The search for a dark sector muonic gauge boson $Z'$ at the Belle experiment.

| 著者 | ハンス・ラベル ラベル |
| 学位授与機関 | 東北大学 |
| 学位授与番号 | 東北大学第 1676 号 |
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The search for a dark sector muonic gauge boson $Z'$ at the Belle experiment

Belle実験における
（ダークセクタームニュオンッゲージ粒子$Z'$の探索）

Thomas Rafael Czank
トマス らファエル キザンキ

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Graduate School of Science, Tohoku University
Department of Physics, Experimental Particle Physics Group
東北大学理学物理学研究科素粒子物理実験グループ
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Chapter 1

Introduction

1.1 Dark Matter Origins

Even before the discovery of the Higgs Boson in 2012 there were and there are still unanswered questions by the Standard Model (SM) of Physics. Some of these questions are a result from cosmological experimental observations, such as: the rotation speed of spiral galaxies, where the observed rotation speed of galaxies was found to be greater than what is expected from the distribution of its visible components mass. This observation provides indirect evidence for the existence of invisible matter, dark matter.

As one of the original motivations for dark matter models, the Galaxy rotation velocity problem was: the velocity of visible astronomical objects farther from their Galaxy center was expected to be smaller than the velocity of those close to the Galaxy center, since the amount of visible objects at edges of galaxies is also smaller than at the center, but the observed result was different. This can be seen in Fig 1-1 in which three differently shaped galaxies were studied, bulge, disk and gas dwarf.

From a historical perspective dark matter was initially proposed by Fritz Zwicky when by using the virial theorem, \( \langle T \rangle = -\langle V \rangle / 2 \), he calculated the gravitational mass from the Coma galaxy cluster. It ended up being at least 400 times greater than
CHAPTER 1. INTRODUCTION

Figure 1-1: Black dots are the observed rotation curves, dark green dotted lines are gas components, red dashed lines are stellar disk, violet dot and dashed lines are the Bulge and the blue continuous line is the visible components together [2].

The mass inferable from the luminosity. This meant that most of the galaxy cluster mass did not come up from its luminous material, but actually from unseen matter, which he called dark matter.

The dark matter content of the universe was measured indirectly by the Wilkinson Microwave Anisotropy Probe (WMAP), which mapped the Cosmic Microwave Background (CMB) radiation accross the universe [4]. CMB photons are scattered across the universe in a very uniform way. The WMAP study, measured perturbations in the CMB distribution. A temperature difference distribution dependent on the angular measurement position. These results are related to the dark matter distribution, since it causes gravitational distortions, Gravitational lensing. These effects are visible in other observations [5]. Such gravitational evidences for dark matter across a wide range of cosmic scales and at many different epochs in cosmic history is a way to obtain the energy budget of the universe at different time ranges. For example the energy budget for the pre BBN (Big Bang Nucleosynthesis) epoch.

The WMAP data is very well fit by a universe dominated by dark energy, 72%, cold dark matter, 23% and 4.6% of baryonic matter in the present day, as in figure 1-2.
1.1. DARK MATTER ORIGINS

Figure 1-2: Total contents of the Universe when it cleared up and photons start to propagate freely (bottom), present (top) [6].

The most recent investigation which also supports the existence of dark matter, was the discovery of a galaxy without dark matter [7], that is, all of its mass corresponds to the luminal components of the galaxy, which paradoxically, would support dark matter as the appropriate cause for the astronomical observations mentioned previously. This is because if there are galaxies that contain dark matter (extra invisible mass) and those which do not, a correction of the physics involved to explain the effects measured is less likely than the existence of some new and yet unknown
These evidences led to the formulation of one of the most popular models for dark matter, the WIMP (Weakly Interacting Massive Particle) [8], its popularity is due to:

- No WIMP candidates in SM, a Beyond the Standard Model (BSM) Physics candidate

- Assuming mass and coupling constant of the weak scale, it naturally leads to the correct relic density (the ‘WIMP miracle’)

- Prediction of signals that may be seen in the current and near-future experiments

In the early Universe, lower part of Fig 1-2 corresponding to the contents of the universe when there was thermal production. It is assumed that WIMPs were produced in collisions between particles in the hot primordial soup (thermal production) during the radiation-dominated era. Production and annihilation of WIMP pairs, $\chi \bar{\chi}$ in particle and antiparticle collisions,

$$\chi \bar{\chi} \leftrightarrow e^+ e^-, \mu^+ \mu^-, q \bar{q}, W^+ W^-, ZZ, HH, ...$$

for temperatures much higher than the WIMP mass, $T \gg m_\chi$, the WIMP mass should be near the Weak Scale (100 GeV to 1 TeV). The colliding particle and antiparticle pairs in the plasma could create WIMP pairs, as well as the inverse reaction, where WIMPs would annihilate into SM particles.

As the Universe expanded, the plasma temperature decreased, getting smaller than the WIMP mass. Annihilation and production reactions were still in equilibrium but the number of WIMPs produced decreased exponentially until they were no longer produced, freeze-out, higher part of Fig 1-2. This is when the WIMP abundance reaches its final value, the thermal relic density.
1.1. DARK MATTER ORIGINS

After freeze-out, there is a constant number of WIMPs in a volume expanding with the universe. WIMP models could naturally explain the dark matter density in the universe, the so called WIMP miracle.

