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THREE ESSAYS ON THE MARKETING STRATEGIES OF A DURABLE GOODS MANUFACTURER

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in the Department of Marketing
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at the University of Central Florida
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ABSTRACT

When purchasing durable goods, consumers not only pay for current but also future consumption; consequently, forward looking behavior is an important consideration in durable goods markets. For example, anticipating that prices will go down in the future, consumers may delay the purchase today; such behavior has a significant impact on the firm's marketing strategies. This dissertation investigates the impact of durability on two marketing strategies: new product introductions and supply chain design.

The first part of this dissertation (Chapter 3) examines a durable goods manufacturer's new product introduction strategy under different market environments where network effects and product compatibility are important. More specifically, this part explores the incentives of a firm to use either a replacement strategy or a skipping strategy—in the former, the firm commercializes the existing technology, while in the latter, it does not; in either case, an improved technology will be available in the future and the firm will introduce a new product at that time. Using a two-period analytical model with network effects, the analysis shows how the level of improvement in the new product, along with the type of compatibility between the products, interacts with network strength to determine the manufacturer's optimal strategy. Under gradual new product improvement, there is a strict preference for replacement. In contrast, under rapid new product improvement, that preference only holds in markets with relatively high levels of the network strength; at lower levels of the network strength, skipping is preferred; interestingly, for moderate values of the network strength, the level of product improvement affects the manufacturer's optimal choice differently under varying types of compatibility.

The second part of this dissertation (Chapters 4 and 5) focuses on the supply chain design decisions of a durable goods manufacturer who is a sole supplier of an essential proprietary component for making the end product. Three different supply chain structures

are considered. In the first, the manufacturer operates as a "component supplier" and sells the component to a downstream firm who then makes the end product. In the second structure, the manufacturer produces the end product using its component but does not make that component available to any other firms; here, the manufacturer operates as a "sole entrant". Finally, the manufacturer can operate as a "dual distributor" who not only makes the end product using its own component, but sells the component to a downstream firm who then competes against the manufacturer in the end product market.

The extant literature on the optimal choice among the above supply chain structures has focused mainly on static settings in a framework of price competition. By contrast, researchers predominantly use quantity competition to examine durable goods markets in dynamic (i.e., multiple time period) settings. Moreover, the literature notes diversity in optimal firm behavior under the two types of (i.e., price and quantity) competition. Therefore, to transition from supply chain design in a static setting to a more dynamic one where consumers are forward-looking, this part utilizes Chapter 4 to analyze the manufacturer's choice using quantity competition in a static setting. This analysis (in Chapter 4) identifies precisely the shift in the manufacturer's choice of supply chain structure when moving from price competition to a quantity competition framework.

With that analysis as a benchmark, the next chapter focuses on the manufacturer's choice in a dynamic setting. More specifically, Chapter 5 investigates the impact of durability on the optimality of the supply chain structures identified above. Using a two period setting, the analysis explores how the manufacturer's preference for different supply chain structures is modified. The findings reveal that, e.g., when durability is taken into account, the manufacturer's preference for the sole entrant role goes up, while the preference for the component supplier role goes down. Further, under certain conditions, the manufacturer may opt to be a dual distributor in the first period and then choose to become only a component supplier in the second period. The underlying rationale for such shifts in preference is directly linked to durability, which creates future competition and substantially reduces the manufacturer's profitability in the long run. Interestingly, this negative impact varies across different supply chain structures.

Overall, this dissertation contributes to the current literature on durable goods and enhances our understanding of the impact of durability on the optimality of distinct marketing strategies, and provides insights that are valuable to both academics and managers.

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CHAPTER 1: INTRODUCTION

Durable goods such as automobiles, aircraft, and consumer electronics are consumed over a long period of time and frequently require sizable investment from both consumers and producers. They also constitute a significant part of the economy, with annual personal consumption expenditures exceeding $1 trillion (The Bureau of Economic Analysis). In particular, since durable goods are often big-ticket items, when purchasing them, consumers take into account not only current but also future consumption. This forward looking behavior has been shown in the literature to be detrimental to the manufacturer's profitability. Therefore, understanding the behavior of consumers and developing suitable marketing strategies are critical for a durable goods manufacturer. This dissertation investigates the impact of durability on two marketing strategies: new product introductions and supply chain design.

The first part of this dissertation (Chapter 3) examines a durable goods manufacturer's new product introduction strategy under different market environments where network effects and product compatibility are important. More specifically, this part explores the incentives of a firm to use either a replacement strategy or a skipping strategy—in the former, the firm commercializes the existing technology, while in the latter, it does not; in either case, an improved technology will be available in the future and the firm will introduce a new product at that time.

The extant literature suggests that when the level of improvement in the newer product is relatively high, the durable goods manufacturer is better off not introducing the older product (Choi 1994, Purohit 1994); the rationale is that the presence of the older product suppresses the margin and subsequently lowers the profit from the newer product. Interestingly, when the durable product has a network effect associated with it as in the smart-phone and tablet-computer categories, research shows that the firm's price can go up over time as the size of the network builds up (see e.g., Liu and Chintagunta 2009). Consequently, when network effects are significant, the impact on the total profit may not be substantial and the manufacturer's incentive to not introduce the older version of the product may be correspondingly attenuated. Further, the products made by a firm using the newer technology are often not fully compatible with that firm's older products; because distinct types of compatibility will generate different network effects, they are likely to influence the manufacturer's preferences differently. In light of this, there is a need for researchers to reevaluate the durable goods manufacturer's product introduction decisions while accounting for the type of compatibility and the associated network effect.

Chapter 3 of this dissertation takes a step towards bridging the above gap in the literature and tackles the following research questions while considering network effects: (1) When is it optimal for a firm to not commercialize an existing technology, given that it will introduce an improved product in the future? (2) What is the role of product compatibility in shaping the firm's preferences in this context?

When the firm follows a replacement strategy, profit accrues from both the older and the newer products. This contrasts with the skipping strategy, where only the newer product contributes to the profit stream. Skipping, however, can help attenuate any effects of intra-brand competition that arise inter-temporally. For instance, in the absence of any network effect, when the older product is not very different from the newer product (as in the case of gradual improvement), conventional wisdom may suggest that there is likely to be more intense intra-brand competition; and consequently, skipping may be preferred. Our goal is to explore whether such intuition is valid and gain a better understanding of how the firm optimally resolves the above trade-off under different market settings.

Using a two-period model, our analysis characterizes when the firm will select skipping over replacement for different levels of product improvement (we consider two levels: rapid and gradual improvement) and compatibility between the new and old products (we consider three types: full, forward and backward compatibility). We find that the relative preference for skipping and replacement indeed depends on the level of product improvement and the type of compatibility. Under gradual improvement, for instance, our finding is contrary to the conventional wisdom suggested above. When consumers are heterogeneous in their valuation for the product, if the difference in the two products is not too large (i.e., as in the case of gradual improvement), then the segment with the relatively higher valuation has less of an incentive to wait for the newer product. Consequently, if the older product were available, such consumers will purchase it. Given the behavior of this higher valuation segment, the firm can charge a relatively higher price for the older product (if it were introduced). That gain in price more than compensates the firm for any loss accruing in the profit from the newer product (due to intra-brand competition). Therefore, under gradual improvement, replacement dominates skipping.

By contrast, if the difference in the two products is relatively large (i.e., as in the case of rapid improvement) and the future is particularly valuable (i.e., the discount factor is relatively high), then the higher valuation segment has an incentive to wait for the new product; consequently, the firm now has an incentive to select skipping to extract a larger surplus from this segment via the new product. In this setting, we characterize the precise conditions when skipping may be preferred to replacement.

We find that product compatibility—by influencing the relative attractiveness of the two products—affects the level of intra-brand competition between the older and the newer products. Under full compatibility, for example, adopters of the older product are able to enjoy the network benefit sooner (since they join the market earlier). When the strength of the network is relatively large, there is a significant raise in the desire to consume the older product, which in turn motivates the firm to commercialize it despite the ensuing impact of intra-brand competition. Therefore, under full compatibility and rapid improvement, the firm will follow a replacement strategy at higher values of the network effect; under gradual improvement, the analogous incentive is even stronger.

Under forward compatibility, as noted earlier, consumers appreciate the older product more; this puts a downward pressure on the newer product's price and adoption—essentially, it weakens the advantage arising from the level of improvement in the newer product. Under rapid improvement, at higher levels of the network effect, the downward pressure affects the firm's profitability significantly and the firm is better off not commercializing the older product.

Under backward compatibility, recall that the newer product's value to consumers is enhanced by the network size of the older product. By the same token, however, consumers are less inclined to buy the older product. Therefore, in order to accrue the benefit from backward compatibility of the newer product, the firm has to offer an incentive to consumers to induce enough purchases of the older product. Such an inducement can prove too costly to the firm, depending on the magnitude of the network effect and the relative improvement level across the two products. Consequently, if the older product were introduced, the firm's profitability under backward compatibility (vis a vis full compatibility) can either be higher or lower depending on the parametric regime. This contrasts with the corresponding shift under the forward compatibility setting noted earlier. In the above sense, our analysis highlights the role of product improvement and compatibility on the product strategies of durable goods manufacturers.

Next, the second part of this dissertation focuses on the supply chain design decisions of a durable goods manufacturer who is a sole supplier of an essential proprietary component for making the end product. Three different supply chain structures are considered. In the first, the manufacturer operates as a "component supplier" and sells the component to a downstream firm who then makes the end product. In the second structure, the manufacturer produces the end product using its component but does not make that component available to any other firms; here, the manufacturer operates as a "sole entrant". Finally, the manufacturer can operate as a "dual distributor" who not only makes the end product using its own component, but sells the component to a downstream firm who then competes against the manufacturer in the end product market.

In practice, we observe different supply chain structures utilized in durable goods markets. Firms such as Intel, Bosch Automotive Group, ARM Holdings, Dolby follow a component supplier structure. A sole entrant structure is employed by other firms like Bose Corporation, and Apple. Finally, firms may follow a dual structure as in the case of Cannon which produces the laser printer using its print engine and supplies that engine to its competitor in the end market HP; or Sony provides its Trinitron TV picture tube to its competitor Toshiba.

Given different structures observed in different industries, extant literature has investigated their optimality in the context of static spacial competition (e.g. Venkatesh et al. 2006, Xu et al. 2010). However, little effort has been dedicated to incorporate the impact of durability on the choice of different supply chain structures despite its detrimental effect on the firm's profitability as noticed by the literature. Chapters 4 and 5 take a step towards examining that impact.

More specifically, we notice that most durable goods are made via significant time consuming production processes. Hence, there is a friction in changing product quantities, i.e., companies are capacity constrained when selecting prices. For this reason, the majority of work in the durable goods literature takes quantity as an important decision variable (e.g., Bulow 1982, 1986, Purohit and Staelin 1994, Purohit 1995, Desai et al. 2004). However, the extant literature on the optimal choice among supply chain structures has focused mainly on price competition (e.g. Venkatesh et al. 2006, Xu et al. 2010). Further, there is a diversity in optimal firm behavior under price and quantity competition (Singh and Vives 1984, Arya et al. 2008c). Consequently, the purposes of Chapter 4 are: (1) to compare and contrast the optimal supply chain design under price vis-a-vis quantity competition; (2) to link the above comparison with the findings from the existing work; (3) to provide a useful benchmark for the design of supply chain in a dynamic setting in Chapter 5.

Extant literature (e.g., Venkatesh et al. 2006, Xu et al. 2010) shows that when the proprietary component manufacturer is a sole entrant, it could avoid the effect of double marginalization but suffers from a cost disadvantage due to lack of expertise in producing and selling the end products (Xu et al. 2010). Alternatively, a component supplier can leverage the expertise of the downstream firm in producing and marketing the end product but its market coverage is restricted due to double marginalization. Finally, the dual structure seems to lower the effect of double marginalization because the end market is more competitive (and has higher coverage); however, channel conflicts may dampen this advantage because the manufacturer has to reconcile between wholesale and end product profits.

Our analysis in Chapter 4 shows that the relative preference for different structures depends on the level of cost disadvantage and the level of product differentiation between the end products of the manufacturer and the downstream firm. More importantly, these preferences change from price to quantity competition, especially between the component supplier role and the dual distributor role. Compared to price competition, the firm's profitability is less disruptive under quantity competition in the following sense: when the firm incurs a higher level of cost disadvantage in producing the end product, there is a steep drop in profitability as the firm transitions to a component supplier structure from a dual distribution structure under price competition. By contrast, under quantity competition, the transition is more gradual. Interestingly, this feature allows the firm to gain higher profits in a certain parametric range of cost disadvantage under quantity competition.

The rationale of the findings is as follows. Under the dual distributor role, managing the wholesale and end product profits forces the manufacturer to lower its component's price offered to the downstream firm (compared to a component supplier structure). This effect is more detrimental under quantity competition; consequently, the dual distributor obtains lower profits here, than under price competition. However, the higher profitability of the dual distribution under price competition goes with an inability to sustain the structure at higher levels of cost disadvantage. In other words, a dual structure can arise for a range of the cost disadvantage under quantity competition but not under price competition; in that range, the quantity competition setting gives the manufacturer higher profits.

Next, we link our results to the existing work by conducting the analysis using their setting but employing quantity competition. We show how our results fit into the context of their models. This exercise helps us reconcile the differences in moving from a horizontal differentiation to a vertical differentiation setting in Chapter 5.

In Chapter 5, we address the research question on how durability impacts the long-term profitability of different supply chain structures by building a two-period analytical model in which the manufacturer selects among being: (a) a sole entrant, (b) a component supplier, and (c) a dual distributor. This two-period setup helps capture the durable nature of the product (i.e., the future competition created by durability). In particular, a new product in the first period becomes a used product in the second period; hence, durability is captured through the valuation of the used product in the second period. If a product is a nondurable, the used product has no value; if the product does not depreciate, the used product's valuation is the same as that of the new product sold in period 2. The competition between the used product and the new product in period 2 is captured via a perfectly competitive secondary market. It is worth noting that when the product in this setting has zero durability, or when the future is not valuable (i.e., the discount factor is zero), the analysis collapses to the one discussed in Chapter 4.

We examine the impact of durability on the profitability of different supply chain structures by conducting comparative statics analysis on key decisions. Our analysis reveals that the optimality of each of the alternatives is changed significantly (from the results in Chapter 4) when the product's durability is taken into account. More specifically, the sole entrant role becomes more preferred to a dual distributor role, while the component supplier role becomes less preferred. Further, under certain conditions, the manufacturer may opt to be a dual distributor in the first period and then become a component supplier in the second period. These results help explain, for instance, why certain companies like Apple may be better off embracing a sole entrant structure in the long-run.

Overall, this dissertation contributes to the current literature on durable goods and enhances our understanding of the impact of durability on the optimality of distinct marketing strategies, and provides insights that are valuable to both academics and managers.

CHAPTER 2: LITERATURE REVIEW

This dissertation generally builds upon the prior literature on durable goods. We explore two major marketing strategies in such markets: new product introductions and supply chain design. The first section will review general issues in durable goods markets, followed by the literature that addresses the topics related to the interest of this dissertation.

The literature on durable goods (see Waldman 2003 for an overview) has highlighted the impact of product durability on the firm's profitability. In a seminal paper published in 1972, Coase conjectured that a durable goods monopolist could lose its market power since the price would go down over time. The rationale is as follows: After serving consumers in a given time period, the monopolist will have an incentive to lower the price of the product to capture the residual demand; anticipating this, consumers will rationally delay their purchase. Successive application of this logic propels the price to the firm's marginal-cost level.

This conjecture is formalized and elaborated in Stokey (1981) using both continuous- and discrete-time models. With the former model, Stokey demonstrates that Coase's conjecture holds in a perfect rational-expectations equilibrium (i.e., consumer's expectations are fulfilled at all times). With the discrete-time version, Stokey shows how the length of the discrete period affects prices (with prices approaching the competitive level as the duration of the period becomes smaller). Using a discrete two-period framework, Bulow (1982) proposes how the monopolist can improve its profit by implementing a leasing policy (also see Purohit 1995, Bhaskaran and Gilbert 2005, Chien and Chu 2008 who examine when such a preference for leasing may be modified). Subsequent work identifies further conditions under which the Coase conjecture may not hold: discrete demand (Bagnoli et al. 1989), decreasing return to scale (Kahn 1986), product depreciation (Bond and Samuelson 1984), and planned obsolescence (Bulow 1986).

2.1 Part I: New Product Introductions and Network Effects

The implications of durability are apparently broader than just the pricing trajectory proposed in Coase (1972). More specifically, as highlighted by Waldman (2003), research on new product introductions in durable goods markets has yielded valuable insights (e.g., Levinthal and Purohit 1989, Purohit 1994, Dhebar 1994, Fudenberg and Tirole 1998, Kornish 2001). Levinthal and Purohit (1989) examine different production strategies (e.g., separate production, joint production, and buy-backs) when the firm introduces an improved product and technological progress leads to the phenomenon of product obsolescence. With a separate production strategy, the innovating firm stops producing the current product when the improved version arrives. Joint production, on the other hand, maintains a product line after the new product launch. Finally, the firm can buy-back the current product when introducing a new one. The authors find that the innovating firm is better-off replacing the current product with an improved version when the improvement level is moderate. By contrast, the innovating firm will follow a buy-back strategy when the improvement level is large. Purohit (1994) extends this framework by endogenizing the innovation level and accommodating entry by a clone into the market. Purohit shows that a product replacement strategy (i.e., separate production) dominates a line extension strategy (i.e., joint production) whenever the firm introduces a new product. This is because maintaining a product line hurts the firm's profit more than the incremental contribution from the old product's sales.

Dhebar (1994) uses an individual level model to investigate the impact of product improvement levels on the firm's ability to commit to future prices. He shows that a relatively high level of product improvement—referred to as rapid improvement (as opposed to gradual improvement)—may lead to disequilibrium outcomes where some consumers feel regret about their purchase. Said differently, under rapid improvement, the firm cannot credibly commit to future prices. The implication is that consumers disfavor rapid improvements that make earlier purchase obsolete; and companies should be cautious when launching such rapidly improved new products. Nevertheless, delaying such commercialization may not be an option for companies at least in some industries (see Ramachandran and Krishnan 2008). Further, Kornish (2001) generalizes the consumer utility function used in Dhebar (1994) and shows that a durable goods monopolist can commit credibly even under rapid improvement. In her model, the firm is not allowed to offer an upgrade price for the new product; consumers, however, can make repeat purchases over time. More recent work (e.g., Ramachandran and Krishnan 2008, Bala and Carr 2009) elaborates on the firm's upgrade policy by considering the roles of product architecture and upgrade costs. It is worth noting that this stream of work does not consider the existence of a secondary market.

Fudenberg and Tirole (1998) focus on pricing the new and improved product when the firm's knowledge of consumer purchase history varies: (1) anonymous consumers (i.e., the firm does not know purchase history) in the presence of a secondary market, (2) no secondary market and identified consumers, and (3) no secondary market and semi-anonymous consumers (i.e., consumers can prove their purchase history if they wish to do so). In (1), they examine pricing under different product policies such as being inactive (i.e., replacement), buybacks, and a product line and show the optimality of these policies under different parametric regimes. In (2) and (3), under gradual improvement, an upgrade price is offered to repeat purchase consumers.

Another important area relates to the excessive R&D investment phenomenon (e.g., Waldman 1996, Nahm 2004) or the excessive number of product upgrades phenomenon in durable goods markets with network effects (e.,g., Ellison and Fudenberg 2000, Sankaranarayanan 2007). This stream of research reveals that a durable good monopolist may face a commitment problem with respect to introducing improved products under certain circumstances. More specifically, a monopolist may be better-off committing to limited R&D or a fewer number of product upgrades. The rationale is similar to the aforementioned Coase's conjecture; that is, the firm has too high of an incentive to introduce new products in later periods that lowers earlier products' value in consumers' eyes, leading to a lower overall profitability.

Research on network effects has recently drawn considerable attention among researchers (see Liu and Chintagunta 2009, Birke 2009, Shy 2011 for detailed reviews). Since several products—such as video game consoles, and software—have a network effect associated with them, it is natural to study the impact of the network in a durable goods framework. In one of the early research efforts, Katz and Shapiro (1985)(KS) conclude that consumer expectations play a crucial role when competing products are incompatible—in the sense that multiple equilibria can arise and if consumers believe a firm to be dominant, then that firm, in fact, will be the winner in equilibrium. KS also argue that firms with a relatively large customer base tend to be against fostering product compatibility; by contrast, firms with smaller networks support product compatibility. Their argument highlights a general trade-off that firms in network industries face: on the one hand, making compatible products raises the products' value to the consumer; on the other hand, incompatibility may increase a firm's monopoly power.

Later work, however, lends more support to the focal firm inviting compatible entry (e.g., Xie and Sirbu 1995, Economides 1996, Baake and Boom 2001, Sun et al. 2004). Xie and Sirbu (1995), for instance, show that an incumbent may be better off if it facilitates entry in the early stages of the diffusion process. Economides (1996) obtains an analogous result using a static model in which the leader firm has an incentive to invite compatible entry at higher levels of the network effect. Such results arise because the incumbent can benefit from a bigger network size; further, the increased competition can signal a higher output level to the consumer. Baake and Boom (2001) conclude that both higher and lower quality firms would agree on compatibility through an adapter, even though the former would prefer an equilibrium without an adapter. In their model, firms choose quality levels prior to selecting compatibility (via consensus). In the absence of compatibility, the lower quality firm selects its quality so that the firms are less differentiated, thus intensifying price competition. Consequently, the higher quality firm prefers the lesser of two evils and agrees on compatibility. More recently, Chen et al. (2009) point out that strategic pricing forces firms to maintain compatibility in the long-run.

Sun et al. (2004) use a static model to characterize the conditions of optimality for various product strategies of an innovator in markets with network effects. The strategies include selling just a single product, licensing, line extension and a combination of licensing and line extension. When there is a strong network effect, they show that the last three strategies mentioned above are all feasible and that the optimal choice depends on the marginal cost of the lower quality product.

Product compatibility is a fundamental issue in markets exhibiting network effects, either for different products of a firm or for the ones from different firms. Apart from the binary options of product compatibility which are usually examined (i.e., compatible vs. incompatible), extant literature alludes to the notion of partial compatibility: "*In reality … compatibility should be regarded as a matter of degree, rather than as an all or nothing decision. Additionally, it is useful to define two kinds of one-way compatibility—backward and forward compatibility …*" (Choi 1994, footnote 7).

Intuitively, backward compatibility helps raise the value of the new technology, whereas, forward compatibility makes the older technology more appreciated. However, it is not obvious how compatibility and technology improvement impact the firm's profitability in a dynamic context when products based on different technologies can be introduced by the same firm. Researchers have been exploring different facets of this issue: Choi (1994), for example, studies a discrete choice of compatibility under new product improvement and shows that incompatibility is desirable when product improvement is relatively large. Ellison and

Fudenberg (2000) and Sankaranarayanan (2007) focus on backward compatibility and do not consider other compatibility settings. Nahm (2008) examines backward compatibility but does not consider the role of product improvement; he finds, for instance, that the profit impact of making hardware backward compatible is always positive, whereas for software the impact depends on the distribution of consumer types.

In Chapter 3 of this dissertation, we look at the choice of introducing a current technology (old) product, given that a new and improved version will be introduced later (i.e., skipping vs. replacement); hence, we complement the literature on both the commitment problem in new product introductions (e.g., Ellison and Fudenberg 2000, Sankaranarayanan 2007), as well as on the rationale for sequential product introduction (e.g., Padmanabhan et al 1997). Sequential innovation happens in our model because of exogenous technological evolution; along with this technological progress, we incorporate different product-compatibility regimes (full-, backward-, and forward-compatibility). Therefore, our work extends the analysis in Purohit (1994) and Choi (1994): Purohit (1994) examines the possibility of skipping but does not focus on network effects; Choi (1994) uses a two-type discrete consumer model with two levels of product compatibility (full- vs. in-compatibility). By contrast, we develop a continuous-type consumer model, consider network effects along with full-, backward- and forward- compatibility. Like Sun et al. (2004), we too look at product strategies, but our model addresses different issues—we study a dynamic setting with different levels of compatibility between the products.

Overall, we draw upon the durable goods framework with disaggregate consumers (e.g., Dhebar 1994, Kornish 2001, Fudenberg and Tirole 1998), as well as the work on network effects and compatibility (e.g., Katz and Shapiro 1985, Choi 1994, Ellison and Fudenberg 2000) to develop our model in Chapter 3.

2.2 Part II: Supply Chain Design

Supply chain design (or channel design) has been a major concern for marketers. Early work focuses on the choice of decentralization vs. centralization in which the firm decides whether to add an intermediary into its channel (e.g., McGuire and Staelin 1983, Coughlan 1985, Moorthy 1988) despite the effect of double marginalization (i.e., the intermediary requires a margin when selling the product to consumers, leading to a higher price). Later work considers more diverse structures (e.g., Choi 1991, Purohit 1995, Purohit and Staelin 1994, Purohit 1997, Desai et al. 2004, Arya and Mittendorf 2006, Bhaskaran and Gilbert 2009). More recently, dual distribution has drawn substantial attention in academia and business, especially with the popularity of e-commerce (see Tsay and Agrawal 2004 for a review). Further, with the advent of international outsourcing, particularly in manufacturing section, firms start to embrace and make it a strategic consideration when designing their supply chain (e.g. Kamien et al. 1989, Shy and Stenbacka 2003, Arya et al. 2008b, 2008c). In what follows, we review each of these research areas.

McGuire and Staelin (1983) consider channel design of duopoly manufacturers with differentiated products under three settings: completely integrated (both do not use an intermediary), completely decentralized (both use their own exclusive intermediary), and mixed (one manufacturer uses an exclusive intermediary while the other does not). It is shown that both manufacturers prefer using an intermediary when the end market is highly competitive. It happens because intense price competition lowers the products' prices, and thus, mitigating double marginalization under decentralization. Coughlan (1985) extends this work with a general demand function and shows support for the above finding via an empirical test using the context of the semiconductor industry.

Later, Moorthy (1988) enhances the understanding of decentralization decisions by examining the combination of demand dependence and strategic dependence. More specifically, demand dependence includes demand complementarity and demand substitutability; strategic dependence includes strategic complementarity and strategic substitutability. For example, with strategic complementarity, a player would reduce his price when the other player does so; one the other hand, with strategic substitutability, a player would reduce his price when the other player increases price. Moorthy shows that decentralization can be preferred under two scenarios. In the first, products are demand substitutes at retail level and strategic complements at the manufacturer or retailer levels. In the second scenario, products are demand complements at the retail level and strategic substitutes at the manufacturer or retailer levels. The implication of this work is that the nature of strategic interaction among firms plays a significant role in determining the optimality of decentralization.

While the above work only considers exclusive retailers under duopoly manufacturers, Choi(1991) examines a different channel structure where there are two competing manufacturers and a common retailer. With this channel, he allows different structures to arise including (i) Manufacturer-Stackelberg, (ii) Vertical Nash, and (iii) Retailer-Stackelberg. He finds, for example, with a linear demand, the manufacturer makes highest profit with (i) while the retailer gets highest profit with (iii). On the other hand, with nonlinear demand, the manufacturer prefers (iii) while the retailer prefers (i). Compared to the exclusive retailers structure of prior work, this channel structure gives the manufacturer lower profit; however, the retailer in his channel obtains higher profit with linear demand and lower profit with nonlinear demand. In general, Choi's work evaluates another alternative channel structure and highlights the important role of the functional form of the demand in determining the optimal channel design.

Next, we review the literature on channel structures with consideration of the durable nature of the product. As mentioned earlier, this consideration implies the Coase's conjecture (i.e., intra-brand competition between current and future sales leads to lower prices over time). Although decentralized channels have been investigated mainly with the focus on the selling vs. leasing policy (e.g., Purohit 1995, Purohit and Staelin 1994, Purohit 1997, Bhaskaran and Gilbert 2009), recently the benefit of decentralization has also been explored (e.g., Desai et. al 2004, Arya and Mittendorf 2006).

Purohit and Staelin (1994) use a two-period model to evaluate three different channel settings: separate, overlapping, and buybacks among a manufacturer and his two retailers called the renter and the dealer. With a separate channel, the renter and the dealer independently make decision on quantities. With an overlapping channel, the renter orders additional cars from the manufacturer and sell the ones (i.e., program cars) bought in period 1 to the secondary market. Finally, the manufacturer buybacks the program cars and resells them to the dealer. Notice that the renter is exogenous in these settings. The paper shows that the dealer's profits in separate and buyback channels are higher than in an overlapping channel. The dealer attains highest sales of new cars in the separate channel, followed by the overlapping and buyback channels. For the manufacturer, the quantities sold are highest in the overlapping channel, followed by the buyback and separate channels. The authors also explore the effect of product substitutability between new and used cars on the profitability of the players. When program cars and new cars are more substitutable, the dealer is better off and the manufacturer is worse off. This is because the manufacturer has to lower its wholesale prices to compensate the dealer's loss in sales due to the increase of product substitutability. The distinct feature of this work is the consideration of retail competition via the existence of the secondary market as time passes. Unlike a static model, here, product durability makes two seemingly (at first) independent channels become competitors when the renter starts to sell his program cars to the secondary market which in turn strengthens competition in the dealer's new cars market.

Purohit (1997) relaxes the assumption of an exogenous rental channel and allows the renter to choose the quantities of new cars to purchase from the manufacturer and on program cars to sell. The paper reveals many interesting results. Firstly, the renter receives the highest wholesale price under the overlapping structure, while the dealer gets the lowest wholesale price. This implies the manufacturer subsidizes the renter when shifting from an overlapping to a buyback channel; and the dealer receives a subsidy under an overlapping option. Secondly, the manufacturer and the renter are better off under the overlapping channel, while the dealer is in the worst situation. This indicates the intense competition under the overlapping channel benefits the former but harms the latter. Further, a separate channel is the least profitable for both the manufacturer and the renter. As a result, this work suggests that the existing buyback channel arrives as a compromise of different players' interest in the automobile industry.

Purohit (1995) considers the choice of selling vs. renting in a decentralized channel. He finds that if the manufacturer is unable to commit to prices in advance, it is better off with selling via a dealer; otherwise, a renter is used to lease the product. Interestingly, a dealer can obtain higher profit than a renter. This conclusion is counter to the typical finding in the durable goods literature which asserts that renting is more profitable than selling. The reason is as follows. When the manufacturer uses a dealer, double marginalization restricts product quantities. Also, the dealer's ability to lower the retail price in the future is constrained by the wholesale price. Consequently, the time inconsistency is mitigated. On the other hand, in the case of renting, time inconsistency is not an issue and the existence of double marginalization lowers channel performance.

Bhaskaran and Gilbert (2009) extend Purohit (1995) by allowing the dealers to choose its own product policy. When the manufacturer chooses to lease the product, dealers have to lease it to customers. Alternatively, when the manufacturer chooses to sell the product, dealers are able to sell, to lease, or to do both. In addition, competition at intermediate level is also taken into account. The authors demonstrate that competitive intensity plays a critical role on selling/leasing decisions. At low and high levels of competition, leasing is the optimal strategy. On the other hand, selling is the optimum at intermediate levels of competition.

Desai et al. (2004) show the benefit of decentralization in durable goods markets from another angle. In particular, without commitment, a decentralized channel can attain the performance of an integrated channel, using two-part tariffs. However, this kind of contract does not solve the Coase problem (i.e., time inconsistency). Subsequently, they suggest a two-part contract with commitment. Interestingly, the result shows that a retailer in a durable goods channel has an incentive to sell a quantity that is too high (proven by the fact that the manufacturer sets wholesale prices higher than his marginal cost in a two-part tariffs contract). This situation contrasts with the incentive to lower the quantity of a typical retailer (by setting higher retail prices). Overall, this work contributes to the literature by showing that decentralization can help soften the Coase problem in durable markets. Later, Arya and Mittendorf (2006) sharpen the benefit of decentralization with a longer product life cycle model. They find that even without commitment, decentralization could provide higher profits than vertical integration. That is because a decentralized channel serves to restrict sales in every period, compared to vertical integration. Though this contains a loss of sales in the first period, it has a commitment benefit in the future (i.e., not 'flooding' the market). It is this benefit that eventually raises the profitability of a decentralized channel for longer time horizons.

While the above channel structures exclude the (horizontally) direct competition between the manufacturer and its own retailer, dual channel triggers such a conflict (see Tsay and Agrawal 2004 for a review). With this structure, the manufacturer sells its end product directly to the consumer and indirectly via an independent intermediary. Moriarty and Moran (1990) observe this structure from many industrial practices as a mean to increase market coverage and reduce costs. Later, Balasubramanian (1998) models the competition phenomenon of the direct marketers (e.g., catalog and Internet marketers) and conventional retailers. Though it only examines the retail competition level (not the whole channel interaction), the paper does show how the direct marketer can navigate the competition in the market via disseminating its product information to the consumers. When information of product from multiple channels are available to all consumers, direct marketers face serious competition from conventional retailers (who otherwise compete against themselves). Hence, by controlling the availability of its product information, the direct marketer may obtain higher profit since retailers will compete against each other for the uninformed consumers.

The literature on dual channels also outlines many benefits of such a structure; such as better reaching a different market segment, benchmarking the performance of an independent intermediary, and influencing the intermediary's behavior (Bell et al. 2002, Coughlan and Soberman 2005, Chiang et al. 2003, Kumar and Ruan 2006, Cattani et al. 2006, Arya et al. 2007). The downside of dual channel includes excessive manufacturer encroachment, and inability to direct traffic in the channel (Vinhas and Anderson 2005, Arya et al. 2008a).

Bell, Wang and Padmanabhan (2002) examine a specific dual channel where the company store and the retailers are located in the same area such as in the same mall. This research explains the benefit of adding a company store aside from market coverage by considering marketing efforts put by different players in the channel to influence the demand of each other. Here, marketing efforts of the company store relax the intensity of price competition and actually benefit the retailers by raising retail prices. In contrast, Coughlan and Soberman (2005) look at a dual channel in which company stores are located far away from the retailer's stores. They show that a dual system is preferred when the service sensitivity across consumers is relatively lower than the price sensitivity. It occurs because the company stores (outlets) attract price-sensitive, no-service-sensitive consumers, while retailers attract high-end consumers. Further, the retailers benefit from the existence of company stores.

Chiang et al. (2003) model a direct online channel parallel with an independent retailer; the authors show how a direct channel helps mitigate the double marginalization problem even if no sales occur in the direct channel. The reason is that the introduction of a direct channel puts a competitive pressure on the retail market, leading to lower retail prices. As a result, the manufacturer compensates the price drop by offering a lower wholesale price. By using the direct channel, the manufacturer is able to increase the channel efficiency. Later, Arya et al. (2007) show when the direct channel is a real 'threat', adding it is still beneficial to both parties, even though products in both direct and indirect channels are identical. This work once again emphasizes the advantage of adding a direct channel: lower wholesale price and increased retail competition help mitigate the double marginalization problem in an indirect channel.

Cattani et al. (2006) examine different equal-pricing strategies to mitigate the conflicts within a dual system. In the first strategy, the manufacturer keeps the same wholesale price (as in without a direct channel) and commit to not undercut the retailer's price. In the second, the manufacturer offers a lower wholesale price. Finally, the manufacturer commits to not undercut the retail price. Surprisingly, the last strategy could be preferred by the manufacturer, the retailer and even the end consumer.

Kumar and Ruan (2006) allow the retailer in the channel to carry a competing product. The retailer sets prices and service levels for his two products. There are two types of consumers in the market: retailer-store loyal and manufacturer-brand loyal; when the direct channel is available, some consumers will buy products directly (here, direct channel is a 'real' threat to the retailer). They compare the proposed dual channel to the one without a direct channel. When it is optimal to follow a dual distribution, having a direct channel allows the manufacturer (1) to price discriminate and enhance profits when the retailer's margin on the competing product is very small, or (2) to increase the level of retail service when the retailer's margin on the competing product is very large.

Despite the aforementioned benefits of a dual system, in reality, retailers commonly oppose the presence of a direct channel (either online or company-owned store). Vinhas and Anderson (2005) test the use of different channel structures using data in B2B market. The authors find that severe channel conflict is unavoidable under certain circumstances and the firm is better off not using the dual channel distribution, such as when the two channels serve the same consumer, or when the customer is able to put the two channels to compete against each other. Furthermore, the paper identifies different conditions favoring dual distribution such as higher growth, greater variability in customers' behavior over purchase occasions, and higher perceived brand differentiation. Most importantly, it is shown that a dual distributor does take effort to coordinate different channels. It does so by increasing product differentiation among different channel offerings, setting rules to mitigate disputes over encroachment, or compensating both channels whenever a sale is made.

When excessive encroachment occurs under dual distribution, Arya et al. (2008a) propose an organizational solution. They show how decentralized control within the direct channel can signal to the indirect channel the incentive to not aggressively encroach retail market. In particular, using an affiliate, the manufacturer has an incentive to charge his affiliate a positive wholesale price, thus restricting the number of products sold by the affiliate. This in turn allows the manufacturer to receive higher wholesale profit from the indirect channel.

Our work is also related to the literature on strategic outsourcing (e.g., Kamien et al. 1989, Shy and Stenbacka 2003, Arya et al. 2008b, 2008c). Our component supplier structure, to some extent, relates to the one discussed in licensing (Katz and Shapiro 1985) and franchising (Lal. 1990, Lafontaine 1992); however, we do not examine the contract settings or any coordination issues. Hence, we exempt from the discussion of this research stream in details here (see Dant et al. 2011 for a review) and focus more on the strategic outsourcing literature.

Kamien et al. (1989) investigate the subcontracting phenomenon found in many industries such as insurance, electronics, and automobiles. In their model, subcontracting reduces costs (since the cost function is strictly convex). By doing so, the firm maintains a smaller capacity. It is showed that the possibility of subcontracting changes the firms' behavior when competing (bidding) for a contract of producing products in the first place. Under the context of Bertrand duopoly, if the loser of the bidding game dictates the terms of the subcontract, then both firms are better off. By contrast, if the winner sets the terms of the subcontract, both firms will receive zero profits. It happens because in the second case, the desire to become the winner in the first place drives profits to zero.

Shy and Stenbacka (2003) show the strategic benefit of outsourcing as well as using a common input supplier when two differentiated manufacturers compete in prices in the end market. First, when both firms outsource, the competitive intensity in the end market is less because outsourcing activities replace fixed costs of producing the input in-house by variable costs (i.e., the wholesale price of the input purchased), which in turn motivates firms to increase their prices. Second, using a common input supplier would exploit economies of scale, and thus increasing social efficiency. The authors also consider horizontal outsourcing (i.e., dual distribution) where one of the manufacturers produces the input in-house and supplies that input to its competitor in the end market. However, this structure is not socially optimal.

Arya et al. (2008b) further analyze the strategic impact of using a common supplier. In their model, the input supplier is a monopolist with constant return to scale production. The incumbent downstream manufacturer could either produce input in-house or outsource the production. With the entry of a downstream competitor who will outsource its production, the incumbent has an incentive to use the supplier's input, even that action may cost him more than in-house production. The rationale is as follows. If the incumbent conducts in-house production, the input supplier may offer the entrant a lower input's price to help it compete with the incumbent. To prevent such a subsidy, the incumbent purchases the input from the supplier in the first place.

The closest work to ours is Venkatesh et al. (2006) and Xu et al. (2010). Using spacial competition, these authors characterize different supply chain structures available to manufacturers of proprietary component brands that possess a proprietary technology of an essential component to make the end product. It is this proprietary nature that allows the manufacturer to dictate how the end market is structured. In particular, the manufacturer could operate as a component supplier and provide its component to a downstream firm who then makes the end product. Alternatively, it could make its own end product as a sole entrant. Finally, it could operate as a co-optor who makes the end product as well as provides the component to a downstream firm who then competes against the co-optor in the end market.

Venkatesh et al. (2006) characterize the optimality of each of the three structures above. It is shown that the quality differential and product substitutability play an important role in determining the optimal structure. In particular, the component supplier role is optimal when the end product of the downstream firm is highly superior to the manufacturer's end product and end products are strong substitutes. The sole entrant role is optimal even when the manufacturer's end product is moderately inferior under the condition that end products are almost perfect substitutes. The optimality of the co-opter role spreads for a wide range of quality differential and product substitutability; e.g., the co-optor role is optimal even under strong substitutability (but not perfect substitutability). Further, the popularity of this role is proven robust in different settings such as when an alternative component available, or when there are two competing downstream firms in the end market under the component supplier role. The reason for this outcome is that the co-optor is able to reduce the competitive intensity in the end market with appropriate pricing. Finally, the implications of the model is testified empirically.

Xu et al. (2010) extend Venkatesh et al. (2006) by considering the impact of different factors such as branding, contracts, product valuation uncertainty on the optimality of the three structures aforementioned. More specifically, they examine the impact of these factors on the basic trade-offs described in Venkatesh et al. (2006): (1) the component supplier structure suffers from double marginalization but the manufacturer can leverage the expertise of the downstream firm in the end market; (2) the monopoly structure (i.e., the sole entrant structure) avoids double marginalization but the manufacturer incurs capability disadvantage when producing the end product; lastly, (3) dual distribution (the cooptor structure) contains higher competition in the end market but the demand for the component is expanded. First, compared to the dual distribution, the monopoly structure is more preferred under component branding, end product branding, and valuation uncertainty; while this structure is less preferred under royalty contract. Second, compared to dual distribution, the component supplier structure is more preferred under end product branding but less under valuation uncertainty. There is no clear cut for the shift of the component supplier structure under component branding and royalty contract.

Focusing on the three structures above, while our model employs quantity competition, the closest work to ours employs price competition (Venkatesh et. al. 2006, Xu et al. 2010). Hence, it is helpful to review the literature that compares the outcomes under price vs. quantity competition. Typically, in a duopoly when products are identical, marginal costs are constant and equal for both firms, then the market price is equal marginal cost under price competition and above it under quantity competition. Hence, price competition is typically thought to be more competitive than quantity competition. Kreps and Scheinkman (1983) show that if production capacity is restricted in advance, the outcome of price competition between firms duplicates the quantity competition outcomes.

In a differentiated duopoly, without capacity constraint, Singh and Vives (1984) demonstrate that compared to quantity competition, price competition yields (1) smaller profits when products are substitutes, (2) larger profits when products are complements, and (3) equal profits when products are independent. It happens because under product substitutability, quantity competition keeps prices high. When products are complements, firms want to maintain higher quantities to reinforce each other's market, hence, a pricing game suits this purpose by offering lower prices.

More recently, Arya et al. (2008c) reverse the above conclusion under dual distribution. Compared to quantity competition, the market prices as well as profits are higher under price competition. Said differently, competition is less intense under price competition. This happens because a dual distributor sets a higher input's price under price competition than under quantity competition. This behavior is derived from the fact that the dual distributor is inclined to set a higher end product's price to protect his wholesale profit, which in turn induces the downstream competitor to increase its price. On the other hand, under quantity competition, the dual distributor does not consider the wholesale profit when setting its end product's quantity. Anticipating this aggressive behavior, the downstream firm orders less input; this forces the dual distributor to offer a much lower wholesale price (vis-a-vis price competition). Consequently, the total profit of a dual distributor is reduced under quantity competition.

Given the diverse outcomes may arise under different types of competition, it is critical to justify the application of one or the other. Extant research suggests that quantity competition is more applicable when there is a friction in changing quantities (Moorthy 1985, Rey and Tirole 2007, Arya and Mittendorf 2011). For example, most durable goods such as automobiles, and electronics are made via significant time consuming production processes. Hence, firms are capacity constrained when marketing the end products. Another example is the wholesale markets, which often require long lead times; and most of the time, orders are made in advance.

The second part of this dissertation examines different supply chain structures in a dynamic context of durable goods. Hence, we complement the literature on channel structures (e.g., Purohit 1995, Desai et al. 2004, Arya and Mittendorf 2006) and strategic outsourcing (e.g. Arya et al. 2008c). Our work extends the analysis in Venkatesh et al. (2006) and Xu et al. (2010) by characterizing the optimal supply chain design of a proprietary component manufacturer in a dynamic setting. While their model is based on horizontal product differentiation with price competition; we develop a vertical differentiation model and focus on quantity competition. Consequently, we devote Chapter 4 of this dissertation to reconcile the changes between price and quantity competition and establish a useful benchmark for a more dynamic model; Chapter 5 then focuses on the dynamic impact of durability on the optimal supply chain design.

It is worth noting that the only work on dual distribution under dynamic context of durable goods markets that we are aware of is Xiong et al. (2012). They consider a dual channel where the manufacturer sells through an online channel and the independent dealer adopts a mix of selling and leasing. They utilize a two-period model to demonstrate the impact of direct channel (i.e., encroachment) on the profitability of both the manufacturer and the dealer in a durable goods context. Here, encroachment happens sequentially in every period; hence, the manufacturer sets his direct sales after observing the dealer's decisions. Compared to a single channel with indirect channel only, the dual structure gives the manufacturer higher profit; while it may or may not be favored by the dealer. Our model is different from theirs in many aspects. We do not allow leasing and sequential encroachment. From Arya et al. (2007), we know that sequential encroachment always gives higher profitability than simultaneous encroachment (which is used in our model). We consider the impact of durability in a continuous scale while the product in their model is completely durable. Finally, we focus more on evaluating the relative strengths of different channel structures rather than dual distribution.

Overall, we draw upon the existing supply chain structures (e.g., Venkatesh et al. 2006, Xu et al. 2010, Arya et al. 2008c) and the durable goods framework with disaggregate consumers (e.g., Desai et al. 2004) to develop our model in Chapters 4 and 5.

CHAPTER 3: SKIPPING VS. REPLACEMENT: THE ROLE OF PRODUCT INNOVATION AND COMPATIBILITY

3.1 Introduction

Rapid technological innovations often serve to accelerate the life-cycle of durable goods, especially in high-tech product categories such as software, smartphones, and tablet computers. Many a time, the products made by a firm using the newer technology are not fully compatible with that firm's older products. When there are network-related benefits that arise from an installed-base of users, such a lack of compatibility can affect consumers' willingness to pay. Therefore, commercializing an existing technology and introducing a product can be a challenging decision for these durable goods firms, particularly when accounting for the value of an installed base of consumers.

Consider the following two examples: (a) Apple introduced the iPad in 2010 and then introduced the improved version, iPad 2, a year later. (b) Nokia announced that its model-7700 smartphone would be introduced in 2003; subsequently, though, retracted that decision and instead introduced model 7710 a year later (see my-symbian.com). While a variety of reasons may help explain the distinct choices of these two durable goods manufacturers, they nevertheless raise questions such as "Would it have been optimal for Nokia to introduce the 7700 and then replace it with the 7710 (as Apple did with the iPad)? Or conversely, optimal for Apple to wait till 2011 to introduce the iPad for the first time?" Such issues will arise in other durable good categories as well, where network effects and compatibility play a role, and help motivate our research questions in this chapter: (1) When is it optimal for the firm to (not) commercialize an existing technology, given that it will introduce an improved product in the future? (2) In that context, what is the role of product compatibility in shaping the firm's preferences?

This chapter takes a step towards addressing these questions by building a two-period analytical model in which the firm chooses whether to commercialize its available technology (by commercializing, we mean that the firm introduces an "old" product) at the beginning of the first period; irrespective of its decision in the first period, the firm will introduce an improved version (referred as the "new" product) at the beginning of the second period. When the firm does not commercialize its old product but only introduces the new product, we say that the firm used a "skipping" strategy[1]; when both products are introduced sequentially, we refer to the strategy as "replacement". Our analysis characterizes when the firm will select skipping over replacement for different levels of product improvement (we consider two levels: *rapid* and *gradual* improvement) and compatibility between the new and old products (we consider three types: *full*, *forward* and *backward* compatibility). This approach helps highlight how our results add to those currently available in the literature (e.g., Purohit 1994, Kornish 2001) on the product strategies of durable goods firms.

A network effect—either direct or indirect—refers to the phenomenon that a given consumer's utility from using a product goes up as the number of consumers using that or other compatible products goes up (see e.g., Liu and Chintagunta 2009). This additional utility essentially raises the amount consumers may be willing to pay for the product. Further, the incremental utility to consumers will likely depend on both the importance of the network as well as the size of the network. Since compatibility affects the effective size of the network (in a manner that we will elaborate below), it affects consumers' incremental utility from the network. In other words, varying levels of compatibility lead to varying levels of network size and consumers' willingness to pay.

[1]In the context of different models, previous research refers to an analogous decision as leapfrogging (e.g., Purohit 1994) or delayed marketing (e.g., Choi 1994). However, leapfrogging is also used in the literature on R&D (e.g. Fudenberg et al. 1983); further, marketing encompasses a wide variety of choices. Therefore, to limit possible confusion, we use this term.

In particular, when a product is backward compatible, all the users of the older product (along with the consumers of the newer product) count towards the installed base of the newer product; however, the newer product users do not count towards the network size of the older product. For example, Microsoft Word 2007 is able to open files created in an earlier version (e.g., Microsoft Word 2003), Blu-ray disc drives are able to play standard DVD discs, and Wii is backward compatible with its earlier system, Nintendo GameCube. With forward compatibility, by contrast, the above roles of the newer and the older products are reversed—that is, the installed base of the newer product does not include the users of the older product, but the newer product users contribute to the network size of the older product. Finally, when the two products are fully compatible, their users contribute fully to each other's installed base.

