Study of High-Order Discontinuous Galerkin Methods for High-Fidelity Aeroacoustic Simulations

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論文内容要約

※題名については，「要約」として下さい。

Rapid growth of air traffic leads to more severe restrictions of aircraft noise around airports, and technologies for reducing the aircraft noise are highly required. Particularly, reduction of the airframe noise is now of great interest in aeronautical engineering areas because its relative contributions become large since jet noise has been substantially reduced. Due to the combination of complexities of the airframe geometry and unsteady turbulent flows, prediction and reduction of the airframe noise are challenging. The experimental measurements and investigations can be first candidate as the tools to investigate and reduce the airframe noise, but there are several problems in the experimental measurements of airframe noise such as the reflections of propagated noise at wind-tunnel walls. The numerical approaches based on the computational fluid dynamics (CFD) can be expected to be a powerful tool for the airframe noise investigations because the unfavorable effects can be eliminated.

In order to accurately predict the airframe noise using CFD, there are two required issues; the accurate predictions of complex flows such as the laminar-to-turbulent transition, the turbulent boundary separation, and its reattachment and the geometrical flexibility for the complicated airframe geometry. For the accurate predictions of turbulent flows, large-eddy simulation (LES) is an attractive numerical approach. In LES, high-order numerical methods that can perform on the regular structured meshes are widely used. However, due to the complexity of the airframe geometry, the structured meshes are unfavorable because mesh generation requires considerable human efforts. On the other hand, the prevailing methodology for handling the complicated geometries is unstructured meshes, but there is a serious problem that the spatial accuracy of the conventional unstructured mesh methods is usually at most of 2nd-order. This 2nd-order accurate method requires a high mesh density in LES to resolve large-scale motions containing most of the energy. Particularly, the wall-bounded turbulent flows associated with high Reynolds number have wide range of energy spectrum, and thus the computational cost of 2nd-order based LES for this kind of flow becomes prohibitive even with

the state-of-the-art computers. Based on these observations, high-order unstructured mesh methods are required for the investigations of airframe noise.

The discontinuous Galerkin (DG) methods are well known to achieve high-order spatial accuracy even on the unstructured meshes. In the DG method, basis functions and degrees of freedom (DOFs) are independently introduced in each computational cell, and thus this method is compact in the sense that information in nearby cells are not required in reconstructions of variables. This compactness provides not only the high-order spatial accuracy but also several advantages such as the high parallel efficiency, straightforward implementation of multigrid techniques, convenient application of boundary conditions, and so on. The high-order spatial accuracy is favorable for LES of the airframe noise, but the literature extending the high-order DG method to such simulations are still few. There are two problems to be solved to realize LES of the airframe noise using the high-order DG methods. Firstly, a required grid spacings for LES of wall-bounded turbulent flows has not been fully clarified in the DG framework. The wall-bounded flows are important physics for the airframe noise, and thus this requirement has to be clarified to realize accurate simulations. Secondly, the implicit time integration requires high computational cost, although this approach is preferred in the simulations of wall-bounded turbulent flows. Particularly, for the high-order DG methods, the computational cost of the implicit time integrations is high because the size of matrix is large due to the introduction of the DOFs. The main objective of this study is to realize accurate simulations of the airframe noise by solving those two problems and developing the high-order implicit DG methods.

In the first part of this thesis, wall-resolved LES of the turbulent channel flow is presented to clarify the grid resolution of the high-order DG methods for wall-bounded turbulent flows. Although there are several guidelines of grid spacings for wall-resolved LES, the required grid parameters based on the degrees of freedom (DOFs) have been still uncleared in the DG framework, and this study attempts to clarify those requirements for the 3rd- and 4th-order DG methods. Firstly, it is demonstrated that the higher-order DG method gives better resolved turbulent flows with the same number of DOFs by comparing the obtained flow statistics of mean velocity, velocity fluctuation, and Reynolds shear stress with the DNS solutions by Moser et al. (*Physics of Fluids*, 1999). Furthermore, comparing the present 2nd-order DG case with the other 2nd-order central difference case, the upwind Riemann solver is shown to give excessive numerical dissipation. It is also found that the high-order DG methods allow us to employ much larger wall-normal minimum grid spacing than the conventional guideline ($y^+_{min} \leq 1$). Finally, the grid resolutions in streamwise and spanwise directions are shown, and it is demonstrated that the required number of DOFs and computational cost for accurate wall-bounded turbulent flows can be reduced for high-order DG methods.