No WIMP signals have been found, and some mass ranges have been ruled out.

Figure 1-3: Plot with many WIMP search experiments and other conditions that constrain the WIMP mass ranges [9].

In Fig 1-3 the lower shaded region in light yellow under the dashed orange line, WIMP signals are ruled out due to the Neutrino Coherent Scattering of Atmospheric and Diffuse Supernova Neutrinos Background (DSNB) that can cause neutrino-induced recoil events which would lower the chance of detecting WIMP induced recoil events [10]. Apart from this region there is also a light red shaded circle that is covered by Minimal Super Symmetric Models (MSSMs), a light blue oval shaded region due
to extra dimensional models, a light green oval shaded region due to the Asymmetric dark matter (DM) models and finally a light violet oval shaded region due to Magnetic DM models. With all this restrictions the upper left region still has very few constraints, this region of mass lower than 10 GeV is probable by high intensity luminosity experiments, such as Belle.

Apart from the previously mentioned evidences for dark matter, there are also some SM fine measurements anomalies that have not yet been resolved, such as the proton size anomaly [11] and the $g-2$, muon magnetic moment anomaly [12]. Besides these inconsistencies from the SM, the absence of direct detection for WIMPs [9] in any of its models for dark matter, remains.

Observing no WIMPs of any models stimulated the development of Dark Sector Models (DSMs). They propose dark (secluded) particles, not in SM, which are neutral under SM force carriers, but charged under Dark Forces. These new dark particles could be in the mass range between 100 MeV and 10 GeV, which would be in the range that could fix the $g-2$ anomaly, the proton size anomaly or the positron fraction increase with cosmic ray energy. These shortcomings will be explained further in the next sections. The energy/mass range of these phenomena is exactly where Belle/BABAR/ Belle II are sensitive.

1.2 The Dark Sector

Positron Fraction in Cosmic Rays

One of the strongest pieces of evidence that points out to models of a Dark Sector is the observation of an increasing positron fraction in cosmic rays composed of electrons and positrons with increasing energy. This was observed at cosmic ray detection experiments in Fig [1-4].

This positron fraction increase with the cosmic rays energy is not expected from the known sources surrounding Earth. Therefore, explanations for the positron frac-
1.2. THE DARK SECTOR

Figure 1-4: Plot with different experiments, AMS-02 $^{[13]}$, PAMELA $^{[14]}$ and Fermi $^{[15]}$ measurements of the positron fraction increase with Cosmic Rays energy $^{[13]}$. The known sources of positrons prediction corresponds to the gray band.

- Positron fraction
- AMS-02
- PAMELA
- Fermi

The positron fraction increase based on the annihilation of dark matter particles producing Dark Photons, $A'$. Where $A'$ is a secluded $U(1)_D$ boson, that is a new gauge boson that barely couples to SM visible particles $^{[16]}$. This secluded boson would then mix kinematically with the SM vector boson, $\gamma$, finally decaying into SM particles, such as leptons.

Regarding the range $> 10$GeV, this positron fraction increase could be related to the heavy neutralino $^{[17]}$ that could decay to leptons at even higher energies. As men-
tioned earlier many flavor experiments are highly sensitive to the dark photon in this region, $1\text{ MeV} \sim 10\text{ GeV}$.

The Anomalous Magnetic Moment of the Muon

Radiative corrections contributions to electron and muon phenomena attributable to self-energy and vacuum polarization have long prevented conciliation between SM and experimental measurements \[18]\.

The magnetic moment of the muon is given by

$$\mu = g \frac{e}{2m} S,$$  \hspace{1cm} (1.2)

where $g$ is the $g$-factor, the dimensionless magnetic moment, predicted by Dirac’s equation to be 2, $e$ is the electron charge, $m$ is the muon mass and $S$ is the spin of the muon, the Feynman diagram corresponding to this calculation is in Fig \[1-5]\.

The anomalous magnetic moment of the muon, $(g - 2)\mu$, measurement precision has increased continuously \[19], \[20], and the same is true for its calculations considering ever more detailed QED contributions \[21], \[22], \[23] and \[24]. However, the discrepancy remains

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{theor}} = (28.8 \pm 8.0) \times 10^{-10},$$  \hspace{1cm} (1.3)

this amounts to $3.6\sigma$ outside of the SM prediction where $a = (g - 2)/2$.

This difference will be further explored by future experiments \[25] and \[26]. The simplest explanation for the $(g - 2)\mu$ discrepancy caused by a dark gauge boson coupling would be the absence of such coupling between the dark gauge boson and other leptons or other particles. In that case the muon would receive a small contribution from the dark gauge boson to its magnetic moment, as visible on the right diagram in Fig \[2-4].
1.2. THE DARK SECTOR

The Proton Size Anomaly

Analogously to the Muon Anomalous Magnetic Moment the proton radius measurement has a discrepancy not accounted for by the SM and is also related to the muon possible coupling to a hidden sector.

\[ \Delta r_p \equiv r_p^H - r_p^{\text{muonic}H} = 0.03496(67)\text{fm}. \]  \hfill (1.4)