Intuitively, in the absence of network effects, existing research (e.g., Dhebhar 1994, Kornish 2001) notes that in making a purchase decision, strategic consumers will take into account the level of product improvement between the older and newer versions of the durable product. When there is a network benefit as well, adopters of the older product are likely to accrue some 'extra' value since they join the market relatively early. Furthermore, the discussion in the previous paragraph suggests that, compared to full-compatibility, backward compatibility will likely make the newer product more attractive; by contrast, forward compatibility makes the older product more appreciated. Therefore, in addition to the level of product improvement between successive generations of the product, strategic consumers will account for the impact of compatibility as well.

Notice that when the firm follows a replacement strategy, profit accrues from both the older and the newer products. This contrasts with the skipping strategy, where only the newer product contributes to the profit stream. Skipping, however, can help attenuate any effects of intra-brand competition that arise inter-temporally. For instance, in the absence of any network effect, when the older product is not very different from the newer product (as in the case of gradual improvement), conventional wisdom may suggest that there is likely to be more intense intra-brand competition; and consequently, skipping may be preferred. Our goal is to explore whether such intuition is valid and gain a better understanding of how the firm optimally resolves the above trade-off under different market settings.

Our analysis shows that the relative preference for skipping and replacement indeed depends on the level of product improvement and the type of compatibility. Under gradual improvement, for instance, our finding is contrary to the conventional wisdom suggested above. When consumers are heterogeneous in their valuation for the product, if the difference in the two products is not too large (i.e., as in the case of gradual improvement), then the segment with the relatively higher valuation has less of an incentive to wait for the newer product. Consequently, if the older product were available, such consumers will purchase it. Given the behavior of this higher valuation segment, the firm can charge a relatively higher price for the older product (if it were introduced). That gain in price more than compensates the firm for any loss accruing in the profit from the newer product (due to intra-brand competition). Therefore, under gradual improvement, replacement dominates skipping.

By contrast, if the difference in the two products is relatively large (i.e., as in the case of rapid improvement) and the future is particularly valuable (i.e., the discount factor is relatively high), then the higher valuation segment has an incentive to wait for the new product; consequently, the firm now has an incentive to select skipping to extract a larger surplus from this segment via the new product. In this setting, we characterize the precise conditions when skipping may be preferred to replacement.

We find that product compatibility—by influencing the relative attractiveness of the two products—affects the level of intra-brand competition between the older and the newer products. Under full compatibility, for example, adopters of the older product are able to enjoy the network benefit sooner (since they join the market earlier). When the strength of the network is relatively large, there is a significant raise in the desire to consume the older product, which in turn motivates the firm to commercialize it despite the ensuing impact of intra-brand competition. Therefore, under full compatibility and rapid improvement, the firm will follow a replacement strategy at higher values of the network effect; under gradual improvement, the analogous incentive is even stronger.

Under forward compatibility, as noted earlier, consumers appreciate the older product more; this puts a downward pressure on the newer product's price and adoption—essentially, it weakens the advantage arising from the level of improvement in the newer product. Under rapid improvement, at higher levels of the network effect, the downward pressure affects the firm's profitability significantly and the firm is better off not commercializing the older product.

Under backward compatibility, recall that the newer product's value to consumers is enhanced by the network size of the older product. By the same token, however, consumers are less inclined to buy the older product. Therefore, in order to accrue the benefit from backward compatibility of the newer product, the firm has to offer an incentive to consumers to induce enough purchases of the older product. Such an inducement can prove too costly to the firm, depending on the magnitude of the network effect and the relative improvement level across the two products. Consequently, if the older product were introduced, the firm's profitability under backward compatibility (vis a vis full compatibility) can either be higher or lower depending on the parametric regime. This contrasts with the corresponding shift under the forward compatibility setting noted earlier. In the above sense, our analysis highlights the role of product improvement and compatibility on the product strategies of durable goods manufacturers.

The rest of this chapter is organized as follows. The next section develops our model. We report the results of our analysis in Section 3 and conclude the chapter in Section 4. All proofs are confined to Appendix A.

3.2 The Model

Our focus here is on an innovating firm, I, that produces durable goods. There are two time periods in our model and at the beginning of the first period, using the available technology, the innovating firm has the option of commercializing an old product (denoted l) of quality θ_l. Irrespective of its decision to (not) commercialize this product, the firm continues to innovate during the first period. Consequently, at the beginning of the second period, the firm is equipped with an improved technology which is commercialized and introduced as a new product (denoted h) with a quality of θ_h ($\theta_h > \theta_l > 0$).

In addition to the intrinsic quality of the durable good, our model incorporates a network effect (as in, e.g., Katz and Shapiro 1985, Choi 1994, Ellison and Fudenberg 2000). In each period, consumers receive a benefit, denoted η, from the installed base (or network) when they consume the product. For analytical convenience, we focus on a linear network function; i.e., $\eta(x_{i,t}) = \omega x_{i,t}$ (where $i = \{l, h\}$, $t = \{1, 2\}$), and ω reflects the strength or impact of the installed-base, $x_{i,t}$, on the consumers' willingness-to-pay. The installed-base $x_{i,t}$ of product i in period t includes all (the consumers who bought) compatible products available in that period after adjusting for the level of compatibility. For instance, if the number of consumers who bought the old and new products in period t are N_l and N_h respectively, then $x_{l,t} = N_l + \mu_h N_h$ and $x_{h,t} = \mu_l N_l + N_h$, where μ_h and μ_l capture the level of compatibility between the new and the old product, with $0 \leq \mu_h, \mu_l \leq 1$. In particular, under full compatibility, $\mu_h = \mu_l = 1$; under backward compatibility, $\mu_h = 0, \mu_l = 1$; and under forward compatibility, $\mu_h = 1, \mu_l = 0$.

Consumers are heterogeneous in terms of their valuation of the product's intrinsic quality. We index consumers by v and assume that it is distributed uniformly along the interval $[-M, 1]$. When M is sufficiently large, the market remains uncovered for a focal range of the network strength, ω (i.e., $\omega \in [0, \overline{\omega}^{i,j}]$; see Assumption 1 a little later in the chapter). Consumers, however, value the network homogeneously. Therefore, a consumer indexed v will derive a surplus $\theta v + \eta - p$ when paying a price p for consuming a product of quality θ, with an associated network benefit η.

We use δ to denote the discount factor, where $\delta \in (0, 1)$; for expositional ease, we assume it to be the same for both the consumers and the firm. Over the two periods, keeping δ in mind, consumers make their purchase decisions based on the prices, qualities and the associated network benefit[2]. We assume that all consumers have the same expectations and invoke sub-game perfection to characterize a consistent set of prices, beliefs, and consumption decisions across time periods. There is no secondary market in our model. Further, to highlight the main issues, we also assume that there is no depreciation and that the marginal costs associated with producing the products are negligible.

Let p_l, p_h be the prices of old and new products respectively. And let $\eta_{l,1}$ and $\eta_{l,2}$ be the network benefit of the old product in periods 1 and 2, while $\eta_{h,2}$ is the network benefit of the new product in period 2. Each consumer, indexed by $v \in [-M, 1]$, maximizes his/her surplus among four alternatives: (1) buy neither of the products and enjoy a surplus denoted W_0 $(= 0)$, (2) purchase the old product only in period 1, with the surplus denoted W_l, where $W_l = \theta_l v + \eta_{l,1} + \delta\eta_{l,2} - p_l$, (3) purchase the new product only in period 2, with a surplus $W_h = \delta(\theta_h v + \eta_{h,2} - p_h)$, and (4) purchase in both periods, with a surplus $W_b = \theta_l v + \eta_{l,1} - p_l + \delta((\theta_h - \theta_l)v + \eta_{h,2} - p_h)$.

Denote S_0, S_l, S_h, and S_b as the four consumer segments corresponding to the above alternatives, and denote N_0, N_l, N_h, and N_b as the respective number of consumers in those segments. Accordingly, the network benefit for the old and new products in periods 1 and 2 can be written as follows: $\eta_{l,1} = \omega(N_l + N_b)$, $\eta_{l,2} = \omega(N_l + \mu_h(N_h + N_b))$, and $\eta_{h,2} = \omega(N_h + N_b + \mu_l N_l)$.

[2]This assumption seems reasonable since with the availability of online information such as news, blogs, social media, nowadays, consumers are more knowledgeable about the state of technological evolution, and the availability of related products.

In this setting, depending on the relative qualities of the two products, two different arrangements of consumer segments can arise. Following the extant literature (e.g. Dhebar 1994, Fudenberg and Tirole 1998, Kornish 2001, Bala and Carr 2009), we refer to the setting in which the first arrangement arises as "rapid improvement," where $\delta\theta_h > \theta_l$, and to the second setting as "gradual improvement," where $\delta\theta_h \leq \theta_l$.

Under either setting, along the $[-M, 1]$ interval, S_0 is always the left most segment, and S_b is always the right most segment, while the other segments are located in between these two. With rapid improvement, segment S_l is to the left of segment S_h, whereas, the opposite order arises under gradual improvement. These arrangements are illustrated in Figures 3.1 and 3.2. Using v_{ij} to denote the marginal consumer who is indifferent between being in segments S_i and S_j, we have:

Under rapid improvement, the demand in period 1 is discontinuous along the $[-M, 1]$ interval, and includes two parts $[v_{0l}, v_{lh}]$ and $[v_{hb}, 1]$. Meanwhile, under gradual improvement, the demand in period 1 is continuous. Subgame perfection requires that the optimal behavior of the innovating firm in period 2 be consistent with the consumer's beliefs and expectations. Hence, the demand structure respects the pattern specified in the following lemma.

Lemma 3.1 *Under rapid improvement, at least one of the two segments S_l and S_h must vanish. Under gradual improvement, however, all segments may coexist in equilibrium.*

In our model, the segmentation structure under rapid improvement arises for reasons that are analogous to the ones discussed in Kornish (2001) and illustrated in Figure 3.3. More specifically, suppose all the segments could coexist; at the beginning of period 2, when maximizing its profit, the firm will price the new product such that the lowest valuation consumer is indifferent between buying and not buying. At the same time, the discontinuity in the demand along the $[-M, 1]$ line in period 1 implies that the expected price of the new product is set to make the lowest valuation consumer indifferent between buying the old and new products. This leads to the nonexistence of segment S_l, which contradicts our earlier supposition. Hence, a subgame perfect pricing strategy cannot simultaneously sustain both S_l and S_h in equilibrium.

Given the above, under rapid improvement, we consider two targeting schemes: (1) the innovating firm does not serve segment S_h, and (2) the innovating firm does not serve segment S_l, denoted T-l and T-h respectively. When the firm adopts one of these schemes, as shown in Figure 3.3, the corresponding prices result in a relatively large installed base in either period 1 or period 2 depending on the selected scheme. The size of that installed-base serves as a credible commitment on the innovating firm's part to behave consistently with the consumer's beliefs and expectations, and thus leads to a subgame-perfect equilibrium.

By contrast, under gradual improvement, the innovating firm is able to implement the targeting scheme illustrated in Figure 3.2 (referred to as T-g hereafter). In particular, the firm may serve all of the segments S_l, S_h and S_b together. Do note that although the above targeting possibilities (under either rapid or gradual improvement) can arise in equilibrium, we are yet to characterize their optimality.

We now consider the firm's decision problem: The innovating firm follows either a product replacement strategy (denoted r)—i.e., only one version of the product is sold in each period—or, it may not commercialize the old product in period 1 and only introduce the new one in period 2. We refer to this latter option as the skipping strategy (denoted s); and by skipping, the innovating firm forgoes the profit of the old product. Our analysis will show, however, that such an option can indeed be an optimal strategy under certain conditions.

The sequence of events in the game is as follows. At the beginning of period 1, the innovating firm selects one of the two product strategies, $k = \{r, s\}$; if it chooses a replacement strategy, then the old product is introduced (under skipping it is not), and the firm will announce the price of the old product. Consumers make purchases based on the current price, and their expectations about the new product's price and the appropriate network sizes (adjusted for compatibility). At the beginning of period 2, irrespective of the decision in the first period, the firm introduces the new product and announces the price; consumers then make their period 2 purchase decisions.

Under each of the strategies (i.e., r or s), we solve the firm's problem in the standard way, starting in the second period to find the optimal price of the new product, and then, deriving the old product's price (if introduced) in period 1 by maximizing the present value of the total profit from both periods; all this is done while taking into account the relevant constraints for the possible targeting schemes. We use an asterisk superscript to indicate the product's optimal price (i.e., p_l^*, p_h^*) in all the considered settings.

Consumers' self-selection criteria are used to derive the demand, and the locations of the marginal consumers are shown in Table 3.2. (Table 3.1 summarizes our notation.) For reasons analogous to the ones in earlier research (e.g., Ellison and Fudenberg 2000, Sankaranarayannan 2007), we confine our analysis to settings where the network effect is not too large (see Ellision and Fudenberge 2000, page 264 and Sankaranarayannan 2007, pages 778-779 for a discussion on the reasonableness of this assumption). More specifically, we focus on a range of ω that ensures the concavity of the new product's profit function, and subsequently, a subgame-perfect pricing equilibrium. Further, our restriction on ω helps streamline the presentation of the results in which the skipping strategy may arise. For instance, we require that (1) the market be not fully covered, and (2) there exists a market for the new product. (We conducted analysis while relaxing both these conditions, but no new qualititative insights arise.)

Assumption 1: $\omega < \overline{\omega}^{i,j}$, where $i = \{h, l, g\}$ with h, l, g corresponding to the different targeting schemes, and $j = \{U, B, F\}$ with U, B, F referring to full, backward, and forward compatibility, respectively.[3]

[3]See Appendix A for specific values.

It is worth noting that the upper bound on ω, that ensures our earlier stated objectives, may change across different targeting schemes and product compatibilities. While one could combine these different values for the upper bound (and only focus on the smallest value), our approach will help isolate and discuss useful pricing practices that the firm can pursue (e.g., the interior solution highlighted in Lemma 3.3, a little later in the chapter).

3.3 The Analysis and Results

3.3.1 Rapid Improvement

In this section, we examine two distinct targeting schemes, T-l and T-h, and consider each of them under different product compatibility settings. Under T-h, three consumer segments may coexist in the market: S_0, S_h, and S_b, whereas, under T-l, segments S_0, S_l, and S_b may coexist (see Figures 3.3a and 3.3b). Our focus is on determining when skipping may arise (i.e., when selling to only S_h is optimal). It is clear that such a segmentation structure only arises as a boundary condition under T-h, where $N_b = 0$. By contrast, when $N_b > 0$, the firm will follow a replacement strategy. Thus, these two strategies are mutually exclusive under T-h.

Notice further that the targeting scheme T-l does not accommodate the skipping strategy, because S_h (where only the new product is purchased in equilibrium) does not exist. Instead, the existence of S_l and S_b imply that the old product is always introduced. Between T-l and T-h, the optimal one is determined by comparing the profitability of the two schemes. Therefore, in order to identify the conditions when skipping arises in equilibrium, we first characterize first its optimality under T-h and then compare that setting with the corresponding parametric regime under T-l.

The innovating firm does not serve segment S_l

Under T-h, if replacement were implemented, then the firm will price the two products such that segment S_b exists. By contrast, under skipping, there are only two consumer segments, S_0 and S_h, in the market. Accordingly, the firm's problem under strategy k ($k \in \{r, s\}$) is specified as follows:

$$\max_{p_l} \Pi_1^{h,x,k} = p_l N_b + \delta \hat{p_h}(N_h + N_b)$$

subject to:

$$\hat{p_h} \in \arg\max_{p_h} \Pi_2^{h,x,k} = p_h(N_h + N_b) \ , \tag{3.1}$$

$$N_h \geq 0 \ ; \ N_b \geq 0 \ ; \ v_{lh} \leq v_{0l} \ , \tag{3.2}$$

$$p_l \geq 0 \ ; \text{ and } p_h \geq 0, \tag{3.3}$$

where $x = \{U, B, F\}$, and U, B, F refer to full, backward, and forward compatibility respectively. We have (using Table 3.2), when $k = r$, $N_h = v_{hb} - v_{0h}$ and $N_b = 1 - v_{hb}$; next, when $k = s$, $N_h = 1 - v_{0h}$ and $N_b = 0$ (i.e., $v_{hb} \geq 1$). Notice that under either strategy, the demand in period 2 is $N_h + N_b = 1 - v_{0h}$. Constraint (3.1) above indicates that the new product's price maximizes the second period's profit. The constraints in (3.2) encompass a nonnegative size for the two segments, S_h and S_b, and the nonexistence of segment S_l. The constraints in (3.3) ensure nonnegative prices.

For expositional ease, let [P-hUk], [P-hBk], and [P-hFk] denote the firm's problem when selecting strategy k with scheme T-h, under full, backward, and forward compatibility settings respectively. The solutions to these problems are summarized in Table 3.3. From that table, notice first that the optimal price of the new product depends primarily on its own quality. This arises because, given the relative locations of the segments, the new product's price is set to attract brand new consumers in period 2, rather than the existing users of the old product (i.e., consumers from segment S_b). Additionally, since segment S_l does not arise, the existence of the buyers of the old product in period 1 puts no downward pressure on the price of the new product. Put differently, there is no impact of intra-brand competition in period 2.

Moreover, to sustain T-h, the old product's price (in period 1) should be set such that segment S_l vanishes, or equivalently, $v_{lh} \leq v_{0l}$. Using Table 3.2, this constraint can be rewritten as:

$$p_l \geq \frac{\theta_l}{\theta_h} * [p_h + \frac{\delta\theta_h}{\theta_l}\eta_{l,2} - \eta_{h,2}] + \eta_{l,1}. \tag{3.4}$$

We also have $\eta_{l,1} = \omega N_b$, $\eta_{l,2} = \eta_{h,2} = \omega(N_h + N_b)$ under full/forward compatibilities, and $\eta_{l,1} = \omega N_b$, $\eta_{l,2} = 0$, $\eta_{h,2} = \omega(N_h + N_b)$ under backward compatibility. Hence, equation (3.4) is equivalent to:

$$p_l \geq \omega + \frac{(\theta_l(1-\delta) - \omega)(\theta_l(\theta_h - 2\omega) + \delta\theta_h\omega)}{2\theta_l(\theta_h - \omega)(1-\delta)} \text{ , and} \tag{3.5}$$

$$p_l \geq \omega + \frac{(\theta_h - 2\omega)(\theta_l(1-\delta) - \omega)}{2(\theta_h - \omega)(1-\delta)}, \tag{3.6}$$

corresponding to full/forward compatibilities and backward compatibility respectively.

These inequalities imply that the old product's price needs to be high enough to keep consumers away from buying in period 1 only. Furthermore, such a high price may lead to a situation in which there is no demand in period 1, i.e., $N_b = 0$ (for $v_{hb} \geq 1$). When that setting arises in equilibrium, the firm follows a skipping strategy. With these above considerations in mind, the following lemma characterizes different product strategies under T-h:

Lemma 3.2 ***(Scheme T-**h**)*** *When the firm does not serve segment S_l,*

(i) Under full compatibility, i.e., $\mu_h = \mu_l = 1$, or forward compatibility, i.e., $\mu_h = 1$ and

$\mu_l = 0$,

- *if either* $\delta \geq 1/2$, *or* $\delta < 1/2$ *and* $\omega \geq \omega_1^{h,U}$, *then it follows a skipping strategy; and*

- *if* $\delta < 1/2$ *and* $\omega < \omega_1^{h,U}$, *then it follows a replacement strategy .*

(ii) Under backward compatibility, i.e., $\mu_h = 0$ *and* $\mu_l = 1$,

- *if* $\delta \geq 1/2$ *and* $\omega \leq \omega_1^{h,B}$, *then it follows a skipping strategy; and*

- *if either* $\delta \geq 1/2$ *and* $\omega > \omega_1^{h,B}$, *or* $\delta < 1/2$, *then it follows a replacement strategy .*

Part (i) of the above lemma states that when the future is relatively more valuable (i.e., $\delta \geq 1/2$), under T-h, there is little to gain from commercializing the old product, since consumers have an incentive to wait for the new product. By contrast, suppose that there is demand in period 1; this can occur when future is not too valuable (i.e., $\delta < 1/2$). With a relatively low stream of discounted profit from period 2, as ω goes up, the firm gains by extracting more surplus in period 1 from segment S_b via a higher price for the old product.

Notice, however, that the size of segment S_b decreases in ω; this occurs because in period 1, the consumers in segment S_b have to pay a higher price for the old product which includes its network benefit in period 2—see the presence of $\eta_{l,2} \geq 0$ on the right-hand side of (3.4). Since consumers from this segment buy again in period 2, they not only give up the old product's quality but also its network benefit. As ω goes up, this burden becomes larger and raises consumers' incentive to wait for the new product. Next, note that the size of segment S_h goes up with ω. Such growth arises from two sources: one is from consumers who would not have bought under a relatively low (or no) network benefit; and the second is from consumers who would otherwise be in S_b. So, a higher network strength ω motivates the firm to extract more from consumers' higher willingness to pay; simultaneously, though, a higher ω can induce switching from S_b to S_h. The latter concern becomes more important when the firm finds the revenue stream in period 1 is significant, i.e., the old product has relatively high quality.

If the quality of the old product is relatively high (i.e., $\theta_l/\theta_h > \frac{3\delta-1-\delta^2}{2\delta(1-\delta)}$), then the firm has an incentive to lower the old product's price—to mitigate the declining size of segment S_b. Otherwise, the old product's price continues to rise up to $\omega_1^{h,U}$. At higher values of ω (i.e., $\omega \geq \omega_1^{h,U}$), all consumers in segment S_b switch to segment S_h, and commercializing the old product is no longer a viable strategy.

The critical value $\omega_1^{h,U}$ is increasing in the relative product quality θ_l/θ_h. That is because when the old product becomes relatively more attractive, consumers are willing to pay more for it, making skipping a less viable strategy. Figure 3.4 illustrates the different regions of the optimal product strategy in the (δ, ω) plane, under T-h when θ_l/θ_h changes from 1/4 to 1/3 (thus shrinking the skipping region).

Next, we examine Part (ii) of Lemma 3.2. Under backward compatibility, the network benefit accruing to the old product in period 2 is zero (i.e., $\eta_{l,2} = 0$); hence, keeping all else the same, the old product's price is not as high as the one under full/forward compatibilities. As a result, the higher valuation consumers, who tend to wait for the new product in the absence of network effects, have an incentive to buy the old product at higher levels of network strength ω (because of a higher network benefit in period 1). Hence, under backward compatibility, skipping is only optimal at lower levels of ω (i.e., $\omega \leq \omega_1^{h,B}$) and higher levels of the discount factor δ (i.e., $\delta \geq 1/2$); otherwise, replacement is the preferred strategy.

Once the old product is commercialized, as ω goes up, the size of S_b goes up, whereas, the size of S_h goes down (contrary to the movement under full/forward compatibilities). Although consumers join the market in period 2 because of a higher network benefit, that increase in S_h cannot cover the decrease due to consumers switching from S_h to S_b. In fact, at higher levels of ω (i.e., $\omega \geq \theta_h/2$), segment S_h vanishes; and the firm only serves S_b.

It is worth noting how prices move under the replacement strategy. In general, the old product's price goes up with ω. However, when the network benefit is relatively low (i.e., $\omega < \omega_2^{h,B}$) and the old product is relatively attractive (i.e., $\theta_l/\theta_h > \frac{2\delta-1}{2\delta(1-\delta)}$ for $\delta \geq 1/2$ or

$\theta_l/\theta_h > \frac{1-2\delta}{1-\delta}$ for $\delta < 1/2$), price may not go up with ω. Notice that the former condition (i.e., $\omega < \omega_2^{h,B}$) implies a lower incentive to raise price (as the network is not particularly beneficial to consumers), while the latter condition generates a stronger incentive to serve segment S_b (due to its higher profitability). Consequently, the old product's price may go down with ω; for instance, at relatively low levels of ω, the size of S_b is small, and the firm may be inclined to lower the old product's price (with ω) to quickly expand S_b.

Figure 3.5 illustrates different regions of product strategies in the (δ, ω) plane under T-h (the numerical example uses $\theta_h = 1$; and the boundary will shift up if θ_h goes up). Compared to Figure 3.4, the region in which skipping arises is smaller under backward compatibility.

The innovating firm does not serve segment S_h

Under T-l, the market segmentation is depicted in Figure 3.3b, where $S_0 = [-M, v_{0l})$, $S_l = [v_{0l}, v_{lb})$ and $S_b = [v_{lb}, 1]$. As mentioned earlier, this structure does not accommodate the skipping strategy; consequently, replacement is always implemented under T-l. The innovator's problem is:

$$\max_{p_l} \Pi_1^{l,x} = p_l(N_l + N_b) + \delta \hat{p_h} N_b$$

subject to:

$$\hat{p_h} \in \arg\max_{p_h} \Pi_2^{l,x} = p_h N_b \ , \tag{3.7}$$

$$N_l \geq 0 \ ; \ N_b \geq 0 \ ; \ v_{hb} \leq v_{lh} \ , \tag{3.8}$$

$$p_l \geq 0 \ ; \ p_h \geq 0, \tag{3.9}$$

where $x = \{U, B, F\}$, and U, B, F refer to full, backward, and forward compatibility respectively. Constraint (3.7) above indicates that the new product's price maximizes the second period's profit. The constraints in (3.8) encompass a nonnegative size for the two segments,

S_l and S_b, and the nonexistence of segment S_h. The constraints in (3.9) ensure nonnegative prices.

Let [P-lU], [P-lB], and [P-lF] denote the firm's problem under full, backward, and forward compatibility settings respectively. The solution to these problems is summarized in Table 3.4. We follow the standard approach to solving such problems (e.g., see Fudenberg and Tirole 1998): that is, first solve the unconstrained problem to obtain an interior solution for the old product's price in period 1. Subsequently, check whether all the constraints are satisfied at that proposed solution; if the constraints are not satisfied, then the firm sets the old product's price based on the appropriate binding constraint, and a corner solution arises. It helps to distinguish between the interior and corner solutions, as summarized in the following lemma.

Lemma 3.3 ***(Scheme T-l)*** *When it does not serve segment S_h, the firm follows a product replacement strategy. More specifically (see Table 3.4 for the optimal prices),*
(i) Under full compatibility (i.e., $\mu_h = \mu_l = 1$), when $\omega < \omega_1^{l,U}$, the old product's price is at a corner solution; and when $\omega \geq \omega_1^{l,U}$, the old product's price is at an interior solution.

(ii) Under backward compatibility (i.e., $\mu_h = 0, \mu_l = 1$), when either $\theta_l/\theta_h \geq \frac{2+\delta}{4+\delta}$, or $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$ and $\omega < \omega_1^{l,B}$, the old product's price is at a corner solution; and when $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$ and $\omega \geq \omega_1^{l,B}$, the old product's price is at an interior solution .

(iii) Under forward compatibility (i.e., $\mu_h = 1, \mu_l = 0$), when either $\theta_l/\theta_h \geq \frac{2}{1+2\delta}$, or $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega < \omega_1^{l,F}$, the old product's price is at a corner solution; and when $\theta_l/\theta_h < \frac{2+\delta}{3+\delta}$ and $\omega \geq \omega_1^{l,F}$, the old product's price is at an interior solution.

Under all the three compatibility settings, a relatively high level of ω is necessary for an interior solution to arise in equilibrium. Under full compatibility, this also serves as a sufficient condition. Forward and backward compatibility, by contrast, require an additional sufficient condition on the relative qualities of the two products θ_l/θ_h (i.e., θ_l/θ_h is relatively small). We now explain the intuition underlying Lemma 3.3 by considering each compatibility setting in turn.

Full Compatibility: When they are fully compatible, both the new and old products share the same installed-base, and thus, provide the same network benefit to consumers in period 2. Consequently, the marginal consumer in period 2 is unaffected by the network size. More specifically, since $\mu_l = \mu_h = 1$, we have $\eta_{l,1} = \eta_{l,2} = \eta_{h,2} = \omega(N_l + N_b)$. In period 2, then, the new product's price is influenced by only the level of quality improvement in the new product. This is because, consumers in segment S_b enjoy the same network benefit across time periods; what matters to them is the quality increment that they may receive when giving up the old product in period 2.

In period 1, in order to eliminate S_h, the firm has to price the old product such that consumers are no better off waiting to buy the new product in period 2 (i.e., ensure $v_{hb} \leq v_{lh}$). This restriction translates into a relatively low price for the old product. Mathematically:

$$p_l \leq \frac{\theta_l[\theta_l(1-\delta) + (1+\delta^2)\omega]}{2(\theta_l - \delta\omega)}. \tag{3.10}$$

Our approach to dealing with the above constraint (as outlined earlier) is to solve problem [P-lU] without (3.10), and then double checking its fulfillment. This leads to a condition on the magnitude of the network strength ω: it should be relatively large (i.e., $\omega \geq \omega_1^{l,U}$) for constraint (3.10) to be satisfied automatically. Put differently, when ω is smaller than the lower bound $\omega_1^{l,U}$, constraint (3.10) is binding; and the firm will price the old product so as to satisfy (3.10) exactly.

Keeping the above in mind, the intuition underlying the firm's optimal behavior can be explained as follows. First, in the absence of network effects, a commitment to not serve segment S_h is stringent (as in Kornish 2001). It forces the firm to penetrate the market in period 1 with a lower price to induce consumers to buy immediately (and not wait for the new product). Though a lower price helps the firm increase sales significantly, profit may be improved only at a higher price (at the expense of a correspondingly lower market size). Secondly, as the network effect comes into existence, consumers begin to benefit from the installed base for the product. This in turn allows the firm to raise the barrier on pricing the old product (cf. equation 3.10). Finally, at a relatively large value of network strength, i.e., $\omega = \omega_1^{l,U}$, segment S_h vanishes without any restriction on the old product's price (and we obtain an interior solution). From that value of ω onwards, the firm prices the old product at the interior solution, which is independent of network effect (market size in period 1, however, is increasing in ω).

The implication here is that the network effect can serve as a tool to eliminate consumers' incentive to wait for the new product (i.e., S_h vanishes at higher levels of ω because those in S_h switch to either S_l or S_b). By joining the market early, consumers are able to enjoy the network benefit sooner. Now the value of $\omega_1^{l,U}$ is increasing with the old product quality θ_l and the discount factor δ; in other words, the region where the old product's price is at a corner solution expands with θ_l and δ. This is because at a higher quality, consumers are willing to pay more, and the unconstrained solution for the price is larger; with a higher discount factor, consumers value the new product in the future more, and thus force the firm to offer an even lower price to induce them to buy the old product. Subsequently, for both these reasons, it is that much harder for an unconstrained solution to arise.

Backward Compatibility: Here, the customer base of the old product contributes to the installed-base for the new product, whereas the customer base of the new product does not contribute to the installed-base of the old, i.e., $\mu_h = 0, \mu_l = 1$. Hence, the network benefits of the product are: $\eta_{l,1} = \eta_{h,2} = \omega(N_l + N_b)$, and $\eta_{l,2} = \omega N_l$.

In period 2, the new product's price is the same as in full compatibility. However, the market size, N_b, is increasing in network strength ω (contrary to the full compatibility setting, where the market size was independent of ω). This is because consumers in segment S_b will have a network size related advantage when making repeat purchases; those who only hold the old product in period 2 are restricted to enjoy a more limited network benefit.

Analogous to the full compatibility setting, here too, the firm will charge a relatively low price for the old product at low levels of ω. This is because the demand-side constraint, $v_{hb} \leq v_{lh}$, forces the firm to lower the price of the old product to induce consumers to buy in period 1 (rather than waiting for the new product). Here, too, the upper bound on the old product's price goes up with ω.

Further, the size of segment S_b increases in ω, whereas, the size of segment S_l may increase or decrease in ω. The former happens because as ω goes up, consumers have an incentive to switch from S_l to S_b; by switching, they are able to maintain the network benefit in period 2. Next, the size of S_l can go up with ω when more consumers join this segment than those who switch to S_b. Conversely, the decreasing trend in the size of S_l occurs either because the number of consumers switching to S_b is larger than the number of those joining segment S_l, or because the increment in the old product's price, at higher levels of ω, surpasses the network benefit from joining S_l; thus shrinking the segment size when ω goes up.

Notice that when the two products are not too different (i.e., $\theta_l/\theta_h \geq \frac{2+\delta}{4+\delta}$), the new product's price is very attractive (as $p_h^* = (\theta_h - \theta_l)/2$ becomes smaller), giving consumers in S_l a strong incentive to make repeat purchases. In fact, the size of segment S_l is rapidly squeezed and vanishes at $\omega = (\theta_h - \theta_l)/2$; at higher levels of ω (i.e., $\omega \geq (\theta_h - \theta_l)/2$), the firm follows a pricing strategy that serves only segment S_b. When the two products are quite different (i.e., $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$), the upper bound on the old product's price approaches the optimal interior price at $\omega = \omega_1^{l,B}$; at higher levels of ω (i.e., $\omega \geq \omega_1^{l,B}$), the constraints on the firm's problem are satisfied automatically, and the old product's price is at the interior solution.

Notice that at higher levels of ω—when either the firm serves only segment S_b, or the old product's price is at an interior solution—the price set in period 1 is not considered "too low". And the firm is inclined to lower the old product's price as ω goes up. This is because under backward compatibility, compared to the new product, the old product is at a relative disadvantage (both from the level of quality and future network benefit). A lower price could help trigger purchases in the first period, which in turn help the profit stream from the new product due to backward compatibility.

Forward Compatibility: Here, $\mu_h = 1, \mu_l = 0$, and the network benefit values are: $\eta_{l,1} = \eta_{l,2} = \omega(N_l + N_b)$, and $\eta_{h,2} = \omega N_b$. In this case, the new product's price depends on the old's price and the network strength. In equilibrium, this price is decreasing in the network strength ω. This is because the new product's purchasers come from the existing consumers; by purchasing the new product, they give up not only the old product but also its bigger network benefit (due to forward compatibility). Consequently, the firm is forced to lower the new product's price to induce existing consumers to purchase again.

The decrement in the new product's price as ω goes up does help expand S_b for low levels of ω. However, the size of S_b goes down with ω when the network benefit becomes larger (and the size of S_l always increases with ω). This is mainly because fewer existing consumers make repeat purchases at higher values of the network strength.

Further, the new product's price is positively related to the old product's price; and this impact is increasing with ω (i.e. $\frac{\partial^2 \hat{p_h}}{\partial\omega\partial p_l} = \frac{\theta_l}{2(\theta_l-(1+\delta)\omega)^2} > 0$). At low levels of ω, the impact of p_l on p_h is very small; further, an interior solution for the old product's price in period 1 is too high. (This combination can induce some consumers in S_l to wait for the new product in period 2.) Consequently, the firm will lower the old product's price; analogous to the other compatibility settings, this restriction on the price of the old product (its upper bound) is increasing in ω and is obtained when the condition $v_{hb} \leq v_{lh}$ holds exactly.

As ω goes up, the impact of p_l on p_h is bigger, leading to a larger increase in the new product's price. In this setting, suppose the two products are not too different (i.e., $\theta_l/\theta_h \geq \frac{2}{1+2\delta}$); this feature, in addition to the more limited network size when buying the new product, results in consumers having a lower incentive to make repeat purchases. Consequently, at higher values of ω, segment S_b shrinks quickly (here, the upper bound of the old product's price never approaches the optimal interior price). By contrast, if the two products are quite different (i.e., $\theta_l/\theta_h < \frac{2}{1+2\delta}$), then consumers still have an incentive to make repeat purchases despite the price increment for the new product. The upper bound of the old product's price then approaches the optimal interior price at $\omega = \omega_1^{l,F}$; the old product's price is at the interior solution.

<u>The Optimal Product Strategy under Rapid Improvement</u>

Up to this point, we have characterized different product strategies for different pricing practices under rapid improvement. Our purpose in this section is to compare the profitability under T-l and the one under skipping (within T-h), when both strategies are feasible. Further, we want to highlight the impact of product compatibility on the above comparison. Accordingly, we first characterize the optimal product strategy under full compatibility. Using those results as a base line, we will explore how the optimal strategy changes for other levels of compatibility.

The optimal product strategy under full compatibility is summarized in the following proposition. Define $\hat{\delta} = \frac{\sqrt{5}-1}{2} \approx 0.62$.

Proposition 3.1 *Under rapid improvement and full compatibility,*

(i) If $\delta < \hat{\delta}$, then the innovating firm follows a product replacement strategy; and

(ii) If $\delta \geq \hat{\delta}$, then the innovating firm follows: (a) a skipping strategy when either $\omega \leq \hat{\omega}$, or $\hat{\omega} < \omega < \omega^*$ and $\theta_h \leq \hat{\theta}_h$; (b) a replacement strategy when either $\omega \geq \omega^*$, or $\hat{\omega} < \omega < \omega^*$ and $\theta_h > \hat{\theta}_h$.

When the firm follows a replacement strategy, profit accrues from both the old and new products. This contrasts with the skipping strategy, where only the new product contributes to the profit stream. Under T-l, the new product's price is constrained by the quality of the old product; in other words, intra-brand competition (read cannibalization) has a significant impact on it. By contrast, under T-h, there is no cannibalization in period 2, and the new product is priced based on its own quality. As a result, though skipping forges the first period profit, the profit from the new product is bigger. Of course, the discount factor plays an important role in a skipping strategy because the total profit is discounted. Consequently, at low levels of the discount factor (i.e., $\delta < \hat{\delta}$), skipping is always dominated.

When the future is more valuable (i.e., $\delta \geq \hat{\delta}$), the skipping strategy becomes more viable. At low levels of the network strength (i.e., $\omega \leq \hat{\omega}$), cannibalization is the main reason that makes a replacement strategy under T-l be dominated by skipping. In contrast, replacement is the dominant strategy when the loss from cannibalization is compensated by a large network benefit (i.e., $\omega \geq \omega^*$). This occurs because the pricing under T-l attracts a larger installed-base of consumers than the one under T-h. Further, the sooner the consumers enter the market, the larger the network benefit they enjoy, and the greater the surplus that the firm is able to extract from them.

When the network strength is moderate (i.e., $\hat{\omega} < \omega < \omega^*$), skipping is the dominant strategy as long as the new product's quality is not too high. This happens because, when $\theta_h \leq \hat{\theta}_h$, the impact of cannibalization in period 2 is relatively large under product replacement; consequently, skipping dominates. This impact of intra-brand competition, however, is mitigated at higher levels of new product's quality, and the preference for replacement goes up accordingly.

Now recall the Series 90 Nokia smartphones (i.e. 7700 and 7710) example mentioned in the Introduction. In November 2003, Nokia announced the release of its model 7700, but never made it available commercially (some 7700s are in circulation, mainly as collectibles). Instead, it cancelled the 7700 in mid-2004 and launched the 7710 at the end of 2004. While Nokia's reasons for its decision are not public, some potential explanations surfaced in the media; for example, *Technology Wire* on November 8th, 2004 noted the unattractive design of 7700.

We note that the two models, 7700 and 7710, were the only available Symbian OS Series 90 smartphones from Nokia, and they did not work with other OS platforms within Symbian 7.0 and 7.0s (e.g., Series 80, Series 60, UIQ). Further, any network benefit consumers may have derived from these phones was likely to be very small—Nokia bundled most of the applications with this platform and the availability of any third-party applications was quite limited. Finally, while the two phones 7700 and 7710 have the same technology platform (suggesting full compatibility[4]), the product designs[5] were quite different.

Thus, in the context of a firm facing little network effects under a setting of full compatibility, if Nokia considered the 7710 to be a significant improvement (in terms of product design), then based on our analysis, Nokia's decision would be optimal. By contrast, in the iPad-iPad 2 example (mentioned in the introduction section), the products share a huge network benefit (there are more than 500,000 iOS applications in the App Store!); so, even though the products are not too different from one another, Apple's decision to employ a product replacement strategy is supported by our analysis.

[4]Typically, newer products contain features that make implementing forward compatibility a challenge; therefore, full compatibility is likely to occur when the two products have the same technological base, as in the case of Nokia 7700 and 7710, iPad and iPad 2, Xbox 360 S and the earlier Xbox 360 Elite, or Play Station 3 Slim and its previous model Play Station 3.

[5]A comparison can be found at http://my-symbian.com.

Figure 3.6 illustrates the product strategies described in Proposition 3.1 in the (δ, ω) plane when $\theta_l = 2$. Notice that ω^* is increasing in θ_l; simulations also show that $\hat{\omega}$ goes up with θ_l.

Next, we examine how the optimal strategy in Proposition 3.1 can change under forward and backward compatibilities. Begin by recalling that the profit stream from skipping remains unchanged across the three levels of compatibility. Consequently, any deviation in the optimal strategy (from full compatibility) revolves around how the profitability from the corner solution at $v_{hb} = v_{lh}$, under T-l, varies with compatibility[6]. When the old product is forward compatible with the new product, we observe the following:

Observation 3.1 *Under rapid improvement and forward compatibility, compared to the full compatibility setting, the innovating firm is more likely to use a skipping strategy.*

Since the profit from skipping is unchanged with compatibility, what makes it more favorable in this setting is due to the change that arises under T-l. In particular, the impact of cannibalization under T-l is now more severe (see the new product's price in Table 3.4): The new product's price is decreasing with ω; further, the new product's profit almost fades out at higher levels of ω because of the decrement in both the price and the market size (i.e., even the 'low' price is not low enough to induce repeat purchases). Though the new product is improved significantly under rapid improvement, its advantages are canceled out by the network effect associated with the old product (which is commercialized). The profit stream from the old product is higher under forward compatibility; however, that cannot compensate for the decrement in the new product's profit. Consequently, at relatively high levels of ω, the profitability under T-l is reduced significantly, and raises the firm's incentive to use skipping.

[6]First, under T-l, an interior solution always (weakly) dominates a corner solution. Next, under full compatibility, at higher values of ω, the corner solution dominates skipping. Since the interior solution arises at even higher values of ω, to conserve space, here we focus on the corner solution under T-l. Other comparisons involving the interior solution are also available, but add little new qualitative insights.

When the new product is backward compatible, the incentive for skipping is summarized in the observation below:

Observation 3.2 *Under rapid improvement and backward compatibility, compared to the full compatibility setting, the innovating firm is less likely to use a skipping strategy when the new product's quality (i.e., θ_h) is relative large; alternatively, it is more likely to use a skipping strategy when the new product's quality is relatively small.*

Interestingly, when compared to a setting of full compatibility, under backward compatibility, a skipping strategy can be either more favorable or less favorable. Here too, notice the change in profitability under T-l when moving from a scenario of full to backward compatibility at the corner solution where $v_{hb} = v_{lh}$ in period 1. From Table 3.4, note that the new product's price is unchanged; meanwhile, the market is expanding with ω. As a result, the new product's profit is unambiguously greater than the one in full compatibility. The old product's profit, however, is smaller due to its lower price and market size (market size goes down because of a decline in N_l). Therefore, if the profit increment in period 2 (which results from market expansion) surpasses the profit decrement in period 1, then replacement becomes more attractive; otherwise, skipping is more preferred.

Intuitively, under backward compatibility, the new product's adoption comes at the expense of the old product's adoption (recall that under T-l, the new product's consumers come from the existing buyers of the old product in period 1). The firm clearly has an incentive to maintain the old product's existence in order to extract more surplus from the buyers of the new product (due to backward compatibility); hence, it lowers the old product's price to induce purchases in period 1. When providing such inducement proves too costly to the firm, it is better off with the skipping strategy.

Using Table 3.4 we can compare, between full and backward compatibility, the solution in period 2 along with the corner solution at $v_{hb} = v_{lh}$ in period 1. Notice that while the old product's price and demands in both periods are dependent on θ_h under backward compatibility, they are independent of θ_h under full compatibility. More specifically, under backward compatibility, as the new product's quality, θ_h, goes up, its market size goes down; hence, in period 2, the new product's profit is increasing slowly when θ_h is large (compared to the one under full compatibility). Further, the old product's price is going up in θ_h; and at higher levels of ω (i.e., $\omega > \frac{\theta_l(1-\delta}{\delta}$), the market size in period 1 also goes up with θ_h. This happens because as θ_h goes up, the decrement in the market size of the new product allows the firm to attract more consumers who are less price-sensitive into segment S_l, which in turn motivates the firm to raise the old product's price. When the network strength is small, this price increment leads to a decrement in the market size in period 1; however, when the network strength is large, more consumers will join the market, and this expansion surpasses the impact of the price increment.

It then follows that under backward compatibility, the old product's profit is decreasing more slowly when θ_h is larger at higher levels of ω (compared to the one under full compatibility). In other words, a relatively large improvement in the new product's quality serves to mitigate the negative impact of backward compatibility on the old product's profit. Consequently, at higher levels of new product improvement and network strength, even though the profit increment in period 2 is not too large, it becomes feasible to overcome the profit decrement in period 1. Under these conditions, skipping becomes less preferred .

On the other hand, at higher levels of network strength, when the new product's quality is not too high, the new product's profit goes up significantly, while the old product's profit goes down sharply (compared to the corresponding profit under full compatibility). Such movements tend to hurt the profitability from replacement, and skipping becomes more preferred.

We use numerical simulations to show how the optimal strategy changes when moving from full to either forward or backward compatibility (see Tables 3.5 and 3.6). Each table includes a column that shows the difference in profit from skipping and replacement under full, forward, and backward compatibility, for distinct values of ω; in that column, a positive number indicates that skipping is optimal, while a negative number implies that replacement is optimal.

For both tables, we set $\delta = 0.8$ and $\theta_l = 2$ (consequently, $\hat{\omega} = 0.25$ and $\omega^* = 0.255$). In Table 3.5 the new product's quality is relatively low, with $\theta_h = 3$ (the requirement for rapid improvement is met); in Table 3.6, θ_h is relatively high with $\theta_h = 10$. In either table, with forward compatibility, the skipping strategy is optimal even when either $\hat{\omega} < \omega < \omega^*$ and $\theta_h > \hat{\theta}_h$, or $\omega > \omega^*$ (under full-compatibility, either of these conditions favors replacement). Next, Table 3.5 also shows that an analogous result arises even under backward compatibility (i.e., when θ_h is not too large); meanwhile, Table 3.6 contrasts with Table 3.5 in the preference for skipping under backward compatibility—when θ_h is relatively large, the replacement strategy is optimal even when either of the conditions $\hat{\omega} < \omega < \omega^*$ and $\theta_h < \hat{\theta}_h$, or $\omega < \hat{\omega}$ is satisfied.

3.3.2 Gradual Improvement

Under gradual improvement, the four consumer segments—S_0, S_l, S_h, and S_b—may co-exist in the market. When product replacement is implemented, the market segmentation (i.e., scheme T-g) is illustrated in Figure 3.2, where $S_0 = [-M, v_{0h})$, $S_h = [v_{0h}, v_{hl})$, $S_l = [v_{hl}, v_{lb})$ and $S_b = [v_{lb}, 1]$. Meanwhile, under skipping only segments S_0 and S_h exist in the market; this requirement makes skipping a special (restricted) case of replacement. Hence, if replacement is viable, then it is also the optimal strategy. The innovating firm's problem, under strategy k ($k \in \{r, s\}$) is specified below.

$$\max_{p_l} \Pi_1^{g,x,k} = p_l(N_b + N_l) + \delta \hat{p}_h(N_h + N_b)$$

subject to:

$$\hat{p_h} \in \arg\max_{p_h} \Pi_2^{g,x,k} = p_h(N_h + N_b) \ , \tag{3.11}$$

$$N_l \geq 0 \ ; \ N_h \geq 0 \ ; \ N_b \geq 0 \ , \tag{3.12}$$

$$p_l \geq 0 \ ; \ p_h \geq 0, \tag{3.13}$$

where $x = \{U, B, F\}$, and U, B, F refer to full, backward, and forward compatibility respectively. We have (using Table 3.2), when $k = r$, $N_h = v_{hl} - v_{0h}$, $N_l = v_{lb} - v_{hl}$, and $N_b = 1 - v_{lb}$; and when $k = s$, $N_h = 1 - v_{0h}$, $N_l = 0$, and $N_b = 0$ (i.e., $\{v_{lb}, v_{hl}\} \geq 1$). Constraint (3.11) above indicates that the new product's price maximizes the second period's profit. The constraints in (3.12) encompass a nonnegative size for the three segments, S_l, S_h and S_b (with the sizes of S_l and S_b becoming zero when $k = s$). The constraints in (3.13) ensure nonnegative prices.

Let [P-gUk], [P-gBk], and [P-gFk] denote the firm's problem under full, backward, and forward compatibility settings respectively. The manufacturer's preference for skipping and replacement in these problem settings is summarized in following proposition:

Proposition 3.2 *Under gradual improvement, product replacement (weakly) dominates the skipping strategy for all the three (i.e., U, B, F) compatibility settings.*

Unlike the rapid improvement setting, here, all consumer segments may coexist and the firm does not face a binding demand-related constraint on pricing. When ω satisfies Assumption 1, the innovating firm's optimal price is at an interior solution and results in a unique subgame perfect equilibrium. The intuition underlying Proposition 3.2 is as follows. Under gradual improvement, compared to the old product, the new product's quality has not improved in terms of its present-value (i.e., $\theta_l \geq \delta\theta_h$); this suggests that the segment with the relatively high consumer valuations has less of an incentive to wait for the new product. Consequently, if the old product were available, then consumers will purchase it. Given the interest of this high-valuation segment, in the absence of a network effect, the firm can charge a relatively high price for the old product (if it were introduced). That gain in price compensates for any loss in the profit from the new product that can arise due to intra-brand competition.

