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Swastik Kumar Brahma
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SPECTRUM SHARING AND SERVICE PRICING IN
DYNAMIC SPECTRUM ACCESS NETWORKS

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
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ABSTRACT

Traditionally, radio spectrum has been statically allocated to wireless service providers (WSPs). Regulators, like FCC, give wireless service providers exclusive long term licenses for using specific range of frequencies in particular geographic areas. Moreover, restrictions are imposed on the technologies to be used and the services to be provided. The lack of flexibility in static spectrum allocation constrains the ability to make use of new technologies and the ability to redeploy the spectrum to higher valued uses, thereby resulting in inefficient spectrum utilization [23, 38, 42, 62, 67]. These limitations have motivated a paradigm shift from static spectrum allocation towards a more ‘liberalized’ notion of spectrum management in which secondary users can borrow idle spectrum from primary spectrum licensees, without causing harmful interference to the latter- a notion commonly referred to as dynamic spectrum access (DSA) or open spectrum access [3], [82]. Cognitive radio [30, 47], empowered by *Software Defined Radio* (SDR) [81], is poised to promote the efficient use of spectrum by adopting this open spectrum approach.

In this dissertation, we first address the problem of dynamic channel (spectrum) access by a set of cognitive radio enabled nodes, where each node acting in a selfish manner tries to access and use as many channels as possible, subject to the interference constraints. We model the dynamic channel access problem as a modified Rubinstein-Ståhl bargaining game.

In our model, each node negotiates with the other nodes to obtain an agreeable sharing rule of the available channels, such that, no two interfering nodes use the same channel. We solve the bargaining game by finding Subgame Perfect Nash Equilibrium (SPNE) strategies of the nodes. First, we consider finite horizon version of the bargaining game and investigate its SPNE strategies that allow each node to maximize its utility against the other nodes (opponents). We then extend these results to the infinite horizon bargaining game. Furthermore, we identify Pareto optimal equilibria of the game for improving spectrum utilization. The bargaining solution ensures that no node is starved of channels.

The spectrum that a secondary node acquires comes to it at a cost. Thus it becomes important to study the ‘end system’ perspective of such a cost, by focusing on its implications. Specifically, we consider the problem of incentivizing nodes to provide the service of routing using the acquired spectrum. In this problem, each secondary node having a certain capacity incurs a cost for routing traffic through it. Secondary nodes will not have an incentive to relay traffic unless they are compensated for the costs they incur in forwarding traffic. We propose a path auction scheme in which each secondary node announces its cost and capacity to the routing mechanism, both of which are considered as private information known only to the node. We design a route selection mechanism and a pricing function that can induce nodes to reveal their cost and capacity honestly (making our auction truthful), while minimizing the payment that needs to be given to the nodes (making our auction optimal). By considering capacity constraint of the nodes, we explicitly support multiple path routing. For deploying our path auction based routing mechanism in DSA networks, we provide polynomial time

algorithms to find the optimal route over which traffic should be routed and to compute the payment that each node should receive.

All our proposed algorithms have been evaluated via extensive simulation experiments. These results help to validate our design philosophy and also illustrate the effectiveness of our solution approach.

Dedicated to,
My Grandparents
Jaya and Radha Krishna Brahma,
My parents
Swapna and Asis Brahma,
My wife
Susmita Brahma,
and
My academic advisor
Dr. Mainak Chatterjee.

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CHAPTER 1

INTRODUCTION

“Any sufficiently advanced technology is indistinguishable from magic.”[16]– Sir Arthur C. Clarke. Wireless communications today indeed portray characteristics of magic– people from far corners of the world can communicate with each other without any apparent link between them; remote devices can be controlled without the need of any physical connection. There are plethora of examples today that would astound someone not familiar with wireless communication technology.

Wireless communications is enabled by the use of electromagnetic spectrum, which has arguably become one of the most precious resources of the modern era. Mobile communications, satellite television, public safety systems, wireless local area networks (WLAN), global positioning system (GPS)– all substantiate the dependency of modern society on this resource.

Similar to any natural resource, like oil and natural gas, land, the usage of spectrum also needs to be managed. This is usually done by a government subsidized agency in various countries. For example, in 1934, the US Congress created the Federal Communications Commission (FCC) to oversee telecommunications. Its responsibilities include management of electromagnetic spectrum within the United States. Traditionally, radio spectrum man-

agement has followed a ‘command-and-control’ approach– regulators like FCC give wireless service providers exclusive rights of using a particular range of frequencies in a particular geographic area. Moreover, restrictions are imposed on the technologies to be used and the services to be provided. Portions of the spectrum managed in this manner are termed as *licensed* spectrum. The ‘command-and-control’ approach results in a static spectrum allocation that is managed manually– geographic areas have to be surveyed and frequency and bandwidth parameters chosen appropriately for each license holder to minimize overall interference.

Broadcast radio and television services, for example, operate using licensed spectrum in most countries, thereby following the command-and-control approach. They were among the first commercially deployed applications using electromagnetic spectrum. Radio and television services are primarily characterized as one-way communication systems. Cellular telephony networks, introduced in 1980s, marked the first widespread use of two-way communication devices. Much of the spectrum used by cellular telephony networks also fall into the category of licensed spectrum usage. Another example of an application that operates using licensed spectrum is the GPS.

Undoubtedly, services provisioned using licensed spectrum have had far reaching impacts on all walks of human lives. However, the ‘static’ nature of the ‘command-and-control’ approach has resulted in inefficient spectrum utilization over the years. While the entire radio spectrum from 6 kHz to 300 GHz is allocated, at any given point in space and time, much of the spectrum remains unused [23, 38, 42, 43, 62, 67], resulting in the existence of

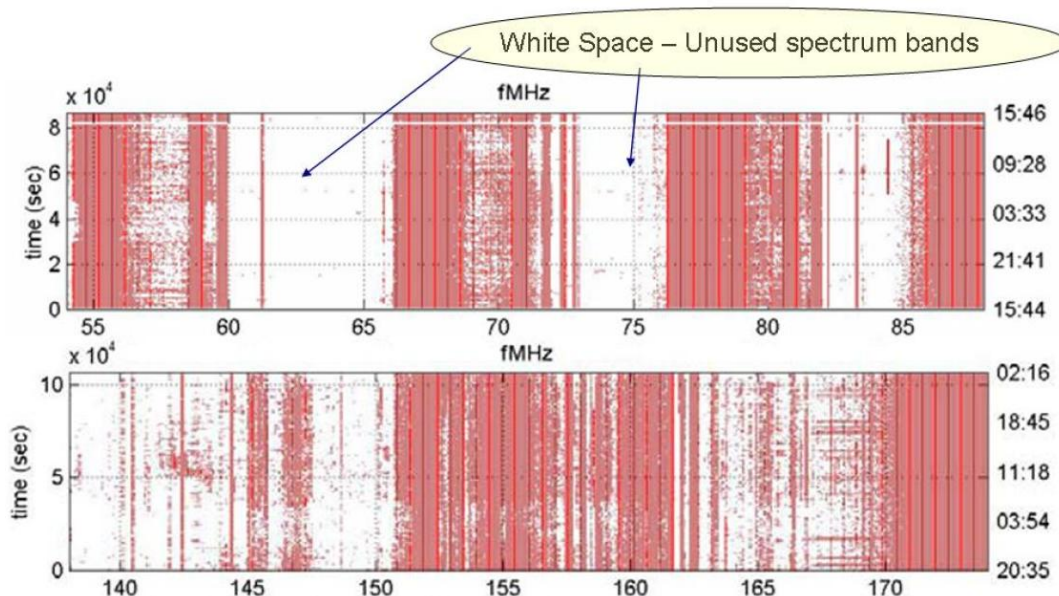


Figure 1.1: White spaces in spectrum usage (Courtesy: Shared Spectrum Company)

‘spectrum holes’ (also called white spaces) in the allocated spectrum. A snapshot of spectrum usage and existence of white spaces is shown in Figure 1.1. The problem of static spectrum allocation also worsens due to the modification of old technologies and adoption of new ones. For example, the allocation of 6 MHz per TV channel in the United States was based on the old analog NTSC system even though better quality video can be now broadcasted with almost 50% less spectrum per channel [9]. After completion of the analog to digital TV transition in the US, most of the spectrum from 54 MHz–698 MHz (channels 2 to 51), which was used for analog TV broadcast, remains unused. These limitations of static spectrum allocation motivated regulatory bodies like FCC to rethink the way in which spectrum is managed and seek a more *dynamic* means of allocating spectrum leading to the concept of *unlicensed* spectrum usage, which we discuss next.

1.1 Unlicensed Spectrum and Dynamic Spectrum Access (DSA)

Intuitively, spectrum utilization can be significantly improved by making it possible for a user to access spectrum unoccupied by its license holder at a given location and time. This is essentially the concept of unlicensed spectrum access. In contrast to static spectrum allocation, for successful sharing of unlicensed spectrum, a dynamic and distributed approach is required. Such a wireless ecosystem would be habited by primarily two classes of spectrum users. The first are the *primary users* who already possess FCC licenses to use particular frequencies. The second class comprises of the unlicensed users, denoted as the secondary users. Primary users would always have full access to their spectrum at their discretion. Secondary users can use the spectrum owned by the primary spectrum licensees, without causing harmful interference to the latter, i.e., in an opportunistic manner. This is known as *dynamic spectrum access* (DSA) or open spectrum access [3], [82]. Cognitive radio [30, 47], empowered by *Software Defined Radio* (SDR) [81], is poised to promote the efficient use of spectrum by adopting this open spectrum approach.

The first step in the paradigm shift from licensed spectrum management towards allowing unlicensed usage was taken in 1985, when the FCC allowed use of direct sequence spread spectrum (DSSS) technology for communications in the Industrial, Scientific, and Medical (ISM) bands at 900 MHz, 2.4 GHz, and 5.8 GHz in the US. The ISM bands have played a key role in facilitating development of various wireless communication systems, including wireless local area networks and cordless phones. More recently, in November 2008, FCC issued a

decision that made the TV white spaces left behind after the analog to DTV transition to be available for unlicensed use. Unlicensed radio transmitters would be able to operate in these white spaces without interfering with other devices. These steps taken by FCC mark the beginning of an era in which devices using unlicensed spectrum will far outnumber devices using licensed spectrum.

Cognitive Radio: Cognitive radio [46], [47], inclusive of software-defined radio, has been proposed as the means to promote the efficient use of the spectrum by exploiting the existence of spectrum holes. Software-defined radio, is a multi-band radio that supports multiple air interfaces and protocols and is reconfigurable through software or general purpose microprocessors[45]. Cognitive radio, built on a software radio platform, is capable of autonomous reconfiguration by learning from and adapting to the communication environment.

1.2 Unlicensed Spectrum Sharing problem

We consider spectrum to be divided into channels. Here we use the term ‘channel’ broadly. A channel can be a frequency band with certain bandwidth, a collection of spreading codes in a code division multiple access (CDMA) network, or a set of tones in an orthogonal frequency division multiplexing (OFDM) system. We assume that cross-channel interference is negligible. Thus, a secondary user transmitting over an available channel does not interfere with primary users using other channels.

If a secondary user (node) uses spectrum without coordinating with the others, then it may cause harmful interference and degrade overall spectrum usage. Nodes in close proximity interfere with each other and cannot use the same channel concurrently, while well separated nodes can reuse the same channel. Each node therefore has to use channels that are orthogonal from its interferers. Such orthogonal channel assignment can be done with either of the following two objectives.

1. **Optimizing System Utility:** Nodes share channels, such that, system wide utility is maximized based on some predefined utility function (e.g., [13, 58, 63]), regardless of individual benefits. This type of optimization techniques primarily corresponds to collaborative schemes among nodes usually deployed by the same network service provider.
2. **Optimizing Individual Benefits:** Nodes share channels, such that, individual benefits are maximized (e.g., [34] [54]). Nodes are considered as rational entities competing with each other to maximize self-gain in a non-collaborative manner. The nodes, for example, can correspond to broadcast access points deployed by competing wireless service providers. By using more channels each provider may intend to support more customers for maximizing its revenue.

When the nodes accessing spectrum are self-gain-motivated rational entities, each would try to to *maximize* its utility by accessing and using as many channels as possible from the set of orthogonal channels not being used by any of the primary incumbents. The

channels that a node selects is, however, subject to the following constraint— nodes within the interference range of each other have to use orthogonal channels to minimize interference. Thus, the nodes will have to agree upon a *sharing rule* of the channels among themselves. The fundamental question that needs to be answered is- *how many* and *which channels* each node should access to *maximize* its gain, given that all nodes are selfish. Clearly, from the above discussion, it is important to study the competition for spectrum among nodes in an interference aware context and investigate self-enforcing spectrum sharing strategies of the nodes, which have not been addressed in previous works.

1.3 The Incentive based Routing Problem

Primary owners of spectrum licenses are mostly numerous, independent self-interested parties. Thus, to motivate such entities to allow usage of their idle spectrum for secondary use, an incentive, generally in the form of monetary benefit, has to be given to them. Such economic considerations have been reflected in recent research [37, 79, 80], which proposes various pricing schemes.

1.3.1 The Routing Problem— from an Economic Perspective

Since secondary users acquire spectrum from the primaries by paying for the same, the former can also be considered as self-gain-motivated independent entities, introducing characteristics of an economy to the secondary user regime. In other words, incentives have to be given

to the secondary users to provide services, such as routing, to outside entities. To forward traffic, each secondary node would incur a *cost* because of using the spectrum it has acquired by paying a price. Moreover, using the acquired spectrum, each node can sustain a certain *capacity*. Clearly, secondary nodes will not have an incentive to relay traffic unless they are compensated for the costs they incur in forwarding traffic. Also, for a routing mechanism to be stable, the amount of traffic that a node forwards should not exceed its capacity.

The routing problem: For a flow from a source s to a destination d in a DSA network, the routing problem (from an economic perspective) is to find a path (or a set of paths) from s to d , that has sufficient capacity to route the given flow, and using which the total payment that needs to be made to relaying nodes is minimized.

1.3.2 Difficulty in Solving the Problem

The complexity in solving the routing problem lies in the fact that the routing mechanism is not aware of the *actual* cost and capacity of a node. Both cost and capacity of a node are considered as its *private* information, which the node is responsible for revealing to the routing mechanism. This gives the nodes an opportunity to *lie* about its cost and/or capacity hoping for an extra profit. For example, a node might understate its cost hoping to attract sufficient additional traffic that countervails its lower than truthful cost announced, or exaggerate its cost that might increase the payment made to the node sufficiently to compensate for any resulting decrease in traffic. Moreover, a node might exaggerate its

capacity hoping to receive additional traffic, which the node may not have sufficient capacity to route, but results in increase in payment made to the node. As will become evident later, there are situations where a node might even expect an extra profit by understating its capacity.

1.3.3 Limitations of Past Work

Most past works have focused on the cost perspective alone and does not consider capacity constraint in their framework [25, 31, 55]. These works consider finding a least cost path (LCP) from s to d with each agent (a node or an edge) having a per packet cost for relaying traffic. To induce the agents to reveal their costs truthfully, the pricing schemes in these works are based on the Vickrey-Clarke-Groves (VCG) class of mechanisms [17, 29, 69]. It is worth emphasizing here that the focus of these works is find a path with the *least cost*, which need not be the path that requires the *least payment*. Minimization of the payment to be made is more attractive to an end user, because price is the actual out-of-pocket expense the end user has to pay. The authors in [39] consider the capacity constraint but do not enforce the nodes to report their capacity truthfully. In [64], the authors also consider the capacity constraint of the nodes. However, to enforce truthful capacity reporting their scheme requires tamper-proof hardware or cryptographic-receipt-based software to be installed in the nodes.

1.4 Solution Approach and Contributions

We first address the question of *how many* and *which* channels each selfish secondary node should access in order to maximize its gain. In order to answer this question, we model the channel access problem as a bargaining game, where nodes ‘haggle’ with each other over their share of channels. The solution to the bargaining game corresponds to self-enforcing equilibrium strategies of the nodes in ‘demanding’ and ‘accepting’ their share of channels which optimizes the utility of each node against all other nodes.

Since the channels that each secondary node acquires comes to it at a cost, these nodes will have to be provided with incentives to provide services such as routing to outside entities. Thus, after investigating the equilibrium share of channels of the nodes, we will delve into economic considerations of providing incentives to nodes to use the channels acquired by them in providing the service of routing to outside entities.

1.4.1 The Spectrum Bargaining Game

We model the competition for spectrum among the nodes using non-cooperative game theory. Specifically, we model the problem of agreeing upon a sharing rule of the channels among the nodes as a Rubinstein-Ståhl [60] [66] bargaining game. In our model, each node “bargains” with the other nodes (opponents) in the network regarding its “share” (*how many* and *which*) of the channels. In the *finite* horizon version of the game, nodes bargain *at most* for a given

time. If the nodes can decide upon a sharing rule of the channels within the given time, each gets its respective share of the channels as per the sharing rule. Otherwise, each node gets *zero* channels. In the *infinite* horizon game, bargaining among the nodes go on until the nodes can agree upon a distribution of the channels. Notice that, until the nodes agree upon the sharing rule, none of the nodes can start data communication. Thus, “waiting” for the bargaining outcome also costs the nodes. We consider this cost by discounting future payoff of the nodes. This discounting represents the *patience* of the nodes in waiting for the bargaining outcome.

We solve the bargaining game by finding Subgame Perfect Nash equilibrium (SPNE) strategies of the nodes. The SPNE strategies that we derive comprise a set of strategies such that, no node in no subgame can deviate from its strategy and thereby gain from the deviation. Formally, the key contribution of our work on spectrum sharing can be considered as follows.

- We model the problem of dynamic spectrum access, in which the nodes have to agree upon a sharing rule of the channels among themselves, as a Rubinstein-Ståhl [60, 66] bargaining game.
- We investigate *finite* horizon version of the game and identify SPNE strategies of the nodes, such that, using their SPNE strategies each node can optimize its utility against all its opponents in any subgame of the bargaining game¹. In other words, considering nodes to be rational, each node can play its SPNE strategy in the very first period

¹Note that, the original game can also be considered a subgame of itself.

of the game to decide on the sharing rule and start data communication. The finite horizon game represents situations when an outside entity limits the time for which the nodes can bargain.

- We also study the *infinite* horizon bargaining game and derive SPNE strategies of the nodes in this game. The infinite horizon version of the game models situations when there is no outside entity to limit the time for which the nodes bargain, so that, nodes can go on bargaining until they can agree upon a sharing rule.
- We identify Pareto optimal equilibrium strategies of the nodes to maximize spectrum utilization. In other words, the spectrum sharing rule that the nodes obtain using their equilibrium strategies is efficient, in the sense that, the channel allocation of a node cannot be improved without hurting the share of another.
- The bargaining solution that we derive is such that no node gets starved of channels when the nodes play their equilibrium strategies. This is an important criteria when analyzing any competition model.
- We conduct simulations to study how the ‘self-gain’ maximizing strategy of the players affect system wide performance. This study reveals how competition for spectrum among nodes influences spectrum usage from a global perspective. We also study how the relative utility of the nodes is affected by their patience factors.

- In our model, each node negotiates with its opponents to agree upon the sharing rule, *without* requiring any centralized controller. Our algorithm thus works in a distributed manner making the system scalable.

Deriving SPNE strategies of the spectrum bargaining game is fundamentally different and much more difficult than solving the conventional Rubinstein-Ståhl bargaining game. This is because of two primary reasons– (i) spectrum can be spatially reused concurrently; two interfering players must not use the same channels simultaneously yet well-separated players can, and, (ii) players can only use whole channels, not fractional channels. We consider both constraints while analyzing the spectrum bargaining game.

1.4.2 Path Auction based Routing Mechanism

As mentioned before, to forward traffic, each secondary node incurs a cost because of using the spectrum it has acquired by paying a price. Moreover, using the acquired spectrum, each node can sustain a certain capacity. Clearly, secondary nodes will not have an incentive to relay traffic unless they are compensated for the costs they incur in forwarding traffic.

In this dissertation, we adopt the approach of Bayesian based algorithmic mechanism design for designing an *optimal* routing mechanism that *minimizes* the payment to be made to the nodes while *theoretically* ensuring that no node has any incentive to dishonestly declare its cost and/or capacity. This is in sharp contrast to previous works that either do not consider the capacity constraint of the nodes, or consider the capacity constraint

but do not enforce truthful capacity reporting, or require special hardware/software to be installed in the nodes for truthful revelation of a node's private information. To the best of our knowledge, ours is the first work that enforces truthfulness on multiple parameters of a node, without requiring any tamper proof software/hardware.

Our routing mechanism is comprised of two components– *a) route selection function*, which determines the route(s) that requires the least *expected* payment. We refer to such a route as the the Expected Least Paid Route (ELPR), and, *b) pricing function*, which determines the payment to be made to each node. The focus of our work is to design these two functions. Formally, the key contributions of our work can be considered as follows.

- We propose a *path auction* based routing mechanism in which nodes announce their cost and capacity, based on which a *multi-path* route is chosen and *payments* are made to the nodes. The route selection mechanism and the payment function ensure that all nodes can *maximize* their profit by truthfully reporting their cost and capacity, while the payment that needs to be made to the nodes is *minimized*.
- In addition to theoretically deriving the functions that determine the route selection and the payments to be made, we provide polynomial time algorithms for deploying the routing mechanism in DSA networks.
- We model cost of a node and its capacity as random variables. This serves a two-fold purpose– *a)* it helps tame the uncertainty regarding the actual cost and capacity of a

node; and, *b*) it helps reflect the heterogenous nature of the nodes in a DSA network in terms of both space and time variation of the cost and capacity of nodes.

In our work on routing, we assume that the interference issue has been taken care of by the spectrum allocation mechanism (such as the bargaining solution), so that interfering nodes have been allocated different channels. Thus, interference is not an issue at the routing layer. In addition, even though a node is only allowed to transmit over channels it has acquired, it can tune to any channel for reception.

Our work is partly based on the optimal auction theory by Myerson [48]. There are however two major differences with Myerson's work that makes our problem significantly more difficult to tame. First, in Myerson's work, each bidder is associated with only his valuation (cost) of the object for sale, while in our scenario, each bidder (node) is associated with a cost and a capacity. This increases the dimensionality of the problem. Secondly, in Myerson's theory, the bidders submit their bids independently, and a single bidder emerges as the winner to whom the object is allocated. On the other hand, in our work, though the nodes bid, the winner(s) corresponds to path(s). The nodes are then allocated traffic in the context of the paths they belong to. Based on the above discussion, clearly, the results in Myerson's work cannot be directly applied to our problem. Thus, our work can also be considered to be expanding the scope of optimal auction design in specific and of mechanism design in general.

1.5 Structure of this Dissertation

The rest of the dissertation is organized as follows. Chapter 2 discusses important game theoretic concepts used in this dissertation. In Chapter 3, we discuss related research in the area of spectrum sharing and also in the area providing incentives to nodes for stimulating cooperation in routing traffic. Chapter 4 first models the channel access problem as a bargaining game, assuming that all nodes are within the interference range of each other. Such a scenario often occurs in urban areas where the distribution of the secondary users is very dense. Chapter 5 extends the bargaining model developed in Chapter 4 by relaxing the assumption on interference constraint. The bargaining solution in Chapters 4 and 5 derive equilibrium strategies of the nodes in ‘demanding’ and ‘accepting’ their share of channels, which primarily answers the question of *how many* and *which channels* each node should access in order to *maximize* its gain. However, each channel that a node acquires comes at a cost and the node has to pay a price for acquiring it. In Chapter 6 we study the ‘end system’ perspective of such a cost by specifically focussing on the problem of providing incentives to nodes for routing traffic in DSA networks. This chapter presents a route selection mechanism and a pricing function that can induce nodes to reveal their cost and capacity honestly (making our auction truthful), while minimizing the payment that needs to be given to the nodes (making our auction optimal). Chapter 7 evaluates the performance of all our proposed algorithms via extensive simulations. Finally, Chapter 8 concludes this dissertation.

CHAPTER 2

A GAME THEORY PRIMER

Game theory is a field of applied mathematics that aims to model situations in which multiple participants interact or affect each other's outcomes. The concepts of game theory provide a language to formulate, analyze, and understand strategic scenarios, and furthermore, predict the outcome of complex interactions among rational entities.

Game theory has been used to analyze problems in economics, political science, sociology, biology and more recently in Computer Science. In this chapter, we formally review some of the fundamental definitions and concepts in game theory [27, 55, 56, 57] that have been used in our analysis.

2.1 Games, Strategies and Payoffs

A game consists of a set \mathcal{N} of players, $\{1, 2, \dots, n\}$. Each player $i \in \mathcal{N}$ has his own set of possible strategies, S_i . To play the game, each player i selects a strategy $s_i \in S_i$. The *vector of strategies* selected by the players comprises the *strategy profile*, $s = (s_1, \dots, s_n)$. Also, $S = \times_{i \in \mathcal{N}} S_i$ denotes the set of all possible ways in which players can pick strategies. The vector of strategies $s \in S$ selected by the players determine the outcome for each player. To formulate the game, a preference ordering on the possible outcomes needs to be specified for

each player. This is accomplished by defining, for each player i , a payoff function $u_i : S \rightarrow \mathbb{R}$.

Next, we formally define a game.

Definition 2.1. A game in strategic or normal form is denoted by the triple (\mathcal{N}, S, u) , where,

- $\mathcal{N} = \{1, 2, \dots, n\}$ denotes the set of players.
- $S = \times_{i \in \mathcal{N}} S_i$ denotes the set of all possible ways in which players can pick strategies. It is the Cartesian product of the strategy spaces S_i of all players.
- $u(s) = (u_1(s), \dots, u_n(s))$ is the n -tuple of payoff functions. $u_i(s)$ is the payoff function of player i that gives his von Neumann-Morgenstern utility for each profile $s = (s_1, \dots, s_n)$.

The most fundamental assumption in game theory is that all players in the game are *rational*. A rational player chooses actions to maximize his payoff. In case the game is not deterministic, a player chooses to maximize his expected utility (payoffs). The idea of maximizing the expected payoff was justified by the seminal work of von Neumann and Morgenstern in 1944 [53]. Since rational players tend to maximize their payoff, they can be regarded as being selfish. It worth emphasizing that being selfish does not necessarily mean “hurting” other players.

2.1.1 Dominant Strategy Solution

After a game is formulated, it is interesting to study whether it is possible to *predict* how the game can be played. Predicting the strategy of the players (and consequently the outcome of the game), is referred to as solving the game. This leads us to the notion of a *dominant strategy solution*.

Let us denote the tuple $(s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$ by s_{-i} , and the tuple (s_1, \dots, s_n) by (s_i, s_{-i}) . We can then define the following two important concepts.

Definition 2.2. *Strategy $s_i \in S_i$ is a **dominant** strategy for player i if for every alternate strategy $s'_i \in S_i$, it holds that,*

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}), \quad \forall s_{-i} \in S_{-i} \quad (2.1)$$

Strategy s_i is the unique dominant strategy for player i if the inequality holds strictly.

Definition 2.3. *A strategy vector $s \in S$ is a **dominant strategy solution**, if for each player i , and each alternate strategy vector $s' \in S$, it holds that,*

$$u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i}) \quad (2.2)$$

Strategy profile s is the unique dominant strategy solution of the game if the inequality holds strictly.

In other words, if each player has a best strategy, independent of the strategies played by the other players then we say that the game has a dominant strategy solution. Having a dominant strategy for each player is a stringent requirement for a game and only few games satisfy it. One game which has a (unique) dominant strategy solution is the famous “Prisoner’s Dilemma” game. We will discuss it next to illustrate the concept of finding a dominant strategy solution.

Prisoner’s Dilemma: In this game, two people (A and B) have been arrested for a crime. However, the police lack sufficient evidence to convict either suspect and thus need them to testify against each other. They have been put in different cells and cannot communicate with each other. The police have asked each to testify against the other. If one testifies (i.e., defects) and the other does not (i.e., cooperates), then the one that defects will be released and given a reward for testifying, while the one that cooperates will go to

Table 2.1: Payoff matrix: Prisoner's Dilemma

	B	
A	Cooperate	Defect
Cooperate	(1, 1)	(-1, 2)
Defect	(2, -1)	(0, 0)

jail. If neither testifies, both prisoners will be released and no rewards will be given. If both testify against the other, both will go to prison, but will however collect rewards for testifying. Also, in this game, both players will have to simultaneously choose between the two options. The payoff of the players for the different strategy profiles have been tabulated in Figure 2.1.

If both players cooperate, they get 1 each. If both defects, each gets 0. If one cooperates while the other does not, the former is punished and gets a payoff of -1, while the latter is rewarded resulting in a payoff of 2. Notice that, each prisoner will do strictly better by defecting, regardless of what the other one does. Thus, *the unique dominant strategy solution of this game is for both prisoners to defect*. It can also be noted that cooperation would actually lead to a better payoff of both players (1 each). However, self-gain-motivated behavior of the players leads to the inefficient outcome with payoffs 0.

2.1.2 Pure Strategy Nash Equilibrium

As noted earlier, only few games possess dominant strategy solutions. Thus, it is desirable to find a less stringent and more widely applicable solution concept. This leads us to the

discussion of a Nash equilibrium, which was proposed by John Nash in his seminal work [50] in 1950.

Definition 2.4. A strategy profile $s \in S$ is said to be a **Nash equilibrium**, if for all players i , and each alternate strategy vector $s'_i \in S_i$, we have that

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \quad (2.3)$$

A Nash equilibrium is strict if each player has a unique best response to his opponents' strategies, i.e., the above inequality is strict for all i and all $s'_i \neq s_i$.

Thus, a Nash equilibrium is a strategy profile, such that, each player's strategy is an optimal response to all the other players' strategies. In other words, no player i can *unilaterally* change his chosen strategy from s_i to s'_i and thereby improve his payoff. Such a solution can also be said to be self-enforcing and stable, in the sense that, given that the players are playing such a solution, it is in every player's best interest not to change his strategy. Notice that, definition (2.4) essentially defines a *pure strategy* equilibrium, since it considers that each player *deterministically* plays a strategy. Furthermore, it can also be observed that a strict Nash equilibrium has to be a pure strategy equilibrium.

Clearly, from definitions (2.3) and (2.4), we can say that a dominant strategy solution is a Nash equilibrium. For example, the (unique) Nash equilibrium in the Prisoner's dilemma game corresponds to both players choosing to defect, since in this case neither player can unilaterally change his strategy to improve his payoff. However, in general, Nash equilibria need not be unique. Coordination games, such as *Battle of the Sexes*, can have multiple Nash equilibria.

Battle of the Sexes: In this game, a couple needs to decide how to spend their evening. Each player considers two possibilities—going to a soccer game or a ballet. The husband (H)

Table 2.2: Payoff matrix: Battle of the sexes

	W		
H		Soccer	Ballet
Soccer		(2, 1)	(0, 0)
Ballet		(0, 0)	(1, 2)

prefers soccer while the wife (W) prefers ballet. However, both prefers spending the evening together rather than separately. The payoff matrix of the game is shown in Table (2.2).

This game has three Nash equilibria— two using pure strategies and one using *mixed* strategy. We will discuss the mixed strategy equilibrium in Section (2.1.3). If both choose to go to the same event, neither can improve his or her utility by *unilaterally* changing his or her strategy. Thus, the two pure strategy equilibria correspond to both attending the same event, with payoffs (2, 1) or (1, 2), depending on whether they both go to the soccer game or the ballet.

2.1.3 Mixed Strategy Nash Equilibrium

Though a pure strategy equilibrium defines a stable solution of a game, all games need not possess one. This leads us to study *mixed strategy* equilibria. A mixed strategy for player i , say σ_i , is a probability distribution over the set of his pure strategies S_i . We will denote the space of probability distributions over S_i by Σ_i . Each player's distribution is assumed to be statistically independent from those of his opponents. The payoffs in a mixed strategy profile are the expected values over the corresponding pure strategy payoffs. When dealing

with mixed strategy equilibria, it is considered that the players are risk neutral and each player seeks to maximize his expected payoff. Formally, a mixed strategy Nash equilibrium can be defined as follows.

Definition 2.5. *A mixed strategy profile σ^* is said to be a **Nash equilibrium**, if for all players i ,*

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*), \quad \forall s_i \in S_i \quad (2.4)$$

In 1951, Nash proved the following important result when the strategies of the players are extended by incorporating randomization [51].

Theorem 2.1. *If a game has a finite set of players and a finite set of strategies, then it necessarily possesses a Nash equilibrium of mixed strategies.*

It is easy to see that if a player uses a mixed strategy in a Nash equilibrium, then he must be indifferent between choosing all pure strategies to which he assigns a positive probability. Let us now consider a game which has no pure strategy equilibrium and find a mixed strategy solution to the game.

Matching Pennies: Two players, 1 and 2, have a penny each, and has to simultaneously announce heads (H) or tails (T). If the announcement matches, player 1 wins and player 2 loses. However, if the announcement differs, player 2 wins and player 1 loses. The payoff matrix is shown in Table (2.3), where 1 indicates win and -1 indicates loss.

Clearly, the game does not have a pure strategy equilibrium, since for all 4 strategy profiles, one player can always improve his payoff by deviating unilaterally. Let us find a mixed strategy equilibrium of the game. Let p_1 and p_2 be the probability of player 1 and 2

Table 2.3: Payoff matrix: Matching Pennies

	2	H	T
1		H	T
	H	(1, -1)	(-1, 1)
	T	(-1, 1)	(1, -1)

playing H respectively. Now, player 1's indifference between choosing H and T implies that,

$$(1) \cdot p_2 + (-1) \cdot (1 - p_2) = (-1) \cdot p_2 + (1) \cdot (1 - p_2)$$

Solving, we get $p_2 = 1/2$. Similarly, for player 2 to be indifferent between choosing H and T, we must have,

$$(-1) \cdot p_1 + (1) \cdot (1 - p_1) = (1) \cdot p_1 + (-1) \cdot (1 - p_1)$$

Solving, we get $p_1 = 1/2$. Thus, the mixed strategy Nash equilibrium in this game is for each player to play each strategy with probability $1/2$.

In a similar fashion, it can be shown that the mixed strategy equilibrium of the battle of the sexes game is for H to choose the soccer game with probability $2/3$ and choose the ballet with probability $1/3$, while the strategy of W is to choose the ballet with probability $2/3$ and choose the soccer game with probability $1/3$.

2.2 Repeated Games

In many cases, games are played repeatedly rather than just one time. Repeated games provide a formal framework to study why self-interested players manage to cooperate in a

long term relationship. In such games, the same set of players repeatedly play the same game, usually referred to as the “stage game”, for a finite or infinite number of times.