The increased accuracy measurement of the proton size was done by using a muonic Hydrogen atom \cite{11}, a Hydrogen atom in which its single electron was replaced by a muon, which is 200 times heavier than the electron. Reducing the Bohr radius and enhancing effects such as the Lamb shift \cite{27}, the energy difference between \( 2S_{1/2} \) and \( 2P_{1/2} \) not expected from Dirac’s equation correction on the Hydrogen Atom energy levels. Again, such a discrepancy could come from the interaction between the muon and a hidden sector gauge boson, specifically for the proton size anomaly an alternate model where a dark sector gauge boson would couple to the muon and the proton.
1.2.1 The Dark Photon Model

These DSMs can explain observed anomalies such as the ones observed by AMS [13], PAMELA and Fermi, where the positron fraction of incoming cosmic rays flux increases with their energy. According to DSMs this anomaly could be due to dark matter annihilation into dark photons, the simplest extension of the SM, a hidden/secluded U(1) vector gauge boson [28], that would mix with the SM photon, of mass lower than 2 GeV/c$^2$, then decay into an electron positron pair. Its decay could produce also other leptons or quarks depending on specific models and couplings, therefore the dark photon would be a portal allowing to probe light dark matter, because of its mixing with the SM and branching ratio into SM particles and dark matter. Other dark sector candidates are scalar or pseudoscalars, such as the dark higgs [29] and the axions, respectively, which may interact with the previously mentioned dark photon [30], changing its lifetime, meaning that it would travel further before decaying into SM or invisible particles. Apart from these portals there is also the sterile neutrino model that recently was investigated by the Ice Cube experiment [31], they could be investigated by highly sensitive collider experiments through dark sector channels. Since there are a lot of the dark sector portals of hidden/secluded candidates that have not been searched or even constrained, except for the dark photon, new investigations over other models or even mixing between the existing ones could be significant to give an initial constraint or try to set a new direction for dark sector investigations.

The first proposal for a new vector boson [16] was not aimed at dark sector candidates but rather at a general possibility of a new vector boson interaction with the SM vector boson, $\gamma$.

New or SM particles gauged by this new U(1) could have their electromagnetic charges shifted by an amount $\epsilon$, as a result of the mixing between the SM electromagnetic mediator, $\gamma$, and the extra vector boson. The mixing parameter, $\epsilon$ is the factor on the interaction in Fig [1-6].
1.2. THE DARK SECTOR

Figure 1-6: Model of the mixing between two vector bosons, for example $A_1^\mu$ as the SM photon, $\gamma$, and $A_2^\mu$ as the dark photon, $A'$.

This initial model then inspired the development of a detailed Lagrangian and Branching Ratio for the dark photon $A'$, an extra vector boson that would allow interactions between dark sector candidates, Light Dark Matter (LDM) for example.

The kinetically mixed dark photon Lagrangian is

$$\mathcal{L}_{A'} = -\frac{1}{4} F'^{\mu\nu} F'^{\rho\sigma} + \frac{1}{2} \epsilon \cos \theta_W B^{\mu\nu} F'^{\rho\sigma} - \frac{1}{2} m_{A'} A'^{\mu} A'^{\nu} \tag{1.5}$$

where $F'^{\mu\nu} \equiv \partial_\mu A'^\nu - \partial_\nu A'^\mu$ is the dark photon field strength, $A'^\mu$ is the dark photon vector field, $B^{\mu\nu} \equiv \partial_\mu B_{\nu} - \partial_\nu B_{\mu}$ is the SM hypercharge field strength, $m_{A'}$ is the dark photon mass and $\epsilon$ is the kinematic mixing factor. After electroweak symmetry breaking, the dominant effect of the kinetic mixing is an electromagnetic field strength $F^{\mu\nu}$ mixing with the dark photon field strength, $F'^{\mu\nu}$ in $\frac{1}{2} \epsilon F^{\mu\nu} F'^{\rho\sigma}$.

It is clear that the kinetically mixed dark photon model allows for a significant decay ratio into $e^+ e^-$ and $\mu^+ \mu^-$ as seen in the top curve in Fig 1-7, the biggest Branching Ratio for $A'$ is the dielectron final state and the second biggest is the dimuon one. These channels are relevant for other hidden gauge boson models, that is why a search strategy and results for the dark photon, $A'$, is suited for other dark sector models, at the analysis strategy level, since the reconstructed candidates are identical.

1.2.2 Previous Searches

Many searches were conducted in low and medium energy colliders for the previously mentioned dark photon, $A'$, in BABAR and also in Belle considering the
coupling with the dark Higgs boson $h'$. Apart from the dark photon searches there was also a search for the $Z'$ (to be explained on the next section) in BABAR conducted in 2016 [36].