In the presence of a network effect, the high-valuation segment's desire to buy the old product is further strengthened under both full and forward compatibility—since, these consumers can enjoy the network benefit over both periods by joining the market early. And given the attractiveness of the old product under gradual improvement, the firm's incentive to employ the replacement strategy is reinforced. Under backward compatibility, some high-valuation consumers may choose to wait for the new product; but the firm can induce such consumers to buy the old product (so as to enhance the value of backward compatibility for the new product). Under gradual improvement, such inducement is never too costly for the firm (given Assumption 1), and replacement is the optimal strategy.

3.4 Conclusion

The focus of our analysis has been on the optimality of skipping under alternative settings for exogenous technological evolution and product compatibility. We examined different product improvement levels and showed that skipping can be optimal only under rapid improvement; further, the incentive to skip an existing product is mitigated by network effects. Interestingly, product compatibility can change the firm's preference for skipping. For ease of exposition, we considered full-compatibility and two types of one-way compatibility (forward and backward compatibility); our analysis reveals that their impact on the optimality of skipping is asymmetric. In particular, compared to full compatibility, forward compatibility favors skipping, whereas, backward compatibility may or may not favor skipping; the relative favorability depends on the magnitude of improvement in the new product.

We also show that for a durable goods manufacturer, the replacement strategy is preferred to skipping at lower levels of product improvement (i.e., as in the case of gradual improvement, where the successive generations are not too different from one another). Further, at relatively high levels of network strength (e.g., as in the case of iPad with over 500,000 iOS applications in the App Store), it is optimal to implement replacement (over skipping) regardless of the level of product improvement.

It is worth pointing out that while product replacement decisions are relatively easy to observe by outsiders, skipping is often an internal consideration for the firm. Essentially, there are limited reasons for the general public to become informed of skipping decisions (unless the firm publicly confirms its decisions). Increased availability of data can certainly help in this case, and in that context, our analysis helps in developing hypotheses for formal empirical testing.

Overall, our purpose has been to underscore the importance of considering network-related issues in the context of new product introduction strategies of durable goods manufacturers. We focused on skipping and product replacement strategies and provided conditions where a focal firm's preferences can change dramatically. Several opportunities seem to exist for further exploration.

For instance, prior research points out that in the absence of any network effects, skipping is less likely when there is an entry by a clone (Purohit 1994); using an aggregate demand function, Chau and Desiraju (2011) show that may not be the case in the presence of network effects. Incorporating competition in the context of an individual level consumer model can help our understanding of such issues; but such a model setting invariably raises the complexity of the analysis significantly. Similarly, it can help to consider the optimal choice of compatibility for the durable goods firm. These types of questions will benefit from increased research attention. We hope that our effort here will spark further work in this area.

Table 3.1: Notation

Symbol	Description
θ_l, θ_h	Qualities of the old (l) and new (h) products
$\eta(.)$	Network benefit function
μ_o, μ_n	Level of product compatibility
ω	Network strength
δ	Discount factor
p_l, p_h	Prices of the old (l) and new (h) products
$W(.)$	Consumer surplus function
S_i	Consumer segment, $i = \{0, l, h, b\}$
N_i	The size of consumer segment i
v_{ij}	Marginal consumer that is indifferent between being in segments S_i and S_j
T-l, T-h	Targeting schemes under rapid improvement
T-g	Targeting scheme under gradual improvement
r, s	Product strategies ($r$eplacement and $s$kipping)
U, B, F	Compatibility regimes (FUll, Backward, and Forward)
$\overline{\omega}^{i,j}$	The upper bound of ω across different targeting schemes and compatibilities, where $i = \{h, l, g\}$ and $j = \{U, B, F\}$; the specific values are defined in the Appendix.
$\Pi_t^{i,j,k}$	The firm's profit in period t ($t \in \{1, 2\}$), under targeting scheme T-i (with $i \in \{l, h, g\}$) under compatibility regime j ($j \in \{U, B, F\}$) and product strategy k ($k \in \{r, s\}$)
$\hat{\omega}, \omega^*$	Critical values that determine the optimal strategies in the setting with rapid improvement and full compatibility.

Table 3.2: Locations of the Marginal Consumers

Segments	S_0	S_l	S_h
S_l	$v_{0l} = \dfrac{p_l - \eta_{l,1} - \delta\eta_{l,2}}{\theta_l}$	-	-
S_h	$v_{0h} = \dfrac{p_h - \eta_{h,2}}{\theta_h}$	$v_{lh} = \dfrac{\delta p_h - p_l + \eta_{l,1} + \delta\eta_{l,2} - \delta\eta_{h,2}}{\delta\theta_h - \theta_l}$	-
S_b	-	$v_{lb} = \dfrac{p_h + \eta_{l,2} - \eta_{h,2}}{\theta_h - \theta_l}$	$v_{hb} = \dfrac{p_l - \eta_{l,1}}{(1-\delta)\theta_l}$

Table 3.3: Rapid Improvement - Scheme T-h

	Full Compatibility ([P-hUk])	Backward Compatibility ([P-hBk])	Forward Compatibility ([P-hFk])
p_h^*	$\theta_h/2$	$\theta_h/2$	same as in [P-hUk]
$N_h + N_b$	$\frac{\theta_h}{2(\theta_h - \omega)}$	$\frac{\theta_h}{2(\theta_h - \omega)}$	same as in [P-hUk]
$v_{lh} = v_{0l}$			
p_l^*	$\omega + \frac{(\theta_l(1-\delta) - \omega)(\theta_l(\theta_h - 2\omega) + \delta\theta_h\omega)}{2\theta_l(\theta_h - \omega)(1-\delta)}$	$\omega + \frac{(\theta_h - 2\omega)(\theta_l(1-\delta) - \omega)}{2(1-\delta)(\theta_h - \omega)}$	same as in [P-hUk]
N_b	$\max\{\frac{\theta_l\theta_h(1-2\delta) + \delta(2\theta_l - \theta_h)\omega}{2\theta_l(1-\delta)(\theta_h - \omega)}, 0\}$	$\max\{\frac{\theta_h(1-2\delta) + 2\delta\omega}{2(1-\delta)(\theta_h - \omega)}, 0\}$	same as in [P-hUk]
$v_{hb} = v_{0h}$			
p_l^*		$\omega + \frac{(\theta_h - 2\omega)(\theta_l(1-\delta) - \omega)}{2(\theta_h - \omega)}$	
N_b		$\frac{\theta_h}{2(\theta_h - \omega)}$	

Scheme T-h with the following segmentation structure:

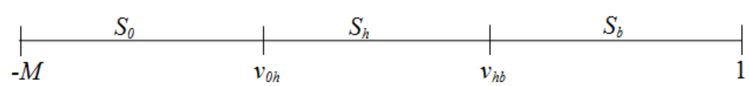

Table 3.4: Rapid Improvement - Scheme T-l

	Full Compatibility ([P-lU])	Backward Compatibility ([P-lB])	Forward Compatibility ([P-lF])
p_h^*	$\frac{\theta_h-\theta_l}{2}$	$\frac{\theta_h-\theta_l}{2}$	$\frac{\theta_h-\theta_l}{2} - \frac{\omega(\theta_l-p_l^*)}{2(\theta_l-(1+\delta)\omega)}$
N_b	$1/2$	$\frac{\theta_h-\theta_l}{2(\theta_h-\theta_l-\omega)}$	$\frac{\theta_h-\theta_l}{2(\theta_h-\theta_l-\omega)} - \frac{\omega(\theta_l-p_l^*)}{2(\theta_h-\theta_l-\omega)(\theta_l-(1+\delta)\omega)}$
a. Interior solution			
p_l^*	$\theta_l/2$	$\frac{\theta_l}{2} - \frac{\delta\omega(\theta_h-\theta_l)}{4(\theta_h-\theta_l-\omega)}$	$\frac{\theta_l}{2} + \frac{\delta\omega(2\theta_l(\theta_h-\theta_l)-(2\theta_h(1+\delta)-\theta_l(1+2\delta))\omega)}{2((4+3\delta)\omega^2+4(\theta_l\delta-\theta_h(1+\delta))\omega+4\theta_l(\theta_h-\theta_l))}$
$N_l + N_b$	$\frac{\theta_l}{2(\theta_l-(1+\delta)\omega)}$	$\frac{2\theta_l(\theta_h-\theta_l)-(\delta\theta_h+\theta_l(2-\delta))\omega}{4(\theta_h-\theta_l-\omega)(\theta_l-(1-\delta)\omega)}$	$\frac{2\theta_l(\theta_h-\theta_l)-(\delta\theta_h+\theta_l(2-\delta))\omega}{(4+3\delta)\omega^2+4(\theta_l\delta-\theta_h(1+\delta))\omega+4\theta_l(\theta_h-\theta_l)}$
b. Corner solution			
$v_{hb} = v_{lh}$			
p_l^*	$\frac{\theta_l(\theta_l(1-\delta)+\omega(1+\delta^2))}{2(\theta_l-\delta\omega)}$	$\{\frac{\theta_l(\theta_l(1-\delta)+\omega(1+\delta^2))}{2(\theta_l-\delta\omega)} - \frac{\omega(\theta_l(1-\delta)(\theta_l-\delta\omega)+\omega(\delta\theta_h-\theta_l))}{2(\theta_l-\delta\omega)(\theta_h-\theta_l-\omega)}\}$	$\{\frac{\theta_l(\theta_l(1-\delta)+\omega(1+\delta^2))}{2(\theta_l-\delta\omega)} - \frac{\omega\theta_l(1-\delta)(\theta_l(1-\delta)-2\delta\omega)(\theta_l-(1+\delta)\omega)}{2(\theta_l-\delta\omega)(2\delta\omega^2-(\theta_l(1-\delta)+2\theta_h\delta)\omega+2\theta_l(\theta_h-\theta_l))}\}$
$N_l + N_b$	$\frac{\theta_l(1+\delta)}{2(\theta_l-\delta\omega)}$	$\frac{\theta_l(1+\delta)}{2(\theta_l-\delta\omega)} - \frac{\omega(\delta\theta_h-\theta_l)}{2(\theta_l-\delta\omega)(\theta_h-\theta_l-\omega)}$	$\frac{\theta_l(1+\delta)}{2(\theta_l-\delta\omega)} + \frac{\omega\theta_l(1-\delta)(\theta_l(1-\delta)-2\delta\omega)}{2(\theta_l-\delta\omega)(2\delta\omega^2-(\theta_l(1-\delta)+2\theta_h\delta)\omega+2\theta_l(\theta_h-\theta_l)))}$
$v_{lb} = v_{0l}$			
p_l^*		$\frac{\theta_l(\theta_h-\theta_l)-\omega(3\theta_l-\theta_h)}{2(\theta_h-\theta_l-\omega)}$	
$N_l + N_b$		$\frac{\theta_h-\theta_l}{2(\theta_h-\theta_l-\omega)}$	

Scheme T-l with the following segmentation structure:

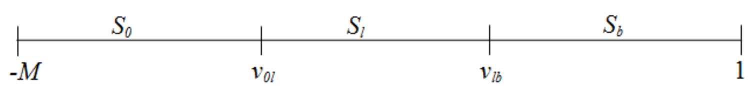

Table 3.5: Numerical Analysis with $\delta = 0.8$, $\theta_l = 2$, and $\theta_h = 3$ (with these values, we obtain $\hat{\omega} = 0.25$ and $\omega^* = 0.255$)

ω	$\hat{\theta}_h$	Full Compatibility $\pi_1^{h,U,s} - \pi_1^{l,U}$	Forward Compatibility $\pi_1^{h,F,s} - \pi_1^{l,F}$	Backward Compatibility $\pi_1^{h,B,s} - \pi_1^{l,B}$
0.100	0.086	0.145	0.164	0.148
0.200	0.049	0.056	0.095	0.069
0.240	-0.807	0.015	0.063	0.036
0.245	-1.926	0.010	0.059	0.032
0.251	11.580	0.003	0.054	0.027
0.253	4.082	0.001	0.052	0.025
0.260	1.465	-0.006	0.046	0.019
0.270	0.913	-0.017	0.037	0.011
0.280	0.737	-0.029	0.028	0.002
0.290	0.654	-0.040	0.019	-0.006
0.300	0.609	-0.052	0.010	-0.015
0.330	0.557	-0.088	-0.019	-0.041
0.350	0.550	-0.113	-0.040	-0.058

Table 3.6: Numerical Analysis with $\delta = 0.8$, $\theta_l = 2$, and $\theta_h = 10$ (with these values, we obtain $\hat{\omega} = 0.25$ and $\omega^* = 0.255$)

ω	$\hat{\theta}_h$	Full Compatibility $\pi_1^{h,U,s} - \pi_1^{l,U}$	Forward Compatibility $\pi_1^{h,F,s} - \pi_1^{l,F}$	Backward Compatibility $\pi_1^{h,B,s} - \pi_1^{l,B}$
0.100	0.086	0.145	0.162	0.135
0.200	0.049	0.054	0.092	0.042
0.240	-0.807	0.012	0.060	0.003
0.245	-1.926	0.007	0.056	-0.002
0.251	11.580	0.000	0.050	-0.008
0.253	4.082	-0.002	0.049	-0.010
0.260	1.465	-0.010	0.043	-0.017
0.270	0.913	-0.021	0.034	-0.028
0.280	0.737	-0.033	0.025	-0.038
0.290	0.654	-0.044	0.016	-0.048
0.300	0.609	-0.056	0.006	-0.059
0.330	0.557	-0.094	-0.023	-0.091
0.350	0.550	-0.120	-0.044	-0.112

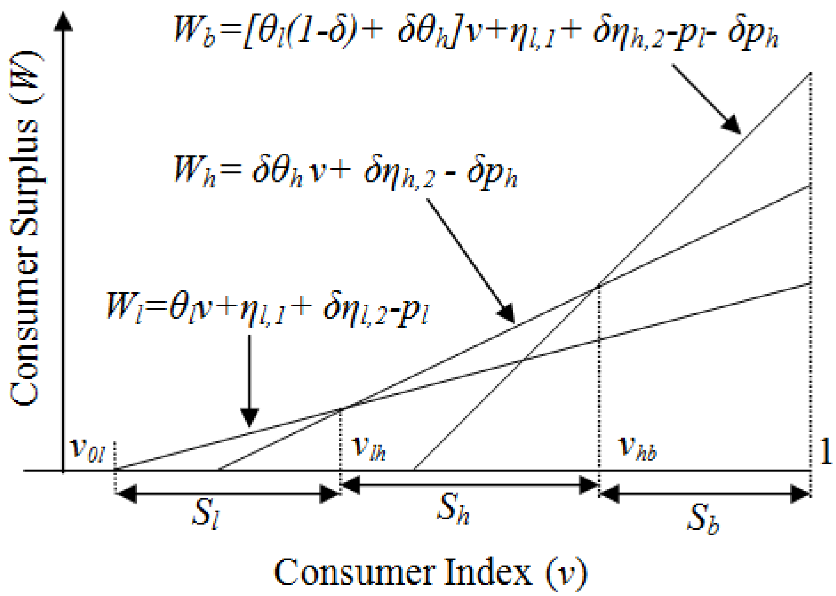

Figure 3.1: Market Segmentation Under Rapid Improvement

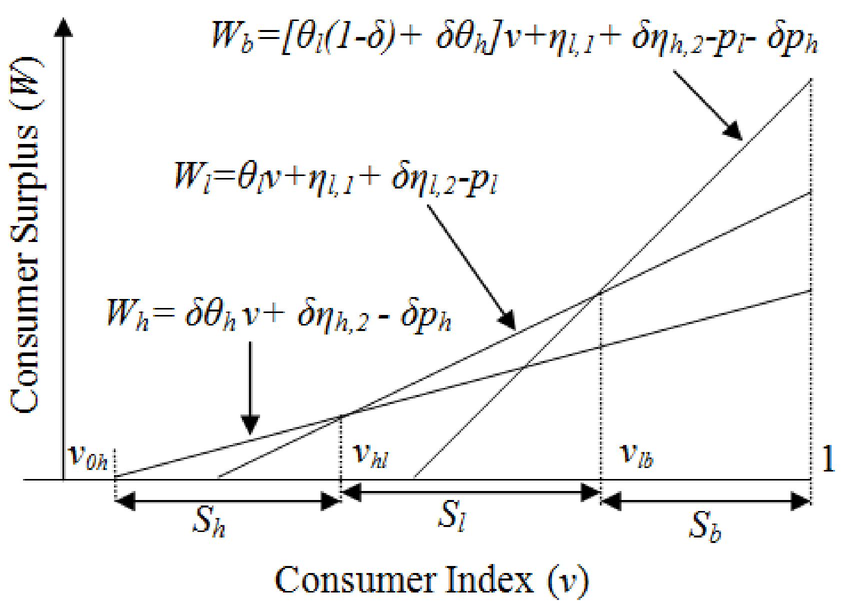

Figure 3.2: Market Segmentation Under Gradual Improvement

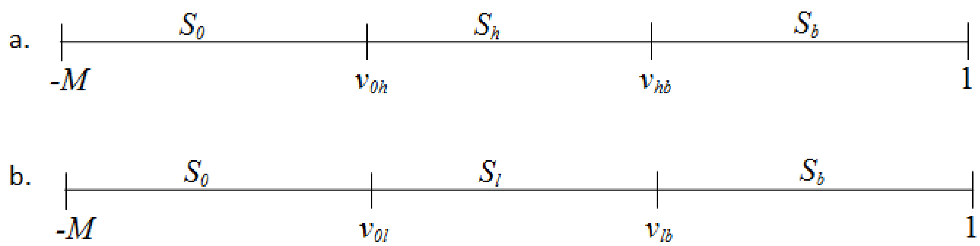

Figure 3.3: Segmentation Structure Under Rapid Improvement

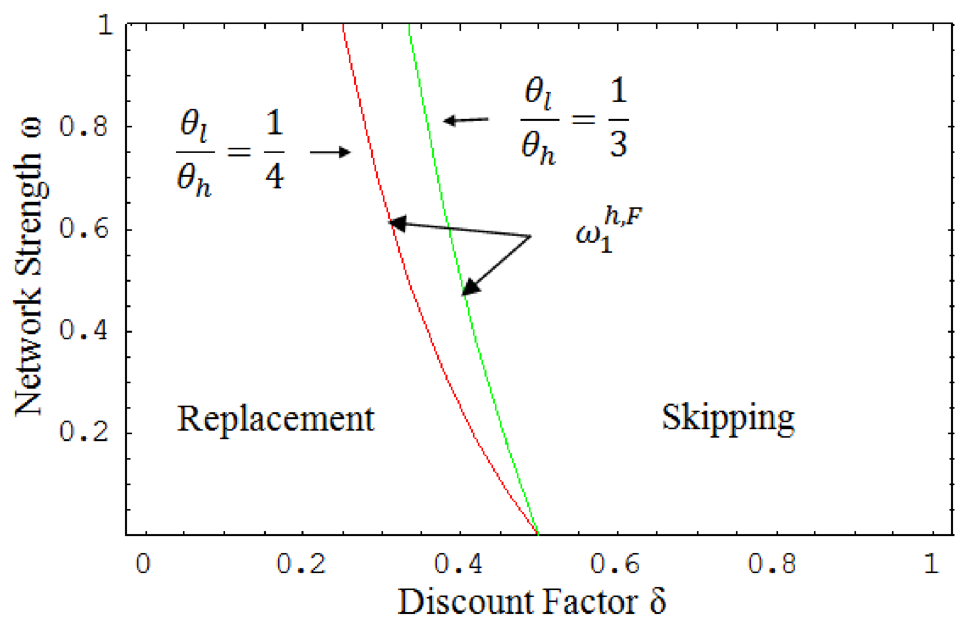

Figure 3.4: Product Strategies Under T-h and Full/Forward Compatibilities

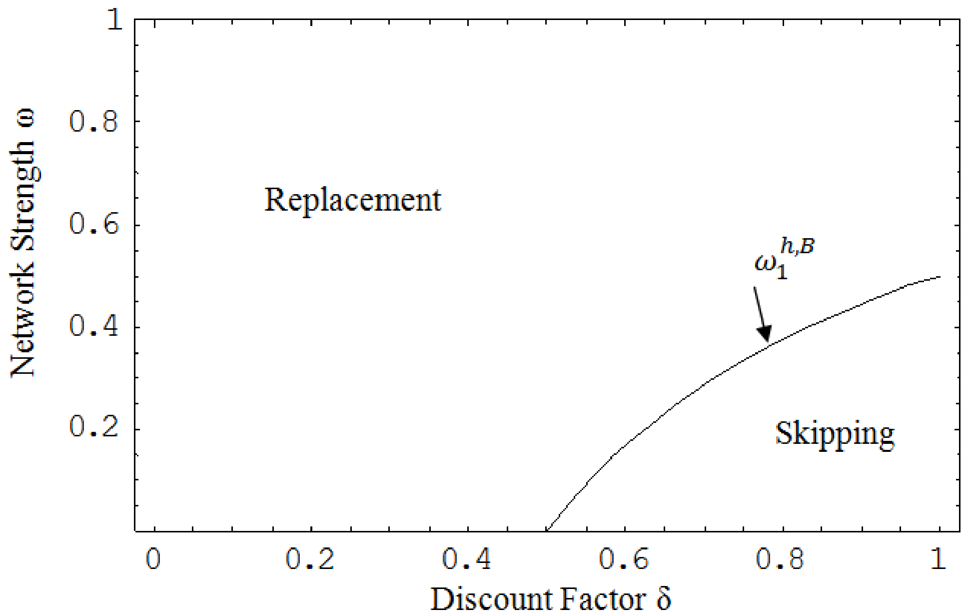

Figure 3.5: Product Strategies Under T-h and Backward Compatibility

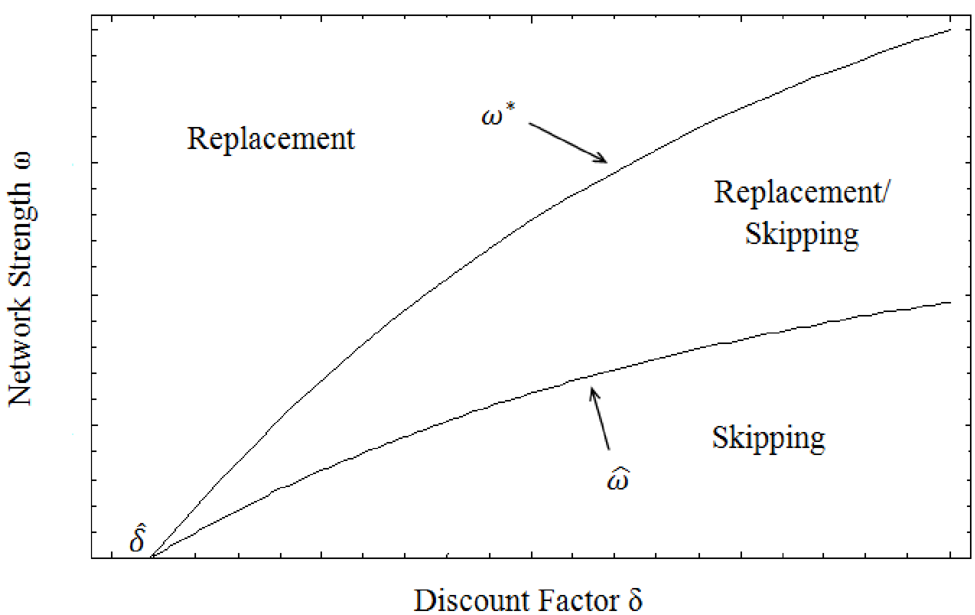

Figure 3.6: Product Strategies Under Rapid Improvement and Full Compatibility

CHAPTER 4: OPTIMAL SUPPLY CHAIN STRUCTURE: IMPACT OF QUANTITY COMPETITION

4.1 Introduction

Durable goods such as automobiles, aircraft, and consumer electronics are consumed over a long period of time and frequently require sizable investment from both consumers and producers. Hence, the impact of market saturation and consumers' forward looking has been long noticed in the durable goods literature. The fact that automakers take actions to influence the secondary market (Purohit and Staelin 1994, Purohit 1997) is a strong indication of how powerful used products can be in affecting the profit of new products. For other product categories like PCs, and iPads, although firms like Apple and HP do not try hard to control the secondary market, the threat of saturation is always around as noted by Wall Street Journal:

> *"The iPad used to be a novelty, but now every Tom, Jill and Nancy has them. Although there has been frenzy and even a riot outside Apple stores in China, the queues for new devices generated by Jobs' aesthetic vision are getting shorter in parts of Europe and the U.S."(March 4, 2012).*

In the extreme case of the Citizen's Band radio, the market was fully saturated after approximately five years, causing a crash in CB radio sales in 1977 (McAfee 2002). Given the unavoidable impact of saturation due to durability, and further, consumers are aware of that when making purchases (i.e., they are strategic), our interest is on how the firm would adapt to it by using supply chain design.

In practice, we observe different supply chain structures utilized in durable goods markets. For example, Intel provides microprocessors to many PC makers such as HP, Dell, Lenovo, Sony, etc.; Bosch Automotive Group, the world's largest supplier of automotive components (Automotive News, June 13, 2011) with sales of 30.4 billion euros in 2011 (www.bosch-press.com), is the component supplier for many auto makers such as Volkswagen, BMW, Audi, Porsche, Ford, GM, Chrysler, etc.; or ARM Holdings is the producer of microchip blueprints found in most mobile phones including Apple's iPhone (Wall Street Journal, April 24, 2012); Dolby develops a variety of audio technologies (www.dolby.com) that are used by many consumer electronics manufacturers, including Nokia, LG, HTC, Acer, Fujitsu, Sony, HP, Lenovo, etc. Obviously, the suppliers in these examples follow a component supplier structure.

Sometimes, the proprietary component manufacturer uses its component to make the end product and enters that market. For example, Bose Corporation produces speakers based on its proprietary technology; and Apple builds smartphones and iPads. Finally, the manufacturer may follow a dual distribution structure as in the case of Cannon which produces the laser printer using its print engine and supplies that engine to its competitor HP to make laser printers; or Sony provides its Trinitron TV picture tube to its competitor Toshiba.

Given different structures observed in different industries, extant literature has investigated their optimality in the context of static spacial competition (e.g. Venkatesh et al. 2006, Xu et al. 2010). However, little effort has been dedicated to incorporate the impact of durability on the choice of different supply chain structures. This chapter and the next take a step towards examining that impact.

More specifically, we notice that most durable goods are made via significant time consuming production processes. Hence, there is a friction in changing product quantities, i.e., companies are capacity constrained when selecting prices. For this reason, the majority of work in the durable goods literature takes quantity as an important decision variable (e.g., Bulow 1982, 1986, Purohit and Staelin 1994, Purohit 1995, Desai et al. 2004). However, the extant literature on the optimal choice among supply chain structures has focused mainly on price competition (e.g. Venkatesh et al. 2006, Xu et al. 2010). Further, there is a diversity in optimal firm behavior under price and quantity competition (Singh and Vives 1984, Arya et al. 2008c). Consequently, the purposes of this chapter are: (1) to compare and contrast the optimal supply chain design under price vis-a-vis quantity competition; (2) to link the above comparison with the findings from the existing work; (3) to provide a useful benchmark for the design of supply chain in a dynamic setting in Chapter 5.

We address these goals by building a static model in which a proprietary component manufacturer chooses among the three alternative supply chain structures outlined above. In the first, the manufacturer operates as a "component supplier" and sells the component to a downstream firm who then makes the end product. In the second structure, the manufacturer produces the end product using its component but does not make that component available to any other firms; here, the manufacturer operates as a "sole entrant." Finally, the manufacturer can operate as a "dual distributor" who not only makes the end product using its own component, but sells the component to a downstream firm who then competes against the manufacturer in the end product market. The reason for using a new setting (instead of existing model) is that we want to transition to utilize a vertical differentiation model in Chapter 5 (this modeling approach is applied in the dynamic context of durable goods) while existing work employs a horizontal differentiation model.

Extant literature (e.g., Venkatesh et al. 2006, Xu et al. 2010) shows that when the proprietary component manufacturer is a sole entrant, it could avoid the effect of double marginalization but suffers from a cost disadvantage due to lack of expertise in producing and selling the end products (Xu et al. 2010). Alternatively, a component supplier can leverage the expertise of the downstream firm in producing and marketing the end product but its market coverage is restricted due to double marginalization. Finally, the dual structure seems to lower the effect of double marginalization because the end market is more competitive (and has higher coverage); however, channel conflicts may dampen this advantage because the manufacturer has to reconcile between wholesale and end product profits.

Our analysis shows that the relative preference for different structures depends on the level of cost disadvantage and the level of product differentiation between the end products of the manufacturer and the downstream firm. More importantly, these preferences change from price to quantity competition, especially between the component supplier role and the dual distributor role. Compared to price competition, the firm's profitability is less disruptive under quantity competition in the following sense: when the firm incurs a higher level of cost disadvantage in producing the end product, under price competition, there is a steep drop in profitability as the firm transitions to a component supplier structure from a dual distribution structure. By contrast, under quantity competition, the change in profitability is more gradual. This feature allows the firm to gain higher profits in a certain parametric range of cost disadvantage under quantity competition.

The rationale of the findings is as follows. Under the distributor role, managing the wholesale and end product profits forces the manufacturer to lower its component's price offered to the downstream firm (compared to a component supplier structure). This effect is more detrimental under quantity competition; consequently, the dual distributor obtains lower profits here, than under price competition. However, the higher profitability of the dual distribution under price competition goes with an inability to sustain the structure at higher levels of cost disadvantage. In other words, a dual structure can arise for a range of the cost disadvantage under quantity competition but not under price competition; in that range, the quantity competition setting gives the manufacturer higher profits.

Finally, we link our results to the existing work by conducting the analysis using their setting but employing quantity competition. We show how our results fit into the context of their models. This exercise helps us reconcile the differences in moving from a horizontal differentiation to a vertical differentiation setting in the next chapter.

The rest of the chapter is organized as follows. We develop our model in the next section. The analysis and results are presented in Section 3. We link our results to the existing work in section 4 and conclude the chapter in Section 5.

4.2 The Model

A firm, m, is the sole supplier of an essential proprietary component for making the end product. To capture the value created by its component, m considers three different supply chain structures, $\{E, C, D\}$. In the first, denoted E, m will produce the end product using its component; when m implements E, we called it a sole entrant. In the second structure, denoted C, m sells the component to a downstream firm n ,who will make the end product; when m implements C, we call it a component supplier. Finally, m will operate as a dual distributor under structure D; here, m makes the end product using its own component as well as sells its component to n, who then becomes m's competitor in the end product market.

Let p_i^j and d_i^j be the end product's price and quantity of firm i ($i = \{m, n\}$) under structure j ($j = \{E, C, D\}$). When m is a sole entrant the end product's inverse demand is given by $p_m^E = a - bd_m^E$. When m is a component supplier, the end product's inverse demand of n is given by $p_n^C = a - bd_n^C$. When m is a dual distributor, the consumer demand of m and n's end products is given by the following inverse demand functions:

$$p_m^D = a - b(d_m^D + kd_n^D), \text{ and } p_n^D = a - b(d_n^D + kd_m^D) \tag{4.1}$$

Where a and b are positive constants. The parameter $k \in [0, 1]$ represents the degree of product homogeneity. Said differently, $(1-k)$ represents the degree of product differentiation;

when $k = 0$, the end products of m and n are independent; on the other hand, when $k = 1$, the two products are identical. [1]

We assume that each unit of the end product requires one unit of the component. The marginal cost of producing the component is assumed constant and normalized to zero. The marginal cost of making the end product is $\tau + c$ and τ for m and n respectively, where $\tau < a$ and $\tau + c < a$. The marginal cost differential between the two firms c can be positive or negative. If $c > 0$, m incurs a cost disadvantage when joining the end market; if $c < 0$, m has a cost advantage; thus, $-\tau \leq c < a - \tau$.

The timing of the game is as follows. There are two stages: (1) In the first stage, m selects the supply chain structure among $\{E, C, D\}$; (2) In the second stage, if m is a sole entrant, it will choose the end product's quantity to sell under quantity competition (or price under price competition) in the end market. If m is a component supplier, it will set the component's price; given that price, n then chooses the end product's quantity to sell under quantity competition (or price under price competition) in the end market. If m is a dual distributor, it will set the component's price; and given that price, n and m simultaneously and noncollusively choose the end product's quantities to sell under quantity competition (or prices under price competition) in the end market. Finally, consumers make purchases and the firms' profits are realized.

[1] In fact, the above inverse demand specification can be derived from an individual model as follows. Let a/b be the number of consumers on the market. Each consumer consumes at most one unit of product. The consumer maximum reservation price x is uniformly distributed with density $1/b$, along the interval $[0, a]$, $0 \leq x \leq a$. The reservation price for a product of firm i ($i = \{m, n\}$) of a consumer indexed x is $x - kbq_j$, where q_j is the quantity from firm j ($j \neq i, j = \{m, n\}$) on the market. Following Martin (2009), we obtain the quantity demanded of i is

$$q_i = \frac{1}{b} \int_{p_i}^{a-kbq_j} dx = \frac{a}{b} - kq_j - \frac{1}{b} p_i. \tag{4.2}$$

Analogously, we have the quantity demanded of j. Solving these two equations leads to the specification in (4.1).

4.3 The Analysis and Results

In this section, we derive the optimal supply chain structure under price competition as the benchmark. The analysis on quantity competition follows and is compared with the benchmark. It is important to notice that under either E or C, the end market is monopolized by the end product providers (m or n). Hence, in those two cases, the nature of competition does not change the equilibrium outcome.

4.3.1 The Benchmark

4.3.1.1 Sole Entrant

Under E, m's problem is specified as follows:

$$\max_{p_m^E} \Pi_m^E = (p_m^E - \tau - c)d_m^E \tag{4.3}$$

$$\text{subject to: } p_m^E \geq 0 \text{ and } d_m^E \geq 0 \tag{4.4}$$

The demand function is given by $d_m^E = \frac{a-p_m^E}{b}$. Taking the first-order condition gives $\frac{\partial \Pi_m^E}{\partial p_m^E} = (p_m^E - \tau - c)(-\frac{1}{b}) + d_m^E = 0 \Leftrightarrow p_m^E = \frac{a+\tau+c}{2}$. This solution is also the optimal one as the second order condition is satisfied ($\frac{\partial^2 \Pi_m^E}{\partial (p_m^E)^2} = -2/b < 0$). Hence, we obtain the following outcome:

$$p_m^E = \frac{a+\tau+c}{2} \tag{4.5}$$

$$d_m^E = \frac{a-\tau-c}{2b} \tag{4.6}$$

$$\Pi_m^E = \frac{(a-\tau-c)^2}{4b} \tag{4.7}$$

4.3.1.2 Component Supplier

Let w_m^C be the component's price set by the component supplier m. The consumer demand is given by $d_n^C = \frac{a-p_n^C}{b}$. Hence, m's problem is:

$$\max_{w_m^C} \Pi_m^C = w_m^C d_n^C \tag{4.8}$$

subject to

$$\max_{p_n^C} \Pi_n^C = (p_n^C - \tau - w_m^C) d_n^C \tag{4.9}$$

$$p_n^C \geq 0; w_m^C \geq 0; d_n^C \geq 0 \tag{4.10}$$

We solve the problem backward, for a given w_m^C, n's problem is solved analogously to the sole entrant's problem. Therefore, we get $p_n^C = \frac{a+\tau+w_m^C}{2}$ and $d_n^C = \frac{a-\tau-w_m^C}{2b}$. The component supplier then sets w_m^C via $\frac{\partial \Pi_m^C}{\partial w_m^C} = d_n^C + w_m^C(-\frac{1}{2b}) \Leftrightarrow w_m^C = \frac{a-\tau}{2}$. Since the second order condition is satisfied (i.e., $\frac{\partial^2 \Pi_m^C}{\partial (w_m^C)^2} = -1/b < 0$), the above solution is indeed the optimal solution. Hence, under C, we obtain:

$$p_n^C = \frac{3a+\tau}{4} \tag{4.11}$$

$$d_n^C = \frac{a-\tau}{4b} \tag{4.12}$$

$$\Pi_m^C = \frac{(a-\tau)^2}{8b} \tag{4.13}$$

$$\Pi_n^C = \frac{(a-\tau)^2}{16b} \tag{4.14}$$

4.3.1.3 Dual Distributor

The demand system under D is obtained via (4.1). It follows that

$$d_m^D = \frac{a(1-k) - p_m^D + kp_n^D}{b(1-k^2)}, \text{ and } d_n^D = \frac{a(1-k) - p_n^D + kp_m^D}{b(1-k^2)}, \ k \neq 1. \tag{4.15}$$

The dual distributor's problem is defined as follows:

$$\max_{w_m^D} \Pi_m^D = w_m^D d_n^D + (p_m^D - \tau - c) d_m^D \tag{4.16}$$

subject to

$$\max_{p_m^D} \Pi_m^D = w_m^D d_n^D + (p_m^D - \tau - c) d_m^D \tag{4.17}$$

$$\max_{p_n^D} \Pi_n^D = (p_n^D - \tau - w_m^D) d_n^D \tag{4.18}$$

$$p_m^D \geq 0; p_n^D \geq 0; w_m^D \geq 0. \tag{4.19}$$

In the above, m's profit includes two parts: the wholesale profit from selling the component to n (called the wholesale profit hereafter) and the end product profit. With backward induction, we solve the pricing game in the end market first. The end product's prices are determined by the following first order conditions (the second order conditions are satisfied, $\frac{\partial^2 \Pi_m^D}{\partial (p_m^D)^2} = \frac{\partial^2 \Pi_n^D}{\partial (p_n^D)^2} = \frac{-2}{b(1-k^2)} < 0$):

$$\frac{\partial \Pi_m^D}{\partial p_m^D} = d_m^D + (p_m^D - \tau - c)\left[\frac{-1}{b(1-k^2)}\right] + w_m^D \left[\frac{k}{b(1-k^2)}\right] = 0, \text{ and} \tag{4.20}$$

$$\frac{\partial \Pi_n^D}{\partial p_n^D} = d_n^D + (p_n^D - \tau - w_m^D)\left[\frac{-1}{b(1-k^2)}\right] = 0, \tag{4.21}$$

or equivalently,

$$a(1-k) + \tau + c - 2p_m^D + kp_n^D + kw_m^D = 0, \text{ and} \tag{4.22}$$

$$a(1-k) + \tau - 2p_n^D + kp_m^D + w_m^D = 0. \tag{4.23}$$

Here, m considers two streams of profit when choosing its end product's price: the wholesale profit and its own end product profit. Subsequently, it has an incentive to raise its end product's price to a higher level than when it only maximizes the end product's profit.

Solving these above equations gives

$$p_m^D = \frac{a(2-k-k^2)+\tau(2+k)+2c+3kw_m^D}{4-k^2}, \tag{4.24}$$

$$p_n^D = \frac{a(2-k-k^2)+\tau(2+k)+kc+(2+k^2)w_m^D}{4-k^2}. \tag{4.25}$$

The quantities sold in the market are

$$d_m^D = \frac{(1-k)((a-\tau)(2+k)-k(1+k)w_m^D)-(2-k^2)c}{b(4-5k^2+k^4)}, \tag{4.26}$$

$$d_n^D = \frac{(1-k)((a-\tau)(2+k)-2(1+k)w_m^D)+kc}{b(4-5k^2+k^4)}. \tag{4.27}$$

It is worth noting that the component's price w_m^D (which is a part of n's cost) negatively affects m's quantity; said differently, m sells less of the product when its competitor incurs a higher cost. It happens because as w_m^D goes up, m obtains higher wholesale profits, which in turn motivates it to increase its end product's price more (see the first order condition above), and subsequently, lowering its own end product's demand.

Next, m selects the component's price offered to n. The price is set by solving $\frac{\partial \Pi_m^D}{\partial w_m^D} = 0$; it follows that $w_m^D = \frac{(a-\tau)(8+k^3)-k^3c}{2(8+k^2)}$ (the second order condition is satisfied, i.e., $\frac{\partial^2 \Pi_m^D}{\partial (w_m^D)^2} = \frac{-2(8+k^2)}{b(4-k^2)^2} < 0$). The equilibrium outcome is provided in Table 4.1.

Notice that the dual distribution is feasible when $d_m^D > 0$ and $d_n^D > 0$; said differently, it requires $c_n^D < c < c_m^D$, where $c_m^D = \frac{(8-6k-k^2-k^4)(a-\tau)}{8-k^2-k^4}$ and $c_n^D = -\frac{(1-k)(a-\tau)}{k}$.

The following proposition characterizes the optimal supply chain structure under price competition

Proposition 4.1 *Under price competition, m is*

(i) a sole entrant when $c \leq c_n^D$*;*

(ii) a component supplier when $c \geq c_m^D$*; and*

(iii) a dual distributor whenever it is feasible, i.e., $c_n^D < c < c_m^D$.

Proof: The profits of m under $\{E, C, D\}$ are provided in Table 4.1. It follows that

$$\Pi_m^D - \Pi_m^E = \frac{(ck + (1-k)(a-\tau))^2}{b(8-k^4-7k^2)} \geq 0 \tag{4.28}$$

$$\Pi_m^D - \Pi_m^C = \frac{2c^2(8-k^4-3k^2) + 4c(8-k(k^3+3k+4))(a-\tau) + (1-k)(k^3+k^2+16)(a-\tau)^2}{8b(8-k^4-7k^2)} \geq 0 \tag{4.29}$$

$$\Pi_m^E - \Pi_m^C = \frac{2c^2 - 4c(a-\tau) + (a-\tau)^2}{8b} \geq 0 \text{ for all } c \leq \frac{(2-\sqrt{2})(a-\tau)}{2}. \tag{4.30}$$

Hence, D is optimal whenever it is feasible. When D is not feasible, E is optimal for all $c \leq c_n^D$ and C is optimal for all $c \geq c_m^D$. ■

The above results which are broadly consistent with those in the literature (e.g., Venkatesh et al. 2006) are illustrated by Figure 4.1. From there, we can see that for a large parametric region, D is the dominant structure. It allows the firm to charge the end product's price even higher than the one of a sole entrant (i.e., $p_m^D \geq p_m^E$), further, m puts the highest number of the component in the market among all supply chain structures. The downside of D is that the component's price is lower than the one under C.

More importantly, the proposition also specifies the feasibility of D. When m incurs a relatively high cost disadvantage, the end product's price is boosted to a level such that the market cannot bear; hence, m operates as a component supplier instead. Or conversely, when m has a relatively high cost advantage, the gain from that advantage can overcome the loss from not selling the component to n; consequently, m forecloses n and becomes a sole entrant.

4.3.2 The Optimal Supply Chain Structure Under Quantity Competition

As noted above, the equilibrium outcome of E or C is unchanged by the nature of competition. We therefore only analyze D under quantity competition. We use the superscript 'q' to denote the results under quantity competition. The inverse demand is:

$$p_m^{D,q} = a - b(d_m^{D,q} + kd_n^{D,q}), \text{ and } p_n^{D,q} = a - b(d_n^{D,q} + kd_m^{D,q}) \tag{4.31}$$

Starting from the last stage, quantities are chosen from the following first order conditions (the second order conditions are satisfied, i.e., $\frac{\partial^2 \Pi_m^{D,q}}{\partial (d_m^{D,q})^2} = \frac{\partial^2 \Pi_n^{D,q}}{\partial (d_n^{D,q})^2} = -2b < 0$):

$$\frac{\partial \Pi_m^{D,q}}{\partial d_m^{D,q}} = (-b)d_m^{D,q} + p_m^{D,q} - \tau - c = 0 \tag{4.32}$$

$$\frac{\partial \Pi_n^{D,q}}{\partial d_n^{D,q}} = (-b)d_n^{D,q} + p_n^{D,q} - \tau - w_m^{D,q} = 0. \tag{4.33}$$

Or equivalently,

$$d_m^{D,q} = \frac{(2-k)(a-\tau) - 2c + kw_m^{D,q}}{b(4-k^2)}, \text{ and} \tag{4.34}$$

$$d_n^{D,q} = \frac{(2-k)(a-\tau) + ck - 2w_m^{D,q}}{b(4-k^2)} \tag{4.35}$$

Notice that m does not consider its wholesale profit when choosing the end product's quantity. Substituting the above into $\Pi_m^{D,q}$ and solving for $w_m^{D,q}$ yields $w_m^{D,q} = \frac{(k^3-4k^2+8)(a-\tau)-ck^3}{16-6k^2}$. The equilibrium outcome is in Table 4.1.

Note that the dual distribution structure is feasible whenever $d_m^{D,q} > 0$ and $d_n^{D,q} > 0$, or equivalently, $c_n^{D,q} < c < c_m^{D,q}$, where $c_m^{D,q} = \frac{8-k^2-2k}{8-k^2}$, and $c_n^{D,q} = -\frac{(1-k)(a-\tau)}{k}$. Define $c_{dc}^{D,q} = \frac{2(k^2-4k+8)-k\sqrt{2(8-3k^2)}}{2(k^2+8)}(a-\tau)$.

The following proposition characterizes the optimal supply chain structure:

Proposition 4.2 *Under quantity competition, m is*

(i) a sole entrant when $c \leq c_n^{D,q}$;

(ii) a component supplier when $c \geq c_{dc}^{D,q}$; and

(iii) a dual distributor when $c_n^{D,q} < c < c_{dc}^{D,q}$.

Proof: The profits of m under $\{E, C, D\}$ are provided in Table 4.1. It follows that

$$\Pi_m^{D,q} - \Pi_m^E = \frac{(ck + (1-k)(a-\tau))^2}{b(8-3k^2)} \geq 0 \tag{4.36}$$

$$\Pi_m^{D,q} - \Pi_m^C = \frac{2c^2(k^2+8) - 4c(k^2-4k+8)(a-\tau) + (5k^2-16k+16)(a-\tau)^2}{8b(8-3k^2)}. \tag{4.37}$$

Combining with the feasibility region of D gives $\Pi_m^{D,q} > \Pi_m^C$ whenever $c < c_{dc}^{D,q}$ (where $c_{dc}^{D,q} \leq c_m^{D,q}$) and the other results. ■

We use Figure 4.2 to illustrate the above proposition. Under quantity competition, the profitability of D decreases at higher levels of c and eventually is dominated by C before it becomes infeasible. This contrasts with what happens under price competition where the dual distribution is optimal whenever it is feasible. Since the profit of C is independent of c, the change occurs because the quantity competition setting lowers the profitability of D. Recall that m ignores its wholesale profit when choosing the end product's quantity, which indicates that m becomes more aggressive in the end market; anticipating this behavior, n lowers its quantity (as quantities are strategic substitutes), causing m to lower the component's price to induce n to buy more of the component ($w_m^{D,q} \leq w_m^D$). Such an inducement proves too costly for m. When m has a high cost advantage, analogous to price competition, m forecloses n and becomes a sole entrant at higher levels of the cost advantage.

Proposition 4.3 *For $c_m^D \leq c \leq c_{dc}^{D,q}$, m implements C under price competition but D under quantity competition; further, m's profitability under quantity competition is higher within this region.*

Proof: We have $c_m^D \leq c_{dc}^{D,q}$ for all $k \in [0, 1]$. The higher profitability of m comes directly from Proposition 4.2. ■

Although the profitability of D is reduced under quantity competition, D is sustained at higher levels of cost disadvantage. In fact, consumers are worse off under price competition and block this structure by not buying the product from m. In particular, m is inclined to raise the end product's price to protect wholesale profit, plus it lowers the wholesale's price (vis-a-vis the one under C); this behavior substantially restricts its own reach in the end market, especially when the cost differential is large (as the impact of c on d_m^D is larger than on $d_m^{D,q}$).

In conclusion, when selecting D under either price or quantity competition, m sets a lower component's price (vis-a-vis the one under C). While raising its end product's price to secure the wholesale profit under price competition, m does not restrict its end product's quantity to protect the wholesale profit under quantity competition. Instead, it sets an even lower component price later under quantity competition (vis-a-vis under price competition). Though m's behavior is different under the two settings, the purpose of its action is to secure the wholesale profit. The consequence of this action is more detrimental under quantity competition (i.e., lower profitability). However, this structure is sustained at higher levels of cost disadvantage; in other words, m obtains higher profits under quantity competition at these levels because m operates as a component supplier under price competition and subsequently obtains lower profits.

4.4 Links to Existing Work

This section revisits the models in existing work (Venkatesh et al. 2006 and Xu et al. 2010) and shows how the optimal supply chain design changes when the nature of competition changes from price to quantity competition. Both papers employ price competition in a horizontal differentiation context. To facilitate the comparison, we adopt most of the notation in their papers. We use the superscript 'V' to denote the values of key variables in Veskatesh et al. (2006). Similarly, we use the superscript 'X' in Xu et al. (2010).

4.4.0.1 The Model from Venkatesh et al. (2006)

The demand structure is given by the following:

$$\text{Sole Entrant: } d_m^{E,V} = \frac{2(R - p_m^{E,V})}{\beta} \tag{4.38}$$

$$\text{Component Supplier: } d_n^{C,V} = \frac{2(R + q - p_n^{C,V})}{\beta} \tag{4.39}$$

$$\text{Dual Distributor: } d_m^{D,V} = \frac{2R - q + f\beta - 3p_m^{D,V} + p_n^{D,V}}{2\beta}, \text{ and} \tag{4.40}$$

$$d_n^{D,V} = \frac{2R + 3q + f\beta - 3p_n^{D,V} + p_m^{D,V}}{2\beta} \tag{4.41}$$

where R is the consumer's reservation of the end product of m; $R + q$ is of n's; β is the transportation cost per unit length; f is the distance between m and n. Notice that the dual distributor is called the 'Co-opter' in Venkatesh et al. (2006).

