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Modeling and Analysis of Automated Storage and Retrievals System with Multiple in-the-aisle Pick Positions

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MODELING AND ANALYSIS OF AUTOMATED STORAGE AND RETRIEVAL SYSTEM WITH MULTIPLE IN-THE-AISLE PICK POSITIONS

by

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ABSTRACT

This dissertation focuses on developing analytical models for automated storage and retrieval system with multiple in-the-aisle pick positions (MIAPP-AS/RS). Specifically, our first contribution develops an expected travel time model for different pick positions and different physical configurations for a random storage policy. This contribution has been accepted for publication in *IIE Transactions* (Ramtin & Pazour, 2014) and was the featured article in the *IE Magazine* (Askin & Nussbaum, 2014). The second contribution addresses an important design question associated with MIAPP-AS/RS, which is the assignment of items to pick positions in an MIAPP-AS/RS. This contribution has been accepted for publication in *IIE Transactions* (Ramtin & Pazour, 2015). Finally, the third contribution is to develop travel time models and to determine the optimal SKUs to storage locations assignment under different storage assignment polies such as dedicated and class-based storage policies for MIAPP-AS/RS.

An MIAPP-AS/RS is a case-level order-fulfillment technology that enables order picking via multiple pick positions (outputs) located in the aisle. We develop expected travel time models for different operating policies and different physical configurations. These models can be used to analyze MIAPP-AS/RS throughput performance during peak and non-peak hours. Moreover, closed-form approximations are derived for the case of an infinite number of pick positions, which enable us to derive the optimal shape configuration that minimizes expected travel times. We compare our expected travel time models with a simulation model of a discrete rack, and the results validate that our models provide good estimates. Finally, we conduct a numerical experiment to illustrate the trade-offs between performance of operating policies and

design configurations. We find that MIAPP-AS/RS with a dual picking floor and input point is a robust configuration because a single command operating policy has comparable throughput performance to a dual command operating policy.

As a second contribution, we study the impact of selecting different pick position assignments on system throughput, as well as system design trade-offs that occur when MIAPP-AS/RS is running under different operating policies and different demand profiles. We study the impact of product to pick position assignments on the expected throughput for different operating policies, demand profiles, and shape factors. We develop efficient algorithms of complexity $O\big(nlog(n)\big)$ that provide the assignment that minimizes the expected travel time. Also, for different operating policies, shape configurations, and demand curves, we explore the structure of the optimal assignment of products to pick positions and quantify the difference between using a simple, practical assignment policy versus the optimal assignment. Finally, we derive closed-form analytical travel time models by approximating the optimal assignment's expected travel time using continuous demand curves and assuming an infinite number of pick positions in the aisle. We illustrate that these continuous models work well in estimating the travel time of a discrete rack and use them to find optimal design configurations.

As the third and final contribution, we study the impact of dedicated and class-based storage policy on the performance of MIAPP-AS/RS. We develop mathematical optimization models to minimize the travel time of the crane by changing the assignment of the SKUs to pick positions and storage locations simultaneously. We develop a more tractable solution approach by applying a Benders decomposition approach, as well as an accelerated procedure for the

Benders algorithm. We observe high degeneracy for the optimal solution when we use chebyshev metric to calculate the distances. As the result of this degeneracy, we realize that the assignment of SKUs to pick positions does not impact the optimal solution. We also develop closed-form travel time models for MIAPP-AS/RS under a class-based storage policy.

This dissertation is dedicated to my compassionate mother, Farzaneh Heshmati, and my supportive father, Tooraj Ramtin.

ACKNOWLEDGMENTS

On Wednesday morning of August 31, 2011, I received this email, which changed the path of my life, “Faraz, Would you have time today or tomorrow to stop by my office briefly to discuss a possible funding opportunity?” This day became the first day I started the journey of my Ph.D. at the University of Central Florida under supervision of **Dr. Jennifer A. Pazour**. I would thank Dr. Pazour for her extreme dedication of her time and energy as well as four years of financial support. I learned a lot during these four years more than any time in my life. I learned to be meticulous researcher, to be hardworking person, and to be moral human. She was not just my academic advisor, she was my teacher, my mentor, my supporter, and my motivator through my research journey.

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CHAPTER ONE: INTRODUCTION

A supply chain includes all of the parties that are involved directly or indirectly to fulfill customer requests. A supply chain does not only include manufacturer and suppliers, but also includes transporters, warehouses, distribution centers, retailers, and customers (Chopra & Meindl, 2007). Distribution centers play a critical role in supporting a company's supply chain success. The mission of a distribution center is to effectively ship products in the requested configuration to the downstream member in the supply chain. Logistics is the indispensable part of any supply chain that manages the flow of materials between the point of origin and the point of end-users. A part of logistics management focuses on activities within facilities and is known as facility logistics. Facility logistics concentrates on facility design, material handling, transportation (Noori *et al.*, 2013, 2014; Kucukvar *et al.*, 2014), and inventory management within manufacturing, distribution, and service facilities. Material handling is the "art and science of moving, storing, protecting, and controlling material". Material handling includes a large variety of manual, semi-automated and automated equipment. The most labor-intensive activity regarding material handling within a distribution center facility is order picking. Order picking is the process of retrieving products from the storage locations to fulfill a specific customer order. Order picking can occur at different levels, which include at the pallet, case, and piece level. In this dissertation, we concentrate on a special type of case-level order fulfillment technology that we call "automated storage and retrieval system with multiple in-the-aisle pick positions" or "MIAPP-AS/RS". MIAPP-AS/RS is a semi-automated order fulfillment technology where the pallet-level put-away and retrieval activities are automated by cranes, but the case-level order picking process is done by human order pickers walking and picking along the aisles.

The global demand for frozen food has grown during the past decade. According to the "Global Frozen Food" report published by Datamonitor (2011) the demand rate for frozen food continues to grow. The global frozen foods market was estimated to be BUSD 165.4 in 2009 and is expected to grow by 21 percent to BUSD 199.5 by 2014. For decades, the majority of deep-freeze distribution centers have been operated manually. Due to the increasing customer demand for frozen products and a highly competitive market, the organizations in deep-freeze supply chain are looking to improve the lead time, order accuracy, and product quality. One of the solutions to decrease the manual operations is turning to "high-bay" deep-freeze distribution centers that are equipped with automated material handling systems such as a MIAPP-AS/RS.

Due to the harsh working conditions and increased safety issues, personnel turnover in deep freeze distribution centers is higher than ambient distribution centers. Also, the majority of cold temperature loss occurs through the roof of a deep freeze distribution center; consequently, effective utilization of vertical space is important (and MIAPP-AS/RSs have been designed as high as 165 feet) (Swisslog Co., 2012). In addition, the MIAPP-AS/RS require less space, which result in further space utilization gains. Therefore, MIAPP-AS/RSs are used to reduce the number of operators who are required to work in the harsh environments, as well as to reduce the amount of space that is required to be temperature controlled (which is both financially and environmentally expensive). Additional benefits include the ability to monitor and control temperature zones and automate the tracking of products.

1-1- Material Handling

The Material Handling Institute of America (MHIA) defines material handling as "the movement, protection, storage and control of materials and products throughout manufacturing, warehousing, distribution, consumption and disposal. As a process, material handling incorporates a wide range of manual, semi-automated and automated equipment and systems that support logistics and make the supply chain work".

Material handling system design is one of the most critical components in any distribution center design. These systems and processes can help extensively to increase the customer service level, reduce inventory levels and order-fulfillment time, and decrease the overall costs in manufacturing and distribution of the products. There is not a single way to ensure that the material handling equipment and processes (including manual, semi-automated and automated) in a distribution center work together as a unified whole. According to the MHIA, in order to reach the material handling goals 10 principles should be considered to properly design a system: 1) Planning, 2) Standardization, 3) Work, 4) Ergonomics, 5) Unit load, 6) Space utilization, 7) System, 8) Environment, 9) Automation, 10) Life cycle cost. Based on these general principles, Tompkins *et al.* (2010) provide the following six-step material handling design process. This process results in the "material handling system equation" (see Figure 1-1), which is the framework that generate solutions for material handling problems. The material handling design process steps are:

1. Define the objectives, goals, and scope.

2. Investigate all of the requirements for moving, storing, securing, and controlling the materials.
3. Create different design alternatives for the requirement of the material handling system.
4. Evaluate the design alternatives.
5. Choose the preferred design alternative.
6. Implement the preferred design.

As shown in Figure 1-2, to complete the material handling system equation, several questions should be answered. These questions (which are listed in Table 1-1) include: *What*, *where?*, *when?*, *how?*, *who* , *which?*. *What* identify the type of material to be moved. *Where* and *when* define the requirements of place and time. *How* and *who* define the methods. The process of answering all of these questions will aid in identifying a recommended material handling system.

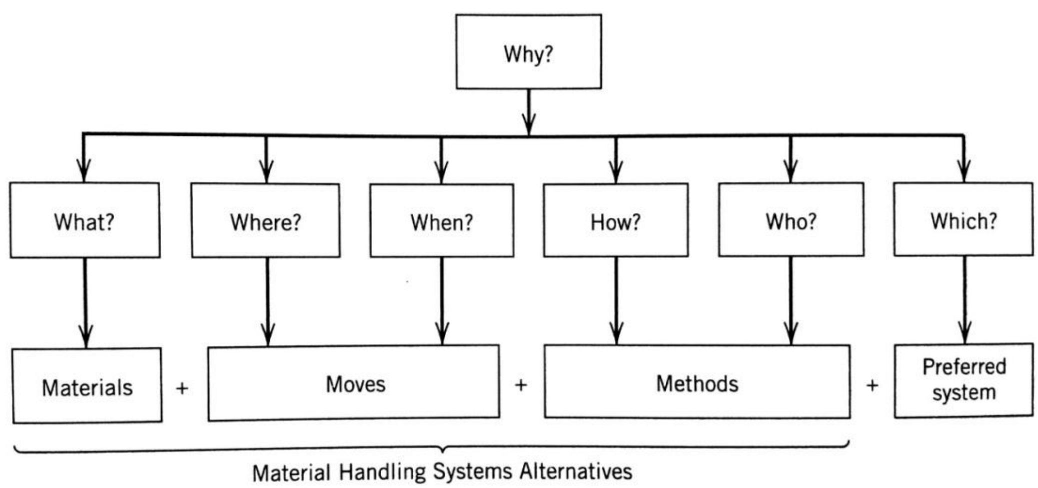


Figure 1-1 : Material handling system equation (Tompkins *et al.*, 2010)

Table 1-1 : Material handling system equation questions (Tompkins *et al.*, 2010)

What Question
- What are the types of materials
- What are the materials characteristics?
- What are the materials amounts to be moved and stored

Where Question
- Where is the material coming from?
- Where is the material going?
- Where is the material stored?
- Where material handling process can be improved?
- Where automation can be used?

When Question
- When is material required?
- When can automation be applied?
- When can material handling performance audit be applied?

How Question
- How is material moved or stored?
- How much inventory of material should be held?
- How should the material be tracked?
- How should the problem be analyzed?

Who Question
- Who have the skills to do material handling tasks?
- Who have the skills to do material handling services?
- Who have the skills to design material handling system?

Which Question
- Which material handling operations are required?
- Which type of material handling tools and equipment are needed?
- Which material handling system is cost efficient?
- Which alternative should be selected?

1-2- Warehouse and distribution center operations

Materials can be handled at different levels. A *stock keeping unit* (*SKU*) is defined as the smallest physical unit of a product that the organization can track. Generally, upstream levels of the supply chain do not carry the small scale of units, and successively when the products flow to downstream levels, they are broken down into smaller units. Figure 1-2 shows how product unit scale can change throughout the supply chain (Bartholdi & Hackman, 2011). In this document the terms *SKU, product*, and *item* are used interchangeably. Typically, retrieval of the products may occur at different levels than the product is received. These levels include pallet, case, or piece-level. If both storage and retrieval of the load are done as one unit, it is denoted as a *unit load*. On the other side, if the items are stored at one level but retrieved in quantities less than unit-load, this retrieval process is generally known as an order picking system (Tompkins *et al.*, 2010).

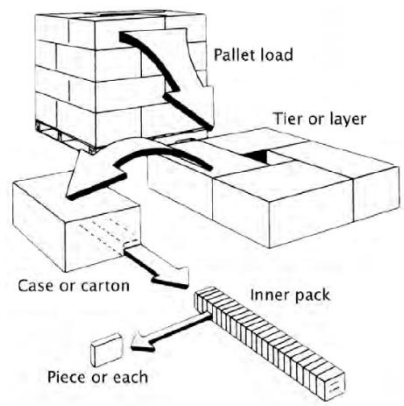


Figure 1-2 : Structure of the stock keeping units (NAVSUP Pub. 529)

The terms *warehouse* and *distribution center* are often used interchangeably. Although these two concepts share many functions in practice, they have some fundamental differences. Specifically, a warehouse receives and fulfills orders at the same product level (i.e., warehouses handle unit loads), whereas distribution centers receive and store products at one level and fulfill customer orders at a less-than-unit load (i.e., distribution centers conduct order picking).

Fulfilling customers' orders requires numerous steps that include coordinating information, labor, material handling equipment, and infrastructure. Hence, designing an effective distribution center requires addressing many key strategic, tactical, and operational questions that impact the company's ability to achieve its mission. The other role of distribution center is to process orders. The orders should be process fast, effectively, and precisely; otherwise, then a company's supply chain planning will suffer. Information technology has a significant role in making distribution center operate more effective, but the best information system will be of little use if the physical systems necessary to get the products out the door are constrained, misapplied, or outdated. There are several metrics to evaluate the performance of distribution centers. These metrics measure the efficiency and effectiveness of labor, equipment, space, financial performance, safety, and sustainability. The common metrics can be categorized as order accuracy metrics, order fill rate, on-time delivery measures, throughput, etc. In practice, an organization takes a collection of these metrics into account to obtain a better understanding of the system.

The criteria to categorize distribution centers is primarily based on the customers who are served. According to Bartholdi & Hackman (2011), important distinctions of distribution centers are as follows.

1. A "*retail distribution center*" which typically provides products to a retail store. Wal-Mart, Publix, and Target are some of the well-known examples. In this type of distribution centers, the primary customer is retail stores. A distribution center generally supplies several retail stores, and the orders include thousands of items. However, planning ahead is possible as typically the orders are known in advance.
2. A "*service parts distribution center*" which generally hold the spare parts of tools or equipment (e.g. airplanes and automobiles). Managing these distribution centers is very challenging because they hold a large number of items with small demands. The variance of the demand, as well as the lead times to replenish is generally large and so high level

of safety stock should be kept. These facts lead to requirement of relatively larger space and consequently more costly order-fulfillment process. Another challenge for these distribution centers is that the demand pattern for service parts are generally unusual as they follow different life cycle patterns. The failure rates for early-, mid-, end-of- life vary product to product. Therefore, it brings hard challenges for demand prediction as well as distribution center design.

3. A "*catalog fulfillment or e-commerce distribution center"* which handles phone, fax, and internet orders placed by the customers. Orders are comprised of a relatively small number of items, but many of these orders are placed during a certain time. Generally, the response time is the most important factor among distribution centers activities.
4. A "*3PL distribution center"* is used for other companies outsourcing warehouse operations. A *3PL distribution center* generally supplies more than one company and thus can gain economies of scale that a single small company cannot gain by its own.
5. A "*perishable products distribution center"*, generally keeps very short shelf life products such as food, flowers, and vaccines that need a temperature-controlled environment. Space efficiency is one of the most challenging parts in these distribution centers as the refrigeration is a very costly process. Other challenges within this type of distribution center may include the handling and shipping of the products that should generally follow the FIFO (First-In-First-Out) or FEFO (First-Expired-First-Out) basis.

Distribution centers can bring different opportunities including order picking, productivity, and value-added services. Any of these opportunities or combination of them can be found in distribution centers (Tompkins *et al.*, 2010). Figure 1-3 shows the schematic view of the opportunities provided by distribution centers. All of these opportunities allow distribution centers to process and ship orders more effectively.

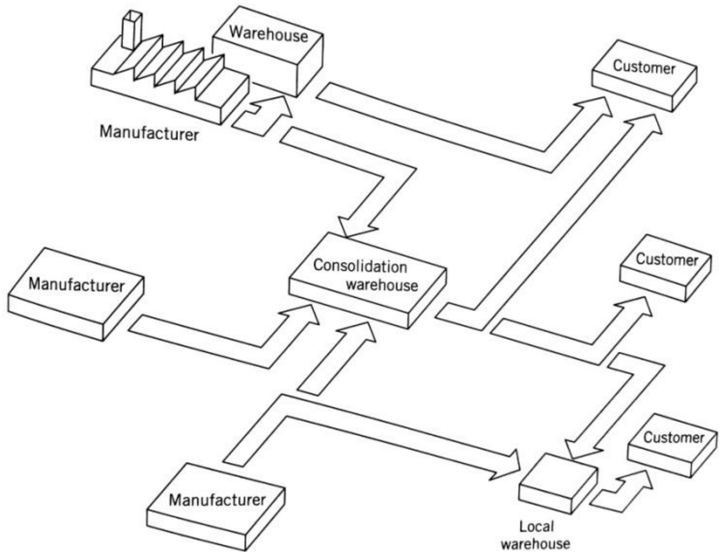


Figure 1-3 : Opportunities provided by distribution centers (Tompkins *et al.*, 2010)

Distribution centers typically are designed to include the following main functional activities: receiving, Put-away (or the term *storage* may use interchangeably), replenishing, picking orders, sort, cross-docking, and shipping (which are displayed in Figure 1-4). *Receiving* activities are comprised of receipt of all incoming materials, making sure the quality and quantity are as ordered, as well as repackaging packages to smaller cases and at the end, preparing materials (such as relabeling them to be identifiable for the organization) to transfer to the next stage. *Put-away* includes the collection of material handling and placement activities to place items in storage locations where the items wait for a demand request. *Order picking* is the action of selecting the right amount of items from storage locations to fulfill customer orders. *Sortation* occurs after the batches of items are picked and the items are sorted and accumulated into individual orders. *Shipping* includes the activities such as finalized the orders for completeness, packing final orders into proper containers, preparing shipping documents, determining shipping charges, and loading trucks. *Replenishment* is the process of movement of SKUs from upstream storage locations (such as reserve storage area) to downstream locations (forward picking area). *Cross-docking* includes the act of transferring the upstream items from the receiving dock to shipping dock directly.

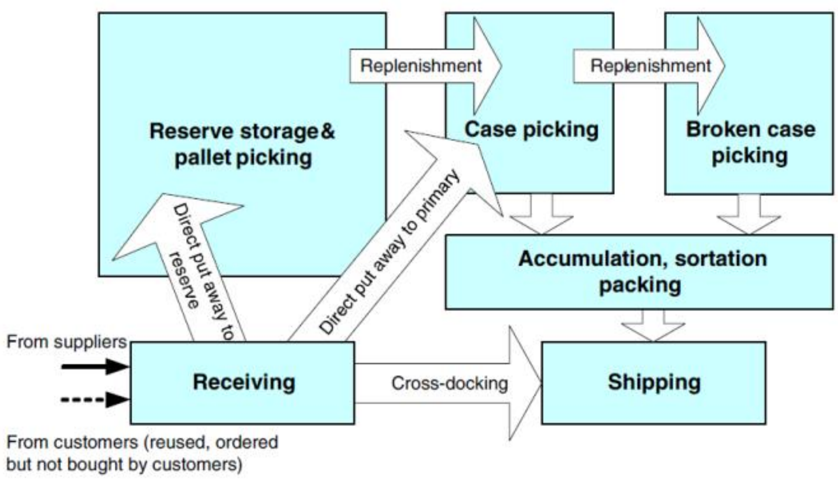


Figure 1-4 : Typical warehouse functions and flows (modified after Tompkins *et al.* (2010))

Gu *et al.*, (2007) identify the problems that can occur, as well as the decision to be made for each of these warehouse functions in Table 1-2. At the viewpoint of warehouse functions, the focus of this dissertation is on *storage* and *order picking* problems. For the storage activity, we study the storage location assignment, and for the order picking function, we look into the sequencing problem.

Table 1-2 : Description of the warehouse operation problems (Gu *et al.*, 2007)

Warehouse operation problems		Decisions
Receiving and shipping		• Truck-dock assignment • Order-truck assignment • Truck dispatch schedule
Storage	SKU-department assignment	• Assignment of items to different warehouse departments • Space allocation
	Zoning	• Assignment of SKUs to zones • Assignment of pickers to zones
	Storage location assignment	• Storage location assignment • Specification of storage classes (for class-based storage)
Order picking	Batching	• Batch size • Order-batch assignment
	Routing and sequencing	• Routing and sequencing of order picking tours • Dwell point selection (for AS/RS)
	Sorting	• Order-lane assignment

1-3- Order Picking

de Koster *et al.* (2007) define order picking as "the process of clustering and scheduling the customer orders, assigning stock locations to order lines, releasing orders to the floor, picking the articles from storage locations and shipment of the packed articles".

Order picking is the highest-priority activity in distribution centers for productivity improvement (Tompkins *et al.*, 2010). There are two major reasons for this concern. First, the order picking process is the most costly activity in the typical distribution center. According to the study of Drury (1988), it is revealed that 55% of all operating costs are associated to order picking in a typical distribution center (see Figure 1-5). Second, as the advent of new operating systems and customers' needs, smaller orders are required to be delivered more frequently and more accurately. Therefore, more SKUs are flowing into order picking systems, which makes the process increasingly difficult to manage. Also, quality control and customer service mean distribution center managers are also focusing on order picking from other points of view such as minimizing product damage, reducing fulfillment times, and improving picking accuracy (Tompkins *et al.*, 2010). Therefore, maximizing the service level subject to resource constraints such as labor, capital, and equipment is a common objective for order picking systems (Goetschalckx & Ashayeri, 1989).

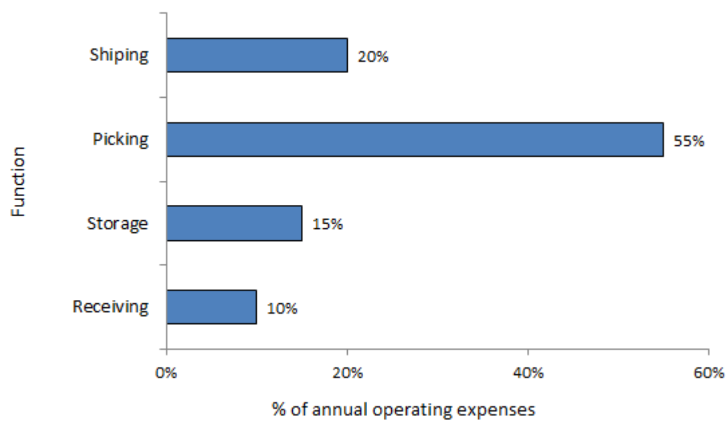


Figure 1-5 : Typical distribution of warehouse operating expenses (Tompkins *et al.*, 2010)

There are several types of order-picking system exist in warehouses. Often, a warehouse uses multiple order picking systems at the time. Van den Berg (1999) classifies order-picking systems into three groups:

1- Picker-to-product systems
2- Product-to-picker systems
3- Picker-less systems

In a *picker-to-product* system, human order-pickers may walk or ride in vehicles to reach the pick positions and pick the items. This picking process can be categorized into two types. First, *low-level* picking occurs when the order pickers travel along the aisle and pick the items from storage racks or bins. Second, *high-level* picking occurs when the order picker travels to the picking spots by riding on board of lifting cranes. This type of the high-level picking is also known as *man-on-board* order picking system. When the order is sufficiently large, each order can be picked in its own single tour individually (*Single or discrete order-picking*). However, when the number of the orders increases and the orders become relatively smaller, there is an opportunity to reduce the travel time by grouping the multiple orders simultaneously in one tour, which is denoted as *batch picking*. In batch picking, sortation is required. If the orders are sorted during the order-picking process, it is known as *sort-while-pick*. Otherwise, if the sortation process is done afterwards, it is called *pick-and-sort* (van den Berg, 1999).

In a *product-to-picker* system, the products are brought to the picker by employing material handling technologies. A*utomated storage and retrieval systems* (AS/RS) and the *carousel* are two well-known examples of product-to-picker technologies. AS/RS consists of high-rise storage racks and the fully automated cranes for handling the loads put-away and retrieval. A carousel consists of storage racks that rotate horizontally or vertically around a closed loop to bring the requested items to order pickers. Finally, *picker-less* systems use robot-technology or automatic dispensers such as *A-frame* systems (van den Berg, 1999; Pazour and Meller, 2011) to fulfill customer orders.

According to Tompkins *et al.* (2010), certain principles exist that are applicable to most order picking processes regardless of material, customer, and warehouse specifications. We mention some of these order picking principles based on relevance to the objectives of our study.

1. Use Pareto's law. Pareto's law is applicable for many cases in business and manufacturing environments. In particular, in warehouses, it is typical to observe a large portion of inventory to be attributed to a small number of SKUs. This idea can be extended to other aspects in warehouses such as demand, throughput, space utilization, and so on.
2. Provide an effective stock location system. Every warehouse should have a consistent stock locating system. The effective system helps as an input to an efficient routing system and prevents tedious non-value-added search for items.
3. Eliminate or combine order picking tasks. Order picking may include activities such that traveling, extracting items, reaching to pick locations, documenting transactions, sorting, searching for pick locations. These activities comprise the order pickers' time. Several opportunities exist to eliminate or combine these elements to reduce the overall picking time. Some of these methods are provided in Table 1-3.
4. Allocate the most demanded items to the most accessible storage locations in the warehouse. As another intuition from Pareto's law, assigning fast-moving items to more accessible locations can reduce the picking times. This rule can be applied for manual and automated order picking. One of the common mistakes that is made in practice is overlooking the size of the product. Therefore, the amount of space required by the items should be considered in any ranking system that identifies the popularity of the items. A

simple ranking system can be the ration of picking frequency per unit to the shipping cube per unit. The items with the highest rank should be assigned to the most accessible locations.

Table 1-3 : Order picking work elements and means for elimination (Tompkins *et al.*, 2010),

Work Element	Method of Elimination	Equipment Required
Traveling	Bring pick locations to picker	Stock-to-picker system Miniload AS/RS Horizontal carousel Vertical carousel
Documenting	Automate information flow	Computer-aided order picking Automatic identification systems Light-aided order picking Radio frequency terminals Headsets
Reaching	Present items at waist level	Vertical carousels Person-aboard AS/RS Miniload AS/RS
Sorting	Assign one picker per order and one order per tour	
Searching	Bring pick locations to picker Take picker to pick locations Illuminate pick locations	Stock-to-picker systems Person-Aboard AS/RS Pick-to-light systems
Extracting	Automated dispensing	Automatic item pickers Robotic order pickers
Counting	Weigh count Prepackage in issue increments	Scales on picking vehicles

5. Balance picking activities across picking locations to reduce congestions. When the most demanded items are assigned to the most accessible locations that are located in a relatively small area, the congestion of order pickers may occur. Therefore, for any design of picking area, ensure that sufficient space exists to avoid congestion.
6. Consider the correlation between item requests to assign similar items to the nearby locations. Besides popularity of the items, correlation of order request can be used for assigning items to locations. It is likely that some of the items are requested together. Assigning correlated items to nearby locations can reduce the travel time. For example,

Frazelle (1989) presents a computerized procedure to consider the popularity and the correlation of the demand of items at the same time.

1-4- Automated Storage/Retrieval System (AS/RS)

An automated storage and retrieval system (AS/RS) is defined by MHIA as "a combination of equipment and controls that handle, store and retrieve materials as needed with precision, accuracy and speed under a defined degree of automation. An AS/RS is used for raw material, work-in-process, and finished goods. A significant increase in the number of AS/RS used in the distribution environments have been seen in United States, and the installations of such systems have become commonplace in all major industries". AS/RSs have had a great impact on manufacturing (especially pull-based systems), warehousing, and different service facilities such as hospitals and libraries (Tompkins *et al.*, 2010). Zollinger (1999) list the major benefits of using AS/RS as follows.

- improve efficiency of operators and storage capacity
- reduce the WIP inventory
- improve the quality and just-in-time performance
- provide make-to-order capability as well as make-to-inventory production
- control the inventory in real-time manner and prompt reporting functionality
- higher inventory security
- less product damage

The major disadvantages associated to AS/RS, which must be considered before justifying acquisition of such a system, are:

- low flexibility in layout
- high initial capital cost
- fixed storage capacity
- lack of visibility

The system design process of implementing an AS/RS should include the following considerations:

- definition of current and future loads to be handled
- number of the loads to be stored in the system
- material flow description (including average and peak rates)
- description of operations
- architectural/engineering considerations

A typical AS/RS includes one or more aisles (each aisle has storage racks on either side), a crane, and an input/output (I/O) queue. The crane can generally access the storage racks on both sides of the aisle. A typical AS/RS storage operation includes the crane picking up a load at the I/O point, moving the load to an empty storage location, depositing the load in the empty storage location, and traveling empty to the I/O point. A similar process can occur for retrieving a load from the AS/RS system. Because such operations are performed by conducting either one storage or one retrieval, they are known as *single command* (SC) operations. A more efficient operation that involves performing both a storage and a retrieval is called a *dual command* (DC) operation. A DC operation consists of the crane picking up the load at the I/O point, travelling to the empty storage location, depositing the load, moving empty to the location of desired retrieval, picking up the load, moving back to the I/O point, and finally depositing the load.

If each aisle has its own crane, the system is known as *aisle-captive*. For some systems, the activity level per aisle may vary during the year due to the seasonal demand. If the activity level of one aisle is low enough that it does not justify dedicating a crane to that aisle, the number of crane can be less than the number of aisles. In this case the system is designed such that the crane(s) can move from one aisle to another.

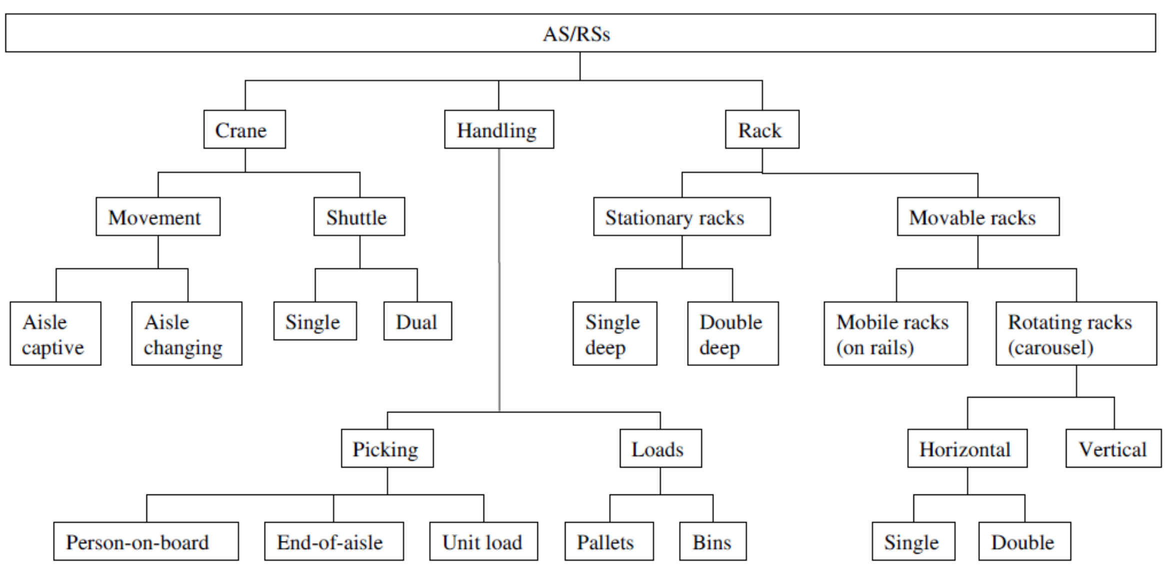


Figure 1-6 : Different AS/RS options (Roodbergen & Vis, 2009)

In AS/RS, horizontal and vertical crane travel occurs simultaneously. Therefore, the time required for the crane to reach any point within the rack is equal to the maximum of the horizontal and vertical travel time. This movement is known as a *chebyshev distant* metric. The horizontal and vertical speeds of a typical crane are up to 600 and 150 feet per minute, respectively (Tompkins *et al.*, 2010).

When the loads have relatively low variety of loads in the system, throughput requirement of each item become relatively high, thereby the number of loads to be stored is high. In this case, storing items in a rack may occur double deep to increase the space utilization. This rack configuration is often referred to double-deep racks (Tompkins *et al.*, 2010).

Roodbergen & Vis (2009) provide an extensive review of 30 years of literature on AS/RS problems. As illustrated in Figure 1-6, they categorize the AS/RSs based on crane and rack configurations as well as type of handling. Also, they categorize the AS/RS design decision problems into: 1) system configuration, 2) storage assignment, 3) batching, 4) sequencing, and 5) dwell-point problems (see Table 1-4). In this dissertation, we only study system configuration, storage assignment, and sequencing aspects of AS/RS problems.

Table 1-4 : Classification of AS/RS design decision problems (Roodbergen & Vis, 2009)

Class of problems	Decisions to be made
System configuration	• Number of aisles • Height of the storage racks • Length of the aisles • Equally sized or modular storage locations • Number and location of the I/O-points • Buffer capacity at the I/O-points • Number of cranes per aisle • Number of order pickers per aisle (if any)
Storage assignment	• Storage assignment method • Number of storage classes • Positioning of the storage classes
Batching	• Type of batching (static or dynamic) • Batch size (capacity or time based) • Selection rule for assignment of orders to batches
Sequencing	• Sequencing restrictions (e.g., due dates) • Type of operation (single or dual command) • Scheduling approach (block or dynamic) • Sequencing method
Dwell-point	• Type of positioning (static or dynamic) • Location where idle cranes will be placed.

1-5- Automated Storage/Retrieval System with multiple in-the-aisle pick positions

Automated storage and retrieval system (AS/RS) with multiple in-the-aisle pick positions (MIAPP-AS/RS) is one type of case-level order-fulfillment technology. As illustrated in Figure 1-7, a MIAPP-AS/RS has two types of aisles. Half of the aisles are dedicated to the movement of human order pickers (denoted as picking aisles) and the other half are dedicated to the movement of storage/retrieval (S/R) machines (denoted as crane aisles). Human order pickers transverse the picking aisles to create mixed or rainbow pallets for customers by picking cases from pallets located on the ground level. The MIAPP-AS/RS supports the case-level order-fulfillment process by performing all pallet storage and replenishment activities. The storage operations performed by the MIAPP-AS/RS originate from end-of-the-aisle input points where a unit-load (typically a pallet) is taken from an input point and stored in a storage position in the rack. A replenishment request is sent from a pick position when the amount of items in the pick position reaches a certain pre-determined level. To replenish pick positions a pallet is retrieved from the storage area and placed into its pick position located in the aisle.

Case-level order-fulfillment with MIAPP-AS/RSs is a semi-automated process because the storage and replenishment of pallets to pick positions is automated using Cranes. However, because building a pallet is difficult to automate due to the varying sizes and weights of cases, the case-fulfillment process is conducted by human order pickers.