The progress of time in repeated games is measured in terms of *time periods*. The game begins in period 0. Let $s^t = (s_1^t, \dots, s_n^t)$ be the strategies that are played in period t . It is assumed that the game begins with the null history h^0 . For $t \geq 1$, $h^t = (s^0, \dots, s^{t-1})$ is the vector of strategy profiles *realized* at all periods before t . $H^t = (A)^t$ denotes the space of all possible period t histories. All players observe history h^t based on which each player can condition his strategy in period t . Thus, a pure strategy s_i for player i in the repeated game can be defined as a sequence of maps s_i^t , one for each period t , that map possible period t histories $h^t \in H^t$ to strategies in S_i . Similarly, a mixed strategy σ_i is a sequence of maps σ_i^t from H^t to the mixed strategy space Σ_i . Note that, the game starting at each play period can be considered as a game in its own right, and is referred to as a *subgame* of the original game.

The concept of a Nash equilibrium can be refined to describe an equilibrium of a repeated game, known as the Subgame Perfect Nash Equilibrium (SPNE).

Definition 2.6. *A repeated game strategy s^* is a **Subgame-Perfect Nash Equilibrium** if at each subgame, for all players i*

$$s_i^* \in \arg \max_{s_i \in S_i} u_i(s_i, s_{-i}^*). \quad (2.5)$$

Also, if h^ is the history generated by s^* , then h^* is the associated **equilibrium path**.*

In other words, a SPNE is an equilibrium such that players’ strategies constitute a Nash equilibrium in every subgame of the original game. In order to verify whether a strategy profile of a repeated game (both finite and infinite horizon) with observed actions constitutes

a SPNE, it is sufficient to check if there exists any history h^t where some player i can gain by deviating from the strategy specified by s_i if h^t occurs and conforming to s_i thereafter.

This is known as the one stage deviation principle.

Theorem 2.2. One Stage Deviation Principle: *A strategy profile s constitutes a SPNE if and only if there is no player i and no strategy s'_i that tallies with s_i except at a single period t and h^t and such that s'_i is a better response to s_{-i} than s_i , conditional on h^t being reached.*

For games with a finite horizon, the set of subgame perfect Nash equilibria can be determined by *backward induction*. However, for infinite horizon games, SPNEs cannot be found using backward induction. Infinite horizon games are more appropriate for modeling situations where players always believe the game can extend one more period with some probability, while the finite horizon game more aptly models situations where the terminal date is well foreseen.

Next, we consider a repeated game that involves *bargaining* between two players to share a piece of pie and discuss its SPNE.

2.2.1 The Rubinstein-Ståhl Bargaining Model

In Rubinstein's model two players must decide how to share a pie of size 1. In periods $0, 2, 4, \dots$ (i.e., in periods $2k$, where $k = 0, 1, 2, \dots$) player 1 proposes a sharing rule $(x, 1 - x)$ that player 2 can accept or reject. If player 2 accepts any offer the game ends. If player 2 rejects player 1's offer in period $2k$, then in period $2k + 1$ player 2 can propose a sharing rule $(x, 1 - x)$ that player 1 can accept or reject. If player 1 accepts one of player 2's offers,

the game ends. Otherwise, if player 1 rejects player 2's offer then the former player can make an offer in the subsequent period and so on. This is clearly an infinite horizon game of perfect information. Note that each period of the game comprises of two stages— in the first stage, one of the players propose a sharing rule, while in the second stage the other player announces his approval or refusal of the sharing rule.

The model specifies that, if $(x, 1-x)$ is accepted in period t , then the payoff of the players are $(\delta_1^t x, \delta_2^t (1-x))$, where x is player 1's share of the pie, and δ_1 and δ_2 are the discount factor of the two players.

The Subgame Perfect Nash Equilibrium (SPNE) in this model is – “Player i always demands a share of $\frac{1-\delta_j}{1-\delta_i\delta_j}$ when making an offer and accepts any share equal to or greater than $\frac{\delta_i(1-\delta_j)}{1-\delta_i\delta_j}$ ”. Note that player i 's demand of:

$$\frac{1-\delta_j}{1-\delta_i\delta_j} = 1 - \frac{\delta_j(1-\delta_i)}{1-\delta_i\delta_j}$$

is the highest share for player i that is accepted by j . Player i cannot gain by making a lower offer, for it too will be accepted. Making a higher (and rejected) offer and waiting to accept his opponent's offer next period hurts player i as:

$$\delta_i \left(1 - \frac{1-\delta_i}{1-\delta_i\delta_j} \right) = \delta_i^2 \frac{1-\delta_j}{1-\delta_i\delta_j} < \frac{1-\delta_j}{1-\delta_i\delta_j} \quad (2.6)$$

Similarly, player i cannot gain by rejecting any offer of at least $\frac{\delta_i(1-\delta_j)}{1-\delta_i\delta_j}$, since if he rejects he receives the share $\frac{1-\delta_j}{1-\delta_i\delta_j}$ next period, which is equivalent to $\frac{\delta_i(1-\delta_j)}{1-\delta_i\delta_j}$ in the current period.

Rubinstein extended the work of Stahl who considered a finite horizon version of the game. With a finite horizon game, the game can be easily solved by backward induction.

The unique SPNE is the last period is for the player who makes the offer (say player 1) to demand the whole pie, and for his opponent to accept this demand. In the second last period, the last offerer (player 1) will refuse all offers that give him less than δ_1 since he can get $\delta_1 \cdot 1$ by refusing and so on.

2.3 Mechanism Design

“The field of mechanism design aims to study how privately known preferences of many people can be aggregated towards a “social choice”. The main motivation of this field is microeconomic, and the tools are game-theoretic.”– Nisan and Ronen [55]. *Mechanism design aims to design games that have a dominant strategy solution such that the solution leads to a desirable outcome.*

In a distributed environment, such as the internet, participants cannot be assumed to follow an algorithm or a protocol, but rather their own self-interest. Since computers on the internet belong to different institutions or persons, they will likely act in a manner that is beneficial to their owner and can potentially manipulate algorithms to this end. It is thus important to design algorithms or protocols considering this kind of behavior in advance. Studying algorithms from such a perspective is the focus of algorithmic mechanism design.

2.3.1 Mechanism Design Problem

A mechanism design problem comprises of two components– *a*) the algorithmic output specification, and, *b*) descriptions of what the participating agents want, defined as utility functions over the set of possible outputs. Consider n agents. Each agent $i \in \{1, \dots, n\}$ has some private information which is referred to as his type $t_i \in T_i$. An output specification maps each type vector $t = (t_1, \dots, t_n)$ to a set of allowed outputs. Depending on his private information, each agent has his own preferences over the possible outputs. The preferences of agent i are given by a valuation function v_i that assigns a real number $v_i(t_i, q)$ to each possible output q .

A mechanism defines for each agent i a set of strategies S_i . Each agent i chooses a strategy $s_i \in S_i$. The mechanism takes as input the vector of strategies $s = (s_1, \dots, s_n)$. Based the vector of announced strategies, s , the mechanism computes an output $q(s)$ and a payment $p_i(s)$ to each of the agents. The payment $p_i(s)$ is used to incentivize agent i to behave in accordance with the mechanism's overall goals. The utility of agent i is $p_i(s) + v_i(t_i, q(s))$. This is what each agent wants to optimize.

Mechanism Design Optimization Problem: This is a mechanism design problem where the output specification is given by a positive real valued objective function $g(q, t)$ and a set of feasible outputs Q . The mechanism should be such that the output $q \in Q$ optimizes g .

2.3.2 Properties of the Mechanism

A mechanism is said to solve a given problem if it can guarantee that the required output occurs when each agent chooses his strategy in a selfish manner for maximizing his own utility. A mechanism thus needs to ensure that the agents' utilities are compatible with the algorithm. The mechanism can influence the utilities of the agents by handing out payments.

2.3.2.1 Dominant Strategy Implementation

Let us denote the tuple $(s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$ by s_{-i} , and the tuple (s_1, \dots, s_n) by (s_i, s_{-i}) . A mechanism is said to be an implementation with *dominant* strategies if,

1. For each agent i and each $t_i \in T_i$, there exists a strategy $s_i \in S_i$ such that for all possible strategies of the other agents s_{-i} , s_i maximizes agent i 's utility. Strategy s_i is said to be a *dominant* strategy for agent i . In other words, for every possible strategy $\hat{s}_i \in S_i$ and $\hat{s}_i \neq s_i$, we must have,

$$v_i(t_i, q(s_i, s_{-i})) + p_i(s_i, s_{-i}) \geq v_i(t_i, q(\hat{s}_i, s_{-i})) + p_i(\hat{s}_i, s_{-i}) \quad (2.7)$$

2. For every possible tuple of dominant strategies $s = (s_1, \dots, s_n)$, the output $q(s)$ satisfies the output specification.

A mechanism is said to be polynomial time computable if the output and payment functions are computable in polynomial time.

2.3.2.2 Truthful Implementation

In a lot of mechanism design problems, such as the shortest path problem and the task scheduling problem, agents' strategies simply involve reporting their types, i.e., $S_i = T_i$. For such problems, the following two properties should be exhibited by the mechanism.

- **Incentive Compatibility:** Each agent should be able to maximize his utility by reporting his true type t_i to the mechanism regardless of what other agents do so that the mechanism is *truthful* or *strategy-proof*. In other words,

$$v_i(t_i, q(t_i, t_{-i})) + p_i(t_i, t_{-i}) \geq v_i(t_i, q(\hat{t}_i, t_{-i})) + p_i(\hat{t}_i, t_{-i}) \quad (2.8)$$

where, $t_i \in T_i$ is the true type of agent i and \hat{t}_i is any other type.

- **Individual Rationality:** The utility of an agent should be non-negative, so that it is rational for him to participate in the game.

2.3.3 Vickrey-Clarke-Groves Mechanisms (VCG)

The Vickrey-Clarke-Groves (VCG) mechanism applies to maximization problems in mechanism design where the objective function is the sum of all agents' valuations, i.e., $g(q, t) = \sum_i v_i(t_i, q)$. The set of possible outputs is assumed to be finite. In general, we can define the class of VCG based mechanisms as follows.

Definition 2.7. A direct revelation mechanism $(p(t), q(t))$ belongs to the VCG family if

1. $q(t) \in \arg \max_Q \sum_{i=1}^n v_i(t_i, q(t))$

$$2. p_i(t) = \sum_{j \neq i} v_j(t_j, q(t)) + h_i(t^{-i}), \text{ where } h_i() \text{ is some function of } t_{-i}$$

It has been proven in [29] that a VCG mechanism is truthful. In short, a VCG mechanism provides a solution to maximization based mechanism design problems.

2.3.3.1 An Example— The Least Cost Path (LCP) Problem

Let us now provide an example of a mechanism design problem that can be solved using VCG mechanism. Consider a communication network represented by a directed graph $G = (E, V)$. For simplicity, assume that the graph is bi-connected. Each edge e in the graph is considered as an agent who has private information regarding the cost of sending a packet along the edge. The cost associated with each agent is considered as its type, $t_e \geq 0$. The objective of the problem is to find the cheapest path for sending a single packet from a given node s to a given node d in the graph G . This is known as the Least Cost Path (LCP) problem.

In this problem, clearly, the set of feasible outputs are all paths from s to d . The objective function is the path's total cost. The valuation of an agent e is 0, if its edge is not included on the chosen path, and $-t_e$, if its edge is included. The following mechanism ensures that the dominant strategy for each agent is to truthfully report its type t_e to the mechanism.

1. Based on the reported cost of each agent, find the LCP from s to d . With the cost of each agent used as its weight in G , the LCP simply corresponds to the shortest path from s to d with respect to the weights of the agents.

2. The payment p^e given to an agent e is 0 if e is not in the LCP and $p^e = d_{G|e=\infty} - d_{G|e=0}$ if it is. Here, $d_{G|e=\infty}$ is the cost of the least cost path that does not contain e , and $d_{G|e=0}$ is the cost of the least cost path when the cost of e is assumed to be zero.

It is easy to see that the shortest path indeed minimizes the total path cost. It can also be verified that the above mentioned mechanism is a VCG mechanism. Here, $d_{G|e=\infty}$ corresponds to $h_i(t^{-i})$ and $d_{G|e=0}$ corresponds to $\sum_{j \neq i} v_j(t_j, q(t))$.

What is the time complexity of finding all the payments? Since all agent (edge) weights are non-negative, Dijkstra's algorithm [19] can be used to find the shortest paths. Using Dijkstra's, a single shortest path can be found in $O(|E| + |V| \log |V|)$ time. Thus, finding all payments would take $O(|E|^2 + |E||V| \log |V|)$ time.

2.4 Auction Theory

An auction can be considered as a mechanism to allocate resources among a group of bidders. Auctions are the one of the oldest surviving classes of economic institutions. The first historical record of an auction is usually attributed to Herodotus, who reported a custom in Babylonia where men bid for women to wed.

Auction theory seeks to study how people act in an auction market and explores designing auction formats for various market scenarios. There is substantial agreement among economists that auctions are the best way to assign resources, such as spectrum [44]. Auc-

tions seek an answer to the question “Who should get the item(s) for sell and at what prices?”. Precisely, an auction can be said to accomplish the following tasks.

- **Price discovery:** In many cases, seller(s) (and even buyers) are not aware of how much an item or service is worth and how much they should sell or buy it for. An auction serves as a “market test” to ascertain the price tag that should be associated with the item or service for sell.
- **Winner determination:** The auction process determines who the object (item, contract etc.) should be allocated to, or who “wins” the auction. An advantage of auction is its tendency to assign the object to the bidder that values it most. This is important in many situations, such as when the government auctions public assets like spectrum, since the buyer that values the object most is usually the one that can use it best.
- **Payment computation:** The process can be used to determine how much the participants should pay.

2.4.1 Common Types of Auction

Traditionally there are four primary types of auction mechanisms that are used for the allocation of a single item.

- **First price sealed-bid auction:** In this auction, bidders submit their bids in sealed envelopes to the auctioneer simultaneously. The envelopes are opened and the person submitting the highest bid wins the object and pays the amount that he or she bid.
- **Second price sealed-bid auctions:** This is also known as the *Vickrey auction*. The bidders submit their bids in sealed envelopes to the auctioneer simultaneously. The envelopes are opened and the person submitting the highest bid wins the object but pays an amount equal to the *second highest bid*.
- **Open ascending-bid auction:** This is also known as the *English auction* [24]. In this auction, the price is steadily raised by the auctioneer, usually in small increments, with bidders dropping out once the price becomes too high for them. The process continues until only one bidder remains who wins the auction at the current price.
- **Open descending-bid auction:** This is also called the *Dutch auction*. The auctioneer begins by calling out a sufficiently high price that can presumably deter all bidders from buying at that price. The price is progressively lowered until a bidder indicates that he is prepared to buy at the current price. He or she wins the auction and pays the price at which they bid.

Designing auctions involve designing games with dominant strategy solutions. We will illustrate this using the example of Vickrey auction.

2.4.2 An Example: The Vickrey Auction

A seller has one indivisible object for sale. There are n potential buyers (bidders). The valuation of bidder i for the object is v_i . The strategy of each player i is to submit his bid $s_i \in [0, \infty)$. Let us consider designing an auction to sell the object, i.e., determine which bidder should get the object and what price he should pay.

A straightforward auction would be to award the painting to the highest bidder and charge him his bid (first price sealed-bid auction). It is easy to see that this game does not have a dominant strategy solution, i.e., a bidder may improve his utility by submitting a bid $s_i \neq v_i$. Vickrey's second price auction, however, can be used to allocate the object so that bidding one's true valuation is a dominant strategy for each bidder. In this case, the highest bidder would win the object and pay a price equal to the second highest bid. Precisely, if i is the highest bidder, i.e., $s_i > \max_{j \neq i} s_j$, his utility would be $v_i - \max_{j \neq i} s_j$, while the other bidders would pay nothing and have an utility of 0.

Clearly, in this auction, for each player i , bidding his true valuation weakly dominates all other strategies. To see why, let $s_i^* = \max_{j \neq i} s_j$. Now suppose first that $s_i > v_i$. If $s_i^* \geq s_i$, then his utility would be 0, which is the same utility he gets by bidding v_i . If $s_i^* \leq v_i$, i would obtain utility $v_i - s_i^*$ regardless of bidding s_i or v_i . If $v_i < s_i^* < s_i$, then i has an utility of 0 by bidding v_i which dominates his strategy of bidding s_i to get an utility of $v_i - s_i^* < 0$. Now consider $s_i < v_i$. In this case, for both $s_i^* \leq s_i$ and $s_i^* \geq v_i$, i 's utility would remain unchanged regardless of bidding s_i or v_i . However, if $s_i < s_i^* < v_i$, then i would have an

utility of $v_i - s_i^*$ by bidding v_i which dominates his strategy of bidding s_i to get an utility of 0.

From the above discussion, clearly, no player can gain by not bidding his true valuation. Moreover, notice that, since bidding one's true valuation is a dominant strategy, it does not matter whether a bidder knows the valuations of other bidders.

CHAPTER 3

RELATED RESEARCH

In this chapter, we review related research on spectrum allocation in DSA networks and on provisioning incentives to stimulate cooperation among nodes in wireless ad-hoc networks to route traffic.

3.1 Related Research on Spectrum Allocation

Coexistence of wireless systems that have to thrive by competing for spectrum has been studied in [34, 54]. Such a competitive scenario typically corresponds to wireless service providers competing for spectrum to maximize their revenue by supporting more customers. In [54], the authors use game theory to analyze strategies of cognitive radio nodes for accessing channels. Their solution approach is based on regret *minimization* of the nodes and propose an iterative learning algorithm using which nodes, that interact in a repeated game, can determine the channels to use. In contrast, we model the competition for spectrum among the nodes by considering the fact that each node will try to *maximize* its own benefit. Such modeling more aptly reflects non-cooperative interactions. Moreover, they assume nodes in their model to be homogenous. However, we study a heterogenous environment and allow nodes to compete for spectrum in a differential manner. In [34], the authors model the

competition among network operators who compete for spectrum. However, their framework is limited to a scenario where only two operators exist.

Spectrum sharing in wireless systems with an objective of maximizing overall system utility has been studied in [9, 10, 13, 58, 59, 63, 76]. These works however do not study power control. Techniques based on optimizing system utility primarily corresponds to collaborative schemes among nodes usually deployed by the same wireless service provider. Buddhikot *et al.* in [9, 10], propose a spectrum access architecture via a regional spectrum broker. In [13], the authors propose a local bargaining approach for mobile ad-hoc networks where users affected by mobility can form bargaining groups and adapt their spectrum assignment to approximate a new optimal assignment, instead of recomputing spectrum assignments for all users after each change in topology due to mobility. Their bargaining approach takes as input a previous spectrum assignment, and performs computations to adapt to recent topology changes. Nodes in their framework bargain to optimize a predefined system utility in contrast to ours where nodes bargain to maximize individual benefits. The authors in [58] formulate the problem of channel assignment, based on optimizing system utility, as a variant of the graph coloring problem by mapping channels into colors, and assigning them to users (nodes in the conflict graph of the network). They propose both a centralized allocation scheme, where a central server calculates an allocation assignment based on global knowledge, and a distributed approach, where devices negotiate local channel assignments towards a global optimization. In [59], the authors propose using a spectrum server to schedule the transmissions of a group of links sharing a common spectrum with an objective

of optimizing network throughput. They assume that the spectrum server knows the link gains in the network. Using a linear programming approach, the server then finds an optimal schedule that maximizes the average sum rate subject to a minimum average rate constraint for each link. To utilize the bandwidth left unused in cellular systems (considered as the primary system), the authors in [63] propose the design of a secondary system in an overlay mode over the primary system. The secondary system operates in a non intrusive manner and does not interact with the primary cellular system. They design a Medium Access Control protocol that enables inter-operation of the primary-secondary systems. The work in [76] proposes a device-centric spectrum management scheme to maximize system utility based on assigning orthogonal channels to interfering nodes. In their scheme, nodes observe local interference patterns and act according to a set of predefined spectrum rules.

Spectrum sharing by making nodes transmit at different power levels for minimizing interference has been studied in [18, 32, 33]. In [18], the authors consider power allocation strategies for radios operating in unlicensed bands. They model radio interaction as a two-player reputation based repeated game and use genetic algorithms to explore the space of possible power allocation strategies. The authors in [32] design auction mechanisms for allocating power among a group of spread spectrum users who share the bandwidth with a licensed user. In these auctions, the spectrum owner charges for SINR and received power. The work in [33] considers a spectrum sharing problem in which each wireless transmitter can select a single channel from a set of available channels, along with the transmission power. In their scheme, users exchange price signals, that indicate the negative effect of interference

at the receivers. Given this set of prices, each transmitter chooses a channel and power level to maximize its net benefit.

3.2 Related Research on Incentive Based Routing

The problem of cooperation in wireless networks has been an area of active research in recent years. Since forwarding a packet will incur a cost to a node, a selfish node will need incentive in order to forward others' packets. One possibility to provide incentive is to use a credit based system for paying nodes that forward traffic.

Buttayan and Hubaux proposed the first credit-based system [11, 12] for wireless ad-hoc networks. In their scheme, they propose the usage of nuglets, a virtual currency, to pay nodes to forward others' packets. Such payments are deducted from the sender or the destination. However, both proposals require a tamper-proof hardware at each node to ensure correct payment. This requirement limits the applicability of their work. Motivated by the nuglet, several other credit-based systems were proposed to stimulate cooperation in packet forwarding. In [77], Zhong et al. proposed Sprite, a credit based system for mobile ad-hoc networks, which provides incentives to selfish nodes for cooperating, without requiring tamper-proof hardware at any node. Their scheme uses a central authority to collect receipts from forwarding nodes based on which charges and rewards are made. They devise a mechanism to make nodes honestly forward packets and report receipts. However, they do not consider capacity constraint of the nodes in their framework. In [61], Salem et

al. proposed a charging and rewarding scheme based on symmetric cryptography to make collaboration rational for selfish nodes.

Nisan and Ronen first introduced the concept of *algorithmic mechanism design* in their seminal work in [55]. In that work, they proposed using an incentive compatibility based approach in conjunction with the more traditional protocol design approach. They also applied the concept of mechanism design to the problem of finding the LCP from s to d in a network with each edge (agent) having a cost of relaying a packet. With the cost of each agent used as its weight in the underlying topology graph of the network, the LCP corresponds to the shortest path from s to d with respect to the weight of the agents. To induce each agent to truthfully report its cost they devise their pricing scheme according to the VCG mechanism. In this scheme, the payment p^e given to an agent e is 0 if e is not in the LCP and $p^e = d_{G|e=\infty} - d_{G|e=0}$ if it is. Here, $d_{G|e=\infty}$ is the cost of the least cost path that does not contain e , and $d_{G|e=0}$ is the cost of the least cost path when the cost of e is assumed to be zero.

Several other works extended the model developed in [55], based on the VCG mechanism, into different networking environments. For example, the authors in [25] tailored the LCP based approach to work with the Border Gateway Protocol (BGP), considering Autonomous Systems (ASs) as the agents. In [1], the authors propose Ad hoc-VCG, which is an adaptation of the basic VCG algorithm to routing in mobile ad hoc networks. VCG has also been extended to work with multicast routing in [72]. A VCG based payment mechanism has also been used for LCP routing in ad-hoc networks in [71]. The work in [31] focusses on

complexity issues of computing VCG payments. In [39], the authors formulated the LCP problem in a multi-path routing scenario, but did not provide a solution to the problem. The authors of [78] proposed Corsac, which integrates VCG and cryptographic techniques to stimulate cooperation among wireless nodes.

As noted earlier, LCP need not be the path that minimizes the total payment that needs to be made to agents for incentivizing them to report their true costs honestly. In fact, VCG based mechanisms in general and LCP in specific suffer from the issue of high overpayment, which was first pointed out by Archer and Tardos [2]. They investigate the “frugal path” problem, which focusses on designing a mechanism that selects a path and induces truthful cost revelation without paying a high premium. Their paper contributes negative results on the frugal path problem and further proves that no reasonable mechanism can always avoid paying a high premium to induce truthtelling. The work in [21] focuses on finding the bounds for the price of truthful shortest path auction based routing mechanisms. In contrast to studying bounds, we focus on explicitly minimizing the payment that needs to be given to the nodes for routing traffic, while ensuring that no node has any incentive to dishonestly report its cost and/or capacity to the routing mechanism.

The authors in [22] studied the minimization of the expected payment that needs to be made to the relaying nodes to maintain truthfulness of their reported costs. But they did not consider the capacity constraint of the nodes, thereby restricting their framework to be applicable only to the single path routing scenario. The authors in [64] extended their framework by considering the capacity constraint of the nodes. However, to enforce the nodes to reveal

their capacity truthfully, they rely on tamper-proof hardware or cryptographic-receipt-based software to be present in the nodes. This is counter intuitive since the very nodes that may have an incentive to lie about their capacity has to install the special software/hardware that will discourage them from lying.

The authors in [68] studies auctions in the context of multi-path routing, but restrict their framework to the special case where the paths are node disjoint. Moreover, instead of investigating strategy-proof solutions (i.e., dominant strategy solutions), they design a mechanism in which there are Nash equilibria for all nodes to honestly bid their true cost. In [70], the authors also relax the dominant strategy criteria of strategy proof solutions to investigate Nash equilibria of honest cost revelation in the context of unicast routing in non-cooperative wireless networks. In general, as noted by the authors in [68, 70], mechanisms that result in Nash equilibria rather than dominant strategy solutions, require less payment than VCG, however at the cost of degenerating strategyproofness. Also, notice that [68, 70] only consider truthful revelation of the cost of a node disregarding capacity constraints. Wu et al. in [74], studies incentive compatibility in the context of opportunistic routing [5, 14] in wireless networks. In opportunistic routing, any node that overhears the transmission of a packet can participate in forwarding the packet. These protocols uses link loss rates to make forwarding decisions. The authors in [74] design a mechanism to prevent cheating in reporting and measuring these loss rates. Feldman et al. in [26] investigates designing contracts to induce cooperation when intermediate nodes can choose to forward or drop

packets, as well as when the nodes can choose to forward packets with different levels of quality of service.

The works we have discussed till now are credit based systems, in the sense that they provide payment as an incentive to nodes for cooperating and forwarding traffic. Another possibility to provide incentive is to use a reputation based system [7, 8, 40]. In general, reputation based systems are usually modeled as a repeated game whose objective is to stimulate cooperation. In [40], Marti et al. proposed a reputation system for ad hoc networks, in which a node monitors the transmission of a neighbor to observe whether the neighbor forwards others' traffic. If the neighbor does not forward others' traffic, it is considered as uncooperative, and this reputation is propagated throughout the network. The authors in [7, 8] proposes and evaluates the CONFIDENT protocol, which detects and isolates misbehaving nodes. A drawback of reputation based systems is that many of them depend on the broadcast nature of wireless networks in order to monitor other nodes. However, such monitoring may not always be accurate due to asymmetric links when nodes use power control. Furthermore, use of directional antennas [65, 73] also increase difficulty in monitoring.

CHAPTER 4

SPECTRUM BARGAINING IN COMPLETE CONFLICT GRAPHS

In this chapter, we model the competition for spectrum among the nodes using non-cooperative game theory. Specifically, we model the problem of agreeing upon a sharing rule of the channels among the nodes as a Rubinstein-Ståhl [60] [66] bargaining game. In our model, each node “bargains” with the other nodes (opponents) in the network regarding its “share” (*how many* and *which*) of the channels. In the *finite* horizon version of the game, nodes bargain *at most* for a given number of periods. If the nodes can decide upon a sharing rule of the channels within the given number of periods, each gets its respective share of the channels as per the sharing rule. Otherwise, each node gets *zero* channels. In the *infinite* horizon game, bargaining among the nodes go on until the nodes can agree upon a distribution of the channels. Notice that, until the nodes agree upon the sharing rule, none of the nodes can start data communication. Thus, “waiting” for the bargaining outcome also costs the nodes. We consider this cost by discounting future payoff of the nodes. This discounting represents the *patience* of the nodes in waiting for the bargaining outcome. We argue that it is the relative patience of the nodes that influence the degree of fairness in the sharing rule. As will become evident later, more patient nodes tend to get a larger fraction of available channels.

We solve the bargaining game depicted above by finding Subgame Perfect Nash equilibrium (SPNE) strategies of the nodes. The SPNE strategies that we derive comprise a set of strategies such that, no node in no subgame can deviate from its strategy and thereby gain from the deviation. In this chapter, we restrict ourselves to a scenario where all secondary nodes are within the interference range of each other. In other words, the conflict graph of the secondary users is a complete graph. Such a scenario often occurs in urban areas where the distribution of the secondary users is very dense. In Chapter 5 we will relax the constraint of all nodes to be in the interference range of each other and extend our work to arbitrary conflict graphs. Formally, the key points of this chapter can be considered as follows.

- We model the problem of dynamic spectrum access, in which the nodes have to agree upon a sharing rule of the channels among themselves, as an infinite horizon Rubinstein-Ståhl [60, 66] bargaining game.
- We derive SPNE strategies of the nodes, such that, using their SPNE strategies each node can optimize its utility against all its opponents in any subgame of the bargaining game¹. In other words, considering nodes to be rational, each node can play its SPNE strategy in the very first period of the game to decide on the sharing rule and start data communication.
- We theoretically study how the discount factor of the nodes impact their relative utility.

¹Note that, the original game can also be considered a subgame of itself.

- In our model, each node negotiates with its neighbors to agree upon the sharing rule, *without* requiring any centralized controller. Our algorithm thus works in a distributed manner making the system scalable.

The rest of the chapter is organized as follows. Section 4.1 describes the system model considered and also illustrates our solution concept using an example. In Section 4.2, we study the bargaining game considering only two nodes for ease of exposition. The N node bargaining game and the proposed spectrum access algorithm is described in Section 4.3. Finally, Section 4.4 concludes the chapter.

4.1 System Model and Solution Concept

4.1.1 System Model

We assume that N nodes (players) in a region are competing for a subset of M separate orthogonal spectrum bands not used by primary incumbents. The nodes are indexed from 1 to N and the spectrum bands (channels) are indexed from 1 to M . The nodes, for example, can correspond to IEEE 802.22 base stations accessing spectrum to connect their subscribers units, cognitive radio based 802.11 network, etc. Each node is equipped with cognitive radio and can communicate using multiple non-contiguous channels. The objective of each node is to acquire the maximum possible number of channels that are orthogonal from its interferers.

We assume that each node can successfully detect the presence of primary users on a channel and maintains a set of channels that it can use without affecting the operations of

any primary user. Interference among the nodes has been modeled using the pair wise binary matrix model [36]. This model basically states that– two nodes either conflict and has to use orthogonal channels, or, they do not conflict and can reuse the same channel. We use the following notations to represent the two system parameters.

- **Interference constraint:** Let $I = \{I_{n,k} | I_{n,k} \in \{0, 1\}\}_{N \times N}$, be a N by N matrix, representing the interference constraint among nodes,

$$I_{n,k} = \begin{cases} 1 & \text{if node } n \text{ and } k \text{ conflict;} \\ 0 & \text{if node } n \text{ and } k \text{ do not conflict} \end{cases} \quad (4.1)$$

Note that, I is the adjacency matrix representing the conflict graph of the network.

- **Channel throughput:** Let $C = \{C_m : 1 \leq m \leq M\}$ be a M element array where C_m represents channel m . Thus, C represents the set of available channels. We consider a static interference environment without considering the impact of fast-scale channel fading, since all the nodes are static. Further, we assume that all channels are homogeneous, i.e., all channels offer the same bandwidth and also have similar interference characteristics.

4.1.2 Solution Concept– The Ultimatum Game

As mentioned earlier, we model the problem of spectrum sharing by a set of cognitive radio enabled nodes as a bargaining game. Let us illustrate the concept of solving such a game, i.e., finding its SPNE using an example. Specifically, let us consider a repeated version of the

ultimatum game, in which two (hungry) players (P_1 and P_2) interact to decide how to divide a sum of money (say \$10) that is given to them (say for the purpose of buying food). The players can meet at most 3 times in a day (finite horizon with $T = 3$ periods)– First in the morning when P_1 makes an offer to P_2 regarding how to divide the sum. If P_2 accepts the offer then the two players buy breakfast using their respective share of the money. However, if P_2 rejects the share, both players stay hungry and meet in the afternoon when P_2 makes an offer to P_1 . If P_1 accepts the offer, the players have lunch using their money. Otherwise, the players do not eat anything and meet in the evening when P_1 makes a final offer to P_2 . If P_2 accepts this offer, the players go for dinner. However, if P_2 rejects P_1 's offer, then neither player gets any money and remains hungry for the night.

Notice that, if the players fail to agree upon a sharing rule in a bargaining period, they have to stay hungry till the next period. Thus, the players *discount* their future payoffs. For example, a player expecting to acquire \$10 in the next period, can be satisfied with only \$7 in the current period, considering a discount factor of 0.7. Also note that, the more hungry a player is, lesser is his discount factor, since he would want to eat something as soon as possible. In this example, we take $\delta_1 = 0.6$ and $\delta_2 = 0.8$. We also assume that the smallest division of the goods available (money) is 1 cent. Let us reason backward from time $t = 2$ to find the SPNE of the 3-period game,

- *Dinner:* At time $t = 2$, the payoff maximizing equilibrium strategy for P_1 is to offer P_2 the smallest amount possible that the latter will accept, which is 1 cent. P_2 will not reject this offer, since then he would get nothing.

Table 4.1: Ultimatum Game: The SPNE strategy for the 3-period game is for P_1 to demand \$6.79 and for P_2 to accept any offer that gives him at least \$3.21.

Period	P_1 's share	P_2 'share
0	<u>6.79</u>	3.21
1	5.99	<u>4.01</u>
2	<u>9.99</u>	0.01

- *Lunch:* At time $t = 1$, the SPNE of the 2-period game is for P_2 to offer P_1 the minimum amount more than which the latter cannot gain by rejecting the offer and for P_1 to accept this offer. Since at $t = 2$, P_1 can acquire at most \$9.99, which is worth $\$9.99 \times \delta_1 = \5.99 now, P_2 should offer P_1 \$5.99 and keep the rest for himself.
- *Breakfast:* At time $t = 0$, the SPNE of the 3-period game is for P_1 to offer P_2 the minimum amount more than which P_2 cannot gain in any subgame by rejecting the offer. Thus, P_1 offers P_2 $\$4.01 \times \delta_2 = \3.21 and keeps the rest of \$6.79 for himself. Clearly, P_2 cannot gain by rejecting this offer.

Table 4.1 shows the SPNE share of the two players in the various subgames. In each row, the underlined player denotes the offerer in that period. Some observations,

- *Discount factor:* Notice that, instead of bargaining over breakfast, lunch and dinner, had the players been bargaining to divide the money in 3 consecutive back-to-back periods for breakfast, the players would become more *patient* while playing the game. In other words, δ_1 and δ_2 will start tending to 1, as the time between two bargaining periods tends to zero. It can be shown that as the discount factors tend to 1 and as $T \rightarrow \infty$ (infinite horizon) the players get an equal share [60].

- *First mover advantage:* The finite horizon bargaining game we discussed has a first mover advantage, since P_1 does better than P_2 . This first mover advantage disappears as T increases and as we take the length of the time periods to be arbitrarily short. This has been discussed in [4].