From the demand structure specified above, we obtain the equivalent inverse demand system:

$$\text{Sole Entrant: } p_m^{E,V} = R - \frac{\beta d_m^{E,V}}{2} \tag{4.42}$$

$$\text{Component Supplier: } p_n^{C,V} = R + q - \frac{\beta d_n^{C,V}}{2} \tag{4.43}$$

$$\text{Dual Distributor: } p_m^{D,V} = R + \frac{f\beta}{2} - \frac{3\beta d_m^{D,V}}{4} - \frac{\beta d_n^{D,V}}{4}, \text{ and} \tag{4.44}$$

$$p_n^{D,V} = R + q + \frac{f\beta}{2} - \frac{3\beta d_m^{D,V}}{4} - \frac{\beta d_n^{D,V}}{4} \tag{4.45}$$

Since the equilibrium outcome of the sole entrant and component supplier roles is unchanged regardless of the nature of competition, we focus on the dual distributor role and compare the new result under quantity competition with the current work. The results under price competition are (from Venkatesh et al. 2006):

Sole entrant:

$$p_m^{E,V} = R/2 \tag{4.46}$$

$$d_m^{E,V} = R/\beta \tag{4.47}$$

$$\Pi_m^{E,V} = \frac{R^2}{2\beta} \tag{4.48}$$

Component supplier:

$$w_m^{C,V} = (R+q)/2 \tag{4.49}$$

$$p_n^{C,V} = \frac{3(R+q)}{4} \tag{4.50}$$

$$d_n^{C,V} = \frac{R+q}{2\beta} \tag{4.51}$$

$$\Pi_m^{C,V} = \frac{(R+q)^2}{4\beta} \tag{4.52}$$

$$\Pi_n^{C,V} = \frac{(R+q)^2}{8\beta} \tag{4.53}$$

Dual distributor:

$$w_m^{D,V} = \frac{432q + 434R + 271f\beta}{876} \tag{4.54}$$

$$p_m^{D,V} = \frac{12q + 77(2R + f\beta)}{292} \tag{4.55}$$

$$p_n^{D,V} = \frac{660q + 293(2R + f\beta)}{876} \tag{4.56}$$

$$d_m^{D,V} = \frac{-81q + 119(2R + f\beta)}{438\beta} \tag{4.57}$$

$$d_n^{D,V} = \frac{19(3q + 2R + f\beta)}{146\beta} \tag{4.58}$$

$$\Pi_m^{D,V} = \frac{162q^2 + 108q(2R + f\beta) + 91(2R + f\beta)^2}{876\beta} \quad (4.59)$$

$$\Pi_n^{D,V} = \frac{361(3q + 2R + f\beta)^2}{31974\beta} \quad (4.60)$$

Notice that for dual distribution to arise, we require a lower bound and an upper bound on $f\beta$. The lower bound guarantees the stability of equilibrium by requiring: at n's location, the consumer's surplus from buying n's product is larger than from buying m's product; and the consumer's surplus from buying m's product is larger than that from buying n's product at m's location. In other words, $R + q - p_n^{D,V} > R - p_m^{D,V} - f\beta$ and $R - p_m^{D,V} > R + q - p_n^{D,V} - f\beta$. It follows that $f\beta > \{\frac{62R-162q}{407}, \frac{126q-62R}{469}\}$. The upper bound requires the distance between m and n cannot exceed the distance at which the indifferent consumer x gets zero surplus, or equivalently, $R - p_m^{D,V} - \beta e = R + q - p_n^{D,V} - \beta(f - e) = 0$ (where e is the distance of x from m). It follows that $f\beta \leq \frac{45q+176R}{350}$. Hence, the feasibility of the dual distributor role outcome is:

$$\max\{\frac{62R - 162q}{407}, \frac{126q - 62R}{469}\} < f\beta \leq \frac{45q + 176R}{350\beta}. \quad (4.61)$$

It is important to point out that with the stability constraints, the firms' demand never approaches zero. So feasibility is not an issue in Venkatesh et al. (2006) (in contrast to Proposition 4.1).

The optimal supply chain structure is summarized in the following lemma:

Lemma 4.1 *Under price competition and* (4.61)*, m will be*

(1) a sole entrant when $f\beta \leq f_{ed}^V$ and $q \leq (\sqrt{2} - 1)R$;

(2) a component supplier when $f\beta \leq f_{cd}^V$ and $q \geq (\sqrt{2} - 1)R$;

(3) a dual distributor when $f\beta > \max\{f_{ed}^V, f_{cd}^V\}$.

Proof: We have

$$\Pi_m^{D,V} - \Pi_m^{E,V} = \frac{162q^2 - 438R^2 + 108q(2R + f\beta) + 91(2R + f\beta)^2}{876\beta} \geq 0 \text{ for all } f\beta \geq f_{ed}^V \tag{4.62}$$

$$\Pi_m^{D,V} - \Pi_m^{C,V} = \frac{162q^2 - 219(R + q)^2 + 108q(2R + f\beta) + 91(2R + f\beta)^2}{876\beta} \geq 0 \text{ for all } f\beta \geq f_{cd}^V \tag{4.63}$$

$$\Pi_m^{E,V} - \Pi_m^{C,V} = -\frac{q^2 + 2qR - R^2}{4\beta} \geq 0 \text{ for all } q \leq (\sqrt{2} - 1)R. \tag{4.64}$$

where $f_{ed}^V = \frac{-54q-182R+\sqrt{438(91R^2-27q^2)}}{91}$, and $f_{cd}^V = \frac{-54q-182R+\sqrt{219(37q^2+182qR+91R^2)}}{91}$. Consequently, the dual distributor role is optimal when $f\beta > \max\{f_{ed}^V, f_{cd}^V\}$. ■

Now, we derive the dual distributor outcome under quantity competition based on the inverse demand described above. m's problem is defined as follows:

$$\max_{w_m^{C,V}} \Pi_m^{D,V} = p_m^{D,V} d_m^{D,V} + w_m^{C,V} d_n^{D,V} \tag{4.65}$$

subject to:

$$\max_{d_m^{D,V}} \Pi_m^{D,V} = p_m^{D,V} d_m^{D,V} + w_m^{C,V} d_n^{D,V} \tag{4.66}$$

$$\max_{d_n^{D,V}} \Pi_n^{D,V} = (p_n^{D,V} - w_m^{C,V}) d_n^{D,V} \tag{4.67}$$

$$d_n^{D,V} \geq 0; d_m^{D,V} \geq 0; w_m^{C,V} \geq 0. \tag{4.68}$$

Under backward induction, the quantities are chosen via the following first-order conditions:

$$\frac{\partial \Pi_m^{D,V}}{\partial d_m^{D,V}} = \Big[-\frac{3\beta}{4}\Big] d_m^{D,V} + \Big[R + \frac{f\beta}{2} - \frac{3\beta d_m^{D,V}}{4} - \frac{\beta d_n^{D,V}}{4}\Big] = 0 \tag{4.69}$$

$$\frac{\partial \Pi_n^{D,V}}{\partial d_n^{D,V}} = \Big[-\frac{3\beta}{4}\Big] d_n^{D,V} + \Big[R + q + \frac{f\beta}{2} - \frac{3\beta d_m^{D,V}}{4} - \frac{\beta d_n^{D,V}}{4} - w_m^{C,V}\Big] = 0 \tag{4.70}$$

Analogous to our model, here, m does not consider the wholesale profit when choosing its end product's quantity. Solving the above gives:

$$d_m^{D,V} = \frac{2(10R - 2q + 5f\beta + 2w_m^{C,V})}{35\beta} \tag{4.71}$$

$$d_n^{D,V} = \frac{2(10R + 12q + 5f\beta - 12w_m^{C,V})}{35\beta}. \tag{4.72}$$

Substituting these expressions into $\Pi_m^{D,V}$ and solving for $w_m^{C,V}$ gives $w_m^{C,V} = \frac{408q+205(2R+f\beta)}{828}$. Hence, the equilibrium outcome is:

$$w_m^{D,V} = \frac{408q + 205(2R + f\beta)}{828} \tag{4.73}$$

$$p_m^{D,V} = \frac{-12q + 65(2R + f\beta)}{276} \tag{4.74}$$

$$p_n^{D,V} = \frac{624q + 277(2R + f\beta)}{828} \tag{4.75}$$

$$d_m^{D,V} = \frac{-12q + 65(2R + f\beta)}{207\beta} \tag{4.76}$$

$$d_n^{D,V} = \frac{8(3q + 2R + f\beta)}{69\beta} \tag{4.77}$$

$$\Pi_m^{D,V} = \frac{144q^2 + 96q(2R + f\beta) + 85(2R + f\beta)^2}{828\beta} \tag{4.78}$$

$$\Pi_n^{D,V} = \frac{16(3q + 2R + f\beta)^2}{1587\beta}. \tag{4.79}$$

The lower and upper bounds of $f\beta$ include:

(1) $R + q - p_n^{D,V} > R - p_m^{D,V} - f\beta$ and $R - p_m^{D,V} > R + q - p_n^{D,V} - f\beta$. It follows that $f\beta > \{\frac{2(42q-41R)}{455}, \frac{2(41R-42q}{373}\}$.

(2) The condition on the distance f where n and m's markets are overlapped is obtained from the indifferent consumer x with zero surplus, or equivalently, $R - p_m^{D,V} - \beta e = R + q - p_n^{D,V} - \beta(f - e) = 0$ (where e is the distance of x from m). It follows that $f\beta \leq \frac{2(30q+89R)}{325}$.

Hence, the feasibility of the dual distributor role outcome is:

$$\max\{\frac{2(42q-41R)}{455}, \frac{2(41R-42q)}{373}\} < f\beta \leq \frac{2(30q+89R)}{325}. \tag{4.80}$$

It is important to point out that with the stability constraints, the firms' demand never approaches zero.

The optimal supply chain structure is summarized in the following lemma:

Lemma 4.2 *Under quantity competition and* (4.80)*, m will be*

(1) a sole entrant when $f\beta \leq f_{ed}^{V,q}$ and $q \leq (\sqrt{2}-1)R$;

(2) a component supplier when $f\beta \leq f_{cd}^{V,q}$ and $q \geq (\sqrt{2}-1)R$;

(3) a dual distributor when $f\beta > \max\{f_{ed}^{V,q}, f_{cd}^{V,q}\}$

Proof: We have

$$\Pi_m^{D,V} - \Pi_m^{E,V} = \frac{144q^2 - 414R^2 + 96q(2R+f\beta) + 85(2R+f\beta)^2}{828\beta} \geq 0 \text{ for all } f\beta \geq f_{ed}^{V,q} \tag{4.81}$$

$$\Pi_m^{D,V} - \Pi_m^{C,V} = \frac{144q^2 - 207(R+q)^2 + 96q(2R+f\beta) + 85(2R+f\beta)^2}{828\beta} \geq 0 \text{ for all } f\beta \geq f_{cd}^{V,q} \tag{4.82}$$

$$\Pi_m^{E,V} - \Pi_m^{C,V} = -\frac{q^2 + 2qR - R^2}{4\beta} \geq 0 \text{ for all } q \leq (\sqrt{2}-1)R. \tag{4.83}$$

where $f_{ed}^{V,q} = \frac{-48q-170R+3\sqrt{46(85R^2-24q^2)}}{85}$, and $f_{cd}^{V,q} = \frac{-48q-170R+3\sqrt{23(37q^2+170qR+85R^2)}}{85}$. Consequently, the dual distributor role is optimal when $f\beta > \max\{f_{ed}^{V,q}, f_{cd}^{V,q}\}$. ■

Next, the following Proposition compares the optimal region of dual distribution under price and quantity competition.

Proposition 4.4 *Under the conditions* (4.61) *and* (4.80)*:*

The optimal region of dual distribution is smaller under quantity competition than under price competition, i.e., $f_{ed}^{V,q} \geq f_{ed}^{V}$ *and* $f_{cd}^{V,q} \geq f_{cd}^{V}$.

Proof: The results come straightforward from $f_{ed}^{V,q} - f_{ed}^{V} \geq 0$ and $f_{cd}^{V,q} - f_{cd}^{V} \geq 0$.

4.4.0.2 The Model from Xu et al. (2010)

The demand structure is given by

$$\text{Sole Entrant: } d_m^{E,X} = \frac{2(V - p_m^{E,X})}{t} \tag{4.84}$$

$$\text{Component Supplier: } d_n^{C,X} = \frac{2(V - p_n^{C,X})}{t} \tag{4.85}$$

$$\text{Dual Distributor: } d_m^{D,X} = \frac{2V + ft - 3p_m^{D,X} + p_n^{D,X}}{2t}, \text{ and} \tag{4.86}$$

$$d_n^{D,X} = \frac{2V + ft - 3p_n^{D,X} + p_m^{D,X}}{2t} \tag{4.87}$$

where V is the consumer's reservation of the end product from both m and n; t is the transportation cost per unit length; f is the distance between m and n. Notice that the sole entrant is called the 'Monopoly' in Xu et al. (2010). m incurs a cost disadvantage c when producing the end product. The equivalent inverse demand system is

$$\text{Sole Entrant: } p_m^{E,X} = V - \frac{td_m^{E,X}}{2} \tag{4.88}$$

$$\text{Component Supplier: } p_n^{C,X} = V - \frac{td_n^{C,X}}{2} \tag{4.89}$$

$$\text{Dual Distributor: } p_m^{D,X} = V + \frac{ft}{2} - \frac{3td_m^{D,X}}{4} - \frac{td_n^{D,X}}{4}, \text{ and} \tag{4.90}$$

$$p_n^{D,X} = V + \frac{ft}{2} - \frac{3td_m^{D,X}}{4} - \frac{td_n^{D,X}}{4} \tag{4.91}$$

Analogous to the model from Venkatesh et al. (2006), we focus on the dual distributor role and compare the new result under quantity competition with the current work. The results under price competition are (from Xu et al. 2010):

Sole entrant:

$$p_m^{E,X} = (V+c)/2 \tag{4.92}$$

$$d_m^{E,X} = (V-c)/t \tag{4.93}$$

$$\Pi_m^{E,X} = \frac{(V-c)^2}{2t} \tag{4.94}$$

Component supplier:

$$w_m^{C,X} = V/2 \tag{4.95}$$

$$p_n^{C,X} = \frac{3V}{4} \tag{4.96}$$

$$d_n^{C,X} = \frac{V}{2t} \tag{4.97}$$

$$\Pi_m^{C,X} = \frac{V^2}{4t} \tag{4.98}$$

$$\Pi_n^{C,X} = \frac{V^2}{8t} \tag{4.99}$$

Dual distributor:

$$w_m^{D,X} = \frac{-2c+217(2V+ft)}{876} \tag{4.100}$$

$$p_m^{D,X} = \frac{150c+77(2V+ft)}{292} \tag{4.101}$$

$$p_n^{D,X} = \frac{74c+293(2V+ft)}{876} \tag{4.102}$$

$$d_m^{D,X} = \frac{-319c+119(2V+ft)}{438t} \tag{4.103}$$

$$d_n^{D,X} = \frac{19(c+2V+ft)}{146t} \tag{4.104}$$

$$\Pi_m^{D,X} = \frac{310c^2 - 256c(2V + ft) + 91(2V + ft)^2}{876t} \quad (4.105)$$

$$\Pi_n^{D,X} = \frac{361(c + 2V + ft)^2}{31974t} \quad (4.106)$$

Analogous to Venkatesh et al. (2006), the outcome of the dual distribution requires a lower bound and an upper bound on ft. The lower bound guarantees the stability of equilibrium by requiring: at n's location, the consumer's surplus from buying n's product is larger than from buying m's product; and the consumer's surplus from buying m's product is larger than that from buying n's product at m's location. In other words, $V - p_n^{D,X} > V - p_m^{D,X} - ft$ and $V - p_m^{D,X} > V - p_n^{D,X} - ft$. It follows that $ft > \{\frac{2(31V-94c)}{407}, \frac{2(94c-31V)}{469}\}$. The upper bound requires the distance between m and n cannot exceed the distance at which the indifferent consumer x gets zero surplus, or equivalently, $V - p_m^{D,X} - te = V - p_n^{D,X} - t(f-e) = 0$ (where e is the distance of x from m). It follows that $ft \leq \frac{176V-131c}{350}$. Hence, the feasibility of the dual distributor role outcome is:

$$\max\{\frac{2(31V - 94c)}{407}, \frac{2(94c - 31V)}{469}\} < ft \leq \frac{176V - 131c}{350}. \quad (4.107)$$

It is important to point out that with the stability constraints, the firms's demand never approaches zero.

The optimal supply chain structure is summarized in the following lemma:

Lemma 4.3 *Under price competition and* (4.107)*, m will be*

(1) a sole entrant when $ft \leq f_{ed}^X$ and $c \leq \frac{(2-\sqrt{2})V}{2}$;

(2) a component supplier when $ft \leq f_{cd}^X$ and $c \geq \frac{(2-\sqrt{2})V}{2}$;

(3) a dual distributor when $ft > \max\{f_{ed}^X, f_{cd}^X\}$.

Proof: The profit comparison under price competition gives: $\Pi_m^{D,X} - \Pi_m^{E,X} \geq 0$ when $ft \geq f_{ed}^X$, where $f_{ed}^X = \frac{128c-182R+\sqrt{438(64c^2-182cV+91V^2)}}{91}$; $\Pi_m^{D,X} - \Pi_m^{C,X} \geq 0$ when $ft \geq f_{cd}^X$, where $f_{cd}^X =$

$\frac{128c-182V+\sqrt{219(91V^2-54c^2)}}{91}$; and $\Pi_m^{E,X} - \Pi_m^{C,X} \geq 0$ when $c \leq \frac{(2-\sqrt{2})V}{2}$. Consequently, the dual distributor role is optimal when $ft > \max\{f_{ed}^X, f_{cd}^X\}$. ■

Now, we derive the dual distributor outcome under quantity competition based on the inverse demand described above. m's problem is defined as follows:

$$\max_{w_m^{D,X}} \Pi_m^{D,X} = p_m^{D,X} d_m^{D,X} + w_m^{D,X} d_n^{D,X} \tag{4.108}$$

subject to:

$$\max_{d_m^{D,X}} \Pi_m^{D,X} = p_m^{D,X} d_m^{D,X} + w_m^{D,X} d_n^{D,X} \tag{4.109}$$

$$\max_{d_n^{D,X}} \Pi_n^{D,X} = (p_n^{D,X} - w_m^{D,X}) d_n^{D,X} \tag{4.110}$$

$$d_n^{D,X} \geq 0; d_m^{D,X} \geq 0; w_m^{D,X} \geq 0. \tag{4.111}$$

The equilibrium outcome is given by:

$$w_m^{D,X} = \frac{-2c + 205(2V + ft)}{828} \tag{4.112}$$

$$p_m^{D,X} = \frac{134c + 65(2V + ft)}{276} \tag{4.113}$$

$$p_n^{D,X} = \frac{70c + 277(2V + ft)}{828} \tag{4.114}$$

$$d_m^{D,X} = \frac{-142c + 65(2V + ft)}{207t} \tag{4.115}$$

$$d_n^{D,X} = \frac{8(c + 2V + ft)}{69t} \tag{4.116}$$

$$\Pi_m^{D,X} = \frac{292c^2 - 244c(2V + ft) + 85(2V + ft)^2}{828t} \tag{4.117}$$

$$\Pi_n^{D,X} = \frac{16(c + 2V + ft)^2}{1587t}. \tag{4.118}$$

Analogous to Venkatesh et al. (2006), the restrictions on ft include

$$\max\{\frac{2(83c - 41V)}{455}, \frac{2(41V - 83c)}{373}\} < ft \leq \frac{2(-59c + 89V)}{325}. \tag{4.119}$$

It is important to point out that with these stability constraints, the firms' demand under dual distribution never approaches zero.

The optimal supply chain structure is summarized in the following lemma:

Lemma 4.4 *Under quantity competition and* (4.119), *m will be*

(1) a sole entrant when $ft \leq f_{ed}^{X,q}$ and $c \leq \frac{(2-\sqrt{2})V}{2}$;

(2) a component supplier when $ft \leq f_{cd}^{X,q}$ and $c \geq \frac{(2-\sqrt{2})V}{2}$;

(3) a dual distributor when $ft > \max\{f_{ed}^{X,q}, f_{cd}^{X,q}\}$

Proof: The profit comparison under quantity competition gives: $\Pi_m^{D,X} - \Pi_m^{E,X} \geq 0$ when $ft \geq f_{ed}^{X,q}$; $\Pi_m^{D,X} - \Pi_m^{C,X} \geq 0$ when $ft \geq f_{cd}^{X,q}$; $\Pi_m^{E,X} - \Pi_m^{C,X} \geq 0$ when $c \leq \frac{(2-\sqrt{2})V}{2}$, where $f_{ed}^{X,q} = \frac{122c-170V+3\sqrt{46(85V^2-170cV+61c^2)}}{85}$, and $f_{cd}^{X,q} = \frac{122c-170V+3\sqrt{23(85V^2-48c^2)}}{85}$. Consequently, the dual distributor role is optimal when $ft > \max\{f_{ed}^{X,q}, f_{cd}^{X,q}\}$. ■

Next, the following Proposition compares the optimal region of dual distribution under price and quantity competition.

Proposition 4.5 *Under the conditions* (4.107) *and* (4.119)*:*

The optimal region of dual distribution is smaller under quantity competition than under price competition, i.e., $f_{ed}^{X,q} > f_{ed}^{X}$ and $f_{cd}^{X,q} > f_{cd}^{X}$.

Proof: The results come straightforward from $f_{ed}^{X,q} - f_{ed}^{X} > 0$ and $f_{cd}^{X,q} - f_{cd}^{X} > 0$.

The results from Propositions 4.4 and 4.5 seem to contradict Proposition 4.3. However, they are consistent. It is because feasibility is not an issue in Venkatesh et al. (2006)'s model as well as Xu et al. (2010)'s; and these results are consistent with the finding that the dual distributor obtains lower profits under quantity competition (vis-a-vis price competition).

4.5 Conclusion

This chapter focuses on showing the changes in the optimal supply chain structure when moving from price to quantity competition in a static setting. We also link our findings to the existing work (Venkatesh et al. 2006, Xu et al. 2010) to highlight the consistency of our results across different settings. Our analysis reveals that compared to price competition, the firm's profitability is less disruptive under quantity competition. In particular, when the firm incurs a higher level of cost disadvantage in producing the end product, under price competition there is a steep drop in profitability as the firm transitions to a component supplier structure from a dual distribution structure under price competition. On the other hand, under quantity competition, the transition is more gradual. More importantly, this feature allows the firm to gain higher profits in a certain range of cost disadvantage under quantity competition.

The framework outlined in this chapter will serve as a benchmark for the next chapter where we incorporate durability into the model.

Table 4.1: The Equilibrium Outcome of Different Supply Chain Structures

Dual Distributor	
Price Competition	Quantity Competition
$w_m^D = \frac{(8+k^3)(a-\tau)-k^3c}{2(8+k^2)}$	$w_m^{D,q} = \frac{(k^3-4k^2+8)(a-\tau)-ck^3}{2(8-3k^2)}$
$d_m^D = \frac{(8-6k-k^2-k^4)(a-\tau)-c(8-k^2-k^4)}{2b(8-7k^2-k^4)}$	$d_m^{D,q} = \frac{(8-k^2-2k)(a-\tau)-c(8-k^2)}{2b(8-3k^2)}$
$d_n^D = \frac{(2+k^2)((1-k)(a-\tau)+kc)}{2b(8-7k^2-k^4)}$	$d_n^{D,q} = \frac{2(1-k)(a-\tau)+2ck}{b(8-3k^2)}$
$p_m^D = \frac{a(8+2k-k^2)+c(8-2k+3k^2)+c(8+3k^2)}{2(8+k^2)}$	$p_m^{D,q} = \frac{a(8-k^2-2k)+\tau(8-5k^2+2k)+c(8-5k^2)}{2(8-3k^2)}$
$p_n^D = \frac{a(12-4k+2k^2-k^3)+\tau(4+4k+k^3)+ck(4+k^2)}{2(8+k^2)}$	$p_n^{D,q} = \frac{a(k^3-4k^2-4k+12)+\tau(4-k^3-2k^2+4k)+ck(4-k^2)}{2(8-3k^2)}$
$\Pi_m^D = \frac{c^2(8-k^4-3k^2)-2c(8-k^4-3k^2-4k)(a-\tau)+(12-k^4-3k^2-8k)(a-\tau)^2}{4b(8-k^4-7k^2)}$	$\Pi_m^{D,q} = \frac{c^2(k^2+8)-2c(k^2-4k+8)(a-\tau)+(k^2-8k+12)(a-\tau)^2}{4b(8-3k^2)}$
$\Pi_n^D = \frac{(k^2+2)^2(ck+(1-k)(a-\tau))^2}{b(1-k^2)(k^2+8)^2}$	$\Pi_n^{D,q} = \frac{4(ck+(1-k)(a-\tau))^2}{b(8-3k^2)^2}$
Sole Entrant	
$p_m^E = \frac{a+\tau+c}{2}$; $d_m^E = \frac{a-\tau-c}{2b}$; $\Pi_m^E = \frac{(a-\tau-c)^2}{4b}$	
Component Supplier	
$w_m^C = \frac{a-\tau}{2}$; $p_n^C = \frac{3a+\tau}{4}$; $d_n^C = \frac{a-\tau}{4b}$; $\Pi_m^C = \frac{(a-\tau)^2}{8b}$; $\Pi_n^C = \frac{(a-\tau)^2}{16b}$	

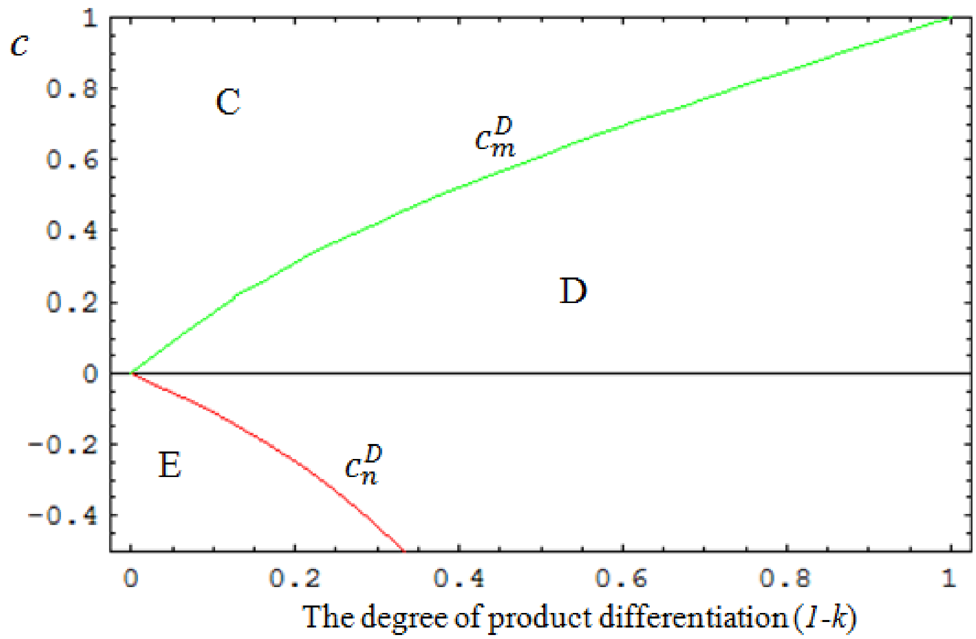

Figure 4.1: Optimal Supply Chain Structure Under Price Competition

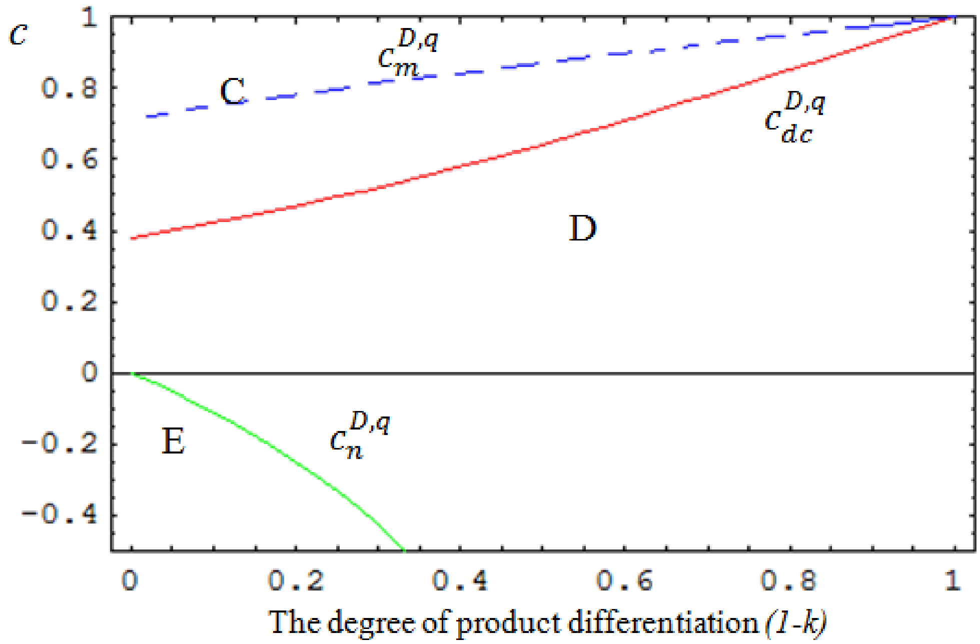

Figure 4.2: Optimal Supply Chain Structure Under Quantity Competition

CHAPTER 5: OPTIMAL SUPPLY CHAIN STRUCTURE: IMPACT OF DURABILITY

5.1 Introduction

This chapter aims to accomplish the goal laid out in the Introduction section of Chapter 4. We examine the impact of product durability on the supply chain design when consumers are forward looking. The three supply chain structures are identified analogous to those considered in Chapter 4. More specifically, our research question is: how does durability impact the long-term profitability of different supply chain structures?

We address the research question by building a two-period analytical model in which the manufacturer selects among being: (a) a sole entrant, (b) a component supplier, and (c) a dual distributor. This two-period setup helps capture the durable nature of the product (i.e., the future competition created by durability). In particular, a new product in the first period becomes a used product in the second period; so durability is captured through the valuation of the used product in the second period. If a product is a nondurable, the used product has no value; if the product does not depreciate, the used product's valuation is the same as that of the new product sold in period 2. The competition between the used product and the new product in period 2 is captured via a perfectly competitive secondary market. It is worth noting that when the product in this setting has zero durability, or when the future is not valuable (i.e., the discount factor is zero), the analysis collapses to the one discussed in Chapter 4.

We examine the impact of durability on the profitability of different structures by conducting comparative statics analysis on key decisions. Our analysis reveals that the optimality of each of the alternatives is changed significantly (from the results in Chapter 4) when the product's durability is taken into account. More specifically, the sole entrant role becomes more preferred to a dual distributor role, while the component supplier role becomes less preferred. Further, under certain conditions, the manufacturer may opt to be a dual distributor in the first period and then become a component supplier in the second period. These results help explain, for instance, why certain companies like Apple may be better off embracing a sole entrant structure in the long-run.

The rest of the chapter is organized as follows. We develop our model in the next section. The analysis and results are presented in Section 3. We conclude the chapter in Section 4. All proofs are confined to Appendix B.

5.2 The Model

There are two periods in our model. 'New' durable goods are sold in every period by the firm(s). These products are assumed to provide a stream of services with a per-period quality of θ. A product sold in period 1 lasts for two periods, while a product sold in period 2 lasts one period (since the game ends at the end of this period). Further, new products in period 1 become used products in period 2; and the used product's quality is downgraded by a factor γ, $\gamma \in [0, 1]$. Here, γ represents the durability of the durable good; in particular, if $\gamma = 0$, products sold in period 1 are nondurables, if $\gamma = 1$, then these products have no depreciation.

Consumers who owned the used product have a choice of reselling it to a perfectly competitive market in the second period. Said differently, only new products are available in the first period; in period 2, however, both used and new products are available in the market. Further, consumers are strategic (or rational) and have rational expectations about future prices of the product when making purchase decisions. In what follows, we describe the consumers' decision problem, the firm's problem, the sequence of events and the equilibrium concept employed. For analytical convenience, we assume both the consumers and the firm(s) discount the future by the same factor δ, $\delta \in [0, 1]$.

5.2.1 Consumers' Decisions

Consumers are heterogeneous in terms of their valuation of product quality and assumed to be distributed uniformly along the interval $[0, 1]$. A consumer indexed v receives a per-period utility θv when consuming a product of per-period quality θ in that period. In particular, the utility of a new product in period 1 includes θv in the first period and $\delta\gamma\theta v$ (discounted value) in the second period; a new product in period 2 has the discounted utility of $\delta\theta v$. This form of consumer utility has been widely employed in the durable goods literature (e.g., Fudenberg and Tirole 1998, Desai and Purohit 1998, Desai et al. 2004).

Denote p_1 and p_2 as the new product's prices in periods 1 and 2 respectively. Since consumers in our model can resell the used product to the secondary market, we denote the price of the used product as p_u. Each consumer consumes at most one unit of product in every period. As mentioned earlier, consumers have rational expectations about the future prices; hence, when making purchase in the first period, consumers are able to predict correctly the used product's price as well as the new product's price in the second period. We use a superscript 'e' to denote the consumer's expected value in describing the consumers' decisions.

Over the two periods, consumers have five purchase alternatives (denoted as b, h, n, u, o):

(b) Buy new product every period with the surplus $W_b = \theta v - p_1 + \delta(\theta v - p_2^e + p_u^e)$,

(h) Buy new product in period 1 and hold with the surplus $W_h = \theta v(1 + \delta\gamma) - p_1$,

(n) Buy new product in period 2 only with the surplus $W_n = \delta(\theta v - p_2^e)$,

(u) Buy used product in period 2 only with the surplus $W_u = \delta(\gamma\theta v - p_u^e)$, and

(o) Buy nothing $W_o = 0$.

Consumers maximize their surpluses across these purchase alternatives

$$\max_{b,h,n,u,o} \{W_b, W_h, W_n, W_u, W_o\} \ . \tag{5.1}$$

Notice that these surplus functions are linear functions of the consumer index v and that their slopes decrease from the first alternative (b) to the last one (o). Let S_i represent the consumer segment that chooses purchase alternative i and v_{ij} represent the indifferent consumer between alternatives i and j, $i \neq j$, $i, j \in \{b, h, n, u, o\}$; the values of v_{ij} are in Table 5.1.

The market segmentation that arises from the above surplus maximization problem has an interesting property which is summarized in the following lemma:

Lemma 5.1 *When consumers maximize their surplus across $\{b, h, n, u, o\}$, both S_h and S_n cannot coexist.*

From the above lemma, it follows that there are two alternative segmentation structures $\mathbb{S}_1$ and $\mathbb{S}_2$ as shown in Figures 5.1 and 5.2. The first structure, $\mathbb{S}_1$ includes $S_o = [0, v_{uo})$, $S_u = [v_{uo}, v_{hu})$, $S_h = [v_{hu}, v_{bh})$, and $S_b = [v_{bh}, 1]$, while the second $\mathbb{S}_2$ consists of $S_o = [0, v_{uo})$, $S_u = [v_{uo}, v_{nu})$, $S_n = [v_{nu}, v_{bn})$, and $S_b = [v_{bn}, 1]$. The reasoning for these structures to arise is as follows. When consumers in the market select purchase alternative (h), option (n) is automatically dominated; or conversely, when consumers choose purchase alternative (n), option (h) becomes an inferior choice. Figures 5.3 and 5.4 illustrate such a rationale.

Let d_1 and d_2 be the demand of new purchases in periods 1 and 2 respectively. It follows that $d_1 = 1 - \frac{p_1 - \delta p_u^e}{\theta}$ and $d_2 = 1 - \frac{p_2^e - p_u^e}{\theta(1-\gamma)}$ under either $\mathbb{S}_1$ or $\mathbb{S}_2$. Since the secondary market is perfectly competitive, the used product's price is determined by the market clearing condition which ensures that supply equals demand. In particular, in Figures 5.1 and 5.2, the length of S_u must equal to the length of S_b. It follows that the used product's price is

$$p_u^e = \frac{\gamma(p_1(1-\gamma) + p_2^e - \theta(1-\gamma))}{1 + \gamma\delta - \delta\gamma^2}. \tag{5.2}$$

Define $\Psi = \frac{(1+\gamma)p_1 - \theta\gamma(1-\delta+\gamma\delta)}{1+2\gamma\delta}$. The following lemma characterizes the condition for different segmentation structures to arise:

Lemma 5.2 *When consumers maximize their surplus across $\{b, h, n, u, o\}$,*

(i) If $p_2^e \geq \Psi$, then the market is segmented as in $\mathbb{S}_1$.

(ii) If $p_2^e < \Psi$, then the market is segmented as in $\mathbb{S}_2$.

The above lemma implies that when consumers expect the future price of the new product to be relatively high, then they will buy the new product in the first period and keep it for using over both periods (instead of say, waiting to purchase a totally new product in period 2). On the other hand, if they expect the future price to be relatively low, then they would wait to buy the new product, rather than buying and holding it over two periods.

Using (5.2) and the definition of d_1 and d_2, we obtain the following inverse demand structure under the condition of fulfilled consumer expectations about future prices (we will revisit this condition later):

$$p_1 = \theta(1 - d_1) + \delta p_u \tag{5.3}$$

$$p_2 = \theta(1 - d_2 - \gamma d_1) \tag{5.4}$$

$$p_u = \theta\gamma(1 - d_1 - d_2) \tag{5.5}$$

5.2.2 The firm's problem

A firm, m, is a sole supplier of an essential proprietary component for making the end product. To capture the value created by its component, m considers three different supply chain structures, $\{C, E, D\}$. In the first, m operates as a 'component supplier' and sells the component to a downstream firm n who then makes the end product. In the second structure, m produces the end product using its component but does not make that component available to any other firms; here, m operates as a 'sole entrant'. Finally, m can operate as a 'dual distributor' who not only makes the end product using its own component, but sells the component to n who then competes against m in the end product market.

We assume that each unit of the end product utilizes one unit of the proprietary component. The marginal cost of making the component is constant and normalized to zero. The marginal cost of producing the end product is $\tau + c$ and τ for m and n respectively, where $0 \leq \tau + c < \theta$. For analytical convenience, we normalize τ to zero. Though firms incur different marginal costs when producing the end product, we assume that these products are viewed identical in the consumers' eye.

Let $p^i_{j,1}$ and $p^i_{j,2}$ be the prices of the new product in periods 1 and 2 respectively of firm j under strategy i, where $j = \{m, n\}$, $i = \{E, C, D\}$; and let p^i_u be the price of the used product in period 2 under strategy i, $i = \{E, C, D\}$. Similarly, denote $d^i_{j,1}$ and $d^i_{j,2}$ as the product quantities sold in periods 1 and 2 respectively by firm j under strategy i, where $j = \{m, n\}$, $i = \{E, C, D\}$.

5.2.3 The Sequence of Events

As noticed earlier, there are two periods in the game. At the beginning of the first period, m selects its supply chain structure from $\{E, C, D\}$.

- If m chooses E, it then selects the quantity of the end product to sell in the first period, $d^E_{m,1}$; and at the beginning of period 2, it sets the quantity provided to the market in period 2, $d^E_{m,2}$.

- If m chooses C, it then sets the component's price $w^C_{m,1}$ offered to n. Given this price, n chooses the quantity to sell in the first period, $d^C_{n,1}$. At the beginning of period 2, m offers n the component at the price $w^C_{m,2}$; given that price, n then selects the quantity to sell in period 2, $d^C_{n,2}$.

- If m chooses D, it then sets the component's price $w^D_{m,1}$ offered to n. Given this price, m and n simultaneously and noncollusively choose the end product's quantities in the first period, $d^D_{m,1}$ and $d^D_{n,1}$. At the beginning of period 2, m offers n the component at the price $w^D_{m,2}$; given that price, m and n then select the end product's quantities in period 2, $d^D_{m,2}$ and $d^D_{n,2}$.

5.2.4 The equilibrium

We seek a sub-game perfect equilibrium in this two period game. At the beginning of period 2, there are two groups of consumers in the market: those (1) who have already bought the product; and (2) who are non-buyers. Consumers in group (1) will choose whether to hold the used product or repurchase a brand new product (and resell the used product to the secondary market). Consumers in group (2) will choose whether to adopt the new product or the used product (from the secondary market). Contingent on the firms' quantities, consumers make their purchase decisions based up on observing the market prices, p_2 and p_u. These prices are determined by the market clearing conditions which ensure that demand equals supply (demand includes the number of products that firms put in the market and the number of used products that repeat purchasers resell in the secondary market).

In period 1, upon observing the market price of the new product p_1, consumers form expectations about future prices (of the used product and the new product in period 2) when they determine whether to adopt the new product. In equilibrium, these expectations are fulfilled, i.e., $p^e_2 = p_2$ and $p^e_u = p_u$.

Given the above process, for each supply chain structure, the inverse demand system is as follows:

Sole Entrant:

$$p^E_{m,1} = \theta(1 - d^E_{m,1}) + \delta p^E_u \tag{5.6}$$

$$p^E_{m,2} = \theta(1 - d^E_{m,2} - \gamma d^E_{m,1}) \tag{5.7}$$

$$p^E_u = \theta\gamma(1 - d^E_{m,1} - d^E_{m,2}) \tag{5.8}$$

Component Supplier:

$$p^C_{n,1} = \theta(1 - d^C_{n,1}) + \delta p^C_u \tag{5.9}$$

$$p^C_{n,2} = \theta(1 - d^C_{n,2} - \gamma d^C_{n,1}) \tag{5.10}$$

$$p^C_u = \theta\gamma(1 - d^C_{n,1} - d^C_{n,2}) \tag{5.11}$$

Dual Distributor:

$$p^D_{m,1} = \theta(1 - d^D_{m,1} - d^D_{n,1}) + \delta p^D_u \tag{5.12}$$

$$p^D_{m,2} = \theta(1 - d^D_{m,2} - d^D_{n,2} - \gamma d^D_{m,1} - \gamma d^D_{n,1}) \tag{5.13}$$

$$p^D_u = \theta\gamma(1 - d^D_{m,1} - d^D_{n,1} - d^D_{m,2} - d^D_{n,2}) \tag{5.14}$$

To solve for the equilibrium, we employ the standard backward induction approach. Starting in the second period, m (and n) maximizes (maximize) the profit of selling the 'new' product in that period; and in the first period, m and n select their actions to maximize the present value of the total profit over two periods.

5.3 The Analysis and Results

In this section, we first examine the change in the profitability of each supply chain structure at varying levels of durability. We then compare the profitability across structures and note any modifications (compared to Chapter 4) in the optimal choice of supply chain structure. For the ease of discussion, let $\Pi^i_{j,1}$ represent the profit of selling products in the first period of firm j under strategy i, $j \in \{m, n\}$ and $i \in \{E, C, D\}$. The specific expressions of all critical values mentioned below are provided in Appendix B.

5.3.1 Sole Entrant

As a sole entrant, m solves the following problem, denoted [m-E]:

$$\max_{d^E_{m,1}} \Pi^E_m = (p^E_{m,1} - c)d^E_{m,1} + \delta(p^E_{m,2} - c)\hat{d}^E_{m,2}$$

subject to

$$\hat{d}^E_{m,2} = \arg\max_{d^E_{m,2}} \Pi^E_{m,2} = (p^E_{m,2} - c)d^E_{m,2} \tag{5.15}$$

$$d^E_{m,1} \geq 0; d^E_{m,2} \geq 0 \tag{5.16}$$

The inverse demand structure is specified in The Model section. Under [m-E], the sole entrant maximizes the present value of its total profit over two periods. Constraint (5.15) indicates that the quantity to sell in period 2 is selected to maximize the sole entrant's profit in that period. The constraints in (5.16) ensure non-negative end product's quantities. The following lemma summarizes the equilibrium outcome of the problem:

Lemma 5.3 *Under the sole entrant role,*
(i) When $c/\theta < t^E$, m will sell new products in both periods; and
(ii) When $c/\theta \geq t^E$, m will sell new products in period 1 only and there is no secondary market in period 2.

More specifically, we solve the problem backward, starting from period 2, the firm selects the quantity to sell $d^E_{m,2}$ by maximizing the profit in this period $\Pi^E_{m,2}$. We obtain

$$\hat{d}^E_{m,2} = \frac{\theta(1 - \gamma d^E_{m,1}) - c}{2\theta}. \tag{5.17}$$

Intuitively, the quantity in period 2 is affected by: (1) the effective leftover demand (i.e., $1 - \gamma d^E_{m,1}$), which is a function of the quantity sold in period 1 adjusted for durability, and (2) the cost disadvantage per quality unit (c/θ). In particular, m sells more in the second period when it provides less of the product in the first period; here, a higher level of the durability factor γ strengthens the impact of the quantity in period 1 on the quantity in period 2.

Substituting $\hat{d}^E_{m,2}$ into Π^E_m and solving for $d^E_{m,1}$ gives $d^E_{m,1} = \frac{2(\theta - c(1-\delta\gamma))}{\theta(4+4\delta\gamma-3\delta\gamma^2)}$. Notice that $d^E_{m,2} > 0$ when $c/\theta < t^E$. When $c/\theta \geq t^E$, the sole entrant will not sell any new product in the second period, i.e. $d^E_{m,2} = 0$. Therefore, m only maximizes the profit in period 1 by choosing $d^E_{m,1}$. Consumers then buy the product in period 1 for using over two periods; as a result, there is no secondary market in period 2. Other details of the solution to [m-E] are provided in Table 5.2.

5.3.1.1 The impact of durability

Lemma 5.4 *For a sole entrant,*
(i) The profit in period 2 is decreasing with both the durability γ and the cost disadvantage c. However, the decrement from γ is less at a higher level of c, i.e., $\frac{\partial^2 \Pi^E_{m,2}}{\partial\gamma\partial c} > 0$.
(ii) The present value of the total profit is increasing with durability γ when either ($\gamma \geq 2/3$) or ($\gamma < 2/3$ and $c/\theta \geq \frac{2-3\gamma}{6-3\gamma+2\delta\gamma}$).

When the product in the first period does not depreciate by much (i.e., higher durability), the used product becomes a stronger competitor in period 2, and induces a lower quantity of the new product to be sold in the second period; further, the new product's price is also lower. Consequently, m's profit in period 2 is suitably reduced at higher levels of durability. Next, as the cost disadvantage increases, m has an incentive to sell less of the product; while this may raise the product's price, the margin nevertheless goes down. In other words, the profit in period 2 is decreasing with the cost disadvantage due to both the decrease in the margin and the quantity sold.

Now, the negative impact of durability on the second period profit is notably mitigated by a higher level of cost disadvantage (i.e. $\frac{\partial^2 \Pi^E_{m,2}}{\partial\gamma\partial c} > 0$). This happens because though a higher level of cost disadvantage lowers the quantity sold, it increases the price insensitivity of the marginal consumer; thus the degree that the price and quantity goes down with γ is mitigated.

In period 1, when the product's durability goes up, consumers are willing to pay more for it, and of course, the used product becomes more valuable in period 2; consequently, both the new product's price and the used product's price (realized in period 2) go up with durability (i.e., $\frac{\partial p^E_u}{\partial\gamma} > 0$ and $\frac{\partial p^E_{m,1}}{\partial\gamma} > 0$). If the increase in new product's price is smaller than the present value of the increase in the used product's price, then more consumers want to buy the new product in period 1; subsequently, the firm sells more in period 1. This occurs at either higher levels of durability (i.e. $\gamma \geq 2/3$) or at higher levels of cost disadvantage (for $\gamma < 2/3$).

Here too, the cost disadvantage serves to increase the price insensitivity of the marginal consumer in period 1. As mentioned earlier, it also increases the price insensitivity of the marginal consumer in period 2; subsequently, the marginal consumer in the secondary market is less price sensitive. The change in the price insensitivity at higher levels of cost disadvantage helps explain the increase differentials in $p^E_{m,1}$ and p^E_u due to γ as described in the previous paragraph.

With the increase of the new product's price, the margin in the first period goes up with γ; but the quantity sold may go down with γ. However, the decrease in the quantity can be compensated by the increase in margin; and consequently, the profit of products sold in period 1 (i.e., $\Pi^E_{m,1}$) goes up with γ.

In summary, as durability goes up, the profit in period 2 goes down, but the profit in period 1 goes up. As a result, if the increase in period 1 surpasses the decrease in period 2, then the present value of the total profit will increase with γ. This occurs at higher levels of durability ($\gamma \geq 2/3$) or higher levels of cost disadvantage (for $\gamma < 2/3$). Though the cost disadvantage lowers profits, its existence turns out to be helpful (via changing the price sensitivity of the marginal consumers) by allowing the momentum for the profit to rise as γ gradually increases from zero.

5.3.2 Component Supplier

Under this structure, in each period, m sets the component's price and given that price, n sets the quantity to sell in that period. In the second period, m's problem is as follows:

$$\max_{w^C_{m,2}} \Pi^C_{m,2} = w^C_{m,2}\hat{d}^C_{n,2} \tag{5.18}$$

subject to

$$\hat{d}^C_{n,2} = \arg\max_{d^C_{n,2}} \Pi^C_{n,2} = (p^C_{n,2} - w^C_{m,2})d^C_{n,2} \tag{5.19}$$

$$d^C_{n,2} \geq 0; w^C_{m,2} \geq 0 \tag{5.20}$$

In (5.18), m maximizes the wholesale profit (from selling its proprietary component) in period 2. Constraint (5.19) indicates that n chooses the end product's quantity to maximize its profit in this period. The constraints in (5.20) ensure non-negative end product's quantity and non-negative component's price.