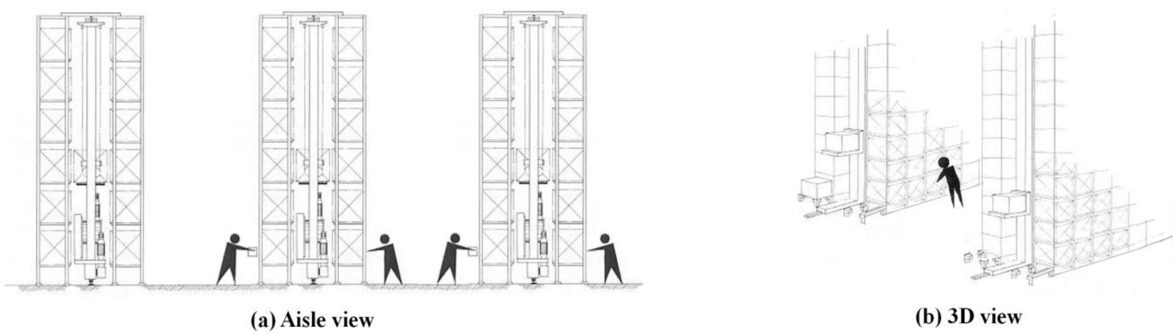


Figure 1-7 : Schematic view of a typical MIAPP-AS/RS

Case-level order-fulfillment is common for company-owned distribution centers that supply individual retail stores with a variety of products. The primary reasons for implementing an MIAPP-AS/RS are to increase space utilization and to eliminate the need for human operators to perform replenishment and storage operations. These factors lead MIAPP-AS/RSs to be common in temperature-controlled distribution centers especially ones that handle frozen items. In the cold supply chain, specific temperature standards are enforced to ensure food quality is maintained: "Chill" (2 to 4 Celsius) is used for fruit and vegetables, "Frozen" (-16 to -20 Celsius) is used for meat, and "Deep Freeze" (-28 to -30 Celsius) is used for seafood and ice-cream (Rodrigue *et al.*, 2009). Due to the harsh working conditions and increased safety issues, personnel turnover in deep freeze distribution centers is higher than ambient distribution centers. Also, the majority of cold temperature loss occurs through the roof of a deep freeze distribution center; consequently, effective utilization of vertical space is important (and MIAPP-AS/RSs have been designed as high as 165 feet) (Swisslog Co., 2012). In addition, Crane aisles require less space than picking aisles, which result in further space utilization gains. Therefore, MIAPP-AS/RSs are used to reduce the number of operators who are required to work in the harsh environments, as well as to reduce the amount of space that is required to be temperature controlled (which is both financially and environmentally expensive). Additional benefits include the ability to monitor and control temperature zones and automate the tracking of products.

The implementation of MIAPP-AS/RSs can be found in numerous grocery distribution centers in the United States (E.g. Publix Super Markets and Wal-Mart), and Europe (E.g. Walkers, Ferrero GmbH and Arla). In these distribution centers a large volume of heavy cases is handled in Chill and Deep Freeze environments (Swisslog Co., 2013). The 2010 global frozen food market is estimated to be worth 192.2 BUSD and the global demand for frozen food is anticipated to grow at a rate of 4 percent annually (Datamonitor, 2011). Therefore, the number of Deep Freeze distribution centers and the use of case-level order-fulfillment technology, in general, and MIAPP-AS/RS in particular, are on the rise.

Because of high infrastructure investment costs and the critical importance of order fulfillment on cost and customer satisfaction, designing and assessing an MIAPP-AS/RS is an important strategic decision in warehouse design. Such systems are commonly constrained by Crane throughput; therefore, estimating the average travel time for different design configurations and operating policies is a fundamental step in designing MIAPP-AS/RSs. To effectively handle a wide range of item requests in the order-fulfillment process, the number of pick positions available is also an important design characteristic of MIAPP-AS/RSs. The number of pick positions can be increased through the use of an additional elevated picking floor on a mezzanine that enables case-level order-fulfillment to be performed at different elevations.

Distribution centers may have different operating policies during peak and non-peak times. For example, many distribution centers have a peak-picking time where a large majority of the distribution center's orders are placed and must be picked before the last truck leaves the dock for shipment. During peak times, the distribution center prioritizes fulfillment of orders over storage requests that can be performed during non-peak times. Also, if a distribution center experiences a balanced number of storage and retrieval requests throughout the day, the Crane can perform a dual command travel that includes both a storage and a retrieval. Consequently, estimating the throughput of an MIAPP-AS/RS is important for different operating strategies.

1-6- Summary of dissertation

This dissertation focuses on deriving analytical expressions to calculate the expected travel time of an MIAPP-AS/RS that applies different operating policies and has different physical configurations. These analytical models are used to find the expected throughput for optimal assignment of SKUs to pick positions located in the aisle. Moreover, closed-form approximations are derived for the case of an infinite number of pick positions that enable us to derive the optimal shape configuration that minimizes travel times. Through comparison with a simulation model, we illustrate that our models provide good estimates and can be used to aid in design and evaluation of real-world systems.

The remainder of the dissertation is organized as follows. In the next chapter we review the literature on travel time and throughput models developed for different aspects of AS/RS design problems such as physical design and storage assignment problems. In Chapter 3, we state the particular areas on which we concentrate as well as review our contributions in each area of our study. In Chapter 4, we present the derivation of expected travel time models for MIAPP-AS/RS under different operating policies and physical configurations. In Chapter 5, we provide the models and procedures to find the optimal SKU assignment to pick positions, as well as developing models to estimate the expected throughput for different SKU assignment cases. In Chapter 6, we study the dedicated and class-based storage policy for MIAPP-AS/RS. Finally, in Chapter 7, we identify the possible future research directions to continue the proposed topics in this dissertation.

CHAPTER TWO: LITERATURE REVIEW

As the main focus of this dissertation is on the AS/RS problems, in this chapter, we focus on reviewing the existing literature of AS/RS problems from different perspectives. First, we review the physical design problems discussed in the literature, as one of the main focuses of our study is to determine the appropriate design for the system. Second, we concentrate on the various travel time models that exist in the literature from different points of view such as command cycles, operating characteristic, and different I/O locations. Finally, we review the storage assignment policy problems of AS/RS.

2-1- Existing Physical Layout Design in an AS/RS

According to Roodbergen & Vis (2009), as shown in Table 1-4, system configuration decisions of the AS/RS involve determining the number of cranes, number of aisle, size of the racks, and so on. However, only few papers consider AS/RS design in combination with other material handling systems. According to Roodbergen & Vis (2009) and Vasili *et al.* (2012), there are two general methods for AS/RS design problems: (1) analytical methods; and (2) simulation.

2-1-1- Analytical Methods

Among the several papers that consider designing and optimization of warehouse and material handling systems problem, Zollinger (1975) is an early study that consider AS/RS design based on cost analysis model.

Karasawa *et al.* (1980) build a deterministic mixed-integer programming (MIP) to minimize the cost of AS/RS problem. They consider the number of the cranes and the size of the rack as the decision variables of the model. The constraints of the model include number of the cranes, storage volume, and throughput.

Ashayeri *et al.* (1985) develop a model to minimize the total investment and warehouse operating costs. Their model identifies the optimal number of the cranes as well as the optimal dimensions of the warehouse subject to system throughput, crane speeds, and size of the building.

Bozer and White (1990) introduce the first analytical stochastic analysis for a mini-load AS/RS that is modeled as a two-server closed queuing network. Bozer & White (1996) have extended Bozer & White (1990) to determine the near-optimal pickers' number and improve the pickers' utilization by considering the sequencing of container retrievals sequence for each order.

Lee (1997) categorizes the techniques of evaluating the performance of AS/RSs into static (Egbelu, 1991; Egbelu and Wu, 1993), computer simulation (Egbelu and Wu, 1993; Linn and Wysk, 1990; Randhawa *et al.*, 1991; Randhawa and Shroff, 1995), and stochastic analysis. He presents a stochastic analysis of unit-load AS/RS for the first time using a continuous time Markov chain. His model is capable of using different formulas for SC and DC of various system configurations such as the case when the I/O point is located other than on the lower left corner of the rack.

Bozer and Cho (2005) extend Lee (1997) by developing analytical closed-form stochastic models to determine if the system meets a desired throughput, as well as identifying the expected

S/R machine utilization. Their model can also apply to alternative I/O point locations or storage methods if E(SC)>E(TB).

Malmborg (2001) generates the modified rule of thumb that does not require the proportion of SC and DC as well as total storage capacity to compare randomized versus dedicated storage. Also, crane utilization is considered in the cost estimation procedure. They propose additional performance measures to evaluate different rack configurations.

Hwang *et al.* (2002) consider the combination of mini-load AS/RS with Automated Guide Vehicle (AGV) to design the assembly line workstation. They propose nonlinear model as well as heuristics to identify the optimal number of AGVs and optimal mini-load AS/RS design.

Le-Duc *et al.* (2006) and de Koster *et al.* (2008) develop the 3D AS/RS . Their focus is on evaluating the performance and optimal dimension of the system. They derive a closed-form expression for the expected travel time model when the system operates under SC basis. Also, travel time approximation is developed for DC cycles.

2-1-2- Simulation and data mining methods

There exist several simulation and data mining techniques in the literature such as Principal component analysis (PCA) (Wu et al., 2014; Yun et al., 2014), discrete-event simulation, agent-based simulation (Beheshti & Sukthankar, 2012, 2013, 2014; Beheshti *et al.*, 2015; Beheshti & Mozayani, 2014), Monte Carlo simulation (Hadian *et al.*, 2012, 2013), and simulation-optimization.

Rosenblatt & Roll (1984) propose a simulation-optimization procedure to find the optimal solution for a particular warehouse design problem that consider there different cost functions (including initial investment cost, shortage cost, and storage cost).

Randhawa *et al.* (1991) analyze the impact of number of the I/O points on mean and maximum waiting time by applying the simulation study. The simulation model investigates the layouts with different number of I/O points per aisle as well as the relationship between the source of storage and retrieval operations. They consider three performance measure (System throughput, mean, and maximum waiting time) as well as three different unit-load AS/RS performing under DC cycles. The results show that introducing two independent I/O points per aisle where the input pallet loads are stored based on *Closest Open Location (COL)* policy, and output pallet retrieval based on a *Nearest Neighbor (NN)* policy.

Randhawa & Shroff (1995) extende the work of Randhawa *et al.* (1991). They perform a comprehensive study that evaluate the performance of six different layouts with single I/O point (but the location varies) performing under three different scheduling policies. Their simulation model considers three different performance measures including system throughput, waiting times, and rejects due of I/O queues. The results show that locating the I/O point at the middle of the rack can obtain higher throughput.

Rosenblatt *et al.* (1993) consider simulation and optimization model simultaneously to determine the design parameters for the system. They capture the dynamic behavior of the system as well as optimize the total cost of the system at the same time. In their model, they assume that number of the crane can be less than the number of the aisles.

2-2- Existing AS/RS Travel time models

2-2-1- Travel time interpretation

As the throughput capacity is the inverse of the average travel time, estimating the average travel time is one of the fundamental steps in AS/RS design. AS/RS systems are often throughput constrained and one way to improve the system throughput is to reduce the travel time. Also, because the total cost of the system is highly dependent to the number of the aisles; it is critical to know the throughput of each aisle to determine the number of the aisles (Sarker & Babu, 1995). To our knowledge, the only survey papers are Sarker & Babu (1995), Roodbergen & Vis (2009) and Manzini (2012) are the only three papers discussing exclusively about AS/RS. Except (Sarker & Babu, 1995) which is totally dedicated to travel time models of AS/RS, the other two papers have the exclusive section about travel time models of AS/RS.

In most AS/RS, as the crane has independent and simultaneous movements in horizontal and vertical directions, the maximum of the horizontal and vertical travel time *Chebyshev* distant metric) is used to calculate the actual travel time. Horizontal and vertical travel speeds are up to

600 and 150 feet per minute, respectively (Tompkins *et al.*, 2010). There are two approaches to estimate the AS/RS travel time, discrete approach (see Egbelu (1991); Thonemann & Brandeau (1998); Sari *et al.* (2005)) or continuous approximation approach (see Table 8 in Roodbergen & Vis (2009)). Simulation studies have shown that there is not much significant difference between the two approaches (see Bozer & White (1984); Hu *et al.* (2005); Sari *et al.* (2005)). Continuous approximation has received much more considerations, as closed-form expressions can be obtained in this case.

2-2-2- Travel time models from the prospective of crane command cycles

An AS/RS crane can have single or multi shuttle. A single shuttle AS/RS can perform single SC or DC cycles. In a SC, either one storage operation or retrieval operation can be performed in each cycle. However, in DC cycles, both storage and retrieval operation can be performed in each cycle. A multi-shuttle AS/RS consists of more than one shuttle, where each shuttle can handle one storage and retrieval of the items in each cycle (Sarker & Babu, 1995; Meller & Mungwattana, 1997; Potrč *et al.*, 2004).

Hausman *et al.* (1976) perform one of the first studies of the travel time model for SC cycle. Graves *et al.* (1977), Bozer & White (1984) and Pan & Wang (1996) consider both SC and DC cycle with some other system configurations. Bozer & White (1984) have presented several closed-form expressions for different I/O point configurations by considering normalized rectangular rack with length of 1.0 and height of *shape factor* in terms of time.

Sarker *et al.* (1991) analyze the double-shuttle AS/RS by considering FC cycle under NN scheduling rule. They show that performing double shuttle system under NN scheduling rule would outperform the throughput performance of single shuttle systems.

Foley & Frazelle (1991) consider end-of-the-aisle mini-load AS/RS with DC cycle. They assume the rack is square-in-time and uniformly distributed, and the pick times are distributed deterministically or exponentially. They derive the closed-form expression for maximum throughput of system.

In order to handle above 20 tons heavy loads, Hu *et al.* (2005) develop a continuous approximation travel time model under SC for a new kind of S/R mechanism which are referred to split-platform AS/RS, or SP-AS/RS. In SP-AS/RS, the horizontal and vertical movements are performed by separate devices.

2-2-3- Travel time models from the prospective of crane operating characteristic

Most of studies have ignored the acceleration and deceleration of the crane, and assumed a constant speed for the crane. Guenov and Raeside (1989) realize by their study that an optimum Chebyshev travel tour may be up to 3% higher than the optimal travel times when model considers the acceleration/deceleration of the crane. Hwang & Lee (1990) derive the continuous travel time model by considering both maximum velocity and the time required to reach the peak velocity. They consider SC and DC cycle under randomized storage policy.

Chang *et al.* (1995) extend the work of Bozer & White (1984) by considering the speed specifications that exist in real-world problems. Chang & Wen (1997) extend Chang *et al.*, 1995) to find out the impact of rack configurations on the crane speed profile. Wen *et al.* (2001) is another extension of Chang *et al.* (1995) which consider different travel speeds ,where the acceleration and deceleration rates are known. They concluded that their exponential travel time model has satisfactory performance.

2-2-4- Travel time models from the prospective of alternative I/O point(s) position

Bozer and White (1984) develop and analyze the expected travel time of five alternative I/O point configurations. They assume that the I/O point can be located at (1) the lower-left corner of the aisle; (2) the opposite ends of the aisle; (3) the same end of the aisle, but at different elevations; (4) the same elevation, but at a midpoint in the aisle; and (5) the end of the aisle, but elevated. All five configurations consider only one input and one output point. The MIAPP-AS/RS has multiple in-the-aisle points that are not necessarily located at the corner of the rack; therefore, their models are not applicable.

Randhawa & Shroff (1995) extende the work of Randhawa *et al.* (1991). They perform a comprehensive study that evaluate the performance of six different layouts with single I/O point (but the location varies) performing under three different scheduling policies. Their simulation model considers three different performance measures including system throughput, waiting times, and rejects of I/O queues. The results show that locating the I/O point at the middle of the rack can obtain higher throughput.

Ashayeri *et al.* (2002) develop geometrical algorithm to derive the travel time and throughput of AS/RS under zone-based storage assignment. They consider one, double (located at two opposite side of floor level), and multiple I/O points.

Vasili *et al.* (2008) develop a novel configuration in split-platform AS/RS (SP-AS/RS) where the I/O point is located at the middle of the rack. They consider a continuous approximation of the rack to model the expected travel time, reduce the mean handling travel time in the system, and validate their model through Monte Carlo simulation. The results show that their proposed configuration, for some particular ranges of shape factor, improve the expected travel time comparing to Chen *et al.* (2003) and Hu *et al.* (2005).

2-3- Existing AS/RS Storage Assignment Models

A storage assignment policy determines the assignment of items to storage locations. The primary goal of a storage policy is to minimize the average travel time subject to satisfying various system constraints (Goetschalckx and Ratliff, 1990). The three most often used storage policies in the literature are randomized storage, dedicated storage, and class-based storage (see e.g., Hausman *et al.* (1976); Graves *et al.* (1977); Schwarz *et al.* (1978); Goetschalckx and Ratliff (1990); Kouvelis and Papanicolaou (1995); Van den Berg (1999); Roodbergen and Vis (2009)). Hausman *et al.* (1976) find that a significant reduction in travel time can be achieved using class-based turnover assignment policies rather than randomized storage policies. Both Rosenblatt and Eynan (1989) and Eynan and Rosenblatt (1994) consider the optimal boundaries for n-class storage racks. They conclude that a storage rack with a limited number of classes (less than 10) can improve the travel time compared to a full-turnover policy. Guenov and Raeside (1992) compare three different zone shapes under DC scheduling. They conclude that performance of the proposed shapes depends on the location of the I/O point. Goetschalckx and Ratliff (1990) consider dedicated storage policies and shared storage policies. They develop a duration-of-stay (DOS) shared policy for unit-load system with balanced input and output. Kulturel *et al.* (1999) compare two shared storage assignment policies with respect to their average travel time by using computer simulation.

2-4- Summary of Literature Review

A great portion of AS/RS literature looks at one I/O point, which is where the crane both initiates an operation and terminates an operation. In MIAPP-AS/RS, a crane can initiate and terminate at any of pick positions or in a storage location in the rack. Therefore, the travel time models that exist in a literature are not applicable for MIAPP-AS/RS because they assume that the travel time of the current operation is independent of the previous operation (which is not the case in a MIAPP-AS/RS). Also, the literature on travel time models for AS/RS with more than one I/O point examines the I/O points at the end-of-the aisle. However, we study the in-the-aisle pick positions which have different travel distance to storage locations on the rack. Moreover, the literature that examines the location of I/O points beyond the end-of-aisle only considers a single I/O point in the middle of the rack. To the best of our knowledge, we are aware of no literature that develops analytical models to analyze AS/RS design issues with multiple I/O points in the aisle.

CHAPTER THREE: PROBLEM STATEMENT

This dissertation focuses on developing analytical models for MIAPP-AS/RS. Specifically, our first contribution develops an expected travel time model for different pick positions and different physical configurations for a random storage policy. This contribution has been accepted for publication in *IIE Transactions* and was the featured article in the *IE Magazine*. The second contribution addresses an important design question associated with MIAPP-AS/RS, which is the assignment of items to pick positions in an MIAPP-AS/RS. This contribution has been accepted for publication in *IIE Transactions*. Finally, the third contribution is to develop travel time models and to determine the optimal SKUs to storage locations assignment under different storage assignment polies such as dedicated and class-based storage policies for MIAPP-AS/RS.

3-1- Contribution 1

An automated storage and retrieval system with multiple in-the-aisle pick positions (MIAPP-AS/RS) is a case-level order-fulfillment technology that enables order picking via multiple pick positions (outputs) located in the aisle. We develop expected travel time models for different operating policies and different physical configurations. These models can be used to analyze MIAPP-AS/RS throughput performance during peak and non-peak hours. Moreover, closed-form approximations are derived for the case of an infinite number of pick positions, which enable us to derive the optimal shape configuration that minimizes expected travel times. We compare our expected travel time models with a simulation model of a discrete rack, and the results validate that our models provide good estimates. Finally, we conduct a numerical experiment to illustrate the trade-offs between performance of operating policies and design configurations. We find that MIAPP-AS/RS with a dual picking floor and input point is a robust configuration because a single command operating policy has comparable throughput performance to a dual command operating policy.

3-2- Contribution 2

As a second contribution, we study the impact of selecting different pick position assignments on system throughput, as well as system design trade-offs that occur when MIAPP-AS/RS is running under different operating policies and different demand profiles. We study the impact of product to pick position assignments on the expected throughput for different operating policies, demand profiles, and shape factors. We develop efficient algorithms of complexity $O\big(nlog(n)\big)$ that provide the assignment that minimizes the expected travel time. Also, for different operating policies, shape configurations, and demand curves, we explore the structure of the optimal assignment of products to pick positions and quantify the difference between using a simple, practical assignment policy versus the optimal assignment. Finally, we derive closed-form analytical travel time models by approximating the optimal assignment's expected travel time using continuous demand curves and assuming an infinite number of pick positions in the aisle. We illustrate that these continuous models work well in estimating the travel time of a discrete rack and use them to find optimal design configurations.

3-3- Contribution 3

As the third and final contribution, we study the impact of dedicated and class-based storage policy on the performance of MIAPP-AS/RS. We develop mathematical optimization models to minimize the travel time of the crane by changing the assignment of the SKUs to pick positions and storage locations simultaneously. We develop a more tractable solution approach by applying a Benders decomposition approach, as well as an accelerated procedure for the Benders algorithm. We observe high degeneracy for the optimal solution when we use chebyshev metric to calculate the distances. As the result of this degeneracy, we realize that the assignment of SKUs to pick positions does not impact the optimal solution. We also develop closed-form travel time models for MIAPP-AS/RS under a class-based storage policy.

CHAPTER FOUR: CONTRIBUTION 1- A PAPER ON "ANALYTICAL MODELS FOR AN AUTOMATED STORAGE AND RETRIEVAL SYSTEM WITH MULTIPLE IN-THE-AISLE PICK POSITIONS

An automated storage and retrieval system with multiple in-the-aisle pick positions (MIAPP-AS/RS) is a case-level order-fulfillment technology that enables order picking via multiple pick positions (outputs) located in the aisle. We develop expected travel time models for different operating policies and different physical configurations. These models can be used to analyze MIAPP-AS/RS throughput performance during peak and non-peak hours. Moreover, closed-form approximations are derived for the case of an infinite number of pick positions, which enable us to derive the optimal shape configuration that minimizes expected travel times. We compare our expected travel time models with a simulation model of a discrete rack, and the results validate that our models provide good estimates. Finally, we conduct a numerical experiment to illustrate the trade-offs between performance of operating policies and design configurations. We find that MIAPP-AS/RS with a dual picking floor and input point is a robust configuration because a single command operating policy has comparable throughput performance to a dual command operating policy.

Keywords: AS/RS, Travel-time models, Multiple I/O points

4-1- Introduction

The most critical and costly task in a distribution center is order-fulfillment. Order-fulfillment is the process of securing a customer order and applying resources to transfer the set of items in the order to the customer (Pazour and Meller, 2011). Case-level order-fulfillment occurs in distribution centers that receive full pallets from upstream entities in their supply chain and fulfill requests for less-than-pallet quantities of a diverse set of items from their downstream customers (de Koster *et al.,* 2007).

Case-level order-fulfillment technology is implemented to reduce space and labor requirements. One such case-level order-fulfillment technology is the automated storage and retrieval system (AS/RS) with multiple in-the-aisle pick positions (MIAPP-AS/RS). As illustrated in Figure 4-1, a MIAPP-AS/RS has two types of aisles. Half of the aisles are dedicated to the movement of human order pickers (denoted as picking aisles) and the other half are dedicated to the movement of storage/retrieval (S/R) machines (denoted as S/R machine aisles). Human order pickers transverse the picking aisles to create mixed or rainbow pallets for customers by picking cases from pallets located on the ground level. The MIAPP-AS/RS supports the case-level order-fulfillment process by performing all pallet storage and replenishment activities. The storage operations performed by the MIAPP-AS/RS originate from end-of-the-aisle input points where a unit-load (typically a pallet) is taken from an input point and stored in a storage position in the rack. A replenishment request is sent from a pick position when the amount of items in the pick position reaches a certain pre-determined level. To replenish pick positions a pallet is retrieved from the storage area and placed into its pick position located in the aisle.

Case-level order-fulfillment with MIAPP-AS/RSs is a semi-automated process because the storage and replenishment of pallets to pick positions is automated using S/R machines.

However, because building a pallet is difficult to automate due to the varying sizes and weights of cases, the case-fulfillment process is conducted by human order pickers.

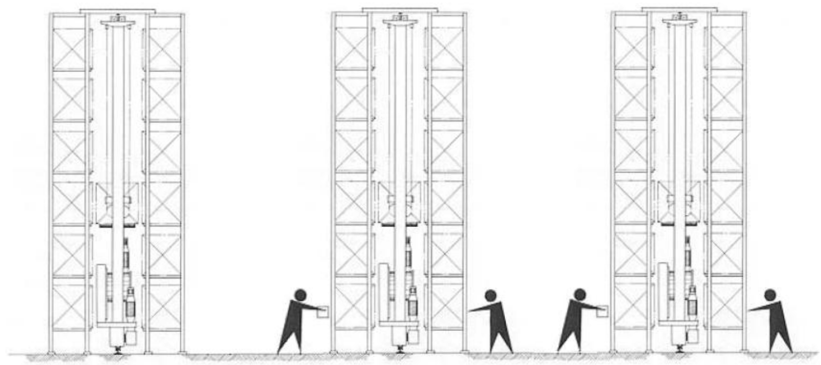


Figure 4-1 : Schematic view of a typical MIAPP-AS/RS

Case-level order-fulfillment is common for company-owned distribution centers that supply individual retail stores with a variety of products. The primary reasons for implementing an MIAPP-AS/RS are to increase space utilization and to eliminate the need for human operators to perform replenishment and storage operations. These factors lead MIAPP-AS/RSs to be common in temperature-controlled distribution centers especially ones that handle frozen items. In the cold supply chain, specific temperature standards are enforced to ensure food quality is maintained: "Chill" (2 to 4 Celsius) is used for fruit and vegetables, "Frozen" (-16 to -20 Celsius) is used for meat, and "Deep Freeze" (-28 to -30 Celsius) is used for seafood and ice-cream (Rodrigue *et al.*, 2009). Due to the harsh working conditions and increased safety issues, personnel turnover in deep freeze distribution centers is higher than ambient distribution centers. Also, the majority of cold temperature loss occurs through the roof of a deep freeze distribution center; consequently, effective utilization of vertical space is important (and MIAPP-AS/RSs have been designed as high as 165 feet) (Swisslog Co., 2012). In addition, S/R machine aisles require less space than picking aisles, which result in further space utilization gains. Therefore, MIAPP-AS/RSs are used to reduce the number of operators who are required to work in the harsh environments, as well as to reduce the amount of space that is required to be temperature controlled (which is both financially and environmentally expensive). Additional benefits include the ability to monitor and control temperature zones and automate the tracking of products.

The implementation of MIAPP-AS/RSs can be found in numerous grocery distribution centers in the United States (eg. Publix Super Markets and Wal-Mart), and Europe (eg. Walkers, Ferrero GmbH and Arla). In these distribution centers a large volume of heavy cases is handled in Chill and Deep Freeze environments (Swisslog Co., 2013). The 2010 global frozen food market is estimated to be worth 192.2 Billion US Dollars and the global demand for frozen food is anticipated to grow at a rate of 4 percent annually (Datamonitor, 2011). Therefore, the number of Deep Freeze distribution centers and the use of case-level order-fulfillment technology, in general, and MIAPP-AS/RS in particular, are on the rise.

Because of high infrastructure investment costs and the critical importance of order fulfillment on cost and customer satisfaction, designing and assessing an MIAPP-AS/RS is an important strategic decision in warehouse design. Such systems are commonly constrained by S/R machine throughput; therefore, estimating the average travel time for different design configurations and operating policies is a fundamental step in designing MIAPP-AS/RSs. To effectively handle a wide range of item requests in the order-fulfillment process, the number of pick positions available is also an important design characteristic of MIAPP-AS/RSs. The number of pick positions can be increased through the use of an additional elevated picking floor on a mezzanine that enables case-level order-fulfillment to be performed at different elevations.

Distribution centers may have different operating policies during peak and non-peak times. For example, many distribution centers have a peak-picking time where a large majority of the distribution center's orders are placed and must be picked before the last truck leaves the dock for shipment. During peak times, the distribution center prioritizes fulfillment of orders over storage requests that can be performed during non-peak times. Also, if a distribution center experiences a balanced number of storage and retrieval requests throughout the day, the S/R machine can perform a dual command travel that includes both a storage and a retrieval. Consequently, estimating the throughput of an MIAPP-AS/RS is important for different operating strategies.

The objective of our research is to understand design decisions and trade-offs that occur in selecting operating policies when implementing an MIAPP-AS/RS. While much research has been conducted on the traditional end-of-aisle AS/RSs, these models are not applicable to in-the-aisle AS/RSs. The difference lies in the location and number of output points, as well as the retrieval operations. Traditional end-of-aisle AS/RSs have a single input/output (I/O) point for each aisle where a S/R machine picks up items to be stored and brings items that are retrieved to order pickers or conveyors. Consequently, travel time models derived for traditional AS/RSs with a single I/O point result in all of the S/R machine trips starting and terminating at a single point and each travel time is independent of the previous trip. Whereas, with MIAPP-AS/RSs, retrievals occur to multiple pick positions (outputs) in the aisle. The S/R machine will start a storage from an input point and terminate a retrieval at one of the in-the-aisle pick positions. Therefore, the travel time depends on where the S/R machine terminated which requires considering the previous trip.

Our contribution includes deriving analytical expressions to calculate the expected travel time of an MIAPP-AS/RS that applies different operating policies and has different physical configurations. Moreover, closed-form approximations are derived for the case of an infinite number of pick positions that enable us to derive the optimal shape configuration that minimizes travel times. Through comparison with a simulation model, we illustrate that our models provide good estimates and can be used to aid in design and evaluation of real-world systems. Through a numerical experiment, we assess throughput performance of MIAPP-AS/RSs during peak and non-peak times for different design configurations. We find that operating policies have different performance impacts depending on the design configuration.

The remainder of this article is organized as follows. In Section 4-2 we briefly review relevant literature in the area of AS/RSs with an emphasis on AS/RSs with more than one I/O point. Section 4-3 provides the description, structure and operations related to an MIAPP-AS/RS, as well as the main assumptions of our models. In Section 4-4 the analytical travel time models for the MIAPP-AS/RS are introduced under the presence of different operating policies and design configurations. Section 4-5 is dedicated to the validation of our analytical models via simulation analysis, as well as numerical experiments to analyze MIAPP-AS/RS system performance that provide design insights into different operating policies and physical design configurations. Finally, in Section 4-6, we review our contributions and provide future research directions.

4-2- Literature review

During the past 30 years AS/RSs have received remarkable attention in the literature. A wide range of issues relating to AS/RSs have been studied. This is illustrated in the following review papers dedicated exclusively to AS/RSs: Roodbergen and Vis (2009), Sarker and Babu (1995) and Vasili *et al.* (2012). AS/RSs have also been addressed in several warehouse design and control review papers such as de Koster *et al.* (2007) and Gu *et al.* (2007).

Two approaches to estimate the AS/RS travel time exist: (1) a discrete approach (see Egbelu (1991), Thonemann and Brandeau (1998) and Sari *et al.* (2005)) or (2) a continuous approximation approach (see Roodbergen and Vis (2009, Table 8)). Simulation studies have shown that the difference between the two approaches is not significant (see Bozer and White (1984), Hu *et al.* (2005) and Sari *et al.* (2005)). Consequently, as closed-form expressions can be obtained, continuous approximation models have received considerably higher attention. We also use the continuous approximation approach in this paper.

AS/RS travel time models have been investigated from different operational perspectives. Detailed information can be found in the survey papers of Sarker and Babu (1995) and Vasili *et al.* (2012). Derivations for several expected travel time models of single command (SC), dual command (DC), and travel between (TB) for different AS/RS configurations exist (Bozer and

White, 1984; Chang *et al.*, 1995; Foley and Frazelle, 1991; Graves *et al.*, 1977; Hu *et al.*, 2005; Kouvelis and Papanicolaou, 1995; Sarker and Babu, 1995).

Researchers have also studied the impact of different AS/RS storage assignment policies. A storage assignment policy determines the assignment of items to storage locations. The primary goal of a storage policy is to minimize the average travel time subject to satisfying various system constraints (Goetschalckx and Ratliff 1990). The five most often used storage policies in the literature are randomized storage; closest-open-location storage assignment; class-based storage; full-turnover-based storage and dedicated storage (see e.g., Hausman *et al.* (1976); Graves *et al.* (1977); Schwarz *et al.* (1978); Goetschalckx and Ratliff (1990); Kouvelis and Papanicolaou (1995); Van den Berg (1999); Roodbergen and Vis (2009)). Hausman *et al.* (1976) find that a significant reduction in travel time can be achieved using class-based turnover assignment policies rather than closest-open-location policies. Both Rosenblatt and Eynan (1989) and Eynan and Rosenblatt (1994) consider the optimal boundaries for n-class storage racks. They conclude that a storage rack with a small number of classes (less than 10) can improve the travel time compared to a full-turnover policy. Guenov and Raeside (1992) compare three different zone shapes under DC scheduling. They conclude that performance of the proposed shapes depends on the location of the I/O point. Goetschalckx and Ratliff (1990) consider dedicated storage policies and shared storage policies. They develop a duration-of-stay (DOS) shared policy for unit-load system with balanced input and output. Kulturel *et al.* (1999) compare two shared storage assignment policies with respect to their average travel time by using computer simulation.

Numerous studies have addressed other operational and control issues such as batching (Elsayed, 1981; Hwang *et al.*, 1988; Egbelu, 1991), dwell point strategies (Bozer and White, 1984; Egbelu, 1991; Peters *et al.*, 1996; van den Berg, 2002), dual-shuttle or multi-shuttle (Keserla and Peters, 1994; Sarker *et al.*, 1994; Meller and Mungwattana, 1997), and sequencing (Schwarz *et al.*, 1978; Han *et al.*, 1987; Eynan and Rosenblatt, 1993).

Lee (1997) categorizes the techniques of evaluating the performance of AS/RSs into static (Egbelu, 1991; Egbelu and Wu, 1993), computer simulation (Egbelu and Wu, 1993; Linn and Wysk, 1990; Randhawa *et al.*, 1991; Randhawa and Shroff, 1995), and stochastic analysis. Bozer and White (1990) introduce the first analytical stochastic analysis for a mini-load AS/RS that is modeled as a two-server closed queuing network. Lee (1997) presents a stochastic analysis of unit-load AS/RS for the first time using a continuous time Markov chain. His model is capable of using different formulas for SC and DC of various system configurations such as the case when the I/O point is located other than on the lower left corner of the rack. Bozer and Cho (2005) extend Lee (1997) by developing analytical closed-form stochastic models to determine if the system meets a desired throughput, as well as identifying the expected S/R machine utilization. Their model can also apply to alternative I/O point locations or storage methods if $E(SC)>E(TB)$.

In addition to operational and control issues, many studies have focused on the physical design and the system configuration of AS/RSs such as the shape of the storage rack, the number of the cranes and aisles, as well as the number and location of the I/O points (see Vasili *et al.* (2012) and references within). The mentioned studies helped extensively to design and operate

AS/RSs. However, there are only a few analytical and simulation studies addressing system configuration with either more than one I/O point or with I/O points not located at the end-of-the aisle.