Another relevant past search was conducted on the CERN SPS by the NA64 collaboration [37]. They performed a direct search for a sub-GeV dark photon $A'$ which could be produced in the reaction $e^-Z \rightarrow e^-ZA'$ via kinetic mixing, by the $\epsilon$ factor, with photons, by 100 GeV electrons incident on an active target. Dark photons would then decay into dark matter particles resulting in events with large missing energy. There was no signal found within $2.75 \times 10^9$ electrons on target. They set up a limit on the $\gamma - A'$ mixing strength and claim to have excluded invisible $A'$ with mass $\lesssim 100$ MeV, it can be seen in Fig 1-8. Again, this was a $A'$ search, but due to its kinetic mixing with the SM $\gamma$ it could also contribute for the $(g - 2)_\mu$ anomaly a model [32].
Due to the absence of right-handed neutrinos the three-generation standard model (SM) Lagrangian is invariant under the three global symmetries of the lepton family number: $U(1)_{L_e}$, $U(1)_{L_\mu}$, and $U(1)_{L_\tau}$ \[38\], three unitary groups. The lepton-number symmetry generated by $L = (L_e + L_\mu + L_\tau)$ and any linear combinations with the Baryon number, $B$, is not anomaly free, therefore it cannot be gauged. However,
there are three other symmetries generated by

\[ L_1 = L_e - L_\mu, \quad L_2 = L_e - L_\tau \quad \text{and} \quad L_3 = L_\mu - L_\tau \]  

(1.6)

which are anomaly free, since leptons have the same SM charge taking their difference cancels out their anomalies which would have opposite signs, \( L_i = L_a - L_b \), therefore, the lepton number differences could be gauged, but not simultaneously.

Three different theories arise as possible gauge groups \( G_{\text{SM}} \otimes U(1)_{L_{1,2,3}} \), where \( G_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \) with \( SU(3)_c \) being the color interaction group, \( SU(2)_L \) is the left-handed doublet group and \( U(1)_Y \) is the hypercharge unitary group. The three additional unitary groups for each of the lepton number differences allow for three additional gauge bosons \( (Z'_1, Z'_2, Z'_3) \).

\( U(1)_{L_i} \) local symmetry is likely to be spontaneously broken. In this occasion the \( Z'_i \) boson would gain mass via the Higgs mechanism, since a Higgs field \( S_i \) which is neutral under \( G_{\text{SM}} \) is not neutral under \( U(1)_{L_i} \). A nonzero vacuum expectation value for \( S_i \) generates mass for \( Z'_i \) by \( M_{Z'_i} = g'_i \langle S_i \rangle \), where \( g'_i \) is the coupling between the leptons and the new neutral gauge boson.

1.4 Thesis Outline

In this study a specific model, \( L_\mu - L_\tau \), [39] for a possible secluded dark sector gauge boson, \( Z' \), which couples only to heavy leptons, leptophilic, is searched in the Belle detector context. This secluded gauge boson, \( Z' \), could explain the \( g - 2 \) problem, the magnetic moment of the muon anomaly or it could be a channel into sterile neutrinos as candidates for dark matter, these motivations will be discussed further in the following chapters. Its similarity to the more well known dark photon[40], also a dark sector gauge boson will be discussed in detail in the following chapter, [2] where a discussion about the first purpose of the study is made, describing the \( Z' \) model parameters. In chapter [3] an overview of the KEKB Accelerator and of
1.4. **THESIS OUTLINE**

the Belle detector is given. Next, in chapter 4 the Analysis outline is described with a flowchart. The procedure is discussed shortly step by step. Following the outline explanation, in chapter 5 the MC study and its results are shown, both signal and background samples described in detail. The real data results are shown in chapter 6 which also contains a summary of the systematic errors. Finally a conclusion followed by the results discussion and future prospects are in chapter 7.
Chapter 2

Purpose of the study

In this chapter the main purpose of this study is explained starting from a brief description of the motivation for this specific $Z'$ model, followed by its definition. Emphasis is given to the direct connection between the $Z'$ gauge boson and a dark matter candidate, the sterile neutrino. Then, the $Z'$ decay channels are discussed along with its previous searches. To conclude this chapter the mass range to be investigated is shortly explained.

2.1 $Z'$ Lagrangian

Motivated by the neutrino trident production, a subweak process consisting of the production of a $\mu^+\mu^-$ pair from the scattering of a muon neutrino off the Coulomb field of a nucleus. A further refinement of the previously discussed gauge boson $Z'_3$ was made looking for the mechanism with which the rate of the neutrino trident production is increased. It has been observed in only a few experiments, such as the Long Baseline Neutrino Experiment (LBNE) [41].

The leading order contribution for the $Z'$ contribution to the neutrino trident production can be seen below in figure [2-1]. Specifically the contribution to the trident production is:
\[ \mathcal{L}_{Z'} = -\frac{1}{4}(Z')_{\alpha\beta}(Z')^{\alpha\beta} + \frac{1}{2}m_{Z'}^2 Z'_{\alpha} Z'^{\alpha} + g'Z'_{\alpha}(\ell_2 \gamma^\alpha \ell_2 - \bar{\ell}_3 \gamma^\alpha \ell_3 + \bar{\mu}_R \gamma^\alpha \mu_R - \bar{\tau}_R \gamma^\alpha \tau_R) \]

(2.1)

where the \( g' \) is the \( U(1) \) gauge coupling, \((Z')_{\alpha\beta} = \partial_\alpha Z'_\beta - \partial_\beta Z'_\alpha \) is the field strength, \( \ell_2 = (\nu_\mu, \mu_L) \) and \( \ell_3 = (\nu_\tau, \tau_L) \) are the electroweak doublets. The first and second terms on the \( Z' \) Lagrangian on equation 2.1 are identical to the first and third terms on equation 1.5 the kinetically mixed dark photon model. Regarding the red part in equation 2.1 it is the \( g' \) coupling the new gauge boson \( Z' \) to the electroweak doublets and the that enhances the rate of neutrino trident production in the \( \nu_\mu N \to N\nu_\mu^+\mu^- \) process.