With backward induction, solving n's problem gives $\hat{d}^C_{n,2} = \frac{\theta(1-\gamma d^E_{m,1})-w^C_{m,2}}{2\theta}$. Substituting $\hat{d}^C_{n,2}$ into $\Pi^C_{m,2}$ and solving for $w^C_{m,2}$ gives $w^C_{m,2} = \frac{\theta(1-\gamma d^C_{n,1})}{2}$; and subsequently $d^C_{n,2} = \frac{1-\gamma d^C_{n,1}}{4}$. Here too, we see a negative impact of the number of product sold in period 1 on $d^C_{n,2}$ (adjusted for durability); and subsequently, m's pricing of the component is affected.

With the above in mind, in period 1, m's problem, denoted [m-C], is:

$$\max_{w_{m,1}^C} \Pi_m^C = w_{m,1}^C \hat{d_{n,1}^C} + \delta \Pi_{m,2}^C$$

subject to

$$\hat{d}_{n,1}^C = \arg\max_{d_{n,1}^C} \Pi_n^C = (p_{n,1}^C - w_{m,1}^C) d_{n,1}^C + \delta \Pi_{n,2}^C \tag{5.21}$$

$$d_{n,1}^C \geq 0; w_{m,1}^C \geq 0 \tag{5.22}$$

Under [m-C], m maximizes the present value of the total profit (from selling its proprietary component) over two periods. Constraint (5.21) indicates that n chooses the quantity to sell in period 1 to maximize the present value of its total profit. The constraints in (5.22) ensure non-negative end product's quantity and non-negative component's price.

Now solving n's problem gives $d_{n,1}^C = \frac{\theta(8+5\delta\gamma)-8w_{m,1}^C}{\theta(16+16\delta\gamma-5\delta\gamma^2)}$. Intuitively, the component's price $w_{m,1}^C$ negatively impacts the quantity sold by n. More importantly, this price helps raise the component's price as well as the quantity sold in period 2 (indirectly through the negative impact on $d_{n,1}^C$). Next, the component's price is set via the first order condition $\frac{\partial \Pi_m^C}{\partial w_{m,1}^C} = 0$. It follows that $w_{m,1}^C = \frac{\theta(128+240\delta\gamma-56\delta(1-2\delta)\gamma^2-45\delta^2\gamma^3)}{32(8+8\delta\gamma-3\delta\gamma^2)}$. Other details of the solution to [m-C] are in Table 5.3.

Here, it is important to point out the severity of double marginalization in problem [m-C]. First, at the end product level, anticipating the downward pressure of its product in period 1 on the profit in period 2, n tends to restrict its quantity in period 1 when maximizing the total profit over two periods (compared to the case when n maximizes the profit of period 1 only). Second, at the upstream level, knowing that its component's price in period 1 can restrict the quantity sold in this period (which in turn mitigates the pressure on the wholesale profit in period 2), m adjusts its component's price in period 1 upward (compared to the case when m maximizes the wholesale profit of period 1 only). Combining these two forces leads to a more severe double marginalization in period 1.

5.3.2.1 The impact of durability

Lemma 5.5 *Under the component supplier structure,*

(i) The profits of m and n in period 2 are decreasing with durability γ.

(ii) The present value of the total profit of m and n is increasing with durability γ when γ is relatively large, i.e., $\frac{\partial \Pi_m^C}{\partial \gamma} \geq 0$ when $\gamma \geq \frac{2}{3(2+\delta)}$, and $\frac{\partial \Pi_n^C}{\partial \gamma} \geq 0$ when $\gamma \geq g(\delta)$.

Analogous to the sole entrant case, in period 2, a higher durability makes the used product a stronger competitor of the new product, thus, n sells less of the product at a lower price; because of this, m charges n a lower component's price. Therefore, in period 2, both m and n obtain less profit at higher levels of durability.

In period 1, when the product's durability goes up, consumers are willing to pay more for it, and of course, the used product becomes more valuable in period 2; consequently, both the new product's price and the used product's price (realized in period 2) go up ($\frac{\partial p_u^C}{\partial \gamma} > 0$ and $\frac{\partial p_{n,1}^C}{\partial \gamma} > 0$). Combining these effects with the forces discussed under problem [m-C], m charges n a higher component's price ($\frac{\partial w_{m,1}^C}{\partial \gamma} > 0$).

Due to the strategic decision of m mentioned above, the market size in period 1 only expands at relatively high levels of durability (i.e., the durability required for $\frac{\partial d_{n,1}^C}{\partial \gamma} \geq 0$ is higher than the one under sole entrant; more specifically, $\frac{\partial d_{n,1}^C}{\partial \gamma} \geq 0$ when $\gamma \geq \frac{2(-4+\sqrt{2(8+5\delta)})}{3\delta}$). It happens because the increase in the component's price under higher durability puts more pressure on the new product's price; subsequently, that the new product's price increases slower than the used product's price when γ goes up (recall the discussion under Lemma 5.4, the movement of prices is the reason causing $\frac{\partial d_{n,1}^C}{\partial \gamma} \geq 0$) is less likely to occur under the component supplier role.

Furthermore, the profits from selling products in period 1, $\Pi^C_{m,1}$ and $\Pi^C_{n,1}$, are increasing with γ. In other words, though the end product's quantity may go down with durability, the increase in the margins (of both firms) more than compensates any decrease in the quantity (if any). The movement of profits here contrasts with the one in period 2. Consequently, when the increase in period 1 exceeds the decrease in period 2, the present value of the total profit is increasing in durability ($\frac{\partial \Pi^C_m}{\partial \gamma} \geq 0$ when $\gamma \geq \frac{2}{3(2+\delta)}$ and $\frac{\partial \Pi^C_n}{\partial \gamma} \geq 0$ when $\gamma \geq g(\delta)$); otherwise, it's decreasing in durability.

5.3.3 Dual Distributor

Under this structure, in each period m sets the component's price, and given that price, n and m simultaneously and noncollusively set the quantities to sell in that period. The following lemma summarizes the equilibrium outcome:

Lemma 5.6 *Under the dual distributor role,*

(i) When $c/\theta < t^D$, m will sell end products in both periods.

(ii) When $t^D \leq c/\theta < t^{D,0}$, m will sell end products in period 1 and act as a component supplier in period 2.

(iii) When $c/\theta \geq \max\{t^D, t^{D,0}\}$, m will act as a component supplier in both periods.

The above results are obtained via backward induction. In period 2, m's problem is

$$\max_{w^D_{m,2}} \Pi^D_{m,2} = w^D_{m,2}\hat{d}^D_{n,2} + (p^D_{m,2} - c)\hat{d}^D_{m,2} \tag{5.23}$$

subject to

$$\hat{d}^D_{m,2} = \arg\max \Pi^D_{m,2} = w^D_{m,2}d^D_{n,2} + (p^D_{m,2} - c)d^D_{m,2} \tag{5.24}$$

$$\hat{d}^D_{n,2} = \arg\max \Pi^D_{n,2} = (p^D_{m,2} - w^D_{m,2})d^D_{n,2} \tag{5.25}$$

$$d^D_{m,2} \geq 0; d^D_{n,2} \geq 0; w^D_{m,2} \geq 0 \tag{5.26}$$

In (5.23), m maximizes the profit from selling the component to n and the end product in period 2. Constraints (5.24) and (5.25) indicate that m and n choose the end product's quantities to maximize their profits in this period. The constraints in (5.26) ensure non-negative end product's quantities and non-negative component's price.

The optimal quantities in period 2 are set by

$$\hat{d}^D_{m,2} = \frac{\theta(1-\gamma(d^D_{m,1}+d^D_{n,1})) - 2c + w^D_{m,2}}{3\theta} \tag{5.27}$$

$$\hat{d}^D_{n,2} = \frac{\theta(1-\gamma(d^D_{m,1}+d^D_{n,1})) - 2w^D_{m,2} + c}{3\theta} \tag{5.28}$$

As expected, each firm's quantity is increasing with the effective leftover demand in period 2 (i.e., $1-\gamma(d^D_{m,1}+d^D_{n,1})$) and decreasing with the marginal cost (i.e., c for m and $w^D_{m,2}$ for n). Intuitively, the dual distributor's quantity increases with $w^D_{m,2}$ because its competitive position is strengthened by the competitor's cost $w^D_{m,2}$; however, due to the cost disadvantage of the dual distributor, n is able to sell to customers outside m's reach (this is confirmed later via the equilibrium value of $d^D_{n,2}$).

Substituting $\hat{d}^D_{m,2}$ and $\hat{d}^D_{n,2}$ into $\Pi^E_{m,2}$ and solving for $w^D_{m,2}$ gives $w^D_{m,2} = \frac{5\theta(1-\gamma(d^D_{m,1}+d^D_{n,1}))-c}{10}$, and subsequently $d^D_{m,2} = \frac{5\theta(1-\gamma(d^D_{m,1}+d^D_{n,1}))-7c}{10\theta}$ and $d^D_{n,2} = \frac{2c}{5\theta}$. Here, for a given level of the effective leftover demand, m charges n a lower component's price when it incurs a higher cost disadvantage. More specifically, when $c = 0$, the wholesale price is the same as the one in the component supplier structure; further, m sets the end product's quantity as a sole entrant and foreclosures n in period 2 (i.e., $d^D_{n,2} = 0$). When the cost disadvantage is higher, m lowers the component's price offered to n, indicating a lower double marginalization effect compared to the component supplier structure. Plus, the market coverage in period 2 (i.e., $d^D_{m,2} + d^D_{n,2} = \frac{1-\gamma(d^D_{m,1}+d^D_{n,1})}{2} - \frac{3c}{10\theta}$) is higher than that under the sole entrant structure (for a given level of the effective leftover demand).

The above discussion shows the benefits of the dual structure for a given level of the effective leftover demand: lower double marginalization and higher market coverage. However, the effective leftover demand of different supply chain structures is determined differently in period 1. In particular, under the dual structure, m's problem in period 1, denoted [m-D], is:

$$\max_{w^D_{m,1}} \Pi^D_m = w^D_{m,1}\hat{d}^D_{n,1} + (p^D_{m,1} - c)\hat{d}^D_{m,1} + \delta\Pi^D_{m,2}$$

subject to

$$\hat{d}^D_{m,1} = \arg\max \Pi^D_m = w^D_{m,1}d^D_{n,1} + (p^D_{m,1} - c)d^D_{m,1} + \delta\Pi^D_{m,2} \tag{5.29}$$

$$\hat{d}^D_{n,1} = \arg\max \Pi^D_n = (p^D_{m,1} - w^D_{m,1})d^D_{n,1} + \delta\Pi^D_{n,2} \tag{5.30}$$

$$d^D_{m,1} \geq 0; d^D_{n,1} \geq 0; w^D_{m,1} \geq 0 \tag{5.31}$$

Under [m-D], m maximizes the present value of the total profit from selling the component to n and the end product over two periods. Constraints (5.29) and (5.30) indicate that m and n choose the end product's quantities to maximize the present value of their profits over two periods. The constraints in (5.31) ensure non-negative end product's quantities and non-negative component's price.

Recall the results in period 2, m's profit including both wholesale and end product is negatively affected by the existence of the product in the first period; however, n's profit (we will see later) is independent of the quantities in period 1. Consequently, when m selects $d^D_{m,1}$ as in (5.29), it tends to lower the quantity in period 1 (compared to when it maximizes the profit in period 1 only). On the other hand, when n chooses $d^D_{n,1}$ as in (5.30), it will pick the quantity as if when it maximizes the profit in period 1 only. Solving the quantity game gives us each firm's quantity as a function of the component's price:

$$\hat{d}^{D}_{m,1} = \frac{5\theta(2+\delta\gamma-\delta^2\gamma^2+\delta^2\gamma^3)+10w^{D}_{m,1}(1+\delta\gamma-\delta\gamma^2)-c(20+7\delta\gamma-\delta(10+13\delta)\gamma^2+5\delta^2\gamma^3)}{5\theta(2+2\delta\gamma-\delta\gamma^2)(3+3\delta\gamma-2\delta\gamma^2)} \tag{5.32}$$

$$\begin{aligned}\hat{d}^{D}_{n,1} = \frac{1}{10\theta(2+2\delta\gamma-\delta\gamma^2)(3+3\delta\gamma-2\delta\gamma^2)}&\big[5\theta(4+8\delta\gamma-4(1-\delta)\delta^2\gamma^2-3\delta^2\gamma^3)\\ &-10w^{D}_{m,1}(4+4\delta\gamma-3\delta\gamma^2)+c(20+16\delta\gamma-2\delta(5+2\delta)\gamma^2-\delta^2\gamma^3)\big].\end{aligned} \tag{5.33}$$

It follows that the total quantity in period 1 is

$$\hat{d}^{D}_{m,1}+\hat{d}^{D}_{n,1} = \frac{5\theta(4+\delta\gamma)-(10-11\delta\gamma)c-10w^{D}_{m,1}}{10(3\theta+3\delta\gamma-2\delta\gamma^2)}. \tag{5.34}$$

Given the outcome of the quantity game, m will select the component's price to resolve the following forces. First, the negative impact of first period quantities on m's profit in period 2 can be mitigated by a higher component's price (via a negative impact of $w^{D}_{m,1}$ on the total quantity in period 1). Second, though raising the component's price lowers the total quantity (in fact, it lowers n's first period quantity), it induces m to sell more in period 1 (see the expression of $\hat{d}^{D}_{m,1}$ above). Lastly, m has an incentive to lower the component's price to maintain the wholesale profit from selling the component to n.

The equilibrium outcome of problem [m-D] is in Table 5.4. Analogous to period 2, the quantity of n in period 1 depends critically on c; in other words, because end products are identical, the reason that n can make a profit is due to the cost disadvantage incurred by m. When $c = 0$, the dual distributor becomes a sole entrant. Notice that $d^{D}_{m,2} > 0$ when $c/\theta < t^{D}$.

When $c/\theta \geq t^{D}$, m does not sell any new product in period 2 (i.e., $d^{D}_{m,2} = 0$); that is, it becomes a component supplier in period 2. Hence, analogous to the case of a component supplier in period 2, n is the monopolist and sets the quantity to sell after receiving the component's price from m. The equilibrium outcome of this case is given in Table 5.4. Denote this special supply chain structure as D_0. Notice that D_0 is feasible only when $t^{D} \leq c/\theta < t^{D,0}$ as $d^{D}_{m,1} > 0 \Leftrightarrow c/\theta < t^{D,0}$. When $c/\theta \geq t^{D,0}$, then $d^{D}_{m,1} = 0$ and the dual distributor becomes a component supplier.

5.3.3.1 The impact of durability

Lemma 5.7 *Under the dual distributor structure,*

(i) In period 2, m's profit decreases with γ; however, n's profit is independent of γ.

(ii) The present value of the total profit of m increases with γ when $\Delta \geq 0$ and $\max\{0, t_1\} \leq c/\theta \leq \min\{t^{D}, t_2\}$ *as well as increases with c when* $c/\theta \leq \min\{t^{D}, t_3\}$.

(iii) The present value of the total profit of n decreases with γ but increases with c.

In period 2, while m's quantity decreases in durability (i.e., $\frac{\partial d^{D}_{m,2}}{\partial \gamma} < 0$), n's quantity is independent of γ (i.e., $\frac{\partial d^{D}_{n,2}}{\partial \gamma} = 0$). In other words, the downstream firm does not internalize the existence of the secondary market; instead, it only focuses on the competition with m (or its relative advantageous position in comparison with m). On the other hand, m absorbs all the negative effect of the secondary market via its lower quantity.

Analogous to other supply chain structures, a higher durability makes the used product a stronger competitor; thus, the market price of the new product is lower at higher levels of durability (i.e., $\frac{\partial p^{D}_{m,2}}{\partial \gamma} < 0$). Facing the decrease in the market's price, m lowers the component's price ($\frac{\partial w^{D}_{m,2}}{\partial \gamma} < 0$) such that n's margin is unaffected by durability. As a result, m's profit decreases with durability, meanwhile, n's profit is independent of γ due to the subsidy from m.

When the cost disadvantage goes up, m's margin goes down; and it sells less of the end product in period 2. Notice that though the market's price goes up, that increment cannot compensate for the surge of marginal cost. On the other hand, the increase in cost disadvantage enhances the relative position of n, allowing it to increase its quantity in the end market. Knowing this, m may raise the component's price, especially at relatively high levels of γ when the wholesale price decreases a lot (recall $\frac{\partial w_{m,2}^D}{\partial \gamma} < 0$). This particular outcome seems to contradict to the discussion on page 113; however, aside from the negative direct effect of c on $w_{m,2}^D$ as mentioned on page 113, there is an indirect effect of c on the effective leftover demand. It is this indirect effect causing the increase in component's price as c goes up; more specifically $\frac{\partial w_{m,2}^D}{\partial c} > 0$ for all $0 < \delta < \frac{40-19\gamma+20\gamma^2-\gamma\sqrt{4681-2860\gamma+400\gamma^2}}{\gamma(35\gamma^2-22\gamma-40)}$.

With the above movements in mind, in period 2, m's profit from the end product goes down while the wholesale profit goes up when c goes up; n, however, benefits from both the margin and the quantity, thus, its profit increases with c. Next, in period 1, as γ goes up, both the new product's price and the used product's price (realized in period 2) go up ($\frac{\partial p_u^D}{\partial \gamma} > 0$ and $\frac{\partial p_{m,1}^D}{\partial \gamma} > 0$). Combining this with the forces discussed earlier (on page 114), in equilibrium, m charges n a higher component's price ($\frac{\partial w_{m,1}^D}{\partial \gamma} > 0$); further, this increase is higher than the increase in the market price. As a result, n purchases less of the component ($\frac{\partial d_{n,1}^D}{\partial \gamma} < 0$); and its profit in period 1 ($\Pi_{n,1}^D$) also goes down with γ due to the decrease in both the quantity and the margin.

On the other hand, m gains in both the wholesale and the end-product markets. In particular, the increase in the component's price surpasses the decrease in the component order from n, and thus, the wholesale profit increases with γ; plus, the increase of the end product's margin offsets any decrease in the quantity sold, subsequently, the profit from selling the end-product also increases in γ.

When the cost disadvantage increases, as in period 2, m's margin and quantity go down. Because of this pressure on the end product, m focuses on the wholesale profit by lowering the component's price to induce n to buy more of the component. As a result, n benefits from the quantity gained by its relative cost advantage as well as the component's price (or the margin); accordingly, its profit in period 1 ($\Pi^D_{n,1}$) goes up. For m, the wholesale profit is going up with c due to the increase in the number of the component sold to n (which more than compensates the price decrease). On the other hand, m's profit from the selling the end-product decreases with c as both the margin and the quantity go down.

Next, the present value of the total profit of m increases in durability under the condition specified in Part (ii). As discussed earlier, when γ goes up, in both markets, m's profit stream in period 1 goes up; while the profit stream in period 2 goes down. Hence, any increase in the present value of the total profit is due to the increase in first period profit stream. With respect to the cost disadvantage, a higher level of c may help increase m's profit due to the increase in the wholesale profit; more specifically, the increase in the wholesale profit helps compensate the loss in the end-product market. Accordingly, the present value of the total profit of m increases when the loss in the end market is not too large as stated in Part (ii).

Finally, n is never better off with higher levels of γ (i.e., $\frac{\partial \Pi^D_n}{\partial \gamma} \leq 0$). It is because both the margin and the quantity sold in the first period is decreasing with durability while the profit in the second period does not change with durability. Further, the discussion earlier shows that the profit of n is increasing with the cost disadvantage ($\frac{\partial \Pi^D_n}{\partial c} \geq 0$) as both margins and quantities increase with c.

Market Segmentation Across Different Supply Chain Structures

As solved in the Model section, the market is segmented by either $\mathbb{S}_1$ or $\mathbb{S}_2$ (see Figures 5.1 and 5.2). Based on the equilibrium outcome of different supply chain structures discussed above, we summarize the optimal segmentation pattern in the following lemma:

Lemma 5.8 ***(Market Segmentation)***

(i) When m is a sole entrant, the market is segmented as in $\mathbb{S}_1$ when either ($\gamma \geq \frac{4\delta-2}{3\delta}$) or

($\gamma < \frac{4\delta-2}{3\delta}$ and $c/\theta \geq t^{E}_{segment}$). Otherwise, $\mathbb{S}_2$ emerges.

(ii) When m is a component supplier, the market is segmented as in $\mathbb{S}_1$ when $\gamma \geq \frac{4(5\delta-2)}{15\delta}$. Otherwise, $\mathbb{S}_2$ emerges.

(iii) When m is a dual distributor, the market is segmented as in $\mathbb{S}_1$ when either ($\gamma \geq \frac{4\delta-2}{3\delta}$) or ($\gamma < \frac{4\delta-2}{3\delta}$ and $c/\theta \geq t^{D}_{segment}$). Otherwise, $\mathbb{S}_2$ emerges.

The above Lemma is illustrated by Figure 5.5. It is shown that $\mathbb{S}_2$ is optimal when the product is not highly durable; further, higher levels of the cost disadvantage will make $\mathbb{S}_2$ less preferable under E and D. As seen in Figure 5.5, region 1 is where $\mathbb{S}_2$ occurs for all structures; region 2 is where $\mathbb{S}_2$ occurs under C and D but not E; in region 3, $\mathbb{S}_2$ occurs under C only; finally, $\mathbb{S}_1$ is optimal for all structures in region 4.

The implication of the above lemma is that a certain segmentation pattern may be pursued by an appropriate choice of the supply chain structure, especially when the product is not highly durable. For example, when the firm incurs a cost disadvantage $c = 0.2$ (given $\theta = 1$), and the durability factor is $\gamma = 0.4$ (in other words, the product's depreciation rate is 0.6), being a component supplier would be m's choice when it wants to pursue $\mathbb{S}_2$.

5.3.4 The Optimal Channel Structure

Before characterizing the optimal structure, we compare the values of the key choice variables. The end product's quantity comparison across structures is provided in Proposition 5.1; and the comparison of the component's price is addressed in Proposition 5.2.

Proposition 5.1 ***(End Product's Quantity)*** *Under the feasibility of all supply chain structures,*

(i) $d^{C}_{n,1} < d^{E}_{m,1}$ when $c/\theta < t^{EC}_{q,1}$; $d^{C}_{n,2} < d^{E}_{m,2}$ when $c/\theta < t^{EC}_{q,2}$; and $d^{C}_{n,1} + d^{C}_{n,2} < d^{E}_{m,1} + d^{E}_{m,2}$ when $c/\theta < t^{EC}_{q}$.

(ii) $d^D_{m,1} < d^E_{m,1} < d^D_{m,1} + d^D_{n,1}$*; and* $d^D_{m,2} < d^E_{m,2} < d^D_{m,2} + d^D_{n,2}$.

(iii) $d^D_{m,1} + d^D_{n,1} > d^C_{n,1}$*; and* $d^D_{m,2} + d^D_{n,2} > d^C_{n,2}$.

Part(i) demonstrates the trade-off of double marginalization effect and the cost disadvantage under C and E. More specifically, when m operates as a component supplier, the end product's price is escalated because of double marginalization; hence, less of the end product is sold. On the other hand, m does not suffer from double marginalization when operating as a sole entrant; however, it incurs a cost disadvantage when entering the end market. Consequently, the pressure on the margin forces m to decrease the end product's quantity. Part(i) indicates that as long as the cost disadvantage is not too high (i.e., the pressure on the margin is not too high), m's can increase its market coverage by being a sole entrant.

Part(ii) states that although m sells less of the end product under D than under E in every period, the total quantity on the market under D is higher. In fact, combining with Part(iii) leads to the conclusion that the dual structure gives the highest market coverage. It happens because the end market under D is more competitive than under other structures, and the market prices decrease accordingly. Here, the competitive pressure is helpful in two ways. First, it erodes the added margin of n in the end product's price. Second, it forces m to produce less of the end product (under a less efficient production process); instead m can benefit from the efficiency of n via the wholesale profit.

Proposition 5.2 ***(The Component's Price)***

In period 2, the optimal component's price under D is smaller than under C. However, in period 1, that price under D is larger than under C as long as $0 \leq c/\theta < t^{CD}_w$.

In a static model, a dual distributor tends to set a lower component's price, compared to a component supplier. This is because the dual distributor compensates n for its entering n's market; said differently, the dual distributor does so to protect its wholesale profit as n

will order less of the component when it faces more competition. That rationale is applicable only in period 2 of our model. In period 1, when m is less inefficient in producing the end product (i.e., $0 \le c/\theta < t_w^{CD}$), it becomes more aggressive in the end market despite the reduction in the wholesale profit. This shift in focus is interesting because m incurs the same level of cost disadvantage in every period. From the discussion under the dual distributor's problem (page 112), there are many forces under m's decision on the component's price in period 1. The result from this Proposition implies that m may raise its component's price even higher than the one under a component supplier structure to restrict n's quantity; in order words, under certain conditions, double marginalization is strengthened under the dual structure for the dual distributor's sake, a result that cannot arise under a static setting.

Next, the comparison of profits in period 2 under different supply chain structures is summarized in the following proposition:

Proposition 5.3 *When $\{E, C, D\}$ are feasible (i.e., $c/\theta < t^D$), in period 2,*
(i) The sole entrant's profit is highest when $c/\theta \le \min\{t_2^{EC}, t_2^{DE}\}$.
(ii) The component supplier's profit is highest when $c/\theta \ge \max\{t_2^{EC}, t_2^{DC}\}$.
(iii) The dual distributor's profit is highest when $t_2^{DE} < c/\theta < t_2^{DC}$

Figure 5.6 illustrates different regions mentioned in the above Proposition. The rationale of this outcome is as follows. From m's problem in period 2 under different supply chain structures, we can obtain

$$\Pi_{m,2}^{E} = \theta\Big[\frac{1}{2}(1-\gamma d_{m,1}^{E}) - \frac{c}{2\theta}\Big]^2 \tag{5.35}$$
$$\Pi_{m,2}^{C} = \frac{\theta(1-\gamma d_{n,1}^{C})^2}{8} \tag{5.36}$$
$$\Pi_{m,2}^{D} = \Big[\frac{c}{5}(1-\gamma(d_{m,1}^{D}+d_{n,1}^{D})) - \frac{2c^2}{50\theta}\Big] + \theta\Big[\frac{1}{2}(1-\gamma(d_{m,1}^{D}+d_{n,1}^{D})) - \frac{7c}{10\theta}\Big]^2 \tag{5.37}$$

Expressions (5.35), (5.36), and (5.37) reveal the importance of the effective leftover demand (i.e., $1 - \gamma d_1$) in determining m's profit in period 2, aside from the cost disadvantage c. For a given level of durability, m's profit in period 2 is depressed by its own product in the past. Further, these effective leftover demands are decreasing with durability; said differently, when the product is more durable, the future profit is squeezed (also see the discussion of the impact of durability under different structures). However, the quantities in the first period are different under different structures; hence, the effective leftover demand has different values under $\{E, C, D\}$.

Let d_l^i be the effective leftover demand under strategy i, $i = \{E, C, D\}$. From Proposition 5.1, the number of the end product in period 1 under D is highest, thus, the effective leftover demand is lowest under D. Furthermore, the negative impact of durability on d_l^D is higher than on d_l^E (similar conclusion is obtained under C when γ is relatively large). That is why when the durability goes up, the profit under D in period 2 goes down significantly; and subsequently, D is completely dominated when the durability is large (see Figure 5.6).

At high levels of durability, when D is dominated, a comparison of profits under C and E shows that the region for E to be higher is decreasing with durability γ. There are two reasons. First, at lower levels of cost disadvantage c, m generally produces more of the end product under E than under C (see Proposition 5.1), leading to a situation where m produces more in period 2 but its effective leftover demand in period 2 is lower. Second, the negative impact of durability on the effective leftover demand under E is stronger than under C. Combining these two effects leads to a more and more restricted region for E to dominate C when γ goes up.

Finally, we characterize the optimal channel structure over two periods in the following proposition:

Proposition 5.4 *When $\{E, C, D\}$ are feasible (i.e., $c/\theta < t^D$), the optimal supply chain structure is: (i) E when $c/\theta \le t^{DE}$, (ii) C when $c/\theta > t^{DC}$, and (iii) D when $t^{DE} < c/\theta < t^{DC}$.*

Figure 5.7 illustrates m's optimal supply chain structure as in the above Proposition. The optimal structure is mainly determined by two factors: the durability γ and m's relative cost disadvantage c/θ. There are three regions by the boundary curves t^{DE} and t^{DC} such that each corresponds to a particular structure being the optimal choice for m; the t^{D} curve ensures the feasibility of all structures.

Comparing Figure 5.7 with Figure 5.6, we can see the inconsistency of the long run (over two periods) vis-a-vis future profitability of different supply chain structures. This inconsistency only arises under durable goods markets with forward-looking consumers. If the future is not valuable at all (or $\delta = 0$), the optimal region coincides with the static model's outcome (as in Chapter 4) and durability has no impact on the optimal region. When the future is more valuable ($\delta > 0$), consumers start considering future benefit of a durable good (when $\gamma > 0$) and the firms' behavior (via expectations on future prices) when making purchase decisions.

Recall the discussion under Proposition 5.1, a dual distributor may benefit from a more competitive end market (i.e., leveraging n's expertise while lowering its margin). Interestingly, these two effects hurt the dual distributor at higher levels of durability. In particular, the feasibility of D reduces (t^{D} goes down with γ), and the region where D is dominated by E is bigger (t^{DE} goes up with γ). This happens because the largest market coverage under D is detrimental in period 2 (see the discussion under Proposition 5.3); and subsequently, D is not a very appealing structure with highly durable goods.

Furthermore, the optimality of E expands as γ goes up, but the optimal region of C tends to be smaller (t^{DC} increases with γ). It indicates that when feasibility is not a problem of D (i.e., $t^{DC} < t^{D}$)[1], the two aforementioned effects inside D (i.e., leveraging n's expertise while lowering its margin) appear to have different directions. More specifically, as durability

[1] It is worth noting that when D is not feasible, C is the optimal structure.

goes up, the first effect does not seem to be helpful as E (with less efficient production) dominates D more; while the latter helps strengthen the optimality of D (compared to C).

It is important to examine that when D is not feasible, whether m can increase its profitability through a different design rather than being a component supplier. The following proposition presents that option. Recall that we denote D_0 as the supply chain structure of a dual distributor when it does not produce the end product in period 2.

Proposition 5.5 *At higher levels of cost disadvantage (i.e., $c/\theta > t^{D}$), m is better off under D_0 when $c/\theta < t^{DC'}$.*

This above proposition proposes an interesting option for m when the products are highly durable (i.e., $t^{D} < c/\theta < t^{DC'}$). That is, m will be a strict dual distributor in period 1 and a component supplier in period 2. This option follows straightforward from the relative attractiveness of D compared to C when the product is more durable. Figure 5.8 illustrates the optimal region of D_0, relative to other structures when feasibility condition (i.e., $c/\theta < t^{D}$) is relaxed.

5.4 Conclusion

The purpose of this chapter has been on the optimality of different supply chain structures in durable goods markets. In particular, we examine three alternatives for a proprietary component manufacturer: being (a) a sole entrant, (b) a component supplier, and (c) a dual distributor. Our analysis reveals that the optimality of each of the alternatives is changed significantly (from the results in Chapter 4) when the product's durability is taken into account. More specifically, the sole entrant role becomes more preferred to a dual distributor role, while the component supplier role becomes less preferred. Further, under certain conditions, the manufacturer may opt to be a dual distributor in the first period and then become a component supplier in the second period. These results help explain, for instance, why certain companies like Apple may be better off embracing a sole entrant structure in the long-run.

Overall, our goal has been to underscore the importance of considering the impact of durability when designing the supply chain for manufacturers of proprietary components. Several opportunities seem to exist for further exploration. For example, companies often introduce improve versions of the product as time passes; hence, it is worth exploring the impact of such improvement on the optimal supply chain design. Another extension would be incorporating product differentiation in the end market (the current analysis assumes identical products). Finally, it is helpful to compare the optimal supply chain design under quantity vis-a-vis price competition.

Table 5.1: Marginal Consumers

	S_b	S_h	S_n	S_u	S_o
S_b	-				$\frac{p_1+\delta p_2^e-\delta p_u^e}{\theta(1+\delta)}$
S_h	$\frac{p_2^e-p_u^e}{\theta(1-\gamma)}$	-			$\frac{p_1}{\theta(1+\delta\gamma)}$
S_n	$\frac{p_1-\delta p_u^e}{\theta}$	$\frac{p_1-\delta p_2^e}{\theta(1-\delta+\delta\gamma)}$	-		$\frac{p_2^e}{\theta}$
S_u	$\frac{p_1+\delta p_2^e-2\delta p_u^e}{\theta(1+\delta-\delta\gamma)}$	$\frac{p_1-\delta p_u^e}{\theta}$	$\frac{p_2-p_u^e}{\theta(1-\gamma)}$	-	$\frac{p_u^e}{\theta\gamma}$

Table 5.2: The Optimal Solution For The Sole Entrant Setting

Solution	Comparative Statics		
	$\frac{\partial}{\partial\gamma}$	$\frac{\partial}{\partial c}$	$\frac{\partial^2}{\partial\gamma\partial c}$
$c/\theta < t^E$			
$d_{m,1}^E = \frac{2(\theta-c(1-\delta\gamma))}{\theta(4+4\delta\gamma-3\delta\gamma^2)}$	+/-	-	+
$p_{m,1}^E = \frac{\theta(4+8\gamma\delta-4\delta(1-\delta)\gamma^2-3\delta^2\gamma^3)+c(4+4\gamma\delta-2\delta\gamma^2-\delta^2\gamma^3)}{2(4+4\delta\gamma-3\delta\gamma^2)}$	+	+	+/-
$d_{m,2}^E = \frac{\theta(4-\gamma(2-4\delta)-3\delta\gamma^2)-c(4-\gamma(2-4\delta)-\delta\gamma^2)}{2\theta(4+4\delta\gamma-3\delta\gamma^2)}$	-	-	+/-
$p_{m,2}^E = \frac{\theta(4-\gamma(2-4\delta)-3\delta\gamma^2)+c(4+2\gamma(1+2\delta)-5\delta\gamma^2)}{2(4+4\delta\gamma-3\delta\gamma^2)}$	-	+	+/-
$p_u^E = \gamma\frac{\theta(2+(4-3\gamma)\delta)+c(8-2\gamma-\delta\gamma^2)}{2(4+4\delta\gamma-3\delta\gamma^2)}$	+	+	+
$c/\theta \geq t^E$			
$d_{m,1}^E = \frac{1}{2} - \frac{c}{2\theta(1+\delta\gamma)}$	+	-	+
$p_{m,1}^E = \frac{\theta(1+\delta\gamma)+c}{2}$	+	+	0

Table 5.3: The Optimal Solution For The Component Supplier Setting

Solution	Comparative Statics w.r.t γ
$d_{n,1}^C = \frac{8+3\delta\gamma}{4(8+8\delta\gamma-3\delta\gamma^2)}$	+/-
$w_{m,1}^C = \frac{\theta(128+240\delta\gamma-56\delta(1-2\delta)\gamma^2-45\delta^2\gamma^3)}{32(8+8\delta\gamma-3\delta\gamma^2)}$	+
$p_{n,1}^C = \frac{\theta(96+180\delta\gamma-4\delta(10-21\delta)\gamma^2-33\delta^2\gamma^3)}{16(8+8\delta\gamma-3\delta\gamma^2)}$	+
$d_{n,2}^C = \frac{32+8(4\delta-1)\gamma-15\delta\gamma^2}{16(8+8\delta\gamma-3\delta\gamma^2)}$	-
$w_{m,2}^C = \frac{\theta(32+8(4\delta-1)\gamma-15\delta\gamma^2)}{8(8+8\delta\gamma-3\delta\gamma^2)}$	-
$p_{n,2}^C = \frac{3\theta(32-(8-32\delta)\gamma-15\delta\gamma^2)}{16(8+8\delta\gamma-3\delta\gamma^2)}$	-
$p_u^C = \frac{\theta\gamma(64+4(2+21\delta)\gamma-33\delta\gamma^2)}{16(8+8\delta\gamma-3\delta\gamma^2)}$	+

Table 5.4: The Optimal Solution For The Dual Distributor Setting

Solution	Comparative Statics $\frac{\partial}{\partial\gamma}$	Comparative Statics $\frac{\partial}{\partial c}$
$c/\theta < t^D$		
$d^D_{m,1} = \frac{2}{4+4\delta\gamma-3\delta\gamma^2} - B*c$	+/-	-
$d^D_{n,1} = \frac{2c(4+4\delta\gamma-3\delta\gamma^2)}{\theta(2+2\delta\gamma-\delta\gamma^2)(10+10\delta\gamma-7\delta\gamma^2)}$	-	+
$w^D_{m,1} = \frac{\theta(4+8\delta\gamma-4\delta(1-\delta)\gamma^2-3\delta^2\gamma^3)}{2(4+4\delta\gamma-3\delta\gamma^2)} - E*c$	+	-
$p^D_{m,1} = \frac{\theta(4+8\delta\gamma-4\delta(1-\delta)\gamma^2-3\delta^2\gamma^3)}{2(4+4\delta\gamma-3\delta\gamma^2)} + H*c$	+	+
$d^D_{m,2} = \frac{4-2(1-2\delta)\gamma-3\delta\gamma^2}{2(4+4\delta\gamma-3\delta\gamma^2)} - F*c$	-	-
$d^D_{n,2} = \frac{2c}{5\theta}$	0	+
$w^D_{m,2} = \frac{\theta(4-2(1-2\delta)\gamma-3\delta\gamma^2)}{2(4+4\delta\gamma-3\delta\gamma^2)} - G*c$	-	+/-
$p^D_{m,2} = \frac{\theta(4-2(1-2\delta)\gamma-3\delta\gamma^2)}{2(4+4\delta\gamma-3\delta\gamma^2)} + K*c$	-	+
$p^D_u = \frac{\theta\gamma^2(2+\delta(4-3\gamma))}{2(4+4\delta\gamma-3\delta\gamma^2)} + L*c$	+	+
$t^D \le c/\theta < t^{D,0}$		
$d^D_{m,1} = \frac{\theta(40+62\delta\gamma+2\delta(11\delta-6)\gamma^2-7\delta^2\gamma^3)-4c(14+14\delta\gamma-5\delta\gamma^2)}{4\theta(4+4\delta\gamma-\delta\gamma^2)(5+5\delta\gamma-2\delta\gamma^2)}$	+	-
$d^D_{n,1} = \frac{\theta\delta\gamma(5\delta\gamma^2+4\gamma(1-2\delta)-8)+c(64+64\delta\gamma-28\delta\gamma^2)}{4\theta(4+4\delta\gamma-\delta\gamma^2)(5+5\delta\gamma-2\delta\gamma^2)}$	-	+
$w^D_{m,1} = \frac{\theta(160+304\delta\gamma-4\delta(19-36\delta)\gamma^2-63\delta^2\gamma^3)-4c(8+8\delta\gamma-5\delta\gamma^2)}{64(5+5\delta\gamma-2\delta\gamma^2)}$	+	-
$p^D_{m,1} = \frac{\theta(80+164\delta\gamma-4\delta(11-21\delta)\gamma^2+39\delta^2\gamma^3)+12c(4+4\delta\gamma-\delta\gamma^2)}{32(5+5\delta\gamma-2\delta\gamma^2)}$	+	+
$d^D_{m,2} = 0$	0	0
$d^D_{n,2} = \frac{5\theta(8-4(1-2\delta)\gamma-5\delta\gamma^2)+12\gamma c}{32\theta(5+5\delta\gamma-2\delta\gamma^2)}$	-	+
$w^D_{m,2} = \frac{5\theta(8-4(1-2\delta)\gamma-5\delta\gamma^2)+12\gamma c}{16(5+5\delta\gamma-2\delta\gamma^2)}$	-	+
$p^D_{m,2} = \frac{3(5\theta(8-4(1-2\delta)\gamma-5\delta\gamma^2)+12c\gamma)}{32(5+5\delta\gamma-2\delta\gamma^2)}$	-	+
$p^D_u = \gamma\frac{\theta(40+4(5+21\delta)\gamma-39\delta\gamma^2)+12c(4-\gamma)}{32(5+5\delta\gamma-2\delta\gamma^2)}$	+	+
$c/\theta \ge \max\{t^D, t^{D,0}\}$: Component Supplier		

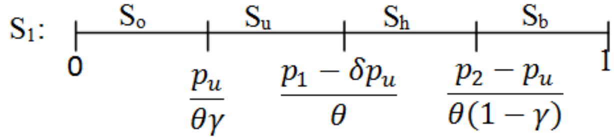

Figure 5.1: Segmentation Structure $\mathbb{S}_1$

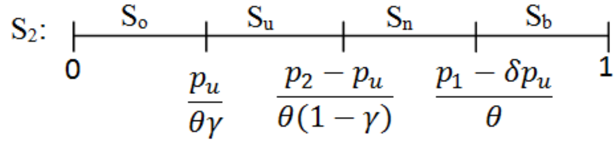

Figure 5.2: Segmentation Structure $\mathbb{S}_2$

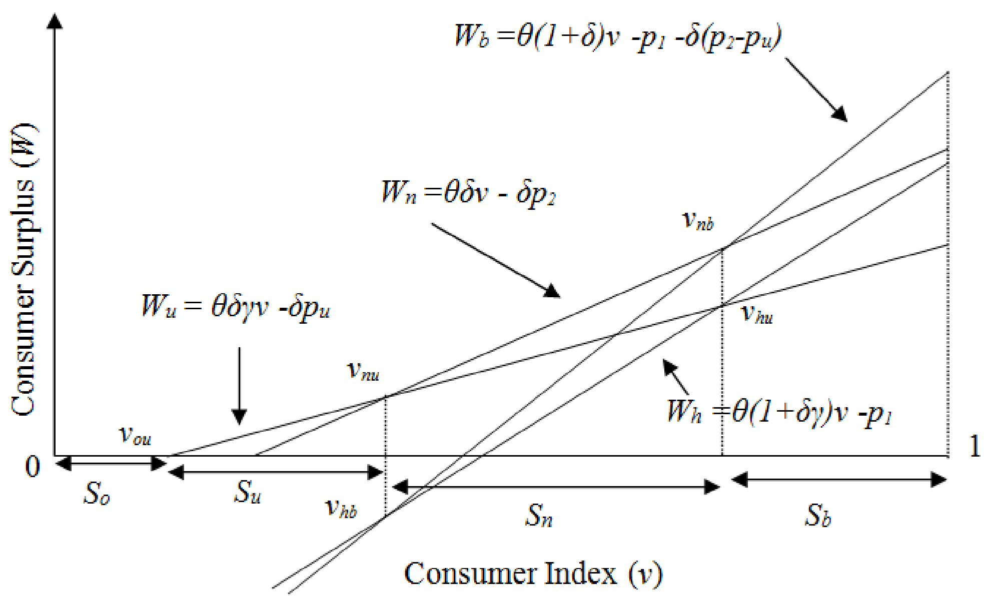

Figure 5.3: Segmentation without S_h

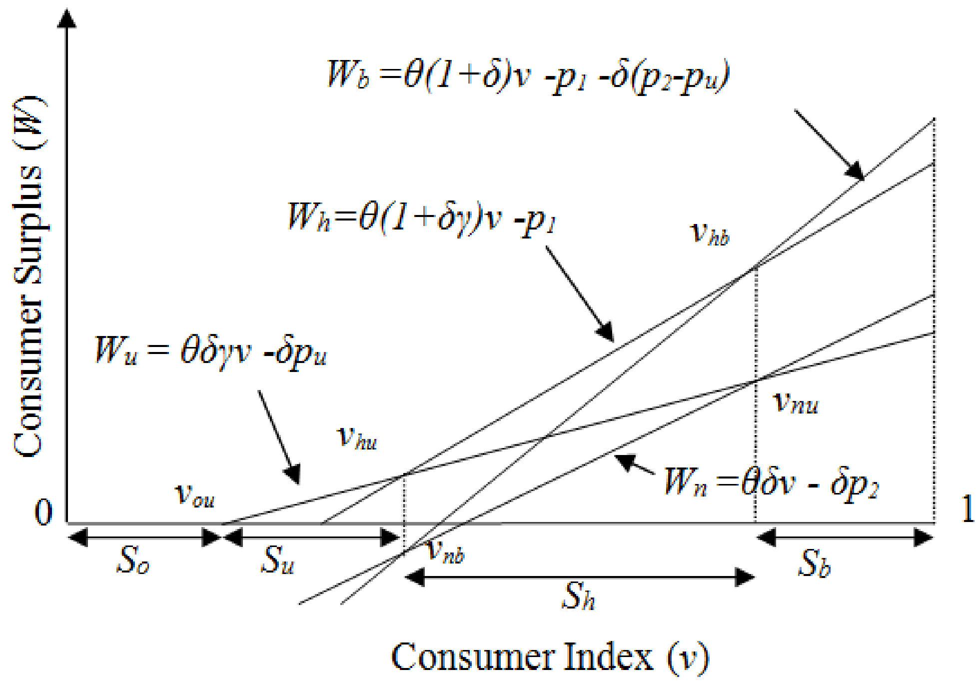

Figure 5.4: Segmentation without S_n

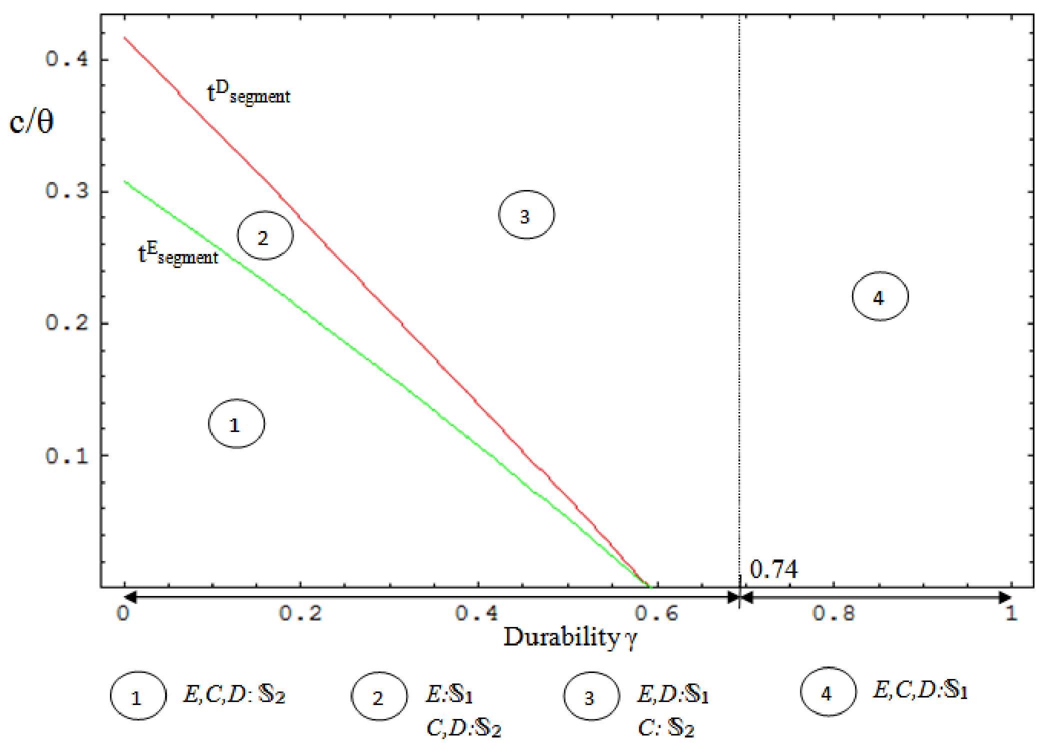

Figure 5.5: Segmentation under Different Channel Structures ($\delta = 0.9$)

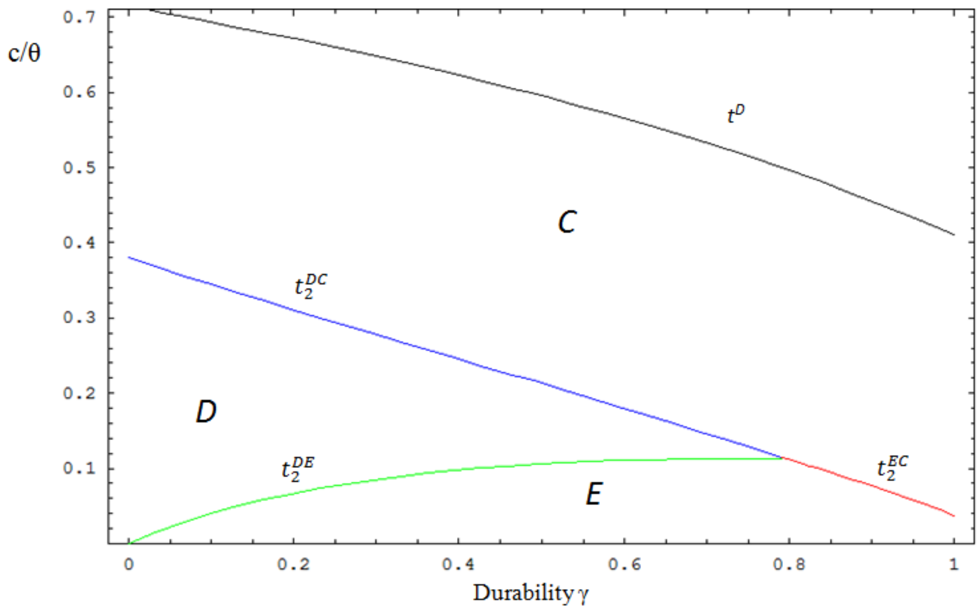

Figure 5.6: Profit Comparison in Period 2 ($\delta = 0.9$)

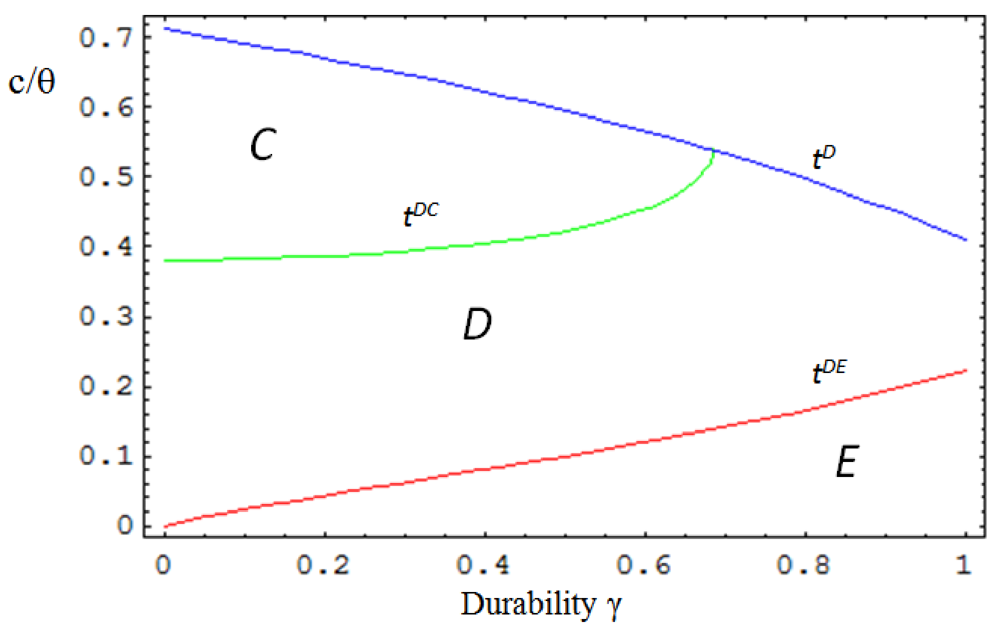

Figure 5.7: Optimal Channel Structure When There Are Product Sales in Both Periods ($\delta = 0.9$)

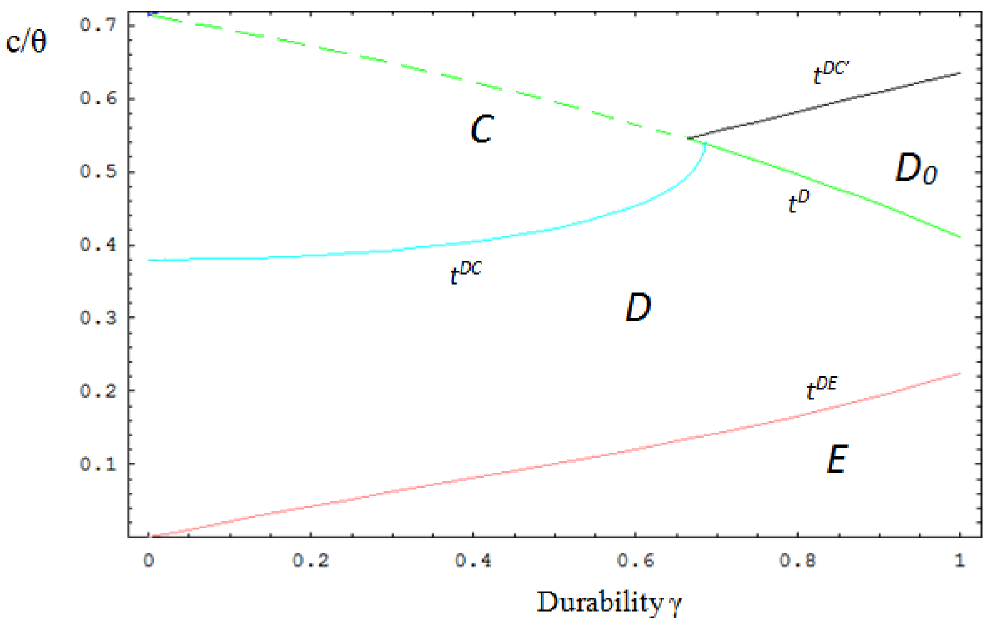

Figure 5.8: Optimal Channel Structure ($\delta = 0.9$)

APPENDIX A: PROOFS OF RESULTS FROM CHAPTER 3

Proof of Lemma 3.1

First, we derive the locations of the marginal consumers v_{0l}, v_{lh}, v_{hb} and v_{lb} from the following indifference conditions: $W_l(v = v_{0l}) = 0$, $W_h(v = v_{0h}) = 0$, $W_l(v = v_{lh}) = W_h(v = v_{lh})$, $W_h(v = v_{hb}) = W_b(v = v_{hb})$, and $W_l(v = v_{lb}) = W_b(v = v_{lb})$. Substituting for $W(.)$ gives:

$$v_{0l}\theta_l + \eta_{l,1} + \delta\eta_{l,2} - p_l = 0 \ , \tag{A1}$$

$$\delta(v_{0h}\theta_h + \eta_{h,2} - p_h) = 0 \ , \tag{A2}$$

$$v_{lh}\theta_l + \eta_{l,1} + \delta\eta_{l,2} - p_l = \delta(v_{lh}\theta_h + \eta_{h,2} - p_h) \ , \tag{A3}$$

$$\delta(v_{hb}\theta_h + \eta_{h,2} - p_h) = v_{hb}\theta_l + \eta_{l,1} - p_l + \delta((\theta_h - \theta_l)v_{hb} + \eta_{h,2} - p_h) \ , \text{ and} \tag{A4}$$

$$v_{lb}\theta_l + \eta_{l,1} + \delta\eta_{l,2} - p_l = v_{lb}\theta_l + \eta_{l,1} - p_l + \delta((\theta_h - \theta_l)v_{lb} + \eta_{h,2} - p_h). \tag{A5}$$

The solution to the above set of equations is summarized in Table 3.2.