Bozer and White (1984) develop and analyze the expected travel time of five alternative I/O point configurations. They assume that the I/O point can be located at (1) the lower-left corner of the aisle; (2) the opposite ends of the aisle; (3) the same end of the aisle, but at different elevations; (4) the same elevation, but at a midpoint in the aisle; and (5) the end of the aisle, but elevated. All five configurations consider only one input and one output point. The MIAPP-AS/RS has multiple in-the-aisle points that are not necessarily located at the corner of the rack; therefore, their models are not applicable. Randhawa *et al.* (1991) develop a simulation study to identify the impact of the number of I/O points located at the end-of-the-aisle for three different unit-load AS/RSs with DC cycles. The AS/RS layouts vary in the number of end-of-aisle I/O points per aisle and the relationship between the storage and retrieval sources. System throughput, mean waiting time and maximum waiting time are considered as criteria for system evaluation. They find that the efficiency of the AS/RS can be improved by the introduction of two end-of-aisle I/O points per aisle. Randhawa and Shroff (1995) extend the work of Randhawa *et al.* (1991) by using simulation to analyze six different layouts with a single I/O point (with varying location) using three different scheduling policies. The results are compared considering the system throughput, storage and retrieval waiting times, and rejects due to the rack or I/O queues being fully utilized. The authors conclude that higher throughput can be obtained by locating the I/O point at middle of the aisle instead of at the end of the aisle. Both of the studies use a simulation approach. Also, Randhawa *et al.* (1991) consider end-of-aisle I/O points only, and Randhawa and Shroff (1995) evaluate a single I/O point in the middle of the aisle. Vasili *et al.* (2008) propose a new configuration for the I/O point in a split-platform AS/RS to reduce average handling time. They develop a continuous travel time model for this new configuration of locating an I/O point at the center of the rack. The travel time model was validated by using Monte Carlo simulation. They assume a single I/O point in the center of the rack and only consider SC cycles.

In summary, the literature on travel time models for AS/RS with more than one I/O point examines the I/O points at the end-of-the aisle. The literature that examines the location of I/O points beyond the end-of-aisle only considers a single I/O point in the middle of the rack. To the best of our knowledge, we are aware of no literature that develops analytical models to analyze AS/RS design issues with multiple I/O points in the aisle.

4-3- Problem definition and assumptions

As illustrated in Figure 4-1 an MIAPP-AS/RS has two types of aisles that we previously defined as picking aisles and S/R machine aisles. The picking aisles are wider than S/R machine aisles to provide space for the pickers to move and conduct case picking. The picking aisle consists of pick positions at both sides of the aisle. The S/R machine aisle consists of an input point, a S/R machine, and storage racks on both sides of the aisle. If there is only one picking floor on the ground, we denote this as a single picking floor configuration. We also study the case where there is a mezzanine located vertically above the ground floor for each picking aisle and the S/R machine services both the ground and mezzanine floors pick positions. We denote this as a dual picking floor configuration. A dual picking floor configuration provides additional pick positions in each aisle for case picking. Both picking floors (ground and mezzanine) are assumed to have the same number of pick positions.

In previous studies on single I/O point AS/RS, all of the S/R machine trips start and terminate at the I/O point; therefore, travel time models are considered cycles, particularly SC and DC cycles. However, in MIAPP-AS/RS, pick-up and drop-off (P/D) points are not the same; consequently, instead of the term cycle we use the term travel. A travel consists of a number of components (denoted as trips). As a dwell point strategy, we assume a stay strategy where the S/R machine stays where it is after the completion of each storage or retrieval and waits for the next operation request. Therefore, the S/R machine can be idle either at a location within the rack after completion of a storage or at one of the pick positions after completion of a retrieval. In other words, where the S/R machine will start a trip depends on the previous trip. Throughout the paper, when we mention that a dwell point is within the rack we are referring to storage locations (excluding pick positions) within the rack.

A travel can be a SC storage travel, a SC retrieval travel, or a DC travel. To perform a SC storage travel the S/R machine moves empty from the dwell point to a requested input point, picks up the load from the input point, travels to the storage location, and deposits the load. To perform a SC retrieval travel the S/R machine moves empty from the dwell point to the storage location, picks up the item, travels to the requested pick position, and deposits the item. A DC travel can be performed if both a storage and a retrieval request are available at the same time. To perform a DC travel the S/R machine returns empty from the dwell point to the requested input point, picks up the load, travels to the storage location, deposits the load, travels empty to the location of the load to be retrieved (called the travel between trip), picks up the load, travels to the requested pick position, and deposits the load.

We study a system where the S/R machine can operate under either SC (storage or retrieval) or DC travels. As the dwell point varies, one travel (SC storage, SC retrieval, and DC travel) can consist of different trips. For example, if the dwell point is at one of the pick positions a SC retrieval consists of two trips: one trip is from the pick position to the storage location of item to be retrieved and the other trip is from the storage location to the requested pick position. However, if the dwell point is at a point within the rack a SC retrieval travel will consist of one travel between trip and one trip from the storage location to the requested pick position. Figure 4-2 illustrates all six possible cases for a SC storage, SC retrieval, and DC travel under a single picking floor configuration ((a) SC storage travel when the dwell point is within the rack, (b) SC retrieval travel when the dwell point is within the rack, and (c) DC travel when the dwell point is within the rack. (d) SC storage travel when the dwell point is at a pick position, (e) SC retrieval travel when the dwell point is at a pick position, and (f) DC travel when the dwell point is at a pick position.). Similarly, for a dual picking configuration travels are also based on the dwell point of the previous travel. For a dual picking configuration there are 24 possible travel cases as there are 2 input points and 2 levels of pick positions ($2 \times 2 \times 6$ cases). In the next section we will develop models that incorporate all of these cases as a way of calculating the expected travel time for SC and DC travels.

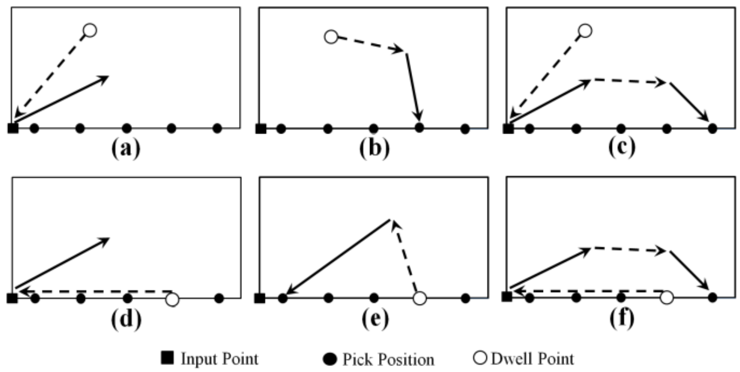


Figure 4-2 : All possible cases of travels under a single picking floor configuration.

During the peak hours the priority is to fulfill customer orders and the AS/RS prefers to perform only SC retrievals. We denote an AS/RS that performs only SC retrievals as a Consecutive Retrievals operating policy. To balance the system the AS/RS will perform only SC storages during non-peak hours (denoted as a Consecutive Storages operating policy). During the periods that the system is more balanced (i.e. the number of storage and retrieval requests, as well as their priority are fairly equal), we prefer to have the AS/RS perform DC travels to decrease empty travels. However, there may not be storage and retrieval requests available at the same time, which results in SC travels to be performed. Consequently, it is common for an AS/RS to perform a mixture of SC and DC travels (denoted as a Mixed operating policy). We consider both balanced and unbalanced periods in our analysis by developing travel time models for a Consecutive Retrievals, Consecutive Storages and different Mixed operating policies in Section 4-4. We also explore trade-offs associated with these policies for different design configurations in more detail in Section 4-5.

We assume that storage locations are continuously and uniformly distributed over the rack (Bozer and Cho, 2005; Bozer and White, 1984; Graves *et al.*, 1977; Hu *et al.*, 2005; Han *et* *al.*, 1987). We analyze MIAPP-AS/RSs where each aisle has a dedicated S/R machine (also known as aisle-captive S/R machines). Therefore, we can analyze each aisle independently. Furthermore, the S/R machine can move in the horizontal and vertical directions simultaneously. Hence, the Chebyshev metric is used to calculate the travel time between two points within the rack, i.e., the travel time between two points within the rack is equal to the maximum time required for horizontal and vertical travel. Based on the approach used in Bozer and White (1984) the rack can be normalized as follows.

Let:

L: length of rack,

H: height of the rack,

S_h: horizontal speed of the S/R machine,

S_v: vertical speed of the S/R machine,

t_h: time required to travel a distance of L horizontally, and

t_v: time required to travel a distance of H vertically.

By definition, $t_h = {}^{L}/_{S_h}$and $t_v = {}^{H}/_{S_v}$. Let $T = max\{t_h, t_v\}$ an $b = min\left\{{}^{t_h}/_{T}, {}^{t_v}/_{T}\right\}$, which refer to the scaling factor and the shape factor, respectively. By definition, $0 \leq b \leq 1$. Assuming that $t_h \geq t_v$, the rack can be normalized as a rectangle with length of 1.0 and height of b. The rack is called square-in-time if $b = 1$; otherwise, it is called rectangular-in-time.

The overall review of the assumptions of this paper is as follows.

1. The S/R machine carries unit-loads under either SC (storage/retrieval) or DC travels.

2. Demand and requested pick position locations for each item type are known and independent.
3. A randomized storage assignment is used, i.e., any point within the rack is equal likely to be selected for a storage.
4. Only the travel time model of the S/R machine is considered. Interactions with human order pickers and the buffers capacity, as well as the P/D times to handle the loads are ignored.
5. The S/R machine follows the Chebyshev metric and moves at a constant horizontal and vertical speed, i.e., acceleration and deceleration are ignored.
6. A stay dwell point strategy is used. In a stay strategy the S/R machine stays where it is after the completion of each storage or retrieval and waits for the next operation request.
7. For a single picking floor configuration the input point is located at the lower-left corner and the pick positions are located on the bottom line of a normalized rack. For a dual picking floor configuration the mezzanine is considered as a line located d time units above the bottom of the normalized rack. The mezzanine floor input point and pick positions are located on the mezzanine line directly above the input point and pick positions on the bottom line. The height of the pick positions and mezzanine structure are ignored.

4-4- Travel time models for MIAPP-AS/RS

First, we present the trips and subsequent notation required to develop our models. Next, travel time models are developed for a single pick position located in the aisle. These models are extended to travel time models with multiple pick positions in the aisle for a Consecutive Retrievals and a Mixed operating policy. Two alternative configurations, a single picking floor and a dual picking floor, are considered for each operating policy. Finally, we develop extreme travel time models that are based on an infinite number of pick positions.

Travels (SC or DC) consist of components that we denote as trips. Figure 4-3 illustrates trips used to develop the travel time models in this paper. All of these trips are considered under the normalized rack with the shape factor of b.

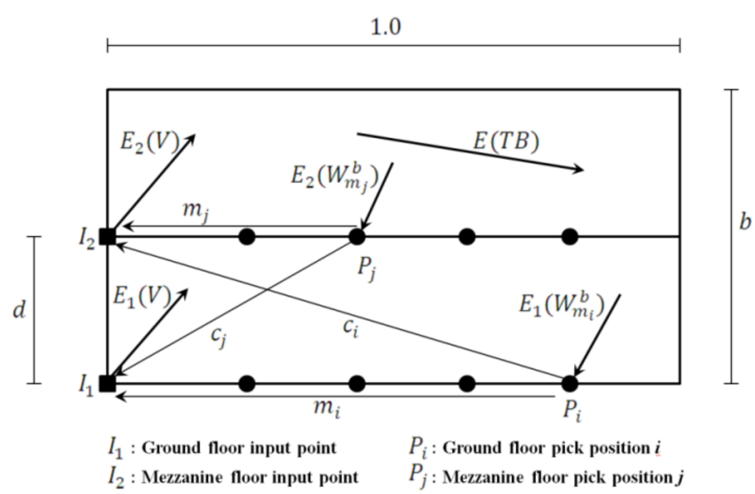


Figure 4-3 : Visual illustrations of the trips

Let:

m_i : Travel time between the ground floor pick position i and the ground floor input point,

m_j : Travel time between the mezzanine floor pick position j and the mezzanine floor input point,

c_i : Travel time between the ground floor pick position i and the mezzanine floor input point,

c_j : Travel time between the mezzanine floor pick position j and the ground floor input point,

$E(TB)$: Expected travel time between any two randomly selected points,

$E_1(V)$: Expected travel time between the ground floor input point and any randomly selected point,

$E_2(V)$: Expected travel time between the mezzanine floor input point and any randomly selected point,

$E_1(W_{m_i}^b)$: Expected travel time between the ground floor pick position i and any randomly selected point that is located vertically between the ground floor and a maximum distance b,

$E_2(W_{m_j}^b)$: Expected travel time between the mezzanine floor pick position j and any randomly selected point that is located vertically between the ground floor and a maximum distance b.

The trips $E_1(V)$ $E_2(V)$, , and $E(TB)$ are developed in the literature (Bozer and White, 1984) and are provided as equations (1), (2), and (3), respectively. The trips m_i, m_j, c_i, and c_j are defined directly for a given input point, output point, and rack dimension. In Section 4-4-1 we derive $E_1(W_{m_i}^b)$ and $E_2(W_{m_j}^b)$. In Section 4-4-2 we combine these trip components to develop travel time models for the above mentioned operating policies and configurations.

$$E_1(V) = {b^2}/{6} + {1}/{2} \tag{1}$$

$$E_2(V) = {b^2}/{6} + {1}/{2} - {1}/{2}\, d(b-d) \tag{2}$$

$$E(TB) = {1}/{3} + {b^2}/{6} - {b^3}/{30} \tag{3}$$

4-4-1- Travel time model for a single in-the-aisle pick position

In this section, first, the travel time model for a single in-the-aisle pick position located on the ground floor is developed. The result is then extended to determine the travel time model for a single in-the-aisle pick position located on the mezzanine floor.

4-4-1-1- on the ground floor

To develop a travel time model for a single in-the-aisle pick position, select a point with the distance of m from the left corner of the normalized rack as a pick position and set the point as the origin of the coordinate system, say $(0,0)$. As illustrated in Figure 4-4 the rack can be divided into two rectangular regions from the point of origin, namely, region I and region II. Any randomly selected point within the rack can be represented as the coordinate of (x_1, y_1) in time if the point belongs to region I, (x_2, y_2)or in time if the point belongs to region II, where $0 \leq x_1 \leq m$ (the positive value of x_1 is considered as x_1 represents time), $0 \leq x_2 \leq 1 - m$, $0 \leq y_1 \leq b$, and $0 \leq y_2 \leq b$. We define $t_{x_1,y_1}$and t_{x_2,y_2} as the travel time of the S/R machine from the origin to the point located within region I or region II, respectively. By definition of the Chebyshev metric, $t_{x_1,y_1} = max\{x_1, y_1\}$ and $t_{x_2,y_2} = max\{x_2, y_2\}$.

The expected travel time is calculated for each of the two regions separately. Then, the results obtained from the two regions are combined based on the probability of being in the two regions. The expected travel time from region I is as follows.

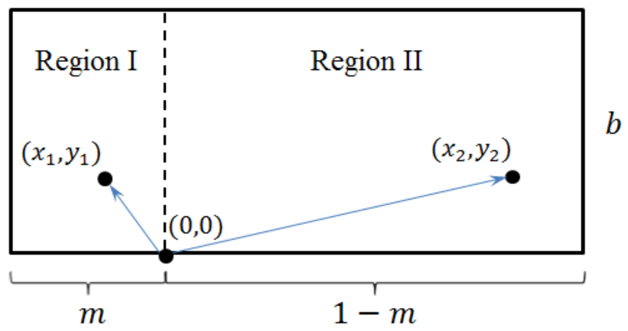


Figure 4-4 : Typical normalized rack in terms of time

Let $F(z_1)$ denote the probability that the travel time to (x_1, y_1) is less than or equal to z_1. The x_1 and y_1 coordinates are assumed to be independent. Therefore,

$$F(z_1) = Pr\left(t_{x_1,y_1} \leq z_1\right) = Pr(x_1 \leq z_1) \times Pr(y_1 \leq z_1)$$

Two cases can occur in region I, specifically, the m value is either greater or smaller than I. The value of z_1 is between 0 and b if $m < b$; otherwisez_1, is between 0 and m. The storage points are randomly and uniformly distributed, therefore, for the case of $m < b$, we have

$$Pr(x_1 \leq z_1) = \begin{cases} {}^{z_1}/_{m} & for \quad 0 \leq z_1 \leq m \\ 1 & for \quad m \leq z_1 \leq b \end{cases}$$

$$Pr(y_1 \leq z_1) = {}^{z_1}/_{b} \qquad for \quad 0 \leq z_1 \leq b$$

Hence,

$$F(z_1) = \begin{cases} {}^{z_1{}^2}/_{mb} & for \quad 0 \leq z_1 \leq m \\ \\ {}^{z_1}/_{b} & for \quad m \leq z_1 \leq b \end{cases}$$

By definition, the probability density function, $f(z_1)$, is

$$f(z_1) = \frac{dF(z_1)}{dz_1} = \begin{cases} {2z_1}/{mb} & for \quad 0 \le z_1 \le m \\ {1}/{b} & for \quad m \le z_1 \le b \end{cases}$$

Le$E_m^b(z_1)$t denote the expected travel time between the origin and any point located in region I. By definition of the expected value $E_m^b(z_1)$ for the case of $m < b$ is obtained as follows.

$$E_m^b(z_1) = \int_0^b z_1 . f(z_1) dz_1 = \int_0^m {2z_1}^2/{mb}\, dz_1 + \int_m^b {z_1}/{b}\, dz_1 = {m^2}/{6b} + {b}/{2} \quad for \quad m < b$$

The derivation of $E_m^b(z_1)$ for the case of $m \ge b$ is similar (see Appendix A). Therefore, the expected travel time between the origin and any point within region I can be calculated by equation (4).

$$E_m^b(z_1) = \begin{cases} {m^2}/{6b} + {b}/{2} & for \quad 0 \le m < b \\ {b^2}/{6m} + {m}/{2} & for \quad b \le m < 1 \end{cases} \tag{4}$$

The derivation for region II is similar to region I (see Appendix B for the derivation). Let $E_m^b(z_2)$ denote the expected travel time between the origin and any point located in region II. $E_m^b(z_2)$ is presented in equation (5).

$$E_m^b(z_2) = \begin{cases} {(1-m)^2}/{6b} + {b}/{2} & for \quad 0 \le (1-m) < b \\ {b^2}/{6(1-m)} + {(1-m)}/{2} & for \quad b \le (1-m) < 1 \end{cases} \tag{5}$$

As any point within the rack is equal likely to be selected, any point can be situated in region I or region II with the probability of m and $(1-m)$, respectively. The expected travel time between a single pick position on the ground floor and any point within the rack, $E_1(W_m^b)$, is obtained as follows.

$$E_1(W_m^b) = mE_m^b(z_1) + (1-m)E_m^b(z_2) \tag{6}$$

The expected travel time for each region, $E_m^b(z_1)$ and $E_m^b(z_2)$, can be calculated for a given value of m and b through equations (4) and (5), respectively. Finally, using equation (6), $E_1(W_m^b)$ can be calculated.

4-4-1-2- on the mezzanine floor

A mezzanine is located d time units above the ground floor (where $d < b$). As illustrated in Figure 4-3, the mezzanine divides the rack into two regions (the region above the mezzanine and below the mezzanine). Equation (6) is valid for a single pick position located on the ground floor. Expected travel time between a single pick position located on the mezzanine floor and any point above the mezzanine and below the mezzanine can be calculated by substituting d and $(b-d)$, respectively, for b in equation (6). Due to a randomized storage policy the probabilities that the point is above the mezzanine or below the mezzanine are $\frac{d}{b}$ and $\frac{(b-d)}{b}$, respectively. Therefore, the expected travel time between a single pick position on the mezzanine floor and any point within the rack, $E_2(W_m^b)$ can be calculated as the following.

$$E_2(W_m^b) = {}^{d}/_{b}\, E_1(W_m^d) + {}^{(b-d)}/_{b}\, E_1(W_m^{b-d}) \tag{7}$$

4-4-2- Travel time model for multiple in-the-aisle pick positions

In this section we extend the results of the previous section to include multiple in-the-aisle pick positions. Let n denote the number of the pick positions in each floor, i denote the corresponding index of each ground pick position, and j denote the corresponding index of each mezzanine pick position, where $1 \leq i \leq n$ and $n + 1 \leq j \leq 2n$.

4-4-2-1- Consecutive Retrievals operating policy

The Consecutive Retrievals operating policy is primarily used during peak hours to maximize the number of retrievals. Thus, it is assumed that only SC retrieval travels are performed consecutively. As a result all of the trips initiate or terminate at in-the-aisle pick positions (see Figure 4-5 and Figure 4-6).

<u>4-4-2-1-1- Travel time model for a single picking floor configuration</u>

Let Pr_i be the percentage of items requested at pick position i (where $\sum_{i=1}^{n} Pr_i = 1$). Therefore, the ratio of operations terminating to pick position i is equal to Pr_i. The travel time between pick position i to any point within the rack, $E_1(W_{m_i}^b)$, can be calculated by equation (6). As the demand of each pick position is independent the expected travel time of a one-way trip (see solid lines in Figure 5-5) from any point within the rack to any pick position, $E_1(W)$, is equal to:

$$E_1(W) = \sum_{i=1}^{n} Pr_i \,.\, E_1(W_{m_i}^b) \tag{8}$$

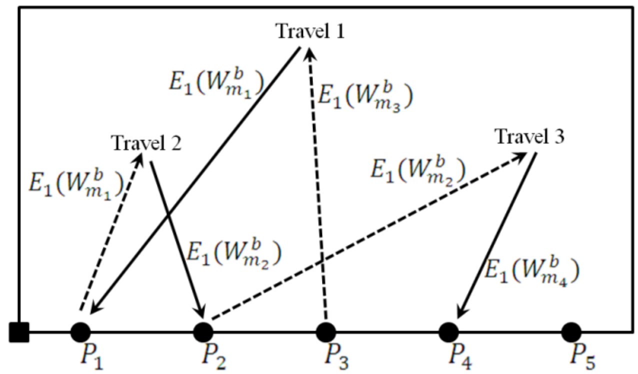


Figure 4-5 : Example of a Consecutive Retrieval operating policy for a single picking floor configuration with three SC retrieval travels

The probability for a trip to originate from a pick position is equal to the probability that a trip terminates to that pick position. This occurs because each trip will originate from the pick position that the previous trip terminated. Therefore, the expected travel time for a trip (see dash lines in Figure 4-5) from any pick position to any point within the rack is also equal to $E_1(W)$. As a result the expected travel time for a Consecutive Retrieval operating policy is equal to $2 \times E_1(W)$.

4-4-2-1-2- Travel time model for a dual picking floor configuration

In this section we consider an alternative design where there are two picking floors. One picking floor is on the ground level and the second picking floor is on the mezzanine level. There are n pick positions on each floor, which results in 2n total pick positions. Note that for a dual picking floor configuration $\sum_{i=1}^{n} Pr_i + \sum_{j=n+1}^{2n} Pr_j = 1$.

As illustrated in Figure 4-6, SC retrievals are performed to pick positions located on both picking floors. The expected travel time of trips from any point within the rack to a pick position located on the ground floor and mezzanine floor, $E_1\left(W_{m_i}^b\right)$ and $E_2\left(W_{m_j}^b\right)$, can be calculated by equation (6) and (7), respectively. Similar to the previous section the expected travel time from any point within the rack to any pick position with a dual picking floor, $E_2(W)$, can be calculated as:

$$E_2(W) = \sum_{i=1}^{n} Pr_i \,.\, E_1\left(W_{m_i}^b\right) + \sum_{j=n+1}^{2n} Pr_j \,.\, E_2\left(W_{m_j}^b\right) \tag{9}$$

The expected travel time for a Consecutive Retrieval operating policy with a dual picking floor is equal to $2 \times E_2(W)$ because the probability of terminating and originating at one pick position is the same.

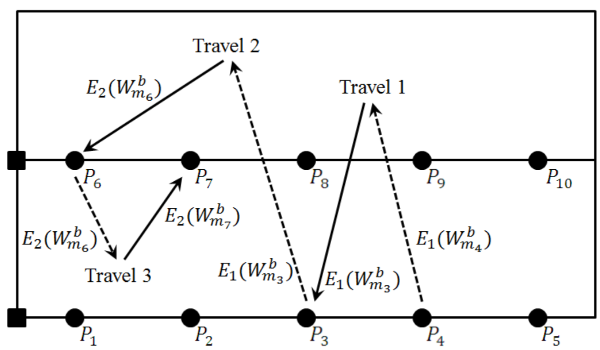


Figure 4-6 : Example of a Consecutive Retrieval operating policy for a dual picking floor configuration

4-4-2-2- Mixed operating policy

For a Mixed operating policy SC storage, SC retrieval, and DC travels can occur during the same period of time. Let N be the total number of operations and α be the percent of storages performed using SC travels. For system stability we assume half of the operations are storages and the other half of the operations are retrievals. Therefore, the number of SC storage travels is equal to $\alpha \frac{N}{2}$, which is also the number of SC retrieval travels. For each DC travel, two operations (one storage and one retrieval) are performed, and $(1-\alpha)\frac{N}{2}$ is the number of DC travels. The total number of travels performed is equal to $\left(\frac{1+\alpha}{2}\right)N$.

Based on the provided number of travels, $\frac{\alpha}{1+\alpha}$ is the probability that a SC storage (or SC retrieval) travel occurs, and $\frac{1-\alpha}{1+\alpha}$ is the probability that a DC travel occurs. In this section, first, we develop the expected travel time 'per travel' for a Mixed operation policy. In many cases, the expected travel time 'per operation' is required to be calculated and can be obtained by multiplying the expected travel time 'per travel' by $\left(\frac{1+\alpha}{2}\right)$.

To illustrate the difference between 'per operation' and 'per travel,' we present the following example. Suppose there are 100 operations, 50 operations are storages and 50 operations are retrievals. If $\alpha = 0.4$, 20 storages and 20 retrievals will be performed under SC travels and 30 storages and 30 retrievals are performed together using DC travels (for a total of 30 DC travels). Consequently, a total of 70 travels (SC or DC) will be performed to accomplish 100 operations. Therefore, $\frac{\alpha}{1+\alpha} = \frac{0.4}{1+0.4} = \frac{20}{70}$, which is the ratio of travels operated using SC storage (or SC retrieval) and $\frac{1-\alpha}{1+\alpha} = \frac{1-0.4}{1+0.4} = \frac{30}{70}$, which is equal to the ratio of travels operated using DC. Additionally, the ratios of operations performed under SC storage (or SC retrieval) and DC are $\frac{\alpha}{2} = \frac{0.4}{2} = \frac{20}{100}$ and $\frac{1-\alpha}{2} = \frac{1-0.4}{2} = \frac{30}{100}$, respectively.

Using the trips we introduced at the beginning of Section 4-4, as well as the developed models from Section 4-4-1 and 4-4-2-1, we develop the Mixed policy travel time models for both single and dual picking floor configurations.

4-4-2-2-1- for a single picking floor configuration

For a single picking floor configuration a travel will originate from the point within the rack if the previous travel was a SC storage. Otherwise, if the previous travel was either a SC retrieval or DC the next travel will originate from one of the in-the-aisle pick positions. Therefore, the probability that a travel starts at a point within the rack is $\frac{\alpha}{1+\alpha}$ and the probability that a travel starts at the pick position i is $Pr_i(\frac{1}{1+\alpha})$.

If a travel starts at some point within the rack the expected travel time to perform a SC storage is equal to $2E_1(V)$, a SC retrieval is equal to $E(TB) + E_1(W)$, and a DC is equal to $2E_1(V) + E(TB) + E_1(W)$. If a travel starts at pick position i the expected travel time to perform a SC storage is equal to m_i+$E_1(V)$, a SC retrieval is equal to $2E_1(W^b_{m_i})$, and a DC is equal to m_i+$E_1(V) + E(TB) + E_1(W)$. Let $E_1'(M)$ denote the expected travel time 'per travel' for a Mixed operating policy with a single picking floor configuration. According to the provided probability of each travel during a Mixed policy, $E_1'(M)$ can be calculated as follows (the first term represents the travel that starts from any point within the rack and the second term the travel that starts from the pick position i).

$$E_1'(M) = \left(\frac{\alpha}{1+\alpha}\right) \times \begin{Bmatrix} \frac{\alpha}{1+\alpha}\big(2E_1(V)\big) + \\ \frac{\alpha}{1+\alpha}\big(E(TB) + E_1(W)\big) + \\ \frac{1-\alpha}{1+\alpha}\big(2E_1(V) + E(TB) + E_1(W)\big) \end{Bmatrix}$$

$$+ \sum_{i=1}^{n} Pr_i\left(\frac{1}{1+\alpha}\right) \times \begin{Bmatrix} \frac{\alpha}{1+\alpha}\big(m_i + E_1(V)\big) + \\ \frac{\alpha}{1+\alpha}\big(2E_1(W_{m_i}^b)\big) + \\ \frac{1-\alpha}{1+\alpha}\big(m_i + E_1(V) + E(TB) + E_1(W)\big) \end{Bmatrix}$$

Let $E_1(R) = \sum_{i=1}^{n} Pr_i \,.\, m_i$ denote the expected return time from a ground floor pick position to a ground floor input point. $E_1'(M)$ can be simplified as follows.

$$E_1'(M) = \left(\frac{\alpha}{1+\alpha}\right)\left\{\left(\frac{2}{1+\alpha}\right)E_1(V) + \left(\frac{1}{1+\alpha}\right)E(TB) + \left(\frac{1}{1+\alpha}\right)E_1(W)\right\} + \left(\frac{1}{1+\alpha}\right)\left\{\left(\frac{1}{1+\alpha}\right)E_1(V) + \left(\frac{1-\alpha}{1+\alpha}\right)E(TB) + E_1(W) + \left(\frac{1}{1+\alpha}\right)E_1(R)\right\} \quad (10a)$$

$E_1(M)$ denotes the expected travel time 'per operation' for a Mixed operating policy with a single picking floor configuration and is calculated by multiplying equation (10a) by $\left(\frac{1+\alpha}{2}\right)$. We present the simplified version in equation (10b).

$$E_1(M) = \left(\frac{1+2\alpha}{2+2\alpha}\right)E_1(V) + \left(\frac{1}{2+2\alpha}\right)E(TB) + \left(\frac{1+2\alpha}{2+2\alpha}\right)E_1(W) + \left(\frac{1}{2+2\alpha}\right)E_1(R) \quad (10b)$$

4-4-2-2-2- for a dual picking floor configuration

Similar to the previous section the probability that a travel starts at a point within the rack is $\frac{\alpha}{1+\alpha}$, at the pick position i (on the ground floor) is $Pr_i(\frac{1}{1+\alpha})$, and at the pick position j (on the mezzanine floor) is $Pr_j(\frac{1}{1+\alpha})$. For a dual picking floor configuration we assume that β percent of storage operations are performed from the ground floor input point and $(1-\beta)$ percent from the mezzanine floor input point. Therefore, $\beta = 1$ represents a system where a single input point is located on the ground floor and $\beta = 0$ represents a system where a single input point is located on the mezzanine floor. To perform a storage, if the S/R machine is at some point within the rack, β percent of time the crane returns to the ground input point and $(1-\beta)$ percent of the time the S/R machine returns to the mezzanine input point. Similarly, if the S/R machine is at pick position i (on ground floor), β percent of time the S/R machine returns m_i time units to the ground input point and $(1-\beta)$ percent of the time the S/R machine returns c_i time units to the mezzanine input point. Finally, if the S/R machine is at pick position j (on the mezzanine floor), β percent of time the crane returns c_j time units to the ground input point and $(1-\beta)$ percent of the time the crane returns m_j time units to the mezzanine input point to perform a storage. By definition of the Chebyshev metric c_i and c_j are calculated as:

$$c_i = \begin{cases} d & for \quad m_i \le d \\ m_i & for \quad m_i > d \end{cases} \tag{11}$$

$$c_j = \begin{cases} d & for \quad m_j \le d \\ m_j & for \quad m_j > d \end{cases} \tag{12}$$

Let $E_2'(M)$ denote the expected travel time 'per travel' for a Mixed operating policy with a dual picking floor configuration. Similar to Section 4-4-2-2-1, $E_2'(M)$ is obtained as follows (the first term represents when the travel starts from any point within the rack, the second term when the travel starts from the pick position i, and the third term when the travel starts from the pick position j).

$E_2'(M) =$

$$\left(\frac{\alpha}{1+\alpha}\right) \times \left\{ \begin{array}{c} \frac{\alpha}{1+\alpha}\left[\beta\left(2E_1(V)\right) + (1-\beta)\left(2E_2(V)\right)\right] + \\ \frac{\alpha}{1+\alpha}\left(E(TB) + E_2(W)\right) + \\ \frac{1-\alpha}{1+\alpha}\left[\beta\left(2E_1(V) + E(TB) + E_2(W)\right) + (1-\beta)\left(2E_2(V) + E(TB) + E_2(W)\right)\right] \end{array} \right\} +$$

$$\sum_{i=1}^{n} Pr_i \left(\frac{1}{1+\alpha}\right) \times$$

$$\left\{ \begin{array}{c} \frac{\alpha}{1+\alpha}\left[\beta\left(m_i + E_1(V)\right) + (1-\beta)\left(c_i + E_2(V)\right)\right] + \\ \frac{\alpha}{1+\alpha}\left(2E_1\left(W_{m_i}^{b}\right)\right) + \\ \frac{1-\alpha}{1+\alpha}\left[\beta\left(m_i + E_1(V) + E(TB) + E_2(W)\right) + (1-\beta)\left(c_i + E_2(V) + E(TB) + E_2(W)\right)\right] \end{array} \right\} +$$

$$\sum_{j=n+1}^{2n} Pr_j \left(\frac{1}{1+\alpha}\right) \times$$

$$\left\{ \begin{array}{c} \frac{\alpha}{1+\alpha}\left[\beta\left(c_j + E_1(V)\right) + (1-\beta)\left(m_j + E_2(V)\right)\right] + \\ \frac{\alpha}{1+\alpha}\left(2E_2\left(W_{m_j}^{b}\right)\right) + \\ \frac{1-\alpha}{1+\alpha}\left[\beta\left(c_j + E_1(V) + E(TB) + E_2(W)\right) + (1-\beta)\left(m_j + E_2(V) + E(TB) + E_2(W)\right)\right] \end{array} \right\}$$

We define$E_2(R) = \sum_{j=n+1}^{2n} Pr_i \,.\, m_i$, $E_1(CR) = \sum_{i=1}^{n} Pr_i \,.\, c_i$, and $E_2(CR) = \sum_{j=n+1}^{2n} Pr_j \,.\, c_j$ as well as $E_1(R)$, which was defined in pervious section, to simplify the result. Hence,

$$E_2'(M) = \left(\frac{\alpha}{1+\alpha}\right)\left\{\left(\frac{2}{1+\alpha}\right)[\beta E_1(V) + (1-\beta)E_2(V)] + \left(\frac{1}{1+\alpha}\right)E(TB) + \left(\frac{1}{1+\alpha}\right)E_2(W)\right\} + \left(\frac{1}{1+\alpha}\right)\left\{\left(\frac{1}{1+\alpha}\right)[\beta E_1(V) + (1-\beta)E_2(V)] + \left(\frac{1-\alpha}{1+\alpha}\right)E(TB) + E_2(W) + \left(\frac{1}{1+\alpha}\right)\left[\beta\left(E_1(R) + E_2(CR)\right) + (1-\beta)\left(E_2(R) + E_1(CR)\right)\right]\right\} \quad (13a)$$

In equation (13b) $E_2(M)$denotes the expected travel time 'per operation' for a Mixed operating policy with a dual picking floor configuration, which is obtained by multiplying equation (13a) by $\left(\frac{1+\alpha}{2}\right)$.

$$E_2(M) = \left(\frac{1+2\alpha}{2+2\alpha}\right)[\beta E_1(V) + (1-\beta)E_2(V)] + \left(\frac{1}{2+2\alpha}\right)E(TB) + \left(\frac{1+2\alpha}{2+2\alpha}\right)E_2(W) + \left(\frac{1}{2+2\alpha}\right)\left[\beta\left(E_1(R) + E_2(CR)\right) + (1-\beta)\left(E_2(R) + E_1(CR)\right)\right] \quad (13b)$$

4-4-3- Travel time model for the case of an infinite number of pick positions

Components of the models derived in Section 4-4-2 require calculation for every pick position. This can be a tedious task as MIAPP-AS/RSs are typically designed with many pick positions per aisle. In this section we develop closed-form travel time models that do not require calculations for each pick position by assuming that an infinite number of pick positions exist in the aisle. We denote these travel time models as extreme models. In Section 4-5 we will show that the travel time models of Section 4-4-2 (hereafter, called base models) can be replaced by their extreme models, as their travel time estimations deviate by only small amounts.