Conclusion: Before breakfast, if P_1 offers P_2 \$3.21 and keeps the rest (\$6.79) for himself, then neither will P_2 have an incentive to reject the offer nor will P_1 have an incentive to demand a larger share (and both players will be able to enjoy their breakfast). In this chapter we seek to find the SPNE strategy of the players in the bargaining game of sharing spectrum which will allow the player making the offer in the *first period to propose a sharing rule such that no player receiving the offer will have an incentive to reject their respective share and neither will the player making the offer have an incentive to demand a larger share.* Thus, assuming players to be rational, all nodes will be able to acquire channels in the very first period and can start communication.

4.2 2 Player Bargaining Game

We first study the channel allocation problem from a game theoretic perspective considering only two nodes (players) for ease of exposition. We model the spectrum allocation problem as an infinite horizon Rubinstein-Ståhl bargaining game. In our model two players must agree on how to share the M available channels among them. The bargaining game proceeds in “time periods” in which one player proposes a sharing rule to the other player who can

either ‘accept’ or ‘reject’ the offer. The bargaining continues until a sharing rule is accepted by both players. In periods $0, 2, 4, \dots$ (i.e., in periods $2k$, where $k = 0, 1, 2, \dots$) player 1 proposes a sharing rule $(x, M - x)$ that player 2 can accept or reject. If player 2 accepts any offer the game ends. If player 2 rejects player 1’s offer in period $2k$, then in period $2k + 1$ player 2 can propose a sharing rule $(x, M - x)$ that player 1 can accept or reject. If player 1 accepts one of player 2’s offers, the game ends. Otherwise, if player 1 rejects player 2’s offer then the former player can make an offer in the subsequent period and so on. This is clearly an infinite horizon game of perfect information.

Payoffs: We specify that if $(x, M - x)$ is accepted in period t , then, the payoffs of players 1 and 2 are $\delta^t x$ and $\delta^t (M - x)$ respectively, where $\delta \in [0, 1]$ is the discount factor of the players. The discount factor δ represents the *delay cost* in achieving the bargaining outcome. Until the players agree upon a sharing rule, none of the players can start communication. Thus, a player values a channel more now than it values the same channel in a future period. This decrease in value of the channels represents the *dissatisfaction* of the players in being unable to start communication immediately. Note that, as the time delay between two bargaining periods decreases, the players become more patient, i.e., δ increases.

Nash Equilibria: Note that there are a great many Nash equilibria (NE) in this game. For example, the strategy profile “player 1 always demands $x = M - 1$ channels and refuses all smaller shares; player 2 always offers $x = M - 1$ channels and accepts any offer giving him at least 1 channel” is a Nash equilibrium. In general, any partition of the M channels between the two players corresponds to a NE strategy profile. However, not all of these

profiles are subgame perfect. For example, in the aforementioned profile if player 2 rejects player 1's first offer, and offers player 1 a share $x > (M - 1)\delta$ in the next period, then player 1 should accept, because the best possible outcome for him if he rejects is to receive $M - 1$ channels in the next period, which is worth only $(M - 1)\delta$.

We next investigate the subgame perfect Nash equilibrium of the bargaining game outlined above. First, we study the finite horizon version of the game and then extend the results to obtain the perfect equilibrium for the infinite horizon version.

4.2.1 Finite Horizon Bargaining Game

The finite horizon bargaining game can be solved by backward induction. The unique subgame-perfect Nash equilibrium in the last period is for the player who makes the offer (let's assume it is player 1) to demand $M - 1$ channels and for his opponent to accept this demand². In the period before that, the offerer in the last period (player 1) will refuse all offers that give him less than $(M - 1)\delta$, because he can ensure this amount by refusing. Thus, the perfect equilibrium in the second last period is for the offerer (player 2) to demand $M - (M - 1)\delta$ channels and for player 1 to accept this demand. This reasoning can be applied backwards to obtain the unique SPNE in the first period.

Table 4.2 shows the SPNE of games (or subgames) with different number of periods.

Row i in the table shows the subgame perfect equilibria in the *first* period of a game with i

²Note that, we assume for a player to accept an offer he has to be given *atleast* one channel. If a player is offered 0 channels, he may not accept the offer deterministically even though such a sharing also corresponds to a NE strategy profile.

Table 4.2: Subgame Perfect Nash Equilibria.

Periods	Offerer	Receiver
1	$M - 1$	1
2	$M - M\delta + \delta$	$M\delta - \delta$
3	$M - M\delta + M\delta^2 - \delta^2$	$M\delta - M\delta^2 + \delta^2$
4	$M - M\delta + M\delta^2 - M\delta^3 + \delta^3$	$M\delta - M\delta^2 + M\delta^3 - \delta^3$
.	.	.
.	.	.

periods. For example, row 3 shows the SPNE of a game with 3 periods. The SPNE in this case is “Player i always demands a share of $M - M\delta + M\delta^2 - \delta^2$ channels when it is his turn to make an offer. He accepts any share equal to or greater than $M\delta - M\delta^2 + \delta^2$ channels and refuses any smaller share”.

It is evident from the table that the SPNE shares demanded by the two players in increasingly larger period games form a pattern. Based on this pattern we find the SPNE shares demanded by the two player in any arbitrary k period game. Depending on whether k is even or odd, we have two cases:

- **k is even:** The SPNE share demanded by player i when it is his turn to make an offer in a k period game is

$$\{M - M\delta + M\delta^2 - M\delta^3 + \dots - M\delta^{k-1}\} + \{\delta^{k-1}\} = \frac{M(1 - \delta^k)}{1 + \delta} + \delta^{k-1} \quad (4.2)$$

Also, player i accepts any share equal to or greater than

$$M - \left[\frac{M(1 - \delta^k)}{1 + \delta} + \delta^{k-1} \right] = \frac{M\delta(1 + \delta^{k-1})}{1 + \delta} - \delta^{k-1} \quad (4.3)$$

- **k is odd:** The SPNE share demanded by player i when it is his turn to make an offer in a k period game is

$$\{M - M\delta + M\delta^2 - M\delta^3 + \dots + M\delta^{k-1}\} - \{\delta^{k-1}\} = \frac{M(1 + \delta^k)}{1 + \delta} - \delta^{k-1} \quad (4.4)$$

Also, player i accepts any share equal to or greater than

$$M - \left[\frac{M(1 + \delta^k)}{1 + \delta} - \delta^{k-1} \right] = \frac{M\delta(1 - \delta^{k-1})}{1 + \delta} + \delta^{k-1} \quad (4.5)$$

Thus, the unique SPNE of a finite horizon bargaining game with k periods is– “Player i always demands a share of $\frac{M\{1-(-\delta)^k\}}{1+\delta} - (-\delta)^{k-1}$ channels when it is his turn to make an offer. He accepts any share greater than or equal to $\frac{M\delta\{1-(-\delta)^{k-1}\}}{1+\delta} + (-\delta)^{k-1}$ and refuses any smaller share.”

4.2.2 Infinite Horizon Bargaining Game

To obtain the SPNE of the infinite horizon game, we make k tends to infinity in the SPNE of the finite horizon game with k periods. Thus, the SPNE share demanded by player i , when it is his turn to make an offer in the infinite horizon game is

$$\lim_{k \rightarrow \infty} \left[\frac{M\{1-(-\delta)^k\}}{1+\delta} - (-\delta)^{k-1} \right] = \frac{M}{1+\delta} \quad (4.6)$$

Similarly, player i accepts any share equal to or greater than

$$\lim_{k \rightarrow \infty} \left[\frac{M\delta\{1-(-\delta)^{k-1}\}}{1+\delta} + (-\delta)^{k-1} \right] = \frac{M\delta}{1+\delta} \quad (4.7)$$

Thus, the SPNE in this model is – “Player i always demands a share of $\frac{M}{1+\delta}$ channels when making an offer and accepts any share equal to or greater than $\frac{M\delta}{1+\delta}$ channels”. Note that

player i 's demand of:

$$\frac{M}{1+\delta} = M - \frac{M\delta}{1+\delta}$$

is the highest share for player i that is accepted by his opponent. Player i cannot gain by making a lower offer, for it too will be accepted. Making a higher (and rejected) offer and waiting to accept his opponent's offer next period hurts player i as:

$$\delta \frac{M\delta}{1+\delta} = \delta^2 \frac{M}{1+\delta} < \frac{M}{1+\delta} \quad (4.8)$$

Similarly, player i cannot gain by rejecting any offer of at least $\frac{M\delta}{1+\delta}$ channels since if he rejects he receives the share $\frac{M}{1+\delta}$ next period, which is equivalent to $\frac{M\delta}{1+\delta}$ in the current period.

Note that in equations (4.6) and (4.7) as $\delta \rightarrow 1$, the players SPNE shares approaches $M/2$. Based on this we make the following important observation.

The more patient players are, higher is the degree of fairness among them, i.e., the channels tend to get equally divided between the players. In other words, lesser the time delay between two consecutive bargaining periods, the channels get more and more equally split between the players.

The SPNE share can be computed from their closed form from equations (4.6) and (4.7) by the player making the offer in the *very first period of the game*. Thus the solution of the bargaining game of apportioning the channels is computationally efficient.

4.2.3 Uniqueness of the Infinite-Horizon Equilibrium

Let us now demonstrate that the subgame perfect equilibrium given in Section 4.2.2 is unique. Continuation payoffs of a strategy profile in a subgame starting at period t is defined to be the payoff in time t units of the outcome induced by that profile. For example, the continuation payoff of player 1 at period 2 of a profile that leads to player 1 getting m channels in period 3 is $m\delta$, whereas this outcome has a payoff of $m\delta^3$ in period 0.

Let v_1 and V_1 be player 1's lowest and highest continuation payoffs in any perfect equilibrium of any subgame that begins with player 1 making an offer. Similarly, let w_1 and W_1 be player 1's lowest and highest perfect equilibrium payoffs in any subgame that begins with an offer by player 2. Also, let v_2 and V_2 be player 2's lowest and highest perfect equilibrium continuation payoffs in any subgames beginning with an offer by player 2, and let w_2 and W_2 be player 2's lowest and highest perfect equilibrium continuation payoffs in any subgame beginning with an offer by player 1.

When player 1 makes an offer, player 2 will accept any x such that player 2's share of $M - x$ exceeds δV_2 , since player 2 cannot expect more than V_2 in the continuation game following his refusal. Thus, we have,

$$v_1 \geq M - \delta V_2 \tag{4.9}$$

Similarly, player 1 will accept any share more than δV_1 . Thus,

$$v_2 \geq M - \delta V_1 \tag{4.10}$$

Now, player 2 will never offer a share greater than δV_1 . Thus, player 1's continued payoff when player 2 makes an offer is

$$W_1 \leq \delta V_1 \quad (4.11)$$

Also, since player 1 can obtain at least v_1 in the continuation game by rejecting player 2's offer, player 1 will reject any x (offered by player 2), such that $x < \delta v_1$. Thus, we have,

$$w_1 \geq \delta v_1 \quad (4.12)$$

Since player 2 can obtain at least v_2 in the continuation game by rejecting player 1's offer, player 2 will reject any x (demanded by player 1), such that, $M - x < \delta v_2$. Hence, player 1's maximum equilibrium payoff when making an offer V_1 , satisfies,

$$V_1 \leq \max(M - \delta v_2, \delta W_1)$$

Using inequality (4.11), we can say,

$$V_1 \leq \max(M - \delta v_2, \delta^2 V_1) \quad (4.13)$$

Next, we will prove that,

$$\max(M - \delta v_2, \delta^2 V_1) = M - \delta v_2 \quad (4.14)$$

We prove this by contradiction. Let us assume that $\max(M - \delta v_2, \delta^2 V_1) = \delta^2 V_1$. Thus, we have, $V_1 \leq \delta^2 V_1$. For this to be true, it must be the case that, $V_1 \leq 0$ (since δ cannot exceed 1). However, this implies that, $M - \delta v_2 > \delta^2 V_1$, since $\delta \in [0, 1]$ and $v_2 \leq M$. This contradicts our assumption. Thus, we can write,

$$V_1 \leq M - \delta v_2 \quad (4.15)$$

Similarly, by symmetry,

$$V_2 \leq M - \delta v_1 \tag{4.16}$$

Now, combining inequalities (4.9) and (4.16), we can write,

$$\begin{aligned} v_1 &\geq M - \delta V_2 = M - \delta(M - \delta v_1) \\ \Rightarrow v_1 &\geq \frac{M}{1 + \delta} \end{aligned} \tag{4.17}$$

Also, combining inequalities (4.10) and (4.15), we can write,

$$\begin{aligned} V_1 &\leq M - \delta v_2 = M - \delta(M - \delta V_1) \\ \Rightarrow V_1 &\leq \frac{M}{1 + \delta} \end{aligned} \tag{4.18}$$

Thus, $v_1 \leq V_1$. This implies that $v_1 = V_1$. Similarly, we can show that,

$$v_2 = V_2 = \frac{M}{1 + \delta} \tag{4.19}$$

Now, since $v_1 = V_1$, from inequality (4.11) and (4.12) we have, $W_1 \leq \delta v_1$ and $w_1 \geq \delta v_1$. This implies that,

$$w_1 = W_1 = \delta \frac{M}{1 + \delta} \tag{4.20}$$

Similarly, we can show that,

$$w_2 = W_2 = \delta \frac{M}{1 + \delta} \tag{4.21}$$

Thus, the perfect equilibrium continuation *payoffs* are unique. Now we will show that the perfect equilibrium *strategy profile* is also unique. Let us consider a subgame that begins

with an offer by player 1. We have already shown that player 1 must demand exactly $x = v_1$. Note that player 2 is indifferent between accepting and rejecting this offer, since even if he rejects the offer, he can acquire the same amount ($\delta \frac{M}{1+\delta}$) in the next period (but cannot gain more than this). However, perfect equilibrium requires that player 2 accept player 1's demand of $x = v_1$ with probability 1. If player 2 accepts all $x < v_1$ with probability 1, but accepts v_1 with probability less than 1, then player 1 does not have any best response. Thus, this randomization by player 2 does not lead to an equilibrium. A similar argument can be applied to subgames that begins with an offer by player 2.

4.3 N Player Bargaining Game

We now investigate the game considering N players, i.e, N nodes, each within the interference range of the others. In this game, N players must decide how to share the M available channels among them. Let P_i ($i \in [1, N]$) denote the player making the offer and let $P_{-i} = \{R_1, R_2, R_3, \dots, R_{N-1}\}$ be the set of players receiving the offer. The subscripts of the players in P_{-i} follows the order in which they will make an offer next, if P_i 's offer is rejected. In periods kN ($k = 0, 1, 2, \dots$), player 1 (P_1) proposes a sharing rule (x_1, x_2, \dots, x_N) that players 2 through N (P_{-1}) can accept or reject. If *all* players in P_{-1} accept their respective offers, the game ends. However, if at least one player in P_{-1} rejects player 1's offer in period kN , then in period $kN + 1$ ($k = 0, 1, 2, \dots$), player 2 (P_2) can propose a sharing rule $(x_1, x_2, x_3, \dots, x_N)$ that players in P_{-2} can accept or reject. And so on. In general player

$i \in [1, N]$ makes an offer in periods $kN + (i - 1)$. The game outlined above is clearly an infinite horizon game of perfect information.

Next, we define the payoff of the players. If $\{x_i | 1 \leq i \leq N\}$ is accepted in period t , then the payoff of player i is $\delta^t x_i$, where $\delta \in [0, 1]$ is the discount factor of the players. Like in the two player case, δ represents the delay cost in achieving the bargaining outcome or in other words reflects the *patience* of the players. Higher the patience of the players, larger is δ .

4.3.1 Subgame Perfection

As mentioned before, let P_i be the player making the offer and $P_{-i} = \{R_1, R_2, R_3, \dots, R_{N-1}\}$ be the set of players receiving the offer. Recall that, the subscripts of the players in P_{-i} follows the order in which they will propose a sharing rule, if the offer proposed by P_i is rejected. Following is the subgame perfect equilibrium of this game— " P_i always demands a share of

$$\frac{M}{\sum_{n=0}^{N-1} \delta^n} \tag{4.22}$$

channels and offers player $R_k \in P_{-i}$ a share of

$$\delta^k \frac{M}{\sum_{n=0}^{N-1} \delta^n}$$

channels. When player $R_k \in P_{-i}$ receives an offer, he accepts any share equal to or greater than,

$$\delta^k \frac{M}{\sum_{n=0}^{N-1} \delta^n} \tag{4.23}$$

channels and rejects all smaller shares". We will now prove that this is indeed a subgame perfect equilibrium of the game. Note that P_i 's demand of,

$$\frac{M}{\sum_{n=0}^{N-1} \delta^n} = M - \left[\sum_{k=1}^{N-1} \delta^k \frac{M}{\sum_{n=0}^{N-1} \delta^n} \right]$$

is the highest share for player i that is accepted by his opponents. Player i cannot gain by making a lower offer, for it too will be accepted. If player i makes a higher offer, then *at least one player* in P_{-i} (say R_k) has to be offered a share lesser than $\delta^k \frac{M}{\sum_{n=0}^{N-1} \delta^n}$. Thus, P_i 's offer will be rejected. After rejection, each of the following offerers in P_{-i} , i.e., R_k for $k \in [1, N - 1]$, will offer player i a share of,

$$\delta^k \left(\delta^{N-k} \frac{M}{\sum_{n=0}^{N-1} \delta^n} \right) = \delta^N \frac{M}{\sum_{n=0}^{N-1} \delta^n} < \frac{M}{\sum_{n=0}^{N-1} \delta^n}$$

Thus, P_i will end up with a smaller share by demanding a share greater than $\frac{M}{\sum_{n=0}^{N-1} \delta^n}$ channels. Also, it is optimal for $R_k \in P_{-i}$ to accept any share greater than equal to $\delta^k \frac{M}{\sum_{n=0}^{N-1} \delta^n}$ channels. This is because even if he rejects, R_k cannot gain more than this share in any subsequent period³.

4.3.2 Discussion

Note that, in equations (4.22) and (4.23), as $\delta \rightarrow 1$, the SPNE shares of the players approach M/N . This corroborates with the observation we made in the two player case. Furthermore, we assert that δ will be close to 1 in our model. This is because, the delay between two bargaining periods is almost negligible. If an offer is rejected, the next bargaining period begins

³For example, if R_1 rejects his share of $\delta \frac{M}{\sum_{n=0}^{N-1} \delta^n}$ in a period, in the *next period* R_1 can demand at most $\frac{M}{\sum_{n=0}^{N-1} \delta^n}$, which is worth only $\delta \frac{M}{\sum_{n=0}^{N-1} \delta^n}$. And so on.

almost immediately. In general, as the delay between two bargaining periods become longer, the players become more impatient (δ decreases). This is because following a rejection, they will have to refrain from communicating over a longer period, until the next opportunity to bargain.

It is also intuitive to think that an impatient player gets a lesser fraction of channels against a more patient player. Since, in our game, all the players will be patient, nobody can outlast the other(s) by waiting to get a larger share. Thus, in equilibrium, the players will tend towards equally dividing the channels among themselves, as suggested by equations (4.22) and (4.23) when $\delta \rightarrow 1$.

Equations (4.22) and (4.23) give the closed form expression of the SPNE share of the players. Thus, even in the N player case, *solution of the bargaining problem can be found efficiently without the need of any iterative process.*

One might suspect that the bargaining game we discussed might have a “first mover advantage”, i.e., a player making an offer earlier may have an advantage over a player who gets to make an offer latter. This happens only in the finite horizon version of the game. In the infinite horizon game, when we take the length of the time periods to be very short, the “first mover advantage” disappears. This is shown in [4]. Since we model the spectrum access problem as an *infinite* horizon bargaining game where the length of each bargaining period will be very short (time taken to exchange messages or packets among nodes), our solution does not suffer from the “first mover advantage” problem. Thus, ordering of the players will not matter in our solution.

4.3.3 Spectrum Allocation Algorithm

Notice that the SPNE strategy discussed in Section 4.3.1 may allocate “fractional” channels to the nodes. However, since we consider each channel as an indivisible unit, such allocation is not possible. Thus, to get the final solution, the SPNE shares of the players has to be rounded to the nearest integer value. More precisely, the revised strategy of the players are—“ P_i always demands a share of $\text{round}(M / \sum_{n=0}^{N-1} \delta^n)$ channels and offers player $R_k \in P_{-i}$ a share of $\text{round}(\delta^k M / \sum_{n=0}^{N-1} \delta^n)$ channels. When player $R_k \in P_{-i}$ receives an offer, he accepts any share equal to or greater than $\text{round}(\delta^k M / \sum_{n=0}^{N-1} \delta^n)$ channels and rejects all smaller shares”. Based on this strategy of the players, Algorithm 1 presents the spectrum allocation procedure that player P_i will invoke when making an offer. In the algorithm, $L(P_i)$ is the set of channels that P_i should demand and $L(R_k)$ is the set of channels that should be offered to $R_k \in P_{-i}$.

Algorithm 1 Allocate Spectrum

Require: Number of players, N ; Set of available channels, $C = \{C_1, \dots, C_M\}$; patience factor, δ

- 1: Channels Allocated, $A = \{\emptyset\}$
- 2: $L(P_i) = \{\text{any round} \left(\frac{M}{\sum_{n=0}^{N-1} \delta^n} \right)$ channels from $C - A\}$
- 3: $A = A \cup L(P_i)$
- 4: **for** $k = 1$ to $N-1$ **do**
- 5: $L(R_k) = \{\text{any round} \left(\delta^k \frac{M}{\sum_{n=0}^{N-1} \delta^n} \right)$ channels from $C - A\}$
- 6: $A = A \cup L(R_k)$
- 7: **end for**

4.4 Summary

This chapter models the problem of dynamic spectrum access by a network of N cognitive radio enabled nodes as a perfect information infinite horizon bargaining game. In the

bargaining game, nodes “haggle” among themselves to agree upon a sharing rule of the channels. The objective of each node is to acquire as many channels as possible from among M orthogonal spectrum bands not in use by any primary incumbents, subject to the interference constraints. The nodes are associated with a *discount factor* which represents the *patience* of the nodes in waiting for the bargaining outcome. We derive SPNE strategies of the bargaining game, using which each node can optimize its utility against all its opponents (interfering neighbors) in any subgame of the bargaining game. A closed form expression of the SPNE share of the nodes has been derived, which makes finding of the bargaining solution computationally efficient. Furthermore, the chapter also investigates the impact of discount factor, δ , on fairness. Also, our approach is distributed in nature, obviating the need of a centralized control.

CHAPTER 5

SPECTRUM BARGAINING IN ARBITRARY CONFLICT GRAPHS

In the previous chapter we assumed all secondary nodes to be within the interference range of each other. This scenario mostly reflects the situation in urban areas. To expand the scope of applicability of our bargaining framework, in this chapter we relax the assumption on interference constraint taken in Chapter 4 and extend our work to arbitrary conflict graphs.

Similar to Chapter 4, we also model the competition for spectrum among nodes in an arbitrary conflict graph as a Rubinstein-Ståhl [60] [66] bargaining game. In this game, each node “bargains” with the other nodes (opponents) in the network regarding its “share” (*how many* and *which*) of the channels. Each node is associated with a *discount factor* which represents the *patience* of the nodes in waiting for the bargaining outcome. The patience of the nodes, for example, can depend on the loss in revenue that a wireless service provider suffers due to delay in being able to provide service.

We solve the bargaining game by deriving Subgame Perfect Nash Equilibrium (SPNE) strategies of the nodes (players) in the game. Specifically, the contributions of this chapter can be considered as follows. *a)* We model the problem of dynamic spectrum access, in which the nodes in an arbitrary conflict graph have to agree upon a sharing rule of the

channels among themselves, as a Rubinstein-Ståhl bargaining game. *b)* First, we investigate *finite* horizon version of the game and identify its SPNE strategies using backward induction. *c)* We then extend the results to the *infinite* horizon bargaining game. *d)* We identify Pareto optimal equilibria of the game for improving spectrum utilization. *e)* We propose polynomial time algorithms to find the SPNE strategies of both the finite and infinite horizon versions of the game.

The rest of the chapter is organized as follows. Section 5.1 formally defines the finite horizon spectrum bargaining game. Section 5.2 derives SPNE strategies of the finite horizon version of the game. These results are extended to the infinite horizon game in Section 5.3. Finally, Section 5.4 concludes the chapter.

5.1 Finite Horizon Game Formulation

We first model the channel access problem as a *finite* horizon Rubinstein-Ståhl bargaining game. In this model, the game is played *at most* for a fixed number of periods. In Section 5.3 we will relax the finite horizon criteria and extend the concept to *infinite* horizon games. The system model considered in this chapter is same as the one discussed in Section 4.1.1.

5.1.1 Game Formulation

Given the conflict graph of a network, we now model the problem of channel access by the nodes (players) as a finite horizon bargaining game. In this game, N players must decide how to share the M available channels among them. The bargaining game proceeds in “time periods” in which one player proposes a sharing rule to the other players. Each of the other players can then either ‘accept’ or ‘reject’ the shares they have been respectively offered. The bargaining continues until a sharing rule has been accepted by all players *or* until the maximum number of allowable periods, T , has been reached (finite horizon game).

Let P_i ($1 \leq i \leq N$) denote the player making the offer in any arbitrary period and let $P_{-i} = \{P_j : 1 \leq j \leq N, j \neq i\}$ be the set of players receiving the offer. P_i makes an offer in the following periods,

$$\begin{cases} kN + (i - 1) & i \leq T, k \in [0, \lfloor (T - i)/N \rfloor] \\ \text{None} & i > T \end{cases} \quad (5.1)$$

Thus, the players make their offer in a round robin fashion. When making an offer in period t , P_i 's strategy is denoted by (x_i^t, x_{-i}^t) , where $x_i^t \subset C$ is the set of channels demanded by P_i in period t . Also, $x_{-i}^t = \{x_j^t \subset C | 1 \leq j \leq N, j \neq i\}$ where x_j^t is the set of channels offered to $P_j \in P_{-i}$ in period t . Also, each player $P_j \in P_{-i}$ chooses some function $f_j^t : [0, |C|] \rightarrow \{\text{‘accept’}, \text{‘reject’}\}$ in period t , i.e, each P_j chooses which offers to accept and which to reject depending on the number of channels he received, $|x_j^t|$.

To illustrate the game, let us consider that $N \leq T^1$. In periods kN (for $k \in [0, \lfloor (T - 1)/N \rfloor]$), player 1 (P_1) proposes a sharing rule (x_1^{kN}, x_{-1}^{kN}) to all players (including himself). After inspecting the offer, each player $P_j \in P_{-1}$ can either accept or reject the respective shares they have been offered. If all players in P_{-1} accept their respective shares, the game ends. However, if at least one player in P_{-1} rejects the share he has been offered by P_1 in period kN , then in period $kN + 1$ (for $k \in [0, \lfloor (T - 2)/N \rfloor]$), player P_2 can propose a sharing rule $(x_2^{kN+1}, x_{-2}^{kN+1})$ that players in P_{-2} can accept or reject. If all players in P_{-2} accept their respective shares, the game ends. And so on until an offer made by P_i is accepted by all players in P_{-i} or until the maximum number of allowable periods, T , has been reached. The game outlined above is clearly a finite horizon game of perfect information.

Payoff: The outcome of the game can correspond to two different cases— all players agree upon a sharing rule of the channels within T periods or they fail to do so within the allocated time. Thus, to define the payoff of the players we need to study the following two cases.

- *An agreeable sharing rule is obtained within T periods:* If $\{x_i^t | 1 \leq i \leq N\}$ is accepted in period t , then the payoff of P_i is $\mathbb{R}_i = \delta_i^t |x_i^t|^2$, where $\delta_i \in [0, 1]$ is the discount factor of P_i . The discount factor represents the *delay cost* in achieving the bargaining outcome. Until the players agree upon a sharing rule, none of the players can start communication. Thus, a player values a channel more today than he values the same channel in a future period. This decrease in value of the channels represents the *dissatisfaction* of the players in being unable to start communication until the agreeable

¹Assuming $N \leq T$ is for illustrating the game now. Our analysis holds for all N and T

²Note that δ_i^t is δ_i raised to the power of t .

sharing rule is achieved. Also, note that, as the time delay between two bargaining periods decreases, the players become more patient, i.e., the discount factor of the players increases.

- *An agreeable sharing rule is not obtained within T periods:* This corresponds to the disagreement outcome of the game. Clearly, if the players are unable to agree upon a sharing rule, the payoff of each player would be *zero*.

5.1.2 Some Definitions

Let us define a few terms in game theory that has been used for the analysis of the bargaining game in this chapter.

- **Pareto Optimality:** Pareto optimality is a measure of efficiency. An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto optimal outcome cannot be improved upon without hurting at least one player.
- **Backward Induction:** Backward induction is an iterative process for solving finite sequential games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next-to-last moving player is determined taking the last player's action as given. This process continues backwards

in time until all players' actions have been determined. Effectively, one determines the Nash equilibrium of each subgame of the original game.

5.1.3 Equilibrium Analysis

We will represent the strategy profile in period t as $\{(x_i^t, x_{-i}^t), f_{-i}^t\}$, where (x_i^t, x_{-i}^t) is the sharing rule as proposed by P_i and $f_{-i}^t = \{f_j^t | 1 \leq j \leq N, j \neq i\}$, where f_j^t is the function used by $P_j \in P_{-i}$. If $f_j^t(|x_j^t|) = \text{'accept'}$ $\forall j \neq i$, then each player gets his respective share as proposed in (x_i^t, x_{-i}^t) . Otherwise all players get zero channels.

The strategy profile $\{(x_i^{T-1}, x_{-i}^{T-1}), f_{-i}^{T-1}\}$ is a Nash Equilibrium in period $T - 1$ (last period) if $f_j^{T-1}(|x_j^{T-1}|) = \text{'accept'}$ $\forall j \neq i$ and there is no set $|y_j^{T-1}| < |x_j^{T-1}|$ such that $f_j^{T-1}(|y_j^{T-1}|) = \text{'accept'}$ $\forall j \neq i$ that leads to the existence of a set $|y_i^{T-1}| > |x_i^{T-1}|$. Here, P_i does not have an incentive to unilaterally increase his demand, because that would be rejected by some $P_j \in P_{-i}$. Also, no $P_j \in P_{-i}$ would want to reject the share offered to him by P_i , since then he would get zero channels.

However, note that all NE's in the last period of the game need not be pareto optimal. Our solution approach identifies and uses those NE's that are pareto optimal to find the SPNE strategy of the players in the first period of the game using backward induction.

In the next section we will first study SPNE of the finite horizon spectrum bargaining game. We will then extend these results for the infinite horizon version of the game in Section 5.3.

5.2 Finite Horizon Bargaining Game

We will now investigate the SPNE of the finite horizon bargaining game (of T periods), where each player bargains with the other players to agree upon a sharing rule of the channels. Finding SPNE involves two main steps – (1) finding equilibrium of the last period of the game and (2) finding equilibrium of the previous periods using backward induction.

5.2.1 Finding Last Period Equilibrium Strategies of the Players

According to the definition of NE in Section 5.1.3, following is a NE strategy for the players in the last period of the game– “ P_i offers a sharing rule $(x_i^{T-1}, x_{-i}^{T-1})$, such that $|x_i^{T-1}|$ is maximized over all possible interference free allocations that assign at least 1 channel to all players in P_{-i} and the players in P_{-i} accept all offers that give them at least 1 channel”. This is a NE because, no $P_j \in P_{-i}$ will have an incentive to reject their respective shares, since doing so will get them zero channels. Also, since P_i 's share has been maximized, P_i will not have an incentive to demand a larger share of channels.

Let us now see when P_i 's share of channels gets maximized. Let us consider players in P_{-i} who are one hop away from P_i , i.e., neighbors of P_i in the conflict graph. Let them be denoted by P_{-i}^{Nbr} , where $P_{-i}^{Nbr} = \{P_j | P_j \in P_{-i} \text{ and } I_{i,j} = 1\}$. P_i will obviously demand all channels not allocated to any of his neighbors. Thus, we can write P_i 's share of channels,

x_i^{T-1} , as,

$$|x_i^{T-1}| = M - \left| \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1} \right| \quad (5.2)$$

Clearly, $|x_i^{T-1}|$ will get maximized when $|\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ has its minimum value over all possible interference free allocations given that each player in P_{-i}^{Nbr} has to be given at least one non-interfering channel. In other words, P_i 's share of channels is maximized in all those sharing rules where the number of *distinct* channels allocated to the players in P_{-i}^{Nbr} taken together has the least value over all possible allocations.

Let $\overline{P_{-i}^{Nbr}} = P_{-i} \setminus P_{-i}^{Nbr}$ denote the players in P_{-i} who are more than one hop away from P_i . Since spectrum can be reused concurrently by players more than one hop away from each other, the channel allocation of the players in $\overline{P_{-i}^{Nbr}}$ do not directly influence x_i^{T-1} . All that is required for P_i is to offer the players in $\overline{P_{-i}^{Nbr}}$ at least one non-interfering channel so that his offer is accepted.

From the above discussion, it can be said that P_i 's strategy, $(x_i^{T-1}, x_{-i}^{T-1})$, corresponds to a NE when the following two conditions hold,

1. $|\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ is minimized over all possible interference free allocations, and,
2. Each player in P_{-i} gets *at least* one non-interfering channel. This condition is to ensure that the sharing rule offered by P_i is accepted, otherwise P_i 's payoff will become zero.

Notice that there can be several NE strategy profiles, $(x_i^{T-1}, x_{-i}^{T-1})$, for P_i that maximizes $|x_i^{T-1}|$. However, not all of those will be *Pareto optimal*. It may be possible to improve the

share of a player, $x_j^{T-1} \in x_{-i}^{T-1}$ without hurting any x_k^{T-1} , $k \in [1, N]$ and $k \neq j$. We need to find those NE's that are pareto efficient to maximize spectrum utilization.