Next, from Figure 3.1, we can see that under rapid improvement (i.e., $\delta\theta_h > \theta_l$), the new product demand in period 2 is made of consumers belonging to the interval $[v_{lh}, v_{hb}]$; consumers on either side of this interval have already purchased in period 1. Consequently, in period 2 (which is the last period in the game), the firm optimally sets the new product's price such that the lowest valuation marginal consumer in this segment receives zero surplus, i.e., $p_h = v_{lh}\theta_h + \eta_{h,2}$. However, over the two periods, the arrangement of consumer segments arises from the following conditions for the marginal consumers at v_{0l} and v_{lh}:

$$v_{0l}\theta_l + \eta_{l,1} + \delta\eta_{l,2} - p_l = 0 \ , \text{ and} \tag{A6}$$

$$v_{lh}\theta_l + \eta_{l,1} + \delta\eta_{l,2} - p_l = \delta(v_{lh}\theta_h + \eta_{h,2} - p_h). \tag{A7}$$

Substituting $p_h = v_{lh}\theta_h + \eta_{h,2}$ into the above equations and solving for p_l gives

$$p_l = v_{0l}\theta_l + \eta_{l,1} + \delta\eta_{l,2} \text{ , and} \tag{A8}$$

$$p_l = v_{lh}\theta_l + \eta_{l,1} + \delta\eta_{l,2}. \tag{A9}$$

Equations (A8) and (A9) require that $v_{lh} = v_{0l}$. This is inconsistent with the starting assumption that segment S_l exists. Hence, under a pricing strategy that constitutes a subgame-perfect equilibrium, both S_l and S_h cannot arise simultaneously.

Under gradual improvement, by contrast, the relative location of the segments (see Figure 3.2) is such that the above type of inconsistency does not arise. In period 2, the firm optimally sets the price of the new product such that the marginal consumer makes zero surplus. And all the segments can co-exist. ■

Proof of Lemma 3.2

Full/Forward Compatibilities

Under rapid improvement with either full or forward compatibility, when the firm does not serve segment S_l (i.e., uses targeting scheme T-h), under replacement strategy, we have $N_h = v_{hb} - v_{0h}$, $N_b = 1 - v_{hb}$, $\eta_{l,1} = \omega(1 - v_{hb})$, and $\eta_{h,2} = \eta_{l,2} = \omega(1 - v_{0h})$. From the main text, we know that skipping arises as a boundary condition on replacement, and under T-h, the new product's price and profitability in period 2 are the same under either replacement or skipping. Consequently, we focus on the segmentation structure under replacement and identify the conditions when skipping can arise.

Substituting the above expressions for the network values into the locations of the marginal consumers (see Table 3.2) gives

$$v_{0l} = \frac{1}{\theta_l}[p_l + \frac{(p_h - \theta_h)\delta\omega}{\theta_h - \omega} + \frac{(p_l - \theta_l(1-\delta))\omega}{\theta_l(1-\delta) - \omega}] \ , \tag{A10}$$

$$v_{lh} = \frac{1}{\delta\theta_h - \theta_l}[\delta p_h - p_l - \frac{(p_l - \theta_l(1-\delta))\omega}{\theta_l(1-\delta) - \omega}] \ , \tag{A11}$$

$$v_{0h} = \frac{p_h - \omega}{\theta_h - \omega} \ , \text{and} \tag{A12}$$

$$v_{hb} = \frac{p_l - \omega}{\theta_l(1-\delta) - \omega}. \tag{A13}$$

Since the solution under forward and full compatibilities is exactly the same under T-h, the solution under full compatibility derived below will also apply for forward compatibility.

The profit function in period 2 is $\Pi_2^{h,U,r} = p_h(1 - \frac{p_h - \omega}{\theta_h - \omega})$. Notice that this function is concave in p_h for all $\omega < \theta_h$ (as $\frac{\partial^2 \Pi_2^{h,U,r}}{\partial p_h^2} = \frac{-2}{\theta_h - \omega}$). Further, the second period price that satisfies the first-order condition $\frac{\partial \Pi_2^{h,U,r}}{\partial p_h} = 0$, is $\hat{p_h} = \frac{\theta_h}{2}$, and the corresponding demand is $N_h + N_b = \frac{\theta_h}{2(\theta_h - \omega)}$. Our focus is on the uncovered market setting, i.e., $N_h + N_b < 1 + M$; this is equivalent to the condition $\omega < \tilde{\omega}^{h,U} = \frac{\theta_h(1+2M)}{2+2M} < \theta_h$, and it follows that $p_h = \theta_h/2$ is the optimal solution for all $\omega < \tilde{\omega}^{h,U}$.

Taking the second period behavior into account, the present value of the total profit in period 1 is

$$\Pi_1^{h,U,r} = p_l(1 - \frac{p_l - \omega}{\theta_l(1-\delta) - \omega}) + \delta\frac{\theta_h^2}{4(\theta_h - \omega)}, \tag{A14}$$

where $\Pi_1^{h,U,r}$ is concave in p_l for all $\omega < \theta_l(1-\delta)$ (since $\frac{\partial^2 \Pi_1^{h,U,r}}{\partial p_l^2} = \frac{-2}{\theta_l(1-\delta) - \omega} < 0$). The unconstrained p_l in this case would be given by solving $\frac{\partial \Pi_1^{h,U,r}}{\partial p_l} = 0$, or equivalently, $p_l = \theta_l(1-\delta)/2$. However, if either ω is too large (i.e., $\omega > \theta_l(1-\delta)$), or the constraints that

p_l has to satisfy to engineer the required segmentation structure are binding, then a corner solution for p_l will arise. We now show that the interior solution will never arise and that the optimal price is indeed at a corner solution.

If $p_l = \theta_l(1-\delta)/2$, then

$$N_h = -\frac{(\theta_h - \theta_l(1-\delta))\omega}{2(\theta_h - \omega)(\theta_l(1-\delta)-\omega)} \leq 0 \text{ , and} \tag{A15}$$

$$v_{lh} - v_{0l} = \frac{-\delta\theta_h(\theta_l^2(1-\delta^2)\omega - 2\theta_l\delta\omega^2 - \theta_h(\theta_l^2\delta(1-\delta) + \theta_l(1-\delta-\delta^2)\omega - \delta\omega^2))}{2\theta_l(\delta\theta_h - \theta_l)(\theta_h-\omega)(\theta_l(1-\delta)-\omega)}. \tag{A16}$$

When $\omega = 0$, $N_h = 0$ and $v_{lh} - v_{0l} > 0$; and when $\omega > 0$, $N_h < 0$. Either of these outcomes violates the constraints required under T-h. Consequently, $p_l = \theta_l(1-\delta)/2$ can never be an optimal solution.

We now need to determine whether $v_{lh} - v_{0l} \leq 0$ or $N_h \geq 0$ is binding on the solution. As shown in period 2 calculations, $N_b + N_h > 0$, thus, at least one of N_b and N_h is strictly positive. Using Table 3.2, the first two constraints from above can be rewritten as $p_h \leq \frac{\theta_h}{\theta_l} * [p_l - \eta_{l,1}] - \frac{\delta\theta_h\eta_{l,2}}{\theta_l} + \eta_{h,2}$ and $p_h \leq \eta_{h,2} + \frac{\theta_h}{\theta_l} * [\frac{p_l - \eta_{l,1}}{1-\delta}]$. If $p_l - \eta_{l,1} \geq 0$, then the right hand side of the first inequality is smaller than the right hand side of the second inequality; and $v_{lh} - v_{0l} \leq 0$ is the critical constraint. However, if $p_l - \eta_{l,1} < 0$, then $v_{lh} - v_{0l} \leq 0$ is never satisfied since the right-hand side of the corresponding condition above is negative. Therefore, $p_l - \eta_{l,1} < 0$ cannot arise in equilibrium; and when $p_l - \eta_{l,1} \geq 0$, $N_h \geq 0$ cannot bind; said differently, N_h is strictly positive.

Substituting the new product's price into $v_{lh} - v_{0l} \leq 0$ gives:

$$p_l - \eta_{l,1} \geq \frac{\theta_l}{\theta_h}[p_h + \frac{\delta\theta_h\eta_{l,2}}{\theta_l} - \eta_{h,2}] = \frac{\omega(\delta\theta_h - \theta_l)}{2(\theta_h - \omega)} + \frac{\theta_l}{2} \tag{A17}$$

Since the right hand side of (A17) is strictly positive, $\hat{p_h}$ derived earlier is consistent with $v_{lh} - v_{0l} \leq 0$ being the critical constraint. Further, any p_l derived from $v_{lh} - v_{0l} = 0$ that satisfies $N_b = 1 - v_{hb} > 0$ will be the optimal price for a replacement strategy; by contrast, at such a price if $1 - v_{hb} \leq 0$ then a skipping strategy would arise.

The old product's price obtained from solving $v_{lh} - v_{0l} = 0$ is provided below:

$$p_l = \omega + \frac{(\theta_l(1-\delta) - \omega)(\theta_l(\theta_h - 2\omega) + \delta\theta_h\omega)}{2\theta_l(\theta_h - \omega)(1-\delta)}. \tag{A18}$$

It follows that $N_h = \frac{\delta(\theta_l(\theta_h - 2\omega) + \theta_h\omega)}{2\theta_l(1-\delta)(\theta_h - \omega)}$ and $N_b = \frac{\theta_l\theta_h(1-2\delta) + \delta(2\theta_l - \theta_h)\omega}{2\theta_l(1-\delta)(\theta_h - \omega)}$. Notice that the numerator of N_h is always positive if $\theta_h \geq 2\theta_l$. If $\theta_h < 2\theta_l$, then there is an upper bound on ω; however, that upper bound is bigger than θ_h, and consequently $N_h > 0$ for all $\omega < \theta_h$. Next, examine the numerator of N_b: if $2\theta_l \geq \theta_h$ (and under rapid improvement $\delta\theta_h > \theta_l$), then we need $\delta > 1/2$; thus, $N_b > 0$ when there is a lower bound on ω (this number is larger than θ_h) which violates the second order condition in period 2; it follows that $N_b = 0$ whenever $2\theta_l \geq \theta_h$. Next, when $2\theta_l < \theta_h$, if $\delta \geq 1/2$, then the numerator is negative and $N_b = 0$; otherwise, if $\delta < 1/2$, then $N_b > 0$ requires an upper bound on ω. Therefore, when $\delta < 1/2$ and $\omega < \omega_1^{h,U}$, where $\omega_1^{h,U} = \frac{\theta_l\theta_h(2\delta-1)}{\delta(2\theta_l-\theta_h)}$, $N_b > 0$; otherwise—i.e., when either $\delta \geq 1/2$, or $\delta < 1/2$ and $\omega \geq \omega_1^{h,U}$—skipping arises.

Next, note that when $\omega \geq \tilde{\omega}^{h,U}$, market is covered and the firm optimally sets the new product's price such that the consumer indexed at $-M$ receives zero surplus, i.e., $\tilde{p_h}^{h,U} = -M\theta_h + (1+M)\omega$. Furthermore, the order of $\omega_1^{h,U}$ and $\tilde{\omega}^{h,U}$ depends on the magnitude of θ_l/θ_h. We then consider two cases when $\delta < 1/2$: (1) $\omega_1^{h,U} < \tilde{\omega}^{h,U}$ and (2) $\omega_1^{h,U} \geq \tilde{\omega}^{h,U}$. When $\delta \geq 1/2$, skipping always occurs; and the market is covered when $\omega \geq \tilde{\omega}^{h,U}$ as mentioned earlier.

If $\omega_1^{h,U} < \tilde{\omega}^{h,U}$, or equivalently $\theta_l/\theta_h < \frac{\delta(1+2M)}{2(1-\delta+M)}$, then segment S_b vanishes when $\omega \geq \omega_1^{h,U}$. Further, the market is covered as ω approaches $\tilde{\omega}^{h,U}$. The new product is provided at $\tilde{p_h}^{h,U} \geq \frac{\theta_h}{2}$. In this case, too, segment S_b can not exist since the price net off the network benefit (i.e., $p_l - \eta_{l,1}$) for the old product is required to be even higher, ensuring that $W_b - W_h$ is negative.

If $\omega_1^{h,U} \geq \tilde{\omega}^{h,U}$, or equivalently $\theta_l/\theta_h \geq \frac{\delta(1+2M)}{2(1-\delta+M)}$, then when $\omega \geq \tilde{\omega}^{h,U}$, the new product is priced higher than $\theta_h/2$ (as argued above). Then the price net off the network benefit (i.e., $p_l - \eta_{l,1}$) of the old product will need to be higher than before, i.e., $p_l - \eta_{l,1} = \frac{\theta_l}{\theta_h}[\tilde{p_h}^{h,U} + \frac{\omega(1+M)(\delta\theta_h - \theta_l)}{\theta_l}]$. Consequently, segment S_b will vanish before ω approaches $\omega_1^{h,U}$. We call that value $\tilde{\omega}_1^{h,U}$. Also notice that $\frac{\delta(1+2M)}{2(1-\delta+M)}\Big|_{M->\infty} = \delta$, and $\theta_l/\theta_h \geq \delta$ will never hold under rapid improvement.

We assume that M is sufficiently large (so that $\omega_1^{h,U} < \tilde{\omega}^{h,U}$), and define $\overline{\omega}^{h,U} = \tilde{\omega}^{h,U}$ for the statement of Assumption 1 in the main text.

The following comparative statics are useful:

$$\frac{\partial N_b}{\partial \omega} = \frac{-\theta_h(\delta\theta_h - \theta_l)}{2\theta_l(1-\delta)(\theta_h - \omega)^2} < 0 \ , \tag{A19}$$

$$\frac{\partial N_h}{\partial \omega} = \frac{\delta\theta_h(\theta_h - \theta_l)}{2\theta_l(1-\delta)(\theta_h - \omega)^2} > 0, \text{ and} \tag{A20}$$

$$\frac{\partial p_l}{\partial \omega} = -\frac{\delta(2\theta_l - \theta_h)\omega^2 - 2\delta\theta_h(2\theta_l - \theta_h)\omega + \theta_l\theta_h(\theta_l(1-\delta) + \theta_h(\delta^2 + \delta - 1))}{2\theta_l(1-\delta)(\theta_h - \omega)^2}. \tag{A21}$$

To sign the derivative in (A21), consider the roots of $\frac{\partial p_l}{\partial \omega} = 0$. We will focus on the case $\delta < 1/2$ (i.e., $2\theta_l - \theta_h < 0$) when replacement arises. $\frac{\partial p_l}{\partial \omega} = 0$ when

$$\omega = \theta_h - \frac{\sqrt{\delta\theta_h(\theta_h - \theta_l(1-\delta))(\delta\theta_h - \theta_l)(\theta_h - 2\theta_l)}}{\delta(\theta_h - 2\theta_l)}, \text{ or} \tag{A22}$$

$$\omega = \theta_h + \frac{\sqrt{\delta\theta_h(\theta_h - \theta_l(1-\delta))(\delta\theta_h - \theta_l)(\theta_h - 2\theta_l)}}{\delta(\theta_h - 2\theta_l)} > \theta_h. \tag{A23}$$

Denote the smaller root as $\omega_2^{h,U}$ (i.e., the first root given in A22). For the uncovered market, recall that $\omega < \tilde{\omega}^{h,U} < \theta_h$; hence, $\frac{\partial p_l}{\partial \omega} \leq 0$ when $\omega > \omega_2^{h,U}$, and $\frac{\partial p_l}{\partial \omega} > 0$ when $\omega < \omega_2^{h,U}$.

Also notice that when $\theta_l/\theta_h \leq \frac{3\delta-1-\delta^2}{2\delta(1-\delta)}$, then $\omega_2^{h,U} \geq \omega_1^{h,U}$; hence, $\frac{\partial p_l}{\partial \omega} > 0$ whenever replacement arises.

Backward Compatibility

Under rapid improvement with backward compatibility and T-h, we have $N_h = v_{hb} - v_{0h}$, $N_b = 1 - v_{hb}$, $\eta_{l,1} = \omega(1 - v_{hb})$, $\eta_{l,2} = 0$, and $\eta_{h,2} = \omega(1 - v_{0h})$. The locations of the marginal consumers are:

$$v_{0h} = \frac{p_h - \omega}{\theta_h - \omega}, \tag{A24}$$

$$v_{hb} = \frac{p_l - \omega}{\theta_l(1-\delta) - \omega}, \tag{A25}$$

$$v_{0l} = \frac{(1-\delta)(p_l - \omega)}{\theta_l(1-\delta) - \omega}, \text{ and} \tag{A26}$$

$$v_{lh} = \frac{1}{\delta\theta_h - \theta_l}\left[\delta p_h - p_l + \frac{\delta\omega(p_h - \theta_h)}{\theta_h - \omega} + \frac{\omega((1-\delta)\theta_l - p_l)}{\theta_l(1-\delta) - \omega}\right]. \tag{A27}$$

Analogous to full compatibility, the profit function in period 2 is $\Pi_2^{h,B,r} = p_h(1 - \frac{p_h - \omega}{\theta_h - \omega})$ and this function is concave in p_h when $\omega < \theta_h$ (as $\frac{\partial^2 \Pi_2^{h,B,r}}{\partial p_h^2} = \frac{-2}{\theta_h - \omega}$). The solution to the first-order condition $\frac{\partial \Pi_2^{h,B,r}}{\partial p_h} = 0$ is $\hat{p_h} = \frac{\theta_h}{2}$; and the corresponding demand is $N_h + N_b = \frac{\theta_h}{2(\theta_h - \omega)}$. The market is uncovered if $N_h + N_b < 1 + M$, or when $\omega < \tilde{\omega}^{h,B} = \frac{\theta_h(1+2M)}{2+2M}$, where $\tilde{\omega}^{h,B} < \theta_h$; in this setting, $p_h = \theta_h/2$ is the optimal solution.

Taking the second period behavior into account, the present value of the total profit in period 1 is

$$\Pi_1^{h,B,r} = p_l(1 - \frac{p_l - \omega}{\theta_l(1-\delta) - \omega}) + \delta\frac{\theta_h^2}{4(\theta_h - \omega)}, \tag{A28}$$

where $\Pi_1^{h,B,r}$ is concave in p_l for all $\omega < \theta_l(1-\delta)$ (since $\frac{\partial^2 \Pi_1^{h,B,r}}{\partial p_l^2} = \frac{-2}{\theta_l(1-\delta) - \omega} < 0$). The unconstrained p_l in this case would be given by solving $\frac{\partial \Pi_1^{h,B,r}}{\partial p_l} = 0$, or equivalently, $p_l =$

$\theta_l(1-\delta)/2$. However, if either ω is too large (i.e., $\omega > \theta_l(1-\delta)$), or the constraints that p_l has to satisfy to engineer the required segmentation structure are binding, then a corner solution for p_l will arise. We now show that the interior solution will never arise and that the optimal price is indeed at a corner solution.

If $p_l = \theta_l(1-\delta)/2$, then

$$N_h = -\frac{(\theta_h - \theta_l(1-\delta))\omega}{2(\theta_h - \omega)(\theta_l(1-\delta) - \omega)} \leq 0 \text{ , and} \tag{A29}$$

$$v_{lh} - v_{0l} = \frac{-\delta\theta_h(\theta_l(1-\delta^2)\omega - 2\delta\omega^2 - \theta_h(\theta_l\delta(1-\delta) + \omega - 2\delta\omega))}{2(\delta\theta_h - \theta_l)(\theta_h - \omega)(\theta_l(1-\delta) - \omega)}. \tag{A30}$$

When $\omega = 0$, $v_{lh} - v_{0l} > 0$; and when $\omega > 0$, $N_h < 0$. Either of these outcomes violates the constraints required under T-h. Consequently, $p_l = \theta_l(1-\delta)/2$ can never be an optimal solution.

Next, we need to determine whether $v_{lh} - v_{0l} \leq 0$ or $N_h \geq 0$ is binding on the solution. From Table 3.2, these two constraints can be rewritten as $p_h \leq \eta_{h,2} + \frac{\theta_h}{\theta_l} * [p_l - \eta_{l,1}]$ and $p_h \leq \eta_{h,2} + \frac{\theta_h}{\theta_l} * [\frac{p_l - \eta_{l,1}}{1-\delta}]$. If $p_l - \eta_{l,1} \geq 0$ then the right hand side of the first inequality is smaller than the right hand side of the second inequality; and $v_{lh} - v_{0l} \leq 0$ is the critical constraint. Otherwise, if $p_l - \eta_{l,1} < 0$, then the second inequality (i.e., $N_h \geq 0$) is the critical constraint. Since $p_h - \eta_{h,2}$ (which equals $\frac{\theta_h(\theta_h - 2\omega)}{2(\theta_h - \omega)}$) has the same sign as $p_l - \eta_{l,1}$ regardless of which constraint is binding, it follows that $p_l - \eta_{l,1} \geq 0$ corresponds to $\omega \leq \theta_h/2$, and $p_l - \eta_{l,1} < 0$ corresponds to $\omega > \theta_h/2$.

If $\omega \leq \theta_h/2$, then $v_{lh} - v_{0l} \leq 0$ is binding; and the old product's price is $p_l = \omega + \frac{(\theta_h - 2\omega)(\theta_l(1-\delta) - \omega)}{2(1-\delta)(\theta_h - \omega)}$; the corresponding demand is $N_b = \frac{\theta_h(1-2\delta) + 2\delta\omega}{2(1-\delta)(\theta_h - \omega)} > 0$ when $\omega > \omega_1^{h,B}$, where $\omega_1^{h,B} = \frac{\theta_h(2\delta - 1)}{2\delta}$. It follows that when $\delta \geq 1/2$ and $\omega \leq \omega_1^{h,B}$, $N_b = 0$, and skipping arises.

If $\omega > \theta_h/2$, then $N_h = v_{hb} - v_{0h} \geq 0$ is binding; and the old product's price is $p_l = \omega + \frac{(\theta_h - 2\omega)(\theta_l(1-\delta)-\omega)}{2(\theta_h-\omega)}$. Under this condition, the firm only serves segment S_b whose size is determined in period 2 as $N_b = \frac{\theta_h}{2(\theta_h-\omega)}$.

We define $\overline{\omega}^{h,B} = \tilde{\omega}^{h,B}$ for the statement of Assumption 1 in the main text.

The following comparative statics results are useful:

When $\omega > \theta_h/2$,

$$\frac{\partial p_l}{\partial \omega} = \frac{\theta_h(\theta_h - \theta_l(1-\delta))}{2(\theta_h-\omega)^2} > 0 \text{ , and} \tag{A31}$$

$$\frac{\partial N_b}{\partial \omega} = \frac{\theta_h}{2(\theta_h-\omega)^2} > 0. \tag{A32}$$

When $\omega \leq \theta_h/2$,

$$\frac{\partial N_b}{\partial \omega} = \frac{\theta_h}{2(1-\delta)(\theta_h-\omega)^2} > 0 \text{ ,} \tag{A33}$$

$$\frac{\partial N_h}{\partial \omega} = \frac{-\delta\theta_h}{2(1-\delta)(\theta_h-\omega)^2} < 0, \text{ and} \tag{A34}$$

$$\frac{\partial p_l}{\partial \omega} = \frac{\theta_h(\theta_h(1-2\delta) - \theta_l(1-\delta)) + 4\theta_h\delta\omega - 2\delta\omega^2}{2(1-\delta)(\theta_h-\omega)^2} \tag{A35}$$

To sign (A35), we need to know the sign of its numerator. We will focus on the case where replacement arises. Now consider the roots of the equation $\theta_h(\theta_h(1-2\delta) - \theta_l(1-\delta)) + 4\theta_h\delta\omega - 2\delta\omega^2 = 0$, which are $\theta_h - \frac{\sqrt{2\delta\theta_h(\theta_h-(1-\delta)\theta_l)}}{2\delta}$ and $\theta_h + \frac{\sqrt{2\delta\theta_h(\theta_h-(1-\delta)\theta_l)}}{2\delta} > \theta_h/2$. Define $\omega_2^{h,B} = \theta_h - \frac{\sqrt{2\delta\theta_h(\theta_h-(1-\delta)\theta_l)}}{2\delta}$.

If $\delta \geq 1/2$ or ($\delta < 1/2$ and $\theta_l/\theta_h > \frac{1-2\delta}{1-\delta}$), the two roots of $\theta_h(\theta_h(1-2\delta) - \theta_l(1-\delta)) + 4\theta_h\delta\omega - 2\delta\omega^2 = 0$ are positive. Hence, when $\omega < \omega_2^{h,B}$, the numerator is negative; and when $\omega_2^{h,B} \leq \omega \leq \theta_h/2$, the numerator is positive. Notice that under $\delta \geq 1/2$, when $\theta_l/\theta_h \leq \frac{2\delta-1}{2\delta(1-\delta)}$, $\omega_1^{h,B} \geq \omega_2^{h,B}$; hence, $\frac{\partial p_l}{\partial \omega} \geq 0$ whenever replacement arises.

If $\delta < 1/2$ and $\theta_l/\theta_h \leq \frac{1-2\delta}{1-\delta}$, then $\theta_h(\theta_h(1-2\delta)-\theta_l(1-\delta))+4\theta_h\delta\omega-2\delta\omega^2=0$ has roots with opposite signs. Hence, the numerator is positive for all $\omega \leq \theta_h/2$.■

Proof of Lemma 3.3

Full Compatibility

Under rapid improvement with full compatibility, when the firm does not serve segment S_h (i.e., uses targeting scheme T-l), we have $N_l = v_{lb} - v_{0l}$, $N_b = 1 - v_{lb}$, $\eta_{l,1} = \eta_{l,2} = \eta_{h,2} = \omega(1 - v_{0l})$. Substituting these into the marginal consumers provided in Table 3.2 gives

$$v_{0l} = \frac{p_l - \omega(1+\delta)}{\theta_l - \omega(1+\delta)}\ , \tag{A36}$$

$$v_{lh} = \frac{\delta p_h(\theta_l - \omega(1+\delta)) - p_l(\theta_l - \delta\omega) + \theta_l\omega)}{(\delta\theta_h - \theta_l)(\theta_l - \omega(1+\delta))}\ , \tag{A37}$$

$$v_{hb} = \frac{p_l(\theta_l - \delta\omega) - \theta_l\omega}{\theta_l(1+\delta)(\theta_l - \omega(1+\delta))}\ , \text{ and} \tag{A38}$$

$$v_{lb} = \frac{p_h}{\theta_h - \theta_l}. \tag{A39}$$

The profit function in period 2 is $\Pi_2^{l,U} = p_h(1 - v_{lb}) = p_h(1 - \frac{p_h}{\theta_h - \theta_l})$. Notice this function is concave in p_h for all $\theta_h > \theta_l$ (as $\frac{\partial^2 \Pi_2^{l,U}}{\partial p_h^2} = \frac{-2}{\theta_h - \theta_l} < 0$). Hence, the optimal price that satisfies the first order condition $\frac{\partial \Pi_2^{l,U}}{\partial p_h} = 0$ is $\hat{p_h} = \frac{\theta_h - \theta_l}{2}$; and the corresponding demand is $N_b = 1/2$.

Taking the second period behavior into account, the present value of the total profit in period 1 is

$$\Pi_1^{l,U} = p_l(1 - \frac{p_l - \omega(1+\delta)}{\theta_l - \omega(1+\delta)}) + \delta\frac{\theta_h - \theta_l}{4}, \tag{A40}$$

where $\Pi_1^{l,U}$ is concave in p_l for all $\omega < \frac{\theta_l}{1+\delta}$ (since $\frac{\partial^2 \Pi_1^{l,U}}{\partial p_l^2} = \frac{-2}{\theta_l - \omega(1+\delta)} < 0$). The unconstrained p_l in this case would be given by solving $\frac{\partial \Pi_1^{l,U}}{\partial p_l} = 0$, or equivalently, $p_l = \theta_l/2$. However, if the constraints that p_l has to satisfy to engineer the required segmentation structure are binding, then a corner solution for p_l will arise. Define $\omega_1^{l,U} = \frac{\delta\theta_l}{1+\delta+\delta^2}$. We now show that when $\omega_1^{l,U} \leq \omega \leq \frac{\theta_l}{1+\delta}$, an interior solution will arise; and when $\omega < \omega_1^{l,U}$, a corner solution will arise.

When $\omega_1^{l,U} \leq \omega \leq \frac{\theta_l}{1+\delta}$, with $p_l = \theta_l/2$, the corresponding demand in period 1 is $N_l + N_b = \frac{\omega(1+\delta)}{2(\theta_l-\omega(1+\delta))} + 1/2$, where $N_l = \frac{\omega(1+\delta)}{2(\theta_l-\omega(1+\delta))} \geq 0$. Also, $v_{hb} - v_{lh} = \frac{\delta(\theta_h-\theta_l)(\delta\theta_l-(1+\delta+\delta^2)\omega)}{2(1-\delta)(\delta\theta_h-\theta_l)(\theta_l-(1+\delta)\omega)} \leq 0$ for all $\omega \geq \omega_1^{l,U}$. Since $p_l = \theta_l/2$ satisfies all the constraints, it is indeed the optimal solution.

Notice that the market is not fully covered when $N_l + N_b < 1 + M$, or equivalently, $\omega < \tilde{\omega}^{l,U} = \frac{\theta_l(1+2M)}{(1+\delta)(2+2M)}$, where $\tilde{\omega}^{l,U} > \omega_1^{l,U}$ for a sufficiently large M (i.e., $M > \frac{1-\delta-\delta^2}{2}$). Therefore, $p_l = \theta_l/2$ is the optimal solution for all $\omega \in [\omega_1^{l,U}, \tilde{\omega}^{l,U})$.

When $\omega < \omega_1^{l,U}$, the constraint $v_{hb} \leq v_{lh}$ is violated and will bind. Solving $v_{hb} = v_{lh}$ gives

$$p_l = \frac{\theta_l(\theta_l(1-\delta) + \omega(1+\delta^2))}{2(\theta_l - \delta\omega)} > 0; \tag{A41}$$

and the corresponding size of segment S_l is $N_l = \frac{\delta(\theta_l+\omega)}{2(\theta_l-\delta\omega)} > 0$.

The following comparative statics are useful:

At the interior solution: $\frac{\partial(N_l+N_b)}{\partial\omega} = \frac{\theta_l(1+\delta)}{2(\theta_l-(1+\delta)\omega)^2} > 0$.

At the corner solution: $\frac{\partial p_l}{\partial\omega} = \frac{\theta_l^2(1+\delta)}{2(\theta_l-\delta\omega)^2} > 0$, $\frac{\partial(N_l+N_b)}{\partial\omega} = \frac{\theta_l(1+\delta)\delta}{2(\theta_l-\delta\omega)^2} > 0$.

Notice that when the market is covered, then the firm sets the old product's price such that the consumer indexed $v = -M$ receives zero surplus, or equivalently, $p_l = -M\theta_l + (1+\delta)(1+M)\omega$. Further, $v_{hb} \leq v_{lh}$ as long as $\omega \leq \tilde{\omega}_f^{l,U} = \frac{\theta_l(1-\delta+2M)}{\delta(2+2M)}$, where $\tilde{\omega}_f^{l,U} > \tilde{\omega}^{l,U}$ for a sufficiently large M (i.e., $M > \frac{\delta^2+\delta-1}{2}$). Hence, at high levels of ω (i.e., $\omega > \tilde{\omega}_f^{l,U}$), the firm will price the product such that $v_{hb} \leq v_{lh}$ binds. It follows that $p_l = \frac{\theta_l(1-\delta)}{2} + \omega(1+M)$. We, however, focus on the $[0, \overline{\omega}^{l,U}]$ range for ω, where $\overline{\omega}^{l,U} = \tilde{\omega}^{l,U}$.

Backward Compatibility

Under rapid improvement with backward compatibility and T-l, we have $N_l = v_{lb} - v_{0l}$, $N_b = 1 - v_{lb}$, $\eta_{l,1} = \omega(1 - v_{0l})$, $\eta_{l,2} = \omega(v_{lb} - v_{0l})$, and $\eta_{h,2} = \omega(1 - v_{0l})$; our approach is similar to the one under full compatibility. The locations of the marginal consumers are:

$$v_{0l} = \frac{p_l - \omega}{\theta_l - (1+\delta)\omega} - \frac{\delta\omega(\theta_h - \omega)}{(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}, \tag{A42}$$

$$v_{lb} = \frac{p_h - \omega}{\theta_h - \theta_l - \omega}, \tag{A43}$$

$$v_{lh} = \frac{p_h\delta((\theta_h - \theta_l)(\theta_l - (1+\delta)\omega) + \omega^2) - p_l(\theta_h - \theta_l - \omega)(\theta_l - \delta\omega)}{(\delta\theta_h - \theta_l)(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}$$
$$+ \frac{\theta_l(\theta_h - \theta_l)(1-\delta)\omega + (\theta_h\delta^2 - \theta_l(1+\delta^2))\omega^2}{(\delta\theta_h - \theta_l)(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}, \text{ and} \tag{A44}$$

$$v_{hb} = \frac{1}{\theta_l(1-\delta)}\Big[p_l - \omega\frac{(\theta_h - \theta_l)(\theta_l - p_l) + (p_l - p_l + (p_h + \theta_l - \theta_h)\delta)\omega}{(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}\Big]. \tag{A45}$$

The profit in period 2 is $\Pi_2^{l,B} = p_h(1 - v_{lb}) = p_h(1 - \frac{p_h - \omega}{\theta_h - \theta_l - \omega})$, and is concave in p_h for all $\omega < \theta_h - \theta_l$ (as $\frac{\partial^2 \Pi_2^{l,B}}{\partial p_h^2} = \frac{-2}{\theta_h - \theta_l - \omega} < 0$). Hence, the optimal price that satisfies the first order condition $\frac{\partial \Pi_2^{l,B}}{\partial p_h} = 0$ is $\hat{p}_h = \frac{\theta_h - \theta_l}{2}$; and the corresponding demand is $N_b = \frac{\theta_h - \theta_l}{2(\theta_h - \theta_l - \omega)}$.

Taking the second period behavior into account, the present value of the total profit in period 1 is

$$\Pi_1^{l,B} = p_l\Big[1 - \frac{p_l - \omega}{\theta_l - (1+\delta)\omega} + \frac{\delta\omega(\theta_h - \omega)}{(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)})\Big] + \delta\frac{(\theta_h - \theta_l)^2}{4(\theta_h - \theta_l - \omega)}, \tag{A46}$$

where $\Pi_1^{l,B}$ is concave in p_l for all $\omega < \frac{\theta_l}{1+\delta}$ (since $\frac{\partial^2 \Pi_1^{l,B}}{\partial p_l^2} = \frac{-2}{\theta_l - \omega(1+\delta)} < 0$). The unconstrained p_l in this case would be given by solving $\frac{\partial \Pi_1^{l,B}}{\partial p_l} = 0$, or equivalently, $p_l = \theta_l/2 - \frac{\delta\omega(\theta_h - \theta_l)}{4(\theta_h - \theta_l - \omega)}$; and the corresponding size of S_l is $N_l = \frac{\omega(\theta_h(2+\delta) - \theta_l(4+\delta))}{4(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}$. However, if the constraints that p_l has to satisfy to engineer the required segmentation structure are binding, then a corner solution for p_l will arise.

Define

$$\omega_1^{l,B} = \frac{\theta_l}{2\delta(3\theta_l\delta + \theta_h(2+\delta))}\big[\theta_h(2+3\delta+2\delta^2) - \theta_l(4-\delta+2\delta^2) -$$
$$\sqrt{8\delta^2(\theta_l-\theta_h)(3\theta_l\delta+\theta_h(2+\delta)) + (\theta_l(-4+\delta-2\delta^2)+\theta_h(2+3\delta+2\delta^2))^2}\big]. \tag{A47}$$

We now show that: (1) when $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$, if $\omega_1^{l,B} \le \omega \le \min\{\theta_h - \theta_l, \frac{\theta_l}{1+\delta}\}$, then an interior solution will arise, and if $\omega < \omega_1^{l,B}$, a corner solution from the binding constraint $v_{hb} = v_{lh}$ will arise; (2) when $\theta_l/\theta_h \ge \frac{2+\delta}{4+\delta}$, a corner solution from either the binding constraint $v_{hb} = v_{lh}$ or $N_l = v_{lb} - v_{0l} = 0$ will arise.

When $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$ and $\omega_1^{l,B} \le \omega \le \min\{\theta_h - \theta_l, \frac{\theta_l}{1+\delta}\}$, with the old product's price at the interior solution, the corresponding size of segment S_l is $N_l = \frac{\omega(\theta_h(2+\delta)-\theta_l(4+\delta))}{4(\theta_h-\theta_l-\omega)(\theta_l-(1+\delta)\omega)} > 0$. Further, we have

$$v_{hb} - v_{lh} = \frac{-\delta(\theta_h-\theta_l)f_1}{4\theta_l(1-\delta)(\delta\theta_h-\theta_l)(\theta_h-\theta_l-\omega)(\theta_l-(1+\delta)\omega)}, \text{ where} \tag{A48}$$

$$f_1 = 2\delta\theta_l^2(\theta_l-\theta_h) + \theta_l(\theta_l(-4+\delta-2\delta^2)+\theta_h(2+3\delta+2\delta^2))\omega - \delta(3\delta\theta_l+\theta_h(2+\delta))\omega^2. \tag{A49}$$

Notice that $f_1 = 0$ has two positive roots, where $\omega_1^{l,B}$ is the smaller root, and the larger root is larger than $\theta_l/(1+\delta)$. Hence, $f_1 \ge 0$ for all $\omega_1^{l,B} \le \omega \le \min\{\theta_h - \theta_l, \frac{\theta_l}{1+\delta}\}$; it follows that $v_{hb} \le v_{lh}$ for all $\omega_1^{l,B} \le \omega \le \min\{\theta_h - \theta_l, \frac{\theta_l}{1+\delta}\}$.

Denote $\tilde{\omega}_1^{l,B}$ as the value of ω at which $N_l + N_b = 1 + M$; when $\omega \ge \tilde{\omega}_1^{l,B}$ (where $\tilde{\omega}_1^{l,B} < \min\{\theta_h - \theta_l, \frac{\theta_l}{1+\delta}\}$), the market is covered, and the firm sets price that makes the surplus of consumer indexed $-M$ equal zero.

When either $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$ and $\omega < \omega_1^{l,B}$, or $\theta_l/\theta_h \ge \frac{2+\delta}{4+\delta}$ and $\omega < \frac{\theta_h-\theta_l}{2}$, the constraint $v_{hb} \le v_{lh}$ is violated at the interior solution. The old product's price is then derived from the condition that $v_{hb} = v_{lh}$:

$$p_l = \frac{\theta_l(\theta_l(1-\delta) + \omega(1+\delta^2))}{2(\theta_l - \delta\omega)} - \frac{\omega(\theta_l(1-\delta)(\theta_l - \delta\omega) + \omega(\delta\theta_h - \theta_l))}{2(\theta_l - \delta\omega)(\theta_h - \theta_l - \omega)}; \tag{A50}$$

and the corresponding size of segment S_l is $N_l = \frac{\delta\theta_l(\theta_h - \theta_l - 2\omega)}{2(\theta_h - \theta_l - \omega)(\theta_l - \delta\omega)} \geq 0$.

Finally, when $\theta_l/\theta_h \geq \frac{2+\delta}{4+\delta}$ and $\frac{\theta_h - \theta_l}{2} \leq \omega < \frac{\theta_l(\theta_h - \theta_l)}{3\theta_l - \theta_h}$, then $N_l = v_{lb} - v_{0l} \geq 0$ is binding; and the old product's price is obtained by setting $v_{lb} - v_{0l} = 0$, or equivalently

$$p_l = \frac{\theta_l(\theta_h - \theta_l) - \omega(3\theta_l - \theta_h)}{2(\theta_h - \theta_l - \omega)} > 0. \tag{A51}$$

At this price, $v_{hb} - v_{lh} = \frac{-\delta^2(\theta_h - \theta_l)(\theta_h - \theta_l - 2\omega)}{2(1-\delta)(\delta\theta_h - \theta_l)(\theta_h - \theta_l - \omega)} \leq 0$.

Notice that the market is not fully covered when $N_b < 1 + M$, or equivalently, when $\omega < \tilde{\omega}_2^{l,B} = \frac{(\theta_h - \theta_l)(1+2M)}{2+2M} < \theta_h - \theta_l$. Further, when $\theta_l/\theta_h < \frac{1+2M}{1+4M}$, then $\frac{\theta_l(\theta_h - \theta_l)}{3\theta_l - \theta_h} > \tilde{\omega}_2^{l,B}$. We assume a sufficiently large M so that $\frac{\theta_l(\theta_h - \theta_l)}{3\theta_l - \theta_h} > \tilde{\omega}_2^{l,B}$.

Define

$$\overline{\omega}^{l,B} = \begin{cases} \tilde{\omega}_1^{l,B} & \text{if } \theta_l/\theta_h < \frac{2+\delta}{4+\delta} \\ \tilde{\omega}_2^{l,B} & \text{if } \theta_l/\theta_h \geq \frac{2+\delta}{4+\delta} \end{cases} \tag{A52}$$

The following comparative statics are useful:

When $\theta_l/\theta_h \geq \frac{2+\delta}{4+\delta}$ and $\frac{\theta_h - \theta_l}{2} \leq \omega < \frac{\theta_l(\theta_h - \theta_l)}{3\theta_l - \theta_h}$,

$$\frac{\partial p_l}{\partial \omega} = -\frac{(\theta_h - \theta_l)(2\theta_l - \theta_h)}{2(\theta_h - \theta_l - \omega)^2} \leq 0 \text{ for all } \theta_l/\theta_h \geq \frac{2+\delta}{4+\delta} \geq 1/2. \tag{A53}$$

When $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$ and $\omega_1^{l,B} \leq \omega \leq \min\{\theta_h - \theta_l, \frac{\theta_l}{1+\delta}\}$,

$$\frac{\partial p_l}{\partial \omega} = \frac{-\delta(\theta_h - \theta_l)^2}{4(\theta_h - \theta_l - \omega)^2} < 0 \ , \tag{A54}$$

$$\frac{\partial N_l}{\partial \omega} = \frac{(\theta_h(2+\delta) - \theta_l(4+\delta))(\theta_l(\theta_h - \theta_l) - (1+\delta)\omega^2)}{4(\theta_h - \theta_l - \omega)^2(\theta_l - (1+\delta)\omega)^2} > 0 \tag{A55}$$

for all $\omega < \theta_l/(1+\delta)$ and $\theta_l/\theta_h < \dfrac{2+\delta}{4+\delta}$, and

$$\frac{\partial N_b}{\partial \omega} = \frac{\theta_h - \theta_l}{2(\theta_h - \theta_l - \omega)^2} > 0. \tag{A56}$$

When either $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$ and $\omega < \omega_1^{l,B}$, or $\theta_l/\theta_h \geq \frac{2+\delta}{4+\delta}$ and $\omega < \frac{\theta_h - \theta_l}{2}$,

$$\frac{\partial N_b}{\partial \omega} = \frac{\theta_h - \theta_l}{2(\theta_h - \theta_l - \omega)^2} > 0, \tag{A57}$$

$$\begin{aligned}\frac{\partial p_l}{\partial \omega} &= \frac{1}{2(\theta_h - \theta_l - \omega)^2(\theta_l - \delta\omega)}\big[\theta_l^2(\theta_h - \theta_l)(\theta_h(1+\delta) - 2\theta_l) \\ &- 2\delta\theta_l(\theta_h - \theta_l)(\delta\theta_l + \theta_h)\omega + \delta(\delta\theta_h^2 - 2\delta\theta_l\theta_h(2-\delta) + \theta_l(2+\delta-\delta^2))\omega^2\big] > 0\end{aligned} \tag{A58}$$

$$\frac{\partial N_l}{\partial \omega} = \frac{\delta\theta_l((\theta_h - \theta_l)(\delta\theta_h - \theta_l(1+\delta) - 2(\theta_h - \theta_l)\delta\omega + 2\delta\omega^2)}{2(\theta_l - \delta\omega)^2(\theta_h - \theta_l - \omega)^2} \lesseqgtr 0. \tag{A59}$$

Note that in order to sign (A58), we need to determine the sign of the numerator. If the numerator has no roots, because the coefficient of ω^2 is strictly positive, then it is positive for all ω. This happens when either $\theta_l/\theta_h > \frac{\delta}{2+\delta}$, or ($\delta < 0.296$ and either $\theta_l/\theta_h < \frac{1+2\delta-\delta^2-\sqrt{1-4\delta+2\delta^2+\delta^4}}{2(2-\delta^2)}$ or $\theta_l/\theta_h > \frac{1+2\delta-\delta^2-\sqrt{1-4\delta+2\delta^2+\delta^4}}{2(2-\delta^2)}$). This observation gives rise to the conclusion that when $\theta_l/\theta_h \geq \frac{2+\delta}{4+\delta} > \frac{\delta}{2+\delta}$, the numerator is positive.