Of the components used in the base models, $E_1(V)$, $E_2(V)$, and $E(TB)$ are independent of the number and location of the pick positions; whereas$E_1(W)$, $E_2(W)E_1(R)$, , $E_2(R)E_1(CR)$, , and $E_2(CR)$, as well as the demand of each pick position are dependent on the value of n. By deriving the extreme value of these dependent components, all base models in Section 4-4-2 can be calculated in the case of an infinite number of pick positions. We assume that demand is equally likely for each pick position (i.e. for the single picking floor configuration, $Pr_i = {}^{1}/_{n}$ and for the dual picking floor configuration, $Pr_i = Pr_j = {}^{1}/_{2n}$). Extreme values of $E_1(W), E_2(W)$, $E_1(R), E_2(R)$, $E_1(CR)$, $E_2(CR)$, $E_1(M)$, and $E_2(M)$ are denoted as $E_1^{\infty}(W)$, $E_2^{\infty}(W)$, $E_1^{\infty}(R)$, $E_2^{\infty}(R)$, $E_1^{\infty}(CR)$, $E_2^{\infty}(CR)$, $E_1^{\infty}(M)$, and $E_2^{\infty}(M)$, respectively.

4-4-3-1- Calculation of the extreme value of the Consecutive Retrievals policy ($\boldsymbol{E_1^{\infty}(W)}$ and $\boldsymbol{E_2^{\infty}(W)}$)

To find the extreme value of $E_1(W)$, let $Pr_i = {}^{1}/_{n}$ in equation (8); therefore, the extreme value of $E_1(W)$ is equal to

$$E_1^{\infty}(W) = lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} E_1\left(W_{m_i}^{b}\right) = \int_0^1 E_1(W_m^b)\, dm \qquad (14a)$$

The derivation of $E_1(W_m^b)$ requires using equations (4) and (5). The right hand side of equation (4) depends on the value of m and b. Equation (5) depends on the value of m and $(1-b)$. Therefore, the integral in (14a) separates into three integrals. See Appendix C for the complete derivation of $E_1^{\infty}(W)$. We present the result in equation (14b).

$$E_1^{\infty}(W) = {}^{1}/_{3} + {}^{b^2}/_{3} - {}^{b^3}/_{12} \qquad (14b)$$

The extreme value of $E_2(W)$ is associated with a dual picking floor configuration. Therefore, let $Pr_i = Pr_j = {}^{1}/_{2n}$ in equation (9), and we have

$E_2^{\infty}(W) = lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2n} E_1\left(W_{m_i}^{b}\right) + \sum_{j=n+1}^{2n} \frac{1}{2n}\left(\frac{d}{b} E_1\left(W_{m_j}^{d}\right) + \frac{(b-d)}{b} E_1\left(W_{m_j}^{b-d}\right)\right) =$

$\frac{1}{2}\left[\int_0^1 E_1(W_m^b)\, dm + \left(\frac{d}{b}\int_0^1 E_1(W_m^d)\, dm + \frac{(b-d)}{b}\int_0^1 E_1(W_m^{b-d})\, dm\right)\right]$ (15a)

Equation (14b) is valid for $0 \le b \le 1$; thus, it can be extended to calculate $\int_0^1 E_1(W_m^d)\, dm$ and $\int_0^1 E_1(W_m^{b-d})\, dm$ in equation (15a) by substituting d and $(b-d)$ for b in equation (14b), respectively. Therefore, $E_2^{\infty}(W)$ is derived as

$E_2^{\infty}(W) = \frac{1}{3} + \frac{(b-d)^3 + b^3 + d^3}{6b} - \frac{(b-d)^4 + b^4 + d^4}{24b}$ (15b)

4-4-3-2- Calculation of $\boldsymbol{E_1^{\infty}(R)}$, $\boldsymbol{E_2^{\infty}(R)}$, $\boldsymbol{E_1^{\infty}(CR)}$, and $\boldsymbol{E_2^{\infty}(CR)}$

First, we calculate $E_1^{\infty}(R)$ for a single picking floor configuration. As we assume that $Pr_i = 1/n$ for the single picking floor configuration, the extreme value of $E_1(R)$ is equal to$lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} m_i = \int_0^1 m\, dm = 1/2$. However, for the dual picking floor configuration, $Pr_i = Pr_j = 1/2n$. Therefore, $lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2n} m_i = \frac{1}{2}\int_0^1 m\, dm = 1/4$.

$E_1^{\infty}(R) = \begin{cases} 1/2 & for \quad single\ picking\ floor \\ 1/4 & for \quad dual\ picking\ floor \end{cases}$ (16)

$E_2(R)$, $E_1(CR)$, and $E_2(CR)$ are required only for a dual picking floor configuration. Therefore, to calculate the extreme values, we let $Pr_i = Pr_j = 1/2n$.

$E_2^{\infty}(R) = lim_{n\to\infty} \sum_{j=n+1}^{2n} \frac{1}{2n} m_j = \frac{1}{2}\int_0^1 m\, dm = 1/4$ (17)

$E_1^{\infty}(CR) = lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2n} c_i = \frac{1}{2}\int_0^d d\, dm + \frac{1}{2}\int_d^1 m\, dm = 1/4 + d^2/4$ (18)

$$E_2^{\infty}(CR) = lim_{n\to\infty} \sum_{j=n+1}^{2n} \frac{1}{2n} c_j = \frac{1}{2}\int_0^d d\ dm + \frac{1}{2}\int_d^1 m\ dm = {}^1\!/_4 + {}^{d^2}\!/_4 \tag{19}$$

4-4-3-3- Calculation of extreme value of the Mixed policy($\boldsymbol{E_1^{\infty}(M)}$ and $\boldsymbol{E_2^{\infty}(M)}$)

By substituting the extreme value of the components of $E_1(M)$ into equation (10b) and using equation (14b) and (16), th$E_1^{\infty}(M)$e in terms of 'per operation' is obtained as:

$$E_1^{\infty}(M) = \left(\frac{1+2\alpha}{2+2\alpha}\right) E_1(V) + \left(\frac{1}{2+2\alpha}\right) E(TB) + \left(\frac{1+2\alpha}{2+2\alpha}\right) E_1^{\infty}(W) + \left(\frac{1}{4+4\alpha}\right) \tag{20}$$

Similarly, to calculate $E_2^{\infty}(M)$ all of the components of $E_2(M)$ in equation (13b) can be substituted by their corresponding extreme values using equations (14b), (15b), (16), (17), (18), and (19). We obtain $E_2^{\infty}(M)$ in the term of 'per operation' as

$$\mathrm{E}_2^{\infty}(\mathrm{M}) = \left(\frac{1+2\alpha}{2+2\alpha}\right)[\beta \mathrm{E}_1(\mathrm{V}) + (1-\beta)\mathrm{E}_2(\mathrm{V})] + \left(\frac{1}{2+2\alpha}\right)\mathrm{E(TB)} + \left(\frac{1+2\alpha}{2+2\alpha}\right)\mathrm{E}_2^{\infty}(\mathrm{W}) + \left(\frac{1}{2+2\alpha}\right)\left({}^1\!/_2 + {}^{\mathrm{d}^2}\!/_4\right) \tag{21}$$

4-4-3-4- Optimal Shape Factors

We can use the extreme models to determine the optimal value of the shape factor,b, that minimizes the expected travel time for different configurations and operating policies. Note that all base and extreme travel time models are for the normalized rack. Denormalized travel times can be obtained by multiplying the normalized travel times by the scaling factor, T. To make equal comparisons, we assume a constant storage area, which we denote as A (in squared time units). $A = bT^2$; therefore, T is obtained as $\sqrt{A/b}$. We find the optimal value of a Consecutive

Retrievals policy with a single picking floor by first denormalizing the time units by multiplying equation (14b) by $\sqrt{A/b}$, then taking the derivative with respect to b, setting the resulting equal to zero, and solving for b. The optimal b value for $E_1^{\infty}(W)$ is derived as 0.6825. Similarly, for the Consecutive Retrievals policy with a dual picking floor, the optimal b value is equal to 0.8965 for $E_2^{\infty}(W)$ when $d = {}^{b}/_{2}$ and $\beta = 0.5$. For either configuration using the Mixed policy ($E_1^{\infty}(M)$ and $E_2^{\infty}(M)$), the optimal b value is 1.0.

4-5- Model validation and numerical discussion

In this section, first, the travel time models developed in Section 4-4 are evaluated for their accuracy. Next, insights and trade-offs between different operating policies under the single and dual picking floor configurations are provided.

4-5-1- Validation of the base and extreme models

In this section the results of the base models presented in Section 4-4-2, as well as their extreme models presented in Section 4-4-3 are compared to results obtained from a discrete-event simulation that enforces a discrete rack. We present the results in this section as denormalized expected travel times 'per operation'. We coded and ran our models and simulation in MATLAB 2010.

An opening in a rack can be either a pick position or a storage location. The number of pick positions is assumed to be equal to the number of columns for a single picking floor configuration and twice the number of columns for a dual picking floor configuration. Openings are assumed to be 4 feet by 4 feet in width and height. We calculate the travel distance between openings using a centroid method. The horizontal and vertical speeds of the S/R machine are equal to 400 and 160 feet per minute, respectively. We consider six different ratios of the number of columns to the number of levels. The ratios are selected such that the number of storage locations remains approximately constant. We ran 5 replications of a sequence of 100,000 operations to simulate each ratio configuration. We test our models assuming that item demand is equally likely for each pick position. To compare the results of our models and those obtained from the simulation, the '% deviation' is used and shown in equation (22).

$$\%Deviation = \frac{base/extreme\ model\ result - simulation\ result}{simulation\ result} \times 100 \qquad (22)$$

We compare the expected travel time from our base and extreme models from Section 4-4 to the average travel time from the simulation. We report our results for a single and dual picking floor configuration in Table 4-1 and Table 4-2, respectively. For the simulation results we report the average and variance of the travel time per operation for each ratio configuration. In Table 4-1, a rack with a single picking floor and approximately 950 storage openings is considered. The results of our base models, as well as their extreme values are compared to the simulation results for both a Consecutive Retrievals and a Mixed policy. For a Consecutive Retrievals policy, the denormalized base and extreme model results are calculated as $2T \times E_1(W)$ and $2T \times E_1^{\infty}(W)$ using equations (8) and (14b), respectively. For a Mixed policy, the denormalized base and extreme model results are equal to $T \times E_1(M)$ and $T \times E_1^{\infty}(M)$ using equations (10b) and (20), respectively. Table 4-2 presents results for a dual picking floor with approximately 900 storage openings. The denormalized base/extreme results for a Consecutive Retrievals policy are calculated as $2T \times E_2(W)$ and $2T \times E_2^{\infty}(W)$ using equations (9) and (15b), respectively. For a Mixed policy, they are calculated as $T \times E_2(M)$ and $T \times E_2^{\infty}(M)$ using equations (13b) and (21), respectively.

Table 4-1 : The denormalized results of the discrete rack simulation versus the base and extreme travel time models for a single picking floor configuration

Rows	20	18	16	14	12	10
Columns	50	56	63	73	86	105
No. of storage opening	950	952	945	949	946	945
T (minutes)	0.5	0.56	0.63	0.73	0.86	1.05
b	1.000	0.804	0.635	0.479	0.349	0.238
Consecutive Retrieval operating policy						
Simulation average travel time	0.578	0.562	0.559	0.582	0.635	0.735
Simulation variance of travel time	0.048	0.038	0.037	0.047	0.071	0.117
Base model travel time	0.583	0.566	0.562	0.585	0.637	0.737
(% deviation)	0.852	0.756	0.691	0.583	0.356	0.279
Extreme model travel time	0.583	0.566	0.562	0.585	0.637	0.737
(% deviation)	0.855	0.759	0.694	0.586	0.358	0.281
Mixed operating policy ($\alpha = 0.4$)						
Simulation average travel time	0.569	0.581	0.607	0.664	0.752	0.896
Simulation variance of travel time	0.223	0.235	0.263	0.327	0.436	0.638
Base model travel time	0.574	0.585	0.611	0.667	0.754	0.897
(% deviation)	0.938	0.728	0.657	0.430	0.280	0.106
Extreme model travel time	0.574	0.585	0.611	0.667	0.754	0.897
(% deviation)	0.939	0.729	0.658	0.430	0.281	0.106

From Table 4-1, the maximum '% deviation' of the simulation results compared to the base model for a Consecutive Retrievals and Mixed policy is equal to 0.852% and 0.938%, respectively. These results indicate our base models provide an extremely accurate representation of a discrete rack. All deviations are positive in value showing that our models overestimate the real discrete rack (albeit the deviation is less than 1%). The reason is that our continuous models ignore the discreteness of the pallet locations in the rack. The maximum '% deviation' between the simulation results and the extreme model results for a Consecutive Retrievals and a Mixed policy are 0.855% and 0.939% respectively.

Table 4-2 : The denormalized results of the discrete rack simulation versus the base and extreme travel time models for a dual picking floor configuration when $d = {}^{b}\!/_{2}$ and $\beta = 0.5$.

Rows	20	18	16	14	12	10
Columns	50	56	64	75	90	112
No. of storage opening	900	896	896	900	900	896
T (minutes)	0.5	0.56	0.64	0.75	0.9	1.12
b	1.000	0.804	0.625	0.467	0.333	0.223
Consecutive Retrieval operating policy						
Simulation average travel time	0.491	0.493	0.513	0.558	0.637	0.767
Simulation variance of travel time	0.036	0.035	0.041	0.056	0.086	0.142
Base model travel time	0.495	0.497	0.516	0.561	0.639	0.769
(% deviation)	0.712	0.707	0.621	0.531	0.272	0.248
Extreme model travel time	0.495	0.497	0.516	0.561	0.639	0.769
(% deviation)	0.716	0.712	0.625	0.534	0.274	0.249
Mixed operating policy ($\alpha = 0.4$)						
Simulation average travel time	0.534	0.554	0.595	0.666	0.776	0.947
Simulation variance of travel time	0.201	0.220	0.261	0.339	0.475	0.724
Base model travel time	0.537	0.557	0.597	0.668	0.777	0.949
(% deviation)	0.592	0.442	0.431	0.282	0.135	0.140
Extreme model travel time	0.537	0.557	0.597	0.668	0.777	0.949
(% deviation)	0.593	0.444	0.432	0.283	0.136	0.141

It is worth mentioning that this small deviation is obtained for the MIAPP-AS/RS with more than 50 pick positions. However, our testing indicates that our extreme models provide errors that are low even when the number of pick positions is less than 50. Because the number of aisles determines the number of cranes required, most MIAPP-AS/RS are designed in practice with more than 50 pick positions per aisle (Swisslog Co., 2013) and the extreme models can be used as a substitute for the base models to estimate the travel time. Similarly, from Table 4-2, we can also see that our models provide an extremely accurate representation of a discrete rack for the dual picking floor configuration. Due to differences in storage locations we observe variance in the individual per operation travel times. However, we also observe that for each ratio configuration the variance of the 5 replications' travel time is extremely small (i.e., the maximum variance was 0.000003).

We use the extreme models to provide further design insights of MIAPP-AS/RS in the following section.

4-5-2- Analysis of operating policies

In this section, we introduce three new policies that utilize the travel time models developed in Section 4-4. The first two policies are extreme cases of the Mixed policy and the last policy is a combination of the Consecutive Retrievals and the Consecutive Storages policies. We note that the expected travel times for a mixed policy with other values of α will fall between the DC and Random Sequence of SC Storages and Retrievals policies.

1. DC Policy, which is a policy where all travels are DC. The expected travel time for this policy is calculated using the model for the Mixed policy with $\alpha = 0$.
2. Random Sequence of SC Storages and Retrievals Policy, which is a policy where all travels are SC and the same number of SC storages and SC retrievals are sequenced randomly (e.g. SRRSSRSRRS). The expected travel time for this policy is calculated using the Mixed policy model with $\alpha = 1$.
3. Consecutive Retrievals then Storages Policy, which is a policy where all travels are SC; however, the SC retrievals are all completed consecutively followed by all of the SC storages (e.g. RRRRRSSSSS). The expected travel time for this policy is calculated as the average of the Consecutive Retrievals policy followed by the Consecutive Storages policy.

In this section we provide denormalized expected travel times for these different policies under a single and dual picking floor configuration. We analyze performance using metrics based on travel times 'per retrieval' and 'per operation'. Denormalized expected travel times 'per operation' for the Consecutive Retrievals then Storages policy under a single and dual picking floor are calculated as $T \times [E_1^{\infty}(W) + E_1(V)]$ and $T \times \left[E_2^{\infty}(W) + \left(\beta E_1(V) + (1-\beta)E_2(V)\right)\right]$, respectively, using equations (1), (2), (14b) and (15b). Also, denormalized expected travel times 'per operation' for the Mixed policies under the single and dual picking floor configuration are equal to $T \times E_1^{\infty}(M)$ and $T \times E_2^{\infty}(M)$, respectively, using equations (20) and (21). Denormalized expected travel times 'per retrieval' for a Consecutive Retrievals policy under the single and dual picking floor configuration are equal to $2T \times E_1^{\infty}(W)$ and $2T \times E_2^{\infty}(W)$ using equations (14b) and (15b), respectively. As only half of the operations performed using a Mixed policy are retrievals, denormalized expected travel times 'per retrieval' for these policies are calculated as $2T \times E_1^{\infty}(M)$ and $2T \times E_2^{\infty}(M)$ under the single and dual picking floor using equations (20) and (21), respectively. The numerical results in this section are created using the same ratio configurations as Table 4-1 and 4-2 in Section 4-5-1. We use our models to provide insights into the design of an MIAPP-AS/RS from four different perspectives.

First, we explore the impact of design criteria associated with the dual picking floor configuration on the expected travel times. Specifically, we evaluate the percent of storage operations being performed from the ground input floor versus the mezzanine input floor and the location of the mezzanine floor. As retrievals are independent of the input point, the β value does not influence the Consecutive Retrievals policy. For the Consecutive Retrievals then Storages policy and the mixed policies increasing β will increase the expected travel time because $E_1(V) \geq E_2(V)$ for any $d \leq b$. Therefore, to minimize the expected travel times for any policy set $\beta = 0$, which represents a design where all storage operations originate from the mezzanine floor input point. Such a design may require additional material handling equipment to transfer the incoming unit loads to the elevated input point; therefore, in subsequent analysis we explore the cases when β is equal to 0.0, 0.5, or 1.0.

To analyze the location of the mezzanine floor, we find the optimal d value that minimizes the expected travel time for any given policy numerically. We coded the numerical search using MATLAB 2010's *fminbnd* function and our results are valid for the performance metrics based on 'per retrieval' and 'per operation'. In Table 4-3 we report the expected value of the best d for the different policies and β values found over all b values given in Table 4-2. The maximum variance for a given policy and β value is 0.0001; therefore, the values for the mezzanine level given in Table 4-3 are good approximations regardless of the value of b. From Table 4-3, we observe that the mezzanine should be located such that $0 \leq d \leq 0.5b$. The reasoning for this result is that three travels influence the value of d: (1) the travel between the mezzanine floor input point and a storage location in the rack, (2) the travel between a storage location and a pick position on the mezzanine floor, and (3) the travel from a pick position on the ground floor back to the mezzanine input point to begin a storage (or from a pick position on the mezzanine level to the ground floor input point). The expected travel between the mezzanine floor input point to a storage location in the rack or from a storage location to a pick position on the mezzanine floor is minimized when $d = 0.5b$ (see equations (2) and (15b)). The expected travel from a pick position on the mezzanine floor to the ground floor input point (or from a pick position on the ground floor to the mezzanine input point) is minimized when $d = 0$ (see equations (11) and (12)).

Table 4-3 : Expected value of the best d found numerically for different operating policies with a given β

	β		
Policy	0.000	0.500	1.000
DC	0.394 b	0.365 b	0.314 b
Random Sequence of SC S/R	0.459 b	0.445 b	0.418 b
Consecutive R then S	0.500 b	0.500 b	0.500 b

Second, we analyze the shape factor of a rack. Tables 4-4 and 4-5 present denormalized expected travel times based on 'per retrieval' and 'per operation', respectively. From Table 4-4 we observe that the shape factor that minimizes travel time for a Consecutive Retrievals policy with a single picking floor for our given ratio of columns to rows is $b = 0.635$. This is close to the unrestricted value found in Section 4-4-3-4 when $b = 0.6825$. This observation illustrates that our derived unrestricted optimal shape factor may not necessarily be achieved when the design specification requires a fixed column and row dimension. The same observation can be seen in Table 4-4 for a Consecutive Retrievals policy with a dual picking floor with $d = {}^{b}/_{2}$ and $\beta = 0.5$. From Table 4-5 the shape factor that minimizes the travel time per operation for a Consecutive Retrievals then Storage Policy with a single picking floor for our given ratio of columns to rows is $b = 0.804$. This is between the optimal shape factor of $b = 1$ for a Consecutive Storage policy (Bozer and White, 1984) and the optimal shape factor of $b = 0.6825$ for a Consecutive Retrievals policy derived in Section 4-4-3-4. For a Mixed policy with either picking floor configuration, the optimal b value per retrieval (see Table 4-4) and per operation (see Table 4-5) is equal to 1.0, which is consistent with the results obtained analytically in Section 4-4-3-4. Therefore, depending on a distribution center operations requirement, systems may be designed differently. If the focus is on retrievals during a peak period, the MIAPP-

AS/RS should be designed with a smaller shape factor than if the focus is on the overall number of operations performed.

Table 4-4 : Denormalized expected 'per retrieval' travel time comparison of different operating policies

Single picking floor configuration						
	b					
Policy	1.000	0.804	0.635	0.479	0.349	0.238
Consecutive R	0.583	0.566	0.562	0.585	0.637	0.737
DC	1.108	1.141	1.201	1.319	1.499	1.788
Random Sequence of SC S/R	1.179	1.194	1.239	1.345	1.515	1.798
Dual picking floor configuration ($d = {}^{b}/_{2}$ and $\beta = 0.5$)						
	b					
Policy	1.000	0.804	0.625	0.467	0.333	0.223
Consecutive R	0.495	0.497	0.516	0.561	0.638	0.769
DC	1.064	1.106	1.190	1.332	1.551	1.896
Random Sequence of SC S/R	1.082	1.119	1.199	1.339	1.555	1.898

Third, we use our model to provide insights into operating policies during peak hours. We are interested in maximizing the number of retrieval operations (rather than the number of total operations) during peak hours; thus, a Consecutive Retrievals policy is used. Table 4-4 illustrates the retrieval performance of the three strategies under a single and dual picking floor configuration. As expected the Consecutive Retrievals policy has higher 'per retrieval' throughput than a DC or Random Sequence of SC Storages and Retrievals policy in both configurations. As an example under a single picking floor configuration with $b = 1$, a Consecutive Retrievals policy has 47.4% less expected travel time than a DC policy to operate one retrieval. For a dual picking floor configuration with $b = 1$, $d = {}^{b}/_{2}$, and $\beta = 0.5$, the reduction is 53.5%. However, a company that conducts only retrievals during peak picking will be required to conduct only storages during non-peak picking. Next, we explore this trade-off in overall throughput efficiencies.

Fourth, we use our models to provide insights when the goal is to maximize the overall number of operations performed. If a Consecutive Retrievals policy is required during peak-picking; then during non-peak hours, the same number of storages will be performed by the Consecutive Storages policy (i.e., the Consecutive Retrievals then Storages policy). Table 4-5 presents the expected 'per operation' travel time for both single and dual picking floor configurations. For a dual picking floor configuration, we analyze the best d values as presented in Table 4-3.

From Table 4-5 we observe that the minimum expected travel time per operation occurs by performing a DC policy when $b = 1$ for the single picking floor and when $b = 1$, $\beta = 0$, and $d = 0.394b$ for the dual picking floor configuration. A DC policy has a lower expected travel time than a Random Sequence of SC Storages and Retrievals or a Consecutive Retrievals then Storages policy because DC travels reduce the amount of empty travel required. Also, a DC policy is more robust to changes in β than the SC-based policies. For example from Table 4-5, a DC policy that changes β from zero to 0.5 and 1 will increase the expected travel time 2.85% and 5.55%, respectively. However, for a Random Sequence of SC Storages and Retrievals policy the increases are 4.49% and 8.93%, and for a Consecutive Retrievals then Storages policy the increases are 6.03% and 12.06%. To make these comparisons, we calculate the policy performance differences using the policy's best d value.

Table 4-5 : Denormalized expected 'per operation' travel time comparison of different operating policies

Single picking floor configuration

Policy	b = 1.000	0.804	0.635	0.479	0.349	0.238
DC	0.554	0.570	0.600	0.659	0.749	0.894
Random Sequence of SC S/R	0.590	0.597	0.619	0.672	0.758	0.899
Consecutive R then S	0.625	0.623	0.639	0.685	0.766	0.904

Dual picking floor configuration

β	d	Policy	b = 1.000	0.804	0.625	0.467	0.333	0.223
0.000	0.394 b	DC*	0.513	0.539	0.585	0.660	0.772	0.946
		Random Sequence of SC S/R	0.518	0.543	0.588	0.662	0.773	0.947
		Consecutive R then S	0.523	0.547	0.591	0.664	0.774	0.947
	0.459 b	DC	0.514	0.540	0.586	0.660	0.772	0.946
		Random Sequence of SC S/R*	0.517	0.542	0.587	0.662	0.773	0.946
		Consecutive R then S	0.519	0.544	0.589	0.663	0.774	0.947
	0.500 b	DC	0.516	0.542	0.587	0.661	0.773	0.946
		Random Sequence of SC S/R	0.517	0.543	0.588	0.662	0.773	0.946
		Consecutive R then S*	0.518	0.544	0.589	0.662	0.773	0.947
0.500	0.365 b	DC*	0.528	0.550	0.593	0.665	0.775	0.947
		Random Sequence of SC S/R	0.541	0.560	0.600	0.670	0.778	0.949
		Consecutive R then S	0.555	0.570	0.607	0.675	0.781	0.951
	0.445 b	DC	0.529	0.551	0.593	0.665	0.775	0.948
		Random Sequence of SC S/R*	0.540	0.559	0.599	0.669	0.777	0.949
		Consecutive R then S	0.550	0.567	0.605	0.673	0.780	0.950
	0.500 b	DC	0.532	0.553	0.595	0.666	0.776	0.948
		Random Sequence of SC S/R	0.541	0.560	0.599	0.669	0.778	0.949
		Consecutive R then S*	0.549	0.566	0.604	0.673	0.780	0.950
1.000	0.314 b	DC*	0.541	0.560	0.600	0.669	0.778	0.949
		Random Sequence of SC S/R	0.564	0.577	0.612	0.677	0.783	0.952
		Consecutive R then S	0.587	0.594	0.623	0.685	0.787	0.955
	0.418 b	DC	0.544	0.561	0.601	0.670	0.778	0.949
		Random Sequence of SC S/R*	0.563	0.576	0.611	0.677	0.782	0.952
		Consecutive R then S	0.582	0.590	0.620	0.683	0.786	0.954
	0.500 b	DC	0.548	0.564	0.603	0.671	0.779	0.950
		Random Sequence of SC S/R	0.564	0.577	0.611	0.677	0.782	0.952
		Consecutive R then S*	0.581	0.589	0.620	0.683	0.786	0.954

Each value of d is optimal for the given β and (*) policy

In addition, Table 4-5 illustrates that the Random Sequence of SC Storages and Retrievals policy has lower expected travel times than the Consecutive Retrievals then Storages policy (even though both are conducting only SC travels). This occurs because there is a lower amount of empty travel required when storages and retrievals arrive in a random manner (versus the Consecutive Retrievals then Storages policy). This observation is valid for both picking floor configurations. However, using a Consecutive Retrieval then Storages policy with an MIAPP-AS/RS with a dual picking floor configuration that utilizes the mezzanine input point has only a small impact on throughput performance (even when compared to a DC policy). This result is not common for traditional end-of-aisle AS/RSs and is also not the case for MIAPP-AS/RS with a single picking floor or a dual picking floor when $\beta = 1$. For example under the single picking floor when $b = 1$, a DC policy has 11.34% less travel time, and a Random Sequence of SC Storages and Retrievals Policy has 5.60% less travel time than a Consecutive Retrievals then Storages policy. For a dual picking floor configuration, when $b = 1$ and $\beta = 1$, these differences are less than 7.79% for a DC policy and less than 3.90% for a Random Sequence of SC Storages and Retrievals Policy. Contrastingly, under the dual picking floor configuration when $\mathrm{b} = 1$ and $\beta = 0$, these differences are less than 1.96% for a DC policy and 0.98% for a Random Sequence of SC Storages and Retrievals Policy. When $b = 1$ and $\beta = 0.5$, these differences are 4.97% for a DC policy and 2.49% for a Random Sequence of SC Storages and Retrievals Policy. To make these comparisons, we calculated the policy performance difference over all d values in Table 4-5 and took the maximum difference.

A reason for the difference in performance of SC-based policies versus DC-based policies is due to the return trip to the input points to service a storage request. In a DC policy, the trip back to the input point always occurs from a pick position. However, in the SC-based policies, the trip back to an input point occurs half of the time from a pick position and the other half of the time from a storage location in the rack. The expected travel time from any point in the rack to an input point is less when the input point is at the mezzanine floor rather than the ground floor. Consequently, the throughput performance of SC-based and DC-based operating policies are closer to each other in a MIAPP-AS/RS with a dual picking floor configuration that utilizes the mezzanine input point than in a single picking floor configuration or a dual picking floor configuration that does not use the mezzanine input point. Consequently, a MIAPP-AS/RS with a dual picking floor configuration that utilizes the mezzanine input point is a more robust configuration than a single picking floor configuration.

In summary, when the concern is about number of retrievals (such as what occurs during peak hours), a Consecutive Retrievals policy performs better than a Mixed policy. However, such a policy sacrifices overall throughput performance by not being able to utilize the efficiencies of DC travels. When the overall number of operations is important, a DC policy performs best as it has the opportunity to reduce the amount of empty travel by doing DC travels. An important insight from our analysis is the impact of operating policies for the different configurations. For a dual picking floor MIAPP-AS/RS that utilizes the mezzanine input point, if the distribution center operations are such that DC travels are not achievable (either due to peak-hour picking requirements or the arrival patterns of inbound or outbound trucks); using a SC-based operating policy does not considerably impact throughput performance. Consequently, by performing only SC retrievals and then only SC storages does not lose much efficiency with a dual picking floor configuration that utilizes the mezzanine input point.

4-6- Conclusion

In this paper, we modeled and analyzed the MIAPP-AS/RS, which is a material handling technology that facilitates semi-automated case-level order picking by automating the storage and replenishment of pallets to multiple in-the-aisle pick positions. We focused on evaluating the expected throughput performance of such systems for a randomized storage policy by developing analytical travel time models. We developed models for different operating policies that can occur during peak and non-peak picking hours, as well as for two design configurations (i.e. the MIAPP-AS/RS with a single picking level and the MIAPP-AS/RS with dual picking levels).We derived base travel time models for n pick positions on each floor, as well as extreme travel time models, which produce closed-form solutions by assuming there are an infinite number of pick positions and the requested demand for each pick position is equal likely. Optimal shape factor for different operating policies were calculated using the extreme travel time models.

A simulation study was conducted for various shape factors showing that the base and extreme models can approximate the discrete rack with less than 1% deviation. Through numerical experiments we provided design insights for MIAPP-AS/RSs. We found that operating policies have different impacts depending on the design configuration. To minimize the expected travel times for any policy we found that the mezzanine should be located such that $0 \leq d \leq 0.5b$ and all storage operations should originate from the mezzanine floor input point (i.e., $\beta = 0$). If the focus is on retrievals during a peak period, the MIAPP-AS/RS should be designed with a smaller shape factor than if the focus is on the overall number of operations performed. Finally, we found that a dual picking floor MIAPP-AS/RS that utilizes the mezzanine input point can use a SC-based operating policy without much loss of efficiency when compared to a DC policy. This is not the case with a single picking floor MIAPP-AS/RS or for traditional end-of-aisle AS/RSs. Therefore, if a distribution center operates in an environment that has peak picking times where only SC retrievals are performed, a dual picking floor configuration is a more robust configuration.

This article is the first study that analyzes AS/RS with multiple in-the-aisle outputs. Therefore, our study could help extensively to design and quantify the performance of systems with in-the-aisle outputs or pick positions. Our study could be extended to consider the following future research directions. The development of analytical models for MIAPP-AS/RS that considers different storage policies, different dwell point strategies, and different sequencing rules would be interesting to understand how policies impact throughput and storage space performance. Also, the development of a systematic model that considers the interaction of human order pickers and the MIAPP-AS/RS would be interesting from an inventory buffer capacity, as well as a staff resource and scheduling perspective.