This study proposes to search for the leptophilic neutral gauge boson, \( Z' \), which couples only to heavy leptons, muons and taus, gauging the \( L_\mu - L_\tau \) symmetry, where \( L_\mu, L_\tau \) are lepton flavor numbers [39], a Standard Model (SM) simplest extension.

The \( Z' \) model described in the Lagrangian on equation 2.1 is the full Lagrangian...
for the $Z'$, its interaction component is the Lagrangian in equation 2.2, which has only the terms relevant to the coupling between $Z'$ the heavy leptons and their neutrinos. This was the $Z'$ model searched in Belle (this study) and in BABAR, its Monte Carlo generator settings were developed by Brian Shuve and presented to us by private communications.

$$\mathcal{L}_{\text{int}} = -g'\bar{\mu}\gamma^\mu Z'_\mu\mu + g'\bar{\tau}\gamma^\mu Z'_\mu\tau - g'\bar{\nu}_{\mu,L}\gamma^\mu Z'_\mu\nu_{\mu,L} + g'\bar{\nu}_{\tau,L}\gamma^\mu Z'_\mu\nu_{\tau,L}$$

(2.2)

where $g'$ is the coupling factor between the gauge boson $Z'$ and the heavy leptons and $\nu_{\mu,L}$ and $\nu_{\tau,L}$ are the left-handed muon and tau neutrinos.

Since there is no Right Handed (RH) neutrino in the SM, the interaction between $Z'$ and the Left Handed (LH) neutrino has to be chiral, while the interaction with the heavy leptons is vector like. As mentioned before at Section 1.3 $Z'$ is anomaly free since the muons and taus have the same charge by SM, taking their difference get same anomalies added with opposite sign, canceling them out.

### 2.2 The sterile neutrino channel

This model is a development of a previous one aiming at gauging lepton number differences interactions [39]. Developments of the $Z'$ were done in [42], the light gauge boson $Z'$ decay could explain the relic abundance of dark matter, as sterile neutrinos, with $m_{Z'}$ in the MeV $\sim$ GeV range and the coupling $g'$ around $10^{-6} \sim 10^{-3}$.

Assuming a sterile neutrino, that mixes weakly with the active $\nu_\mu$ or $\nu_\tau$ states, is added to the SM.

$$\begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Where $\nu_1$ and $\nu_2$ are the mass eigenstates, and $\nu_a$ and $\nu_s$ are the active and sterile “flavor” eigenstates. And finally $\theta_0$ is the vacuum mixing angle in the very early Universe.
20

CHAPTER 2. PURPOSE OF THE STUDY

Figure 2-2: Feynman diagrams corresponding to the lowest order channels that would produce sterile neutrinos through the $Z'$ decay \[42\].

\[
\Gamma_{Z'\to\nu_S} = \frac{g'^2 M_{Z'}}{12\pi} \frac{\sin^2\theta_m}{4}(1 + \tan^2\theta_m)
\]  

(2.3)

The decay channel of $Z'$ into sterile neutrinos is in Fig 2-2 and its decay width is in equation 2.3 where $\theta_m$ is the mixing angle.

Finally in figure 2-3 the relationship between the $Z'$ mass and its coupling, $g'$, to the heavy leptons and their neutrinos is plotted. Each sterile neutrino mass plotted corresponds to the dark matter relic abundance observed ($Y_{DM} = 4.7 \times 10^{-4}\text{keV/}m_s$) for different mixing angles between the active and sterile neutrinos.

### 2.3 $Z'$ decay channels

This newest development for the $Z'$ model considers heavy leptons, $\mu$, $\tau$ and their neutrinos coupled by $g'$ to the $Z'$, however, $Z'$ could also be coupled indirectly to the sterile neutrinos (via neutrino mixing) or light dark matter (LDM), but since Belle is not exactly sensible enough at this lower mass range, the LDM channel will not be discussed here.

Explicitly these are the decay possibilities for $Z'$:
2.3. $Z'$ DECAY CHANNELS

Figure 2-3: $M_{Z'} - g'$ plane with the magnetic moment of the muon anomaly favored region (red dashed/continuous lines), as well as different masses sterile neutrino candidates (blue dashed/continuous lines). For $m_s = 7.1\text{keV} \sin 2\theta_0 = 8 \times 10^{-6}$, $m_s = 30\text{keV} \sin 2\theta_0 = 2.2 \times 10^{-6}$, $m_s = 50\text{keV} \sin 2\theta_0 = 3.5 \times 10^{-8}$ and for $m_s = 100\text{keV} \sin 2\theta_0 = 5 \times 10^{-9}$ [42].

- $m_{Z'} < m_{\mu^+\mu^-}$: $Z' \rightarrow \text{invisible, } \nu_\ell \bar{\nu}_\ell$ where $\ell = \mu, \tau$
- $m_{Z'} > m_{\mu^+\mu^-}$: $Z' \rightarrow \nu_\ell \bar{\nu}_\ell$ and $Z' \rightarrow \mu^+\mu^-$
- $m_{Z'} > m_{\tau^+\tau^-}$: $Z' \rightarrow \nu_\ell \bar{\nu}_\ell$, $Z' \rightarrow \mu^+\mu^-$ and $Z' \rightarrow \tau^+\tau^-$
- model dependent $Z' \rightarrow \text{invisible}$, meaning a decay into the dark sector, Light
CHAPTER 2. PURPOSE OF THE STUDY

Dark Matter

![Diagram](image)

Figure 2-4: The diagram on the left is the channel available at Belle, a $e^-e^+$ collision producing a $\mu$ pair and a $Z'$ that promptly decays into another $\mu$ pair. The right diagram shows the main motivation for the $Z'$ which is its role in a loop correction that could explain the $g_\mu - 2$ anomaly.