Algorithm 2 Find Last Period SPNE

Require: Number of Players, N ; Interference Constraint, I ; Set of Available Channels, C ; Number of Periods, T

- 1: $i \leftarrow \text{findLastOfferer}(N, T)$
- 2: Sort the players in P_{-i}^{Nbr} in non-increasing order according to their degree in g_{-i}^{Nbr} .
- 3: **for all** $P_j \in P_{-i}^{Nbr}$ **do**
- 4: $x_j^{T-1} = \{C_m | C_m \notin \bigcup_{I_{j,q}=1} x_q^{T-1} \text{ and } C_k \in \bigcup_{I_{j,q}=1} x_q^{T-1} \forall k < m\}$
- 5: **end for**
- 6: $x_i^{T-1} \leftarrow \{C \setminus \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}\}$
- 7: Sort the players in P_{-i}^{Nbr} in non-increasing order according to their degree in g_{-i} .
- 8: **for all** $P_j \in P_{-i}^{Nbr}$ **do**
- 9: $x_j^{T-1} = \{C_m | C_m \notin \bigcup_{I_{j,q}=1} x_q^{T-1} \text{ and } C_k \in \bigcup_{I_{j,q}=1} x_q^{T-1} \forall k < m\}$
- 10: **end for**
- 11: **while true do**
- 12: assigned \leftarrow false
- 13: **for all** $P_j \in P_{-i}$ **do**
- 14: **if** $\exists C_m : C_m \notin \bigcup_{I_{j,q}=1} x_q^{T-1} \text{ and } C_k \in \bigcup_{I_{j,q}=1} x_q^{T-1} \forall k < m$ **then**
- 15: $x_j^{T-1} = \{x_j^{T-1} \cup C_m\}$
- 16: assigned \leftarrow true
- 17: **end if**
- 18: **end for**
- 19: **if** !assigned **then**
- 20: break
- 21: **end if**
- 22: **end while**
- 23: **return** $(x_i^{T-1}, x_{-i}^{T-1})$

Algorithm 2 finds the pareto efficient NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ of the player P_i making the offer in the last period of the game. The algorithm does three primary tasks.

1. **Find the last offerer:** First we need to find the player, P_i , who will make the offer in the last period. Algorithm 3 does this. It takes as input N and T and returns the ID of the player making the last offer.

Algorithm 3 findLastOfferer(N, T)

Require: Number of players, N ; Number of Periods, $T \geq 1$

```
1: if  $T \leq N$  then
2:    $i \leftarrow T$ 
3: else
4:   if  $T \% N == 0$  then
5:      $i \leftarrow N$ 
6:   else
7:      $i \leftarrow T \% N$ 
8:   end if
9: end if
10: return  $i$ 
```

2. **Find equilibrium strategy of P_i :** In order to find pareto optimal NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ of P_i , where $|x_i^{T-1}|$ is maximized, our algorithm minimizes $|\cup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ such that each $P_j \in P_{-i}$ receives at least one channel and no $x_j^{T-1} \in x_{-i}^{T-1}$ can be improved without hurting any $x_k^{T-1}, k \in [1, N]$ and $k \neq j$.

First we will describe how the algorithm finds a NE $(x_i^{T-1}, x_{-i}^{T-1})$ that need not be pareto efficient. Minimizing $|\cup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ is equivalent to the problem of coloring the subgraph induced by the players in P_{-i}^{Nbr} with the minimum number of colors. We will use degree ordered graph coloring for this purpose. Let the subgraph of the conflict graph induced by the players in P_{-i}^{Nbr} be g_{-i}^{Nbr} (need not be connected) and the subgraph induced by the players in P_{-i} be g_{-i} . g_{-i}^{Nbr} is also a subgraph of g_{-i} . To maximize $|x_i^{T-1}|$, g_{-i}^{Nbr} has to be colored with the least possible number of colors. Note that this is different from coloring g_{-i} , because a minimum color assignment of g_{-i} does not necessarily minimize the color assignment of g_{-i}^{Nbr} .

In line 2, the algorithm sorts the players in P_{-i}^{Nbr} in non-increasing order based on their degree in g_{-i}^{Nbr} . In lines 3 to 5, the algorithm considers each player $P_j \in P_{-i}^{Nbr}$ in non-increasing order of their degree, and assigns P_j the first channel in C that has not been assigned to any of P_j 's neighbors. This process essentially intends to minimize $|\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$. After the **for** loop in lines 3 to 5 ends, we can thus assign P_i his maximizing share of $M - |\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ channels. This is done in line 6, which assigns,

$$x_i^{T-1} = \{C \setminus \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}\} \quad (5.3)$$

Next, we are left with assigning a single channel to each player in $P_{-i}^{\overline{Nbr}}$ to find P_i 's NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ that need not be pareto optimal. To do this, the algorithm first sorts the players in $P_{-i}^{\overline{Nbr}}$ in non-increasing order according to their degree in g_{-i} . Then in lines 8 to 10, each player $P_j \in P_{-i}^{\overline{Nbr}}$ is considered in non-increasing order of their degree and assigned a channel that has not been assigned to any of P_j 's neighbors.

$(x_i^{T-1}, x_{-i}^{T-1})$ obtained after the **for** loop in lines 8 to 10 ends is a NE strategy³ for P_i even though it may not be *pareto optimal*. Since each $P_j \in P_{-i}$ receives only one channel it may be possible to improve the share of some players in P_{-i} without decreasing the share of any other player. Notice that the share of P_i cannot possibly be improved further since it has already been optimized.

³ $|x_i^{T-1}|$ has been maximized, and all players in P_{-i} receives a channel each – so no $P_j \in P_{-i}$ has an incentive to reject his share and also P_i does not have an incentive to demand more channels

3. **Find pareto optimal NE strategy of P_i :** Improvement of P_i 's NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ obtained so far to get a pareto optimal NE strategy is called pareto improvement. This pareto improvement is done by the **while** loop in lines 11 through 22. At each iteration of the **while** loop, the algorithm checks each player $P_j \in P_{-i}$ to see if a channel can be added to x_j^{T-1} . The **while** loop iterates till no more channels can be assigned to any player in P_{-i} . Clearly, after the **while** loop terminates, $(x_i^{T-1}, x_{-i}^{T-1})$ produced will correspond to a pareto optimal NE strategy of P_i . Also note that the pareto improvements are done by trying to assign a single channel to a player $P_j \in P_{-i}$ at a time, instead of assigning all $C \setminus \{\{\bigcup_{l,j,q=1} x_q^{T-1}\} \cup x_j^{T-1}\}$ channels to P_j at the same time. This has been done to improve fairness.

5.2.2 Finding Equilibrium of the Previous Periods using Backward Induction

Let P_i be the offerer in period t and P_l ($l \neq i$) be the offerer in period $t + 1$. Given the SPNE strategy of P_l in period $t + 1$, SPNE strategy of P_i in period t can be found based on the following fact— if a player $P_j \in P_{-i}$ gets $|x_j^{t+1}|$ channels in period $t + 1$, then in period t , P_j will accept any offer that gives him greater than equal to $\lceil \delta_j |x_j^{t+1}| \rceil$ channels. This is because x_j^{t+1} channels in period $t + 1$ is worth only $\delta_j |x_j^{t+1}|$ in period t to P_j . Thus P_j can be “satisfied” with only $\delta_j |x_j^{t+1}|$ channels in period t . However, since a player cannot get fractional channels, hence $\lceil \delta_j |x_j^{t+1}| \rceil$ channels has to be offered to P_j in period t . Formally, following is the SPNE strategy in period t .

- **SPNE strategy of P_i :** For each $P_j \in P_{-i}^{Nbr}$, P_i chooses a set of channels $c_j \subset x_j^{t+1}$ such that $|c_j| \leq (|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil)$ and for $C_s \in \bigcup_{P_j \in P_{-i}^{Nbr}} c_j$ it holds that $C_s \notin \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{t+1} \setminus c_j, \forall C_s \in \bigcup_{P_j \in P_{-i}^{Nbr}} c_j$. Also, $|\bigcup_{P_j \in P_{-i}^{Nbr}} c_j|$ should be the largest such set possible so that $x_i^t = \{x_i^{t+1} \cup \{\bigcup_{P_j \in P_{-i}^{Nbr}} c_j\}\}$ is maximized. Each $P_j \in P_{-i}^{Nbr}$ is offered the set of channels $x_j^t = x_j^{t+1} \setminus c_j$. In other words, P_i offers at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels to each $P_j \in P_{-i}^{Nbr}$, taking at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from each $P_j \in P_{-i}^{Nbr}$ such that x_i^t is maximized over all possible interference free allocations that allows P_i to take at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ from each $P_j \in P_{-i}^{Nbr}$.
- **SPNE strategy of $P_j \in P_{-i}$:** Each $P_j \in P_{-i}$ accepts all offers in which they get at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels.

Clearly, no player will have an incentive to unilaterally deviate from his strategy. If P_i makes a larger demand of channels than x_i^t defined above, then some $P_j \in P_{-i}^{Nbr}$ has to be given a smaller share of channels than $\lceil \delta_j |x_j^{t+1}| \rceil$ in period t , and thus P_i 's offer will be rejected. If rejected, in no subsequent period can P_i hope to get a share of channels which in period t is worth more than $|x_i^t|^4$. Also, no player $P_j \in P_{-i}$ in period t can hope to get a share of channels in any subsequent period which in period t is worth more than $\lceil \delta_j |x_j^{t+1}| \rceil$. Thus, the above mentioned strategy of the players comprises a SPNE in period t . In other words, *no player can deviate from his above mentioned strategy in period t and subsequently gain from his deviation in any sub-game starting from period t .*

⁴Recall that if a player $P_j, j \in [1, N]$ gets the set of channels x_j^{t+2} in period $t+2$, then in period t it is worth only $\delta_j^2 |x_j^{t+2}|$ to P_j .

Algorithm 4 Find SPNE by backward induction

Require: Number of Players, N ; Interference Constraint, I ; Set of Available Channels, C ; Number of Periods, T

```
1:  $(x_i^{T-1}, x_{-i}^{T-1}) \leftarrow$  Find Last Period SPNE( $N, I, C, T$ )
2: for  $t = T - 2$  to 0 do
3:   if  $l = 1$  then
4:      $i \leftarrow N$ 
5:   else
6:      $i \leftarrow l - 1$ 
7:   end if
8:    $\widehat{P}_{-i}^{Nbr} \leftarrow \{\phi\}$ 
9:   for all  $P_j \in P_{-i}^{Nbr}$  do
10:     $s \leftarrow |x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ 
11:    if  $s > 0$  then
12:       $Q_j \leftarrow \mathcal{P}_s(x_j^{t+1}) \setminus \mathcal{P}_{s-1}(x_j^{t+1})$ 
13:       $\widehat{P}_{-i}^{Nbr} \leftarrow \widehat{P}_{-i}^{Nbr} \cup P_j$ 
14:    end if
15:  end for
16:  if  $\widehat{P}_{-i}^{Nbr}$  is null then
17:     $x_i^t \leftarrow x_i^{t+1}$ 
18:     $x_j^t \leftarrow x_j^{t+1}, \forall P_j \in P_{-i}$ 
19:     $l \leftarrow i$ 
20:    continue
21:  else
22:     $Q \leftarrow \prod_{j=1}^N Q_j : P_j \in \widehat{P}_{-i}^{Nbr}$ 
23:  end if
24:  for  $r = 1$  to  $r = |Q|$  do
25:     $index \leftarrow 1$ 
26:     $S \leftarrow \{\cup_{P_j \in P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}} x_j^{t+1}\}$ 
27:    for all  $j = 1$  to  $j = N : P_j \in \widehat{P}_{-i}^{Nbr}$  do
28:       $s \leftarrow |x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ 
29:       $c_j^r \leftarrow \overset{index+s-1}{index} q_r$  {Elements  $index$  to  $index + s - 1$  of set  $q_r$ }
30:       $S \leftarrow S \cup \{x_j^{t+1} \setminus c_j^r\}$ 
31:       $index \leftarrow index + s$ 
32:    end for
33:     $profit(q_r) \leftarrow \{q_r \setminus S\}$ 
34:  end for
35:  Select  $q_m \in Q : |profit(q_m)| = \max_{q_r \in Q; 1 \leq r \leq |Q|} (|profit(q_r)|)$ 
36:   $x_i^t \leftarrow x_i^{t+1} \cup profit(q_m)$ 
37:   $x_j^t \leftarrow x_j^{t+1} \setminus profit(q_m), \forall P_j \in \widehat{P}_{-i}^{Nbr}$ 
38:   $x_j^t \leftarrow x_j^{t+1}, \forall P_j \in P_{-i} \setminus \widehat{P}_{-i}^{Nbr}$ 
39:  while true do
40:    assigned  $\leftarrow$  false
41:    for all  $P_j \in P_{-i}$  do
42:      if  $\exists C_m \in C : C_m \notin \cup_{I_{j,q}=1} x_q$  and  $C_k \in \cup_{I_{j,q}=1} x_q \forall k < m$  then
43:         $x_j^t = \{x_j^t \cup C_m\}$ 
44:        assigned  $\leftarrow$  true
45:      end if
46:    end for
47:    if !assigned then
48:      break
49:    end if
50:  end while
51:   $l \leftarrow i$ 
52: end for
53: return  $(x_1^0, x_{-1}^0)$ 
```

Algorithm 4 finds the SPNE strategy of the offerer $P_i, i \in [1, N]$ in period t given the equilibrium strategy of the offerer $P_l, l \in [1, N], l \neq i$ in period $t + 1$ to finally find the SPNE strategy (x_1^0, x_{-1}^0) of P_1 in the first period of the game. The algorithm first invokes algorithm 2 to find the equilibrium strategy, $(x_l^{T-1}, x_{-l}^{T-1})$, of offerer P_l in period $T - 1$ (last period) and works in iterations of decreasing period number, using backward induction to find the SPNE strategy of the offerer in the respective period at each iteration. The algorithm finally outputs the SPNE strategy (x_1^0, x_{-1}^0) of offerer P_1 in the *first* period of the game. As discussed earlier, this is such a strategy that no $P_j \in P_{-1}$ can gain in any sub-game by rejecting his respective share in x_{-1}^0 . Also P_1 's share, x_1^0 , is maximized so that P_1 does not have an incentive to demand a larger share of channels.

Let us now delve into the details of algorithm 4. We will explain how the algorithm finds the equilibrium strategy (x_i^t, x_{-i}^t) of offerer P_i in period t given the equilibrium strategy $(x_l^{t+1}, x_{-l}^{t+1})$ of offerer $P_l (l \neq i)$ in period $t + 1$. Algorithm 4 does the following tasks:

1. **Find offerer P_i in period t :** First the algorithm finds the offerer P_i in period t . This is done in lines 3 to 7.
2. **Find SPNE strategy of P_i in period t :** P_i 's SPNE strategy in period t will correspond to P_i taking the maximum possible number of channels from his neighbors in order to maximize his share of channels in period t . In order to maximize the number of channels that P_i can acquire from his neighbors in P_{-i}^{Nbr} , P_i will have to consider all *interference free* allocations that allow him to take at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels

from $P_j \in P_{-i}^{Nbr}$ and use the one that allows P_i to take the maximum number of channels from his neighbors. Steps 8 to 38 basically does this. Let us look at these steps in more detail.

Note that, P_i can potentially take channels only from those $P_j \in P_{-i}^{Nbr}$ for whom $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil > 0$. Let $\widehat{P}_{-i}^{Nbr} \subset P_{-i}^{Nbr}$ be the set of neighbors of P_i satisfying this criteria. The `for` loop in lines 9 to 15 finds the set \widehat{P}_{-i}^{Nbr} and also generates the set Q_j for each $P_j \in \widehat{P}_{-i}^{Nbr}$ where Q_j is the set of all subsets of x_j^{t+1} with cardinality $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$. Note that, it is possible that the set \widehat{P}_{-i}^{Nbr} is null. In this case P_i cannot take channels from any of his neighbors. Thus, the SPNE strategy for P_i will be to demand the set of channels, $x_i^t = x_i^{t+1}$, and offer each $P_j \in P_{-i}$ the set of channels $x_j^t = x_j^{t+1}$. The case of \widehat{P}_{-i}^{Nbr} being null is taken care of in lines 16 to 20 at the end of which the algorithm *continues* onto the next iteration to find the strategy of the offerer in period $t - 1$.

For the case when \widehat{P}_{-i}^{Nbr} is not null, we define set Q as the cartesian product of all Q_j for $P_j \in \widehat{P}_{-i}^{Nbr}$. Thus, element $q_r \in Q, r \in [1, |Q|]$, is a set of channels that P_i can acquire from his neighbors in \widehat{P}_{-i}^{Nbr} taken together, taking $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from neighbor $P_j \in \widehat{P}_{-i}^{Nbr}$. Set Q has all such combination of channels that P_i can take from his neighbors in \widehat{P}_{-i}^{Nbr} . Notice that in line 22, the cartesian product of all Q_j for $P_j \in \widehat{P}_{-i}^{Nbr}$ considers the Q_j 's in ascending order of their subscripts. According to our definition of Q and rules of cartesian product, if P_j is the lowest

numbered player in \widehat{P}_{-i}^{Nbr5} , then the first $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels of $q_r \in Q$ belong to $P_j \in \widehat{P}_{-i}^{Nbr}$. Likewise, if P_k is the second lowest numbered player in \widehat{P}_{-i}^{Nbr} then the next $|x_k^{t+1}| - \lceil \delta_k |x_k^{t+1}| \rceil$ channels belong to $P_k \in \widehat{P}_{-i}^{Nbr}$. And so on. Let $c_j^r \subset q_r$ be the set of channels belonging to $P_j \in \widehat{P}_{-i}^{Nbr}$. Notice that P_i can use a channel $C_k \in q_r$, iff $C_k \notin \left\{ \bigcup_{P_j \in \widehat{P}_{-i}^{Nbr}} \{x_j^{t+1} \setminus c_j^r\} \right\} \cup \left\{ \bigcup_{P_j \in P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}} x_j^{t+1} \right\}$. In other words, P_i can use a channel $C_k \in q_r$ if and only if *no* $P_j \in \widehat{P}_{-i}^{Nbr}$ has C_k *after* the set of channels c_j^r has been taken from P_j *and* neither does any $P_j \in P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}$ have the channel C_k . Let $\text{profit}(q_r) \subset q_r$ be the set of channels that P_i can use from among the channels in q_r . Thus,

$$\text{profit}(q_r) = q_r \setminus \left\{ \left\{ \bigcup_{P_j \in \widehat{P}_{-i}^{Nbr}} \{x_j^{t+1} \setminus c_j^r\} \right\} \cup \left\{ \bigcup_{P_j \in P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}} x_j^{t+1} \right\} \right\} \quad (5.4)$$

where, $c_j^r \subset q_r$ is the set of channels belonging to $P_j \in \widehat{P}_{-i}^{Nbr}$. The `for` loop in lines 24 to 34 finds the set $\text{profit}(q_r) \subset q_r$ for $q_r \in Q, \forall r \in [1, |Q|]$. Trying to maximize the number of channels P_i can acquire from his neighbors, P_i will choose set $q_m \in Q, m \in [1, |Q|]$ such that,

$$|\text{profit}(q_m)| = \max_{q_r \in Q; 1 \leq r \leq |Q|} (|\text{profit}(q_r)|) \quad (5.5)$$

and take the set of channels $\text{profit}(q_m)$ from his neighbors. Thus, in period t , P_i 's share of channels will be (line 36),

$$x_i^t \leftarrow x_i^{t+1} \cup \text{profit}(q_m) \quad (5.6)$$

⁵Let $Q_1 = \{\{C_1, C_2\}, \{C_3, C_4\}\}$ and $Q_2 = \{\{C_3, C_5\}, \{C_6, C_7\}\}$. Then $Q = Q_1 \times Q_2 = \{\{C_1, C_2, C_3, C_5\}, \{C_1, C_2, C_6, C_7\}, \{C_3, C_4, C_3, C_5\}, \{C_3, C_4, C_6, C_7\}\}$. For any $q \in Q$, the first two elements of q belong to Q_1 and the next two elements belong to Q_2 . Also we define Q to be $Q_1 \times Q_2$ and not $Q_2 \times Q_1$

Also, clearly in period t , each $P_j \in \widehat{P}_{-i}^{Nbr}$ will be left with the set of channels $\{x_j^{t+1} \setminus \text{profit}(q_m)\}$. Since each $P_j \in \widehat{P}_{-i}^{Nbr}$ will have at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels⁶ in period t , no P_j will have an incentive to reject his share of (line 37),

$$x_j^t \leftarrow x_j^{t+1} \setminus \text{profit}(q_m), \quad \forall P_j \in \widehat{P}_{-i}^{Nbr} \quad (5.7)$$

All other players, i.e., $P_j \in P_{-i} \setminus \widehat{P}_{-i}^{Nbr}$ ⁷, can have the same share of channels in period t as they had in period $t+1$, and thus no P_j will not have an incentive to reject their share of (line 38),

$$x_j^t \leftarrow x_j^{t+1}; \forall P_j \in P_{-i} \setminus \widehat{P}_{-i}^{Nbr} \quad (5.8)$$

The strategy (x_i^t, x_{-i}^t) obtained for P_i is a SPNE strategy for P_i in period t . Clearly, no $P_j \in P_{-i}$ can gain in any sub-game (play after period t) by rejecting their respective shares in x_{-i}^t . Also, P_i cannot make a “successful” demand of a larger share of channels than x_i^t , since the number of channels that P_i can take from his neighbors has been optimized over all possible interference free allocations that allows P_i to take at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from $P_j \in P_{-i}^{Nbr}$. However, (x_i^t, x_{-i}^t) strategy for P_i obtained so far *may not be pareto optimal*. It may be possible to improve the share of some players in P_{-i} without decreasing the share of any player. We deal with this next.

3. Find pareto optimal SPNE strategy of P_i : Pareto improvement of P_i 's SPNE

strategy, (x_i^t, x_{-i}^t) , obtained so far to obtain a pareto optimal strategy for P_i is done

⁶ $q_r \in Q$ has been computed taking $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ from $P_j \in \widehat{P}_{-i}^{Nbr}$. Thus, each $P_j \in \widehat{P}_{-i}^{Nbr}$ will have at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels in the share offered to them in period t .

⁷Note that this includes neighbors of P_i from whom P_i could not take any channels ($P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}$), as well as players who are more than a hop away from P_i (P_{-i}^{Nbr})

by the `while` loop in steps 39 through 50. At each iteration of the `while` loop, the algorithm checks each player $P_j \in P_{-i}$ to see if a channel can be added to x_j^t . The `while` loop iterates till no more channels can be assigned to any player in P_{-i} . Clearly, after the while loop terminates, (x_i^t, x_{-i}^t) produced will correspond to a pareto optimal SPNE strategy for P_i in period t .

When algorithm (4) terminates, it finds a SPNE strategy, (x_1^0, x_{-1}^0) , for P_1 in the first period, such that P_1 cannot make a larger demand of channels than x_1^0 that will be accepted by all players in P_{-1} . If rejected, P_1 cannot hope to get a share of channels in any subsequent period which in period 0 is worth more than $|x_1^0|$. Also, no player in P_{-1} can hope to get a share of channels in any subsequent period which in period 0 is worth more than his respective share in x_{-1}^0 . Thus, the players in P_{-1} does not have an incentive to reject their share in x_{-1}^0 .

5.2.2.1 A more efficient implementation

Though algorithm 4 lays down the basic idea of finding SPNE strategies of the spectrum bargaining game using backward induction, it checks all combinations of channels that allow an offerer P_i in period t to take $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from neighbor $P_j \in \widehat{P}_{-i}^{Nbr}$. To do away with checking all such combinations, algorithm 5 presents a more computationally efficient procedure for finding the SPNE strategy of offerer P_i in period t (that need not be Pareto optimal).

Algorithm 5 Find SPNE strategy of offerer P_i in period t

```

1:  $\mathcal{S} \leftarrow \{\phi\}$ 
2: for all  $C_m \in C : C_m \notin \cup_{P_j \in P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}} x_j^{t+1}$  do
3:    $s \leftarrow \sum_{P_j \in \widehat{P}_{-i}^{Nbr}} |C_m \cap x_j^{t+1}|$ 
4:   if  $s > 0$  then
5:      $\mathcal{S} \leftarrow \mathcal{S} \cup C_m^s$ 
6:   end if
7: end for
8: Sort the channels in  $\mathcal{S}$  in non-decreasing order according to their superscripts in  $\mathcal{S}$ .
9: Mark all  $P_j \in \widehat{P}_{-i}^{Nbr}$  as available.
10:  $r_j \leftarrow |x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil \forall P_j \in \widehat{P}_{-i}^{Nbr}$ 
11:  $x_j^t \leftarrow \{x_j^{t+1}\} \forall j \in [1, N]$ 
12: for all  $C_m \in \mathcal{S}$  do
13:   if all  $P_j \in \widehat{P}_{-i}^{Nbr}$  for which  $|x_j^t \cap C_m| = 1$  is marked available then
14:      $r_j \leftarrow r_j - 1 \forall P_j \in \widehat{P}_{-i}^{Nbr} : |x_j^t \cap C_m| = 1$ 
15:      $x_j^t \leftarrow x_j^t \setminus C_m \forall P_j \in \widehat{P}_{-i}^{Nbr} : |x_j^t \cap C_m| = 1$ 
16:      $x_i^t \leftarrow x_i^t \cup C_m$ 
17:     Mark all  $P_j \in \widehat{P}_{-i}^{Nbr}$  as unavailable for which  $r_j = 0$ .
18:   end if
19: end for

```

The critical task is to find the maximum set of channels that offerer P_i can acquire in period t . Recall that, P_i can potentially take channels only from those $P_j \in P_{-i}^{Nbr}$ for whom $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil > 0$ (denoted as \widehat{P}_{-i}^{Nbr}). Keeping this in mind, algorithm 5 finds the *maximum* set of channels that P_i can *take* from this neighbors. The basic idea of the algorithm is to sort the channels that the players in \widehat{P}_{-i}^{Nbr} has (in period $t + 1$) in ascending order based on the number of players that possess each channel. The algorithm then considers taking each channel in this ascending order, ensuring that no more than $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels is taken from each $P_j \in \widehat{P}_{-i}^{Nbr}$.

Notice that the strategy (x_i^t, x_{-i}^t) found by algorithm 5 for offerer P_i in period t need not yield a Pareto optimal equilibrium. It may be possible to improve the share of some players

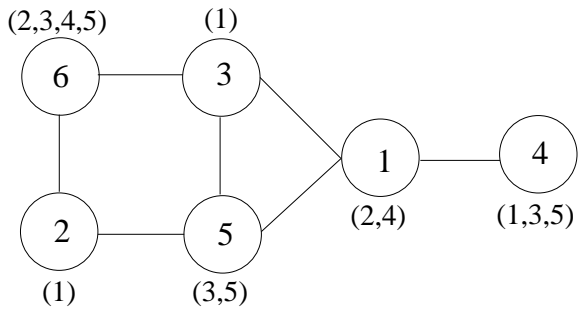
in P_{-i} without decreasing the share of any player. The Pareto optimal SPNE strategy of P_i can be found by a `while` loop similar to the one in lines 39 to 50 of algorithm 4.

It can be easily verified that the **worst** case complexity of finding the SPNE strategy of offerer P_i in period t is $O(M^2D)$, where D is the highest degree of a node in the conflict graph.

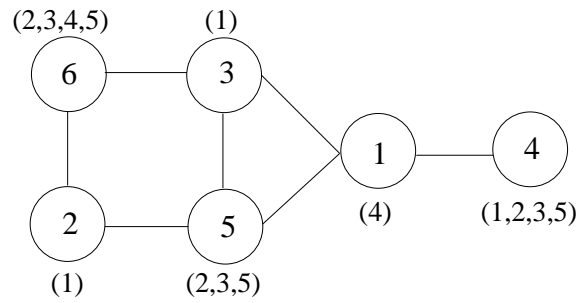
5.2.3 An Illustrative Example

Figure 5.1 shows an example of how to find the SPNE strategy (x_1^0, x_{-1}^0) of P_1 in the first period of the game. We consider a network of 6 nodes. The graphs in Figure 5.1 depict the conflict graph of the network. The number of channels available, M , has been assumed to be 5. Each node has a discount factor of 0.5. The game is played for 6 periods. The channels assigned to the nodes in each period has been shown in brackets beside the node.

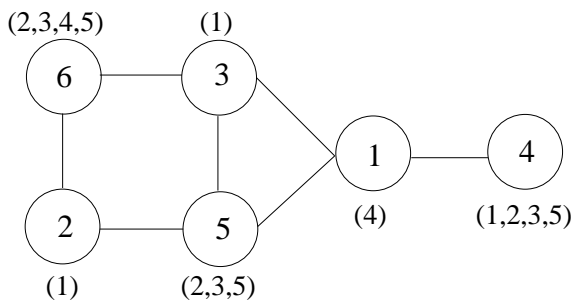
Figure 5.1(a) shows the pareto optimal NE strategy of P_6 in the last period of the game as obtained by using algorithm 2. The offerer in this period is P_6 . First, a NE strategy of P_6 in the last period (which need not be pareto optimal) is found using lines 2 to 10 of the algorithm. In lines 2 to 6, P_6 colors his neighbors (P_2 and P_3) with the least possible number of colors (channels) and keeps rest of the channels for himself (thereby maximizing his share). In lines 7 to 10, the players in $(P_{-6}^{\overline{Nbr}})$ (i.e, P_1, P_4 and P_5) are given a channel each by graph coloring them. This is done by considering P_1, P_4 and P_5 in non-increasing order of



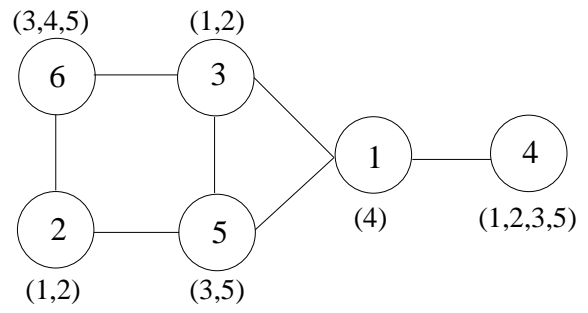
(a) Period 5; Offerer P_6



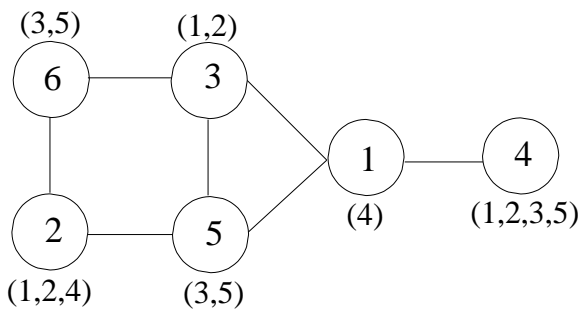
(b) Period 4; Offerer P_5



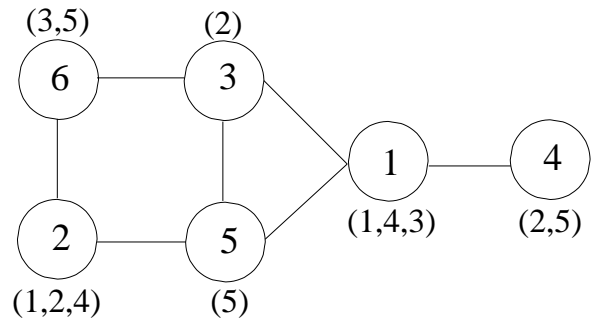
(c) Period 3; Offerer P_4



(d) Period 2; Offerer P_3



(e) Period 1; Offerer P_2



(f) Period 0; Offerer P_1

Figure 5.1: Pareto optimal SPNE of the respective offerer in different periods.

their degree in the subgraph induced by P_1, P_2, P_3, P_4 and P_5 . Thus, P_1 is considered first⁸ and gets C_2 , next P_5 gets C_3 and finally P_4 gets channel C_1 . Clearly, the channel assignment obtained so far corresponds to a NE strategy of P_1 in the last period, but one that may not be pareto optimal, since the shares of some players (P_1, P_4 and P_5) can be improved without hurting the share of any other player.

The pareto optimal NE strategy of P_6 is obtained using lines 11 to 22 of algorithm 2. This is done by considering the players in P_{-6} and checking to see if more channels can be assigned to the player. In the first iteration of the while loop, P_1 receives C_4 , P_2 and P_3 does not get any more channels, P_4 gets C_3 and P_5 gets C_5 . In the second iteration of the while loop only P_4 gets C_5 . The channel assignment obtained now is shown in Figure 5.1(a), and corresponds to the pareto optimal NE strategy of P_6 in the last period of the game.

We will exemplify the concept of finding the SPNE strategy of an offerer P_i in period t from the SPNE strategy of offerer P_l ($l \neq i$) in period $t + 1$ (refer algorithm (5)) by showing how to find the SPNE strategy of the offerer P_3 in period 2 from the SPNE strategy of P_4 in period 3. Note that $P_{-3}^{Nbr} = \{P_1, P_5, P_6\}$. The four channels that P_6 has in period 3 is equivalent to having $\lceil 4 \times 0.5 \rceil = 2$ channels in period 2. Thus, P_3 can potentially take 2 channels from P_6 . Similarly, the 3 channels that P_5 has in period 3 is equivalent to having $\lceil 3 \times 0.5 \rceil = 2$ channels in period 2. Thus, P_3 can potentially take 1 channel from P_5 . However, P_3 will not be able to take any channel from P_1 , since the latter has only one channel in the previous period. In fact, this also implies that P_3 will not be able to take channel 4 (due to

⁸If two nodes have the same degree then the lower numbered node is considered first

interference constraints). Thus $\widehat{P}_{-3}^{Nbr} = \{P_6, P_5\}$. Now, algorithm (5) will sort the channels that P_3 can potentially take from this neighbors in \widehat{P}_{-3}^{Nbr} in ascending order based on the number of players in \widehat{P}_{-3}^{Nbr} that possess each channel (line 8 algorithm (5)). This yields the set $S = \{C_2, C_3, C_5\}$. First, C_2 is considered. Since the channel is possessed by both players P_5 and P_6 and a channel can be taken from both the players, P_3 takes channel C_2 from P_5 and P_6 . Next, channel C_3 is considered, which is also possessed by both players P_5 and P_6 . However, since no more channels can now be taken from P_5 , P_3 cannot take C_3 . Similarly, C_5 cannot be taken also. Thus, P_3 only takes C_2 from both P_5 and P_6 , making $x_3^2 = \{1, 2\}$. It can be easily seen that the maximum number of channels that P_3 can acquire from his neighbors is indeed 1.

The channel assignment obtained so far is a SPNE strategy for P_3 in period 2. However, it may not be pareto optimal. To obtain the pareto optimal SPNE strategy of P_3 , a **while** loop of the form discussed earlier can be used which would assign C_2 to P_2 . The channel assignment obtained subsequently would correspond to the pareto optimal SPNE strategy of P_3 in period 2 of the game. This is shown in Figure 5.1(d).

Following the same line of reasoning, we finally obtain the SPNE strategy (x_1^0, x_{-1}^0) of P_1 in the first period of the game. This strategy of P_1 is shown in Figure 5.1(f).

5.3 Infinite Horizon Bargaining Game

Until now we have studied the bargaining game for accessing channels as a *finite* horizon game of T periods. In this game, if the players can agree upon a sharing rule within T periods then each player gets his respective share of channels. However, if the players fail to agree on a sharing rule within T periods, then the outcome of the game corresponds to the disagreement outcome, and all player gets zero channels. We have studied SPNE strategies in this game and have also presented an algorithm to find the SPNE strategy of the player (P_1) making the offer in the first period.