Appendix A: The derivation of $\boldsymbol{E_m^b(z_1)}$ for the case of $\boldsymbol{m \geq b}$

$$Pr(x_1 \leq z_1) = {z_1}/{m} \quad for \quad 0 \leq z_1 \leq m$$

$$Pr(y_1 \leq z_1) = \begin{cases} {z_1}/{b} & for \quad 0 \leq z_1 \leq b \\ 1 & for \quad b \leq z_1 \leq m \end{cases}$$

$$F(z_1) = Pr(x_1 \leq z_1) \times Pr(y_1 \leq z_1) = \begin{cases} {z_1}^2/{mb} & for \quad 0 \leq z_1 \leq b \\ {z_1}/{m} & for \quad b \leq z_1 \leq m \end{cases}$$

$$f(z_1) = \frac{dF(z_1)}{dz_1} = \begin{cases} {2z_1}/{mb} & for \quad 0 \le z_1 \le b \\ {1}/{m} & for \quad b \le z_1 \le m \end{cases}$$

$$E_m^b(z_1) = \int_0^m z_1 . f(z_1) dz_1 = {b^2}/{6m} + {m}/{2} \qquad for \quad m \ge b$$

Appendix B: The derivation of $\boldsymbol{E_m^b(z_2)}$for the cases of $\boldsymbol{(1-m) < b}$ and$\boldsymbol{(1-m) \ge b}$

Let $F(z_2)$ be the probability that the travel time between the origin and (x_2, y_2) in region II is less than or equal to z_2. As the x_1 and $y_1$coordinates are assumed to be independent, $F(z_2)$ is equal to

$$F(z_2) = Pr\left(t_{x_2,y_2} \le z_2\right) = Pr(x_2 \le z_2) . Pr(y_2 \le z_2)$$

Based on randomized storage, when $(1-m) < b$,

$$Pr(x_2 \le z_2) = \begin{cases} {z_2}/{(1-m)} & for \qquad 0 \le z_2 \le (1-m) \\ 1 & for \qquad (1-m) \le z_2 \le b \end{cases}$$

$$Pr(y_2 \le z_2) = {z_2}/{b} \quad for \quad 0 \le z_2 \le b$$

Hence,

$$F(z_2) = \begin{cases} {z_2}^2 / {(1-m)b} & for \qquad 0 \le z_2 \le (1-m) \\ {z_2}/{b} & for \qquad (1-m) \le z_2 \le b \end{cases}$$

$$f(z_2) = \frac{dF(z_2)}{dz_2} = \begin{cases} {2z_2}/{(1-m)b} & for \quad 0 \le z_2 \le (1-m) \\ {1}/{b} & for \quad (1-m) \le z_2 \le b \end{cases}$$

$E_m^b(z_2)$ denotes the expected travel time between the origin and any point located in region II. By definition, $E_m^b(z_2)$ is obtained as follows.

$$E_m^b(z_2) = \int_0^b z_2.f(z_2)dz_2 = {(1-m)^2}/{6b} + {b}/{2} \quad for \quad 0 \le (1-m) < b$$

Similarly, when $(1-m) \ge b$,

$$Pr(x_2 \le z_2) = {z_2}/{(1-m)} \quad for \quad 0 \le z_2 \le (1-m)$$

$$Pr(y_2 \le z_2) = \begin{cases} {z_2}/{b} & for \quad 0 \le z_2 \le b \\ 1 & for \quad b \le z_2 \le (1-m) \end{cases}$$

Hence,

$$F(z_2) = \begin{cases} {z_2{}^2}/{(1-m)b} & for \quad 0 \le z_2 \le b \\ {z_2}/{(1-m)} & for \quad b \le z_2 \le (1-m) \end{cases}$$

$$f(z_2) = \frac{dF(z_2)}{dz_2} = \begin{cases} {2z_2}/{(1-m)b} & for \quad 0 \le z_2 \le b \\ {1}/{(1-m)} & for \quad b \le z_2 \le (1-m) \end{cases}$$

$$E_m^b(z_2) = \int_0^{1-m} z_2.f(z_2)dz_2 = {b^2}/{6(1-m)} + {(1-m)}/{2} \quad for \quad 0 \le b < (1-m)$$

Appendix C. Calculation of $E_1^\infty(W)$

According to equation (14a), $E_1^\infty(W) = \int_0^1 E_1(W_m^b)\, dm$. By substituting the right hand side of equation (6) for $E_1(W_m^b)$, we have

$$E_1^{\infty}(W) = \int_{m=0}^{1} [mE_m^b(z_1) + (1-m)E_m^b(z_2)]\, dm \tag{C.1}$$

To calculate $E_m^b(z_1)$ and $E_m^b(z_2)$, equations (4) and (5) should be used respectively based on the value of b and m. According to the right hand side of equations (4) and (5), *m* depends on b and $(1-b)$. Therefore, to break the integral on the right hand side of equation (C.1) we need to consider two cases, $0 \le b < 1/2$ and $1/2 \le b \le 1$.

When $0 \le b < 1/2$,

$$E_1^{\infty}(W) = \int_{m=0}^{b} \left\{ m\left(m^2/6b + b/2\right) + (1-m)\left[b^2/6(1-m) + (1-m)/2 \right] \right\} dm + \int_{m=b}^{1-b} \left\{ m\left(b^2/6m + m/2\right) + (1-m)\left[b^2/6(1-m) + (1-m)/2 \right] \right\} dm + \int_{m=1-b}^{1} \left\{ m\left(b^2/6m + m/2\right) + (1-m)\left[(1-m)^2/6b + b/2 \right] \right\} dm = 1/3 + b^2/3 - b^3/12 \tag{C.2}$$

When $1/2 \le b \le 1$,

$$E_1^{\infty}(W) = \int_{m=0}^{1-b} \left\{ m\left(m^2/6b + b/2\right) + (1-m)\left[b^2/6(1-m) + (1-m)/2 \right] \right\} dm + \int_{m=1-b}^{b} \left\{ m\left(m^2/6b + b/2\right) + (1-m)\left[(1-m)^2/6b + b/2 \right] \right\} dm + \int_{m=b}^{1} \left\{ m\left(b^2/6m + m/2\right) + (1-m)\left[(1-m)^2/6b + b/2 \right] \right\} dm = 1/3 + b^2/3 - b^3/12 \tag{C.3}$$

Based on (C.2) and (C.3), for $0 \le b \le 1$, we have

$$E_1^{\infty}(W) = 1/3 + b^2/3 - b^3/12 \tag{C.4}$$

CHAPTER FIVE: CONTRIBUTION 2- A PAPER ON "PRODUCT ALLOCATION PROBLEM FOR AN AS/RS WITH MULTIPLE IN-THE-AISLE PICK POSITIONS"

An automated storage/retrieval system (AS/RS) with multiple in-the-aisle pick positions (MIAPP-AS/RS) is a semi-automated case-level order fulfillment technology that is widely used in distribution centers. We study the impact of product to pick position assignments on the expected throughput for different operating policies, demand profiles, and shape factors. We develop efficient algorithms of complexity O(nlog(n)) that provide the assignment that minimizes the expected travel time. Also, for different operating policies, shape configurations, and demand curves, we explore the structure of the optimal assignment of products to pick positions and quantify the difference between using a simple, practical assignment policy versus the optimal assignment. Finally, we derive closed-form analytical travel time models by approximating the optimal assignment's expected travel time using continuous demand curves and assuming an infinite number of pick positions in the aisle. We illustrate that these continuous models work well in estimating the travel time of a discrete rack and use them to find optimal design configurations.

Keywords: Automated Storage/Retrieval System, Case-level Order Picking, Closed-form Travel-time, Multiple input/output (I/O) points

5-1- Introduction

According to the U.S. Roadmap for Material Handling & Logistics, currently 350,000 distribution centers operate in the Unites States. A distribution center's most important, as well as most costly and labor-intensive operation is to fulfill customer order requests. Thus, careful design and control of order-fulfillment operations can result in considerable improvements in distribution center performance. In this work, we are interested in case-level order-fulfillment when non-identical demand is experienced for different products (i.e., item demand varies among the set of items managed). A typical phenomenon exists where a small percentage of stock-keeping-units (SKUs) will make up a large percentage of the total demand. This demand skewness is captured in a well-known method called *ABC* analysis, which ranks the SKUs based on their contribution to total demand. The "A" class represents the fast-moving SKUs, the "B" class represents the medium-moving SKUs, and the "C" class represents the slow-moving SKUs.

An automated storage/retrieval system (AS/RS) with multiple in-the-aisle pick positions (MIAPP-AS/RS) is an example of a case-level order-fulfillment technology that is frequently used in distribution centers (See Fig. 5-1). These systems are common in temperature-controlled distribution centers because of their ability to increase the space utilization and energy efficiency, and to reduce the number of employees that have to work in the harsh environment.

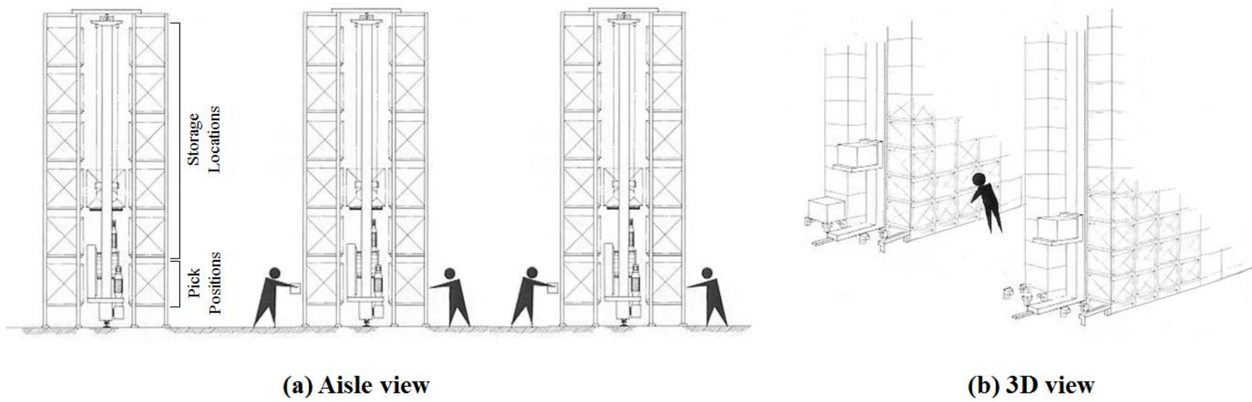


Figure 5-1 : Typical MIAPP-AS/RS

The MIAPP-AS/RS consists of crane aisles and picking aisles. The crane aisles are dedicated to the movement of the cranes, and the picking aisles to the travel of the order pickers. Distribution centers that utilize MIAPP-AS/RS receive full pallets from an upstream member of their supply chain and are tasked with supplying a variety of products fulfilled at the case-level to a downstream member of their supply chain. In these systems, each opening can store one pallet (in single-deep systems) or two pallets (in double-deep systems). As shown in Fig. 5-1, the openings in the first row of the rack are for pick positions and the other openings in the rack are for storage locations. Pallets are put-away and stored in full-pallet quantities in the storage openings. When all of the cases of a particular SKU are picked from a pick position, the crane replenishes another full-pallet of that SKU to the pick position. Generally, for the convenience of the picking process, each pick position in the aisle is dedicated to a particular type of SKU. Therefore, an MIAPP-AS/RS is a semi-automated order-fulfillment strategy because the full-pallet put-away-to-storage and replenishment-to-picking position movements are completed with cranes. On the other hand, the picking of cases from pick positions in the aisle is done via manual order pickers.

In this research, we are interested in the assignment of the SKUs to pick positions for MIAPP-AS/RS. We call this problem as the *SKU Assignment Problem (SAP)*. Two key research questions regarding the SAP include:

1. How should SKUs be assigned to pick positions such that the maximum expected crane throughput occurs?

2. What is the impact of different SKU to pick position assignments on throughput performance under different operating policies, demand curves, and system configurations?

The goal of our study is to understand the impact of different SKU assignments on the MIAPP-AS/RS throughput, as well as system design trade-offs that occur when MIAPP-AS/RS are operating using different policies and demand profiles. To do so, we begin by reviewing the relevant AS/RS and SKU assignment problem literature in Section 5-2. We describe the MIAPP-AS/RS physical configuration, demand curve models, operating policies, and key assumptions in Section 5-3. Next, in Section 5-4, we develop mathematical optimization models to find the optimal assignment minimizing the expected travel time, and provide an optimal assignment algorithm of complexity $\boldsymbol{O}\big(\boldsymbol{nlog}(\boldsymbol{n})\big)$. We also provide structural properties of the optimal assignment of SKUs to pick positions in MIAPP-AS/RS's for different operating policies, demand profiles, and shape factors. We quantify the performance difference between optimal assignments and a common assignment used in practice that assigns the most demanded SKUs to the pick positions that are closest to the input point. In Section 5-5, we derive and validate analytical closed-form models to approximate the expected travel time for different assignments; also, we use these models to obtain the optimal shape factors for different policies. These closed-form analytical travel time models are derived by using continuous demand curves and assuming an infinite number of pick positions in the aisle to approximate the optimal assignment's expected travel time. In Section 5-6, we illustrate that these models work well in estimating the travel time of a discrete rack and use them to derive optimal design configurations and provide managerial insights. We also provide numerical results and managerial insights by exploring the trade-offs that occur while implementing different assignments. Finally, in Section 5-7, we review our main contributions, provide insights from our study, and direction for future research.

5-2- Literature Review

During the past three decades AS/RS have received notable attention from different perspectives that can be categorized into: system configuration, travel time estimation, storage assignment, dwell-point, and batching/sequencing (Roodbergen & Vis, 2009). In addition, some AS/RS research has been addressed in warehouse design papers; interested readers may refer to reviews papers such as de Koster *et al.* (2007) and Gu *et al.*, (2007). In fact, AS/RS literature is the exclusive focus in the following review papers: Sarker & Babu (1995), Roodbergen & Vis (2009), Vasili *et al.* (2012), and Gagliardi *et al.* (2012). Among these review papers, Sarker and Babu (1995) focus specifically on travel time models. According to the AS/RS review papers, AS/RS with multiple I/O points is a subject that deserves further study.

Similar topics to the SAP include the storage location assignment problem (SLAP) and the product allocation problem (PAP), which have been widely studied in traditional order-picking and warehouse management literature. The SLAP represents a notable portion of the AS/RS literature. A storage assignment policy identifies how items are assigned to storage locations. Three main storage assignment policies have received the most attention in the literature: randomized storage, turnover-based storage, and class-based storage. Generally, randomized storage has the best space-utilization; turnover-based storage has the shortest expected travel time; and class-based storage is a compromise between these two policies (Pohl *et al.*, 2011). The turnover-based and class-based storage policies incorporate that different SKUs have different demand profiles when determining the assignment of SKUs to storage locations. For the special cases of 2 or 3 classes in class-based storage, explicit travel time analytical expressions have been developed by Hausman *et al.* (1976), Kouvelis & Papanicolaou (1995) and Eynan & Rosenblatt (1994). Rosenblatt & Eynan (1989) and Eynan & Rosenblatt (1994) provide a procedure to obtain optimal boundaries for *n*-classes under a specific layout. Van den Berg (1996) develops a dynamic programming algorithm that optimally assigns products and locations into a certain number of classes. He considers travel time minimization under *single command* cycles. Goetschalckx & Ratliff (1990) consider a duration-of-stay shared policy for unit-load AS/RS and show the optimality of this policy under an assumption of perfectly balanced I/O flows. All of the above studies focus on the assignment of the SKUs to storage locations in traditional AS/RS that have a single I/O point, and thus no SKU to pick positions assignment problem exists. In contrast, we focus on the assignment of the non-identical demanded SKUs to multiple in-the-aisle pick positions.

Also, the PAP has been widely studied in traditional order-picking systems literature where a manual order picker traverses aisles to pick a set of items (Francis *et al.*, 1992; de Koster *et al.*, 2007). The PAP determines which SKUs should be assigned to which picking locations to minimize a given performance measure, such as travel time. For example, Francis *et al.* (1992) analyze various assignment models in a traditional manual picking warehouse subject to different space and throughput constraints. Jarvis & McDowell (1991) consider the PAP in different types of order picking warehouses. They present a stochastic model to obtain the optimal results. Petersen & Aase (2004) build a simulation model to evaluate the picking, storage, and routing effect on order picking travel time. They show that order batching of products results in the highest savings, particularly when the order sizes are smaller. Heragu *et al.* (2005) develop a mathematical model and a heuristic procedure that simultaneously determines the allocation of the products, as well as the size of the functional areas. Pazour & Meller (2011) analyze the SKU assignment and allocation problem in an A-frame dispenser system, which is a fully automated order picking technology. Guerriero *et al.* (2013) develop a nonlinear integer mathematical model to minimize the handling cost considering compatibility constraints among classes. They use an Iterated Local Search based algorithm to solve large scale instances. Our work is similar to this area of research, in that we are also interested in assigning SKUs to pick position locations. However, the focus of our study is on the assignment of SKUs to pick positions in a MIAPP-AS/RS, which have different travel time dynamics than manual order picking operations, including the need to consider simultaneous vertical and horizontal travel movements and retrieving, as well as storage operations.

Only a few studies exist that study an AS/RS with either multiple I/O points or non-end-of-aisle I/O points. Bozer & White (1984) derive and analyze the expected travel time for different alternative I/O point locations; however, all of the alternatives considered a single input and single output point. Randhawa *et al.* (1991) and their extended study by Randhawa & Shroff (1995) focus on simulation analysis to identify the impact of changing the location and the number of I/O points on travel time. They consider different layouts and performance measures to evaluate their simulation studies. Vasili *et al.* (2008) develop a continuous travel time model for the split-platform AS/RS, where an I/O point is located at the center of the rack. Ramtin & Pazour (2014) are the first to study the AS/RS with multiple in-the-aisle pick position, and develop expected travel time models for different operating policies and physical configurations. For their closed-form travel time models and numerical analysis, they assume that the demand of SKUs is identical and thus do not consider the assignment of SKUs to pick positions. To our knowledge, no study exists that addresses the assignment of SKUs to multiple pick positions in an AS/RS.

5-3- Problem Definition and Assumptions

In this section, we provide a description of the environment that we model. To do so, we describe the MIAPP-AS/RS's physical configuration, demand models, crane travels and operational policies, as well as the key modeling assumptions.

Physical configuration

We assume the system has a single picking level and each crane aisle has its own crane that is working independently from other cranes, as illustrated in Fig. 5-1 Each rack has one input point located at the lower-left corner and the pick positions are located on the bottom line of the rack. We develop our models considering a single rack.

Most of the existing AS/RS travel time model studies have used continuous approximation of the rack to derive expected value models for different configurations (Graves *et al.*, 1977; Bozer & White, 1984; Foley & Frazelle, 1991; Chang *et al.*, 1995; Kouvelis & Papanicolaou, 1995; Sarker & Babu, 1995; Hu *et al.*, 2005). We also use continuous approximation of the rack for the travel time models in Section 5-4 and 5-5. The storage locations are assumed to be continuously and uniformly distributed over the rack. We follow the Bozer & White (1984) methodology used to 'normalize' the rack as follows. Let:

t_h: time required to travel horizontally from the input point to the most distant point,

t_v: time required to travel vertically from the input point to the most distant point,

T: scaling factor, where $T = max\{t_h, t_v\}$

b: shape factor, where $b = min\left\{{t_h}/{T}, {t_v}/{T}\right\}$

By assuming $t_h \geq t_v$, the rack can be normalized as a rectangle with length of 1.0 and height of b, where $0 < b \leq 1$.

Demand curve representation

We denote I and K as the sets of all pick positions and SKUs to be assigned to those pick positions, respectively, such that $I = \{1,2, \ldots, n\}$ and $K = \{1,2, \ldots, n\}$. Let $i \in I$ and $k \in K$ be the index of pick positions and SKUs, respectively. Let Pr_i denote the probability that pick position i is visited such that $\sum_{i=1}^{n} Pr_i = 1$. Let Pr_k denote the probability that a storage/retrieval request is for SKU k among a total of n SKUs, where $\sum_{k=1}^{n} Pr_k = 1$.

We use the continuous demand model in Hausman *et al.* (1976) to represent the *ABC* analysis that ranks the items in monotonically decreasing order based on their contribution to total demand. In this demand model, x is the percentage of inventoried SKUs, $x \in [0,1]$, s represents the skewness of the demand curve, and sx^{s-1} is the demand of the 100*x* percentage SKU. We consider 20/20, 20/40, 20/60, 20/80, and 20/90 demand curves, which correspond to $s = 1$, $s = 0.5693$, $s = 0.3174$, $s = 0.1386$, $s = 0.0655$, respectively. For example, a 20/60 curve represents the case that 20% of inventoried SKUs contribute 80% of the total demand. SKUs with identical demand can be represented by 20/20 curves. To calculate Pr_k, the continuous demand curve model can be discretized by the approach used in Pohl *et al.* (2011) as shown in equation (1).

$$Pr_k = \int_0^{k/n} sx^{s-1}\,dx - \int_0^{(k-1)/n} sx^{s-1}\,dx \qquad \forall k \in K \tag{1}$$

Crane travels and operational policies structure

In a MIAPP-AS/RS, the crane can carry a single unit-load and travel between the input point, storage locations, and pick positions. In a single command (SC) storage operation, full-pallets of different SKUs originate from an input point and are transferred to a storage location in the rack by the crane. A SC retrieval operation is performed by the crane when it moves a SKU stored in a rack location to a pick position located in the aisle. A dual command (DC) operation involves the crane conducting a storage operation followed by a retrieval operation. All of the travels performed by a MIAPP-AS/RS can be broken into four elements denoted as *trips* (see Fig. 5-2). All of the trips and travels notation, as well as their expressions are as follows (for detailed derivation of equation (2) and (3) please refer to Bozer &White (1984), and for equations (4) to (7) refer to Ramtin & Pazour (2014)):

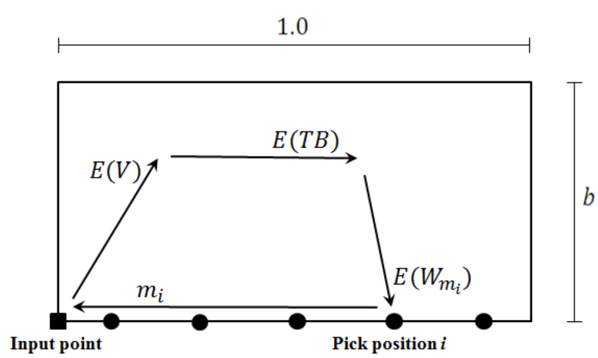


Figure 5-2 : Trips in MIAPP-AS/RS

m_i : Travel time between pick position i and the input point (Hereafter, we refer to m_i as the location of pick position i),

$E(V)$: Expected travel time between the input point and any randomly selected point in the rack, where

$$E(V) = {b^2}/{6} + {1}/{2} \tag{2}$$

$E(TB)$: Expected travel time between any two randomly selected points in the rack, where

$$E(TB) = {}^{1}/_{3} + {}^{b^2}/_{6} - {}^{b^3}/_{30} \tag{3}$$

$E(W_{m_i})$: Expected travel time between pick position i and any randomly selected point in the rack, where

$$E(W_{m_i}) = \frac{m_i[Min(m_i,b)]^2}{6Max(m_i,b)} + \frac{(1-m_i)[Min(1-m_i,b)]^2}{6Max(1-m_i,b)} \tag{4}$$

We consider operating policies for the MIAPP-AS/RS that are common for peak and non-peak hours. The first operating policy is a *Consecutive Retrievals* (*CR*) policy, where the AS/RS performs only SC retrievals. This policy is used during the peak hours when the priority is to fullfil the customer orders. In the case of a *CR* policy, the crane travels between storage locations and pick positions only. Thus all trips are $E(W_{m_i})$ trips, and the overall expected travel time can be calculated as $2T \times E(W)$ using equation (5). If a *CR* policy is used during peak hours, then the AS/RS will need to perform consecutive storages (*CS*) during the non-peak hours. We call this policy as *CR then CS* policy, and the overall expected travel time for this policy can be calculated as $T \times (E(W) + E(V))$, which is the average of the *CR* and *CS* policies. Another policy we consider during non-peak hours is a *Mixed* policy. In a *Mixed* policy, the crane performs both SC (storage/retrieval) and DC. In this policy the proportion of the number of SC to DC operations is defined with the parameter α. Let α denote the percentage of storage (or retrieval) operations that are performed under SC basis. For this policy the combination of all $E(V)$, $E(TB)$, $E(W_{m_i})$, and m_i trips may occur, and the overall expected travel time can be obtained as $T \times E(M)$, where $E(M)$ is defined in equation (7).

$E(W)$: Expected one-way travel time between any pick position and any randomly selected point in the rack, where

$$E(W) = \sum_{i=1}^{n} Pr_i \,.\, E(W_{m_i}) \tag{5}$$

$E(R)$: Expected one-way travel time between any pick position and input point, where

$$E(R) = \sum_{i=1}^{n} Pr_i \,.\, m_i \tag{6}$$

$E(M)$: Expected 'per operation' travel time for the *Mixed* policy, where

$$E(M) = \left(\frac{1+2\alpha}{2+2\alpha}\right) E(V) + \left(\frac{1}{2+2\alpha}\right) E(TB) + \left(\frac{1+2\alpha}{2+2\alpha}\right) E(W) + \left(\frac{1}{2+2\alpha}\right) E(R) \tag{7}$$

Finally, a review of the assumptions of this work follows.

1. Each normalized rack has one input point located at the lower-left corner and the pick positions are located on the bottom line of a normalized rack.
2. Demand for each item type (and thus each pick position location) is known and independent.
3. For assigning SKUs to storage locations, a randomized storage assignment is used, i.e., any point within the rack is equally likely to be selected as a storage location. However, assigning SKUs to pick positions is a decision discussed in Section 5-4.
4. Each pick position is dedicated to particular type of SKU.
5. We consider a single-deep rack, where each opening can contain a single pallet.
6. The storage and retrieval rate is assumed to be equal in steady state, and operation requests are processed on a first-come-first-serve basis.

7. Only the travel time model of the crane is considered. Human order pickers travel time and the pick/deposit times to handle the loads are ignored.
8. The crane carries unit-loads under either single command (SC) (storage/retrieval) or dual command (DC) travels.
9. The crane travel time follows the *Chebyshev* metric and moves at a constant horizontal and vertical speed, i.e., acceleration and deceleration are ignored.
10. A *stay* dwell point strategy is considered, i.e., after completing any storage or retrieval operation, the crane stays where it is, and waits for the next operation request.

5-4- SKU Assignment Problem (SAP)

In this section, we analyze the SAP for the MIAPP-AS/RS, which determines the assignment of SKUs to pick positions to minimize the expected travel time of the crane. When SKUs' demands are identical, changing the assignment of SKUs to pick positions does not impact the expected travel time for any policy (see equations (5) and (7)). However, in the case of non-identical demand, as the expected travel time to each pick position is different (see equation (4)), assigning higher demanded SKUs to pick positions with less expected travel time will result in less overall expected travel time.

5-4-1- Optimal Assignment Problem

Our objective is to minimize the expected travel time for different operating policies by changing the assignment of SKUs to pick positions. Referring to equation (5) and (7), the expected travel time models for *CR* and *Mixed* policy are linear functions of SKUs' demands and the locations of the pick positions. Therefore, we formulate our problems as a *Linear*

Assignment Problem (LAP) for each policy. Let x_{ik} be the binary variable where 1 denotes that the pick position i is assigned to SKU k, and is 0 otherwise. We formulate the assignment problem for the *CR* policy and denote it as Model 1. The objective function is to minimize the expected travel time for a *CR* policy, which is expressed in equation (8). Equation (9) provides the constraints that each SKU must be assigned to one pick position, and equation (10) provides the constraints that each pick position must be assigned to one SKU.

Model 1 (for a *CR* Policy)

$$Min\ \sum_{i=1}^{n}\sum_{k=1}^{n} Pr_k \, . \, E(W_{m_i}) . \, x_{ik} \tag{8}$$

$$\sum_{i=1}^{n} x_{ik} = 1 \quad \forall k \in K \tag{9}$$

$$\sum_{k=1}^{n} x_{ik} = 1 \ \forall i \in I \tag{10}$$

$$x_{ik} \in \{0,1\} \quad \forall i \in I, \forall k \in K \tag{11}$$

Model 2 is the SAP for the *Mixed* policy. In this case, the assignment problem depends only on the elements of the *Mixed* policy expected travel time in equation (7) that are a function of m_i (the location of the pick positions). Consequently, $E(V)$ and $E(TB)$ are not impacted. Therefore, the objective function of the assignment problem for the *Mixed* policy is formulated as equation (12) subject to constraints (9), (10), and (11).

Model 2 (for a *Mixed* Policy)

$$Min\ \sum_{i=1}^{n}\sum_{k=1}^{n} Pr_k \left(\frac{1+2\alpha}{2+2\alpha} E(W_{m_i}) + \frac{1}{2+2\alpha} m_i\right) x_{ik} \tag{12}$$

Because of the structure of Model 1 and 2, we are able to provide an algorithm for the optimal solution that is computationally tractable.

In order to simplify Model 1 and 2, we define $f_i = E(W_{m_i})$ and $g_i = \frac{1+2\alpha}{2+2\alpha} E(W_{m_i}) + \frac{1}{2+2\alpha} m_i$. Therefore, the objective functions of Model 1 and 2 become $Min\ \sum_{i=1}^{n} \sum_{k=1}^{n} f_i \times Pr_k\, x_{ik}$ and $Min\ \sum_{i=1}^{n} \sum_{k=1}^{n} g_i \times Pr_k\, x_{ik}$, respectively.

Proposition 1a. The optimal solution to Model 1 is obtained by the assignment permutation that has the pick positions sorted in ascending order based on their f_i values and the SKUs sorted in descending order based on their Pr_k values.

Proof. f_i is independent of a SKU's demand. Hence, the cost of assigning SKU k to pick position i is obtained by multiplying two independent parameters (f_i and Pr_k). Therefore, by introducing permutation functions W and P, Model 1 can be represented as its combinatorial formulation, $Min_{W,P \epsilon Z}\ \sum_{i'=1}^{n} f_{W(i')} \times Pr_{P(i')}$. Let Z be the set of all permutations on $W: I \rightarrow I$ and $P: K \rightarrow K$. According to the *rearrangement inequality,* the minimum value to this model is obtained by the optimal permutations (W^* and P^*) such that $f_{W^*(1)} \leq f_{W^*(2)} \leq \cdots \leq f_{W^*(n)}$ and $Pr_{P^*(1)} \geq Pr_{P^*(2)} \geq \cdots \geq Pr_{P^*(n)}$ (For an extended proof, please refer to Hardy *et al.* (1988)). Therefore, the optimal assignment for Model 1 can be obtained by bijective mapping of permutation W^* and P^*, which is represented as equation (13).

$$x_{ik}^* = \begin{cases} 1 & \text{for } \mathrm{i} = W^*(i') \text{ and } k = P^*(i'), \\ 0 & \text{otherwise.} \end{cases} \qquad i' = 1,2, \ldots, n \tag{13}$$

Proposition 1b. The optimal solution to Model 2 is obtained by the assignment permutation that has the pick positions sorted in ascending order based on their g_i values and the SKUs sorted in descending order based on their Pr_k values.

Proof. Following directly from Proposition 1a's proof. Let Z be the set of all permutations on $M: I \to I$ and $P: K \to K$. The combinatorial formulation of Model 2 becomes as $Min_{M,P \epsilon Z} \sum_{i'=1}^{n} g_{M(i')} \times Pr_{P(i')}$. The minimum value to this model is obtained by the permutations (M^* and P^*) such that $g_{M^*(1)} \leq g_{M^*(2)} \leq \cdots \leq g_{M^*(n)}$ and $Pr_{P^*(1)} \geq Pr_{P^*(2)} \geq \cdots \geq Pr_{P^*(n)}$. Therefore, the optimal assignment for Model 2 can be obtained by bijective mapping of permutation M^* and P^*, which is represented as equation (14).

$$x_{ik}^* = \begin{cases} 1 & \text{for } i = M^*(i') \text{ and } k = P^*(i'), \\ 0 & \text{otherwise.} \end{cases} \qquad i' = 1,2,\dots,n \tag{14}$$

Corollary 1. The minimum expected travel time 'per operation' with respect to the optimal assignment of SKUs to pick positions is equal to equations (15) and (16) for a *CR* and a *Mixed* policy, respectively.

$$E^*(W) = \sum_{i'=1}^{n} E(W_{m_{W^*(i')}}) \times Pr_{P^*(i')} \tag{15}$$

$$E^*(M) = \frac{1+2\alpha}{2+2\alpha} E(V) + \frac{1}{2+2\alpha} E(TB) + \sum_{i'=1}^{n} Pr_{P^*(i')} \left(\frac{1+2\alpha}{2+2\alpha} E(W_{m_{M^*(i')}}) + \frac{1}{2+2\alpha} m_{M^*(i')} \right) \tag{16}$$

According to Proposition 1a and 1b, the optimal algorithm for the SKU assignment problems is of complexity $O(nlog(n))$, as it sorts two sets of cardinality *n*. The optimal assignment can be obtained using the following procedure:

STEP 1: Calculate f_i and g_i for $\forall i \in I$, as well as Pr_k for $\forall k \in K$.

STEP 2: Find permutations W^* and M^* by sorting f_i and g_i in monotonically increasing order, respectively. Find permutation P^* by sorting Pr_k in monotonically decreasing order.

STEP 3: Provide the optimal assignment for Model 1 and 2 by equations (13) and (14), respectively.

Example: Consider a square-in-time system ($b = 1$) where 40% of storages are performed by SC in a Mixed policy ($\alpha = 0.4$). Consider there are 5 pick positions ($n = 5$) whose distance from the input point are 0.1, 0.3, 0.5, 0.7, 0.9; and 5 different SKUs which follows the 20-60 demand curve ($s = 0.3174$). Therefore, based on the definitions, $f_i = \{0.62, 0.56, 0.54, 0.56, 0.62\}$, $g_i = \{0.44, 0.47, 0.53, 0.61, 0.72\}$, and by using equation (1), $Pr_k = \{0.60, 0.15, 0.10, 0.08, 0.07\}$. We obtain $W^* = \{3,2,4,1,5\}$, $M^* = \{1,2,3,4,5\}$, and $P^* = \{1,2,3,4,5\}$ by sorting f_i, g_i, and Pr_k as defined. According to Proposition 1a and 1b, the optimal assignment of SKUs to pick positions for a *CR* and a *Mixed* policy is obtained by bijective mapping W^* and M^*, respectively, on P^*. Based on equations (13) and (14), the optimal assignment for *CR* policy has $x_{31} = x_{22} = x_{43} = x_{14} = x_{55} = 1$ and for the *Mixed* policy has $x_{11} = x_{22} = x_{33} = x_{44} = x_{55} = 1$. Also, according to equations (15) and (16), $E^*(W) = 0.5586$ and $E^*(M) = 1.0869$, respectively.

Proposition 2. When all of the storage and retrieval operations are performed by DC (a $Mixed$ policy with $\alpha = 0$), the assignment of SKUs ordered based on monotonically decreasing demand to pick positions ordered from nearest to farthest from the input point is optimal.

Proof. Appendix A.

For the sake of convenience, we call 'the assignment of SKUs ordered based on monotonically decreasing demand to pick positions ordered from nearest to farthest from the input point' as the *monotonically decreasing demand (MDD)* assignment and is shown in Fig. 5-4(c). The insight behind Proposition 2, as well as the other assignment characteristics will be discussed in Section 5-4-2.