In this study the search for the $Z'$ is done through its coupling to muons, as seen on the left in Fig 2-4 and on the right it is one of the main motivations for this model the diagram for the $(g - 2)_\mu$, the magnetic moment of the muon anomaly. This is an attempt to find a $Z'$ signal or to place limits on its coupling constant, $g'$.

Mainly, the merits for searching for $Z'$ through its muon channel, $Z' \rightarrow \mu^+\mu^-$ are:

- it is the visible channel, different from $Z' \rightarrow \nu_{\ell}\nu_{\ell}$, which is invisible
- very high sensitivity in Belle
- broad mass coverage different from the $\tau\tau$ decay channel, the Branching Ratio (Br) as visible in Fig 2-5 stretches from the dimuon threshold (0.212 GeV/$c^2$) to higher masses

Partial widths and BR for $Z'$ were derived from Equation 2.12, in [45] and are still unpublished by Brian Shuve.

The rate of the $Z'$ decay into leptons is related to the coupling constant $g'$ by the following equations:
2.3. $Z'$ DECAY CHANNELS

\[ \Gamma(Z' \to \ell^+ \ell^-) = \frac{(g')^2 M_{Z'}}{12\pi} \left(1 + \frac{2M^2_\ell}{M^2_{Z'}}\right) \sqrt{1 - \frac{4M^2_\ell}{M^2_{Z'}}} \theta(M_{Z'} - 2M_\ell) \]  

(2.4)

\[ \theta(M_{Z'} - 2M_\ell) = 1 \text{ for } M_{Z'} \geq 2M_\ell \]  

(2.5)

\[ \theta(M_{Z'} - 2M_\ell) < 1 \text{ for } M_{Z'} < 2M_\ell \]  

(2.6)

\[ \Gamma(Z' \to \nu_\ell \bar{\nu}_\ell) = \frac{(g')^2 M_{Z'}}{24\pi} \]  

(2.7)

For $M_{Z'} \gg M_\ell$ the branching fraction to one neutrino flavor is half of that to a lepton. This is due to the fact that the $Z'$ only couples to left-handed neutrinos, but couples to both right and left handed leptons.

The visible branching fraction for muons ($Z' \to \mu^+ \mu^-$) is

\[ \text{Br}(Z' \to \mu^+ \mu^-) = \frac{\Gamma(Z' \to \mu^+ \mu^-)}{2\Gamma(Z' \to \nu_\ell \bar{\nu}_\ell) + \Gamma(Z' \to \mu^+ \mu^-) + \Gamma(Z' \to \tau^+ \tau^-)} \]  

(2.8)

which is identical to the one for taus ($Z' \to \tau^+ \tau^-$), just replacing the decay width ($\Gamma$) with the appropriate channel. As for the invisible branching fraction,

\[ \text{Br}(Z' \to \text{invisible}) = \frac{2\Gamma(Z' \to \nu_\ell \bar{\nu}_\ell)}{2\Gamma(Z' \to \nu_\ell \bar{\nu}_\ell) + \Gamma(Z' \to \mu^+ \mu^-) + \Gamma(Z' \to \tau^+ \tau^-)} \]  

(2.9)

For $M_{Z'} < 2M_\mu$, the invisible branching fraction is 100%. For $2M_\mu < M_{Z'} < 2M_\tau$, the invisible branching fraction is close to 50%. For $M_{Z'} > 2M_\tau$, the invisible branching fraction is almost 1/3.

Considering light dark matter (LDM) as the only decay channel, completely invisible, $Z' \to \chi \bar{\chi}$, with $M_{Z'} > m_\chi m_{\bar{\chi}}$ the Branching Ratio (BR) to this channel is
1.

Figure 2-5: This plot shows the Branching Ratio as a function of the $Z'$ mass.

Therefore a limit on the $\sigma \times \text{Br}$ will be set to extract a limit on the $Z'$ cross section which then allows to set a limit on the $g'$ coupling for different values of $M_{Z'}$. The $Z'$ cross section was studied as a function of its mass and with different couplings $g'$ using Monte Carlo samples simulated by MadGraph 5 [46], as the event generator that will be discussed in detail in chapter 5, but for now the results for the $Z' \rightarrow \mu^+\mu^-$ cross section can be seem below in figure 2-6.
Figure 2-6: $Z' \rightarrow \mu^+\mu^-$ cross section as a function of its mass for different couplings, $g' = 0.1, 0.01$ and 0.001. Produced at a center of mass energy of 10.58 GeV.
2.4 Previous Searches