When $\theta_l/\theta_h < \frac{2+\delta}{4+\delta}$, $\frac{\partial p_l}{\partial \omega}$ may be positive or negative. However, we will show that the numerator of $\frac{\partial p_l}{\partial \omega}$ is positive for all $\omega < \theta_l/2$, where $\theta_l/2 > \omega_1^{l,B}$. Consequently, $\frac{\partial p_l}{\partial \omega}$ is positive for all $\omega < \omega_1^{l,B}$. Below is a sketch of that proof.

We need to show when either $\theta_l/\theta_h \leq \frac{\delta}{2+\delta}$, or $\delta < 0.296$ and $\frac{1+2\delta-\delta^2-\sqrt{1-4\delta+2\delta^2+\delta^4}}{2(2-\delta^2)} \leq \theta_l/\theta_h \leq \frac{1+2\delta-\delta^2-\sqrt{1-4\delta+2\delta^2+\delta^4}}{2(2-\delta^2)}$, the numerator of $\frac{\partial p_l}{\partial \omega}$ is positive for all $\omega < \theta_l/2$. We first evaluate the value of the expression $\theta_l^2(\theta_h - \theta_l)(\theta_h(1+\delta) - 2\theta_l) - 2\delta\theta_l(\theta_h - \theta_l)(\delta\theta_l + \theta_h)\omega +$

$\delta(\delta\theta_h^2 - 2\delta\theta_l\theta_h(2-\delta) + \theta_l(2+\delta-\delta^2))\omega^2$ at $\omega = \theta_l/2$ and show that value is positive in the above region of θ_l/θ_h. This result implies that $\omega = \theta_l/2$ is either smaller or larger than the two roots of the expression. The next step helps resolve the ambiguity. We show that the critical value of the expression (where the first derivative w.r.t ω vanishes) is larger than $\theta_l/2$. This indicates $\theta_l/2$ has to be smaller than both the two roots (imagine a convex parabola cutting the x-axis; $\theta_l/2$ is to the left of the smaller root). Consequently, the expression is positive for all $\omega < \theta_l/2$.

Next, the sign of (A59) resolves around the sign of the quadratic function of ω in the numerator. If $\theta_l/\theta_h < \frac{\delta}{2+\delta}$, then the quadratic function has no root; and $\frac{\partial N_l}{\partial \omega} > 0$. If $\frac{\delta}{2+\delta} \leq \theta_l/\theta_h \leq \frac{\delta}{1+\delta}$, the quadratic function has two positive roots, then $\frac{\partial N_l}{\partial \omega} \geq 0$ for all $\omega \leq \frac{\theta_h - \theta_l}{2} - \frac{\sqrt{\delta(\theta_h-\theta_l)(\theta_l(2+\delta)-\delta\theta_h)}}{2\delta}$ and $\frac{\partial N_l}{\partial \omega} < 0$ for all $\frac{\theta_h-\theta_l}{2} - \frac{\sqrt{\delta(\theta_h-\theta_l)(\theta_l(2+\delta)-\delta\theta_h)}}{2\delta} < \omega < \frac{\theta_h-\theta_l}{2} + \frac{\sqrt{\delta(\theta_h-\theta_l)(\theta_l(2+\delta)-\delta\theta_h)}}{2\delta}$. If $\theta_l/\theta_h > \frac{\delta}{1+\delta}$, one root is negative while the other is positive, then $\frac{\partial N_l}{\partial \omega} \leq 0$ for all $\omega \leq \frac{\theta_h-\theta_l}{2} + \frac{\sqrt{\delta(\theta_h-\theta_l)(\theta_l(2+\delta)-\delta\theta_h)}}{2\delta}$. Notice that $\frac{\theta_h-\theta_l}{2} + \frac{\sqrt{\delta(\theta_h-\theta_l)(\theta_l(2+\delta)-\delta\theta_h)}}{2\delta} > \omega_1^{l,B}$.

Forward Compatibility

Under rapid improvement with forward compatibility and T-l, we have $N_l = v_{lb} - v_{0l}$, $N_b = 1 - v_{lb}$, $\eta_{l,1} = \eta_{l,2} = \omega(1-v_{0l})$, and $\eta_{h,2} = \omega(1-v_{lb})$. The locations of the marginal consumers are:

$$v_{0l} = \frac{p_l - (1+\delta)\omega}{\theta_l - (1+\delta)\omega}, \tag{A60}$$

$$v_{lb} = \frac{p_h(\theta_l - (1+\delta)\omega) - (p_l - (1+\delta)\omega)\omega}{(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}, \tag{A61}$$

$$v_{lh} = \frac{p_h\delta(\theta_h - \theta_l)(\theta_l - (1+\delta)\omega) + p_l(\theta_l^2 - \delta\omega^2 - \theta_l(\theta_h - \omega))}{(\delta\theta_h - \theta_l)(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)} - \omega\frac{\theta_l^2 - \delta\theta_h(1+\delta)\omega - \theta_l(\theta_h - (1+\delta+\delta^2)\omega)}{(\delta\theta_h - \theta_l)(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}, \text{ and} \tag{A62}$$

$$v_{hb} = \frac{p_l(\theta_l - \delta\omega) - \omega\theta_l}{\theta_l(1-\delta)(\theta_l - (1+\delta)\omega)}. \tag{A63}$$

The profit in period 2 is $\Pi_2^{l,F} = p_h(1 - v_{lb}) = p_h\big[1 - \frac{p_h(\theta_l-(1+\delta)\omega)-(p_l-(1+\delta)\omega)\omega}{(\theta_h-\theta_l-\omega)(\theta_l-(1+\delta)\omega)}\big]$, which is concave in p_h for all $\omega < \theta_h - \theta_l$ (as $\frac{\partial^2\Pi_2^{l,F}}{\partial p_h^2} = \frac{-2}{\theta_h-\theta_l-\omega} < 0$). Hence, the optimal price that satisfies the first order condition $\frac{\partial\Pi_2^{l,F}}{\partial p_h} = 0$ is $\hat{p_h} = \frac{\theta_h-\theta_l}{2} - \frac{\omega(\theta_l-p_l)}{2(\theta_l-(1+\delta)\omega)}$; and the corresponding demand is $N_b = \frac{(\theta_h-\theta_l)(\theta_l-(1+\delta)\omega)+(\theta_l-p_l)\omega}{2(\theta_h-\theta_l-\omega)(\theta_l-(1+\delta)\omega)}$.

Taking the second period behavior into account, the present value of the total profit in period 1 is

$$\Pi_1^{l,F} = p_l\Big[1 - \frac{p_l - (1+\delta)\omega}{\theta_l - (1+\delta)\omega}\Big] + \delta\Big[\frac{\theta_h - \theta_l}{2} - \frac{\omega(\theta_l - p_l)}{2(\theta_l - (1+\delta)\omega)}\Big]\frac{(\theta_h - \theta_l)(\theta_l - (1+\delta)\omega) + (\theta_l - p_l)\omega}{2(\theta_h - \theta_l - \omega)(\theta_l - (1+\delta)\omega)}. \tag{A64}$$

Notice that

$$\frac{\partial^2\Pi_1^{l,F}}{\partial p_l^2} = -\frac{(4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l)}{2(\theta_h - \theta_l - \omega)(\theta_l - \omega(1+\delta))^2}, \tag{A65}$$

and $\frac{\partial^2 \Pi_1^{l,F}}{\partial p_l^2} \leq 0$ when $(4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l) \geq 0$; this is satisfied for all $\omega \leq \omega_{soc}^{l,F}$, where

$$\omega_{soc}^{l,F} = \frac{2(\theta_h(1+\delta) - \delta\theta_l - \sqrt{\theta_l(\theta_l - \theta_h)(4+3\delta) + (\theta_l\delta - \theta_h(1+\delta))^2})}{4+3\delta}, \tag{A66}$$

and $\omega_{soc}^{l,F} < \min\{\theta_h - \theta_l, \theta_l/(1+\delta)\}$.

The unconstrained p_l in this case would be given by solving $\frac{\partial \Pi_1^{l,F}}{\partial p_l} = 0$, or equivalently

$$p_l = \theta_l/2 + \frac{\delta\omega(2\theta_l(\theta_h - \theta_l) - (2\theta_h(1+\delta) - \theta_l(1+2\delta))\omega)}{2((4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l))}. \tag{A67}$$

However, if the constraints that p_l has to satisfy to engineer the required segmentation structure are binding, then a corner solution for p_l will arise.

Define

$$\begin{aligned}
\omega_1^{l,F} &= \frac{\theta_l}{2\delta(\delta\theta_h - 2\theta_l(1+2\delta))}\big[\theta_l(3 - 2\delta + 2\delta^2) - \theta_h(2 + \delta + 2\delta^2) + \\
&\quad \sqrt{8\delta^2(\theta_h - \theta_l)(\delta\theta_h - 2\theta_l(1+2\delta)) + (\theta_h(2+\delta+2\delta^2) - \theta_l(3 - 2\delta + 2\delta^2))^2}, && \text{(A68)} \\
\omega_2^{l,F} &= \frac{2\theta_l(\theta_h - \theta_l)}{2\theta_h(1+\delta) - \theta_l(1+2\delta)}, \text{ and} && \text{(A69)} \\
\omega_3^{l,F} &= \frac{\theta_l(\theta_h - \theta_l)}{\delta\theta_h}. && \text{(A70)}
\end{aligned}$$

We now show that when $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega_1^{l,F} \leq \omega \leq \omega_2^{l,F}$, the interior solution arises; and when either $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega < \omega_1^{l,F}$, or $\theta_l/\theta_h \geq \frac{2}{1+2\delta}$ and $\omega < \omega_3^{l,F}$, the corner solution at $v_{hb} = v_{lh}$ arises. (The above regions of ω cover all the feasible values according to Assumption 1; see equation (A80) below.)

When $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega_1^{l,F} \leq \omega \leq \omega_2^{l,F}$, the old product's price at the interior solution gives

$$N_l = \frac{\omega(\theta_h(2+\delta) - \theta_l(3+\delta))}{(4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l)} \geq 0, \tag{A71}$$

$$N_b = \frac{2\theta_l(\theta_h - \theta_l) - (2\theta_h(1+\delta) - \theta_l(1+2\delta))\omega}{(4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l)} \geq 0, \tag{A72}$$

$$p_h = \frac{(\theta_h - \theta_l - \omega)(2\theta_l(\theta_h - \theta_l) - (2\theta_h(1+\delta) - \theta_l(1+2\delta))\omega)}{(4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l)} > 0 \tag{A73}$$

$$v_{hb} - v_{lh} = \frac{-(\theta_h - \theta_l)f_2}{\theta_l(1-\delta)(\delta\theta_h - \theta_l)((4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l))}, \text{ where} \tag{A74}$$

$$f_2 = \delta(\delta\theta_h - 2\theta_l(1+2\delta))\omega^2 + \theta_l(\theta_h(2+\delta+2\delta^2) - \theta_l(3-2\delta+2\delta^2))\omega + 2\delta\theta_l^2(\theta_l - \theta_h). \tag{A75}$$

It is easy to show that $f_2 = 0$ has two roots; when $\theta_l/\theta_h \geq \frac{\delta}{2(1+2\delta)}$, $\omega_1^{l,F}$ is the smaller root among the two positive roots; and when $\theta_l/\theta_h < \frac{\delta}{2(1+2\delta)}$, $\omega_1^{l,F}$ is the positive root while the other is negative. It then follows that $f_2 \geq 0$, and subsequently, $v_{hb} - v_{lh} \leq 0$ for all $\omega_1^{l,F} \leq \omega \leq \omega_2^{l,F}$.

When either $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega < \omega_1^{l,F}$, or $\theta_l/\theta_h \geq \frac{2}{1+2\delta}$ and $\omega < \omega_3^{l,F}$, then the constraint $v_{hb} \leq v_{lh}$ is violated; the old product's price is then obtained by solving $v_{hb} = v_{lh}$:

$$\begin{aligned} p_l = & \frac{\theta_l(\theta_l(1-\delta) + \omega(1+\delta^2))}{2(\theta_l - \delta\omega)} \\ & - \frac{\omega\theta_l(1-\delta)(\theta_l(1-\delta) - 2\delta\omega)(\theta_l - (1+\delta)\omega)}{2(\theta_l - \delta\omega)(2\delta\omega^2 - (\theta_l(1-\delta) + 2\theta_h\delta)\omega + 2\theta_l(\theta_h - \theta_l))}. \end{aligned} \tag{A76}$$

Hence, the new product's price and the demand are obtained as follows

$$p_h = \frac{(\theta_h - \theta_l - \omega)(\theta_l(\theta_h - \theta_l) - \delta\omega\theta_h)}{2\delta\omega^2 - (\theta_l(1-\delta) + 2\theta_h\delta)\omega + 2\theta_l(\theta_h - \theta_l)} > 0, \tag{A77}$$

$$N_l = \frac{\delta(\theta_l(\theta_h - \theta_l) - (2\theta_l - \theta_h)\omega)}{2\delta\omega^2 - (\theta_l(1-\delta) + 2\theta_h\delta)\omega + 2\theta_l(\theta_h - \theta_l)} > 0, \text{ and} \tag{A78}$$

$$N_b = \frac{\theta_l(\theta_h - \theta_l) - \delta\theta_h\omega}{2\delta\omega^2 - (\theta_l(1-\delta) + 2\theta_h\delta)\omega + 2\theta_l(\theta_h - \theta_l)} > 0. \tag{A79}$$

Define

$$\overline{\omega}^{l,F} = \begin{cases} \omega_2^{l,F} & \text{if } \theta_l/\theta_h < \frac{2}{1+2\delta} \\ \omega_3^{l,F} & \text{if } \theta_l/\theta_h \geq \frac{2}{1+2\delta} \end{cases} \tag{A80}$$

Notice that when $\omega \geq \overline{\omega}^{l,F}$, the firm will not introduce the new product. Under that scenario, it sets the old product's price to maximize the profit from the old product in period 1, $\Pi_1^{l,F} = p_l(1 - v_{0l}) = \frac{p_l(\theta_l - p_l)}{\theta_l - (1+\delta)\omega}$. When $\omega < \theta_l/(1+\delta)$ (i.e., $\frac{\partial^2 \Pi_1^{l,F}}{\partial \theta_l^2} = \frac{-2}{\theta_l - (1+\delta)\omega} < 0$), the optimal price that satisfies the first order condition $\frac{\partial \Pi_1^{l,F}}{\partial \theta_l} = 0$ is $p_l = \theta_l/2$. Consequently, the profit obtained is $\frac{\theta_l^2}{4(\theta_l - (1+\delta)\omega)}$.

The following comparative statics are useful:

When $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega_1^{l,F} \leq \omega \leq \omega_2^{l,F}$, we have

$$\frac{\partial N_b}{\partial \omega} = \frac{(4+3\delta)((2\theta_h(1+\delta) - \theta_l(1+2\delta))\omega^2 - 4\theta_l(\theta_h - \theta_l)\omega) + 4\theta_l^2(\theta_h - \theta_l)}{((4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l))^2} < 0, \tag{A81}$$

$$\frac{\partial N_l}{\partial \omega} = \frac{(\theta_h(2+\delta) - \theta_l(3+\delta))(4\theta_l(\theta_h - \theta_l) - (4+3\delta)\omega^2)}{((4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l))^2} > 0, \text{ and} \tag{A82}$$

$$\frac{\partial p_h}{\partial \omega} = -N_b + (\theta_h - \theta_l - \omega)\frac{\partial N_b}{\partial \omega} < 0, \text{ and} \tag{A83}$$

$$\frac{\partial p_l}{\partial \omega} = \frac{\delta(N_b + \omega\frac{\partial N_b}{\partial \omega})}{2} \lesseqgtr 0. \tag{A84}$$

(A84) is equivalent to

$$\frac{\partial p_l}{\partial \omega} = \frac{\delta}{((4+3\delta)\omega^2 + 4(\theta_l\delta - \theta_h(1+\delta))\omega + 4\theta_l(\theta_h - \theta_l))^2}\big[4\theta_l^2(\theta_h - \theta_l)^2 +$$
$$4\theta_l(\theta_h - \theta_l)(\theta_l(1+2\delta) - 2\theta_h(1+\delta))\omega +$$
$$(4\theta_h^2(1+\delta)^2 + \theta_l^2(4+\delta(5+4\delta)) - \theta_l\theta_h(6+\delta(13+8\delta)))\omega^2\big], \tag{A85}$$

whose sign depends on the quadratic function of ω in the numerator. This function has two positive roots; one of the roots is smaller than $\omega_2^{l,F}$ and the other is larger than $\omega_2^{l,F}$. It follows that when ω is smaller than the smaller root, $\frac{\partial p_l}{\partial \omega} > 0$, and when ω is larger than the smaller root, $\frac{\partial p_l}{\partial \omega} < 0$.

When either $\theta_l/\theta_h < \frac{2}{1+2\delta}$ and $\omega < \omega_1^{l,F}$, or $\theta_l/\theta_h \geq \frac{2}{1+2\delta}$ and $\omega < \omega_3^{l,F}$, we have

$$\frac{\partial p_l}{\partial \omega} = \frac{1}{(-2\delta\omega^2 + (\theta_l(1-\delta) + 2\theta_h\delta)\omega - 2\theta_l(\theta_h - \theta_l))^2}\big[\theta_l(2\delta(2\theta_l - \theta_h(1-\delta^2))\omega^2 -$$
$$4\delta(1+\delta)\theta_l(\theta_h - \theta_l)\omega + 2\theta_l(\theta_h - \theta_l)(2\theta_h(1+\delta) - \theta_l(3+\delta^2)))\big] > 0, \tag{A86}$$

$$\frac{\partial N_l}{\partial \omega} = \frac{\delta(2\delta(2\theta_l - \theta_h)\omega^2 - 4\delta\theta_l(\theta_h - \theta_l)\omega + \theta_l(\theta_h - \theta_l)(2\theta_h(1+\delta) - \theta_l(3+\delta^2)))}{(-2\delta\omega^2 + (\theta_l(1-\delta) + 2\theta_h\delta)\omega - 2\theta_l(\theta_h - \theta_l))^2} > 0, \tag{A87}$$

$$\frac{\partial p_h}{\partial \omega} = \theta_l\frac{(\theta_h(1-\delta) - 2\theta_l)\delta\omega^2 + 4\theta_l(\theta_h - \theta_l)\delta\omega - \theta_l(\theta_h - \theta_l)^2(1+\delta)}{(-2\delta\omega^2 + (\theta_l(1-\delta) + 2\theta_h\delta)\omega - 2\theta_l(\theta_h - \theta_l))^2} < 0, \text{ and} \tag{A88}$$

$$\frac{\partial N_b}{\partial \omega} = \frac{2\theta_h\delta^2\omega^2 - 4\delta\theta_l(\theta_h - \theta_l)\omega + \theta_l^2(1-\delta)(\theta_h - \theta_l)}{(-2\delta\omega^2 + (\theta_l(1-\delta) + 2\theta_h\delta)\omega - 2\theta_l(\theta_h - \theta_l))^2} \lesseqgtr 0. \tag{A89}$$

The sign of (A89) resolves around the sign of the quadratic function of ω in the numerator. When $\omega < \frac{\theta_l(\theta_h - \theta_l)}{\delta\theta_h} - \frac{\theta_l\sqrt{2(\theta_h - \theta_l)(\theta_h(1+\delta) - 2\theta_l)}}{2\delta\theta_h}$, $\frac{\partial N_b}{\partial \omega} \geq 0$; otherwise $\frac{\partial N_b}{\partial \omega} < 0$. ■

Proof of Proposition 3.1

Begin by recalling (a) the present value of the total profit under replacement with full compatibility and T-l where a corner solution arises (i.e., $\Pi_1^{l,U}$); and (b) the profit under skipping with full compatibility and T-h (i.e, $\Pi_1^{h,U,s}$). Note that under T-l, the corner solution arises at low levels of ω (i.e., $\omega < \omega_1^{l,U}$), whereas, skipping occurs when either $\delta \geq 1/2$, or $\delta < 1/2$ and $\omega \geq \omega_1^{h,U}$. Let $\Delta = \Pi_1^{h,U,s} - \Pi_1^{l,U}$. We will examine the sign of Δ to determine which product strategy is optimal.

We have $\Delta = \frac{\delta\theta_h^2}{4(\theta_h-\omega)} - \frac{\delta(\theta_h-\theta_l)}{4} - \Gamma$, where Γ is the old product's profit, and

$$\Gamma = \frac{\theta_l^2(1+\delta)(\theta_l(1-\delta)+\omega(1+\delta^2))}{4(\theta_l-\delta\omega)^2}. \tag{A90}$$

If $\Delta > 0$, or equivalently, $(4\Gamma - \delta\theta_l)\omega + \theta_h(\delta\theta_l + \delta\omega - 4\Gamma) > 0$, then it follows that: (1) $\theta_h > \hat{\theta}_h$ when $\delta\theta_l + \delta\omega - 4\Gamma \geq 0$, or (2) $\theta_h < \hat{\theta}_h$ when $\delta\theta_l + \delta\omega - 4\Gamma < 0$, where $\hat{\theta}_h = \frac{(\delta\theta_l - 4\Gamma)\omega}{\delta\theta_l+\delta\omega-4\Gamma}$, $\delta\theta_l + \delta\omega - 4\Gamma = \frac{f}{(\theta_l-\delta\omega)^2}$, and $f = \delta^3\omega^3 - (2-\delta)\theta_l\delta^2\omega^2 - (1+3\delta^2+\delta^3)\theta_l^2\omega + (\delta^2+\delta-1)\theta_l^3$.

Since f and $(\delta\theta_l + \delta\omega - 4\Gamma)$ have the same sign, we will consider the sign of f instead of the sign of $(\delta\theta_l + \delta\omega - 4\Gamma)$. Notice that f is a cubic function of ω. A tedious calculation from the critical values (at which the first derivative of f w.r.t ω vanishes) shows that $f = 0$ has three real roots and the smallest root is negative, while the largest root is positive and the middle root has the same sign as that of $\delta^2 + \delta - 1$. Denote $\hat{\omega}$ and $\hat{\omega}_1$ as the middle root and the largest one respectively. If $\delta^2 + \delta - 1 \geq 0$, or equivalently, $\delta \geq \hat{\delta}$, where $\hat{\delta} = \frac{\sqrt{5}-1}{2} \approx 0.62$, then (1) $f \geq 0$ when $\omega \leq \hat{\omega}$ or $\omega \geq \hat{\omega}_1$; and (2) $f < 0$ when $\hat{\omega} < \omega < \hat{\omega}_1$. If $\delta^2 + \delta - 1 < 0$, or equivalently, $\delta < \hat{\delta}$, then (1) $f \geq 0$ when $\omega \geq \hat{\omega}_1$; and (2) $f < 0$ when $\omega < \hat{\omega}_1$. Furthermore, the value of f at $\omega = \theta_l/(1+\delta)$ is $\frac{-\theta_l^3(2+4\delta+5\delta^2+2\delta^3)}{(1+\delta)^3} < 0$. This implies $\frac{\theta_l}{1+\delta} < \hat{\omega}_1$. Thus, we will focus on the case where $\omega < \hat{\omega}_1$. That is, $f \geq 0$ for $\omega \leq \hat{\omega}$ and $\delta \geq \hat{\delta}$. Otherwise, $f < 0$.

By the definition of rapid improvement, $\delta\theta_h > \theta_l$. We then need to compare $\hat{\theta}_h$ and θ_l/δ to derive the condition that dictates the sign of Δ. We have

$$\hat{\theta}_h - \frac{\theta_l}{\delta} = \frac{\theta_l(\theta_l-\delta\omega)(\delta^2(1-\delta)\omega^2 + \theta_l(1+3\delta^2+\delta^3)\omega + \theta_l^2(1-\delta-\delta^2))}{\delta f}. \tag{A91}$$

Let $\Lambda = \delta^2(1-\delta)\omega^2 + \theta_l(1+3\delta^2+\delta^3)\omega + \theta_l^2(1-\delta-\delta^2)$. If $\Lambda \geq 0$, then $\omega > \omega^*$, where ω^* is the larger root of $\Lambda = 0$ (the other root is negative), and

$$\omega^* = \frac{-1-3\delta^2-\delta^3+\sqrt{1+2\delta^2+10\delta^3+9\delta^4+2\delta^5+\delta^6}}{2(1-\delta)\delta^2}\theta_l. \tag{A92}$$

When $\delta < \hat{\delta}$, ω^* is negative; otherwise, it is nonnegative. In addition, f is negative at $\omega = \omega^*$ [2]; if $\omega^* > 0$, then $\hat{\omega} < \omega^*$.

When $\delta \geq \hat{\delta}$ and $\omega \leq \hat{\omega}$, we have $f \geq 0$ and $\Lambda < 0$. Subsequently, $\hat{\theta}_h < \theta_l/\delta$. This indicates that $\theta_h > \hat{\theta}_h$ for all $\delta\theta_h > \theta_l$, and thus, $\Delta > 0$ for all $\delta\theta_h > \theta_l$. In other words, skipping is the optimal strategy.

When $\delta < \hat{\delta}$, we have $f < 0$, $\hat{\omega} < 0$ and $\omega^* < 0$, leading to $\Lambda > 0$, and subsequently, $\hat{\theta}_h < \theta_l/\delta$. It follows that $\Delta < 0$ for all $\delta\theta_h > \theta_l$; and product replacement is the dominant strategy.

When $\delta \geq \hat{\delta}$ and $\omega > \hat{\omega}$, we have $\omega^* > \hat{\omega} > 0$ and $f < 0$. If $\hat{\omega} < \omega < \omega^*$, then $\Lambda < 0$ and thus $\hat{\theta}_h > \theta_l/\delta$. Hence, $\Delta > 0$ when $\theta_h < \hat{\theta}_h$. Otherwise, i.e., $\theta_h > \hat{\theta}_h$, $\Delta < 0$. If $\omega \geq \omega^*$, then $\Lambda \geq 0$ and $\hat{\theta}_h \leq \theta_l/\delta$, subsequently, $\Delta \leq 0$ for all $\delta\theta_h > \theta_l$.

It is important to notice that $\omega^* < \omega_1^{l,U}$, at higher levels of ω (i.e., $\omega \geq \omega_1^{l,U}$), when an interior solution arises, replacement strategy continues dominating skipping since profits under this solution are always higher than the ones under a corner solution (which have already dominated skipping).

Extension to a fully covered market setting: Since $\tilde{\omega}^{l,U} < \tilde{\omega}^{h,U}$, the market under T-$l$ is covered before the one under T-h. If the market under T-l is covered, then the old product's pricing allows the firm to extract more surplus from consumers than before, and subsequently, it will continue dominating skipping strategy (from scheme T-h). If the market under both schemes is covered, then it is easy to show that the replacement strategy

[2]This can be shown by considering the value of f at ω^*, rearranging the terms involving the θ's, and inspecting the expression for different values of δ

under scheme T-l is also the dominant strategy. ■

Proof of Observations 3.1 and 3.2

The present value of the new product's profit under skipping is $\frac{\delta\theta_h^2}{4(\theta_h-\omega)}$. From Table 3.4, the present value of the total profit in period 1 when a corner solution at $v_{hb} = v_{lh}$ arises is:

Under full compatibility:

$$\Pi_1^{l,U} = \frac{\theta_l(\theta_l(1-\delta)+\omega(1+\delta^2))}{2(\theta_l-\delta\omega)} * \frac{\theta_l(1+\delta)}{2(\theta_l-\delta\omega)} + \delta\frac{\theta_h-\theta_l}{4} \tag{A93}$$

Under backward compatibility:

$$\Pi_1^{l,B} = \Big[\frac{\theta_l(\theta_l(1-\delta)+\omega(1+\delta^2))}{2(\theta_l-\delta\omega)} - \frac{\omega(\theta_l(1-\delta)(\theta_l-\delta\omega)+\omega(\delta\theta_h-\theta_l))}{2(\theta_l-\delta\omega)(\theta_h-\theta_l-\omega)}\Big]* \\ \Big[\frac{\theta_l(1+\delta)}{2(\theta_l-\delta\omega)} - \frac{\omega(\delta\theta_h-\theta_l)}{2(\theta_l-\delta\omega)(\theta_h-\theta_l-\omega)}\Big] + \delta\frac{(\theta_h-\theta_l)^2}{4(\theta_h-\theta_l-\omega)} \tag{A94}$$

Under forward compatibility:

$$\Pi_1^{l,F} = \frac{1}{(2\theta_l^2+2\delta\omega(\theta_h-\omega)-\theta_l(2\theta_h-\omega+\delta\omega))^2}\big[\theta_l^2(\theta_h-\theta_l)^2(\delta\theta_h+\theta_l(1-\delta-\delta^2))- \\ \theta_l(\theta_h-\theta_l)(2\theta_h^2\delta^2-\theta_l\theta_h(1+3\delta^2+\delta^3)+\theta_l^2(2+\delta(2-2\delta+\delta^2)))\omega+ \\ \delta(2\theta_l-\theta_h)(2\theta_l^2-2\delta\theta_l\theta_h-(\theta_h-\theta_l)(2\theta_l+\theta_h)\delta^2)\omega^2+(4\theta_l^2-\theta_h^2)\delta^3\omega^3\big] \tag{A95}$$

Given the complexity of these profit functions, we use simulations to identify the optimal regions for skipping and replacement under forward and backward compatibility; subsequently, we compare these regions with the ones obtained from Proposition 3.1. In particular, we compare profitability of the corner solution at $v_{hb} = v_{lh}$ under T-l and the skipping solution under T-h when they are both feasible. We conducted the simulation by setting $\theta_h = 1$; θ_l is selected from 0.001 to 1; $\delta \in (0,1)$, and $\theta_l/\theta_h \in (0, \delta)$.

Under backward compatibility with T-l and a corner solution at $v_{hb} = v_{lh}$, these comparative statics are useful:

$$\frac{\partial N_b}{\partial \theta_h} = \frac{-\omega}{2(\theta_h - \theta_l - \omega)^2} < 0, \tag{A96}$$

$$\frac{\partial p_l^*}{\partial \theta_h} = \frac{\omega\theta_l(1-\delta)(\theta_l - (1+\delta)\omega)}{2(\theta_l - \delta\omega)(\theta_h - \theta_l - \omega)^2} > 0, \text{ and} \tag{A97}$$

$$\frac{\partial N_l + N_b}{\partial \theta_h} = \frac{\omega(\delta\omega - \theta_l(1-\delta))}{2(\theta_l - \delta\omega)(\theta_h - \theta_l - \omega)^2} > 0 \text{ for all } \omega > \theta_l(1-\delta)/\delta. \tag{A98}$$

■

Proof of Proposition 3.2

We will show that a product replacement strategy weakly dominates a skipping strategy when $\omega < \overline{\omega}^{g,j}$, $j = \{U, B, F\}$. As mentioned in the main text, skipping is a more restricted case of replacement; hence, if the unconstrained prices, p_l and p_h (which are the solution to the first order conditions), yield positive product sales in period 2 and the location of the marginal consumer $v_{lb} \leq 1$, then a replacement strategy is weakly viable and is the optimal strategy. In what follows, we will show the viability of a replacement strategy for all $\omega < \overline{\omega}^{g,j}$, $j = \{U, B, F\}$.

Full Compatibility

Under full compatibility, the network value is $\eta_{l,1} = \omega(1 - v_{hl})$, and $\eta_{h,2} = \eta_{l,2} = \omega(1 - v_{0h})$. In period 2, given v_{hl}, the demands are $N_l = \frac{p_h}{\theta_h - \theta_l} - v_{hl}$, $N_h = v_{hl} - \frac{p_h - \omega}{\theta_h - \omega}$, and $N_b = 1 - \frac{p_h}{\theta_h - \theta_l}$. The profit in period 2 is $\Pi_2^{g,U,r} = p_h(v_{hl} - \frac{p_h - \omega}{\theta_h - \omega} + 1 - \frac{p_h}{\theta_h - \theta_l})$.

Notice that the profit in period 2 is concave in p_h for all $\omega < \theta_h$ (as $\frac{\partial^2 \Pi_2^{g,U,r}}{\partial p_h^2} = \frac{-2(2\theta_h - \theta_l - \omega)}{(\theta_h - \omega)(\theta_h - \theta_l)} < 0$). Hence, the optimal price that satisfies the first order condition $\frac{\partial \Pi_2^{g,U,r}}{\partial p_h} = 0$ is $\hat{p_h} = \frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(2\theta_h - \theta_l - \omega)}$; and the corresponding demand is $N_h + N_b = \frac{1}{2}[v_{hl} + \frac{\theta_h}{\theta_h - \omega}] > 0$ for all $\hat{p_h} > 0$. This result shows that the product's sales in period 2 are positive.

In period 1, the firm sets the old product's price such that the consumer indexed v_{hl} is indifferent between buying in periods 1 and 2. We then obtain

$$v_{hl} = \frac{\delta\theta_h(\theta_h - \theta_l) + 2\omega(2\theta_h - \theta_l) - 2\omega^2 - 2p_l(2\theta_h - \theta_l - \omega)}{2\theta_l^2 - \theta_l\theta_h(4+\delta) + 3\delta\theta_h^2 + \omega(\theta_h(4-\delta) - \delta\theta_l) - 2\omega^2}, \tag{A99}$$

where $v_{0h} \leq v_{hl} \leq v_{lb}$ (as depicted in Figure 3.2).

The present value of the total profit in period 1 is $\Pi_1^{g,U,r} = p_l(N_b + N_l) + \delta\hat{p}_h(N_h + N_b)$, or equivalently,

$$\Pi_1^{g,U,r} = p_l(1 - v_{hl}) + \delta\frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(2\theta_h - \theta_l - \omega)} * \frac{1}{2}[v_{hl} + \frac{\theta_h}{\theta_h - \omega}]. \tag{A100}$$

Since the shape of this function dictates how the optimal solution is derived, we first examine its concavity through the second derivative w.r.t p_l.

$$\frac{\partial^2\Pi_1^{g,U,r}}{\partial p_l^2} = -2\frac{\partial v_{hl}}{\partial p_l} + \frac{\delta(\theta_h - \theta_l)(\theta_h - \omega)}{2(2\theta_h - \theta_l - \omega)}[\frac{\partial v_{hl}}{\partial p_l}]^2 \tag{A101}$$

Hence, $\frac{\partial^2\Pi_1^{g,U,r}}{\partial p_l^2} \leq 0$ when $0 \leq \frac{\partial v_{hl}}{\partial p_l} \leq \frac{4(2\theta_h - \theta_l - \omega)}{\delta(\theta_h - \theta_l)(\theta_h - \omega)}$. It follows that $2(2\theta_l^2 - \theta_l\theta_h(4+\delta) + 3\delta\theta_h^2 + \omega(\theta_h(4-\delta) - \delta\theta_l) - 2\omega^2) \leq -\delta(\theta_h - \theta_l)(\theta_h - \omega) \Leftrightarrow 4\theta_l^2 + 7\delta\theta_h^2 - \theta_l\theta_h(8+3\delta) + (\theta_h(8 - 3\delta) - \delta\theta_l)\omega - 4\omega^2 \leq 0 \Leftrightarrow \omega \leq \overline{\omega}^{g,U}$ where

$$\overline{\omega}^{g,U} = \frac{1}{8}\big[\theta_h(8-3\delta) - \delta\theta_l - \sqrt{64(\theta_h - \theta_l)^2 + 64(\theta_h - \theta_l)\delta\theta_h + \delta^2(3\theta_h + \theta_l)^2}\big], \tag{A102}$$

and $\overline{\omega}^{g,U} < \theta_h$.

In the regime $\omega \in [0, \overline{\omega}^{g,U})$, the unconstrained price p_l is derived from the first-order condition $\frac{\partial \Pi_1^{g,U,r}}{\partial p_l} = 0$:

$$\frac{\partial \Pi_1^{g,U,r}}{\partial p_l} = N_l + N_b + \delta \frac{\partial \Pi_2^{g,U,r}}{\partial p_l} - p_l \frac{\partial v_{hl}}{\partial p_l} = 0 \tag{A103}$$

$$\Leftrightarrow \qquad \underbrace{N_l + N_b + \delta \frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(2\theta_h - \theta_l - \omega)} \frac{\partial v_{hl}}{\partial p_l}}_{+} - \underbrace{p_l \frac{\partial v_{hl}}{\partial p_l}}_{+} = 0 \tag{A104}$$

or equivalently,

$$p_l = \frac{1}{2}\big[\theta_l(1-\delta) + \frac{\delta\theta_l(\theta_h - \theta_l)}{2\theta_h - \theta_l - \omega} + \frac{\delta(\theta_h - \theta_l)(\theta_l - \delta\theta_h)(\theta_h - \omega)}{4\theta_l^2 + 7\delta\theta_h^2 - \theta_l\theta_h(8+3\delta) + (\theta_h(8-3\delta) - \delta\theta_l)\omega - 4\omega^2}\big]. \tag{A105}$$

Demand in period 1 is

$$N_l + N_b = 1 - v_{hl} = \frac{2(\theta_l - \delta\theta_h)(\theta_l - 2\theta_h + \omega)}{4\theta_l^2 + 7\delta\theta_h^2 - \theta_l\theta_h(8+3\delta) + (\theta_h(8-3\delta) - \delta\theta_l)\omega - 4\omega^2} \geq 0. \tag{A106}$$

Backward Compatibility

Under backward compatibility, the network value is $\eta_{l,1} = \omega(1 - v_{hl})$, $\eta_{l,2} = \omega(v_{lb} - v_{hl})$, and $\eta_{h,2} = \omega(1 - v_{0h})$. Our analysis follows similar steps as in the full compatibility setting.

In period 2, given v_{hl}, the demands are $N_l = \frac{p_h\theta_h - \omega(\theta_h + (\theta_h - \omega)v_{hl})}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)} - v_{hl}$, $N_h = v_{hl} - \frac{p_h - \omega}{\theta_h - \omega}$, and $N_b = 1 - \frac{p_h\theta_h - \omega(\theta_h + (\theta_h - \omega)v_{hl})}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)}$. The profit in period 2 is

$$\Pi_2^{g,B,r} = p_h(v_{hl} - \frac{p_h - \omega}{\theta_h - \omega} + 1 - \frac{p_h\theta_h - \omega(\theta_h + (\theta_h - \omega)v_{hl})}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)}). \tag{A107}$$

Now examine the shape of this profit function through the second derivative w.r.t p_h: $\frac{\partial^2 \Pi_2^{g,B,r}}{\partial p_h^2} = \frac{-2(2\theta_h - \theta_l - \omega)}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)}$. It follows that if $\omega < \theta_h - \theta_l$, then the profit function is concave; and the optimal price that satisfies the first order condition $\frac{\partial \Pi_2^{g,B,r}}{\partial p_h} = 0$ is $\hat{p_h} = \frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(2\theta_h - \theta_l - \omega)}$. The corresponding demand in period 2 is $N_h + N_b = \frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(\theta_h - \omega)(\theta_h - \theta_l - \omega)} > 0$ for all $\hat{p_h} > 0$. This result shows that the product's sales in period 2 are positive whenever the firm introduces the new product.

In period 1, the firm sets the old product's price such that consumer indexed v_{hl} is indifferent between buying in periods 1 and 2. We then obtain $v_{hl} = \frac{X_{bwd}}{Y_{bwd}}$, where

$$\begin{aligned} X_{bwd} &= \delta\theta_h^2(\theta_h - \theta_l)^2 + 2\theta_h(2\theta_h - \theta_l)(\theta_h - \theta_l)(1-\delta)\omega - 2\theta_l(\theta_l^2 + \theta_h(5-\delta)(\theta_h - \theta_l))\omega^2 \\ &\quad + 4(2\theta_h - \theta_l)\omega^3 - 2\omega^4 - 2p_l(\theta_h - \omega)(\theta_h - \theta_l - \omega)(2\theta_h - \theta_l - \omega), \text{ and} \end{aligned} \tag{A108}$$

$$\begin{aligned} Y_{bwd} &= (\theta_h - \omega)\big[(\theta_h - \theta_l)(2\theta_l^2 - \theta_l\theta_h(4+\delta) + 3\delta\theta_h^2) \\ &\quad + 2(\theta_h^2(2-\delta) - \delta\theta_l\theta_h - \theta_l^2(1-\delta))\omega + 2(\theta_l(1+\delta) - 3\theta_h)\omega^2 + 2\omega^3\big]. \end{aligned} \tag{A109}$$

and $v_{0h} \leq v_{hl} \leq v_{lb}$ (as depicted in Figure 3.2).

The present value of the total profit in period 1 is $\Pi_1^{g,B,r} = p_l(N_b + N_l) + \delta\hat{p_h}(N_h + N_b)$, or equivalently,

$$\Pi_1^{g,B,r} = p_l(1 - v_{hl}) + \delta\frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(2\theta_h - \theta_l - \omega)} * \frac{(\theta_h - \theta_l)(\theta_h + (\theta_h - \omega)v_{hl})}{2(\theta_h - \omega)(\theta_h - \theta_l - \omega)}. \tag{A110}$$

This function is concave in p_l when the following second derivative is non-positive.

$$\frac{\partial^2 \Pi_1^{g,B,r}}{\partial p_l^2} = -2\frac{\partial v_{hl}}{\partial p_l} + \frac{\delta(\theta_h - \theta_l)^2(\theta_h - \omega)}{2(2\theta_h - \theta_l - \omega)(\theta_h - \theta_l - \omega)}\Big[\frac{\partial v_{hl}}{\partial p_l}\Big]^2 \tag{A111}$$

Hence, $\frac{\partial^2 \Pi_1^{g,B,r}}{\partial p_l^2} \leq 0$ when $0 \leq \frac{\partial v_{hl}}{\partial p_l} \leq \frac{4(2\theta_h - \theta_l - \omega)(\theta_h - \theta_l - \omega)}{\delta(\theta_h - \theta_l)^2(\theta_h - \omega)}$. It follows that $2\big[(\theta_h - \theta_l)(2\theta_l^2 - \theta_l\theta_h(4+\delta) + 3\delta\theta_h^2) + 2(\theta_h^2(2-\delta) - \delta\theta_l\theta_h - \theta_l^2(1-\delta))\omega + 2(\theta_l(1+\delta) - 3\theta_h)\omega^2 + 2\omega^3\big] \leq$

$-\delta(\theta_h-\theta_l)^2(\theta_h-\omega) \Leftrightarrow (\theta_l-\theta_h)(4\theta_l^2+7\delta\theta_h^2-\theta_l\theta_h(8+3\delta))-(\theta_h^2(8-5\delta)-2\delta\theta_l\theta_h-\theta_l^2(4-3\delta))\omega+4(3\theta_h-\theta_l-\delta\theta_l)\omega^2-4\omega^3 \geq 0 \Leftrightarrow \omega \leq \omega_1^{g,B}$, where $\omega_1^{g,B}$ is the smallest root of the cubic function and $\omega_1^{g,B} < \theta_h - \theta_l$. This is obtained by showing that the cubic function has two critical points (where the first derivative w.r.t ω vanishes) and the values of the cubic function at these points have opposite sign; combining this with a positive value at $\omega = 0$ leads to the conclusion that the cubic function has three positive roots. Further, the fact that the two critical points are larger than $\theta_h - \theta_l$ and the value of the cubic function is negative at $\theta_h - \theta_l$ indicates $\omega_1^{g,B} < \theta_h - \theta_l$ (imagine the graph of a cubic function with negative coefficient on the term involving ω^3, and two critical points cutting the x-axis at three positive values).

Hence, the unconstrained price p_l is derived from the first-order condition $\frac{\partial \Pi_1^{g,B,r}}{\partial p_l} = 0$:

$$\frac{\partial \Pi_1^{g,B,r}}{\partial p_l} = N_l + N_b + \delta \frac{\partial \Pi_2^{g,B,r}}{\partial p_l} - p_l \frac{\partial v_{hl}}{\partial p_l} = 0 \tag{A112}$$

$$\Leftrightarrow \quad \underbrace{N_l + N_b + \delta \frac{(\theta_h-\theta_l)^2(\theta_h+(\theta_h-\omega)v_{hl})}{2(2\theta_h-\theta_l-\omega)(\theta_h-\theta_l-\omega)} \frac{\partial v_{hl}}{\partial p_l}}_{+} - \underbrace{p_l \frac{\partial v_{hl}}{\partial p_l}}_{+} = 0 \tag{A113}$$

The solution p_l to the above equation is always nonnegative. Demand in period 1 is $N_l + N_b = \frac{Q_{bwd}}{Z_{bwd}}$, where

$$\begin{aligned} Q_{bwd} &= 2\theta_h(2\theta_h-\theta_l)(\theta_h-\theta_l)(\theta_l-\delta\theta_h) - 2(\theta_l^3 - 5\theta_l^2\theta_h - 2\delta\theta_h^3 + (5+\delta)\theta_l\theta_h^2)\omega \\ &\quad - (\theta_l^2(4-\delta) - 4\theta_l\theta_h(2-\delta) + \delta\theta_h^2)\omega^2 - 2\theta_l(1-\delta)\omega^3, \text{ and} \qquad \text{(A114)} \\ Z_{bwd} &= (\theta_h-\omega)((\theta_l-\theta_h)(4\theta_l^2+7\delta\theta_h^2-\theta_l\theta_h(8+3\delta)) - (\theta_h^2(8-5\delta)-2\delta\theta_l\theta_h-\theta_l^2(4-3\delta))\omega \\ &\quad + 4(3\theta_h-\theta_l-\delta\theta_l)\omega^2 - 4\omega^3). \qquad \text{(A115)} \end{aligned}$$

We have $Z_{bwd} > 0$ for all $\omega < \omega_1^{g,B}$ and $Q_{bwd} \geq 0$ for all $\omega \leq \omega_2^{g,B}$, where $\omega_2^{g,B}$ is the smallest root of $Q_{bwd} = 0$ and $\omega_2^{g,B} < \theta_h - \theta_l$. Define $\overline{\omega}^{g,B} = \min\{\omega_1^{g,B}, \omega_2^{g,B}\}$. We then conclude that sales in period 1 (i.e., $1 - v_{hl}$) are nonnegative for all $\omega < \overline{\omega}^{g,B}$.

Forward Compatibility

Under forward compatibility, the network value is $\eta_{l,1} = \omega(1 - v_{hl})$, $\eta_{l,2} = \omega(1 - v_{0h})$, and $\eta_{h,2} = \omega(1 - v_{lb} + v_{hl} - v_{0h})$. In period 2, given v_{hl}, the demands are $N_l = \frac{p_h - \omega v_{hl}}{\theta_h - \theta_l - \omega} - v_{hl}$, $N_h = v_{hl} - \frac{(\theta_h - \theta_l)(p_h - \omega(1+v_{hl})) + \omega^2}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)}$, and $N_b = 1 - \frac{p_h - \omega v_{hl}}{\theta_h - \theta_l - \omega}$. The profit function in period 2 is

$$\Pi_2^{g,F,r} = p_h(v_{hl} - \frac{(\theta_h - \theta_l)(p_h - \omega(1 + v_{hl})) + \omega^2}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)} + 1 - \frac{p_h - \omega v_{hl}}{\theta_h - \theta_l - \omega}). \tag{A116}$$

Now examine the shape of this profit function through the second derivative w.r.t θ_h: $\frac{\partial^2 \Pi_2^{g,F,r}}{\partial p_h^2} = \frac{-2(2\theta_h - \theta_l - \omega)}{(\theta_h - \omega)(\theta_h - \theta_l - \omega)}$. It follows that if $\omega < \theta_h - \theta_l$, then the profit function is concave; and the optimal price that satisfies the first order condition $\frac{\partial \Pi_2^{g,F,r}}{\partial p_h} = 0$ is $\hat{p_h} = \frac{\theta_h((\theta_h - \theta_l)(1+v_{hl}) - \omega)}{2(2\theta_h - \theta_l - \omega)}$. The corresponding demand in period 2 is $N_h + N_b = \frac{\theta_h((\theta_h - \theta_l)(1+v_{hl}) - \omega)}{2(\theta_h - \omega)(\theta_h - \theta_l - \omega)} > 0$ for all $\hat{p_h} > 0$.