According to the definition of the *MDD* assignment, the most demanded SKU is assigned to the pick position closest to the input point, and the next demanded SKU to the next pick position closest to input point, and so forth. The expected travel time for *CR* and *Mixed* policy under *MDD* assignment can be calculated by substituting any monotonically decreasing demand for Pr_i into equation (5) and (7), respectively.

5-4-2- Structure of the optimal assignment

In this section, we explore the structure of the optimal assignment of SKUs to pick positions introduced in Section 5-4-1. Recall that the set of f_i is used to find the optimal assignment for a *CR* policy and the set of g_i for a *Mixed* policy. If we denote m as the location of a pick position from the input point ($0 \leq m \leq 1$) these sets can be defined as functions of m, such that $f = E(W_m)$ and $g = \frac{1+2\alpha}{2+2\alpha}E(W_m) + \frac{1}{2+2\alpha}m$. To observe the behavior of the expected travel time, Fig. 5-3 plots f and g as a function of the location of pick positions from the input point for different values of α and b.

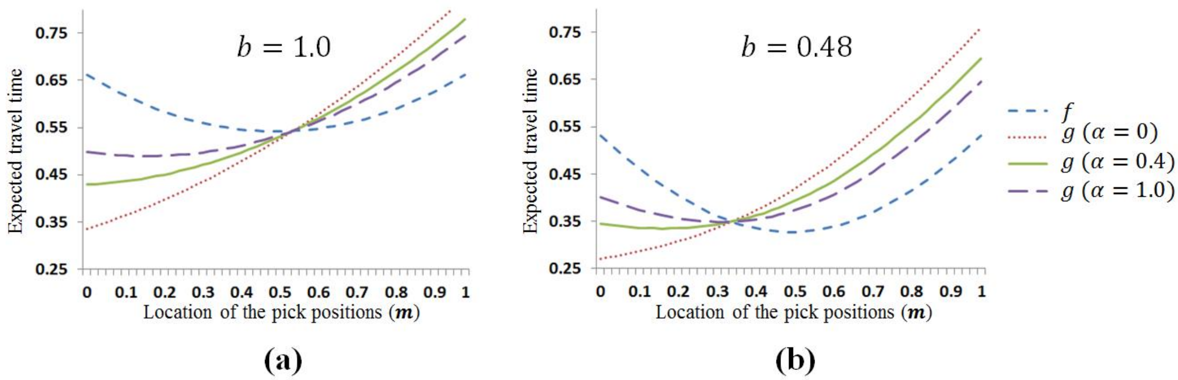
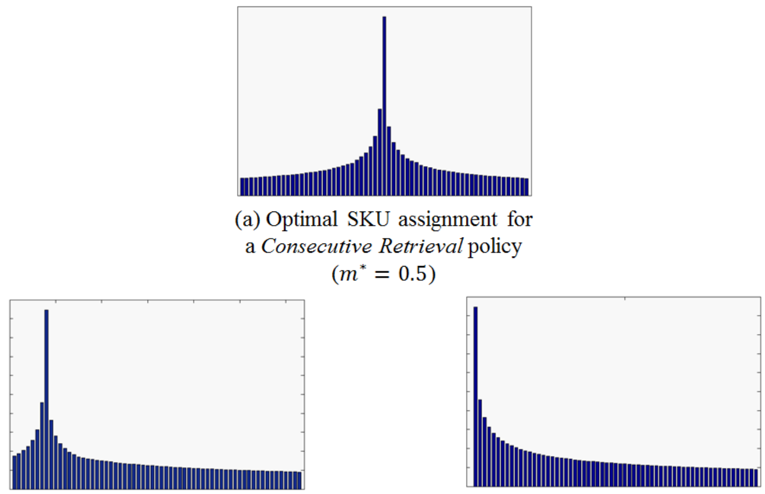


Figure 5-3 : Pick positions' expected travel time for different policies

Let m^* denote the location of a pick position to which the expected travel time is minimum for each f and g function. Function f has $m^* = 0.5$ for any shape factor (Proof is provided in the supplement A). The intuitive reason for this result is that a *CR* policy performs only SC retrievals; thus, the crane never travels to the input point. For a randomized storage policy, storage location decisions are made independent of pick positions. Therefore, the pick position located in the middle of the rack has the shortest expected travel time to conduct retrievals. Comparing Fig. 5-3(a) with 5-3(b), another observation for function f is that the difference between the largest and smallest pick position location's expected travel times increases by decreasing the b value. The optimal assignment of the SKUs to the pick positions for a *CR* policy are illustrated in Fig. 4(a), where the most demanded SKU is assigned to the pick position in the middle of the rack.

In contrast, a *Mixed* policy performs both SC and DC operations. Therefore, to perform a storage operation after a retrieval operation, the crane is required to travel from a pick position to the input point. Consequently, m^* shifts to the left and will be located between the input point location and the middle of the rack ($0 \leq m^* < 0.5$). For this case, we show the optimal assignment of SKUs in Fig. 5-4(b) (e.g. when $\alpha = 0.4$ and $b = 0.48$). When the function g is monotonically increasing, then m^* is equal to zero and the *MDD* assignment is optimal for the *Mixed* policy. We illustrate such a case (e.g. when $\alpha = 0.4$ and $b = 1$) in Fig 5-4(c). The *MDD* assignment is optimal for several combination of α and b; however, in Proposition 2, we prove this is always true for $\alpha = 0$ regardless of the shape factor. This occurs because when all of the operations are performed by DC ($\alpha = 0$), the crane has to travel back from a pick position to the input point at the end of each cycle. However, as α increases, the number of these returns to the input point decreases because the chance of consecutive SC storages or consecutive SC retrievals is higher with higher α values. As an example for the case of $\alpha = 0 \; or \; 0.4$ and $b = 0.48$, as shown in Fig. 5-3(b), m^* shifts to the right (from the location of the input point to a point towards the middle of the rack).



(a) Optimal SKU assignment for a *Consecutive Retrieval* policy ($m^* = 0.5$)

(b) Optimal SKU assignment for a *Mixed* policy (when $0 < m^* < 0.5$)

(c) Optimal SKU assignment for a *Mixed* policy (when $m^* = 0$)

Figure 5-4 : Typical optimal SKU assignment for different policies (X-axis represents the location of the pick positions and Y-axis represent the SKUs' demand)

5-5- Extreme travel time models for non-identical demand curves

In Section 5-4-1, we derived the expected travel time for discrete pick position locations for *CR* and *Mixed* policies under the *optimal* and *MDD* assignment (which we denote as *base models*). In this section, we derive closed-form travel time models for the *CR* and *Mixed* policies (which we denote as *extreme models)*. We derive extreme models by assuming 1) there are an infinite number of pick positions in the aisle, and 2) demand curves are continuous. The motivation for deriving extreme models is twofold. First, the closed-form solutions provide a computational efficient way to evaluate the expected travel time in an MIAPP-AS/RS with non-identical demand. Second, we use the models to derive optimal design parameters of an MIAPP-AS/RS with non-identical demand.

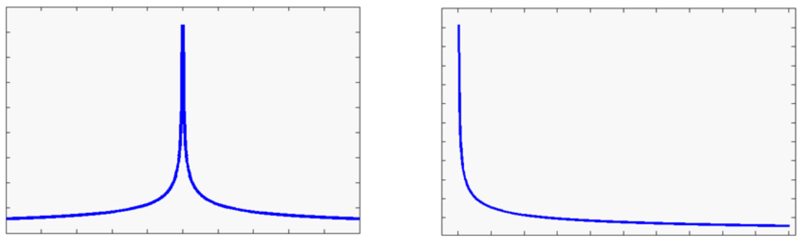
In the previous section, we showed that $m^* = 0.5$ is optimal for the *CR* policy regardless of the shape factor. Therefore, to approximate the optimal assignment for the *CR* policy shown in Fig. 5-4(a), we introduce a new continuous demand curve defined in equation (17) and denoted as $D_{OPT}(m, s)$. Let m and s denote the location of the pick position and demand curve skewness factor, respectively. As illustrated in Fig. 5-5(a), this curve assigns the highest demanded SKUs to locations in the middle of the rack. We use this curve to represent the optimal assignment for the *CR* policy.

$$D_{OPT}(m, s) = \begin{cases} s(1-2m)^{s-1} & for \quad 0 \leq m < {}^{1}\!/_{2} \\ s(2m-1)^{s-1} & for \quad {}^{1}\!/_{2} \leq m \leq 1 \\ 0 & otherwise \end{cases} \tag{17}$$

This demand curve is obtained by truncating and shifting the demand curve expressed in Hausman *et al.* (1976). Therefore, to represent the different demand curves (e.g. 20/20, 20/40, 20/60, 20/80, and 20/90), the same s values provided in Section 5-3 can be used. $D_{OPT}(m, s)$ is a valid probability distribution function (PDF) for $0 \leq m \leq 1$ and $0 \leq s \leq 1$ (Proof is provided in the supplement B).

We use the continuous monotonically decreasing demand curve defined in equation (18) to represent the *MDD* assignment. As illustrated in Fig. 5-5(b), this curve is the continuous representation of the *MDD* assignment shown in Fig. 5-4(c). This curve will be used for representing the *MDD* assignment and we derive extreme models for both *CR* and *Mixed* policies with an *MDD* assignment.

$$D_{MDD}(m, s) = sm^{s-1} \tag{18}$$



(a) Continuous approximate demand curve of optimal assignment for CR policy

(b) Continuous approximate demand curve of *MDD* assignment

Figure 5-5 : Continuous demand curves for different policies (based on 20/40 curve)

In Section 5-5-1 and 5-5-2, we will develop extreme travel time models for *CR* policy under *optimal* and *MDD* curves mentioned above. For the *Mixed* policy, m^* varies based on the value of α and b. In section 5-4-2, we showed that for several combination of α and b where $m^* = 0$, and the *MDD* assignment for a *Mixed* policy is the optimal assignment. Also, later in Section 5-6, we will show that for several cases where $m^* \neq 0$ (i.e., applying the *MDD assignment* is not optimal), we do not lose much efficiency compared to the optimal assignment due to the small *% differences* (see Section 5-6 - See Table 5-7). Therefore, in Section 5-5-3, we will only develop extreme travel time model for *Mixed* policy under *MDD* curve.

5-5-1- Extreme travel time models for *CR* policy under optimal assignment

We denote the extreme value of $E(W)$ under the optimal assignment as $E^{\infty}_{OPT}(W)$, which is calculated as in equation (19a). This extreme value is based on letting n go to infinity in equation (5), as well as using equation (17), which is an optimal assignment. Each of the integrals at the right hand side of the equation (19a) separate into two integrals as the value of

$E(W_m)$ changes for different m and b based on equation (4). The final closed-form result is given in (19b). See Appendix B for the complete derivation of $E_{OPT}^{\infty}(W)$ for the *CR* policy.

$$E_{OPT}^{\infty}(W) = \lim_{n\to\infty} \sum_{i=1}^{n} Pr_i \,.\, E(W_{m_i}) = \int_0^1 D_{OPT}(m,s)\, E(W_m) dm \tag{19a}$$

$$E_{OPT}^{\infty}(W) =$$

$$\begin{cases} \dfrac{3-3(1-2b)^s+6(1-2b)^s b\left(3-6b+4b^2\right)+2b(3+s)\left(3+3b(2+s)+6s(2+s)+2b^2(1+s)(2+s)\right)}{24b(1+s)(2+s)(3+s)} & for\ \ 0 \le b < {}^{1}\!/_{2} \\ \dfrac{-6b(-1+2b)^s\left(3-6b+4b^2\right)+3\left(1+(-1+2b)^s\right)+2b(3+s)\left(3+3b(2+s)+6s(2+s)+2b^2(1+s)(2+s)\right)}{24b(1+s)(2+s)(3+s)} & for\ \ {}^{1}\!/_{2} \le b \le 1 \end{cases} \tag{19b}$$

5-5-2- Extreme travel time models for *CR* policy under *MDD* assignment

Let $\boldsymbol{E_{MDD}^{\infty}(W)}$ denote the extreme value of $\boldsymbol{E(W)}$ under the *MDD* assignment. Following the same approach as in the previous section, by using the *MDD* assignment curve (given in equation (18)), $\boldsymbol{E_{MDD}^{\infty}(W)}$ can be calculated as equation (20a), which results in the closed-form expression given in equation (20b). Appendix C shows the complete derivation of $\boldsymbol{E_{MDD}^{\infty}(W)}$.

$$E_{MDD}^{\infty}(W) = \lim_{n\to\infty} \sum_{i=1}^{n} Pr_i \,.\, E(W_{m_i}) = \int_0^1 D_{MDD}(m,s) E(W_m)\, dm \tag{20a}$$

$$E_{MDD}^{\infty}(W) =$$

$$\frac{6-6(1-b)^s+b\left[18(1-b)^s-6b^{2+s}+3s(1+s)(3+s)+3b\left(6-6(1-b)^s+s(5+s)\right)+b^2\left(6\left(1+(1-b)^s\right)+s\left(11+s(6+s)\right)\right)\right]}{6b(1+s)(2+s)(3+s)} \tag{20b}$$

Note that $E_{OPT}^{\infty}(W)$ derived as equation (19b) varies from $E_{MDD}^{\infty}(W)$ in equation (20b) and are <u>not</u> replaceable because $E_{OPT}^{\infty}(W)$ is derived based on the continuous demand curve used for approximating optimal assignment for *CR* policy (equation (17)) and $E_{MDD}^{\infty}(W)$ is derived based on the *MDD* assignment continuous demand curve (equation (18)).

5-5-3- Extreme travel time models for *Mixed* policy under *MDD* assignment

To derive the extreme travel time model for the *Mixed* policy under an *MDD* assignment, note that $E(W)$ and $E(R)$ are the only components of $E(M)$ impacted by varying the location of the pick positions and their assigned SKU's demand (See equation (7)). Therefore, the extreme value of $E(M)$ under an MDD assignment can be obtained by replacing $E(W)$ and $E(R)$ with their extreme values. Let $E_{MDD}^{\infty}(W)$, $E_{MDD}^{\infty}(R)$, and $E_{MDD}^{\infty}(M)$ denote the extreme values of $E(W)$, $E(R)$, and $E(M)$, respectively. The extreme values are derived using the continuous demand curve for the *MDD* assignment given in equation (18). $E_{MDD}^{\infty}(W)$ is derived in the previous section as equation (20b), and $E_{MDD}^{\infty}(R)$ is derived as equation (21). Finally, by substituting $E_{MDD}^{\infty}(W)$ and $E_{MDD}^{\infty}(R)$ for $E(W)$ and $E(R)$, respectively, into equation (7), the extreme value of $E(M)$ under an *MDD* assignment is obtained as equation (22).

$$E_{MDD}^{\infty}(R) = \lim_{n\to\infty} \sum_{i=1}^{n} Pr_i m_i = \int_0^1 s m^{s-1} m \, dm = {}^{s}\!/_{s+1} \tag{21}$$

$$E_{MDD}^{\infty}(M) = \left(\frac{1+2\alpha}{2+2\alpha}\right) E(V) + \left(\frac{1}{2+2\alpha}\right) E(TB) + \left(\frac{1+2\alpha}{2+2\alpha}\right) E_{MDD}^{\infty}(W) + \left(\frac{1}{2+2\alpha}\right)\left(\frac{s}{s+1}\right) \tag{22}$$

5-5-4- Validation of the extreme travel time models

In this section the results of the extreme models developed in Section 5-5-1 to 5-5-3 are compared to results from a discrete-event simulation. The purpose is to understand the impact of the extreme models' assumptions (i.e., a continuous rack, an infinite number of pick positions, and continuous demand curves) on the expected travel time. We use MATLAB 2013a to code and run the simulation.

In the discrete-event simulation, we assume that the number of pick positions, as well as the number of SKUs are equal to the number of the columns. The openings in the first row of the rack are for pick positions and the other openings in the rack are for storage locations. We calculate the distance between any two openings' centroids using the *chebyshev* distance metric. We consider six different configurations as shown in Table 5-1. To make an equal comparison, we set the number of storage locations in each configuration to be approximately equal to 950. To simulate the expected travel time, we ran 5 replications of a sequence of 100,000 storage and retrieval operations for each 6 configurations, 5 demand curves, and 2 operating policies. In the simulation, we model discrete pick positions and SKU demand. The demand for each SKU is obtained by equation (1). To simulate the *CR* policy under optimal assignment, the optimal assignment of a discrete number of SKUs to pick positions is obtained using Proposition 1a, for each shape configuration. Also, we apply a *MDD* assignment to simulate the *CR* and *Mixed* policy under *MDD* assignment. These results are used as inputs to the simulation model. To compare the accuracy of the results obtained from the continuous extreme models versus the simulation, we calculate the '% deviation' as shown in equation (23).

$$\%Deviation = \frac{extreme\ model\ result - simulation\ model\ result}{simulation\ model\ result} \times 100 \tag{23}$$

Table 5-1: Structure of shape configurations

Rows	20	18	16	14	12	10
Columns	50	56	63	73	86	105
Openings	950	952	945	949	946	945
T (minutes)	0.5	0.56	0.63	0.73	0.86	1.05
b	1.000	0.804	0.635	0.479	0.349	0.238

We report the results obtained for the *CR* policy under an optimal assignment, the *CR* policy under an *MDD* assignment and the *Mixed* policy under the *MDD* assignment in Table 5-2, Table 5-3 and Table 5-4, respectively. For the simulation results, we report the average and variance of the travel time 'per operation' for each configuration (as 'Sim. Mean' and 'Sim. variance' in Tables 5-2 to 5-4). The denormalized expected travel time for the extreme models are calculated as $2T \times E_{OPT}^{\infty}(W)$ using equations (19b) for the *CR* policy under an optimal assignment; as $2T \times E_{MDD}^{\infty}(W)$ using equation (20b) for the *CR* policy under an *MDD* assignment, and as $T \times E_{MDD}^{\infty}(M)$ using equation (22) for the *Mixed* policy under the *MDD* assignment.

Table 5-2 : Simulation vs. extreme travel time results for a CR Policy under an optimal assignment

	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	Optimal	0.583	0.566	0.562	0.585	0.637	0.737
	MDD	0.583	0.566	0.562	0.585	0.637	0.737
	% diff.	0.000	0.000	0.000	0.000	0.000	0.000
20-40	Optimal	0.569	0.547	0.536	0.549	0.591	0.680
	MDD	0.596	0.583	0.586	0.616	0.677	0.789
	% diff.	4.657	6.727	9.317	12.180	14.472	16.040
20-60	Optimal	0.559	0.532	0.516	0.522	0.557	0.636
	MDD	0.614	0.608	0.619	0.660	0.734	0.862
	% diff.	9.856	14.358	20.079	26.573	31.894	35.639
20-80	Optimal	0.550	0.519	0.498	0.498	0.527	0.598
	MDD	0.636	0.639	0.660	0.715	0.804	0.954
	% diff.	15.674	23.009	32.471	43.484	52.708	59.375
20-90	Optimal	0.546	0.514	0.490	0.487	0.513	0.581
	MDD	0.648	0.656	0.683	0.745	0.844	1.005
	% diff.	18.825	27.738	39.320	52.969	64.520	72.979

Table 5-3 : Simulation vs. extreme travel time results for a CR Policy under an optimal assignment

	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	Sim. mean	0.579	0.562	0.559	0.582	0.634	0.735
	Sim. variance	0.048	0.039	0.037	0.047	0.071	0.117
	Extreme model	0.583	0.566	0.562	0.585	0.637	0.737
	%dev.	0.824	0.797	0.691	0.486	0.486	0.273
20-40	Sim. mean	0.591	0.579	0.582	0.613	0.675	0.786
	Sim. variance	0.048	0.041	0.043	0.058	0.089	0.146
	Extreme model	0.596	0.584	0.586	0.616	0.677	0.789
	%dev.	0.843	0.728	0.732	0.593	0.370	0.352
20-60	Sim. mean	0.609	0.604	0.616	0.658	0.732	0.860
	Sim. variance	0.048	0.046	0.054	0.077	0.120	0.198
	Extreme model	0.615	0.609	0.620	0.662	0.735	0.864
	%dev.	0.886	0.807	0.712	0.596	0.441	0.359
20-80	Sim. mean	0.631	0.635	0.657	0.712	0.802	0.952
	Sim. variance	0.050	0.053	0.068	0.102	0.160	0.263
	Extreme model	0.638	0.642	0.663	0.718	0.808	0.957
	%dev.	1.071	0.998	0.916	0.854	0.660	0.541
20-90	Sim. mean	0.644	0.652	0.680	0.743	0.842	1.004
	Sim. variance	0.051	0.057	0.076	0.116	0.182	0.298
	Extreme model	0.652	0.660	0.688	0.751	0.849	1.011
	%dev.	1.254	1.173	1.120	0.993	0.879	0.715

Table 5-4 : Simulation vs. extreme travel time for the Mixed Policy (α $=0.4$) under an MDD assignment

	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	Sim. Mean	0.569	0.581	0.608	0.664	0.753	0.895
	Sim. Variance	0.223	0.235	0.263	0.328	0.438	0.639
	Extreme model	0.574	0.585	0.611	0.667	0.754	0.897
	%dev.	0.904	0.697	0.595	0.439	0.207	0.152
20-40	Sim. Mean	0.549	0.559	0.584	0.638	0.722	0.861
	Sim. Variance	0.211	0.221	0.249	0.308	0.411	0.602
	Extreme model	0.554	0.564	0.588	0.641	0.725	0.862
	%dev.	0.875	0.730	0.590	0.474	0.365	0.129
20-60	Sim. Mean	0.533	0.543	0.568	0.621	0.704	0.838
	Sim. Variance	0.202	0.212	0.239	0.297	0.397	0.582
	Extreme model	0.538	0.547	0.571	0.624	0.706	0.840
	%dev.	0.948	0.803	0.609	0.472	0.304	0.204
20-80	Sim. Mean	0.520	0.530	0.555	0.608	0.691	0.825
	Sim. Variance	0.194	0.205	0.232	0.290	0.390	0.573
	Extreme model	0.524	0.534	0.558	0.611	0.693	0.826
	%dev.	0.887	0.795	0.636	0.500	0.298	0.127
20-90	Sim. Mean	0.513	0.524	0.549	0.603	0.685	0.818
	Sim. Variance	0.191	0.202	0.229	0.288	0.387	0.571
	Extreme model	0.518	0.528	0.553	0.606	0.688	0.820
	%dev.	0.940	0.820	0.699	0.519	0.336	0.232

From Tables 5-2, 5-3, and 4, the '% deviation' measures between the simulation results and extreme models are less than 1.3% in all cases. Therefore, the extreme models can be used to represent a discrete rack with a finite number of pick positions without substantial loss of accuracy. We also calculate the % differences between the extreme and discrete models for both the *CR* and *Mixed* policies. The differences are extremely small and less than 0.1% in all cases; therefore, the continuous approximations of the number of the pick positions and demand profiles do not have much impact on the expected travel times compared to the case of a discrete number of pick positions and discrete demand profiles. Based on the mentioned facts, for the *CR* policy, the extreme models expressed in equation (19b) and (20b), can be substituted for the discrete models to estimate the expected travel time with an *optimal* and *MDD* assignment, respectively. Also, for the *Mixed* policy, the extreme model developed as equation (22) can be used to estimate the expected travel time of *MDD assignment*.

5-5-5- Optimal Shape factors

We use the extreme models developed in the Section 5-5-1 to 5-5-3 to derive the optimal shape factor that minimizes the expected travel time for different combinations of operating and assignment policies. We consider three operating policies: *CR* (with *optimal* and *MDD* assignment), *CR then CS* (with optimal and *MDD* assignment), and *Mixed* (with *MDD* assignment). The expected travel time for these policies can be calculated as $2T\times\ E_{OPT}^{\infty}(W)$, $2T\times\ E_{MDD}^{\infty}(W)$, $T\times(\ E_{OPT}^{\infty}(W)+E(V))$, $T\times(\ E_{MDD}^{\infty}(W)+E(V))$, and $T\times\ E_{MDD}^{\infty}(M)$, respectively. Note that the optimal assignment for *CR then CS* policy is the same as optimal assignment for *CR* policy because the performance of a *CS* policy is not impacted by the SKU assignment. To find the optimal shape factor for each policy, we consider a system that has constant area space (denoted as A) and the equality constraint of $A = bT^2$. By substituting T as $\sqrt{A/b}$ in the extreme travel time models, we can find the optimal shape factor, denoted as b^*, which achieves the minimum expected travel time for the range of b ($0 < b \leq 1$). Because of the complexity of the extreme models, we use the '*fminbnd*' function of MATLAB 2013a, which applies numerical search methods to find the minimum of a function for a given range of a variable. The results of b^* for different policies and demand curves are shown in Table 5-5. For the *CR* policy and the *CR then CS* policy using an optimal assignment, increasing the skewness of the demand curve results in decreasing the optimal shape factor. However, for these policies with an *MDD* assignment, the optimal shape factor increases when the skewness of the demand curve increases. For the same demand skewness the corresponding optimal shape factor for *CR then CS* policy is greater than a *CR* policy. This occurs because when we are only performing storages (a *CS* policy), the optimal shape factor is equal to 1.0 (Bozer and White (1984)); therefore, for a *CR then CS* policy $b^*$is between the optimal shape of a *CR* policy and 1.0. Also, we observe that $b^* = 1$ for a *Mixed* policy under a *MDD* assignment for any demand curve skewness.

Table 5-5 : Optimal Shape factor for different policies

Policy	Demand curve	b^*	
		optimal assignment	*MDD* assignment
CR policy			
	20-20	0.682	0.682
	20-40	0.617	0.737
	20-60	0.568	0.814
	20-80	0.528	0.905
	20-90	0.514	0.953
CR then CS policy			
	20-20	0.853	0.853
	20-40	0.823	0.879
	20-60	0.799	0.914
	20-80	0.777	0.955
	20-90	0.767	0.977
Mixed policy			
	20-20	N/A	1.000
	20-40	N/A	1.000
	20-60	N/A	1.000
	20-80	N/A	1.000
	20-90	N/A	1.000

5-6- Numerical Results and Managerial Insights

In this section, we conduct a numerical study and provide the numerical results for the *CR* and *Mixed* policies under *optimal* and *MDD* assignments using the models derived in Section 5-4 and 5-5. We also provide the percentage difference between the *optimal* and *MDD* assignments expected travel time to identify how much efficiency is lost by adapting the *MDD* assignment over the *optimal* assignment. We consider the same six different shape configurations and five different demand curves as in Section 5-5-4.

In Tables 5-6 and 5-7, we calculate the expected 'per operation' travel times with respect to the optimal assignment of SKUs to pick positions versus the *MDD* assignment for different demand curves and different shape configurations. In Table 5-6, the expected 'per operation' travel time for *CR* policy is calculated by the extreme models as $2T \times E_{OPT}^{\infty}(W)$ and $2T \times E_{MDD}^{\infty}(W)$ for the *optimal* and *MDD* assignments (as 'OPT extreme' and 'MDD extreme'), respectively. In Table 5-7, the expected 'per operation' travel time for the *Mixed* policy (for $\alpha = 0.4$, 0.6, and 1.0) with optimal assignment is calculated by the discrete base model as $T \times E^{*}(M)$ using equation (16) (as 'OPT base'), and for the *Mixed* policy with the *MDD* assignment is calculated by the extreme model as $2T \times E_{MDD}^{\infty}(M)$ using equation (22) (as 'MDD extreme'). Note that all of the extreme models are based on continuous demand curves that approximate the *optimal* and *MDD* assignments. In Table 5-7, we do not provide the results for $\alpha = 0$, as we proved in Proposition 2 that results for the *optimal* and *MDD* assignment are equal for the *Mixed* policy with $\alpha = 0$. For demand profiles that have identical demand (i.e., 20/20 curves), the assignment problem does not impact the expected travel times and thus the difference between the *optimal* and *MDD* assignment is zero for both *CR* and *Mixed* policies.

Table 5-6 and 5-7 show that when an *optimal* assignment is adopted, increasing the skewness of the demand curve improves the expected travel time for any policy or configuration. Intuitively, when demand curve skewness increases, the pick positions with shorter expected travel time are visited more frequently, because the SKUs assigned to these pick positions have higher levels of demand. However, when the *MDD* assignment is applied for the *CR* policy, the expected travel degrades by increasing the demand curve skewness. The reason results from $m^{*} = 0.5$ for the *CR* policy (as mentioned in section 5-4-2). That is, the pick position located in the middle of the rack has the shortest expected travel time. Thus, the *MDD* assignment is far from optimal because it allocates the highest demanded SKUs to pick positions nearest to the input point (which as shown in Fig. 5-4(c)) have the highest expected travel time in a *CR* policy).

For all operating and assignment policies, the expected travel time increases as the shape of the rack become more rectangular (i.e. b decreases). This occurs because the difference between the highest and shortest expected travel time increases by decreasing the b value (see Fig. 5-3(a) and Fig. 5-3(b)). For these reasons, when a distribution center adopts a *MDD* assignment instead of the *optimal* assignment and uses a *CR* policy, it loses more efficiency by increasing the skewness of the demand and decreasing the shape factor.

From Table 5-7, the same observation can be observed for a *Mixed* policy. However, the % differences between the *optimal* and *MDD* assignment are much smaller compared to the *CR* policy (i.e., all are less than 8%). For the *Mixed* policy, we have $0 \leq m^* < 0.5$. As m^* becomes closer to zero, the % difference between the optimal assignment and the *MDD* assignment becomes smaller as the pick positions with the shortest expected travel time are located closer to the input point where the highest demanded SKUs are assigned in a *MDD* assignment (See Fig. 5-4(a) and (b)). We observe that the % difference increases by increasing α, increasing the skewness of the demand (s), and decreasing the shape factor (b). Therefore, the high % differences occur in extreme cases of system parameters (e.g. for highly skewed demand curves or relatively small shape factor configuration), and for most of the practical cases of α, b, and s, the % differences are relatively small (less that 5%). In these cases, a system will not lose considerable throughput by implementing a *MDD* assignment instead of an *optimal* assignment for *Mixed* policies.

Table 5-6 : Expected travel time for CR Policy (optimal vs. MDD assignment)

	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	OPT extreme	0.583	0.566	0.562	0.585	0.637	0.737
	MDD extreme	0.583	0.566	0.562	0.585	0.637	0.737
	% diff.	0.0	0.0	0.0	0.0	0.0	0.0
20-40	OPT extreme	0.569	0.547	0.536	0.549	0.591	0.680
	MDD extreme	0.596	0.584	0.586	0.616	0.677	0.789
	% diff.	4.7	6.8	9.4	12.2	14.5	16.1
20-60	OPT extreme	0.559	0.532	0.516	0.522	0.556	0.636
	MDD extreme	0.615	0.609	0.620	0.662	0.735	0.864
	% diff.	10.0	14.5	20.3	26.8	32.1	35.8
20-80	OPT extreme	0.550	0.519	0.498	0.498	0.527	0.598
	MDD extreme	0.638	0.642	0.663	0.718	0.808	0.957
	% diff.	16.1	23.5	33.1	44.1	53.4	60.0
20-90	OPT extreme	0.546	0.514	0.490	0.487	0.513	0.581
	MDD extreme	0.652	0.660	0.688	0.751	0.849	1.011
	% diff.	19.5	28.5	40.3	54.0	65.6	74.0

In Fig. 5-6, we illustrate the impact of demand curve skewness on the performance of different policies. Both for the *optimal* and *MDD* assignments, a *CR then CS* policy has lower performance on a per operation basis than a *Mixed* policy for all of the demand curves. Also, for the mixed policy, decreasing the α value results in performance improvement because decreasing α decreases the probability of empty trips that occur from a pick position back to the input point. However, we observe that increasing the demand curve skewness increases the difference between policies' performance. As an example, for the *optimal* assignment, the difference between a *CR* then *CS* policy and a *Mixed* policy with $\alpha = 0$ is 18.1% for a 20-40 curve, but the difference is equal to 31.1% for a 20-90 curve. Moreover, the differences between policies are even higher for the *MDD* assignment, e.g., the difference between the *CR then CS* policy and *Mixed* policy with $\alpha = 0$ is 20.7% for a 20-40 curve, but the difference is equal to 42.2% for a 20-90 curve.