In the older version of the model, $Z'$ was a gauge boson that could also gauge $L_e - L_\mu$ or $L_e - L_\tau$ lepton number differences, however, any interaction involving electrons have been well constrained by dark photon searches through the channel $A' \rightarrow e^+e^-$. The dark photon, $A'$, is coupled to electrons and muons by a different mechanism than the one coupling $Z'$, but since both of them are $U(1)$ gauge bosons a search/analysis strategy designed for one is suited for the other. Such was the case for the $A' \rightarrow e^+e^-$ channel, which found nothing [34], [43] and [44], thus ruling out $A'$ and $Z'$ couplings to $e^+e^-$. The $Z'$ search at BABAR from 2016 [36] scanned the $Z'$ masses in the range of 0.212-10 GeV using the $e^-e^+ \rightarrow Z'\mu^+\mu^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ channel and 514 fb$^{-1}$ of data collected by BABAR, no significant signal was observed in the selected mass range, and limits on the coupling parameter $g'$, between $Z'$ and the $\mu$, as low as $7 \times 10^{-4}$ were set, an improvement compared to the previous limits set from neutrino experiments.

The samples used by BABAR were taken at the $\Upsilon(4S)$, as well as $\Upsilon(3S)$ and $\Upsilon(2S)$. Monte Carlo samples were simulated by MadGraph 5 [46], as the event generator, with the hadronization performed by Pythia 6 [47]. The background arises mainly from QED processes, such as $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ generated by Diag36 [48], which includes all lowest order diagrams. Other processes such as $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and $e^+e^- \rightarrow \tau^+\tau^- (\gamma)$ were generated using the KK generator [49]. Remaining background channels investigated were $e^+e^- \rightarrow q\bar{q}(q = u, d, s, c)$ continuum production through JETSET [50], and finally the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ channel was produced using EvtGen [51] with a phase-space model. The detector acceptance and reconstruction efficiencies are determined using a Monte Carlo (MC) GEANT4 [52] simulation.

Their selection criteria were:

- 2 pairs of oppositely-charged tracks, where both positive or both negative were identified as muons by PID, maintaining a high signal efficiency while rejecting
most background channels except \( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- \) events

- the sum of energies of electromagnetic clusters above 30 MeV not associated with any charged track must be less than 200 MeV to remove background containing neutral particles

- to suppress specific background coming from the off-resonance samples, such as \( \Upsilon(3S,2S) \rightarrow \pi^+\pi^-\Upsilon(1S), \, \Upsilon(1S) \rightarrow \mu^+\mu^- \) events taken from \( \Upsilon(2S,3S) \) peaks containing any pair of oppositely charged tracks with dimuon invariant mass within 100 MeV of the nominal \( \Upsilon(1S) \) mass are rejected

- aiming at the most significant channel \( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- \), events are selected requiring a four-muon invariant mass within 500 MeV of the nominal Center of Mass System (CMS) energy, allowing for Initial State Radiation (ISR) emissions, that can distort the mass conservation

From the plot on the right in Fig 2-7, it is visible that the BABAR collaboration did not observe any significant signal in the reduced mass distribution, \( m_R = \sqrt{m_{\mu^+\mu^-}^2 - 4m_\mu^2} \). After the cuts described above, a kinematic fitter is imposed constraining the four muon system center of mass (CM) energy to be within the beam energy spread, and, the tracks have to originate from the interaction point (IP), within its uncertainty. This is to improve the \( Z' \) mass resolution of the events near the \( \Upsilon(4S) \) resonance peak, 10.58 GeV, since it is not possible to tell which of the muon pairs in the final state decayed from the \( Z' \) candidate, all 4 possible combinations were considered.

The right plot on Fig. 2-7 is the reduced dimuon mass, \( m_R = \sqrt{m_{\mu^+\mu^-}^2 - 4m_\mu^2} \) distribution from the following collection of channels, dominated by \( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- \), but also with \( e^+e^- \rightarrow \pi^+\pi^-\rho, \rho \rightarrow \pi^+\pi^- \) and \( e^+e^- \rightarrow \pi^+\pi^-J/\psi, J/\psi \rightarrow \mu^+\mu^- \) where one or many pions were misidentified as muons. There is a peak from the \( \rho \) decay at
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Figure 2-7: The plot on the left is the four-muon invariant mass distribution for all the background channels taken at the $\Upsilon(4S)$ sample normalized to data luminosity, where the $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ channel does not include ISR corrections. And the one on the right is the reduced dimuon mass for the data and MC samples for all the considered channels, also normalized to data luminosity, four different muon pairings are considered per event. The ratio between reconstructed and generated events is on the lower right in a light blue dashed line, it was used later on as a correction factor.

Apart from $J/\psi$ (3.1 GeV/$c^2$) there is no significant signal or narrow resonance.

Due to the 4 combinations and $\rho$ resonance peak, the alternative pairings contribution result in a peak around the remaining mass. So considering $m_\Upsilon(4S) - m_\rho = 9.8$ GeV/$c^2$, this peak is visible on the lower right of Fig 2-7.

The detection efficiency in their case increased from around 35% at low masses to 50% at $m_R = 6 - 7$ GeV, then decreasing at higher masses. The BABAR analysis signal efficiencies include a correction factor of 0.82 due not only to the absence of ISR interactions, which were not simulated, differences between data and simulated trigger efficiency, charged PID and track or photon reconstruction efficiencies. The correction factor was derived from the ratio of the $m_R$ distribution between the simulated and observed $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ events, in the 1 – 9 GeV mass region, excluding $J/\psi$.