However, it is more realistic to consider that players will go on bargaining until all players can agree on a sharing rule of the channels. Such a bargaining game is called an infinite horizon bargaining game. To study the infinite horizon game, we will first extend the definition of the finite horizon game defined in Section 5.1. Specifically, in the infinite horizon game, each P_i for $i \in [1, N]$ makes an offer in periods $kN + (i - 1)$ where $k \in \{0, 1, 2, \dots\}$. We will denote the SPNE strategy of P_1 in the first period of a T period finite horizon game as $(x_1^0, x_{-1}^0)^T$. Also, let $(x_1^0, x_{-1}^0)_j^T$ denote set x_j^0 in $(x_1^0, x_{-1}^0)^T$ for $j \in [1, N]$ ⁹.

Our solution of the *infinite* horizon game is based on the following fact— for the *finite* horizon game, there exists a T such that the number of channels each player receives in the SPNE strategy of P_1 , $(x_1^0, x_{-1}^0)^T$, of a T period game, is *equal to* the number of channels each player receives in the SPNE strategy of P_1 , $(x_1^0, x_{-1}^0)^{T'}$, in a T' period game, for all $T' > T$.

⁹Recall that $(x_1^0, x_{-1}^0) = (x_1^0, \{x_2^0, x_3^0, \dots, x_N^0\})$

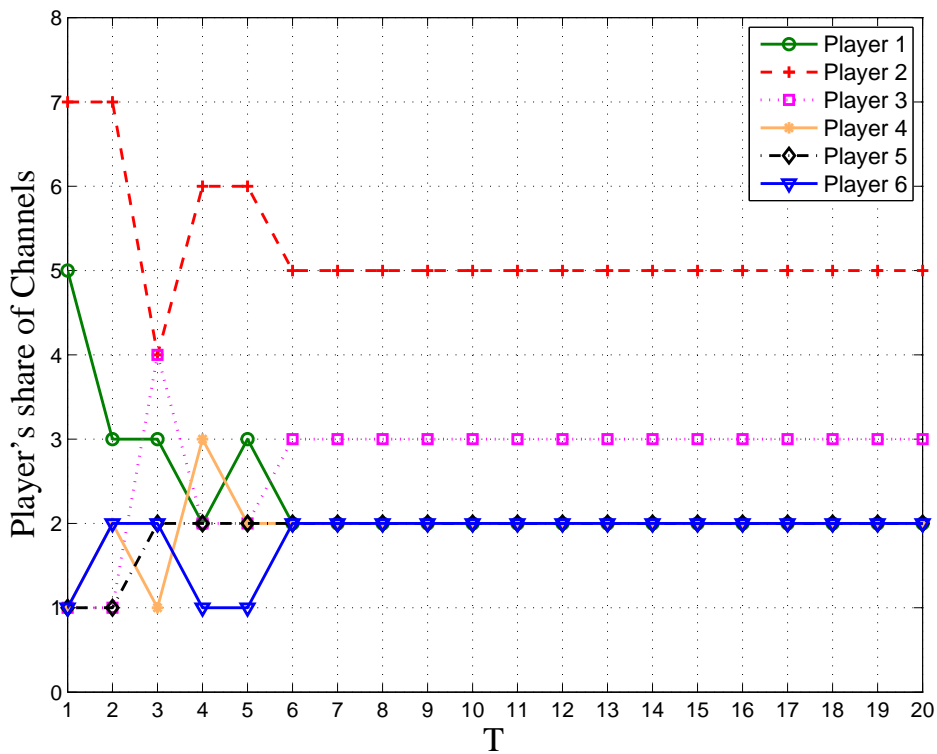


Figure 5.2: Pareto optimal SPNE strategy $(x_1^0, x_{-1}^0)^T$ of a 6 player finite horizon game of varying number of periods, T . Here $M = 8$ and $\delta_i = 0.7, \forall i \in [1, 6]$

In other words, $\exists T$ such that for all $T' > T$ we have $|(x_1^0, x_{-1}^0)_j^{T'}| = |(x_1^0, x_{-1}^0)_j^T|, \forall j \in [1, N]$. We will show this via simulation. Figure 5.2 shows the SPNE strategy $(x_1^0, x_{-1}^0)^T$ of a 6 player finite horizon game with varying number of periods, T . The conflict graph has been randomly generated. The number of channels available, M , has been taken to be 8 and the discount factor of all players is 0.7. As can be clearly seen from the figure, for $T > 6$, $|(x_1^0, x_{-1}^0)_j^T| = |(x_1^0, x_{-1}^0)_j^6|, \forall j \in [1, N]$. Thus, to find the SPNE strategy, $(x_1^0, x_{-1}^0)^\infty$, of the infinite horizon game, we simply need to find the SPNE strategy, $(x_1^0, x_{-1}^0)^T$, of a finite horizon game of T periods, such that for all $T' > T$ we have $|(x_1^0, x_{-1}^0)_j^{T'}| = |(x_1^0, x_{-1}^0)_j^T|, \forall j \in [1, N]$. Based on this, algorithm 6, gives the procedure that P_1 will invoke to find his SPNE strategy, $(x_1^0, x_{-1}^0)^\infty$, in the infinite horizon game.

Algorithm 6 Find Infinite Horizon SPNE

Require: Number of players, N ; Interference Constraint, I ; Set of Available Channels, C ;
Number of Periods, T ;
1: $(x_1^0, x_{-1}^0)_j^{T-1} \leftarrow \{\phi\}, \forall j \in [1, N]$
2: **while** true **do**
3: $(x_1^0, x_{-1}^0)^T \leftarrow$ Find SPNE by B.I(N, I, C, T)
4: **if** $|(x_1^0, x_{-1}^0)_j^T| = |(x_1^0, x_{-1}^0)_j^{T-1}|, \forall j \in [1, N]$ **then**
5: break
6: **else**
7: $T \leftarrow T + 1$
8: **end if**
9: **end while**
10: **return** $(x_1^0, x_{-1}^0)^T$

5.3.1 Discussion

- Value of T , the starting period:** Algorithm 6 finds the SPNE of P_1 in the first period, starting from a T period game, until it finds a T' period game, such that, $|(x_1^0, x_{-1}^0)_j^{T'}| = |(x_1^0, x_{-1}^0)_j^{T'-1}|, \forall j \in [1, N]$. The algorithm then outputs the SPNE of P_1 in the infinite horizon game as $(x_1^0, x_{-1}^0)^{T'}$. The period T' at which the SPNE strategy of P_1 converges depends on several factors– the number of players, N ; the number of available channels, M ; and the average discount factor of the players, δ . More precisely, we have,

$$T' \propto \frac{N}{M \cdot (1 - \delta)} \quad (5.9)$$

Thus, T' – (i) *increases* as the number of player increases, (ii) *decreases* as more channels become available and (iii) *increases* as the discount factor of the players increases, i.e., as the players become more *patient* in waiting for the bargaining outcome. To minimize the number of iterations required by algorithm 6 to find T' , T should be made proportional to $\frac{N}{M \cdot (1 - \delta)}$.

- First (last) Mover Advantage:** One might suspect that the bargaining game we discussed suffers from the “first (last) mover advantage” problem, i.e., a player making an offer earlier (later) may have an advantage over a player who gets to make an offer later (earlier). This happens only in the finite horizon version of the game. In the infinite horizon game, when we take the length of the time periods to be very short, the “first (last) mover advantage” disappears [4]. Since we model the spectrum access

problem as an *infinite* horizon bargaining game where the length of each bargaining period will be very short (in the order of time taken to exchange messages or packets among nodes), our solution does not suffer from the “first (last) mover advantage” problem. Thus, ordering of the players will also not matter in our solution.

- **Building the conflict graph:** In our solution, we consider that each player knows the interference constraint, I , i.e., is aware of the conflict graph of the network. Players can know the conflict graph of the network over time by periodically sharing their *perceived* conflict graph with their interferers (neighbors) using control channel. Let us assume time is divided into slots. Initially, at time $t = 0$, the players only know of themselves. At $t = 1$, the players exchange their conflict graph with their neighbors to know their 1-hop away interferers. At $t = 2$, the players again exchange their conflict graph with their neighbors to know of nodes that are 2-hop away. And so on. For a conflict graph of N vertices, following this procedure, a player can know the conflict graph of the network in at most $N - 1$ time steps.

5.4 Summary

This chapter models the problem of dynamic spectrum access by a set of cognitive radio enabled nodes as an infinite horizon bargaining game where the nodes bargain among themselves to agree upon a sharing rule of the channels. First, the chapter explores the finite horizon version of the bargaining game and presents polynomial time algorithms to find the

Pareto optimal SPNE strategy of the player making the offer in the first period of the game. This is a strategy, such that, neither can the player making the offer increase his utility by making any other offer, nor can the players receiving the offer gain in any subsequent period by rejecting this offer. Next, we extend the results from the finite horizon game to find the Pareto optimal SPNE strategies of the infinite horizon game. Using their SPNE strategies, no player gets starved of channels. Finally, since in the bargaining model nodes negotiate among themselves, no centralized controller is needed.

CHAPTER 6

INCENTIVE BASED ROUTING

The channels that a secondary node acquires come to it at a price. Thus, to forward traffic, each secondary node would incur a *cost* because of using the spectrum it has acquired by making a payment. Moreover, using the acquired spectrum, each node can sustain a certain *capacity*. Clearly, secondary nodes will not have an incentive to relay traffic unless they are compensated for the costs they incur in forwarding traffic. Also, for a routing mechanism to be stable, the amount of traffic that a node forwards should not exceed its capacity.

In this chapter, we adopt the approach of Bayesian based algorithmic mechanism design for designing an *optimal* routing mechanism that *minimizes* the payment to be made to the nodes while *theoretically* ensuring that no node has any incentive to dishonestly declare its cost and/or capacity. Our routing mechanism is comprised of two components– *a) route selection function*, which determines the route(s) that requires the least *expected* payment. We refer to such a route as the the Expected Least Paid Route (ELPR), and, *b) pricing function*, which determines the payment to be made to each node. The focus of this chapter is to design these two functions. Formally, the key contributions of this chapter can be considered as follows.

- **Optimal Routing Mechanism Design:** We propose a *path auction* based routing mechanism in which nodes announce their cost and capacity, based on which a *multi-path* route is chosen and *payments* are made to the nodes. The route selection mechanism and the payment function ensure that all nodes can *maximize* their profit by truthfully reporting their cost and capacity, while the payment that needs to be made to the nodes is *minimized*.
- **Polynomial Time Implementation:** In addition to theoretically deriving the functions that determine the route selection and the payments to be made, we provide polynomial time algorithms for deploying the routing mechanism in DSA networks.
- **Modeling Uncertainty in DSA Networks:** We model cost of a node and its capacity as random variables. This serves a two-fold purpose– *a)* it helps tame the uncertainty regarding the actual cost and capacity of a node; and, *b)* it helps reflect the heterogenous nature of the nodes in a DSA network in terms of both space and time variation of the cost and capacity of nodes.

The remainder of the chapter is organized as follows. Section 6.1 defines the system model and also formally presents the path auction problem. Section 6.2 develops the theory of optimal path auction design. For simplicity and clarity of exposition, first Section 6.3 considers how to deploy such a path auction mechanism in the case of single-path routing, by assuming that each node in the network has sufficient capacity to route the sender's

traffic. Next, Section 6.4 discusses algorithms to deploy the path auction mechanism in the multi-path routing scenario. Finally, Section 6.5 concludes the chapter.

6.1 Network Model and Problem Formulation

6.1.1 Network Model

We consider a DSA network whose topology is depicted by the graph $G = (V, E)$. Here V is the set of secondary nodes and E is the set of links between the secondary nodes. Each node $j \in V$ has the following two *private* information, that comprises its type.

- **Capacity c_j :** Each node j acquires a certain amount of spectrum which can sustain a capacity c_j , measured in packets per unit time.
- **Cost m_j :** Each node j pays a cost m_j per unit time, for the time it uses the spectrum, to the primary owner.

Thus, the *type* of node $j \in V$ is defined by the 2-tuple,

$$t_j = \{m_j, c_j\} \tag{6.1}$$

We refer to the parameters of cost and capacity of a node as the *dimensions* of its type. It is worth emphasizing that, even though we consider a packet to be the smallest granularity of data transfer, our analysis would also hold true even if we consider other granularities such as bits.

Consider a traffic flow that originates from source node s and terminates at destination node d . Let N represent the set of candidate secondary nodes available for routing the flow. Thus,

$$N = V \setminus \{s, d\} \quad (6.2)$$

We label the nodes in N from 1 to n , i.e., $N = \{1 \cdots n\}$. Before the flow begins, each node $j \in N$ reports its type to the routing mechanism as the 2-tuple,

$$\hat{t}_j = \{\hat{m}_j, \hat{c}_j\} \quad (6.3)$$

A truthful mechanism has to guarantee that no node $j \in N$ has any incentive to lie about its type, i.e., no node can earn an extra profit by falsely reporting its type. In other words, the mechanism has to ensure that the reported type $\hat{t}_j = t_j, \forall j \in N$ ($\hat{m}_j = m_j$ and $\hat{c}_j = c_j$).

6.1.2 Bayesian based Modeling and Assumptions

The routing mechanism, however, is not aware of the true type of an intermediate node $j \in N$ willing to route the sender's traffic. To this end, we will tackle the problem using the conventional economics approach of *Bayesian* mechanism design, modeling m_j and c_j as random variables. We assume that the uncertainty about the type of node $j \in N$ can be described by two continuous probability density functions (p.d.f)– one for the cost, m_j and another for the capacity, c_j – each defined over some finite interval and that the functions are known to the mechanism.

- **p.d.f of m_j :** We let a_j and b_j represent the lowest and highest possible cost respectively that node j might incur per unit time for transmitting packets over its spectrum. We let $f_j^M : [a_j, b_j] \rightarrow \mathbb{R}_+$ be the p.d.f for j 's cost m_j and assume that: $-\infty < a_j < b_j < \infty$; $f_j^M(m_j) > 0, \forall m_j \in [a_j, b_j]$; and $f_j^M(\cdot)$ is a continuous function on $[a_j, b_j]$.
- **p.d.f of c_j :** We let v_j and w_j represent the lowest and highest possible capacity respectively that node j can sustain. We let $f_j^C : [v_j, w_j] \rightarrow \mathbb{R}_+$ be the p.d.f for j 's capacity c_j and assume that: $-\infty < v_j < w_j < \infty$; $f_j^C(c_j) > 0, \forall c_j \in [v_j, w_j]$; and $f_j^C(\cdot)$ is a continuous function on $[v_j, w_j]$.

The two density functions and their corresponding domains can be constructed in practise based on historical reports of the cost m_j and the capacity c_j . Now, cost, m_j , as well as the capacity, c_j , depend on the supply demand dynamics of spectrum availability in the vicinity of node j . Thus, we consider m_j and c_j to be dependent random variables. Let f_j^{MC} be their continuous joint probability density function defined over the domain, T_j . Thus,

$$T_j = [a_j, b_j] \times [v_j, w_j] \quad (6.4)$$

Let T denote the set of all possible combinations of types of the nodes in N . Thus,

$$T = T_1 \times \dots \times T_n \quad (6.5)$$

Now, if $t_j = \{m_j, c_j\}$ is the type of j , then the price that j pays to the primary spectrum owner for transmitting a packet over its spectrum is m_j/c_j . We denote this price associated with the type of j as t_j^p . Let f_j^P be the probability density function of the continuous random

variable t_j^p over the domain,

$$T_j^p = \left[\frac{a_j}{w_j}, \frac{b_j}{v_j} \right] \quad (6.6)$$

Let T^p denote the set of all possible combinations of the per packet prices that can be associated with the types of the nodes in N . Thus,

$$T^p = T_1^p \times \cdots \times T_n^p \quad (6.7)$$

Also, for node $j \in N$, let T_{-j}^p denote the set of all possible combinations of the per packet prices that can be associated with the types of the nodes in N other than j , so that

$$T_{-j}^p = \times_{\substack{i \in N \\ i \neq j}} T_i^p \quad (6.8)$$

We assume that the types of the nodes in N are stochastically independent from each other. Thus, the joint density function on T^p for the vector $t^p = (t_1^p, \dots, t_n^p)$ of individual per packet prices associated with the vector $t = (t_1, \dots, t_n)$ of individual types is,

$$f^P(t^p) = \prod_{j \in N} f_j^P(t_j^p) = \prod_{j \in N} f_j^P(m_j/c_j) \quad (6.9)$$

Also, we assume that a node $j \in N$ assesses the uncertainty about the types of the other nodes in N in the same way as the routing mechanism does. Consequently, both the mechanism and node j assess the joint density function on T_{-j}^p for the vector $t_{-j}^p = (t_1^p, \dots, t_{j-1}^p, t_{j+1}^p, \dots, t_n^p)$ associated with the vector $t_{-j} = (t_1, \dots, t_{j-1}, t_{j+1}, \dots, t_n)$ of individual types of all nodes in N except j as,

$$f_{-j}^P(t_{-j}^p) = \prod_{\substack{i \in N \\ i \neq j}} f_i^P(t_i^p) \quad (6.10)$$

6.1.3 Problem Formulation

The *optimal* routing mechanism design problem is to devise a path auction mechanism such that the *expected* payment made by the *end-user* to the nodes in N to have its traffic routed from s to d is *minimized*, while enforcing nodes to reveal their type to the mechanism truthfully. The end-user can correspond to either s or d . We consider the *direct revelation* mechanism class, where the nodes in N simultaneously and confidentially announce their respective type $t_j = \{m_j, c_j\}$, $j \in N$, to the routing mechanism. The mechanism then determines on which path(s) to route the traffic from s to d and how much each node in N should be paid, as some functions of the vector of per packet prices $t^p = (t_1^p, \dots, t_n^p)$ associated with the vector of announced types $t = (t_1, \dots, t_n)$. Next, we formally define these functions.

6.1.3.1 The outcome functions— $p(\cdot)$ and $q(\cdot)$

Let $P^{s \rightarrow d}$ denote the set of all paths from s to d , with each path consisting of nodes defined in G , and let $K = |P^{s \rightarrow d}|$ with the paths in $P^{s \rightarrow d}$ labeled from 1 to K . The routing mechanism can then be described by a pair of *outcome functions* (p, q) such that, $p_j(t^p)$ ($j \in [1, n]$) is the amount of money paid to node j for each packet delivered to d by the nodes in N (or, routes in $P^{s \rightarrow d}$) and $q_i(t^p)$, ($i \in [1, K]$), is the fraction of traffic routed over path i . The function $q(\cdot)$ is thus the route selection function and $p(\cdot)$ is the pricing function. Two points to be observed are,

- In our definitions we allow for the possibility that a node might receive payment without routing any traffic, though later we will show that this will never be the case.
- We allow multiple path routing.

6.1.3.2 Expected Utility functions

We consider expected utility of the end-user in terms of the price that the end-user needs to pay for each packet from s that is routed by the underlying network and delivered to d . In Section (6.4.3) we will discuss who actually makes the payments. The expected utility of the end-user from the routing mechanism described by (p, q) is,

$$U_e(p, q) = \int_{T^p} \left(\sum_{j \in N} p_j(t^p) \right) f^P(t^p) dt^p. \quad (6.11)$$

where, $dt^p = dt_1^p \cdots dt_n^p$ and $dt_j^p = d(\frac{m_j}{c_j})$, $j \in [1, n]$. Similarly, we consider the expected utility of node $j \in N$ in terms of its profit per packet delivered to d by the routes in $P^{s \rightarrow d}$.

If the type of j is $t_j = \{m_j, c_j\}$, then its expected utility from the auction mechanism is,

$$U_j(p, q, t_j^p) = \int_{T_{-j}^p} \left(p_j(t^p) - t_j^p \sum_{i \in P^{s \xrightarrow{j} d}} q_i(t^p) \right) f_{-j}^P(t_{-j}^p) dt_{-j}^p \quad (6.12)$$

where, $P^{s \xrightarrow{j} d}$ is the subset of paths in $P^{s \rightarrow d}$ that pass via node j .

6.1.3.3 The Optimization problem

The objective of the path auction design problem is to shape the pair of outcome functions (p, q) , such that, the expected utility of the end-user as defined by equation (6.11) is minimized. However, there are certain constraints that must be imposed on (p, q) to be able to describe a feasible auction mechanism. We describe these constraints below.

1. Individual Rationality Constraint: No node in N can be forced to participate in the auction if its utility from routing traffic is negative. If a node does not participate in the auction, then clearly its utility would be zero. Thus, to ensure that all nodes in N participate in the auction, the following individual rationality constraint must hold.

$$U_j(p, q, t_j^p) \geq 0, \quad \forall j \in N, \forall t_j^p \in T_j^p \quad (6.13)$$

2. Incentive Compatibility Constraint: The type announced by a node $j \in N$ to the mechanism is always susceptible to have been dishonestly reported if j expected to gain from lying. Thus, to make the nodes in N reveal their respective type truthfully, no node should be able to expect an extra profit from lying about its type. In other words, honestly revealing its own type should form a *dominant strategy* for every node in N in the auction game. If node $j \in N$ reports its type as $\hat{t}_j = \{\hat{m}_j, \hat{c}_j\}$ while its true type is $t_j = \{m_j, c_j\}$, then its expected utility is given by,

$$\int_{T_{-j}^p} \left(p_j(\hat{t}_j^p, t_{-j}^p) - t_j^p \sum_{i \in P^{s \rightarrow d}} q_i(\hat{t}_j^p, t_{-j}^p) \right) f_{-j}^P(t_{-j}^p) dt_{-j}^p \quad (6.14)$$

where, $(\hat{t}_j^p, t_{-j}^p) = (t_1^p, \dots, t_{j-1}^p, \hat{t}_j^p, t_{j+1}^p, \dots, t_n^p)$. Thus, to ensure that no node in N has an incentive to lie about its type, the following incentive compatibility condition must hold,

$$U_j(p, q, t_j^p) \geq \int_{T_{-j}^p} \left(p_j(\hat{t}_j^p, t_{-j}^p) - t_j^p \sum_{i \in P^{s \rightarrow d}} q_i(\hat{t}_j^p, t_{-j}^p) \right) f_{-j}^P(t_{-j}^p) dt_{-j}^p, \\ \forall j \in N, \forall t_j^p \in T_j^p, \forall \hat{t}_j^p \in T_j^p \quad (6.15)$$

Notice that if $\hat{t}_j^p = t_j^p$, i.e., $\frac{\hat{m}_j}{\hat{c}_j} = \frac{m_j}{c_j}$, then the expected utility of j by reporting type \hat{t}_j is equal to $U_j(p, q, t_j^p)$. So node j cannot expect to gain by falsely reporting a type \hat{t}_j if $\hat{t}_j^p = t_j^p$.

3. Traffic Allocation Constraint: Clearly, sum of the fraction of traffic allocated over all paths is 1. Thus,

$$\sum_{i=1}^K q_i(t^p) = 1 \text{ and } q_i(t^p) \geq 0, \quad \forall i \in [1, K], \forall t^p \in T^p \quad (6.16)$$

4. Node Capacity Constraint: The total traffic rate that a node routes cannot exceed its capacity. Thus, the following constraint must hold on the aggregate traffic rate that traverses a node,

$$R \sum_{i \in P^{s \rightarrow d}} q_i(t^p) \leq c_j, \quad \forall j \in N, \forall t^p \in T^p \quad (6.17)$$

where, R is the rate at which s injects its traffic into the network.

6.2 Optimal Path Auction Design

Let $Q_j(q, t_j^p)$ denote the *expected* fraction of traffic that node $j \in N$ will be made responsible for forwarding due to a path auction mechanism (p, q) , given that j 's type is $t_j = \{m_j, c_j\}$.

Thus, we can define,

$$Q_j(q, t_j^p) = \int_{T_{-j}^p} \left(\sum_{i \in P_{s \rightarrow d}^j} q_i(t^p) \right) f_{-j}^P(t_{-j}^p) dt_{-j}^p \quad (6.18)$$

Based on the above definition, we first present a simplified incentive compatibility constraint.

Lemma 6.1. *The incentive compatibility constraint presented in (6.15) holds if the following two conditions holds for $\forall j \in N$, $\forall m_j, n_j \in [a_j, b_j]$, $\forall c_j, d_j \in [v_j, w_j]$.*

1. if $m_j \leq n_j$ and $c_j \leq d_j$, then,

$$Q_j(q, \frac{n_j}{c_j}) \leq Q_j(q, \frac{m_j}{d_j}) \quad (6.19)$$

2.
$$U_j(p, q, \frac{m_j}{c_j}) = \int_{m_j/c_j}^{b_j/v_j} Q_j(q, \psi_j) d\psi_j + U_j(p, q, \frac{b_j}{v_j}) \quad (6.20)$$

Proof. To simplify the incentive compatibility constraint we need to consider all the ways in which a node can lie about its type. A node can do so by increasing or decreasing the value of one or both the dimensions of its type. Thus, we need to consider four cases– a node can either (i) exaggerate its cost but understate its capacity, or, (ii) exaggerate its capacity but understate its cost, or, (iii) exaggerate both its cost and capacity, or, (iv) understate both its cost and capacity.

Case (i) and (ii): *The value of one of the dimensions is exaggerated while the other one is understated.*

Let us first assume that m_j is the true cost and d_j is the true capacity of j ($t_j = \{m_j, d_j\}$), while the node dishonestly announces n_j and c_j as its cost and capacity respectively ($\hat{t}_j =$

$\{n_j, c_j\}$). The expected utility of j because of adopting this strategy is,

$$\begin{aligned}
& \int_{T_{-j}^p} \left[p_j\left(\frac{n_j}{c_j}, t_{-j}^p\right) - \frac{m_j}{d_j} \sum_{i \in P^{s \rightarrow d}} q_i\left(\frac{n_j}{c_j}, t_{-j}^p\right) \right] f_{-j}^P(t_{-j}^p) dt_{-j}^p \\
&= \int_{T_{-j}^p} \left[p_j\left(\frac{n_j}{c_j}, t_{-j}^p\right) - \frac{n_j}{c_j} \sum_{i \in P^{s \rightarrow d}} q_i\left(\frac{n_j}{c_j}, t_{-j}^p\right) \right] f_{-j}^P(t_{-j}^p) dt_{-j}^p + \\
& \int_{T_{-j}^p} \left[\left(\frac{n_j}{c_j} - \frac{m_j}{d_j}\right) \sum_{i \in P^{s \rightarrow d}} q_i\left(\frac{n_j}{c_j}, t_{-j}^p\right) \right] f_{-j}^P(t_{-j}^p) dt_{-j}^p \\
&= U_j\left(p, q, \frac{n_j}{c_j}\right) + \left(\frac{n_j}{c_j} - \frac{m_j}{d_j}\right) Q_j\left(q, \frac{n_j}{c_j}\right) \tag{6.21}
\end{aligned}$$

The incentive compatibility constraint (6.15) states that, for any node in N , the expected utility obtained by reporting its true type must be greater than or equal to the expected utility obtained by lying about its type. Thus, we have,

$$U_j\left(p, q, \frac{m_j}{d_j}\right) \geq U_j\left(p, q, \frac{n_j}{c_j}\right) + \left(\frac{n_j}{c_j} - \frac{m_j}{d_j}\right) Q_j\left(q, \frac{n_j}{c_j}\right) \tag{6.22}$$

Similarly, considering n_j as the true cost and c_j as the true capacity of j ($t_j = \{n_j, c_j\}$), while the node reports m_j and d_j as its cost and capacity respectively ($\hat{t}_j = \{m_j, d_j\}$), we get,

$$U_j\left(p, q, \frac{n_j}{c_j}\right) \geq U_j\left(p, q, \frac{m_j}{d_j}\right) + \left(\frac{m_j}{d_j} - \frac{n_j}{c_j}\right) Q_j\left(q, \frac{m_j}{d_j}\right) \tag{6.23}$$

From inequalities (6.22) and (6.23), we have,

$$\left(\frac{n_j}{c_j} - \frac{m_j}{d_j}\right) Q_j\left(q, \frac{n_j}{c_j}\right) \leq U_j\left(p, q, \frac{m_j}{d_j}\right) - U_j\left(p, q, \frac{n_j}{c_j}\right) \leq \left(\frac{n_j}{c_j} - \frac{m_j}{d_j}\right) Q_j\left(q, \frac{m_j}{d_j}\right) \tag{6.24}$$

Since, we know $\frac{n_j}{c_j} > \frac{m_j}{d_j}$, condition (1) of lemma (6.1) follows.

Case (iii) and (iv): *The value of both the dimensions is either exaggerated or understated.* Next, let us consider that m_j is the true cost and c_j is the true capacity of j ($t_j = \{m_j, c_j\}$), while the node announces n_j and d_j as its cost and capacity respectively ($\hat{t}_j = \{n_j, d_j\}$). The

expected utility of j because of adopting this strategy is,

$$\begin{aligned} & \int_{T_{-j}^p} \left[p_j \left(\frac{n_j}{d_j}, t_{-j}^p \right) - \frac{m_j}{c_j} \sum_{i \in P^{s \rightarrow d}} q_i \left(\frac{n_j}{d_j}, t_{-j}^p \right) \right] f_{-j}^P(t_{-j}^p) dt_{-j}^p \\ &= U_j(p, q, \frac{n_j}{d_j}) + \left(\frac{n_j}{d_j} - \frac{m_j}{c_j} \right) Q_j(q, \frac{n_j}{d_j}) \end{aligned} \quad (6.25)$$

Because of incentive compatibility constraint (6.15), we have,

$$U_j(p, q, \frac{m_j}{c_j}) \geq U_j(p, q, \frac{n_j}{d_j}) + \left(\frac{n_j}{d_j} - \frac{m_j}{c_j} \right) Q_j(q, \frac{n_j}{d_j}) \quad (6.26)$$

Again, considering n_j as the true cost and d_j as the true capacity of j ($t_j = \{n_j, d_j\}$), while the node reports m_j and c_j as its cost and capacity respectively ($\hat{t}_j = \{m_j, c_j\}$), we get,

$$U_j(p, q, \frac{n_j}{d_j}) \geq U_j(p, q, \frac{m_j}{c_j}) + \left(\frac{m_j}{c_j} - \frac{n_j}{d_j} \right) Q_j(q, \frac{m_j}{c_j}) \quad (6.27)$$

From inequalities (6.26) and (6.27), we have,

$$\left(\frac{n_j}{d_j} - \frac{m_j}{c_j} \right) Q_j(q, \frac{n_j}{d_j}) \leq U_j(p, q, \frac{m_j}{c_j}) - U_j(p, q, \frac{n_j}{d_j}) \leq \left(\frac{n_j}{d_j} - \frac{m_j}{c_j} \right) Q_j(q, \frac{m_j}{c_j}) \quad (6.28)$$

In inequality (6.28) we can distinguish two cases— if $\frac{n_j}{d_j} > \frac{m_j}{c_j}$, then we can say that $Q_j(q, \frac{n_j}{d_j}) \leq Q_j(q, \frac{m_j}{c_j})$. Otherwise, we have $Q_j(q, \frac{m_j}{c_j}) \leq Q_j(q, \frac{n_j}{d_j})$.

Notice that, essentially both inequalities (6.24) and (6.28) imply the same fact— the fraction of traffic that a node expects to forward follows a *non-increasing* trend as the per packet cost associated with the reported type increases. Also, for any node $j \in N$, if we consider $\{n_j, c_j\}$ and $\{m_j, d_j\}$ as two points (types) in the domain T_j , instead of considering $\{m_j, c_j\}$ and $\{n_j, d_j\}$ as two points, then we know definitively that the fraction of traffic that j expects to forward by reporting type $\{n_j, c_j\}$ is less than or equal to that expected by reporting type $\{m_j, d_j\}$. Thus, to shape the expected utility function of a node, for conforming it to the incentive compatibility constraint, we can consider only inequality (6.24).

Now, let $\frac{n_j}{c_j} - \frac{m_j}{d_j} = \delta_j$. Then, we can rewrite inequality (6.24) as,

$$\delta_j Q_j(q, \frac{n_j}{c_j}) \leq U_j(p, q, \frac{n_j}{c_j} - \delta_j) - U_j(p, q, \frac{n_j}{c_j}) \leq \delta_j Q_j(q, \frac{n_j}{c_j} - \delta_j)$$

Since $Q_j(\cdot)$ decreases as the per packet cost associated with the type of a node increases, we can write $Q_j(\cdot)$ as a Riemann integral,

$$\int_{n_j/c_j}^{b_j/v_j} Q_j(q, \psi_j) d\psi_j = U_j(p, q, \frac{n_j}{c_j}) - U_j(p, q, \frac{b_j}{v_j}) \quad (6.29)$$

which holds for all type $t_j \in T_j$ for any node $j \in N$. Thus, condition (2) of lemma (6.1) follows. \square

Thus, (p, q) can be said to describe an optimal path auction if and only if it minimizes $U_e(p, q)$ subject to the incentive compatibility constraints (6.19)-(6.20), individual rationality constraint (6.13), traffic allocation constraint (6.16), and node capacity constraint (6.17). Next, we simplify the optimization problem presented in Section (6.1.3.3).