Next, the fast implicit time integration method is explored for the high-order DG methods. The high computational cost in the implicit time integration comes from the constructions of the large matrix and solving the resulted large linear system. This study focuses on the former, especially the quadratures, and proposes the simple and effective quadrature simplification method to realize the fast implicit DG methods for solving Navier-Stokes equations. In the proposed method, flux Jacobians are assumed to be constant in each cell so as to utilize orthogonal properties of basis functions. This assumption allows the quadratures to be simplified, drastically reducing their computational cost. Based on those favorable features, this method is named quadrature simplification by orthogonality (QSO). Naturally, there is a concern that the assumption of constant flux Jacobians may affect the computation of flowfields including steep gradients. Therefore, the basic performances of QSO are first assessed through problems including shock waves and boundary layers using two-dimensional regular meshes. It is shown that QSO gives substantial speedup in the part of the implicit time integration on stable computations without any deterioration in convergence properties and numerical solutions. The obtained speedup ratios are roughly same as the analytical estimations and increase when three-dimensional computational meshes and higher-order DG methods are employed. Moreover, the required memory storage in QSO is also estimated, and the feasibility of QSO for huge number of computational cells and very high-order DG cases is ensured. It is also shown that QSO works well on 3D irregular unstructured meshes, and unsteady flow simulations are also successfully carried out using QSO without deteriorations of the time accuracy, which indicates that QSO can be applied to LES of complex turbulent flows. Finally, typical aerospace problems such as the vortical flowfield around a delta-wing and aerodynamics predictions of the NASA common research model are computed by the high-order implicit DG methods with QSO to demonstrate the capability of the developed methods. In those simulations, good agreements with experimental data are obtained with reasonable computational cost.

To realize the implicit time integration with high performances on massive parallel computations, the efficient numerical method for the large linear system in the implicit time integration is next explored. In the DG framework, the generalized minimal residual (GMRES) linear system solver is widely employed because of its fast and stable properties. The GMRES approach, however, requires sweeps referring to up-to-date solutions of nearby cells, which is unfavorably against the compactness of the DG method itself. On the other hand, the cellwise relaxation implicit (CRI) scheme proposed by Yasue et al. (*Communications in Computational Physics*, 2010) solves the linear system without any sweeps by ignoring off-diagonal components, but it is highly possible that those neglects cause large errors in time evolutions of unsteady turbulent flows. Based on these observations, this study extends the block Jacobi (BJ) scheme to ensure both the time accuracy and the cellwise features. The BJ scheme, which is one of the classical linear system solvers, solves the large linear system with referring to previously obtained solutions, not up-to-date solutions, in nearby cells. The sweeps are thus not required, which is well suited with the compactness of the DG methods, with maintaining the time accuracy by considering the off-diagonal components. The performances of the BJ scheme are assessed through the canonical unsteady flow problems such as the isentropic vortex advection, and it is well shown that the robustness and time accuracy of this scheme are ensured. Furthermore, we find an advantage of the BJ-based implicit time integration that the number of sub-iterations to ensure accurate time evolutions can be small for the high-order DG cases. This interesting advantage comes from the fact that the high-order scheme gives the well-resolved continuous flowfield which is supposed in the Taylor expansion used in the flux linearization. The small number of sub-iterations provides fast computations due to a small number of right-hand-side computations. The developed BJ-based implicit DG method is finally extended to the turbulent channel flow to demonstrate the capabilities for unsteady turbulent flows. Compared with an explicit time integration, the BJ scheme gives 2.5 times faster computation while maintaining not only the time accuracy but also the simplicity and cellwise feature.

Finally, high-fidelity aeroacoustics simulations for the three-element airfoil are presented using LES with the developed high-order implicit DG methods. The developed high-order implicit DG methods are first validated in terms of the aeroacoustics simulations through the problems of sound scattering and flow-generated sound around a cylinder. Through the former problem, it is found that the required grid spacing for the sound propagation is 3 cells per sound wave length. In the later validation, the computed mean velocity and velocity fluctuations profiles are compared with the solutions by 6th-order compact differential scheme reported by Khalighi et al. (*AIAA Journal*, 2010), and good agreements are obtained. The same trends of the velocity and sound spectrum are also obtained, and the capability of the high-order DG methods with LES for aeroacoustics simulations is well demonstrated. Finally, the developed high-order implicit DG method is applied to the aeroacoustics simulations of the high-lift device. The instantaneous characteristic flows such as the turbulent shear layer from the leading edges, its impingement, and laminar-to-turbulent transition and characteristic aeroacoustics generated by such flows are well simulated by employing the developed solver. Although the computed turbulent flows are relatively two dimensional due to the narrow spanwise domain size, it is shown that the trends of the flow statistics are reasonably predicted. To obtain the quantitative fidelity, the computation with the wider spanwise domain size than the present case is required. The feasibility of such computation is finally estimated and it is found that it can be realized using the state-of-the-art supercomputer such as the K computer with reasonable computer resources. We would like to emphasize that this feasibility is achieved by the numerical method for implicit time integration such as the QSO and BJ scheme developed in this study.