Table 5-7 : Expected travel time for Mixed Policy (optimal vs. MDD assignment)

$\alpha = 0.4$	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	OPT base	0.574	0.585	0.611	0.667	0.754	0.897
	MDD extreme	0.574	0.585	0.611	0.667	0.754	0.897
	% diff.	0.0	0.0	0.0	0.0	0.0	0.0
20-40	OPT base	0.554	0.564	0.588	0.640	0.723	0.858
	MDD extreme	0.554	0.564	0.588	0.641	0.725	0.862
	% diff.	0.0	0.0	0.0	0.2	0.3	0.4
20-60	OPT base	0.538	0.547	0.571	0.621	0.700	0.831
	MDD extreme	0.538	0.547	0.571	0.624	0.706	0.840
	% diff.	0.0	0.0	0.2	0.5	0.8	1.1
20-80	OPT base	0.525	0.534	0.557	0.606	0.682	0.808
	MDD extreme	0.525	0.534	0.558	0.611	0.693	0.826
	% diff.	0.1	0.0	0.3	0.9	1.5	2.1
20-90	OPT base	0.518	0.528	0.550	0.599	0.674	0.799
	MDD extreme	0.518	0.528	0.553	0.606	0.688	0.820
	% diff.	0.0	0.0	0.4	1.2	2.0	2.7
$\alpha = 0.7$	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	OPT base	0.583	0.592	0.616	0.670	0.756	0.898
	MDD extreme	0.583	0.592	0.616	0.670	0.756	0.898
	% diff.	0.0	0.0	0.0	0.0	0.0	0.0
20-40	OPT base	0.568	0.575	0.597	0.648	0.729	0.865
	MDD extreme	0.568	0.576	0.599	0.652	0.736	0.874
	% diff.	0.0	0.1	0.3	0.6	0.9	1.0
20-60	OPT base	0.556	0.563	0.584	0.632	0.710	0.841
	MDD extreme	0.556	0.565	0.588	0.642	0.725	0.862
	% diff.	0.1	0.3	0.8	1.5	2.1	2.5
20-80	OPT base	0.546	0.553	0.573	0.619	0.695	0.822
	MDD extreme	0.547	0.556	0.581	0.636	0.721	0.859
	% diff.	0.1	0.7	1.6	2.7	3.7	4.5
20-90	OPT base	0.542	0.548	0.568	0.614	0.688	0.813
	MDD extreme	0.543	0.553	0.579	0.634	0.720	0.859
	% diff.	0.2	0.9	2.0	3.4	4.7	5.7
$\alpha = 1.0$	b	1.000	0.804	0.635	0.479	0.349	0.238
20-20	OPT base	0.590	0.597	0.619	0.673	0.758	0.899
	MDD extreme	0.590	0.597	0.619	0.673	0.758	0.899
	% diff.	0.0	0.0	0.0	0.0	0.0	0.0
20-40	OPT base	0.577	0.582	0.603	0.652	0.733	0.868
	MDD extreme	0.577	0.584	0.607	0.659	0.743	0.882
	% diff.	0.1	0.3	0.6	1.0	1.4	1.6
20-60	OPT base	0.567	0.572	0.591	0.638	0.716	0.846
	MDD extreme	0.569	0.577	0.600	0.654	0.739	0.878
	% diff.	0.3	0.8	1.6	2.5	3.2	3.8
20-80	OPT base	0.560	0.564	0.581	0.626	0.701	0.828
	MDD extreme	0.563	0.572	0.598	0.653	0.740	0.882
	% diff.	0.6	1.5	2.8	4.3	5.6	6.5
20-90	OPT base	0.556	0.560	0.577	0.621	0.694	0.820
	MDD extreme	0.560	0.571	0.597	0.655	0.743	0.886
	% diff.	0.7	1.9	3.5	5.4	7.0	8.1

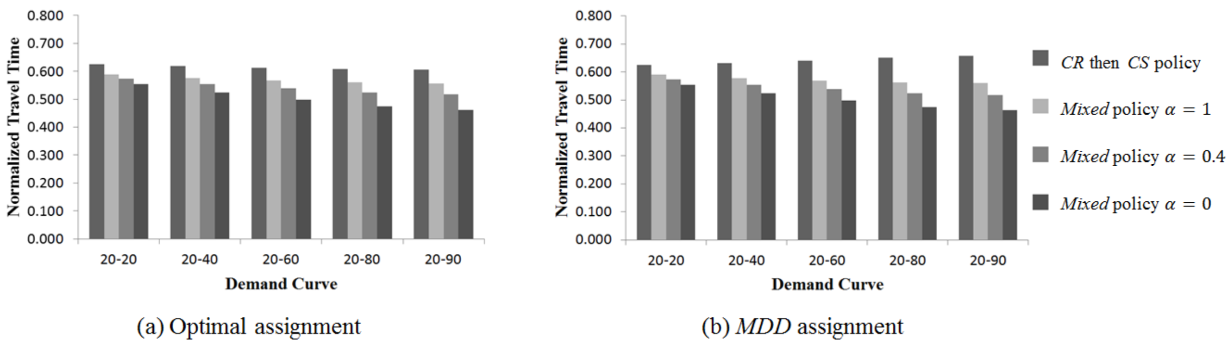


Figure 5-6: Normalized travel time per operation for Mixed policy ($b = 1$)

5-7- Conclusion and future research directions

This paper investigated the effect of assigning the most-active SKUs to the best pick positions in an automated storage/retrieval system (AS/RS) with multiple in-the-aisle pick positions (MIAPP-AS/RS). We presented mathematical models to find the optimal assignment of SKUs with non-identical demand to pick positions that minimizes the expected travel time for MIAPP-AS/RS under different operating policies. We provided an optimal SKU to pick position procedure of complexity $\boldsymbol{O(nlog(n))}$**.** We also derived a continuous demand curve that can be used to model the optimal assignment in a *CR* policy. Based on the proposed continuous demand curves, we derived closed-form expressions for a *CR* and a *Mixed* policy that approximate the expected travel time by assuming there are an infinite number of pick positions, as well as continuous demand curves. We validated these continuous extreme models through a set of discrete-event simulations that enforce the discreteness of the rack and observed that all of *% deviations* were less that 1%. This showed that our continuous extreme models can approximate a discrete rack accurately. Also, by comparing the results obtained from the extreme models versus the discrete models under the same assignment, we observed less than 0.1 *% difference* for all cases. Using extreme models, we calculated the optimal shape configurations for each demand curve and operating policy.

We analyzed structural results of the optimal assignment problem. For a *CR* policy with an optimal assignment, we observed that regardless of the shape factor the most demanded SKU is assigned to the pick position located in the middle of the rack, and the next most demanded SKU is assigned to the next closest pick position to the middle of the rack and so on. For the *Mixed* policy the smallest expected travel time occurs at a pick position located between the input point and the middle of the rack.

We explored the impact of different SKU assignments on the expected throughput rates that can be obtained under different system configurations, demand curves, and operating policies for peak and non-peak hours. We compared the results obtained from the *optimal* assignment with an easy-to-implement assignment that is often seen in practice -- the *MDD* assignment, which assigns the highest demanded SKUs to the locations closest to the input point. We observed that maximum % difference between the optimal and the *MDD* assignment for a *Mixed* policy was less that 8%, and in most practical MIAPP-AS/RS configurations less than 5%.

As future research directions, this work can be extended by considering different storage policies (such as turnover-based storage and class-based storage policy), different dwell point strategies, and different sequencing rules. Applying these directions can help to increase understanding of the impact of different storage policies and demand skewness on the throughput and space utilization of MIAPP-AS/RS.

Appendix A. *MDD* assignment optimality condition for *Mixed* policy with $\alpha = 0$

According to Proposition1b, the *MDD* assignment is an optimal assignment for the *Mixed* policy if function g is a monotonically increasing function. $g = \frac{1}{2}[E(W_m) + m]$ when $\alpha = 0$. The sufficient condition for g to be monotonically increasing is $\frac{dg}{dm} \geq 0$, which is equivalent to $\frac{dE(W_m)}{dm} + 1 \geq 0$. According to equation (4), obtaining the values for $Min(m, b)$, $Min(1 - m, b)$, $Max(m, b)$, and $Max(1 - m, b)$ to calculate $E(W_m)$ requires finding the relation between m and $(1 - m)$, and b. To do so, we consider two ranges of b value and calculate the $E(W_m)$ as equations (A.1) and (A.2). We checked that the sufficiency condition mentioned above holds for any relation between b and m in equations (A.1) and (A.2).

For $0 < b < 1/2$,

$$E(W_m) = \begin{cases} m^3/6b + mb/2 + b^2/6 + (1-m)^2/2 & for \quad m < b \\ b^2/3 + m^2/2 + (1-m)^2/2 & for \quad b \leq m < 1 - b \\ b^2/6 + m^2/2 + (1-m)^3/6b + (1-m)b/2 & for \quad 1 - b \leq m \leq 1 \end{cases} \tag{A.1}$$

For $1/2 \leq b \leq 1$,

$$E(W_m) = \begin{cases} m^3/6b + mb/2 + b^2/6 + (1-m)^2/2 & for \quad m < 1 - b \\ m^3/6b + (1-m)^3/6b + b/2 & for \quad 1 - b \leq m < b \\ b^2/6 + m^2/2 + (1-m)^3/6b + (1-m)b/2 & for \quad b \leq m \leq 1 \end{cases} \tag{A.2}$$

Appendix B. Derivation of $E_{OPT}^{\infty}(W)$ for the CR policy

We use equation (17) to substitute for $D_{OPT}(m, s)$, and equations (A.1) and (A.2) to substitute for $E(W_m)$ in equation (19a).

For $0 < b < 1/2$,

$E_{OPT}^{\infty}(W) =$

$$\int_{m=0}^{b} s(1-2m)^{s-1}\left\{m\left(\frac{m^2}{6b}+\frac{b}{2}\right)+(1-m)\left[\frac{b^2}{6(1-m)}+\frac{(1-m)}{2}\right]\right\} dm +$$

$$\int_{m=b}^{0.5} s(1-2m)^{s-1}\left\{m\left(\frac{b^2}{6m}+\frac{m}{2}\right)+(1-m)\left[\frac{b^2}{6(1-m)}+\frac{(1-m)}{2}\right]\right\} dm +$$

$$\int_{0.5}^{1-b} s(2m-1)^{s-1}\left\{m\left(\frac{b^2}{6m}+\frac{m}{2}\right)+(1-m)\left[\frac{b^2}{6(1-m)}+\frac{(1-m)}{2}\right]\right\} dm +$$

$$\int_{m=1-b}^{1} s(2m-1)^{s-1}\left\{m\left(\frac{b^2}{6m}+\frac{m}{2}\right)+(1-m)\left[\frac{(1-m)^2}{6b}+\frac{b}{2}\right]\right\} dm =$$

$$\frac{3-3(1-2b)^s+6(1-2b)^s b\left(3-6b+4b^2\right)+2b(3+s)\left(3+3b(2+s)+6s(2+s)+2b^2(1+s)(2+s)\right)}{24b(1+s)(2+s)(3+s)}$$

For $1/2 \leq b \leq 1$,

$E_{OPT}^{\infty}(W) =$

$$\int_{m=0}^{1-b} s(1-2m)^{s-1}\left\{m\left(\frac{m^2}{6b}+\frac{b}{2}\right)+(1-m)\left[\frac{b^2}{6(1-m)}+\frac{(1-m)}{2}\right]\right\} dm +$$

$$\int_{m=1-b}^{0.5} s(1-2m)^{s-1}\left\{m({m^2}/{6b}+{b}/{2})+(1-m)\left[{(1-m)^2}/{6b}+{b}/{2}\right]\right\} dm +$$

$$\int_{0.5}^{b} s(2m-1)^{s-1}\left\{m({m^2}/{6b}+{b}/{2})+(1-m)\left[{(1-m)^2}/{6b}+{b}/{2}\right]\right\} dm +$$

$$\int_{m=b}^{1} sm^{s-1}\left\{m({b^2}/{6m}+{m}/{2})+(1-m)\left[{(1-m)^2}/{6b}+{b}/{2}\right]\right\} dm =$$

$$\frac{-6b(-1+2b)^s(3-6b+4b^2)+3(1+(-1+2b)^s)+2b(3+s)\left(3+3b(2+s)+6s(2+s)+2b^2(1+s)(2+s)\right)}{24b(1+s)(2+s)(3+s)}$$

Appendix C. Derivation of $E_{MDD}^{\infty}(W)$ for the *Mixed* policy

We use equation (18) to substitute for $D_{MDD}(m,s)$, and equations (A.1) and (A.2) to substitute for $E(W_m)$ in equation (20a).

For $0 < b < {1}/{2}$,

$$E_{MDD}^{\infty}(W) = \int_{m=0}^{b} sm^{s-1}\left\{m\left({m^2}/{6b}+{b}/{2}\right)+(1-m)\left[{b^2}/{6(1-m)}+{(1-m)}/{2}\right]\right\} dm +$$

$$\int_{m=b}^{1-b} sm^{s-1}\left\{m\left({b^2}/{6m}+{m}/{2}\right)+(1-m)\left[{b^2}/{6(1-m)}+{(1-m)}/{2}\right]\right\} dm +$$

$$\int_{m=1-b}^{1} sm^{s-1}\left\{m\left({b^2}/{6m}+{m}/{2}\right)+(1-m)\left[{(1-m)^2}/{6b}+{b}/{2}\right]\right\} dm =$$

$$\int_{m=0}^{b} s\left\{{m^{s+2}}/{6b}+{m^s b}/{2}+{m^{s-1}b^2}/{6}+{m^{s-1}(1-m)^2}/{2}\right\} dm + \int_{m=b}^{1-b} sm^{s-1}\left\{m\left({b^2}/{6m}+\right.\right.$$

$$\left.{}^{m}/_{2}\right) + (1-m)\left[{}^{b^2}/_{6(1-m)} + {}^{(1-m)}/_{2}\right]\Big\} dm + \int_{m=1-b}^{1} sm^{s-1}\left\{m\left({}^{b^2}/_{6m} + {}^{m}/_{2}\right) + \right.$$

$$\left.(1-m)\left[{}^{(1-m)^2}/_{6b} + {}^{b}/_{2}\right]\right\} dm =$$

$$\frac{6-6(1-b)^s+b\left[18(1-b)^s-6b^{2+s}+3s(1+s)(3+s)+3b\left(6-6(1-b)^s+s(5+s)\right)+b^2(6(1+(1-b)^s)+s(11+s(6+s)))\right]}{6b(1+s)(2+s)(3+s)}$$

For ${}^{1}/_{2} \leq b \leq 1$,

$$E_{MDD}^{\infty}(W) = \int_{m=0}^{1-b} sm^{s-1}\left\{m({}^{m^2}/_{6b} + {}^{b}/_{2}) + (1-m)\left[{}^{b^2}/_{6(1-m)} + {}^{(1-m)}/_{2}\right]\right\} dm +$$

$$\int_{m=1-b}^{b} sm^{s-1}\left\{m({}^{m^2}/_{6b} + {}^{b}/_{2}) + (1-m)\left[{}^{(1-m)^2}/_{6b} + {}^{b}/_{2}\right]\right\} dm +$$

$$\int_{m=b}^{1} sm^{s-1}\left\{m({}^{b^2}/_{6m} + {}^{m}/_{2}) + (1-m)\left[{}^{(1-m)^2}/_{6b} + {}^{b}/_{2}\right]\right\} dm =$$

$$\frac{6-6(1-b)^s+b\left[18(1-b)^s-6b^{2+s}+3s(1+s)(3+s)+3b\left(6-6(1-b)^s+s(5+s)\right)+b^2(6(1+(1-b)^s)+s(11+s(6+s)))\right]}{6b(1+s)(2+s)(3+s)}$$

Supplement A. Optimal m for CR Policy

$E(W_m)$ is calculated as equations (A.1) and (A.2) for two cases of b ($0 < b < {}^{1}/_{2}$ and ${}^{1}/_{2} \leq b \leq 1$). Equations (A.1) and (A.2) are continuous piecewise functions. To find the optimal m, we follow this procedure for each range of b.

STEP 1) Find m^* for each range of m by setting $\frac{dE(W_m)}{dm} = 0$, then solve for m and calculate the value of $E(W_{m^*})$ if the m^* is in the given range.

STEP 2) Calculate the value of $E(W_m)$ for the beginning and ending of each range of m.

STEP 3) Compare the results from STEP 1 and 2 to find the m^*.

The results of both ranges of b ($0 < b < {}^1/_2$ and ${}^1/_2 \leq b \leq 1$) are shown in Table A1 and A2, respectively. For both cases, for the first and third range of m, the calculated m^* is not in the range; therefore, we calculate the $E(W_{m^*})$ only for the second range of m, and calculate the boundaries value for $E(W_m)$ for the first and third ranges. Finally, we compare the results obtained in the previous steps. The minimum $E(W_m)$ is equal to ${}^{b^2}/_3 + {}^1/_4$, corresponding to $m^* = {}^1/_2$.

Table A1. Results for calculating m^* for $0 < b < {}^1/_2$

Interval	m^*		Boundaries value for $E(W_m)$		$E(W_{m^*})$
$0 \leq m < b$	$m^* = \sqrt{2b} - b$ (*not* in range)	$m^* = -\sqrt{2b} - b$ (*not* in range)	$E(W_0) = +\infty$	$E(W_b) = {}^{5b^2}/_6 + {}^{(1-b)^2}/_2$	−
$b \leq m < 1 - b$	$m^* = {}^1/_2$		$E(W_b) = {}^{5b^2}/_6 + {}^{(1-b)^2}/_2$	$E(W_{1-b}) = {}^{5b^2}/_6 + {}^{(1-b)^2}/_2$	$E(W_{0.5}) = {}^{b^2}/_3 + {}^1/_4$
$1 - b \leq m < 1$	$m^* = 1 - \sqrt{2b} + b$ (*not* in range)	$m^* = 1 + \sqrt{2b} + b$ (*not* in range)	$E(W_{1-b}) = {}^{5b^2}/_6 + {}^{(1-b)^2}/_2$	$E(W_1) = {}^{b^2}/_6 + {}^1/_2$	−

Table A2. Results for calculating m^* for ${}^{1}/{}_{2} \leq b \leq 1$

Interval	m^*		Boundaries value for $E(W_m)$		$E(W_{m^*})$
$0 \leq m < 1-b$	$m^* = \sqrt{2b} - b$ (*not* in range)	$m^* = -\sqrt{2b} - b$ (*not* in range)	$E(W_0) = +\infty$	$E(W_{1-b}) = {}^{5b^2}/{}_{6} + {}^{(1-b)^2}/{}_{2}$	$-$
$1-b \leq m < b$	$m^* = {}^{1}/{}_{2}$		$E(W_{1-b}) = {}^{5b^2}/{}_{6} + {}^{(1-b)^2}/{}_{2}$	$E(W_b) = {}^{5b^2}/{}_{6} + {}^{(1-b)^2}/{}_{2}$	$E(W_{0.5}) = {}^{b^2}/{}_{3} + {}^{1}/{}_{4}$
$b \leq m < 1$	$m^* = 1 - \sqrt{2b} + b$ (*not* in range)	$m^* = 1 + \sqrt{2b} + b$ (*not* in range)	$E(W_b) = {}^{5b^2}/{}_{6} + {}^{(1-b)^2}/{}_{2}$	$E(W_1) = {}^{b^2}/{}_{6} + {}^{1}/{}_{2}$	$-$

Supplement B. Proof of $D_{OPT}(m, s)$ is a Valid Probability Distribution Function

Any continuous probability distribution function (PDF), $f(x)$, must satisfy the two following conditions. $f(x) \geq 0 \; \forall x \epsilon \mathbb{R}$, and $\int_{-\infty}^{+\infty} f(x)dx = 1$ (Mendenhall & Sincich, 2006). We show that both conditions hold for $D_{OPT}(m, s)$ as follows.

1) $D_{OPT}(m, s) \geq 0$ for $0 \leq m \leq 1$ and $0 \leq s \leq 1$.

2) $\int_{-\infty}^{+\infty} D_{OPT}(m, s)dx = \int_{0}^{0.5} s(1-2m)^{s-1}\, dm + \int_{0.5}^{1} s(2m-1)^{s-1}\, dm = {}^{1}/{}_{2} + {}^{1}/{}_{2} = 1.$

CHAPTER SIX: CONTRIBUTION 3- “ANALYTICAL MODELS FOR MIAPP-AS/RS UNDER DEDICATED AND CLASS –BASED STORAGE POLICY”

We study the impact of dedicated and class-based storage policies on the throughput performance of MIAPP-AS/RS. We develop mathematical optimization models to minimize the travel time of the crane by changing the assignment of the SKUs to pick positions and storage locations simultaneously. We develop a more tractable solution approach by applying a Benders decomposition approach, as well as an accelerated procedure for Benders algorithm. We observe high degeneracy in the optimal solution when a chebyshev metric is used to calculate the distances. As a result of this degeneracy, we realize that the assignment of SKUs to pick positions does not impact the optimal solution in a dedicated storage policy. We also developed closed-form travel time models for MIAPP-AS/RS under a class-based storage policy.

6-1- Introduction

Distribution centers and warehouses are generally operated under three storage policies: randomized storage, dedicated storage, and class-based storage. In randomized storage, SKUs are assigned to the storage locations on a random basis (e.g., SKUs are stored based on availability of storage location at the time of storage request). The advantage of a randomized storage policy is efficient space utilization. In Chapter 4 and Chapter 5, we developed several travel time and optimization models when the system is operated under randomized storage. In a dedicated storage policy, the specific storage locations are assigned to a specific SKU. If high demanded SKUs are dedicated to the most convenient locations, a dedicated storage policy is expected to result in minimum travel time and material handling cost required for storage and retrieval operations (Lee & Elsayed, 2005)). A class-based storage policy is a hybrid policy that shares the advantages of the two other policies. Based on some criteria (such as demand, correlation, size, etc), each SKU is assigned to a class, where each class occupies a given set of storage locations. Storage of SKUs within each class is based on a randomized storage policy (Larson *et. al*, 1997). In this chapter, we study the dedicated and class-based storage policies for MIAPP-AS/RS. The goal of our study is to develop models to find the assignment of SKUs to pick positions and storage locations that minimize the travel time of the crane under the *CR* operating policy. The remainder of this chapter is as follows. In Section 6-2, we develop the mathematical optimization models to find the optimal assignment of the SKUs when the system is operated under dedicated storage. In Section 6-3, we develop the closed-form travel time models to estimate expected travel time of the crane under class-based storage policy. Finally, in Section 6-4, we review the main contributions and the practical insights as well as the future research directions.

6-2- Dedicated Storage Policy for MIAPP-AS/RS

In this section, we consider the dedicated storage policy for MIAPP-AS/RS. In a dedicated storage policy each storage location in the rack is dedicated to one specific SKU. We are interested in the assignment of the SKUs to the storage locations, as well as the assignment of SKUs to pick positions. The key research goals regarding this problem are as follows.

1- Find the optimal assignment of SKUs to storage locations that minimizes the expected travel time of the crane, given the assignment of SKUs to pick positions.

2- Simultaneously, find the optimal assignment of SKUs to storage locations, as well as to pick positions that minimizes the expected travel time of the crane.

We use the following notations for this section.

Indices and sets:

i : SKU's index, $i \in \{1,2,\dots,n\}$

j : Pick Position's index, $j \in \{1,2,\dots,n\}$

k : Storage Location's index, $k \in \{1,2,\dots,N\}$

Parameters:

n : Number of SKUs and Pick Positions

N : Number of Storage Locations

P_i : Normalized Demand of SKU i, $P_i \in [0,1]$

I_i : Number of SKU i's copies stored in the rack

d_{jk} : Distance between Pick Position j and Storage Location k

Decision Variables:

$$x_{ijk} = \begin{cases} 1 & \text{if SKU } i \text{ is assigned to Pick Position } j \text{ and Storage Location } k \\ 0 & \text{Otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if SKU } i \text{ is assigned to Pick Position } j \\ 0 & \text{Otherwise} \end{cases}$$

$$x'_{jk} = \begin{cases} 1 & \text{if Pick Position } j \text{ is assigned to Storage Location } k \\ 0 & \text{Otherwise} \end{cases}$$

We consider that there are n SKUs and each of the n SKUs have a single, dedicated pick position; thus, there are also n pick positions. The normalized demand of each SKU i is denoted as P_i where $P_i \geq 0$, $\sum_i P_i = 1$. The pick positions are replenished from storage locations. In a dedicated storage policies, locations in the rack are dedicated to a specific SKU. The number of storage locations dedicated to a specific SKU, say SKU i, is equal to the number of SKU i's copies, which is denoted as I_i. Note that for a dedicated storage policy, the total storage locations that exist in the rack is equal to the sum of SKU's copies (i.e. $\sum_i I_i = N$). We assume that the number of each SKU's copies is equal to maximum inventory of that SKU. Defining the inventory level of each SKU depends on the warehousing policies. In this work, we consider that the inventory level of each SKU follows the EOQ inventory model; therefore, the inventory of each SKU in the rack is proportional to the square root of each SKU's demand (i.e. $I_i \propto \sqrt{P_i}$). In Fig. 6.1, we plot the normalized demand versus the normalized inventory level (inventory of each SKU divided by total inventory) for the fixed number of SKUs for a 20/80 demand curve. Fig. 6-1 shows that the skewness of the corresponding inventory curve is much less than the skewness of the demand. As an example for the 20/80 curve, 80% of the total demand is attributed to 20% of SKUs; however, for the inventory curve obtained by the described method, only 50% of the total inventory is attributed to the top 20% of SKUs.

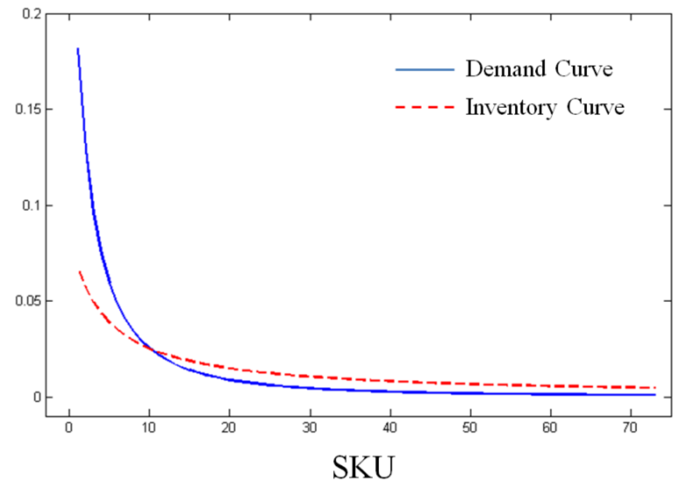


Figure 6-1 : Typical 20/80 demand curve versus corresponding EOQ inventory curve

Our objective in this section is to minimize the expected travel time of the crane by changing the assignment of SKUs to storage locations; first, by assuming that the assignment of the SKUs to pick position is known; second, by assuming that the assignment of the SKUs to pick position is changing as well. We call these problems as *SKU Assignment Problems (SAP)*.

6-2-1- SKU Storage Assignment Problem, when the assignment of SKUs to pick positions is known

The objective of this model is to minimize the expected travel time for different operating policies by changing the assignment of the SKUs to storage locations. When the assignment of SKUs to pick positions is known, we know the picking position location where all of the copies of a specific SKU are retrieved to. We define the set O to identify the assignment of SKUs to pick positions, where $O' = \{(i,j)|\text{SKU } i \text{ is assigned to Pick Position } j\}$. First, we formulate the assignment problem for a *CR* policy as Model1. We assume that all of the storage locations in the rack are occupied and the crane will perform only retrieval operations. A *CR* policy consists of two types of trips. The first trip is from storage locations to the pick positions. For this trip type, the crane is restricted to move to only the pick position that is dedicated to the certain SKU stored in the storage location. The latter trip is from pick positions to storage locations. In this case, the crane can move to any storage locations based on the next retrieval request; therefore, we assume that each storage location in the rack is visited with the probability of ${}^{P_i}/_{I_i}$. According to the mentioned properties of the crane movement in the *CR* policy, the model can be formulated as follows. We call this model as SAP1.

SAP1:

$$Min \;\; \sum_j \sum_k d_{jk} x'_{jk} + \sum_i \sum_j \sum_k {}^{P_i}/_{I_i}\, d_{jk} x'_{jk}$$

$$\sum_k x'_{jk} = I_i \quad \forall (i,j) \in O'$$

(1)

$$\sum_j x'_{jk} = 1 \quad \forall k \tag{2}$$

$$x'_{jk} \in \{0,1\} \tag{3}$$

Constraint (1) guarantees that the total number of storage locations assigned to each pick position must be equal to the maximum inventory level of the SKU assigned to that pick position. Equation (2) provides the constraint that each storage location must be assigned to just one pick position. The structure of Model1 shares the same structure as the well-known

Transportation Problem. We can transform our problem into a Transportation Problem, by imagining that there are n pick positions that supplies I_i SKUs and k storage locations that each have demand for 1 SKU. Consequently, according to the *unimodular* property of the transportation problem, the linear programming relaxing of constraint (3) by considering x'_{jk} as a positive continuous variable will always result in x'_{jk} being integer (given I_i will always be integer).

6-2-2- SKU Assignment Problem, when SKUs are assigned simultaneously to storage locations as well as pick positions

The objective of this model is to minimize the expected travel time for different policies by determining the assignment of the SKUs to storage locations, as well as determining the assignment of SKUs to pick positions. This problem is similar to the problem described in Section 6-2-1; however, the assignment of SKUs to pick positions is a decision, and it is <u>not</u> known like in Section 6-2-1. Therefore, the number of storage locations that must be assigned to each pick position is not known and depends on which SKU is assigned to each pick position. To resolve this issue, we introduce the new binary variable, y_{ij}, that controls the assignment of SKUs to pick positions, and x_{ijk} controls the assignment of SKUs to storage locations. The formulation of the assignment problem for this section is given as follows. We call this model as SAP2.

SAP2:

$$Min\ \sum_i \sum_j \sum_k d_{jk} x_{ijk} + \sum_i \sum_j y_{ij} \sum_k {}^{P_i}\!/_{I_i}\, d_{jk} \tag{4}$$

$$\sum_j y_{ij} = 1 \quad \forall i \tag{5}$$

$$\sum_i y_{ij} = 1 \quad \forall j \tag{6}$$

$$\sum_k x_{ijk} = I_i y_{ij} \quad \forall i, j \tag{7}$$

$$\sum_i \sum_j x_{ijk} = 1 \quad \forall k \tag{8}$$

$$x_{ijk}, y_{ij} \in \{0,1\} \tag{9}$$

Constraint (5) ensures that each SKU type is assigned to only one pick position. Constraint (6) enforces that each pick position must be assigned to only one SKU type. Constraint (7) ensures that if a SKU is assigned to a pick position, the number of storage locations assigned to that pick position must be equal to the maximum inventory level of the assigned SKU. Constraint (8) enforces that each storage location must be assigned to just one set of SKU type and pick position.

The SAP2 is a binary integer programming which cannot be solved by optimization solver packages (e.g. CPLEX) for real world instances due to large number of variables and constraints. However, SAP2 has a special structure that allows us to decompose this problem into more tractable instances. Specifically, SAP2 has the dual block angular structure, which means some variables exist in most of the constraints (complicating variables). By considering this structure, we use a *Benders Decomposition* method, introduced by Benders (1962), to reformulate the problem. Particularly, for any feasible solution to constraints (5) and (6) that only include y_{ij}, SAP2 (problem (4)-(9)) transforms to a Transportation Problem.

6-2-2-1- Benders Decomposition Overview

Benders Decomposition is a large scale optimization method used to reduce the size of the large problem to easier-to-solve problems. This method was introduced by Benders in 1962. In this method, the main problem is broken into a subproblem and master problem. The method is an iterative process that iteratively solves the subproblem and master problem. In each iteration, after solving the subproblem, cuts are added to the master problem using the subproblem extreme points and extreme rays. The master problem is solved with the new cuts, and the master problem optimal solution is used to reset the parameters of the subproblem. This process is repeated until the termination condition is met. Mostly, when the upper bound and lower bound is close enough (less than a desired limit), the process is terminated.

Original Problem (OP):

$Min\ \ C^T X + f(w)$

$AX + w = b$

$X \geq 0$

$w\ \in W$

Where $X, C \in \mathbb{R}^n$, $f(.)$ is a real valued function, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $W \subseteq \mathbb{R}^m$, and 0 is the $m -$ dimensional null vector (where m is the number of constraints). Let $\bar{w}$ be denoted as the fixed value of w. By fixing the value of vector w as $\bar{w}$, for any given $\bar{w}$ vector, the above problem can be decomposed to the following Benders subproblem.

Benders Subproblem:

$Min \;\; z(X) = C^T X$

$AX = b - \bar{w}$

$X \geq 0$

Let's refer the dual of Benders subproblem as Benders dual, which is given as the following.

Benders Dual Problem:

$Max \;\; z(u) = (b - \bar{w})^T u$

$A^T u \leq C$

Where u is the dual variable corresponding to $AX = b - \bar{w}$ constraint. By looking to the above problem, the space of $A^T u \leq C$ does not depend on the value of the $\bar{w}$ vector. Let U denote the polyhedron defined by $A^T u \leq C$ constraints. Let $u^1, u^2, \ldots, u^I$ and $r^1, r^2, \ldots, r^J$ denote the extreme points and extreme rays of U, respectively. Using the extreme points and extreme rays of the dual of Benders subproblem, the Benders Master Problem is formulated as the following.

Benders Master Problem:

$Min \quad z$

$(b-w)^T u^i + f(w) \leq z \quad \forall i = 1, \dots, I$

$(b-w)^T r^j \leq 0 \quad \forall j = 1, \dots, J$

When the size of the subproblem increases, the numbers of extreme points and rays increase exponentially. Therefore the number of constraints increases exponentially in the master problem; however, only a limited number of these constraints are active at the optimal solution. Hence, Benders suggests to solve the restricted master problem with a small subset of the constraints and iteratively add new constraints in each step. As the number of extreme points and rays are finite, this process finds the optimal solution in a finite number of steps (or reports the result as infeasible).

6-2-2-2- Magnanti-Wong acceleration method for Benders algorithm

Magnanti & Wong (1981) identified that the efficiency of Benders algorithm is low when the Benders Subproblem is degenerate. A degenerate subproblem means that there does not exist a unique solution to the subproblem; consequently, there are several potential cuts to be added to the master problem in each step. Therefore, they develop the procedure to generate the strongest possible cut (which they call a Pareto-optimal cut). Based on their definition, cut1 dominates cut2 if $(b-w)^T u^1 \geq (b-w)^T u^2$ for $\forall w \in W$, where $u^1 \in U$ and $u^2 \in U$ are the corresponding extreme points for cut1 and cut2, respectively. In order to find the Pareto-optimal cut, they use the concept of a *core point*. Let a point $w^0 \in ri(W^c)$ denote a core point of W, where W^c and $ri(W^c)$ are the convex hull and the relative interior of set W, respectively. Let $\bar{u}$ be the optimal solution of the dual Benders subproblem. Using the mentioned notations, the *Magnanti-Wong Problem (M-W problem)* is formulated as following.

M-W Problem:

$$Max \quad (b - w^0)^T u$$

$$(b - \overline{w})^T u = z(\bar{u})$$

$$A^T u \leq C$$

Let u^0 denote the optimal solution for this problem, and $(b - w)^T u^0$ is the resulting *Pareto-optimal* cut. Finding the *Pareto-optimal* cut, requires solving the additional *M-W Problem.* There is a trade-off between the performance of the *Pareto-optimal* cut on convergence of the problem and total computational effort. Therefore, the *M-W Problem* may not be solved in each iteration. In many problems, finding a core point ($w^0 \in ri(W^c)$) is reported as a very difficult task (Papadakos, 2008). Different methods are introduced in the literature to approximate the core point (Mercier *et al.*,2005). One of these methods includes finding an arbitrary subset of extreme points of the prior master problem and forming the linear convex combination of those extreme points to use as a core point approximation. Choosing no core point may affect the strength of the cut (the cut can be not a Pareto-optimal cut). Mercier *et al.* (2005) suggest choosing the arbitrary coefficient for variable u in *M-W Problem.* By applying this method, the space of the *M-W Problem* remains the same; therefore, this method still generates a valid Benders cut.

Benders subproblem may have some special structures. In these cases, the Benders subproblem becomes a better candidate to solve instead of the Benders subproblem dual. A similar approach can be used for *M-W Problem,* meaning the dual of the *M-W Problem* can be solved instead of *M-W Problem*. Let δ be the dual variable corresponding to first constraint of M-W Problem. Let $\bar{X}$ denote the optimal solution to the Benders subproblem. Using δ and $\bar{X}$, the dual of *M-W Problem* is formulated as follows.

M-W Dual Problem:

$$Min \quad z(X) = C^T X + z(\bar{X})\, \delta$$

$$AX + (b - \bar{w})\delta = b - w^0$$

$$X \geq 0$$

6-2-2-3- Benders decomposition formulation for SAP2

Let Y be the set of all binary vectors of y_{ij} that satisfy constraints (5) and (6). Let $\bar{y} \in Y$ be any arbitrary vector of set Y, where $\bar{y}_{ij}$ are the components of $\bar{y}$. For any given $\bar{y}$, the main problem reduces to the following *primal subproblem (PSP)*, that only includes x_{ijk} variables. Equation (11) has n^2 constraints and equation (12) includes N constraints; in result, PSP has $n^2 + N$ constraints and $n^2 N$ variables.

PSP:

$$Z_{PSP} = Min\ \sum_i \sum_j \sum_k d_{jk} x_{ijk} + f(\bar{y}) \tag{10}$$

$$\sum_k x_{ijk} = I_i \bar{y}_{ij} \quad \forall i,j \tag{11}$$

$$\sum_i \sum_j x_{ijk} = 1 \quad \forall k \tag{12}$$

$$x_{ijk} \in \{0,1\} \tag{13}$$

We observe two special structures for PSP. First, according to the structure of constraints (5) and (6), any given $\bar{y}$ vector can be considered as a solution of an *Assignment Problem*. In particular, any given $\bar{y}$ is an assignment of SKUs to pick positions, and set Y involves all possible $n!$ permutations of such assignments. Therefore, for any given $\bar{y}$, one of the i or j indices of x_{ijk} variable is redundant. We remove index i and substitute the new x'_{jk} variable for x_{ijk} in PSP. Second, according to constraints (5) and (6), any $\bar{y}$ vector has only n zero values and n one values. Therefore, the right hand sides of n constraints included in equation (11) are equal to zero. Also, as x_{ijk} is a binary variable, we can remove these n constraints from equation (11). Let O be the set of i,j pairs, where $O = \{(i,j) \mid \bar{y}_{ij} = 1\}$. Based on these mentioned observations, we reformulate the PSP, and call the new model as *reduced primary subproblem (RPSP)*. The reformulation (problem (14)-(17)) is given as follows.