Lemma 6.2. *(p, q) represents an optimal path auction mechanism if q minimizes*

$$\int_{T^p} \left[\sum_{j \in N} \left(t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)} \right) \sum_{i \in P^{s \rightarrow d}} q_i(t^p) \right] f^P(t^p) dt^p \quad (6.30)$$

subject to constraints (6.16) and (6.17), **and**,

$$p_j(t^p) = \frac{m_j}{c_j} \sum_{i \in P^{s \rightarrow d}} q_i(t^p) + \int_{m_j/c_j}^{b_j/v_j} \sum_{i \in P^{s \rightarrow d}} q_i(\psi_j, t_{-j}^p) d\psi_j, \quad \forall j \in N, \forall t^p \in T^p \quad (6.31)$$

Proof. We can write the end-user's utility function (6.11) as,

$$U_e(p, q) = \sum_{j \in N} \int_{T^p} t_j^p \sum_{i \in P^{s \rightarrow d}} q_i(t^p) f^P(t^p) dt^p + \sum_{j \in N} \int_{T^p} \left[p_j(t^p) - t_j^p \sum_{i \in P^{s \rightarrow d}} q_i(t^p) \right] f^P(t^p) dt^p \quad (6.32)$$

Also, we know that,

$$\begin{aligned}
& \int_{T^p} \left[p_j(t^p) - t_j^p \sum_{i \in P^{s \xrightarrow{j} d}} q_i(t^p) \right] f^P(t^p) dt^p \\
&= \int_{a_j/w_j}^{b_j/v_j} U_j(p, q, \frac{m_j}{c_j}) f_j^P(t_j^p) dt_j^p \\
&= \int_{a_j/w_j}^{b_j/v_j} \left[\int_{m_j/c_j}^{b_j/v_j} Q_j(q, \psi_j) d\psi_j + U_j(p, q, \frac{b_j}{v_j}) \right] f_j^P(t_j^p) dt_j^p \\
&= \int_{a_j/w_j}^{b_j/v_j} \int_{a_j/w_j}^{\psi_j} f_j^P(t_j^p) Q_j(q, \psi_j) dt_j^p d\psi_j + U_j(p, q, \frac{b_j}{v_j}) \\
&= \int_{a_j/w_j}^{b_j/v_j} F_j^P(\psi_j) Q_j(q, \psi_j) d\psi_j + U_j(p, q, \frac{b_j}{v_j}) \\
&= \int_{a_j/w_j}^{b_j/v_j} F_j^P(t_j^p) \int_{T_{-j}^p} \sum_{i \in P^{s \xrightarrow{j} d}} q_i(t^p) f_{-j}^P(t_{-j}^p) dt_{-j}^p dt_j^p + U_j(p, q, \frac{b_j}{v_j}) \\
&= \int_{T^p} \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)} \sum_{i \in P^{s \xrightarrow{j} d}} q_i(t^p) f^P(t^p) dt^p + U_j(p, q, \frac{b_j}{v_j}) \tag{6.33}
\end{aligned}$$

Substituting (6.33) into (6.32), we get,

$$U_e(p, q) = \int_{T^p} \left[\sum_{j \in N} \left(t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)} \right) \sum_{i \in P^{s \xrightarrow{j} d}} q_i(t^p) \right] f^P(t^p) dt^p + \sum_{j \in N} U_j(p, q, \frac{b_j}{v_j}) \tag{6.34}$$

Notice that, we have accounted for the incentive compatibility constraint while simplifying the utility function of the end-user. So the routing mechanism should be such that the end-user's utility as formulated in (6.34) is minimized subject to the individual rationality constraint (6.13), traffic allocation constraint (6.16), and node capacity constraint (6.17). In the formulation of (6.34), notice that p appears *only* in the last term ($\sum_{j \in N} U_j(p, q, \frac{b_j}{v_j})$) of

the objective function. Now, rewriting (6.20) we get,

$$U_j(p, q, \frac{b_j}{v_j}) = \int_{T_{-j}^p} \left[p_j(t^p) - \frac{m_j}{c_j} \sum_{i \in P^{s \rightarrow d}} q_i(t^p) - \int_{m_j/c_j}^{b_j/v_j} \sum_{i \in P^{s \rightarrow d}} q_i(\psi_j, t_{-j}^p) d\psi_j \right] f_{-j}^P(t_{-j}^p) dt_{-j}^p$$

Thus, if the end-user pays a node $j \in N$ according to equation (6.31), then the individual rationality constraint (6.13) is satisfied, *as well as* the best possible value of the last term in (6.34) is obtained, which is zero. The individual rationality constraint is satisfied since the payment made to a node j , $p_j(t^p) \geq \frac{m_j}{c_j} \sum_{i \in P^{s \rightarrow d}} q_i(t^p)$. Intuitively, this can also be understood as follows. The per packet price associated with the type of a node j is highest if its type is $\{b_j, v_j\}$. The incentive compatibility constraint dictates that as the per packet cost associated with the type of a node decreases, the fraction of traffic that a node expects to route follows a non-decreasing trend. Thus, if the mechanism sets the expected utility of node j because of type $\{b_j, v_j\}$ to zero, then the expected utility corresponding to any type $t_j \in T_j$ will be greater than or equal to zero.

So we can simplify the objective function of our optimization problem to (6.30) subject to the traffic allocation constraint and the node capacity constraint, which are the only two constraints left to be satisfied. Thus, lemma (6.2) follows. \square

Next, we will formulate lemma (6.2) in a deterministic manner.

Lemma 6.3. *For any given t in T , the optimal traffic allocation over different routes in $P^{s \rightarrow d}$ is obtained if q minimizes the following function for the corresponding t^p ,*

$$\Phi(t^p) = \sum_{i=1}^K \left[\sum_{j \in P_i^{s \rightarrow d}} \left(t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)} \right) \right] q_i(t^p) \quad (6.35)$$

subject to constraints (6.16) and (6.17). Here, $P_i^{s \rightarrow d}$ denotes the set of nodes on the i^{th} path in $P^{s \rightarrow d}$. Also, the payment made to a node $j \in N$ is given by,

$$p_j(t^p) = \frac{m_j}{c_j} \sum_{i \in P^{s \rightarrow d}} q_i(t^p) + \int_{m_j/c_j}^{b_j/v_j} \sum_{i \in P^{s \rightarrow d}} q_i(\psi_j, t_{-j}^p) d\psi_j, \quad \forall j \in N \quad (6.36)$$

Proof. Clearly, for the minimization of the *expected* objective function given by (6.30), q should be such that the following function gets minimized for all t^p in T^p (subject to (6.16) and (6.17)),

$$\sum_{j \in N} \left(t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)} \right) \sum_{i \in P^{s \rightarrow d}} q_i(t^p) \quad (6.37)$$

Now, (6.37) has been expressed as a summation over the candidate nodes available for routing a session's traffic, i.e., over the nodes in N . However, q has to assign traffic to the nodes in the context of the paths in $P^{s \rightarrow d}$; not by considering the nodes individually. So it is more appropriate to express (6.37) as a summation over the paths in $P^{s \rightarrow d}$ as has been formulated in (6.35). Thus, lemma (6.3) follows. \square

6.3 Single Path Routing

Based on the theory of truthful optimal path auction design developed in the last section, we will now devise algorithms to deploy such an auction mechanism in a network. For simplicity of exposition we first consider a special case of the auction design problem in which each node $j \in N$ has sufficient capacity to route the sender's traffic, i.e., we consider $c_j \geq R, \forall j \in N$, so that the traffic from s to d can always be routed over a *single* path. In Section 6.4 we will explore implementation of the path auction mechanism in the generalized case, where a node $j \in N$ can have any arbitrary capacity, with s sending data at any arbitrary rate, which would result in multi-path routing. Let us first define the following function,

$$\zeta_j(t_j^p) = t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)}, \quad \forall j \in N \quad (6.38)$$

For any given $t \in T$, notice that $\zeta_j(t_j^p)$ is a constant for each $j \in N$. We refer to $\zeta_j(t_j^p)$ as the *competence factor* of node j . Also, the competence factor of path $i \in P^{s \rightarrow d}$ can be defined as,

$$\sum_{j \in P_i^{s \rightarrow d}} \zeta_j(t_j^p) \quad (6.39)$$

Based on the above definitions and lemma (6.3), q should be such that the following function is minimized,

$$\Phi(t^p) = \sum_{i=1}^K \left[\sum_{j \in P_i^{s \rightarrow d}} \zeta_j(t_j^p) \right] q_i(t^p) \quad (6.40)$$

for all t^p in T^p , subject to constraint (6.16)¹.

The function $q(\cdot)$: Clearly, (6.40) would get minimized for any $t^p \in T^p$ while satisfying constraint (6.16) if q assigns all traffic to the path in $P^{s \rightarrow d}$ having the *least competence factor, which we consider as the most competent path*. In essence, *the path with the least competence factor is a path that minimizes the expected payment that the end-user has to make to have a packet routed from s to d* . We refer to such a path as the **ELPR (Expected Least Paid Route)**.

Formally, q should assign all traffic to path k in $P^{s \rightarrow d}$, such that,

$$\sum_{j \in P_k^{s \rightarrow d}} \zeta_j(t_j^p) = \min_{1 \leq i \leq K} \sum_{j \in P_i^{s \rightarrow d}} \zeta_j(t_j^p)$$

One can find the path to which all traffic should be assigned by using the competence factor $\zeta_j(t_j^p)$ as the weight of node $j \in N$ in the graph G and using Dijkstra's algorithm to find the path from s to d having the minimum competence factor, which is the shortest path w.r.t node weights.

The function $p(\cdot)$: As suggested by the payment formula (6.36), a node that does not route any traffic, i.e., is not included in the most competent path, does not receive any payment.

¹Note that constraint (6.17) need not be considered in this section since it will be always satisfied because of the assumption on node capacity.

This is because if $t_j = \{m_j, c_j\}$ is the type of node j not on the most competent path, then we have $\sum_{i \in P_{s \rightarrow d}^j} q_i(\psi_j, t_{-j}^p) = 0, \forall \psi_j \in [\frac{m_j}{c_j}, \frac{b_j}{v_j}]$.

On the other hand, (6.36) dictates that node j on the most competent path has to be paid an amount equal to the cost that j would have incurred for forwarding a packet from s if the type of j had been associated with the highest possible per packet cost that allowed j to be included on the most competent path. Specifically, let t_j be the type of node j on the most competent path. Also, let the difference between the competence factor of the most competent path in the graph G and that of graph $G \setminus j$ be ϵ . To be included on the most competent path in the graph G , j 's type could have corresponded to all those types in T_j for which its competence factor did not become more than or equal to $\zeta_j(t_j^p) + \epsilon$. Clearly, the highest possible per packet price that can be associated with these types is $\zeta^{-1}(\zeta_j(t_j^p) + \epsilon)$. Thus, j has to be paid the minimum of $\zeta^{-1}(\zeta_j(t_j^p) + \epsilon)$ and $\frac{b_j}{v_j}$.

Based on the above argument, algorithm (7) presents the procedure for computing the most competent path (ELPR) in G on which all traffic should be routed and the payment to be made to each node in N .

Algorithm (7) requires finding the shortest path from s to d w.r.t node weights. This can be done using Dijkstra's algorithm in $O(|E| + |V| \log |V|)$ time. Clearly, the overall running time of the algorithm is $O(|E||V| + |V|^2 \log |V|)$.

Algorithm 7 Single Path Routing– Computing Optimal Path and Payments

Require: $G = (V, E)$; source s ; destination d ; $t = (t_1, \dots, t_n)$

Ensure: Most competent path P (ELPR); Pay p_j , $\forall j \in N$

- 1: **for all** $j \in N$ **do**
 - 2: $t_j^p \leftarrow \frac{m_j}{c_j}$
 - 3: $\zeta_j(t_j^p) \leftarrow t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)}$
 - 4: **end for**
 - 5: Using $\zeta_j(t_j^p)$ as the weight of node $j \in N$, find a path in G from s to d that has the least competence factor. Denote the set of nodes in the selected path as P .
 - 6: $p_j \leftarrow 0$, $\forall j \in \{N \setminus P\}$.
 - 7: **for all** $j \in P$ **do**
 - 8: Using $\zeta_k(t_k^p)$ as the weight of node $k \in N \setminus j$, find a path in $G \setminus j$ from s to d with the least competence factor. Denote the set of nodes in the selected path as S .
 - 9: $\epsilon \leftarrow \sum_{j \in S} \zeta_j(t_j^p) - \sum_{j \in P} \zeta_j(t_j^p)$
 - 10: $p_j \leftarrow \min(\zeta^{-1}(\zeta_j(t_j^p) + \epsilon), \frac{b_j}{v_j})$
 - 11: **end for**
-

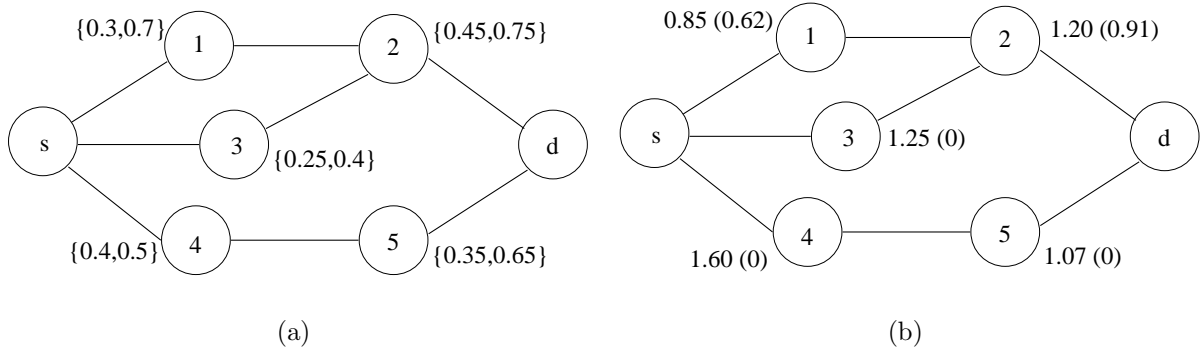


Figure 6.1: Single path routing example.

6.3.1 An Illustrative Example

Let us look at an example to gain insight into the working of algorithm (7). The graph of the network is shown in Figure (6.1). For the example we assume that the cost and capacity of a node are uniformly distributed over $[0, 1]$. Also, we consider that they are independent random variables, so that $f_j^{MC}(m_j, c_j) = f_j^M(m_j) \cdot f_j^C(c_j)$. The sender is assumed to be sending data at 0.2 pkts/sec.

Given two random variables X and Y , the ratio distribution of $Z = X/Y$ can be obtained by integration of the following form [20],

$$f^Z(z) = \int_{-\infty}^{+\infty} |y| f^{XY}(zy, y) dy \quad (6.41)$$

where $f^{XY}(x, y)$ is the joint distribution of X and Y . Based on (6.41), it can be shown that the distribution of t_j^p in our example is,

$$f_j^P(t_j^p) = \begin{cases} 1/2 & 0 < t_j^p < 1 \\ \frac{1}{2(t_j^p)^2} & t_j^p \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (6.42)$$

In Figure (6.1(a)) the cost and capacity reported by each node is shown in brackets beside it. The values next to a node in Figure (6.1(b)) show the competence factor (weight) of the node (outside the bracket) and the payment received by it (inside the bracket) for each packet served by the network. For example, the weight of node 1 is,

$$t_1^p + \frac{F_1^P(t_1^p)}{f_1^P(t_1^p)} = \frac{0.3}{0.7} + \frac{\int_0^{0.3/0.7} 1/2 d\psi_j}{1/2} = 0.85$$

Likewise, the weights of the other nodes has been calculated. Clearly, the ELPR comprises of the route, $s \rightarrow 1 \rightarrow 2 \rightarrow d$, since it is the path with the least competence factor in the graph.

As noted earlier, nodes not on the ELPR does not receive any payment. The payment of node 1 can be found as follows. With node 1 removed, the path with the lowest competence factor in the graph is $s \rightarrow 3 \rightarrow 2 \rightarrow d$, with a competence factor of 2.45. So node 1 can report all types that does not increase its weight by more than $2.45 - 2.05 = 0.4$. Now, $\min(\zeta^{-1}(0.85 + 0.4), \infty) = 0.62$, which is the payment received by node 1. The payment of node 2 can be similarly calculated.

6.4 Multiple Path Routing

We will now study the deployment of the path auction mechanism by relaxing the assumption on node capacity. In this generalized case, based on lemma (6.3) and the definition of node competence factor, q should be such that the following function is minimized,

$$\Phi(t^p) = \sum_{i=1}^K \left[\sum_{j \in P_i^{s \rightarrow d}} \zeta_j(t_j^p) \right] q_i(t^p) \quad (6.43)$$

for all t^p in T^p subject to constraints (6.16) and (6.17).

6.4.1 The Function $q(\cdot)$:

For any given $t^p \in T^p$, to find a feasible traffic allocation (one that satisfies constraint (6.16) and (6.17)) over the paths in $P^{s \rightarrow d}$ that minimizes (6.43) we resort to a modified version of the Edmonds-Karp algorithm [19]. In our modified version of the algorithm we use the competence factor of each node as its weight to find the path with the least competence factor while searching for the augmenting path.

Algorithm 8 Multiple Path Routing– Computing Paths

Require: $G = (V, E)$; source s ; destination d ; traffic rate R , $t = (t_1, \dots, t_n)$

Ensure: Optimal traffic allocation over the paths in G (ELPR)

- 1: $f \leftarrow 0$
- 2: $i \leftarrow 1$
- 3: **for all** $j \in N$ **do**
- 4: $t_j^p \leftarrow \frac{m_j}{c_j}$
- 5: $\zeta_j(t_j^p) \leftarrow t_j^p + \frac{F_j^P(t_j^p)}{f_j^P(t_j^p)}$
- 6: **end for**
- 7: **while** $f < R$ AND \exists a path from s to d in G^f **do**
- 8: Using $\zeta_j(t_j^p)$ as the weight of node $j \in N$, find a path in G^f from s to d that has the least competence factor. Denote the set of nodes in the selected path as $\mathcal{P}_i^{s \rightarrow d}$.
- 9: Find node $k \in \mathcal{P}_i^{s \rightarrow d}$, such that,

$$c_k^f = \min_{j \in \mathcal{P}_i^{s \rightarrow d}} c_j^f$$

- 10: $r \leftarrow \min(c_k^f, R - f)$
 - 11: $\tilde{q}_i \leftarrow r/R$
 - 12: $c_j^{f+r} \leftarrow c_j^f - r, \forall j \in \mathcal{P}_i^{s \rightarrow d}$
 - 13: $c_j^{f+r} \leftarrow c_j^f, \forall j \in N \setminus \mathcal{P}_i^{s \rightarrow d}$
 - 14: $f \leftarrow f + r$
 - 15: $i \leftarrow i + 1$
 - 16: **end while**
-

Algorithm (8) is our modified Edmonds-Karp algorithm that computes the optimal traffic allocation over the paths in G . Before we delve into the details of algorithm (8) let us go over some definitions. Given a DSA network represented by the graph $G = (V, E)$ with sender s and destination d and a flow f from s to d , we define the **residual capacity** c_j^f of a node $j \in N$ as the amount of *additional* flow that can pass via node j before exceeding its capacity c_j . Based on this we can define the **residual network** of G induced by f as $G^f = \{V^f, E^f\}$, where

$$V^f = \{v \in V : c_v^f > 0\}$$

$$E^f = \{(u, v) \in V \times V : c_u^f > 0 \text{ and } c_v^f > 0\}$$

The working of algorithm (8) can be explained as follows,

1. **Compute node weight:** Compute the weight of each node j in N as its competence factor $\zeta_j(t^p)$, based on the vector of individual types t announced by the nodes. Also, initialize flow f from s to d to 0 and path i to 1.
2. **Find an augmenting path in G^f :** Find an augmenting path from s to d in G^f that has the least competence factor and denote the set of nodes in the selected path as $\mathcal{P}_i^{s \rightarrow d}$. This path can be computed by using Dijkstra's algorithm starting from node s . If no path can be found, then there is no feasible traffic allocation that satisfies constraints (6.16) and (6.17) and the algorithm exits.
3. **Allocate traffic to the selected path i :** Find node k in $\mathcal{P}_i^{s \rightarrow d}$ that has the *least* residual capacity because of flow f from s to d . Node k is said to be the bottleneck node

of the selected path. Clearly, $\min(c_k^f, R - f)^2$ is the amount of flow that can be assigned to path i and thus the fraction of traffic allocated to path i is, $\tilde{q}_i = \min(c_k^f, R - f)/R$.

4. **Compute new residual capacities:** Update the residual capacity of each node $j \in \mathcal{P}_i^{s \rightarrow d}$ by subtracting $\min(c_k^f, R - f)$ from its residual capacity c_j^f in G^f . The residual capacity of all nodes not on path i is same as their residual capacity in G^f . Then increment f by the amount of flow assigned to path i .
5. **Flow allocation:** If $f = R$, i.e., there is a flow rate R from s to d , then the optimal traffic allocation that minimizes (6.43) satisfying constraints (6.16) and (6.17) has been found. The optimal allocation is over the set of paths, $\mathcal{P}^{s \rightarrow d} = \{\mathcal{P}_1^{s \rightarrow d}, \dots, \mathcal{P}_i^{s \rightarrow d}\}$, with \tilde{q}_1 to \tilde{q}_i being the respective fraction of traffic over the paths. Otherwise, if $f < R$, then increment i by 1 and go to STEP 2.

The augmenting paths in algorithm (8) can be found using Dijkstra's algorithm in $O(|E| + |V| \log |V|)$ time. Since, in every iteration of the while loop in lines 7-16 at least one node becomes saturated, the overall running time of the algorithm is $\Theta(|E||V| + |V|^2 \log |V|)$.

Notice that, (6.43) can be minimized by using linear programming. However for such a technique to navigate through the space of feasible solutions requires enumeration of all paths from s to d , which may be computationally infeasible in practise. To reduce the complexity of finding the optimal traffic allocation, algorithm (8) essentially progresses by minimizing subproblems of the original optimization problem, that can be solved in polyno-

²Note that, if $\min(c_k^f, R - f) = R - f$, then i is necessarily the *last* augmenting path needed to allocate the flow rate R from s to d .

mial time using Dijkstras, and finally combines the individual solutions to obtain the feasible traffic allocation that minimizes (6.43). We will next state and prove the following theorem regarding the optimality of the allocation provided by algorithm (8).

Theorem 6.1. *Given the existence of feasible traffic allocations, the solution provided by algorithm (8) minimizes (6.43) among all such feasible allocations.*

Proof. Without loss of generality let us assume that the set of paths $P^{s \rightarrow d} = \{P_1^{s \rightarrow d}, \dots, P_K^{s \rightarrow d}\}$ is indexed in non-decreasing order of their competence factors. Then, for a given $t \in T$, let the traffic allocation over the K paths in $P^{s \rightarrow d}$ as computed by algorithm (8) be represented by the vector $q = (q_1, \dots, q_K)^3$. Also, let $q' = (q'_1, \dots, q'_K)$ denote *any* feasible traffic allocation over the set of K paths in $P^{s \rightarrow d}$ for the same vector of individual types t . Further, let k be the highest index of a path in $P^{s \rightarrow d}$ which has a non-zero fraction of traffic assigned to it in q and let k' be the highest index of a path in $P^{s \rightarrow d}$ which has a non-zero fraction of traffic assigned to it in q' . Notice that, in the representation of algorithm (8)'s traffic allocation as the vector q , we consider the case of algorithm (8) not encountering a path while assigning traffic, due to the path's bottleneck capacity being zero, by considering that the algorithm assigns 0 traffic to it.

To show that q minimizes (6.43) we will show that the value of the function $\Phi(\cdot)$ obtained using q' cannot be less than that obtained using q . Clearly, for q' to provide a lower value of the function $\Phi(\cdot)$, k' has to be *less than* or equal to k . Given that q is a feasible allocation, we will first prove that k' can never be less than k . Next, we will show that, when $k' = k$, even then q' cannot yield a lower value of the function $\Phi(\cdot)$ than q .

Case 1- $k' < k$: We will prove that $k' < k$ is not possible by contradiction. For the sake of argument assume that $k' < k$. This implies that there has to be *at least* one path, say w , in $[2, k']$ such that $q'_w > q_w \geq 0$. Since $q'_w > q_w$, it has to be the case that both the following are true,

- Path w intersects at some node(s) with *at least* one path in $[1, w - 1]$.
- The bottleneck node(s) on path w , after algorithm (8) assigned traffic to path $w - 1$, must correspond to one (or some) of the nodes that w has in common with the paths in $[1, w - 1]$.

Based on the above arguments, clearly, the residual capacity of at least one node that w has in common with the path(s) in $[1, w - 1]$ must be 0 after algorithm (8) assigns traffic to w^4 . Let us consider one such node and denote it as j^* . Also, let W be the set of paths in

³For clarity, if path x in $\mathcal{P}^{s \rightarrow d}$ corresponds to path x' in $P^{s \rightarrow d}$, i.e., $\mathcal{P}_x^{s \rightarrow d} = P_{x'}^{s \rightarrow d}$ then $q_{x'} = \tilde{q}_x$

⁴It is worth emphasizing here that the residual capacity of a node on path w , that is *not* in common with any path in $[1, w - 1]$, *cannot* be zero after algorithm (8) assigns traffic to w .

$[1, w - 1]$ that traverse via node j^* and let $q'_w - q_w = \epsilon$. For q' to be a feasible allocation, the following constraint must hold,

$$\sum_{i \in W} q_i - \sum_{i \in W} q'_i \geq \epsilon \quad (6.44)$$

Next we claim that, $\sum_{i=1}^w q_i \geq \sum_{i=1}^w q'_i$. We will prove this by mathematical induction.

Base Case: Let w' be the lowest indexed path in $[1, k']$ such that $q'_{w'} > q_{w'}$ (note that w' cannot be 1). Let $q'_{w'} - q_{w'} = \epsilon'$. Because of constraint (6.44) we can state that,

$$\begin{aligned} \sum_{i=1}^{w'-1} q_i &\geq \sum_{i=1}^{w'-1} q'_i + \epsilon' \\ \implies \sum_{i=1}^{w'} q_i &\geq \sum_{i=1}^{w'} q'_i \end{aligned}$$

This proves the base case.

Inductive Step: Let w' be the index of a path in $[1, k']$ such that $q'_{w'} > q_{w'}$. Also, let w'' in $[1, k']$ be the index of the first path *after* w' such that $q'_{w''} > q_{w''}$ and let $q'_{w''} - q_{w''} = \epsilon''$. Let the hypothesis hold for w' , i.e., assume that $\sum_{i=1}^{w'} q_i \geq \sum_{i=1}^{w'} q'_i$. Then, we have,

$$\sum_{i=1}^{w'} q_i + \sum_{i=w'+1}^{w''-1} q_i \geq \sum_{i=1}^{w'} q'_i + \sum_{i=w'+1}^{w''-1} q'_i + \epsilon'' \quad (6.45)$$

The validity of (6.45) can be explained as follows. The first term on the LHS is greater than or equal to the first term on the RHS since we have assumed that the hypothesis holds for w' . The second term on the LHS is greater than or equal to the second term on the RHS since $q'_i \leq q_i, \forall i \in [w' + 1, w'' - 1]$. Also, (6.44) dictates that there has to be a set of paths, say W'' , which is a subset of the paths in $[1, w'' - 1]$, for which the total fraction of traffic allocated to the paths in W'' in q' is *at least* ϵ'' less than that allocated to the paths in W'' in q . Thus inequality (6.45) holds. Adding $q_{w''}$ to both sides of (6.45) yields,

$$\sum_{i=1}^{w''} q_i \geq \sum_{i=1}^{w''} q'_i \quad (6.46)$$

This completes the induction step and proves our hypothesis. Notice that, (6.46) implies that $\sum_{i=1}^{k'} q_i \geq \sum_{i=1}^{k'} q'_i$. However, this is clearly a contradiction. Thus, k' can never be less than k .

Case 2 - $k' = k$: Notice that, when $k' = k$ it is not possible that every $q'_i < q_i, \forall i \in [1, k]$ or every $q'_i > q_i, \forall i \in [1, k]$. Thus only two cases are possible— either every $q'_i = q_i, \forall i \in [1, k]$ or $q'_i > q_i$ for some (not all) $i \in [1, k]$. For the latter case, if there is a path w in $[1, k]$ for which $q'_w > q_w$, then because of (6.44) the value of function (6.43) yielded by q' will be clearly greater than or equal to that yielded by q . Thus, we can say that q minimizes (6.43). \square

6.4.2 The Function $p(\cdot)$

According to (6.36), a node that does not route any traffic does not receive any payment. In computing the payment for node j that forwards a non-zero fraction of traffic, the difficulty lies in evaluating the integral term of (6.36). To compute this integration we will make use of the fact that, given t_{-j} , the fraction of traffic forwarded by j is a (decreasing) *step function* of per packet cost associated with its type domain. Thus, to compute the integral, the fraction of traffic that j forwards has to be evaluated only at the points of discontinuity of the step function.

For a given t_{-j} , algorithm (9) computes the payment to be received by node j in an iterative manner by finding a partial payment to be received by the node in each iteration and summing the partial payments progressively to finally compute the total payment to be received by j . Let Z_i denote the competence factor of path i in G . Let us consider a node j that routes traffic. To compute j 's payment the algorithm iterates on the while loop in lines 6-16. Before the iterations start, the algorithm initializes j 's payment, p_j , to $t_j^p \cdot \sum_{i \in \mathcal{P}^{s \rightarrow d}} \tilde{q}_i$, which corresponds to the first term of (6.36). Also, the set $\mathcal{P}^{s \rightarrow d}$ is initialized to $\mathcal{P}^{s \rightarrow d}$ and the set \tilde{q}' to \tilde{q} . Let us now consider how the algorithm computes the partial payment of j in an iteration. Let,

$$\epsilon \leftarrow \min_{i \in \mathcal{P}^{s \rightarrow d}} (Z_{i^*} - Z_i),$$

where i^* is the first path in G having a competence factor greater than that of i and that does not pass via j . Clearly, j will receive $\sum_{i \in \mathcal{P}^{s \rightarrow d}} \tilde{q}'_i$ fraction of traffic as long as it does

not report a type that increases its competence factor, $\zeta_j(t_j^p)$, by ϵ or more. Since j does not report a type whose per packet cost is more than b_j/v_j , thus, the highest per packet cost that can be associated with the type of j so that j still receives the same fraction of traffic is,

$$\hat{t}_j^p \leftarrow \min(\zeta^{-1}(\zeta_j(t_j^p) + \epsilon), \frac{b_j}{v_j})$$

Thus, the partial payment of j computed in the iteration is,

$$(\hat{t}_j^p - t_j^p) \sum_{i \in \mathcal{P}^{s \rightarrow d}} \tilde{q}'_i$$

Notice that, \hat{t}_j^p and t_j^p are essentially two consecutive points of discontinuity of the step function mentioned before. The algorithm then updates the payment of j by adding the partial payment calculated to the previously computed payment for j . Now, if $\hat{t}_j^p = \frac{b_j}{v_j}$, then we have completed computing the integral term of (6.36) and the algorithm breaks from the iterative loop for calculating j 's payment. Otherwise, t_j^p is updated to \hat{t}_j^p and algorithm (8) is invoked with the type vector (\hat{t}_j, t_{-j}) , such that the per packet price associated with \hat{t}_j is \hat{t}_j^p (computed above), to find the new set of optimal paths as $\mathcal{P}^{s \rightarrow d}$ and its corresponding traffic allocation vector as \tilde{q}' . The iterations continue until the highest possible per packet price (less than equal to b_j/v_j) that can be associated with the type of j for j to receive a non-zero fraction of traffic is found.

For each node j that routes traffic, if the fraction of traffic function of j has d points of discontinuity, then algorithm (8) has to be invoked d times for computing the payment of j .

Algorithm 9 Multiple Path Routing– Computing Payments

Require: Optimal set of paths $\mathcal{P}^{s \rightarrow d}$ over which traffic should be routed; vector \tilde{q} of traffic fraction over the paths in $\mathcal{P}^{s \rightarrow d}$.

Ensure: $p_j, \forall j \in N$

- 1: $p_j \leftarrow 0, \forall j \in \{N \setminus \bigcup_{i \in \mathcal{P}^{s \rightarrow d}} \mathcal{P}_i^{s \rightarrow d}\}$
 - 2: **for all** $j \in \bigcup_{i \in \mathcal{P}^{s \rightarrow d}} \mathcal{P}_i^{s \rightarrow d}$ **do**
 - 3: $p_j \leftarrow t_j^p \cdot \sum_{i \in \mathcal{P}^{s \rightarrow d}} q_i$
 - 4: $\mathcal{P}^{s \rightarrow d} \leftarrow \mathcal{P}^{s \rightarrow d}$
 - 5: $\tilde{q}' \leftarrow \tilde{q}$
 - 6: **while** $\sum_{i \in \mathcal{P}^{s \rightarrow d}} \tilde{q}'_i > 0$ **do**
 - 7: $\epsilon \leftarrow \min_{i \in \mathcal{P}^{s \rightarrow d}} (Z_{i^*} - Z_i)$, where i^* is the first path in G having a competence factor greater than or equal to that of i and that does not pass via j .
 - 8: $\hat{t}_j^p \leftarrow \min(\zeta^{-1}(\zeta_j(t_j^p) + \epsilon), \frac{b_j}{v_j})$
 - 9: $p_j \leftarrow p_j + (\hat{t}_j^p - t_j^p) \sum_{i \in \mathcal{P}^{s \rightarrow d}} \tilde{q}'_i$
 - 10: **if** $\hat{t}_j^p = \frac{b_j}{v_j}$ **then**
 - 11: break;
 - 12: **else**
 - 13: $t_j^p = \hat{t}_j^p$
 - 14: Invoke algorithm (8), with t_j^p as the per packet cost associated with the type of node j , to find the optimal set of paths as $\mathcal{P}'^{s \rightarrow d}$ and the corresponding fraction of traffic as the vector \tilde{q}' .
 - 15: **end if**
 - 16: **end while**
 - 17: **end for**
-

Thus, the time complexity of finding j 's payment is $O(d|E||V| + d|V|^2 \log|V|)$. Consequently, the payment of all nodes that route traffic can be found in $O(d|E||V|^2 + d|V|^3 \log|V|)$ time.

6.4.3 Who Pays the Nodes?

We have established that in our path auction mechanism no node has an incentive to dishonestly report its cost or capacity. However, there still exists the following loophole in the mechanism if it is the sender who pays the nodes— node j honestly announces its type, receives payment for the fraction of traffic it has been made responsible for forwarding but simply drops the packets instead of forwarding them. To discourage nodes from exhibiting such dropping behavior we make the destination node make the payment to each intermediate node for every packet it receives. In this case, if a node drops a packet the destination will not receive the packet and the node will consequently not receive payment for it. Thus, no node will have an unilateral incentive to drop any fraction of traffic.

6.4.4 An Illustrative Example

To gain insight into how algorithm (8) and (9) finds the optimal set of paths over which to route traffic and calculates payment of the nodes that route traffic, let us study an example. Figure (6.2) shows the graph of the network. We again assume that cost and capacity of the nodes are independent random variables, with each being uniformly distributed over

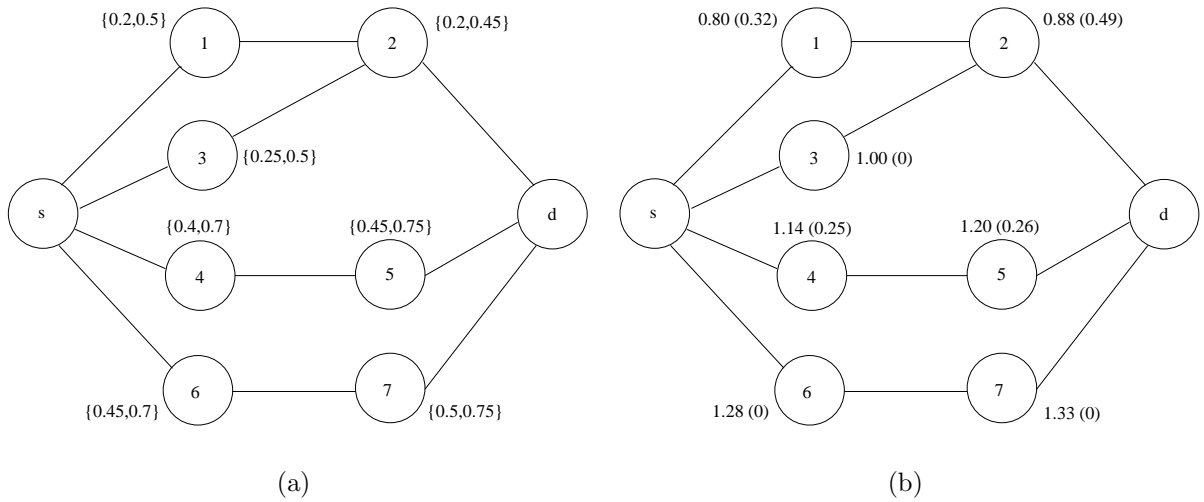


Figure 6.2: Multi-path routing example.