In period 1, the firm sets the old product's price such that consumer indexed v_{hl} is indifferent between buying in periods 1 and 2. We then obtain $v_{hl} = \frac{X_{fwd}}{Y_{fwd}}$, where

$$X_{fwd} = (\theta_h - \theta_l - \omega)(2\omega^2 - 2(2\theta_h - \theta_l)\omega - \delta\theta_h(\theta_h - \theta_l) + 2p_l(2\theta_h - \theta_l - \omega)), \text{ and} \tag{A117}$$

$$\begin{aligned} Y_{fwd} = {} & (\theta_l - \theta_h)(2\theta_l^2 + 3\theta_h^2\delta - \theta_l\theta_h(4 + \delta)) + 2(\theta_l^2(1 - \delta) - \theta_h^2(2 - \delta) + \delta\theta_l\theta_h)\omega \\ & + 2(3\theta_h - \theta_l - \delta\theta_l)\omega^2 - 2\omega^3. \end{aligned} \tag{A118}$$

and $v_{0h} \leq v_{hl} \leq v_{lb}$ (as depicted in Figure 3.2).

The present value of the total profit in period 1 is $\Pi_1^{g,F,r} = p_l(N_b + N_l) + \delta\hat{p_h}(N_h + N_b)$, or equivalently,

$$\Pi_1^{g,F,r} = p_l(1 - v_{hl}) + \delta\frac{\theta_h((\theta_h - \theta_l)(1 + v_{hl}) - \omega)}{2(2\theta_h - \theta_l - \omega)} * \frac{\theta_h((\theta_h - \theta_l)(1 + v_{hl}) - \omega)}{2(\theta_h - \omega)(\theta_h - \theta_l - \omega)}. \tag{A119}$$

This function is concave when the second derivative w.r.t p_l is nonpositive. We have

$$\frac{\partial^2 \Pi_1^{g,F,r}}{\partial p_l^2} = -2\frac{\partial v_{hl}}{\partial p_l} + \frac{\delta(\theta_h-\theta_l)^2\theta_h^2}{2(\theta_h-\omega)(2\theta_h-\theta_l-\omega)(\theta_h-\theta_l-\omega)}\Big[\frac{\partial v_{hl}}{\partial p_l}\Big]^2. \tag{A120}$$

Hence, $\frac{\partial^2 \Pi_1^{g,F,r}}{\partial p_l^2} \leq 0$ when $0 \leq \frac{\partial v_{hl}}{\partial p_l} \leq \frac{4(\theta_h-\omega)(2\theta_h-\theta_l-\omega)(\theta_h-\theta_l-\omega)}{\delta(\theta_h-\theta_l)^2\theta_h^2}$. It follows that $2(\theta_h - \omega)Y_{fwd} \geq \delta\theta_h^2(\theta_h-\theta_l)^2 \Leftrightarrow (\theta_h-\omega)Y_{fwd} \geq \frac{\delta\theta_h^2(\theta_h-\theta_l)^2}{2}$. This inequality holds for all $\omega \leq \omega_1^{g,F}$, where $\omega_1^{g,F}$ is the smallest solution of $(\theta_h-\omega)Y_{fwd} = \frac{\delta\theta_h^2(\theta_h-\theta_l)^2}{2}$ and $\omega_1^{g,F} < \theta_h-\theta_l$. This result is obtained by showing that $Y_{fwd}=0$ has three positive roots, and that $(\theta_h-\theta_l)$ is larger than the smallest root but smaller than the other roots, and that $(\theta_h-\omega)Y_{fwd}\Big|_{\omega=0} > \frac{\delta\theta_h^2(\theta_h-\theta_l)^2}{2}$.

The unconstrained price p_l is derived from the first-order condition $\frac{\partial \Pi_1^{g,F,r}}{\partial p_l} = 0$:

$$\frac{\partial \Pi_1^{g,F,r}}{\partial p_l} = N_l + N_b + \delta\frac{\partial \Pi_2^{g,F,r}}{\partial p_l} - p_l\frac{\partial v_{hl}}{\partial p_l} = 0 \tag{A121}$$

$$\Leftrightarrow \qquad \underbrace{N_l + N_b + \frac{(\theta_h-\theta_l)\theta_h^2((\theta_h-\theta_l)(1+v_{hl})-\omega)}{2(\theta_h-\omega)(2\theta_h-\theta_l-\omega)(\theta_h-\theta_l-\omega)}\frac{\partial v_{hl}}{\partial p_l}}_{+} - p_l\underbrace{\frac{\partial v_{hl}}{\partial p_l}}_{+} = 0 \tag{A122}$$

The solution p_l to the above equation is always nonnegative. Demand in period 1 is $N_l + N_b = \frac{Q_{fwd}}{Z_{fwd}}$, where

$$Q_{fwd} = 2\theta_h(2\theta_h-\theta_l)(\theta_h-\theta_l)(\theta_l-\delta\theta_h) - 2(\theta_l^3 - 5\theta_l\theta_h^2 - 2\delta\theta_h^3 - (5-\delta)\theta_l^2\theta_h)\omega$$
$$- (2\theta_l^2(2-\delta) - \theta_l\theta_h(8-5\delta) + \delta\theta_h^2)\omega^2 - 2\theta_l(1-\delta)\omega^3 \text{ ,and} \tag{A123}$$

$$Z_{fwd} = 2(\theta_h-\omega)Y_{fwd} - \delta\theta_h^2(\theta_h-\theta_l)^2. \tag{A124}$$

We have $Z_{fwd} > 0$ for all $\omega < \omega_1^{g,F}$ and $Q_{fwd} \geq 0$ for all $\omega \leq \omega_2^{g,F}$, where $\omega_2^{g,F}$ is the smallest root of $Q_{fwd} = 0$ and $\omega_2^{g,B} < \theta_h - \theta_l$. Define $\overline{\omega}^{g,F} = \min\{\omega_1^{g,F}, \omega_2^{g,F}\}$. We then conclude that sales in period 1 (i.e., $1 - v_{hl}$) are nonnegative for all $\omega < \overline{\omega}^{g,F}$. ■

APPENDIX B: PROOFS OF RESULTS FROM CHAPTER 5

Proof of Lemmas 5.1

Solving the surplus maximization problem of consumers, we obtain the sets of consumers choosing different purchase alternatives as follows:

$$S_b = \{v : 0 \leq v \leq 1, v \geq \max\{v_{bh}, v_{bn}, v_{bu}, v_{bo}\}\}, \tag{B1}$$

$$S_h = \{v : 0 \leq v \leq 1, \max\{v_{ho}, v_{hn}, v_{hu}\} \leq v \leq v_{bh}\}, \tag{B2}$$

$$S_n = \{v : 0 \leq v \leq 1, \max\{v_{no}, v_{nu}\} \leq v \leq \min\{v_{bn}, v_{hn}\}\}, \tag{B3}$$

$$S_u = \{v : 0 \leq v \leq 1, v_{uo} \leq v \leq \min\{v_{bu}, v_{hu}, v_{nu}\}\}, \tag{B4}$$

$$S_o = \{v : 0 \leq v \leq 1, v \leq \min\{v_{bo}, v_{ho}, v_{no}, v_{uo}\}\} \tag{B5}$$

where v_{ij}'s values are in Table 5.1.

Now suppose S_h exists in the market, from (B2), it is required that $v_{hn} < v_{bh}$, since $v_{bh} = v_{nu}$ (from Table 5.1), it follows that $v_{hn} < v_{nu}$. From (B3), the condition $v_{hn} < v_{nu}$ excludes the existence of S_n. In other words, both S_n and S_h cannot coexist.■

Proof of Lemmas 5.2

Under $\mathbb{S}_1$, we need $v_{hu} \leq v_{bh}$ or $\frac{p_1 - \delta p_u^e}{\theta} \leq \frac{p_2^e - p_u^e}{\theta(1-\gamma)}$; on the other hand, $\mathbb{S}_2$ requires $v_{nu} \leq v_{bn}$, or $\frac{p_2^e - p_u^e}{\theta(1-\gamma)} \leq \frac{p_1 - \delta p_u^e}{\theta}$. Hence, the critical constraint determining which structure to arise is the relative magnitude of $\frac{p_2^e - p_u^e}{\theta(1-\gamma)}$ and $\frac{p_1 - \delta p_u^e}{\theta}$. Using (5.2) in the main text, we have

$$\frac{p_1 - \delta p_u^e}{\theta} \leq \frac{p_2^e - p_u^e}{\theta(1-\gamma)} \Leftrightarrow \; p_2^e \geq \Psi. \tag{B6}$$

Under the above condition, $\mathbb{S}_1$ arises; otherwise, $\mathbb{S}_2$ arises. ■

Proof of Lemma 5.3

(i) When $c/\theta < t^E$

The inverse demand structure is specified in The Model section. We solve the problem backward, starting from period 2, the firm selects the quantity sold $d_{m,2}^E$ by maximizing the profit in this period $\Pi^E_{m,2}$. The first order condition is $\frac{\partial \Pi^E_{m,2}}{\partial d^E_{m,2}} = 0 \Leftrightarrow d^E_{m,2} = \frac{\theta(1-\gamma d^E_{m,1})-c}{2\theta}$. Since the second order condition is satisfied (i.e., $\frac{\partial^2 \Pi^E_{m,2}}{\partial (d^E_{m,2})^2} = -2\theta < 0$), the solution to the first order condition is indeed the optimal solution. We then have

$$\hat{d}^E_{m,2} = \frac{\theta(1-\gamma d^E_{m,1})-c}{2\theta}. \tag{B7}$$

Substituting $\hat{d}^E_{m,2}$ into Π^E_m and solving for $d^E_{m,1}$ gives $d^E_{m,1} = \frac{2(\theta - c(1-\delta\gamma))}{\theta(4+4\delta\gamma-3\delta\gamma^2)}$. Checking the second order condition also confirms that the above is indeed the optimal solution ($\frac{\partial^2 \Pi^E_{m,2}}{\partial (d^E_{m,2})^2} = -\theta(4+4\delta\gamma-3\delta\gamma^2)/2 < 0$). Subsequently, the optimal quantity in period 2 is $d^E_{m,2} = \frac{\theta(4-\gamma(2-4\delta)-3\delta\gamma^2)-c(4-\gamma(2-4\delta)-\delta\gamma^2)}{2\theta(4+4\delta\gamma-3\delta\gamma^2)}$. The equilibrium outcome is in Table 5.2.

(ii) When $c/\theta \geq t^E$

The sole entrant will not sell any new product in the second period. The first period quantity is set by solving the following problem:

$$\Pi^E_0 = \max_{d^E_{m,1}} (p^E_{m,1} - c) d^E_{m,1} = (\theta(1-d^E_{m,1}) + \delta\theta\gamma(1-d^E_{m,1}) - c) d^E_{m,1} \tag{B8}$$

Solving the first order condition gives $d^E_{m,1} = \frac{1}{2} - \frac{c}{2\theta(1+\delta\gamma)}$; this is also the optimal solution as the second order condition is satisfied ($\frac{\partial^2 \Pi^E_0}{\partial (d^E_{m,1})^2} = -2\theta(1+\delta\gamma) < 0$). In this case, there is no secondary market, and consumers who buy in period 1 will hold the product in period 2. The price of the product is $p^E_{m,1} = \frac{\theta(1+\delta\gamma)+c}{2}$ and the corresponding profit is $\Pi^E_0 = \theta(1+\delta\gamma)\left(\frac{1}{2} - \frac{c}{2\theta(1+\delta\gamma)}\right)^2$. ■

Proof of Lemma 5.4

(i) In period 2, we obtain

$$\frac{\partial \Pi^E_{m,2}}{\partial \gamma} = -2\theta d^E_{m,1}\frac{\theta(4+3\delta\gamma^2) - c(4-8\gamma\delta+\delta(3-4\delta)\gamma^2)}{\theta(4+4\delta\gamma-3\delta\gamma^2)^2} < 0 \tag{B9}$$

$$\frac{\partial \Pi^E_{m,2}}{\partial c} = -2\theta d^E_{m,1}\frac{\gamma^2(-\delta)+4\gamma\delta-2\gamma+4}{2\theta(4\gamma\delta-3\gamma^2\delta q+4)} < 0 \tag{B10}$$

$$\frac{\partial^2 \Pi^E_{m,2}}{\partial \gamma \partial c} = \frac{1}{\theta(\gamma(3\gamma-4)\delta-4)^3}\Big[2c(\gamma((\gamma-4)\delta+2)-4)(\gamma\delta(\gamma(4\delta-3)+8)-4)+$$
$$4\theta(\gamma(\gamma\delta(\gamma(\delta(-3\gamma(\delta-1)+4\delta-14)+3)+12\delta-6)+4)-8)\Big] > 0 \tag{B11}$$

(ii) The comparative static of the present value of the total profit w.r.t γ is:

$$\frac{\partial \Pi^E_m}{\partial \gamma} = \frac{2\delta(\theta - c(1-\delta\gamma))(\theta(3\gamma-2)+c(6-3\gamma+2\delta\gamma))}{\theta(4+4\delta\gamma-3\delta\gamma^2)^2}. \tag{B12}$$

Thus, $\frac{\partial \Pi^E_m}{\partial \gamma} \geq 0$ when $c/\theta \geq \frac{2-3\gamma}{6-3\gamma+2\delta\gamma}$. More specifically, $\frac{\partial \Pi^E_m}{\partial \gamma} \geq 0$ when either ($\gamma \geq 2/3$) or ($\gamma < 2/3$ and $c/\theta \geq \frac{2-3\gamma}{6-3\gamma+2\delta\gamma}$). ■

Proof of Lemma 5.5

The derivation of the equilibrium outcome is given below

In period 2, since the second order condition is satisfied (i.e., $\frac{\partial^2 \Pi^C_{n,2}}{\partial (d^E_{m,2})^2} = -2\theta < 0$), the optimal solution of n is the solution of the first order condition $\frac{\partial \Pi^C_{n,2}}{\partial d^E_{m,2}} = 0 \Leftrightarrow d^C_{n,2} = \frac{\theta(1-\gamma d^E_{m,1})-w^C_{m,2}}{2\theta}$. In other words,

$$\hat{d}^C_{n,2} = \frac{\theta(1-\gamma d^E_{m,1}) - w^C_{m,2}}{2\theta}. \tag{B13}$$

Substituting $\hat{d}^C_{n,2}$ into $\Pi^C_{m,2}$ and solving for $w^C_{m,2}$ gives $w^C_{m,2} = \frac{\theta(1-\gamma d^C_{n,1})}{2}$, and subsequently $d^C_{n,2} = \frac{1-\gamma d^C_{n,1}}{4}$. Checking the second order condition confirms that the above is indeed the optimal solution (i.e., $\frac{\partial^2 \Pi^E_{m,2}}{\partial (w^C_{m,2})^2} = -1/\theta < 0$).

Following the same sequence, in period 1: Solving the first-order condition of the downstream firm's problem gives $d^C_{n,1} = \frac{\theta(8+5\delta\gamma)-8w^C_{m,1}}{\theta(16+16\delta\gamma-5\delta\gamma^2)}$. This is the optimal solution as the second-order condition is satisfied (i.e., $\frac{\partial^2 \Pi_n^C}{\partial (d_{n,1}^C)^2} = -\theta(2+2\delta\gamma - \frac{5\delta\gamma^2}{8}) < 0$). Consequently, the component's price in period 1 is $w_{m,1}^C = \frac{\theta(128+240\delta\gamma-56\delta(1-2\delta)\gamma^2-45\delta^2\gamma^3)}{32(8+8\delta\gamma-3\delta\gamma^2)}$, a solution to $\frac{\partial \Pi_m^C}{\partial w_{m,1}^C} = 0$ (with $\frac{\partial^2 \Pi_m^C}{\partial (w_{m,1}^C)^2} = -\frac{32(8+8\delta\gamma-3\delta\gamma^2)}{\theta(16+16\gamma\delta-5\delta\gamma^2)^2} < 0$).

Comparative Statics

In period 2, we obtain

$$\frac{\partial \Pi_{m,2}^C}{\partial \gamma} = -2\theta d_{n,2}^C \frac{8+6\delta\gamma+3\delta(1+\delta)\gamma^2}{2(8+8\delta\gamma-3\delta\gamma^2)^2} < 0 \tag{B14}$$

$$\frac{\partial \Pi_{n,2}^C}{\partial \gamma} = -\theta d_{n,2}^C \frac{8+6\delta\gamma+3\delta(1+\delta)\gamma^2}{2(8+8\delta\gamma-3\delta\gamma^2)^2} < 0 \tag{B15}$$

Finally, the changes in the present value of the total profit of both firms with respect to γ are given below:

$$\frac{\partial \Pi_m^C}{\partial \gamma} = \frac{\theta\delta(8+3\delta\gamma)(3(2+\delta)\gamma-2)}{8(8+8\delta\gamma-3\delta\gamma^2)^2} \tag{B16}$$

$$\frac{\partial \Pi_n^C}{\partial \gamma} = \frac{\theta\delta(8+3\delta\gamma)(6(4-\gamma)\gamma^2\delta^2+\gamma(8+15\gamma(4-\gamma))\delta+56\gamma-16)}{16(8+8\delta\gamma-3\delta\gamma^2)^3} \tag{B17}$$

Thus, $\frac{\partial \Pi_m^C}{\partial \gamma} \geq 0$ when $\gamma \geq \frac{2}{3(2+\delta)}$. And $\frac{\partial \Pi_n^C}{\partial \gamma} \geq 0$ when $\gamma \geq g(\delta)$, where $g(\delta) = \arg \frac{-8-15\gamma(4-\gamma)+\sqrt{1600-4800\gamma+4704\gamma^2-1800\gamma^3+225\gamma^4}}{12\gamma(4-\gamma)}$, and $g(\delta)$ is decreasing in δ.

Proof of Lemma 5.6

The optimal quantities in period 2 are the solution to the following first order conditions (since the second order conditions are satisfied, i.e., $\frac{\partial^2 \Pi_{m,2}^D}{\partial (d_{m,2}^D)^2} = \frac{\partial^2 \Pi_{n,2}^D}{\partial (d_{n,2}^D)^2} = -2\theta < 0$)

$$\frac{\partial \Pi_{m,2}^D}{\partial d_{m,2}^D} = \theta - c - \theta(2d_{m,2}^D + d_{n,2}^D + \gamma(d_{m,1}^D + d_{n,1}^D)) = 0 \tag{B18}$$

$$\frac{\partial \Pi_{n,2}^D}{\partial d_{n,2}^D} = \theta - w_{m,2}^D - \theta(d_{m,2}^D + 2d_{n,2}^D + \gamma(d_{m,1}^D + d_{n,1}^D)) = 0 \tag{B19}$$

It follows that

$$\hat{d}^D_{m,2} = \frac{\theta(1-\gamma(d^D_{m,1}+d^D_{n,1})) - 2c + w^D_{m,2}}{3\theta} \tag{B20}$$

$$\hat{d}^D_{n,2} = \frac{\theta(1-\gamma(d^D_{m,1}+d^D_{n,1})) - 2w^D_{m,2} + c}{3\theta} \tag{B21}$$

Substituting $\hat{d}^D_{m,2}$ and $\hat{d}^D_{n,2}$ into $\Pi^E_{m,2}$ and solving for $w^D_{m,2}$ gives $w^D_{m,2} = \frac{5\theta(1-\gamma(d^D_{m,1}+d^D_{n,1}))-c}{10}$, and subsequently $d^D_{m,2} = \frac{5\theta(1-\gamma(d^D_{m,1}+d^D_{n,1}))-7c}{10\theta}$ and $d^D_{n,2} = \frac{2c}{5\theta}$. Checking the second order condition also confirms that the above is indeed the optimal solution (i.e., $\frac{\partial^2\Pi^D_{m,2}}{\partial(w^D_{m,2})^2} = -10/9\theta < 0$).

The optimal quantities in period 1 are obtained by the following first-order conditions (since the second-order conditions are satisfied, i.e., $\frac{\partial^2\Pi^D_m}{\partial(d^D_{m,1})^2} = -\frac{\theta(4+4\delta\gamma-3\delta\gamma^2)}{2} < 0$ and $\frac{\partial^2\Pi^D_n}{\partial(d^D_{n,1})^2} = -\theta(2+2\delta\gamma-\delta\gamma^2) < 0$)

$$\frac{\partial\Pi^D_m}{\partial d^D_{m,1}} = \theta - \frac{c(5-4\delta\gamma)}{5} - \frac{\theta(4+4\delta\gamma-3\delta\gamma^2)}{2}d^D_{m,1} - \theta(1+\delta\gamma-\delta\gamma^2)d^D_{n,1} = 0 \tag{B22}$$

$$\frac{\partial\Pi^C_m}{\partial d^D_{n,1}} = \frac{2+\delta\gamma}{2}\theta - w^D_{m,1} + \frac{3c\delta\gamma}{10} - \frac{\theta(2+2\delta\gamma-\delta\gamma^2)}{2}d^D_{m,1} - \theta(2+2\delta\gamma-\delta\gamma^2)d^D_{n,1} = 0 \tag{B23}$$

It follows that

$$\hat{d}^D_{m,1} = \frac{5\theta(2+\delta\gamma-\delta^2\gamma^2+\delta^2\gamma^3) + 10w^D_{m,1}(1+\delta\gamma-\delta\gamma^2) - c(20+7\delta\gamma-\delta(10+13\delta)\gamma^2+5\delta^2\gamma^3)}{5\theta(2+2\delta\gamma-\delta\gamma^2)(3+3\delta\gamma-2\delta\gamma^2)} \tag{B24}$$

$$\begin{aligned}\hat{d}^D_{n,1} = \frac{1}{10\theta(2+2\delta\gamma-\delta\gamma^2)(3+3\delta\gamma-2\delta\gamma^2)}\big[&5\theta(4+8\delta\gamma-4(1-\delta)\delta^2\gamma^2-3\delta^2\gamma^3) \\ &- 10w^D_{m,1}(4+4\delta\gamma-3\delta\gamma^2) + c(20+16\delta\gamma-2\delta(5+2\delta)\gamma^2-\delta^2\gamma^3)\big]\end{aligned} \tag{B25}$$

Next, the component's price in period 1 is

$$w^D_{m,1} = \frac{\theta(4 + 8\delta\gamma - 4\delta(1-\delta)\gamma^2 - 3\delta^2\gamma^3)}{2(4 + 4\delta\gamma - 3\delta\gamma^2)} - E * c, \text{ where} \tag{B26}$$

$$E = \frac{40 + 120\delta\gamma - 20\delta(5-6\delta)\gamma^2 - 2\delta^2(59-20\delta)\gamma^3 + 2\delta^2(25-9\delta)\gamma^4 - 7\delta^3\gamma^5}{10(4 + 4\delta\gamma - 3\delta\gamma^2)(10 + 10\delta\gamma - 7\delta\gamma^2)}. \tag{B27}$$

The above is a solution to $\frac{\partial \Pi^D_m}{\partial w^D_{m,1}} = 0$ (with $\frac{\partial^2 \Pi^D_m}{\partial (w^D_{m,1})^2} < 0$). Subsequently, the component's price in period 2 is

$$w^D_{m,2} = \frac{\theta(4 - 2(1-2\delta)\gamma - 3\delta\gamma^2)}{2(4 + 4\delta\gamma - 3\delta\gamma^2)} - G * c, \text{ where} \tag{B28}$$

$$G = \frac{40 - (60 - 80\delta)\gamma - 2\delta(19 - 20\delta)\gamma^2 + 2\delta(20 + 11\delta)\gamma^3 - 35\delta^2\gamma^4}{10(4 + 4\delta\gamma - 3\delta\gamma^2)(10 + 10\delta\gamma - 7\delta\gamma^2)}. \tag{B29}$$

And the optimal quantities chosen by m are

$$d^D_{m,1} = \frac{2}{4 + 4\delta\gamma - 3\delta\gamma^2} - B * c, \tag{B30}$$

$$d^D_{m,2} = \frac{4 - 2(1-2\delta)\gamma - 3\delta\gamma^2}{2(4 + 4\delta\gamma - 3\delta\gamma^2)} - F * c, \text{ where} \tag{B31}$$

$$B = \frac{2(140 + 200\delta\gamma - 10\delta(19 + 2\delta)\gamma^2 - 2\delta^2(47 + 40\delta)\gamma^3 + \delta^2(65 + 96\delta)\gamma^4) - 28\delta^3\gamma^5)}{5\theta(2 + 2\delta\gamma - \delta\gamma^2)(4 + 4\delta\gamma - 3\delta\gamma^2)(10 + 10\delta\gamma - 7\delta\gamma^2)}, \tag{B32}$$

$$F = \frac{280 - 20(3 - 28\delta)\gamma - 2\delta(193 - 140\delta)\gamma^2 + 2\delta(20 - 163\delta)\gamma^3 + 91\delta^2\gamma^4}{10\theta(4 + 4\delta\gamma - 3\delta\gamma^2)(10 + 10\delta\gamma - 7\delta\gamma^2)}. \tag{B33}$$

Next, the optimal quantities set by n are $d^D_{n,2} = \frac{2c}{5\theta}$, $d^D_{n,1} = \frac{2c(4+4\delta\gamma-3\delta\gamma^2)}{\theta(2+2\delta\gamma-\delta\gamma^2)(10+10\delta\gamma-7\delta\gamma^2)}$.

Notice that $d^D_{m,2} > 0$ when $c/\theta < t^D$, where

$$t^D = \frac{5(10 + 10\delta\gamma - 7\delta\gamma^2)(4 - 2(1-2\delta)\gamma - 3\delta\gamma^2)}{280 - 20(3 - 28\delta)\gamma - 2\delta(193 - 140\delta)\gamma^2 + 2\delta(20 - 163\delta)\gamma^3 + 91\delta^2\gamma^4}. \tag{B34}$$

When $c/\theta \geq t^D$, m does not sell any new product in period 2 (i.e., $d^D_{m,2} = 0$); that is, it becomes a component supplier in period 2. Hence, analogous to the case of a component supplier in period 2, n is the monopolist and sets the quantity sold after receiving the component's price from m. We have the following problem of m in period 2:

$$\max_{w^D_{m,2}} \Pi^{D,0}_{m,2} = w^D_{m,2}\hat{d}^D_{n,2} \tag{B35}$$

such that

$$\hat{d}^D_{n,2} = \arg\max_{d^D_{n,2}} \Pi^{D,0}_{n,2} = (p^D_{m,2} - w^D_{m,2})d^D_{n,2} \tag{B36}$$

$$w^D_{m,2} \geq 0; d^D_{n,2} \geq 0 \tag{B37}$$

Solving the downstream firm's problem yields $\hat{d}^D_{n,2} = \frac{\theta(1-\gamma(d^D_{m,1}+d^D_{n,1}))-w^D_{m,2}}{2\theta}$ (the solution to the first-order condition $\frac{\partial \Pi^{D,0}_{n,2}}{\partial d^D_{n,2}} = 0$ is indeed the optimal solution as the second order condition is satisfied $\frac{\partial^2 \Pi^{D,0}_{n,2}}{\partial (d^D_{n,2})^2} = -2\theta < 0$). Subsequently, the component's price is obtained by solving $\frac{\partial \Pi^{D,0}_{m,2}}{\partial w^D_{m,2}} = 0 \Leftrightarrow w^D_{m,2} = \theta(1 - \gamma(d^D_{m,1} + d^D_{n,1}))/2$, and thus, the quantity sold in period 2 is $d^D_{n,2} = (1 - \gamma(d^D_{m,1} + d^D_{n,1}))/4$.

In period 1, m sets the component's price, and given that, both firms simultaneously and noncollusively choose the quantities sold:

$$\max_{w^D_{m,1}} \Pi^{D,0}_{m} = w^D_{m,1}\hat{d}^D_{n,1} + (p^D_{m,1} - c)\hat{d}^D_{m,1} + \delta\Pi^D_{m,2} \tag{B38}$$

subject to

$$\hat{d}^D_{m,1} = \arg\max \Pi^{D,0}_{m} = w^D_{m,1}d^D_{n,1} + (p^D_{m,1} - c)d^D_{m,1} + \delta\Pi^D_{m,2} \tag{B39}$$

$$\hat{d}^D_{n,1} = \arg\max \Pi^{D,0}_{n} = (p^D_{m,1} - w^D_{m,1})d^D_{n,1} + \delta\Pi^D_{n,2} \tag{B40}$$

$$d^D_{m,1} \geq 0; d^D_{n,1} \geq 0; w^D_{m,1} \geq 0 \tag{B41}$$

The first order conditions are

$$\frac{\partial \Pi_m^{D,0}}{\partial d_{m,1}^D} = \frac{\theta(2+\delta\gamma)}{2} - c - \theta(2+2\delta\gamma-3\delta\gamma^2/4)d_{m,1}^D - (1+\delta\gamma-\delta\gamma^2/2)d_{n,1}^D = 0, \text{ and} \tag{B42}$$

$$\frac{\partial \Pi_n^{D,0}}{\partial d_{n,1}^D} = \frac{\theta(8+5\delta\gamma)}{8} - w_{m,1}^D - \theta(1+\delta\gamma-3\delta\gamma^2/8)d_{m,1}^D - (2+2\delta\gamma-5\delta\gamma^2/8)d_{n,1}^D = 0. \tag{B43}$$

The optimal solution is

$$\hat{d}_{m,1}^D = \frac{4\big(\theta(8+11\delta\gamma+\delta(3\delta-1)\gamma^2) - c(16+16\delta\gamma-5\delta\gamma^2) + 4(2+2\delta\gamma-\delta\gamma^2)w_{m,1}^D\big)}{3\theta(4+4\delta\gamma-\delta\gamma^2)(8+8\delta\gamma-3\delta\gamma^2)}, \text{ and} \tag{B44}$$

$$\hat{d}_{n,1}^D = \frac{\theta(4+3\delta\gamma)+4c-8w_{m,1}^D}{3\theta(4+4\delta\gamma-\delta\gamma^2)}. \tag{B45}$$

Subsequently, the component's price is

$$w_{m,1}^D = \frac{\theta(160+304\delta\gamma-4\delta(19-36\delta)\gamma^2-63\delta^2\gamma^3) - 4c(8+8\delta\gamma-5\delta\gamma^2)}{64(5+5\delta\gamma-2\delta\gamma^2)}. \tag{B46}$$

It follows that the optimal solution is:

$$w_{m,2}^D = \frac{5\theta(8-4(1-2\delta)\gamma-5\delta\gamma^2)+12\gamma c}{16(5+5\delta\gamma-2\delta\gamma^2)}, \tag{B47}$$

$$d_{n,2}^D = \frac{5\theta(8-4(1-2\delta)\gamma-5\delta\gamma^2)+12\gamma c}{32\theta(5+5\delta\gamma-2\delta\gamma^2)}, \tag{B48}$$

$$d_{m,1}^D = \frac{\theta(40+62\delta\gamma+2\delta(11\delta-6)\gamma^2-7\delta^2\gamma^3) - 4c(14+14\delta\gamma-5\delta\gamma^2)}{4\theta(4+4\delta\gamma-\delta\gamma^2)(5+5\delta\gamma-2\delta\gamma^2)}, \text{ and} \tag{B49}$$

$$d_{n,1}^D = \frac{\theta\delta\gamma(5\delta\gamma^2+4\gamma(1-2\delta)-8) + c(64+64\delta\gamma-28\delta\gamma^2)}{4\theta(4+4\delta\gamma-\delta\gamma^2)(5+5\delta\gamma-2\delta\gamma^2)}. \tag{B50}$$

Notice that $d_{m,1}^D > 0 \Leftrightarrow c/\theta < t^{D,0} = \frac{\theta(40+62\delta\gamma+2\delta(11\delta-6)\gamma^2-7\delta^2\gamma^3)}{4(14+14\delta\gamma-5\delta\gamma^2)}$. It follows that when $c/\theta \geq t^{D,0}$, then $d_{m,1}^D = 0$ and the dual distributor becomes a component supplier.■

Proof of Lemma 5.7

In period 2, we obtain

$$\frac{\partial \Pi^{D}_{m,2}}{\partial \gamma} = d^{D}_{n,2}\frac{\partial w^{D}_{m,2}}{\partial \gamma} + 2\theta d^{D}_{m,2}\frac{\partial d^{D}_{m,2}}{\partial \gamma} < 0 \tag{B51}$$

$$\frac{\partial \Pi^{D}_{n,2}}{\partial \gamma} = 2\theta d^{D}_{n,2}\frac{\partial d^{D}_{n,2}}{\partial \gamma} = 0 \tag{B52}$$

where $\frac{\partial w^{D}_{m,2}}{\partial \gamma} < 0$ and $\frac{\partial d^{D}_{m,2}}{\partial \gamma} < 0$.

Next, in period 1, we have $\frac{\partial \Pi^{D}_{m}}{\partial \gamma} \geq 0$ when $\Delta \geq 0$ and $\max\{0, t_1\} \leq c/\theta \leq \min\{t^{D}, t_2\}$, where $\Delta = 196\delta^4\gamma^8 - 1344\delta^4\gamma^7 + \delta^2(3424\delta^2 - 1344\delta - 1575)\gamma^6 + 4\delta^2(-960\delta^2 + 1712\delta + 1725)\gamma^5 + 4\delta(400\delta^3 - 2880\delta^2 - 1919\delta + 1050)\gamma^4 + 20\delta(320\delta^2 - 186\delta - 635)\gamma^3 + 20(380\delta^2 + 438\delta - 135)\gamma^2 + 2400(\delta + 2)\gamma - 400$

$$t_1 = -\frac{5}{4Y}(3((\gamma-2)\gamma\delta-2)^2(\gamma(2\gamma\delta-5)+6)(\gamma(7\gamma-10)\delta-10)^2 + \sqrt{((\gamma-2)\gamma\delta-2)^2(-3\delta\gamma^2+4\delta\gamma+4)^2(\gamma(7\gamma-10)\delta-10)^2\delta}) \tag{B53}$$

$$t_2 = -\frac{5}{4Y}(3((\gamma-2)\gamma\delta-2)^2(\gamma(2\gamma\delta-5)+6)(\gamma(7\gamma-10)\delta-10)^2 - \sqrt{((\gamma-2)\gamma\delta-2)^2(-3\delta\gamma^2+4\delta\gamma+4)^2(\gamma(7\gamma-10)\delta-10)^2\delta}) \tag{B54}$$

$Y = 49\delta^5(8\delta - 15)\gamma^{10} + 28\delta^4(-96\delta^2 + 208\delta + 75)\gamma^9 + 2\delta^4(3424\delta^2 - 10452\delta - 5195)\gamma^8 + 2\delta^3(-3840\delta^3 + 20896\delta^2 + 7557\delta - 6300)\gamma^7 + 5\delta^3(640\delta^3 - 8880\delta^2 + 1434\delta + 12381)\gamma^6 + 3\delta^2(6400\delta^3 - 12940\delta^2 - 37548\delta + 9525)\gamma^5 + 10\delta^2(2780\delta^2 + 8424\delta - 11387)\gamma^4 - 40\delta(420\delta^2 - 3921\delta + 725)\gamma^3 + 1200(68 - 61\delta)\delta\gamma^2 + (11100 - 61600\delta)\gamma - 17000.$

With respect to the cost disadvantage, we obtain $\frac{\partial \Pi^{D}_{m}}{\partial c} \leq 0$ when $c/\theta \leq \min\{t^{D}, t_3\}$, where $t_3 = \frac{1}{Z}[5(\delta\gamma^2 - 2\delta\gamma - 2)(7\delta\gamma^2 - 10\delta\gamma - 10)(5\gamma(3\gamma - 4)\delta^2 + 4(4\gamma - 5)\delta - 20)]$, and $Z = \gamma^3(497\gamma^3 - 2964\gamma^2 + 5740\gamma - 3600)\delta^4 + 4\gamma^2(280\gamma^3 - 1701\gamma^2 + 3670\gamma - 2700)\delta^3 - 20\gamma(80\gamma^3 - 48\gamma^2 - 427\gamma + 540)\delta^2 + 400(12\gamma^2 - 10\gamma - 9)\delta - 3600.$

Next, the comparative static of the present value of the total profit of n w.r.t γ is:

$$\frac{\partial \Pi_n^D}{\partial \gamma} = \frac{\partial \Pi_{n,1}^D}{\partial \gamma} + \delta \frac{\partial \Pi_{n,2}^D}{\partial \gamma} = \frac{\partial \Pi_{n,1}^D}{\partial \gamma} \tag{B55}$$
$$= \frac{\partial (p_{m,1}^D - w_{m,1}^D)}{\partial \gamma} d_{n,1}^D + (p_{m,1}^D - w_{m,1}^D) \frac{\partial d_{n,1}^D}{\partial \gamma} \leq 0 \tag{B56}$$

where $\frac{\partial (p_{m,1}^D - w_{m,1}^D)}{\partial \gamma} < 0$ and $\frac{\partial d_{n,1}^D}{\partial \gamma} < 0$.

Finally $\frac{\partial \Pi_n^D}{\partial c} = \frac{\partial \Pi_{n,1}^D}{\partial c} + \delta \frac{\partial \Pi_{n,2}^D}{\partial c}$, where

$$\frac{\partial \Pi_{n,1}^D}{\partial c} = \frac{\partial (p_{m,1}^D - w_{m,1}^D)}{\partial \gamma} d_{n,1}^D + (p_{m,1}^D - w_{m,1}^D) \frac{\partial d_{n,1}^D}{\partial \gamma} \geq 0 \tag{B57}$$
$$\frac{\partial \Pi_{n,2}^D}{\partial c} = 2\theta d_{n,1}^D \frac{\partial d_{n,1}^D}{\partial \gamma} \geq 0 \tag{B58}$$

where $\frac{\partial (p_{m,1}^D - w_{m,1}^D)}{\partial \gamma} \geq 0$, $\frac{\partial d_{n,1}^D}{\partial \gamma} \geq 0$, and $\frac{\partial d_{n,1}^D}{\partial \gamma} > 0$. ■

Proof of Lemma 5.8

(i) Market segmentation under Sole Entrant

We have $d_{m,1}^E - d_{m,2}^E = \gamma \frac{\theta(2-(4-3\gamma)\delta)+c(\delta(8-\gamma)-2)}{2\theta(4+4\delta\gamma-3\delta\gamma^2)}$. Therefore, $d_{m,1}^E \geq d_{m,2}^E$ when either $(\gamma \geq \frac{4\delta-2}{3\delta})$ or $(\gamma < \frac{4\delta-2}{3\delta}$ and $c/\theta \geq t_{segment}^E)$, where $t_{segment}^E = \frac{\delta(4-3\gamma)-2}{\delta(8-\gamma)-2}$.

(ii) Market segmentation under Component Supplier

We have $d_{n,1}^C - d_{n,2}^C = \gamma \frac{8-5(4-3\gamma)\delta)}{16(8+8\delta\gamma-3\delta\gamma^2)}$. Therefore, $d_{n,1}^C \geq d_{n,2}^C$ when $\gamma \geq \frac{4(5\delta-2)}{15\delta}$.

(iii) Market segmentation under Dual Distribution

$$(d_{m,1}^D + d_{n,1}^D) - (d_{m,2}^D + d_{n,2}^D) = \frac{\gamma}{X}\big[5\theta(2-(4-3\gamma)\delta)(10+10\delta\gamma-7\delta\gamma^2)$$
$$- c(60 - 280\delta + 2\delta(37-140\delta)\gamma - 2\delta(20-103\delta)\gamma^2 - 7\delta^2\gamma^3)\big]. \tag{B59}$$

where $X = 10\theta(4+4\delta\gamma-3\delta\gamma^2)(10+10\delta\gamma-7\delta\gamma^2)$. Therefore, $d_{m,1}^D + d_{n,1}^D \geq d_{m,2}^D + d_{n,2}^D$ when either $(\gamma \geq \frac{4\delta-2}{3\delta})$ or $(\gamma < \frac{4\delta-2}{3\delta}$ and $c/\theta \geq t_{segment}^D)$, where $t_{segment}^D = \frac{5(2-(4-3\gamma)\delta)(10+10\delta\gamma-7\delta\gamma^2)}{60-280\delta+2\delta(37-140\delta)\gamma-2\delta(20-103\delta)\gamma^2-7\delta^2\gamma^3}$.

■

Proof of Proposition 5.1

The comparisons of quantities follow directly from Tables 5.2, 5.3, and 5.4. Here we define the values of all critical values.

(i) $d^C_{n,1} < d^E_{m,1}$ when $c/\theta < t^{EC}_{q,1}$, where $t^{EC}_{q,1} = \frac{9\gamma^3\delta^2-12\gamma^2\delta^2+20\gamma\delta+32}{8(3\gamma^3\delta^2-\gamma^2\delta(8\delta+3)+8)}$.

$d^C_{n,2} < d^E_{m,2}$ when $c/\theta < t^{EC}_{q,2}$, where $t^{EC}_{q,2} = \frac{27\gamma^4\delta^2-132\gamma^3\delta^2+24\gamma^3\delta+128\gamma^2\delta^2-228\gamma^2\delta+256\gamma\delta-96\gamma+128}{8(\gamma^2\delta-4\gamma\delta+2\gamma-4)(3\gamma^2\delta-8\gamma\delta-8)}$.

$d^C_{n,1}+d^C_{n,2} < d^E_{m,1}+d^E_{m,2}$ when $c/\theta < t^{EC}_{q}$, where $t^{EC}_{q} = \frac{27\gamma^4\delta^2+24\gamma^3(1-4\delta)\delta+4\gamma^2\delta(20\delta-57)+48\gamma(7\delta-2)+256}{8(\gamma^2\delta+2\gamma-8)(3\gamma^2\delta-8\gamma\delta-8)}$.

(ii) $d^D_{m,1} < d^E_{m,1} < d^D_{m,1} + d^D_{n,1}$ and $d^D_{m,2} < d^E_{m,2} < d^D_{m,2} + d^D_{n,2}$ for all $c/\theta < t^D$.

(iii) $d^D_{m,1} + d^D_{n,1} > d^C_{n,1}$ and $d^D_{m,2} + d^D_{n,2} > d^C_{n,2}$ for all $c/\theta < t^D$. ■

Proof of Proposition 5.2

The comparisons of the component's prices follow directly from Tables 5.3, and 5.4, where $t^{CD}_w = -\frac{5\gamma\delta(\gamma(7\gamma-10)\delta-10)(\gamma(\delta(\gamma(3\gamma((3\gamma-20)\delta+8)+64\delta-156)+128)-96)+64)}{16(\gamma(3\gamma-8)\delta-8)(\gamma\delta(\gamma(\gamma(\gamma+4)(7\gamma-10)\delta^2-2(\gamma(25\gamma-59)+60)\delta+100)-120)-40)}$.

Notice that $t^{CD}_w > 0$ when $\delta > \frac{2(-32+39\gamma-6\gamma^2+\gamma\sqrt{3(107-84\gamma+12\gamma^2)})}{\gamma(64-60\gamma+9\gamma^2}$. ■

Proof of Proposition 5.3

The comparisons of profits follow directly from Tables 5.2, 5.3, and 5.4, where

$t^{EC}_2 = -\frac{\gamma(\delta(\gamma(3\gamma-4)(3(5\sqrt{2}-8)\gamma-32(\sqrt{2}-2))\delta+4\gamma(6(\sqrt{2}-2)\gamma-47\sqrt{2}+104)+256(\sqrt{2}-2))-32(\sqrt{2}-4))+128(\sqrt{2}-2)}{8(\gamma(3\gamma-8)\delta-8)(\gamma((\gamma-4)\delta+2)-4)}$

$t^{DE}_2 = -\frac{T_1}{T_2}$, $t^{DC}_2 = -\frac{T_3}{T_4}$

$T_1 = 5\gamma(\gamma(7\gamma-10)\delta-10)(\gamma((3\gamma-4)\delta+2)-4)(\gamma\delta(7\gamma^2\delta-5\gamma(2\delta+3)+10)+20)$

$T_2 = \gamma(\gamma(\delta(\gamma(-(10-7\gamma)^2\gamma(\gamma(51\gamma-140)+80)\delta^3+5(7\gamma-10)(\gamma(\gamma(7\gamma+332)-984)+640)\delta^2+5(\gamma(\gamma(165\gamma-1006)-2924)+12720)\delta-2300\gamma+9400)+1200(11-40\delta))+1600)-4000(8\delta+1))-8000$

$T_3 = 5(\sqrt{2}\sqrt{T_3^1}-8((8-3\gamma)\gamma\delta+8)^2(\gamma(7\gamma-10)\delta-10)(\gamma((3\gamma-4)\delta+2)-4)(\gamma(\delta(\gamma(\gamma(7(7\gamma-30)\delta+40)+200\delta-270)+400)-60)+200)$

$T_3^1 = ((8-3\gamma)\gamma\delta+8)^2(32((8-3\gamma)\gamma\delta+8)^2(\gamma(7\gamma-10)\delta-10)^2(\gamma((3\gamma-4)\delta+2)-4)^2(\gamma(\delta(\gamma(\gamma(7(7\gamma-30)\delta+40)+200\delta-270)+400)-60)+200)^2-((10-7\gamma)\gamma\delta+10)^2(\gamma(\gamma(\delta(\gamma((4-3\gamma)^2\gamma(9\gamma(7\gamma-64)+1024)\delta^3+8(3\gamma-4)(3\gamma(\gamma(18\gamma-245)+800)-2048)\delta^2+16(\gamma(36(\gamma-30)\gamma+6079)-10944)\delta-4608\gamma+47360)+3072(32\delta-35))+7168)+8192(8\delta-3))+$

$16384)(\gamma(\gamma(\delta(\gamma((10-7\gamma)^2\gamma(\gamma(229\gamma-760)+720)\delta^3+20(7\gamma-10)(\gamma(\gamma(28\gamma-617)+1764)-1440)\delta^2+20(\gamma(2\gamma(40\gamma-687)+12529)-30120)\delta-4800\gamma+48400)+800(540\delta-311))+3600)+24000(12\delta-1))+72000))$.

$T_4 = 8((8-3\gamma)\gamma\delta+8)^2(\gamma(\gamma(\delta(\gamma((10-7\gamma)^2\gamma(\gamma(229\gamma-760)+720)\delta^3+20(7\gamma-10)(\gamma(\gamma(28\gamma-617)+1764)-1440)\delta^2+20(\gamma(2\gamma(40\gamma-687)+12529)-30120)\delta-4800\gamma+48400)+800(540\delta-311))+3600)+24000(12\delta-1))+72000)$. ■

Proof of Proposition 5.4

The comparisons of profits follow directly from Tables 5.2, 5.3, and 5.4, where $t^{DC}=\frac{T_5}{T_6}$

$T_5=20((\gamma-2)\gamma\delta-2)(\gamma(3\gamma-8)\delta-8)(\gamma(7\gamma-10)\delta-10)(\delta(\gamma(5(3\gamma-4)\delta+16)-20)-20)-5\sqrt{T_5^1}$

$T_5^1=((\gamma-2)\gamma\delta-2)(\gamma(3\gamma-8)\delta-8)(\gamma(3\gamma-4)\delta-4)(\gamma(7\gamma-10)\delta-10)(\delta(\gamma((\gamma-2)\gamma^3(7\gamma-10)(\gamma(1257\gamma-3916)+1280)\delta^5-4\gamma^2(\gamma(\gamma(\gamma(1764\gamma+11147)-68602)+89780)-25600)\delta^4-4\gamma(\gamma(\gamma(16\gamma(617\gamma-2612)+2759)+92980)-38400)\delta^3+16(\gamma(2\gamma(2\gamma(80\gamma+2649)-11405)-3215)+6400)\delta^2+80(\gamma(16\gamma(35\gamma-48)-2675)+1520)\delta-19200(4\gamma-1))+25600(\delta+2))+25600)$

$T_6=4(\gamma(3\gamma-8)\delta-8)(\delta(\gamma(\gamma(\delta((\gamma-2)\gamma(7\gamma-10)(71\gamma-180)\delta^2+4(\gamma(7\gamma(40\gamma-243)+3670)-2700)\delta+20(16(3-5\gamma)\gamma+427))+4800)-400(27\delta+10))-3600)-3600)$

$t^{DE}=\frac{-10\gamma\delta((\gamma-2)\gamma\delta-2)(\gamma(7\gamma-10)\delta-10)}{\delta(\gamma(\gamma(\delta(4(\gamma-2)\gamma(6\gamma-5)(7\gamma-10)\delta^2-2(\gamma(7\gamma(5\gamma+34)-860)+600)\delta+15(56-15\gamma)\gamma+160)+600)-200(6\delta+5))-400)-400}$.

■

Proof of Proposition 5.5

The comparisons of profits follow directly from Tables 5.2, 5.3, and 5.4, where

$t^{DC'}=\frac{(\gamma(3\gamma-8)\delta-8)(\gamma\delta(\gamma((33\gamma-96)\delta+52)-256)-160)-\sqrt{2}\sqrt{(3\gamma\delta+8)^2((\gamma-4)\gamma\delta-4)(\gamma(2\gamma-5)\delta-5)(\gamma(3\gamma-8)\delta-8)^2}}{36(\gamma(3\gamma-8)\delta-8)^2}$.

■

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