RPSP:

$$Z_{RPSP} = Min\ \sum_j \sum_k d_{jk} x'_{jk} + f(\bar{y}) \tag{14}$$

$$\sum_k x'_{jk} = I_i \quad \forall (i,j) \in O \tag{15}$$

$$\sum_j x'_{jk} = 1 \quad \forall k \tag{16}$$

$$x'_{jk} \in \{0,1\} \tag{17}$$

The structure of RPSP is identical to the problem we defined before for Case1. Therefore, the RPSP is a well-known transportation problem, and the constraint (17) can be discarded with no effect on integral optimality.

Let w_{ij} and ν_k be the dual variables corresponding to constraints (15) and (16), and $u = (w_{ij}, \nu_k)$ be the dual vector constructed by these variables. Let Δ denote the set of all extreme points defined by the polyhedron of RPSP dual problem. Note that the RPSP is a transportation problem; therefore, the polyhedron space of the RPSP is always bounded. Consequently, the RPSP needs to only generate the optimality cuts, as the feasibility cuts are not required to be added to the master problem. Using the defined notion, the MIP relaxation of SAP2 can be reformulated as the *master problem (MP)*(problem (18)-(22)).

MP:

$$Z_{MP} = Min\ \ z \tag{18}$$

$$-z + \sum_{(i,j) \in O} \sum \left(w_{ij} I_i + {}^{P_i}/_{I_i} \sum_k d_{jk} \right) y_{ij} + \sum_{(i,j) \notin O} \sum \left({}^{P_i}/_{I_i} \sum_k d_{jk} \right) y_{ij} \le -\sum_k \nu_k$$

$$u = (w_{ij}, \nu_k) \in \Delta \tag{19}$$

$$\sum_j y_{ij} = 1 \quad \forall i \tag{20}$$

$$\sum_i y_{ij} = 1 \quad \forall j \tag{21}$$

$$y_{ij} \in \{0,1\} \tag{22}$$

Intuitively, the MP solves the SKUs-to-pick-positions assignment problem, and passes this assignment to the RPSP. On the other hand, given the assignment of SKUs to pick positions passed by the MP, the RPSP solves the SKUs to storage locations assignment problem. Then, by using the dual solutions to the RPSP, a new cut restricts the MP space to improve the quality of the MP solution space. In each iteration, the value of the MP objective function gives the lower bound to the problem. As in each iteration a new constraint (optimality cut) is added to the MP, therefore, the value of the lower bound (LB) is monotonically increasing. The value of the RPSP may fluctuate in each iteration as the right hand sides of the constraints are updated by the solution that is passed by the MP. Therefore, in each iteration, the best current value of RPSP is updated as the upper bound (UB) of the problem. We terminate the algorithm when the difference between the upper bound and the lower bound is smaller than the desired limit (i.e. $UB - LB \leq \varepsilon$).

As we mentioned earlier, the RPSP is a transportation problem which is a well-known for its degeneracy. Therefore, implementation of M-W acceleration method can produce the stronger cuts for MP in our case. We expect implementation of this acceleration method results in faster convergence.

In order to formulate the M-W dual problem, an auxiliary variable associated with first constraint of M-W problem is required. Let π be an arbitrary variable, where $\pi \in \mathbb{R}$. Also, let vector y^0=(y_{ij}^0) define the core point of MP in each iteration. Z_{RPSP}^* denotes the optimal value of RPSP objective function. Using the defined notations, the M-W dual formulation for SAP2 (problem (23)-(27)) is given as follows.

M-W dual:

$$Min\ \sum_i \sum_j \sum_k d_{jk} x_{ijk} + Z_{RPSP}^* \pi \tag{23}$$

$$\sum_k x_{ijk} - I_i \bar{y}_{ij} \pi = I_i y_{ij}^0 \quad \forall (i,j) \in O \tag{24}$$

$$\sum_k x_{ijk} = I_i y_{ij}^0 \quad \forall (i,j) \notin O \tag{25}$$

$$\sum_i \sum_j x_{ijk} = 1 \quad \forall k \tag{26}$$

$$x_{ijk} \in \{0,1\} \tag{27}$$

6-2-3- Benders Algorithm Implementation and Numerical Result

We solve the SAP2 problem by the Benders decomposition procedure. We code and run the Benders algorithm using ILOG CPLEX 12.6. We consider two types of distances (d_{jk}) for the problem instances 1) Rectilinear distances 2) Chebyshev distances. We apply two types of implementation of Benders algorithm. First, we use the classical Benders decomposition procedure, which the RPSP and MP are solved in each iteration. The steps of the classic Benders algorithm are as follows.

Classic Benders Algorithm for SAP2:

Step 1) $\Delta \leftarrow \emptyset$, $UB \leftarrow +\infty$

Step 2) Solve the MP

Step 3) Use $\bar{y}_{ij}$ to solve RPSP

Step 4) Calculate $\bar{u} = (\bar{w}_{ij}, \bar{v}_k)$ using RPSP optimal solution

Step 5) Update optimality cuts set, $\Delta \leftarrow \Delta \cup \bar{u}$

Step 6) Update $UB \leftarrow \min\{UB, Z^*_{RPSP}\}$, $LB \leftarrow Z^*_{MP}$

IF $UB - LB > \varepsilon$ THEN GOTO Step 2, ELSE STOP

Second, we implement the M-W acceleration method for Benders algorithm. The procedure is given as following steps.

M-W Accelerated Benders Algorithm for SAP2:

Step 1) $\Delta \leftarrow \emptyset$, $UB \leftarrow +\infty$

Step 2) Solve the MP

Step 3) Find the y^0_{ij} (core point)

Step 4) Use $\bar{y}_{ij}$ to solve RPSP

Step 5) Calculate $\bar{u} = (\bar{w}_{ij}, \bar{v}_k)$ using RPSP optimal solution

Step 6) Use y^0_{ij} (Step 3) and Z^*_{RPSP} (Step 4) to solve M-W dual

Step 7) Calculate $\bar{\bar{u}} = (\bar{\bar{w}}_{ij}, \bar{\bar{v}}_k)$ (M-W cut) using M-W dual optimal solution

Step 8) Update optimality cuts set, $\Delta \leftarrow \Delta \cup \bar{u} \cup \bar{\bar{u}}$

Step 9) Update $UB \leftarrow \min\{UB, Z^*_{RPSP}\}$, and $LB \leftarrow Z^*_{MP}$

IF $UB - LB > \varepsilon$ THEN GOTO Step 2, ELSE STOP

In order to find the core point in Step 3, we form the linear convex combination with identical weights of the binary solutions found in Step 2 (Solving MP). For example, suppose y^1,y^2,and y^3 are the three binary solutions found in Step 2. The core point is calculated as $y^0 = \frac{1}{3}\, y^1 + \frac{1}{3} y^2 + \frac{1}{3} y^3$. We set ε equal to 0.1 for both the Classic and M-W acceleration method implementations of Benders algorithm. We consider the instances with two problem sizes; first, 20 SKU types and 5 pick positions (SAP2 problem has 525 binary variables). Second, 60 SKU types and 10 pick positions (SAP2 problem has 6100 binary variables). In Table 6.1, we calculate the maximum inventory level of each SKU for demand curve based on EOQ model (as described in Section 6-2 introduction).

Table 6-1 : Maximum inventory level

Problem Size	Demand Curve	Max. Inventory Level
$n = 5$ $N = 20$	20/20	$I_i = (4,4,4,4,4)$
	20/40	$I_i = (6,5,4,3,2)$
	20/60	$I_i = (8,5,3,2,2)$
	20/80	$I_i = (10,4,3,2,1)$
$n = 10$ $N = 60$	20/20	$I_i = (6,6,6,6,6,6,6,6,6,6)$
	20/40	$I_i = (9,8,7,6,6,6,5,5,4,4)$
	20/60	$I_i = (14,10,7,6,5,4,4,4,3,3)$
	20/80	$I_i = (20,10,7,5,4,4,3,3,2,2)$

Fig. 6.2 shows the convergence of the classic Benders algorithm. The classic Benders algorithm finds the optimal solution after 70 iterations for the case of $n = 5$ and $N = 20$ (Small size rack). However, as we illustrate in Fig. 6.3, for the case of $n = 10$ and $N = 60$ (Medium size rack), the classic Benders algorithm does not converge as the gap between the UB and LB after 500 iteration is considerable. Therefore, we implement the M-W acceleration method for this case. As we show in Fig. 6.3, the M-W acceleration method improves the convergence of Benders algorithm, and the optimal solution is found after 150 iterations. We still cannot solve large size real world problem instances by our proposed decomposition algorithm.

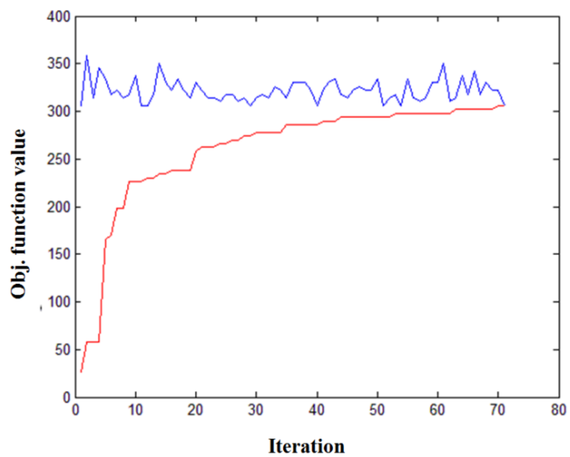


Figure 6-2: Classic Benders algorithm convergence for $n = 5$ and $N = 20$

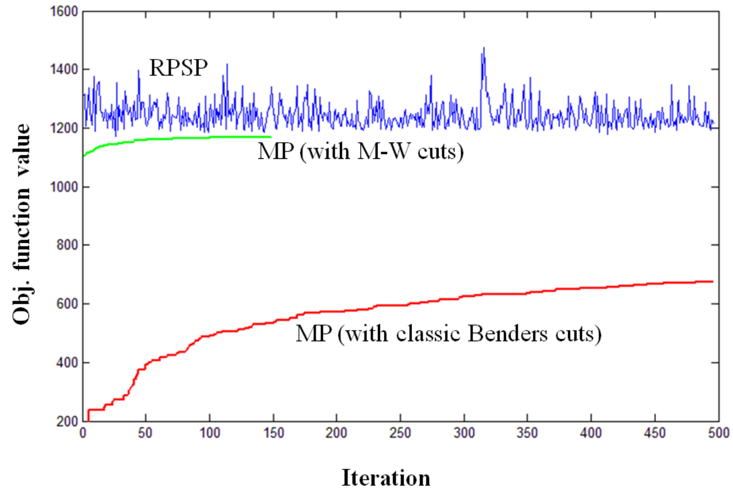


Figure 6-3 : Classic and M-W accelerated Benders algorithm convergence for $n = 10$ and $N = 60$

We show the layout of the rack with the optimal assignment of SKUs to pick positions and storage locations in Fig. 6.4 and 6.5 for the small and medium size rack (Problem sizes of $n = 5$ / $N = 20$ and $n = 10$ / $N = 60$, respectively) with rectilinear distances. We have three major observations from the optimal layouts. First, SKUs are assigned to the storage locations vertically above the pick position dedicated to the same SKU. Second, the pick position assignment alternates between allocating a high demanded SKU next to a pick position that is allocated to a low demanded SKU. For example in Fig. 6.4 and 6.5, in all layouts the pick positions assigned to most demanded and least demanded SKUs are located next to each other. As a result, the total inventory assigned to the pick positions located at the half left side of the rack (from the middle of the rack to left, i.e. $0 \leq m_i \leq 0.5$) is balanced to the pick positions located at the half right side of the rack (from the middle of the rack to right, i.e. $0.5 \leq m_i \leq 1.0$). As an example for the rack with $N = 60$, the total inventory assigned to the 5 pick positions at the left side of the rack is around 30 (i.e. $\sum_i I_i/_2$). These observations can be helpful in guiding the development of heuristic approaches to solve the real world size problems by considering which SKUs can be located next to each other in optimal solution. These valid inequalities may be able to decrease the solution time of the MP significantly. Finally, from the second observation, we are expecting that designing the class-based storage system with 2 classes will result in similar performance as dedicated storage. Additionally, the complexity of SAP for a 2-class storage system is considerably lower than dedicated storage, because with a class-based storage policy the decision is whether a SKU belongs to class 1 or 2 (versus assigning a SKU to a specific storage location in a dedicated policy).

1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5

20/20 curve

1	5	4	2	3
1	1	4	2	3
1	1	4	2	3
1	5	2	2	3
1	5	4	2	3

20/40 curve

1	1	5	2	2
1	1	4	2	3
1	1	4	2	3
1	1	5	2	3
1	5	4	2	3

20/60 curve

1	1	5	3	2
1	1	4	1	2
1	1	4	3	2
1	1	1	3	2
1	5	4	3	2

20/80 curve

Figure 6-4 : Optimal SKU assignment for $n = 10$ and $N = 60$ (for rectilinear distances)

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10

20/20 curve

1	1	7	6	4	5	2	9	3	8
1	1	7	6	4	5	2	9	3	8
1	1	7	6	4	5	2	9	3	8
1	10	10	6	4	5	2	9	3	8
1	10	7	6	4	5	2	2	3	8
1	10	7	6	4	5	2	2	3	3
1	10	7	6	4	5	2	9	3	8

20/40 curve

1	1	1	7	8	2	6	4	5	3
1	1	1	7	8	2	6	4	3	3
1	1	9	1	8	2	2	4	5	3
1	10	9	7	8	2	2	4	5	3
1	10	9	7	2	2	6	4	5	3
1	1	10	1	2	2	6	4	5	3
1	10	9	7	8	2	6	4	5	3

20/60 curve

1	1	1	1	7	3	4	2	2	2
1	10	1	1	1	3	4	2	2	5
1	1	9	1	1	3	4	6	2	5
1	1	1	1	7	3	4	6	2	2
1	1	9	8	7	3	4	6	2	5
1	10	1	8	8	3	3	6	2	5
1	10	9	8	7	3	4	6	2	5

20/80 curve

Figure 6-5 : Optimal SKU assignment for $n = 10$ and $N = 60$ (for rectilinear distances)

As the final observation, we observe that the optimal solution is highly degenerate. Several different assignments to pick positions and storage locations exist with the same optimal objective function value. We also implement the classic and M-W accelerated Benders algorithm for our problem with the chebyshev distances. We observe that the degeneracy for the assignment of SKUs to pick position is very high. Therefore, we generate 100 million random assignments of SKUs to pick positions and solve the RPSP. The results show that for all randomly generated assignments, RPSP gives the same optimal objective function value. We run the same experiment for all the small, medium, and real world size problem instances, and have not found a single counter example. We have explored several extreme problem instances (extremely skewed demand curve), and have identified a few counter-examples where solutions exist whose objective function values are not equal to the optimal objective function value. The reasoning associated to the large number of multiple optimal solutions is that, several elements in the chebyshev distance matrix, d_{jk}, are identical. Therefore, several different storage locations have the same distance to particular pick positions. As a result, although RPSP finds the different optimal assignment of SKUs to storage locations, the optimal objective function value remains the same, which means changing the assignment of SKUs to pick positions do not impact the optimal objective function value.

6-3- Class-based Storage Policy for MIAPP-AS/RS

In this section, we are interested to develop models to analyze class-based storage policies for MIAPP-AS/RS. In the previous section, for dedicated storage policy, the distances from storage locations to pick positions are fixed. However, in a class-based storage policy, the SKUs assigned to a particular class are randomly stored in any location within that class. Therefore, to have fixed distance between each class and each pick position, we consider the expected travel time between each class and each pick position to use for an optimization model.

6-3-1- Expected Travel time between a storage region and a pick position

In this section, we are interested in deriving expected travel time models between a storage region and an in-the-aisle pick position, and will use these travel times models as an input to optimization models for systems with a class-based storage policy. In Section 4-4-1, we developed models to derive the expected travel time between a pick position and any random point within the rack. In this section, we intend to develop models that enable us to calculate the expected travel time between a pick position and any random point within the given specific vertical region in the rack (See Fig. 6.6).

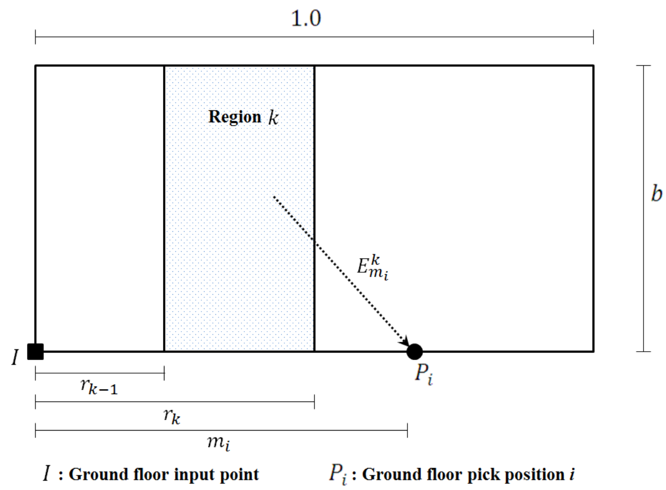


Figure 6-6 : Expected travel time trip in a normalized rack for class-based storage policy

To model this case, we need to consider that a crane may travel from a pick position in one region to a storage location in the same region or to a storage location in another region (See Fig. 6-6). Although we use the same approach as Section 4-4-1, we cannot use Section 4-4-1 models directly for the current mentioned case due to the travel between regions. Let's consider the following notations for our models.

m_i: Distance of pick position i from the lower left corner of the rack ($i: 0, \dots, n$)

r_j: Distance of vertical boundary j from the left corner of the rack (such that the region k is bounded by boundary line j and $(j-1)$) ($j: 0, \dots, K$ and $k: 1, \dots, K$)

$E^k_{m_i}$: Expected Travel time between pick position i and region k

We assume that the storage locations are continuously and uniformly distributed in each region (class). We consider the vertical regions in this case. Therefore, the boundaries of the region k can be defined with two points (r_{k-1} and r_k) on the bottom of the rack (See Fig. 6-6). We consider three cases for the locations of a pick position; 1) the pick position is located within the boundary of a given region ($r_{k-1} \leq m_i \leq r_k$); 2) the pick position is located at the left side of the region ($m_i \leq r_{k-1}, r_k$), and 3) the pick position is located at the right side of the region ($m_i \geq r_{k-1}, r_k$).

6-3-1-1- Pick Position $\boldsymbol{i}$ is in Region $\boldsymbol{k}$ ($\boldsymbol{r_{k-1} \leq m_i \leq r_k}$)

In this case, we can use the models in Section 4-4-1 directly as the pick position is located within the region. In Section 4-4-1, the normalized shape of the rack in terms of time is a rectangle with a width of 1 and height of b. In this section, the given region has the same height of b, but a width of $(r_k - r_{k-1})$. Therefore, similar to Cases 1 and 2 in Section 4-4-1, the distance of the pick position i to the left side of the region k is equal to $(m_i - r_{k-1})$, and to the right side of the region k is equal to $(r_k - m_i)$. Similar to Section 4-4-1, using equation (4) and (5), we calculate the expected travel time between the pick position i and any random point within the left part of region k and right part of region k as equation (28) and (29), respectively. The probability that a point is selected at the left side of region k is equal to $\frac{m_i - r_{k-1}}{r_k - r_{k-1}}$, and at the right side of region k is equal to $\frac{r_k - m_i}{r_k - r_{k-1}}$. Hence, the overall expected travel time between pick position i and any random point within region k, $E_{m_i}^k$, is calculated as equation (30).

$$E_{m_i}^k(z_1) = \begin{cases} {(m_i - r_{k-1})^2}/{6b} + {b}/{2} & for \quad 0 \le (m_i - r_{k-1}) < b \\ {b^2}/{6(m_i - r_{k-1})} + {(m_i - r_{k-1})}/{2} & for \quad b \le (m_i - r_{k-1}) < 1 \end{cases} \tag{28}$$

$$E_{m_i}^k(z_2) = \begin{cases} {(r_k - m_i)^2}/{6b} + {b}/{2} & for \quad 0 \le (r_k - m_i) < b \\ {b^2}/{6(r_k - m_i)} + {(r_k - m_i)}/{2} & for \quad b \le (r_k - m_i) < 1 \end{cases} \tag{29}$$

$$E_{m_i}^k = \frac{m_i - r_{k-1}}{r_k - r_{k-1}} E_{m_i}^k(z_1) + \frac{r_k - m_i}{r_k - r_{k-1}} E_{m_i}^k(z_2) \tag{30}$$

6-3-1-2- Pick Position $\boldsymbol{i}$ is Located at the Left Side of Region $\boldsymbol{k}$ ($\boldsymbol{m_i \le r_{k-1}, r_k}$)

For this case, we follow the same methodology we used in Section 4-4-1 to derive the travel times. The distance between the pick position and the boundaries are equal to $(r_{k-1} - m_i)$ and $(r_k - m_i)$. We follow the approach Bozer & White (1984) applied to normalized the rack and derive the travel time (Similar to Section 4-4-1). All of the notations and assumptions are similar to what we defined in Section 4-4-1. In this case, the horizontal travel time follows the uniform distribution between $(r_{k-1} - m_i)$ and $(r_k - m_i)$. Also, the vertical travel time follows the uniform distribution between 0 and b. Hence, we have

$t_x \sim Uniform(r_{k-1} - m_i, r_k - m_i)$

$t_y \sim Uniform(0, b)$

We consider the three possible cases based on the relation between the distances to the boundaries and the shape factor, b. Recall from Section 4-4-1, the joint cumulative probability distribution function (CDF) of crane travel time is given as following.

$$F(z) = \Pr\left(t_{x,y} \le z\right) = \Pr(t_x \le z) \times \Pr(t_y \le z)$$

For the first case, we consider that the shape factor is smaller than the distance of the pick position to both boundaries (i.e. $b \le r_{k-1} - m_i \le r_k - m_i$). Based on the mentioned distribution of horizontal and vertical travel time, the CDFs of horizontal and vertical travel time are given as follows.

for $b \le r_{k-1} - m_i \le r_k - m_i$, we have

$$\Pr(t_x \le z) = \begin{cases} \frac{z - r_{k-1} + m_i}{r_k - r_{k-1}} & for \quad r_{k-1} - m_i \le z \le r_k - m_i \\ 1 & for \quad z \ge r_k - m_i \end{cases}$$

$$\Pr\left(t_y \le z\right) = 1 \qquad for \quad z \ge b$$

Hence, the joint CDF for crane travel time is

$$F(z) = \begin{cases} \frac{z - r_{k-1} + m_i}{r_k - r_{k-1}} & for \quad r_{k-1} - m_i \le z \le r_k - m_i \\ 1 & for \quad z \ge r_k - m_i \end{cases}$$

According to the definition of probability distribution function (PDF) and the expected value of a random variable, the PDF and expected travel time of the crane is as follows.

$$f(z) = \frac{dF(z)}{dz} = \begin{cases} \frac{1}{r_k - r_{k-1}} & for \quad r_{k-1} - m_i \le z \le r_k - m_i \\ 0 & for \quad z \ge r_k - m_i \end{cases}$$

$$E_{m_i}^k = \int_{r_{k-1}-m_i}^{r_k-m_i} zf(z)dz = \int_{r_{k-1}-m_i}^{r_k-m_i} \frac{z}{r_k-r_{k-1}} dz = \frac{r_k+r_{k-1}-2m_i}{2} \tag{31}$$

For the second case we consider that the value of the shape factor is greater than the distance of the pick position to the closer boundary, but smaller than the distance to the farther boundary (i.e. $r_{k-1} - m_i \leq b \leq r_k - m_i$). Following a similar procedure as the one used to derive the previous case, we derive the CDF, PDF, and expected value of the travel time for this case as follows.

for $r_{k-1} - m_i \leq b \leq r_k - m_i$, we have

$$\Pr(t_x \leq z) = \begin{cases} \frac{z-r_{k-1}+m_i}{r_k-r_{k-1}} & for \quad r_{k-1} - m_i \leq z \leq b \\ \frac{z-r_{k-1}+m_i}{r_k-r_{k-1}} & for \quad b \leq z \leq r_k - m_i \end{cases}$$

$$\Pr\left(t_y \leq z\right) = \begin{cases} \frac{z}{b} & for \quad 0 \leq z \leq b \\ 1 & for \quad z \geq b \end{cases}$$

$$F(z) = \begin{cases} \frac{z(z-r_{k-1}+m_i)}{b(r_k-r_{k-1})} & for \quad r_{k-1} - m_i \leq z \leq b \\ \frac{z-r_{k-1}+m_i}{r_k-r_{k-1}} & for \quad b \leq z \leq r_k - m_i \end{cases}$$

$$f(z) = \frac{dF(z)}{dz} = \begin{cases} \frac{2z-r_{k-1}+m_i}{b(r_k-r_{k-1})} & for \quad r_{k-1} - m_i \leq z \leq b \\ \frac{1}{r_k-r_{k-1}} & for \quad b \leq z \leq r_k - m_i \end{cases}$$

$$E_{m_i}^k = \int_{r_{k-1}-m_i}^{r_k-m_i} zf(z)dz = \frac{1}{b(r_k-r_{k-1})} \int_{r_{k-1}-m_i}^{b} 2z^2 - z(r_{k-1}+m_i)dz + \frac{1}{r_k-r_{k-1}} \int_{b}^{r_k-m_i} zdz =$$

$$\frac{\frac{2b^2}{3} - (r_{k-1}-m_i)\frac{3b^2-(r_{k-1}-m_i)^3}{6}}{b(r_k-r_{k-1})} + \frac{\frac{(r_k-m_i)^2-3b^2}{6}}{r_k-r_{k-1}} \tag{32}$$

For the third case, we consider that the value of the shape factor is greater than the distance of the pick position to both boundaries (i.e. $r_{k-1} - m_i \leq r_k - m_i \leq b$). Similar to previous cases, the derivation are given as following.

for $r_{k-1} - m_i \leq r_k - m_i \leq b$, we have

$$\Pr(t_x \leq z) = \begin{cases} \frac{z - r_{k-1} + m_i}{r_k - r_{k-1}} & for \quad r_{k-1} - m_i \leq z \leq r_k - m_i \\ 1 & for \quad r_k - m_i \leq z \leq b \end{cases}$$

$$\Pr\left(t_y \leq z\right) = \frac{z}{b} \quad for \quad 0 \leq z \leq b$$

$$F(z) = \begin{cases} \frac{z(z - r_{k-1} + m_i)}{b(r_k - r_{k-1})} & for \quad r_{k-1} - m_i \leq z \leq r_k - m_i \\ \frac{z}{b} & for \quad r_k - m_i \leq z \leq b \end{cases}$$

$$f(z) = \frac{dF(z)}{dz} = \begin{cases} \frac{2z - r_{k-1} + m_i}{b(r_k - r_{k-1})} & for \quad r_{k-1} - m_i \leq z \leq r_k - m_i \\ \frac{1}{b} & for \quad r_k - m_i \leq z \leq b \end{cases}$$

$$E_{m_i}^k = \int_{r_{k-1}-m_i}^{b} z f(z) dz = \frac{1}{b(r_k - r_{k-1})} \int_{r_{k-1}-m_i}^{r_k - m_i} 2z^2 - z(r_{k-1} + m_i) dz + \frac{1}{b} \int_{r_k - m_i}^{b} z dz =$$

$$\frac{4(r_k - m_i)^3 - 3(r_{k-1} - m_i)(r_k - m_i)^2 - (r_{k-1} - m_i)^3}{6b(r_k - r_{k-1})} + \frac{b^2 - (r_k - m_i)^2}{2b} \tag{33}$$

6-3-1-3- Pick Position $\boldsymbol{i}$ is Located at the Right Side of Region $\boldsymbol{k}$ ($\boldsymbol{m_i \geq r_{k-1}, r_k}$)

Similar to the previous case, the horizontal travel time follows the uniform distribution between $(m_i - r_k)$ and $(m_i - r_{k-1})$. Also, the vertical travel time follows the uniform distribution between 0 and b. Hence, we have $t_x \sim Uniform(m_i - r_k,\, m_i - r_{k-1})$ and

$t_y \sim Uniform(0, b)$. Also, similarly, we consider the three possible cases based on the relation between the distances to the boundaries and the shape factor, b. Following the same approach for the three cases in Case 2, the derivations for expected travel time are given as follows.

for $b \le m_i - r_k \le m_i - r_{k-1}$,

$$E_{m_i}^k = \frac{2m_i - r_k - r_{k-1}}{2} \tag{34}$$

for $m_i - r_k \le b \le m_i - r_{k-1}$,

$$E_{m_i}^k = \frac{\frac{2b^2}{3} - (m_i - r_k)\frac{b^2}{2} - \frac{(m_i - r_k)^3}{6}}{b(r_k - r_{k-1})} + \frac{\frac{(m_i - r_{k-1})^2}{6} - \frac{b^2}{2}}{r_k - r_{k-1}} \tag{35}$$

for $m_i - r_k \le m_i - r_{k-1} \le b$,

$$E_{m_i}^k = \frac{\frac{2(m_i - r_{k-1})^3}{3} - \frac{(m_i - r_k)(m_i - r_{k-1})^2}{2} - \frac{(m_i - r_k)^3}{6}}{b(r_k - r_{k-1})} + \frac{b^2 - (m_i - r_{k-1})^2}{2b} \tag{36}$$

6-3-2- Expected travel time between a storage region and an input point

In this section, we use the models we developed in the previous section to derive the expected travel time between an input point (which is located at the lower left corner of the rack) and any random point with a given region (class). Let E_0^k denote the expected travel time between the input point and any randomly selected point within region k. An input point can be considered as a pick position which is located at the lower left corner of the rack. As shown in Fig. 6.7, an input point is located at the left side of any possible region. Therefore, any E_0^k can be simply calculated by letting $m_i = 0$ in equations of Case 2. The distance between an input point and region k's boundaries are equal to r_{k-1} and r_k. Similar to Case 2, there are three cases based on the relation between these distances to boundaries and the value of the shape factor. We derive the E_0^k using the equations (31), (32), and (33) for cases of $b \leq r_{k-1} \leq r_k$, $r_{k-1} \leq b \leq r_k$, and $r_{k-1} \leq r_k \leq b$, respectively, as follows.

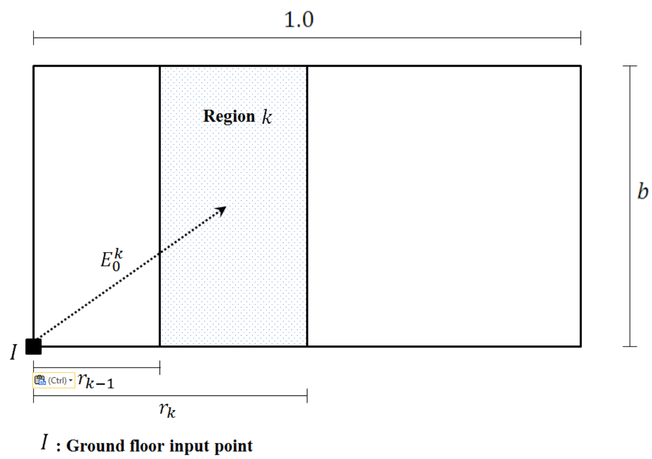


Figure 6-7 : Expected travel time between an Input point and given region (Class) in a normalized rack

for $b \leq r_{k-1} \leq r_k$, we have

$$E_0^k = \frac{r_k + r_{k-1}}{2} \tag{37}$$

for $r_{k-1} \leq b \leq r_k$, we have

$$E_0^k = \frac{\frac{2b^2}{3} - \frac{b^2 r_{k-1}}{2} - \frac{{r_{k-1}}^3}{6}}{b(r_k - r_{k-1})} + \frac{\frac{{r_k}^2}{6} - \frac{b^2}{2}}{r_k - r_{k-1}} \tag{38}$$

for $r_{k-1} \leq r_k \leq b$, we have

$$E_0^k = \frac{\frac{2{r_k}^3}{3} - \frac{r_{k-1}{r_k}^2}{2} - \frac{{r_{k-1}}^3}{6}}{b(r_k - r_{k-1})} + \frac{b^2 - {r_k}^2}{2b} \tag{39}$$

6-4- Conclusion and future research directions

In this chapter, we developed mathematical binary linear programming models to minimize the travel time by changing the assignment of SKUs to pick positions and storage locations under the dedicated storage policy. Because of the special structure of the problem, we developed a solution approach based on the Benders decomposition algorithm to break the original problem into the smaller tractable instances. Through this decomposition, the subproblem is a well-known transportation problem that iteratively finds the assignment of the SKUs to storage locations as well as the master problem that finds the assignment of the SKUs to pick positions. Due to the degeneracy of the subproblem, we applied the Magnanti-Wong acceleration method for Benders algorithm to speed up the convergence of the algorithm. We implemented both algorithms for small and medium size problem instances. We observed some intuitive results from the optimal solutions: 1) the layout of the optimal assignment includes vertical SKUs stored above their dedicated pick positions 2) the pick positions dedicated to high demanded SKUs are mostly assigned close to pick positions dedicated to low demanded SKUs 3) The optimal assignment is highly degenerate for rectilinear and chebyshev distance metrics. Also by applying the chebyshev metrics to realistic instances, changing the assignment of the SKUs to pick positions does not impact the optimal objective function value for the original problem. Finally, we developed the closed-form statements that can estimate the expected travel time of the crane under the class-based storage policy.

As the future research directions, we propose to develop heuristic algorithms based on the idea that the pick positions dedicated to high demanded SKUs are mostly assigned close to pick positions dedicated to low demanded SKUs.

CHAPTER SEVEN: FUTURE STUDY DIRECTIONS

Contribution 1 is the first study that analyzes AS/RS with multiple in-the-aisle outputs. Therefore, our study could help extensively to design and quantify the performance of systems with in-the-aisle outputs or pick positions. In Contribution 2, we consider more realistic aspects of the environment of MIAPP-AS/RS by incorporating the idea of non-identical SKUs' demand, which relaxed some of the assumptions in Contribution 1. This brings more practical design decisions such as optimal SKU assignment problem, which is discussed in Contribution 2. In Contribution 3, we relaxed the assumption of randomized storage and developed travel time and optimization models based on dedicated and class-based storage policies. Our study could be extended to consider the following future research directions. The development of analytical models for MIAPP-AS/RS that considers different dwell point strategies and different sequencing rules would be interesting to understand how policies impact throughput and storage space performance. Also, the development of a systematic model that considers the interaction of human order pickers and the MIAPP-AS/RS would be interesting from an inventory buffer capacity, as well as a staff resource and scheduling perspective.

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