$[0,1]$. The sender inputs data at 0.7 pkts/sec. The distribution of t_j^p for each node j is given by (6.42).

The cost and capacity of a node has been shown beside it in Figure 6.2(a). Figure 6.2(b) shows the competence factor (weight) of a node (outside bracket) and the payment received by it (inside bracket) for each packet served by the network. The weights of the nodes have been found similar to the example in Section 6.3.1 and we will not discuss it here. After the weight calculations, algorithm 8 finds the path with the least competence factor (shortest path w.r.t node weights) in the network and saturates the path. This path is $s \rightarrow 1 \rightarrow 2 \rightarrow d$, which is allocated $0.45/0.7=0.64$ fraction of traffic. Notice that, node 2 saturates in the process. Next, node 2 is disconnected from the network and the path with the least competence factor is found in the residual network. This yields the path $s \rightarrow 4 \rightarrow 5 \rightarrow d$, which is allocated $0.25/0.7=0.36$ fraction of traffic.

Next, let us consider node 2 and see how its payment is calculated. Notice that, node 2 will receive 0.64 fraction of traffic until it reports a type which makes the competence factor of path $s \rightarrow 1 \rightarrow 2 \rightarrow d$ (1.68) greater than or equal to that of path $s \rightarrow 4 \rightarrow 5 \rightarrow d$ (2.34), in which case it will get 0 traffic. Now, the maximum per packet price that can be associated with the reported type of node 2 for it to receive 0.64 fraction of traffic is $\min(\zeta^{-1}(0.88 + 0.66), \infty) = 0.77$. So the payment received by node 2 is, $\frac{0.2}{0.45} \cdot 0.64 + (0.77 - \frac{0.2}{0.45}) \cdot 0.64 + 0 = 0.49$. The payment of the other nodes is similarly calculated.

6.5 Summary

The problem of routing in DSA networks, where each secondary node has a privately known cost and capacity, is complicated by the fact that each node can potentially improve its utility by dishonestly reporting its type to the routing mechanism. In the absence of techniques to induce nodes for truthfully reporting their cost and capacity to the routing mechanism, there may be an unnecessary overpayment of incentives provided to the nodes for routing traffic. We design a path auction based routing scheme for DSA networks that can enforce honest revealing of cost and capacity to be a dominant strategy for every node in the *path auction game*. Furthermore, our mechanism also minimizes the payment that needs to be given to the nodes that forward traffic, making our scheme attractive to the end-user. By considering capacity constraint of the nodes we explicitly support multiple path routing. For deploying our routing mechanism in DSA networks, we present polynomial time algorithms

for computing the optimal route over which to route traffic and the payment to be received by each node.

CHAPTER 7

SIMULATION RESULTS

In this chapter, we conduct simulations to validate the design philosophy and study the effectiveness of all our proposed algorithms.

7.1 Spectrum Bargaining in Complete Conflict Graphs

In this section, we will analyze the results obtained in Chapter 4 using numerical analysis to get a better insight into the equilibrium channel share of the players.

7.1.1 Channel Share Convergence

We will show numerically that the SPNE shares of a 2-player *finite* horizon bargaining game indeed converges when the game is played with a large horizon (refer Section 4.2). Figure 7.1 shows the SPNE shares of a 2-player finite horizon game with varying number of game periods. The number of available channels M have been taken to be 25, while the discount factor of the players δ has been set at 0.95. The SPNE share of the players have been computed using the perfect equilibrium strategies derived in Section 4.2.1 for a finite horizon game. As can be seen from the figure, as the number of period increases, each player

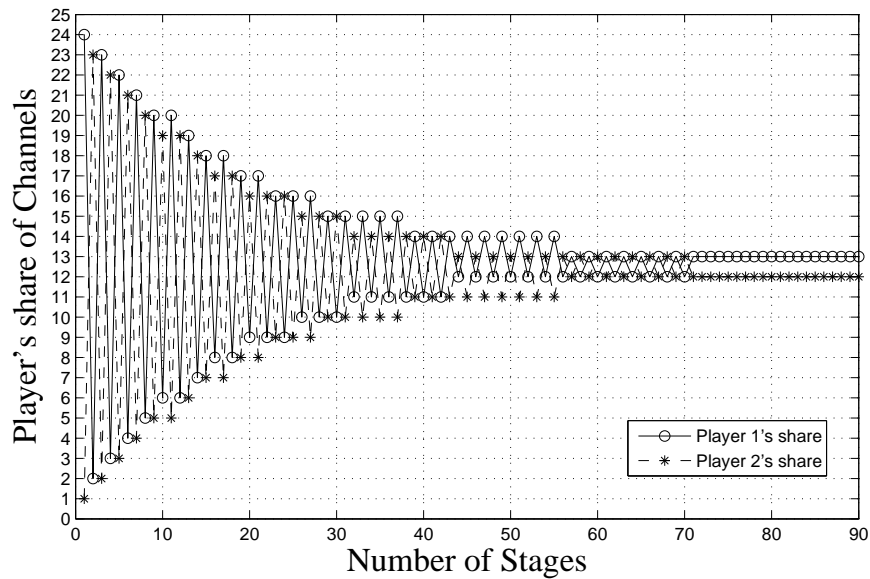


Figure 7.1: SPE shares of a 2 player game against varying number of game periods. Here $M = 25$ and $\delta = 0.95$.

starts getting a more *fair* share of the channels. For example, when the game is played for only one period, one of the players (the one making the offer) gets 24 channels while the other one gets only 1 channel. In the 2 period game, however, one player gets 23 channels (the player receiving the offer in the first period) and the other one gets 2 channels. And so on. Finally, beyond period 70, the SPNE shares of the two players converges to an equal split of the channel between the players. This validates the observation we made in Section 4.3.2, that for δ close to one, each player tends to get an equal share of the channels in the *infinite horizon* game. Note that, when channel share of the players converge, one of the players gets 13 channels, while the other one gets 12 channels. This is because the number of channels available in this case ($M = 25$) is not equally divisible between the 2 players. However, if M/N is an integer, then each player gets an exactly equal share as the others in the infinite

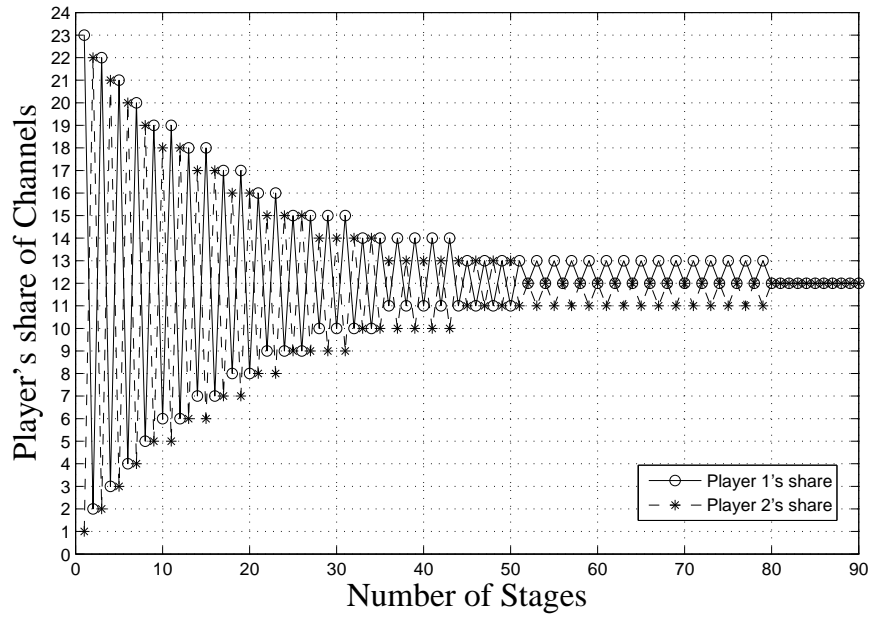


Figure 7.2: SPE shares of a 2 player game against varying number of game periods. Here $M = 24$ and $\delta = 0.95$.

horizon game. This is shown in Figure (7.2), which considers that there are 24 available channels.

It is worth mentioning here that, as noted earlier, the 2 players **do not** have to play all the periods to find what the SPNE share of the players converges to. It can be found from their closed form expressions in equations (4.22) and (4.23). Knowing their infinite horizon equilibrium shares in the very first period, rational players will play their equilibrium strategies and play will also end in the very first period.

7.1.2 Variation of Fairness

Fairness is concerned with the relative utility of the nodes competing for the M channels. To study the relative utility the players get as a result of the competition process, from a system wide perspective, we use Jain's fairness index [35]. Recall that, in the infinite horizon game, if $\{x_i | 1 \leq i \leq N\}$ is accepted in period t , then the utility (payoff) of player i is $\delta^t x_i$, where $\delta \in [0, 1]$ is the discount factor of the players¹. Thus, if $\{x_i^t | 1 \leq i \leq N\}$ is accepted in period t , then we can define fairness in terms of the utilities (payoffs) of the players as follows.

$$F = \frac{(\sum_{i=1}^N \delta^t x_i)^2}{N \cdot \sum_{i=1}^N (\delta^t x_i)^2} \quad (7.1)$$

Now, recall that we derive the infinite horizon SPNE strategy of the players (refer Section 4.3), which all rational players will play in the *first* period ($t = 0$) of the game. Thus, the payoff of P_i ($i \in [1, N]$) will be $\delta^0 x_i = x_i$. Based on this we simplify (7.1) as follows.

$$F = \frac{(\sum_{i=1}^N \delta^0 x_i)^2}{N \cdot \sum_{i=1}^N (\delta^0 x_i)^2} = \frac{(\sum_{i=1}^N x_i)^2}{N \cdot \sum_{i=1}^N (x_i)^2} \quad (7.2)$$

The fairness index defined above is bounded between 0 and 1. More the players tend to equally distribute the channels among themselves, fairness approaches 1. Also, if one node acquires all the channels, fairness becomes $1/N$, which is 0 in the limit $N \rightarrow \infty$.

Figure 7.3 shows how fairness varies with the discount factor, δ . The number of nodes N have been taken to be 30, while M is set at 55. Fairness has been calculated using the index defined in (7.2). As can be seen from the figure, as δ increases, i.e, as the players

¹Note that, here x_i denotes the number of channels that P_i gets in the sharing rule.

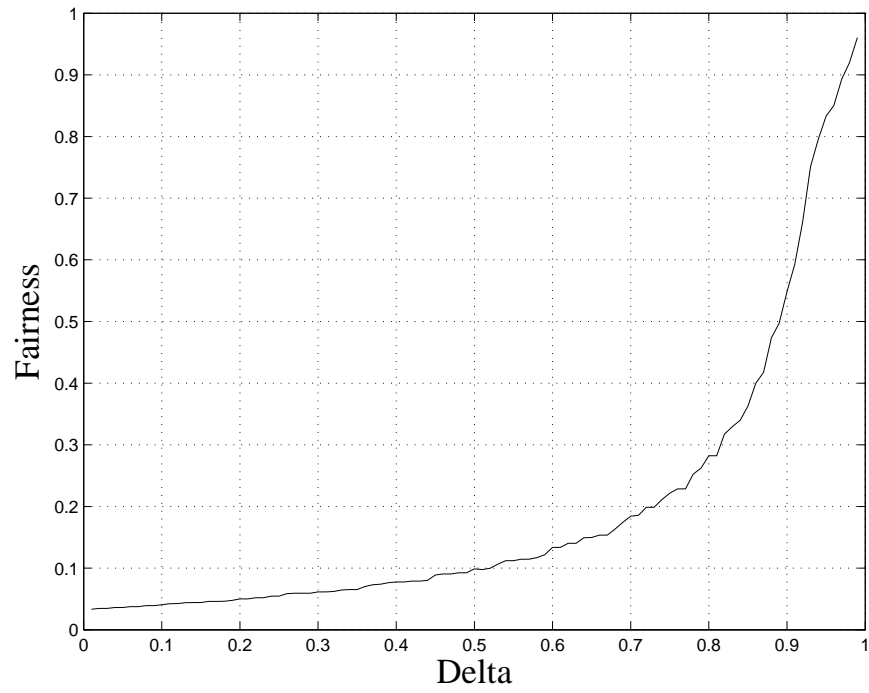


Figure 7.3: Variation of fairness with discount factor δ . Here, $N = 30$, $M = 55$

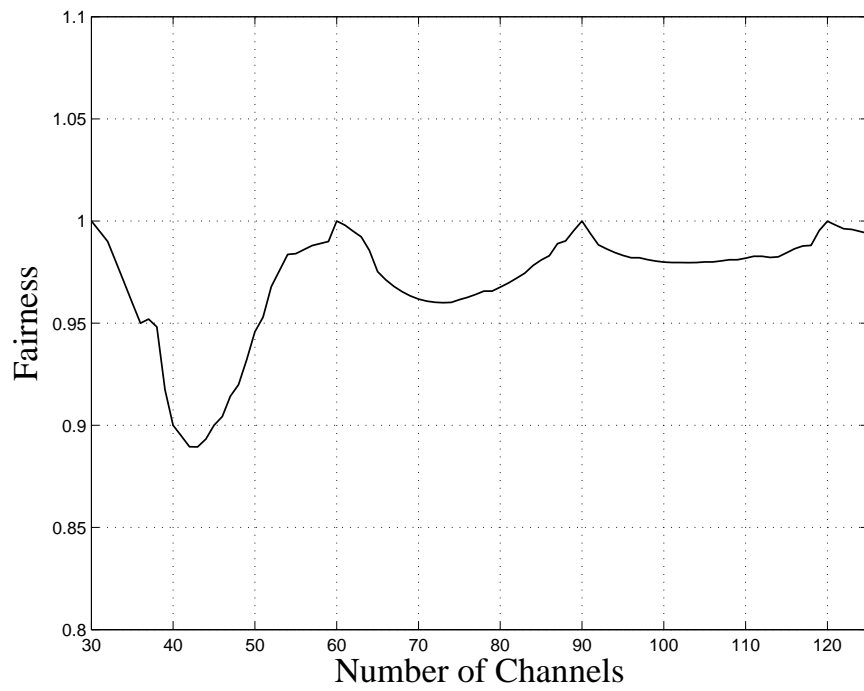


Figure 7.4: Variation of fairness with number of channels. Here, $N = 30$, $\delta = 0.95$.

become more patient, system level fairness increases. This is because, as the players become increasingly more patient, no player can outlast the other players by waiting to get a larger share. Thus, the channels start getting more and more *fairly* apportioned among the players.

Figure 7.4 shows the variation of fairness with number of channels. The number of nodes N have been taken to be 30, while the discount factor of the players δ is set at 0.95. Notice that, at values of M where M/N is an integer value (i.e, at $M = xN$, where $x = 1, 2, 3, \dots$ etc), fairness becomes very close to 1. This is because all players being patient in the infinite horizon game ($\delta = 0.95$), the players' SPNE strategies tend to equally divide the available channels among them. At values of M where M/N is not an integer, fairness falls below 1, since all players cannot possibly get an equal share. Between xN and $(x + 1)N$ fairness falls to a local minimum and then again starts approaching 1 as M approaches $(x + 1)N$. Moreover, note that, as M increases, the local minimum of fairness between xN and $(x + 1)N$ starts increasing. This is because as more channels get available, the relative discrepancy among the channel allocation of different nodes start decreasing.

7.2 Spectrum Bargaining in Arbitrary Conflict Graphs

In this section, we conduct simulations to study how the “self-gain” maximizing strategy of the players impact system wide performance. For simulations, we assume a noiseless, immobile radio network. The conflict graph of the network has been generated randomly for a given graph density. Note that, conflict graph density depends on the transmission

power (and hence interference range) of the secondary users. Higher the transmit power of nodes, larger will be their interference range. Hence, graph density of the conflict graph will increase. Also, in our simulations we assume that all players have an equal discount factor.

7.2.1 System Utility

If $\{x_i^t | 1 \leq i \leq N\}$ is accepted in period t , then we can define system utilities in terms of the payoffs of the players as follows.

- **Sum Utility:** This considers the total system utility regardless of fairness.

$$U_{sum} = \sum_{i=1}^N \mathbb{R}_i = \sum_{i=1}^N \delta_i^t |x_i^t| \quad (7.3)$$

- **Minimum Utility:** This considers the utility of the player with the least payoff.

$$U_{min} = \min_{1 \leq i \leq N} \mathbb{R}_i = \min_{1 \leq i \leq N} \delta_i^t |x_i^t| \quad (7.4)$$

- **Proportional Fairness based Utility** [49]:

$$U_{fair} = \sum_{i=1}^N \log(\mathbb{R}_i) = \sum_{i=1}^N \log(\delta_i^t |x_i^t|) \quad (7.5)$$

To make it comparable to U_{min} and U_{sum} , we modify the fairness utility to:

$$U_{fair} = \left(\prod_{i=1}^N \mathbb{R}_i \right)^{1/N} = \left(\prod_{i=1}^N \delta_i^t |x_i^t| \right)^{1/N} \quad (7.6)$$

Now, Algorithm 6 finds the infinite horizon SPNE strategy of P_1 in the *first* period ($t = 0$) of the game, which all rational players in P_{-1} will accept. Thus, the payoff of P_i ($i \in [1, N]$) will be $\mathbb{R}_i = \delta_i^0 |x_i^0| = |x_i^0|$. Based on this we simplify the metrics defined above as follows.

- U_{fair} : We use proportional fairness based system utility as defined in (7.6). Based on the above argument U_{fair} becomes,

$$U_{fair} = \sqrt[1/N]{\prod_{i=1}^N \delta_i^0 |x_i^0|} = \sqrt[1/N]{\prod_{i=1}^N |x_i^0|} \quad (7.7)$$

Notice that, $U_{fair} = 0$ if there is any $|x_i^0| = 0, i \in [1, N]$. Thus, this metric will also help capture whether any node gets starved of channels.

- U_{mean} : We use mean utility instead of sum utility (7.3) in our simulations, so that all three utilities are within the same scale,

$$U_{mean} = \frac{1}{N} \sum_{i=1}^N \mathbb{R}_i = \frac{1}{N} \sum_{i=1}^N \delta_i^0 |x_i^0| = \frac{1}{N} \sum_{i=1}^N |x_i^0| \quad (7.8)$$

- U_{min} : We use minimum utility as defined in equation (7.4). U_{min} becomes,

$$U_{min} = \min_{1 \leq i \leq N} \mathbb{R}_i = \min_{1 \leq i \leq N} |x_i^t| \quad (7.9)$$

7.2.2 Impact of the Number of Channels (M)

Figures 7.5, 7.6 and 7.7 show how the three utilities vary with the number of available channels, M . We consider a 8 node (player) network, with each player having a discount factor of 0.7. Figure 7.5 shows how U_{fair} varies with M . As can be seen from the figure, proportional fairness increases with the number of available channels. The graph also shows the impact of graph density on U_{fair} . For a given M (and N), U_{fair} decreases as graph density increases. Increasing graph density creates additional interference constraints. Thus,

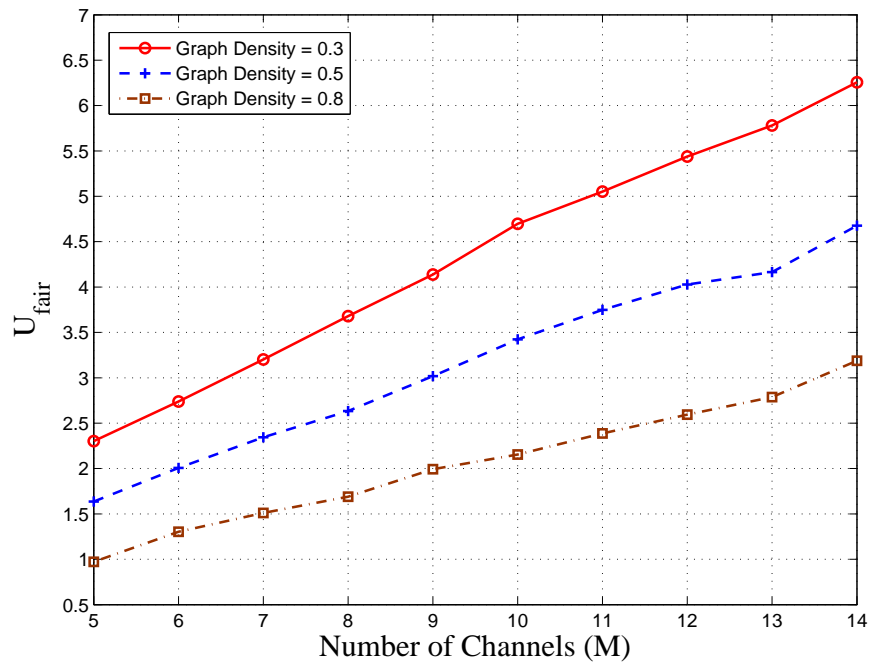


Figure 7.5: Fairness with varying number of available channels.

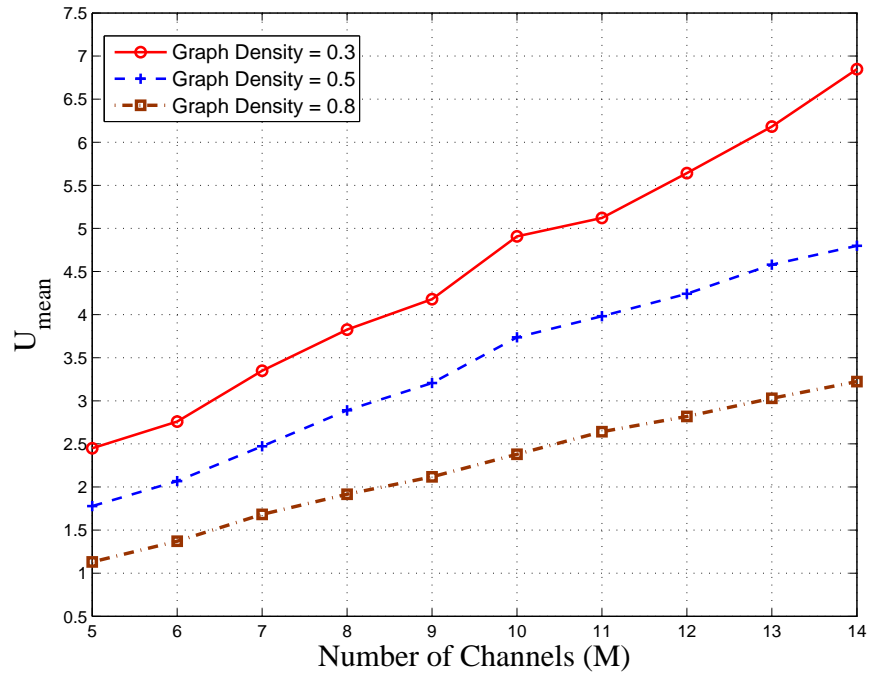


Figure 7.6: Mean utility with varying number of available channels.

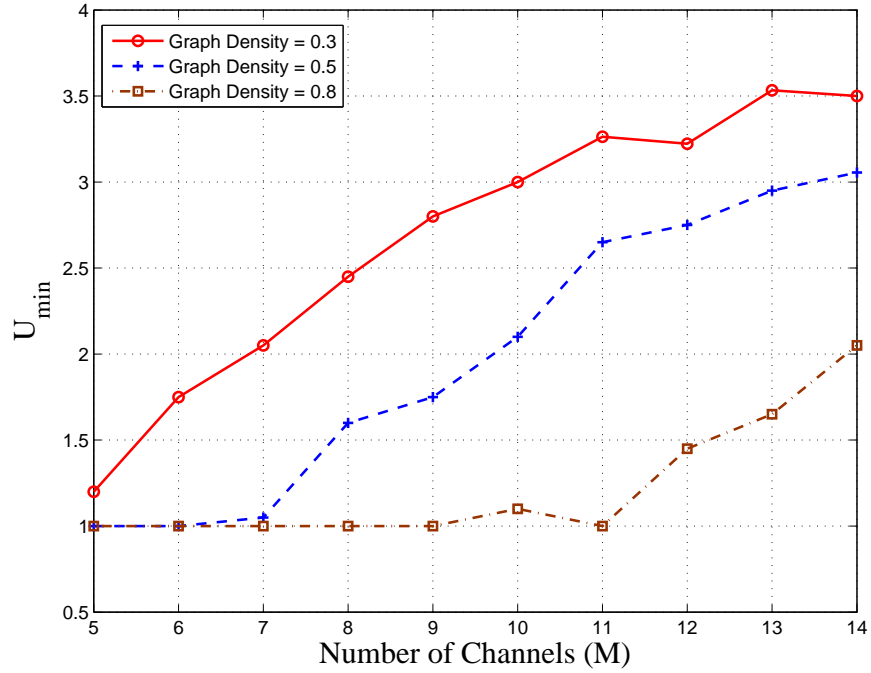


Figure 7.7: Minimum utility with varying number of available channels.

the average vertex degree in the conflict graph increases and each node tends to get lesser number of channels. Therefore, U_{fair} scales inversely with graph density.

Figure 7.6 shows the average number of channels received by the nodes. As M increases, U_{mean} increases. Also, as graph density increases, for a given M and N , U_{mean} decreases due to the increase in average vertex degree. Figure 7.10 shows the minimum number of channels received by a node (U_{min}) as M increases. U_{min} increases as more channels become available. Since, U_{min} never falls below 1, no node is ever “starved” in the solution produced by our bargaining approach. This can also be noted from Figure (7.5). Moreover, the minimum value of U_{min} is 1 because we have considered that a player has to be offered at least 1 channel to make him accept an offer in the last period (Section 5.2). Generally speaking,

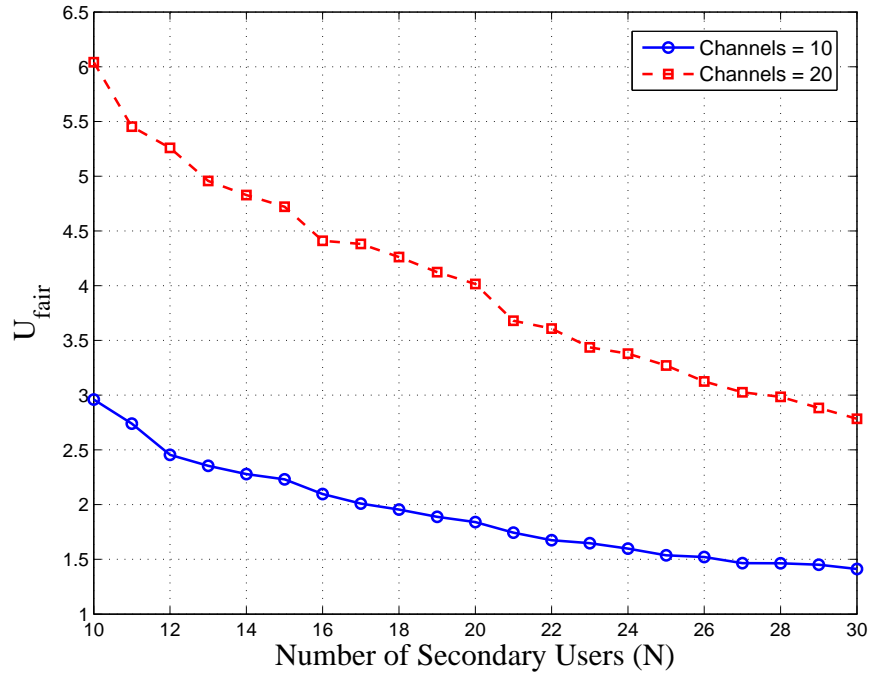


Figure 7.8: Fairness with varying number of Secondary Users.

our bargaining framework can be tailored to provide application specific minimum level of QoS.

7.2.3 Impact of the Number of Secondary users (N)

Figures 7.8, 7.9 and 7.10 show how the three utilities degrade with increasing number of secondary nodes, N . The conflict graphs has been randomly generated with a graph density of 0.5. The discount factor of all players is 0.8. Figure 7.8 shows how proportional fairness based system utility, U_{fair} , vary with N . As N increases, U_{fair} decreases. This is because, as N increases (for a given graph density), more interference constraints are produced, thereby

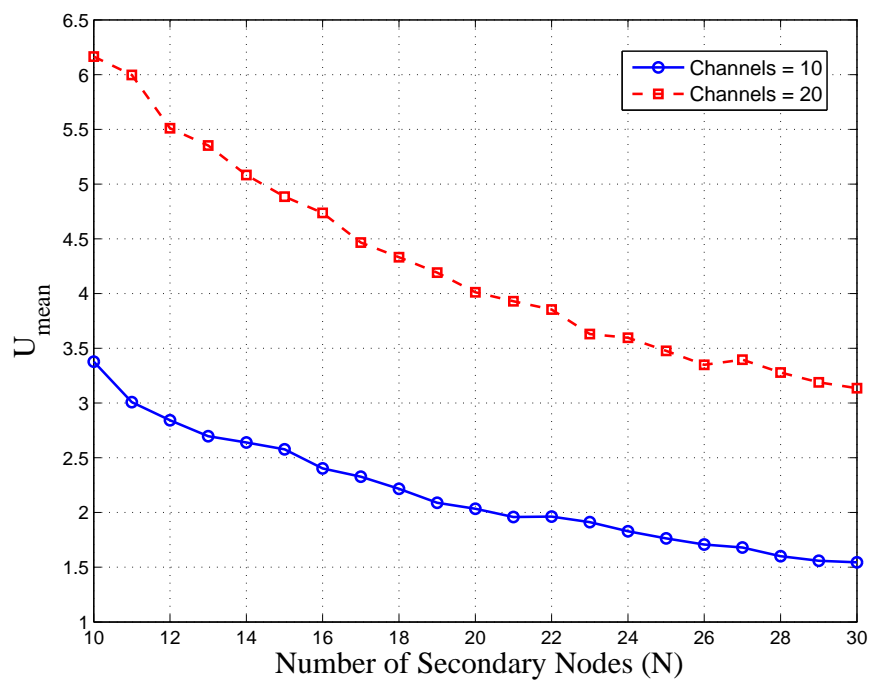


Figure 7.9: Mean utility with varying number of Secondary Users.

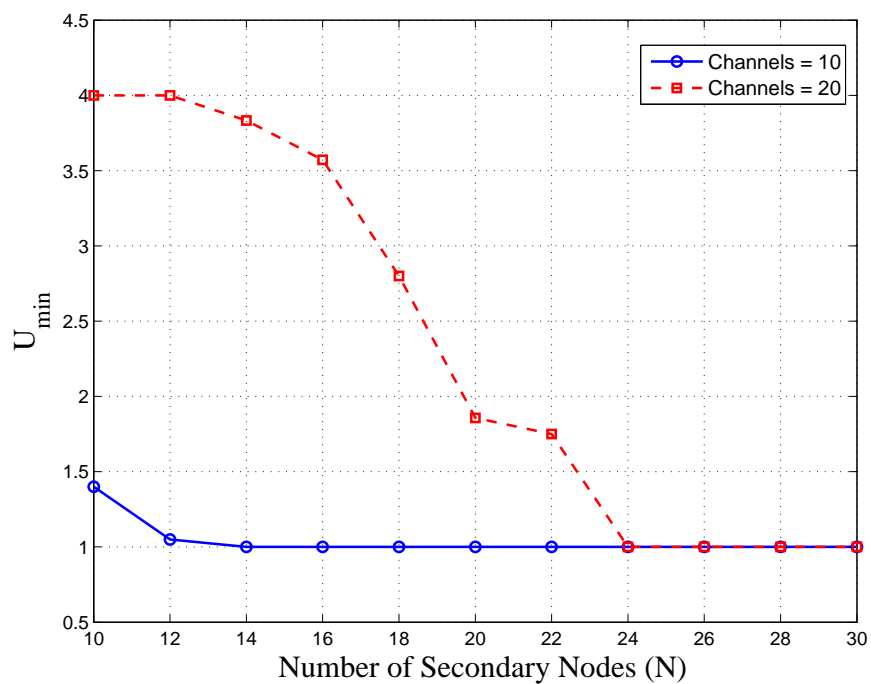


Figure 7.10: Minimum utility with varying number of Secondary Users.

increasing average vertex degree in the conflict graph. As average vertex degree increases, and M remains fixed, each node tends to get lesser number of channels. Thus, U_{fair} is inversely proportional to the number of secondary users, N . As expected, for a given N and graph density, when more channels become available, U_{fair} increases.

Figure 7.9 plots the mean number of channels received by a node, U_{mean} , with varying number of secondary users, N . U_{mean} degrades with increasing N , due to the increase in average vertex degree in the conflict graph. As M increases, for a given N and graph density, each node on an average gets more channels. Note that, the rate of decrease of U_{mean} reduces with increasing N , i.e., U_{mean} tends to saturate around a minimum value for large N . This behavior can be more pronouncedly seen when the number of channels available is 10.

Figure 7.10 shows how the minimum number of channels received by any node, U_{min} , degrades with increasing N . As can be seen, U_{min} never falls below 1 (because of reasons explained for Figure 7.7) for any number of secondary users.

7.2.4 Impact of Graph Density

As noted earlier, conflict graph density depends on the transmission power (and hence interference range) of the secondary users. Higher the transmit power of nodes, larger will be their interference range and thus larger will be the number of interference constraints. Hence, graph density of the conflict graph will increase. Higher transmit power leads to improved spectrum utilization for secondary users who are distant from primary users. However, this

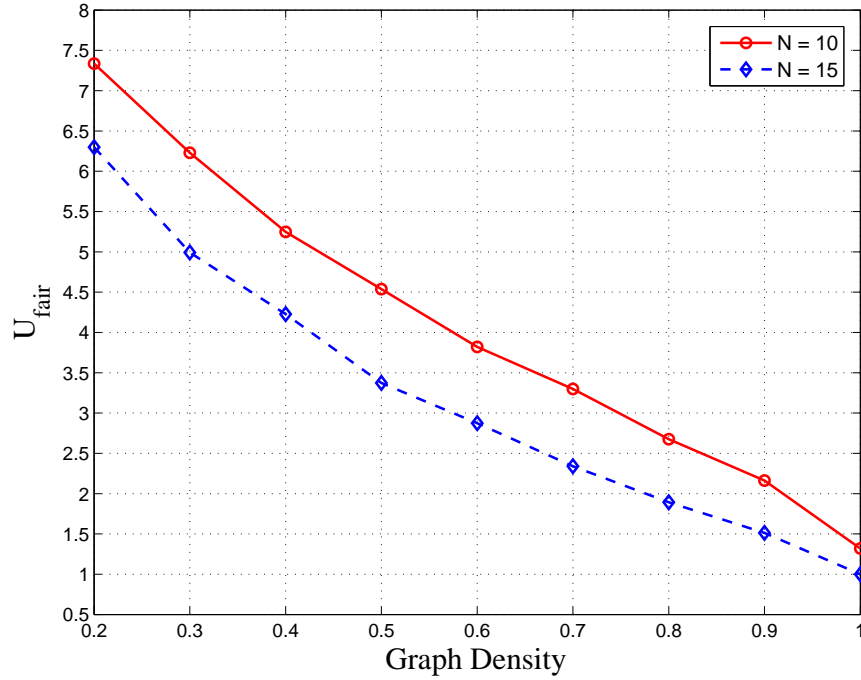


Figure 7.11: Fairness with varying conflict graph density.

also leads to additional interference constraints and reduced possibility of spectrum sharing. Hence, there exists a tradeoff between improving spectrum utilization and degrading spectrum sharing. We will see how conflict graph density (or in effect the transmission power of nodes) affect spectrum sharing by the secondary users by studying the three system utilities.

