \frac{1}{r_1} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}} \frac{1 - \frac{1}{r_i} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}}{\frac{1}{r_i} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}} = \frac{r_i[1 - \frac{1}{r_i} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}]}{r_1[1 - \frac{1}{r_1} \times \frac{n-1}{\sum_{j=1}^{n} 1/r_j}]} = \frac{r_i - \frac{n-1}{\sum_{j=1}^{n} 1/r_j}}{r_1 - \frac{n-1}{\sum_{j=1}^{n} 1/r_j}}$$

$$\therefore w_i = \frac{r_i - \dfrac{n-1}{\sum_{j=1}^{n} 1/r_j}}{r_1 - \dfrac{n-1}{\sum_{j=1}^{n} 1/r_j}} \qquad \forall i \neq 1 \tag{10.8}$$

∎

The weightages given by Proposition 10.1 ensure that the Nash Equilibrium given by (8.11) or (10.2) will result in fairness and maximize the throughput. This precludes the necessity for CNE. Moreover, as the NE is unique and self-enforcing, no user benefits by deviating from equilibrium point. Therefore, the equilibrium point is stable. However, the drawback with this approach is the existence problem of the weightages as evidenced from (10.8). In cases where the existence condition is satisfied this approach is recommended.

We now consider a second approach, designated as the punishment model for achieving the same goal.

10.2 Punishment Model

The previous mechanism proposed choosing proper weightages to achieve the appropriate NE. However, the existence of the suitable weightages is not guaranteed. Therefore, we propose another mechanism in this sub-section so that the NE of the Access Game coincides with the desirable transmission strategies as in (10.1).

In order to do so, we adopt a punishment model whereby if the transmission probability of users is more than their optimal transmission probability, some punishment is meted out to the users. Let the optimal transmission probabilities be $(p_1^* ... p_n^*)$. The utility functions are now modified as follows:

$$u_i = p_i \times \prod_{j \neq i}^{n} (1 - p_j) \qquad p_i < p_i^* \tag{10.9a}$$

$$u_i = p_i \times \prod_{j \neq i}^{n} (1 - p_j) - k_i \times (p_i - p_i^*) \qquad p_i \geq p_i^* \tag{10.9b}$$

k_i denotes the cost of transmitting at a higher probability. Using this modifies utility functions, we have the following result.

Proposition 10.2 Using (10.9), a sufficient condition for $(p_1^* ... p_n^*)$ *to be the NE of the modified Access Game is that* $k_i > 1 \quad \forall i$

Proof: In the modified game, user i can be thought to have three possible actions:

1. transmitting with a lower probability than p_i^*, designated as action "1"
2. transmitting at p_i^*, designated as action "2"
3. transmitting with a higher probability than p_i^*, designated as action "3"

$$u_i(2, a_{-i}) - u_i(1, a_{-i}) = (p_i^* - p_i) \times \prod_{j \neq i}^{n} (1 - p_j) > 0$$

$$u_i(2, a_{-i}) - u_i(3, a_{-i}) = (p_i^* - p_i) \times \prod_{j \neq i}^{n} (1 - p_j) - k_i (p_i^* - p_i) = (p_i - p_i^*) \times (k_i - \prod_{j \neq i}^{n} (1 - p_j))$$

Clearly, a sufficient condition for $u_i(2, a_{-i}) - u_i(3, a_{-i}) > 0$ is that $k_i > 0$

Therefore, if $k_i > 0 \quad \forall i$

$$u_i(2, a_{-i}) > u_i(3, a_{-i}), u_i(3, a_{-i}) \qquad \forall i$$

Hence, action “2” for all users is the unique pure strategy Nash Equilibrium for the Game considered. Therefore, $(p_1^* ... p_n^*)$ is the unique Nash Equilibrium of the game with the modified utility functions as in (10.9).

$\blacksquare$

In this approach, there is a cost associated with transmitting at a higher transmission probability. Therefore, it is in the best interest of the users to transmit at the desirable transmission probability. The question is how the punishment should be administered. It is plausible that the registration authority (RA) will monitor the activities of the users and will charge the deviating users if they transmit at a higher transmission probability. However, the intervention by the RA makes the system dependent on centralized intervention; thus violating the distributed property of system to certain extent.