Figures 7.11, 7.12 and 7.13 show how the three utilities degrade as conflict graph density increases. The number of channels available, M , was set to 15 and the discount factor of all players in 0.8. From Figure 7.11 it can be clearly seen that U_{fair} degrades with increasing graph density for a fixed number of secondary users, N . This is again because the average vertex degree of the conflict graph increases with increasing graph density. For a given graph density, as N decreases, average vertex degree decreases, and thus U_{fair} increases for fixed

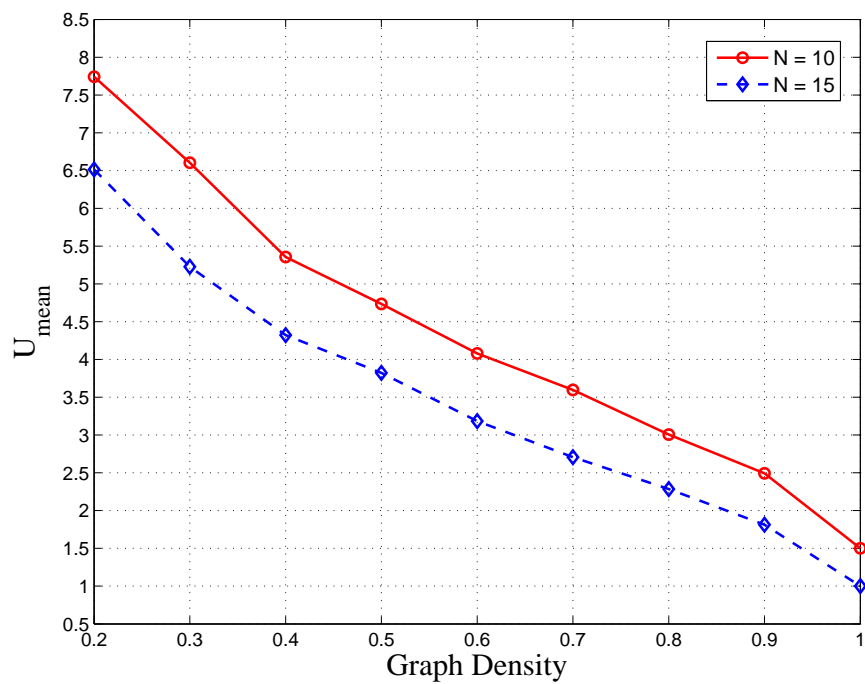


Figure 7.12: Mean utility with varying conflict graph density.

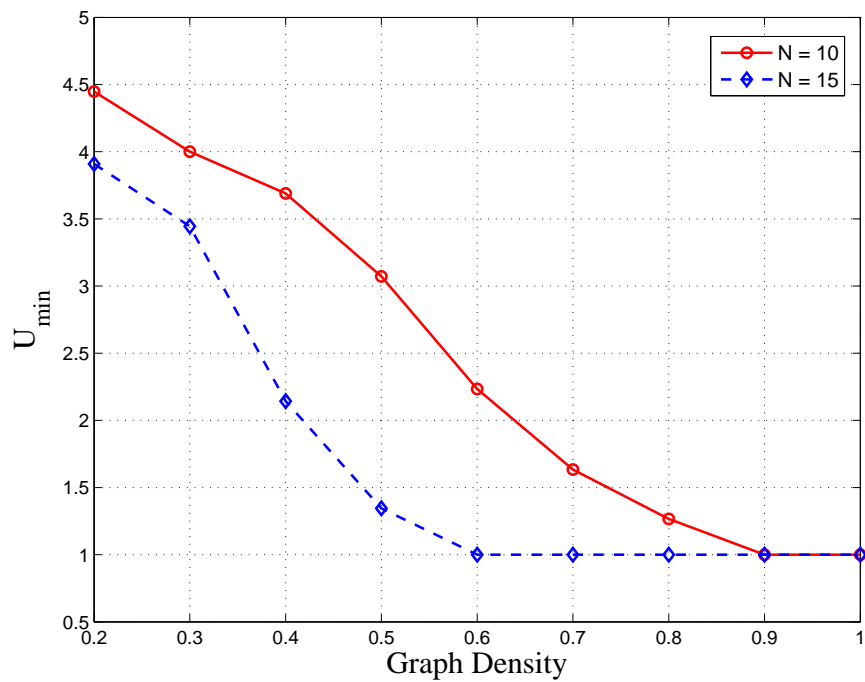


Figure 7.13: Minimum utility with varying conflict graph density.

M . Note that, when graph density is 1, we have a complete graph on N vertices, i.e, all nodes are within the interference range of each other. In this case, when N is 15, all nodes get a channel each (recall $M = 15$) and thus U_{fair} becomes 1.

Figure 7.12 plots U_{mean} with varying graph density. As can be seen, U_{mean} decreases with increasing graph density due to the increase in average vertex degree. Again, when N is 15 with a graph density of 1, U_{mean} becomes 1. Figure 7.13 shows the degradation of U_{min} with increasing graph density.

Note that the spectrum reusing capability of the network deteriorates more rapidly with increasing graph density (transmit power) for a fixed number of secondaries than it degrades with increasing number of secondary users for a fixed graph density. This can be seen by comparing the graphs showing the variation of U_{fair} and U_{mean} with graph density with their respective counterparts in Section 7.2.3.

7.2.5 Impact of Discount Factor

In Figure 7.14, the y -axis corresponds to the number of periods, T , of a finite horizon bargaining game of N players at which algorithm 6 finds $|(x_1^0, x_{-1}^0)_j^T| = |(x_1^0, x_{-1}^0)_j^{T-1}|, \forall j \in [1, N]$, i.e., the period T at which the SPNE strategy of P_1 converges, thus corresponding to his strategy in the infinite horizon game, $(x_1^0, x_{-1}^0)^\infty$. The x -axis corresponds to varying discount factor, δ , of the players. Conflict graph density has been taken to be 0.6 and the number of channels available, M , is 14. As can be seen from the figure, T scales inversely

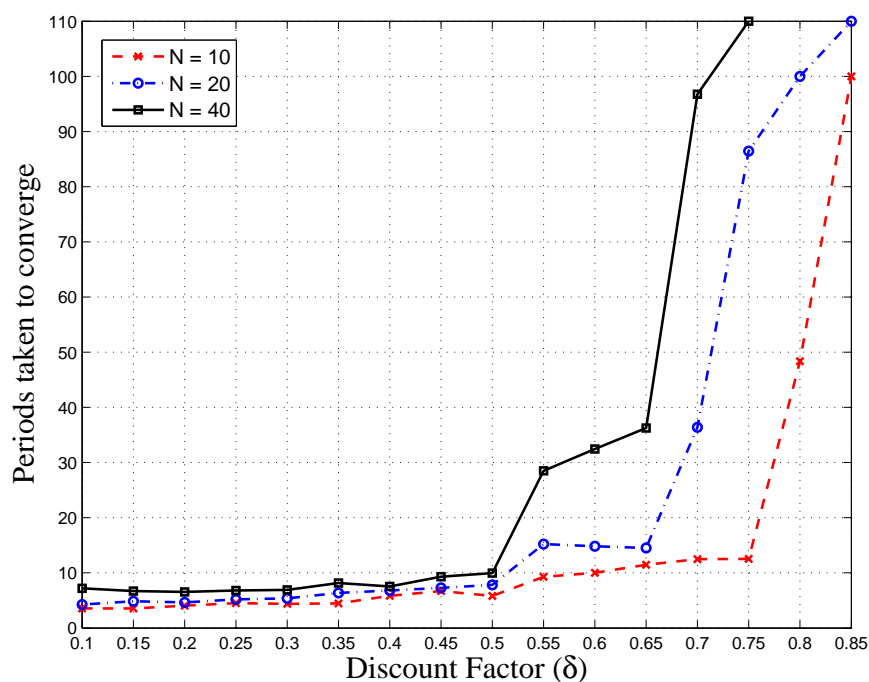


Figure 7.14: Convergence period T of first period offerer's SPNE strategy with varying discount factor of players.

with $1 - \delta$. This is because, as the discount factor of the players increases, the players become more *patient* in waiting for the bargaining outcome. When δ is 1, the players can wait infinitely long for the bargaining outcome. Moreover, it can also be noted that T increases as the ratio of $N : M$ increases. This can be clearly seen, since, T for any given discount factor is least for $\frac{N}{M} = \frac{10}{14}$, increases when $\frac{N}{M} = \frac{20}{14}$ and highest when $\frac{N}{M} = \frac{40}{14}$. These observations corroborate (5.9).

7.3 Incentive Based Routing

We will now evaluate the performance of the incentive based routing mechanism proposed in Chapter 6 via simulations. We will primarily focus on the effectiveness of our proposed algorithms in saving payment, since it is the central design criteria of our path auction mechanism. For the purpose of simulations, nodes (in N) have been randomly placed in a 1000×1000 m^2 area, with the source s and destination d being placed near two opposite corners of the square. In the simulations we assume that m_j and c_j are independent random variables, i.e., $f_j^{MC} = f_j^M \cdot f_j^C$. For each node $j \in N$, its cost m_j has been assumed to be uniformly distributed over $[1, 4]$ and its capacity c_j has been assumed to be uniformly distributed over $[2, 7]$. All results have been averaged over 1000 independent runs.

We will first compare the performance of our algorithm with LCP routing (refer Section 2.3.3.1). However, since LCP routing has been developed only for the single path routing scenario, we will use our algorithms for the same from Section 6.3 for the purpose of comparison. Moreover, in LCP the type of a node corresponds to the per packet cost it owes to its primary spectrum holder for transmitting a packet over its spectrum. Thus, we consider that in LCP routing a node declares its type to the routing mechanism as t_j^p . In Section 7.3.2 we will delve into the dynamics of the generalized (multi-path) routing scenario.

7.3.1 Single Path Routing

In the single path routing scenario, it has been assumed that s injects data at 2 pkts/sec for all the experiments.

7.3.1.1 Varying Transmission Radius

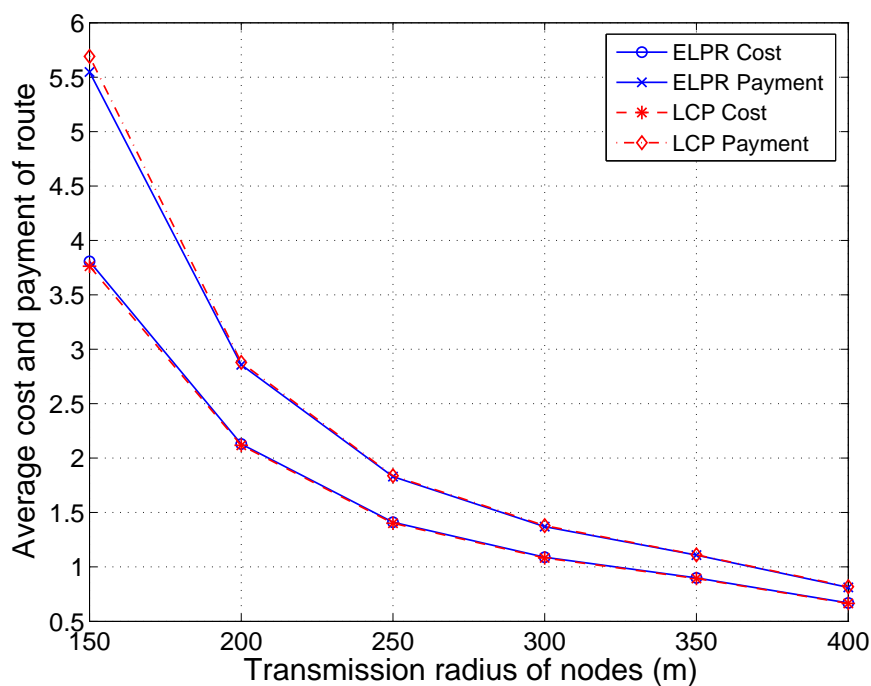


Figure 7.15: Variation of cost and payment with transmission radius of the nodes.

For Figures 7.15, 7.16 and 7.17, the number of nodes have been taken to be 200. Figure 7.15 shows how the cost of ELPR and LCP route and their corresponding payment decreases with increasing transmission radius of the nodes. The cost of a route refers to the summation of the costs *incurred* by the nodes on the route for each packet routed by the

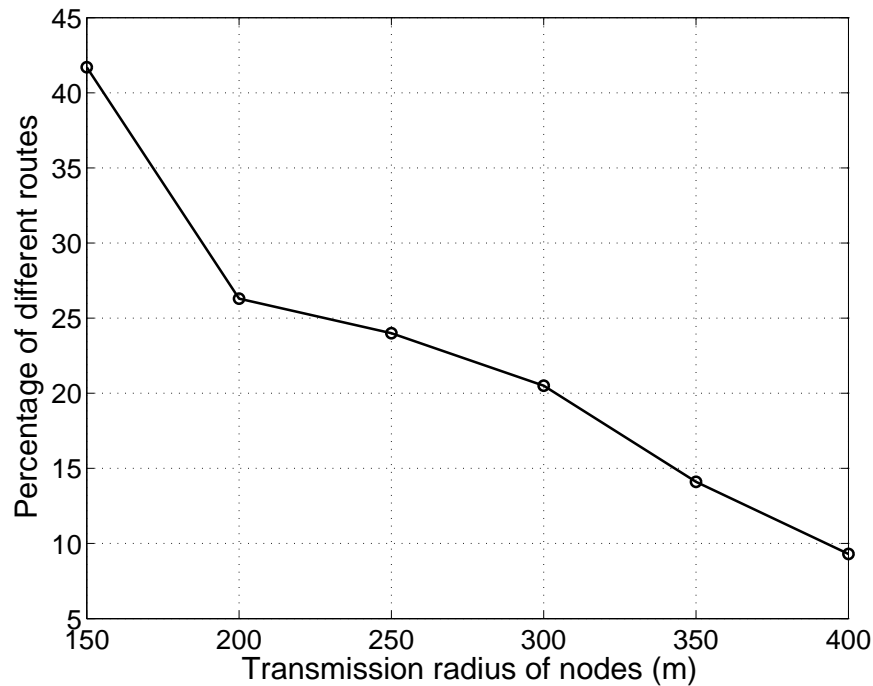


Figure 7.16: Percentage of routes for which ELPR and LCP are different.

network. Similarly, the payment of a route is the summation of the payments received by the nodes on the route (for each packet routed by the network). As the transmission radius of the nodes increase, the path length (hop count) tends to decrease, thereby resulting in paths of lower cost to which lesser payments has to be made. Note that, the cost of ELPR is only slightly larger than that of LCP. This implies that, as expected, ELPR is a route of relatively low cost, but not necessarily the route with the lowest cost. Further, the payment needed for ELPR is slightly less than that needed for LCP. This shows that *ELPR can enforce truthfulness on **both** cost and capacity reporting of a node while requiring a payment that is less than that required by LCP, which only ensures truthful cost reporting.* Note that the saving in payment by ELPR against LCP is more pronounced at lower transmission

radius. This is because the total savings over the intermediate hops become more and more significant as the number of hops increases.

Figure 7.16 shows the percentage of cases (in the 1000 independent runs) in which ELPR and LCP were different routes, with varying transmission radius of the nodes. As can be seen from the figure, the percentage of cases in which ELPR and LCP were different routes decrease with increasing transmission radius of the nodes. This implies that, as the transmission radius increases, or in other words as the path length decreases, the chances of the path with the minimum cost being also the path that minimizes the expected payment to be made per packet increases.

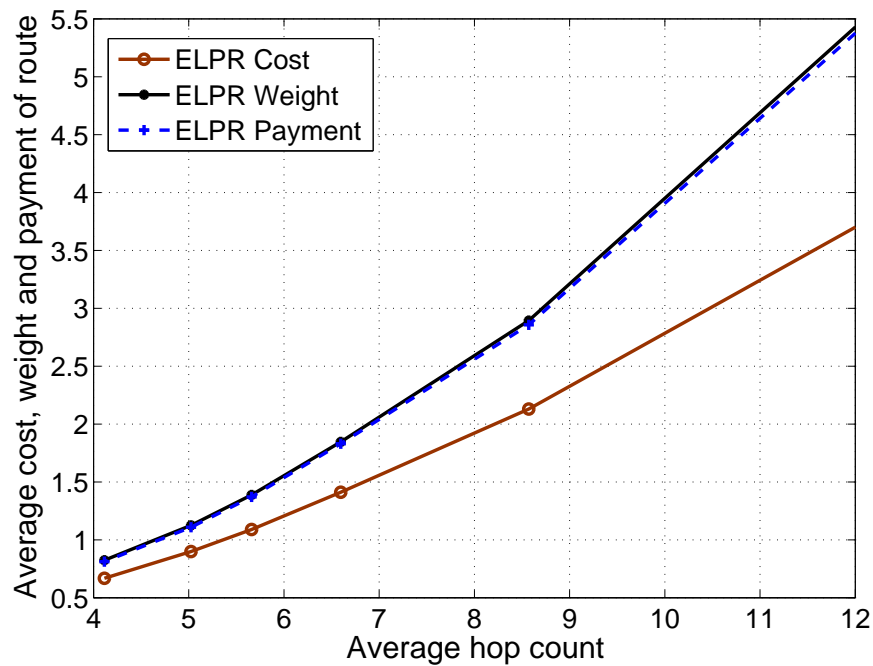


Figure 7.17: Variation of cost, weight and payment of ELPR with average path length.

In Figure 7.17 we plot how the cost, weight and payment of ELPR varies with average number of hops. The number of hops has been affected by varying the transmission radius of the nodes. Here, the weight of ELPR refers to its competence factor. As expected, all three factors increase with the number of hops.

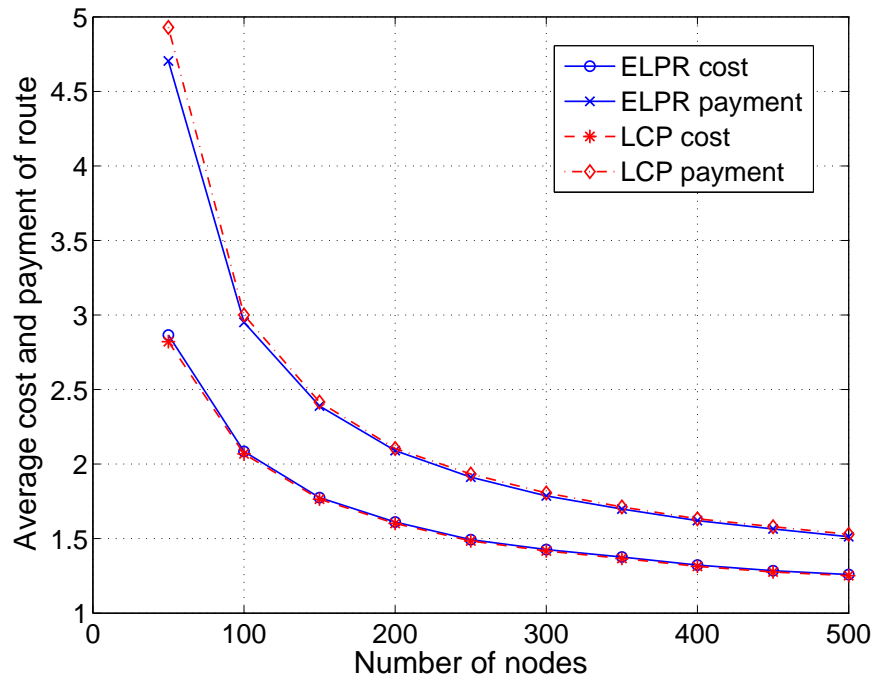


Figure 7.18: Variation of cost and payment with number of nodes in the network.

7.3.1.2 Varying number of nodes

Figure 7.18 shows how the cost of ELPR and LCP route and their corresponding payments decrease as the total number of nodes is increased. The transmission radius of a node was set at 250 m. As the number of nodes is increased, the number of alternative paths between sender and destination increases and thus both ELPR and LCP starts corresponding to

lower and lower cost routes. Accordingly, the payment needed for the routes also follows a decreasing trend. It can again be noted that the cost of ELPR is slightly *larger* than that of LCP, while the payment required for ELPR is slightly *lower* than that required by LCP. This fact indicates that there is a tradeoff between achieving social efficiency and minimizing the payment required for a route to maintain truthfulness. LCP attains the socially desirable solution by corresponding to the path having the minimum cost, while ELPR sacrifices a little social efficiency in order to minimize the payment to be made by the end-user. Note that the saving in payment by ELPR is more significant when the number of nodes is less. This is because of fewer alternative paths at lower number of nodes. When the number of alternative paths is less, the difference of cost (competence factor) between the best and the second best path for LCP (ELPR) tends to be higher than when the number of alternative paths is more. Now, since VCG does not have an upper limit for payment to a node, LCP requires a *more* higher payment than ELPR, when the number of alternative paths is relatively lesser.

7.3.2 Multi-path Routing

We now consider our path auctions in a generalized setting, i.e., s can inject data at any arbitrary rate, which would result in multiple path routing. To gain insight into the advantage of using ELPR we will again weigh it against LCP. However, as noted earlier, LCP routing can only be used in a network where every node has sufficient capacity to forward the sender's

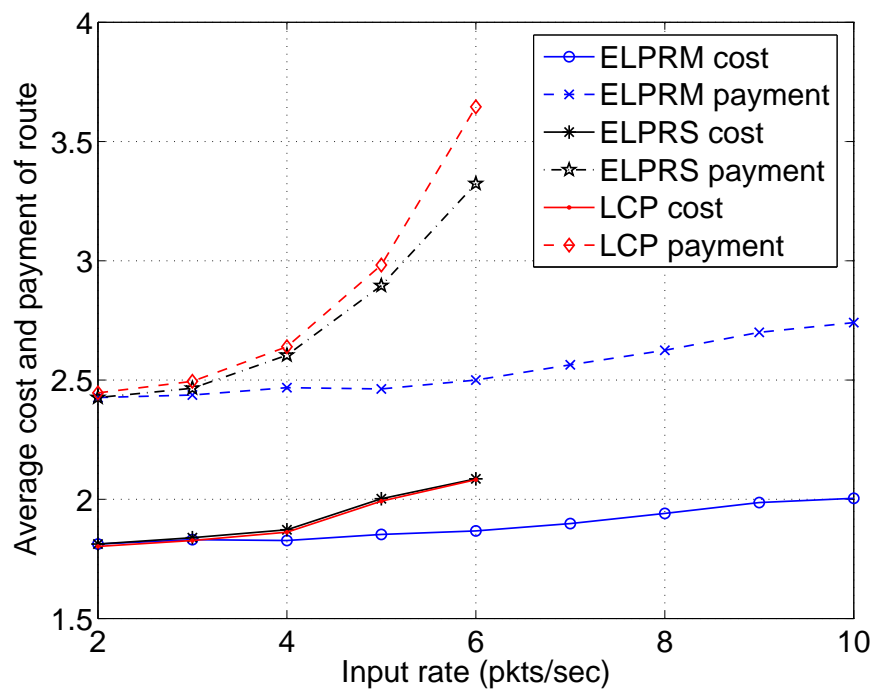


Figure 7.19: Variation of cost and payment with input rate R .

traffic. Thus, to use LCP routing we will disconnect each node j from the network at hand for which $c_j < R$. For completeness, we will also use ELPR routing in such a scenario (nodes with insufficient capacity removed). To distinguish between the cases of single path and multi-path routing for ELPR, we will refer to the former as ELPRS and to the latter as ELPRM.

7.3.2.1 Varying input rate R

Figure 7.19 shows how the cost of ELPRM, ELPRS and LCP route and their corresponding payments increase with increasing data input rate R . The number of nodes was taken to be

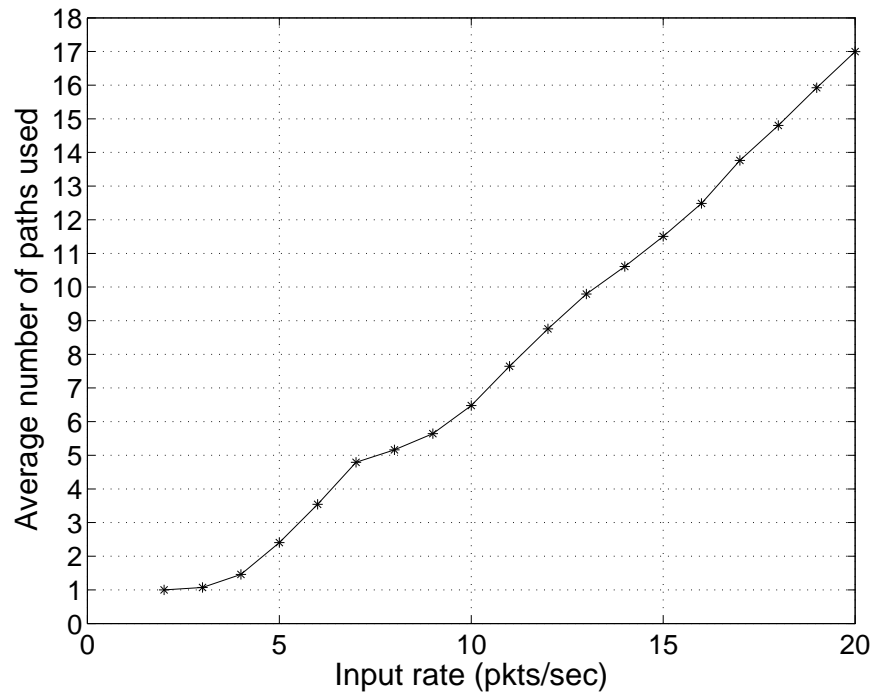


Figure 7.20: Average number of paths used for different input rates.

150 and the transmission radius of the nodes was set at 250 m. As can be clearly seen from the figure, the cost and the payment of ELPRM is considerably lesser than the other two. Also, note that, after disconnecting nodes with insufficient capacity, the network was unable to accommodate flows demanding a rate greater than 6 pkts/sec. This emphasizes the fact that ELPRM can support higher data rate applications by distributing the input load over multiple paths. It is also worth noting that the increase in cost and payment of ELPRM with R is much more gradual than those of ELPRS and LCP.

Intuitively, as the input rate R is increased, ELPRM would start distributing the load over more number of paths. This is shown in Figure 7.20, which plots the average number

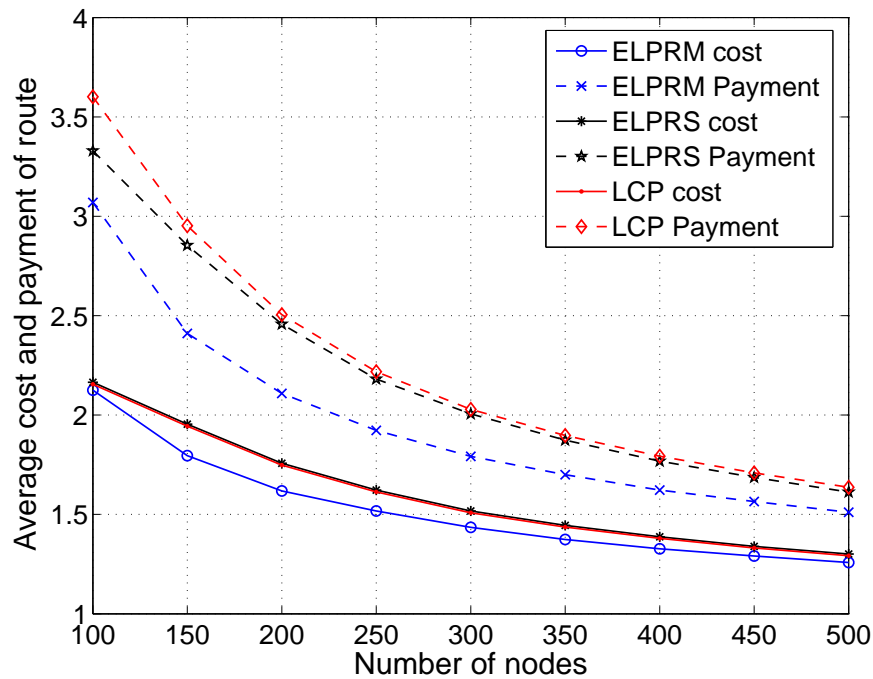


Figure 7.21: Variation of cost and payment with number of nodes in the network.

of paths used by ELPRM (over the 1000 runs) with varying R . As can be seen from the figure, the number of paths used by ELPRM increases with the input rate.

7.3.2.2 Varying number of nodes

Figure 7.21 shows how the cost of ELPRM, ELPRS and LCP route and their corresponding payments decrease with increasing number of nodes. The transmission radius of the nodes was set at 250 m with s sending data at 5 pkts/sec. The decrease in cost of the three different types of routes and their corresponding payment can be attributed to the increase in number of alternative paths as the number of nodes increases. Clearly, the figure suggests that the

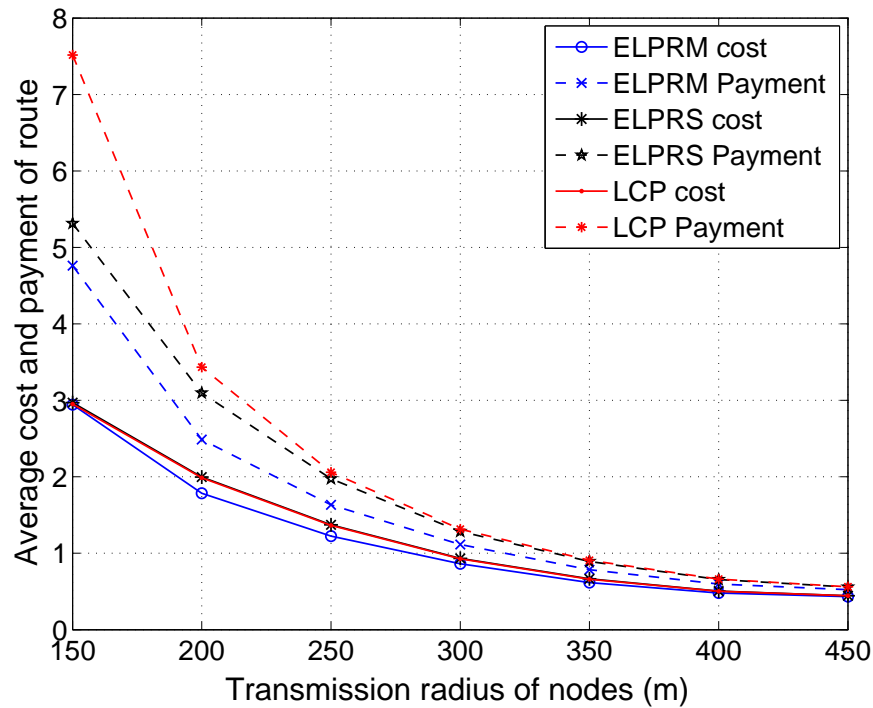


Figure 7.22: Variation of cost and payment with transmission radius of nodes.

cost and payment of ELPRM fare best among the three. Also, the saving in payment by ELPRM is more pronounced at lower number of nodes which is in line with our observation and reasoning for Figure 7.18. Note that, the decreasing nature of the cost of the three different types of routes and their associated payments tend to saturate when the number of nodes becomes high, or in other words as the number of alternative paths become large. Thus, though increase in competition (with increase in number of alternative paths) tends to bring down both the cost and required payment (for all three type of routes) the net utility of the end-user tends to level off when the number of competitors is considerably large— a phenomenon typically seen in a market with perfect competition.

7.3.2.3 Varying transmission radius

Figure 7.22 shows how the cost of ELPRM, ELPRS and LCP route and their corresponding payments decrease with increasing transmission radius of the nodes. The number of nodes was taken to be 150 with the source sending data at 5 pkts/sec. As can be noted from the figure, ELPRM again fares best among the three. The saving in payment by ELPR (for both single and multi path cases) is more pronounced at lower transmission radius, which corroborates our observation in Figure 7.15.

CHAPTER 8

CONCLUSIONS

In this dissertation, we first addressed the problem of dynamic channel access by a set of cognitive radio enabled nodes, where each node acting in a selfish manner tries to access and use as many channels as possible, subject to the interference constraints. We modeled this problem as an infinite horizon bargaining game where the nodes bargain among themselves regarding their share of channels. First, we explored the finite horizon version of the bargaining game and presented polynomial time algorithms to find the Pareto optimal SPNE strategy of the player making the offer in the first period of the game. This is a strategy, such that, neither can the player making the offer increase his utility by making any other offer, nor can the players receiving the offer gain in any subsequent period by rejecting this offer. Next, we extended the results from the finite horizon game to find the Pareto optimal SPNE strategies of the infinite horizon game. Using simulations we studied how the selfish strategies of the players affect spectrum usage from a system wide perspective. As suggested by theory as well as simulations, in our bargaining solution, no player gets starved of channels. Furthermore, since in the bargaining model nodes negotiate among themselves, no centralized controller is needed.

Secondly, in this dissertation, we also studied the implications of the price the secondary nodes have to pay to primary spectrum owners for acquiring channels. To forward traffic, each secondary node would incur a cost because of using the spectrum it has acquired by paying a price. Moreover, using the acquired spectrum, each node can sustain a certain capacity. Clearly, secondary nodes will not have an incentive to relay traffic unless they are compensated for the costs they incur in forwarding traffic. Also, for a routing mechanism to be stable, the amount of traffic that a node forwards should not exceed its capacity. The problem of routing is further complicated by the fact that each node can potentially improve its utility by dishonestly reporting its type to the routing mechanism. In the absence of techniques to induce nodes for truthfully reporting their cost and capacity to the routing mechanism, there may be an unnecessary overpayment of incentives provided to the nodes for routing traffic. We designed a path auction based routing scheme for DSA networks that can enforce honest revealing of cost and capacity to be a dominant strategy for every node in the path auction game. Furthermore, our mechanism also minimizes the payment that needs to be given to the nodes that forward traffic, making our scheme attractive to the end-user. By considering capacity constraint of the nodes we explicitly support multiple path routing. For deploying our routing mechanism in DSA networks, we present polynomial time algorithms for computing the optimal route over which to route traffic and the payment to be received by each node. Simulation results suggest that our path auction mechanism can ensure truthfulness on both cost and capacity while making a payment less than that required for VCG based LCP routing.

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