CHAPTER ELEVEN: CONCLUSIONS

In this dissertation, our primary objective was to devise a transmission strategy for distributed MAC scenarios such that users get their fair share of bandwidth. Our approach to this problem is based on the following constructs. First, we propose that the users should have autonomy over choosing their own transmission strategies based on the state of the network. Secondly, we model the medium access with independent and selfish users as a non-cooperative game. We designate this game as the Access Game. Therefore, the fairness problem is mapped to a non-cooperative game problem and the solution of the game is analyzed for its fairness characteristics. In this dissertation, we have computed and discussed two solution concepts for the Access Game.

Theses solutions are Nash Equilibrium and Constrained Nash Equilibrium. We prove that both these solutions are unique for the Access Game. The advantage of the Constrained Nash Equilibrium is that the solution is guaranteed to satisfy the fairness conditions. However, the Constrained Nash Equilibrium has some stability problems. On the other hand, the Nash Equilibrium is stable in nature but does not guarantee fairness. Therefore, we propose two techniques that result in a Nash Equilibrium that satisfies fairness and optimizes the system throughput. One of these techniques involves choosing proper weightages for the user and the other uses a punishment model.

In Chapter Three we model the distributed medium access as a non-cooperative game and designated the game as the Access Game. It is proposed that all the users use *p-CSMA* type MAC protocol for medium access. For analytical tractability, we assumed the game to be a complete information game. We also assumed that the umber of users playing the game remains and unchanged and each user has a packet to transmit at the beginning of each transmission slot. Later in Chapter Nine, we relax all of these assumptions. Each user receives a payoff after playing the game. We have argued that the payoff has two components, Quality of Service (QoS) and Battery Power (BP). The analysis presented in Chapters following Chapter Three, it is seen that the relative importance of QoS and BP plays an important role in the computation of the equilibrium for the Access Game.

Before presenting our main body of work for the complete information Access Game in Chapters Five-Eight, we provide an analysis of incomplete information Access Game in Chapter Four. We compute equilibrium access strategies for simple cases and compare these strategies with the corresponding complete information access strategies. It is observed that the access strategies of complete information game give users a higher chance of success. This is due to the fact that if the users have complete information about the other users, they can a make a more efficient choice.

In Chapter Five, we analyze the special case of the wired networks. Wired networks are special because the BP component of the payoff vanishes in this case. The analysis shows that there is an infinite number of Nash Equilibrium for the wired networks. However, all these Nash Equilibriums are inefficient in nature. Therefore, we propose the Constrained Nash Equilibrium as a solution concept. It is shown that the Constrained Nash Equilibrium in this case in unique. This Constrained Nash Equilibrium satisfies fairness and maximizes the system throughput.

In Chapter Six we show that for the existence of Nash Equilibrium in the Access Game, the payoff ratios of users should not be greater than “1” or less than “0”. In Chapter Seven, we analyze two special Cases. In Case1, the payoff ratio of all the users is “0” and in Case 2, the payoff ratio of all the users is “1”. The general case where the payoff ratio lies between “0’ and “1” is analyzed in Chapter Eight and we show that the Nash Equilibrium for this case is unique. However, certain existence conditions should be satisfied for the existence of the Nash Equilibrium. Moreover, the Nash Equilibrium does not satisfy the fairness conditions in general. Therefore, we compute the Constrained Nash Equilibrium for the general case also and show that the Constrained Nash Equilibrium is unique for the general case.

In Chapter Nine, we present an approximation scheme so that the assumptions made for the previous analysis can be relaxed. A central Registration Authority plays a key role in the approximation scheme. Consequently, more realistic scenarios can also be analyzed through the techniques presented in this dissertation. Finally, in Chapter Ten we address the stabilization concerns arising out of the solutions computed in the previous Chapters.

Our contributions can be summarized as follows. We have proposed and analyzed a novel framework for medium access, where each user chooses its transmission strategy to maximize its utility functions. We have shown that it is possible to achieve through a non-cooperative fashion. This is achieved explicitly in Constrained Nash Equilibrium. Moreover, we have shown that by designing the system parameters suitably, fairness can be achieved through the Nash Equilibrium also.

Finally, we note the following area of future research. Let us consider that the players are playing the Access Game at some mutually beneficial equilibrium point. What happens if some of the users start cheating i.e. deviate from the equilibrium point? Can some optimal control strategy be adopted by the other users such that the Access Game reverts back to the original equilibrium point? This question presents an interesting situation and we suggest that a discrete-time system dynamics approach be used to solve this problem.

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