The light blue line in the right lower plot on Fig. 2-7. An uncertainty of 5% was
propagated as systematic, covering data taking periods and uncertainties in the cross-section.

They extracted the signal yield as a function of $m_{Z'}$ by unbinned maximum likelihood fits to the reduced dimuon mass spectrum, covering the range of $m_R < 10$ GeV for the $\Upsilon(4S)$ peak, and up to 9 GeV for the other peaks, $\Upsilon(2S, 3S)$. The search was done in varying steps dependent on the dark boson mass resolution. The fitting occurs in an interval 50 times broader than the signal mass resolution at that mass for $m_R > 0.2$ GeV, or fixed interval as $0.3$ GeV for $m_R < 0.2$ GeV. To define the signal resolution Gaussian fits to different $Z'$ samples was used to set the scanning steps, interpolation the results to all other masses. The resolution varies from $1-9$ MeV, due to experimental effects. 2219 mass hypotheses were probed. The bias due to fitting values, the step used when scanning for different values of $m_{Z'}$ has a negligible bias due to the large number samples.

Figure 2-8: On the left is the measured $e^+e^- \to Z'\mu^+\mu^-$ cross section above its statistical significance, $S_S$ (defined in the text) as function of the $Z'$ mass. The uncertainty on each point is shown as light gray error bands, the black band covered region is the excluded (the $J/\psi$ resonance peak). As for the plot on the right, 90% CL for the cross section of the signal channel as function of the $Z'$, where the black band is the region excluded [36].

The background was described by $\arctan(ax + bx^2 + cx^3)$ for fits in the low mass
region, with $a$, $b$ and $c$ as free parameters. And a second order polynomial for masses above $m_R = 0.2$ GeV. Contributions from the $J/\psi$ peak are rejected by a range of $\pm 30$ MeV around its nominal mass.

Their cross section for $e^+e^- \rightarrow \mu^+\mu^-Z', Z' \rightarrow \mu^+\mu^-$ is calculated by dividing the signal yield by the efficiency times the luminosity, $\sigma = N/(\epsilon \times \mathcal{L})$, where $N$ is the signal yield, $\epsilon$ the detection efficiency and $\mathcal{L} = 514 \text{ fb}^{-1}$. The uncertainties on the luminosity (0.6%) \cite{53} and the limited MC sample (1–3%) are propagated as systematic. All the uncertainties except the luminosity and efficiency corrections are considered uncorrelated, and the statistical significance of each fit as a function of the $m_Z'$ is in Fig 2-8 was taken as $S = \text{sign}(N_{\text{sig}}) \sqrt{2 \log(\mathcal{L}/\mathcal{L}_0)}$, $N_{\text{sig}}$ is the fitted signal yield sign, that is positive or negative, and $\mathcal{L}(\mathcal{L}_0)$ is the maximum likelihood values for a fit. The global significance was of $1.6\sigma$, compatible with the null hypothesis.

A 90% confidence level (CL) on the cross-section $\sigma(e^+e^- \rightarrow \mu^+\mu^-Z', Z' \rightarrow \mu^+\mu^-)$, assuming a uniform prior in the cross section and then integrating the likelihood from zero to 90% of its area, resulted on the right of Fig 2-8 as a function of the $Z'$ mass.

From the 90% CL of the signal cross section the upper limit (UL) on the coupling parameter $g'$ assuming an equal $Z'$ coupling to muons, taus and neutrinos, is obtained visible in Fig 2-9.

Since the BABAR analysis could set an upper limit for the $g'$ coupling factor as low as $7 \times 10^{-4}$ near the dimuon threshold for a luminosity of 514 fb$^{-1}$. Thus the expectation of this study is to find a $Z'$ signal or set a lower limit with Belle data, performing the same analysis of the same model but almost twice the integrated luminosity and higher sensitivity due to a larger drift chamber. Essentially BABAR analysis parameters and cuts tuned for Belle will be used in an attempt to improve the search of the dark gauge boson $Z'$ and impose a better upper limit for its coupling factor $g'$. 

2.5. MASS RANGES

The dark gauge boson \( Z' \) will be searched in the Belle detector, using the channel \( e^-e^+ \to Z'^+\mu^-\mu^- \to \mu^+\mu^-\mu^+\mu^- \) in the range of \( 0.212 < M_{Z'} < 10 \text{ GeV}/c^2 \) in 0.01\text{GeV}/c\(^2\) steps from 0.212 to 0.25 as well as from 0.41 to 0.45, and 0.1\text{GeV}/c\(^2\) steps on all other candidates, summing up to 107 points in the MC sample to be used. The analysis strategy will be developed in detail in chapters 4 and 5.

Figure 2-9: 90 % CL on the new gauge boson coupling \( g' \) results as function of the \( Z' \) mass, along with the constraints from the production of a \( \mu^+\mu^- \) pair in a \( \nu_\mu \) scattering ("Trident/Borexino" production) \[41\], \[54\], the anomalous magnetic moment of the muon discrepancy is in red \[36\]
Chapter 3

Experimental Apparatus

In this chapter, the experimental apparatus of the KEK $B$ factory is described, it consists of the KEKB accelerator and the Belle detector. The experiment is located at the High Energy Accelerator Research Organization (KEK) in Tsukuba-city, Japan.

3.1 KEKB Accelerator

KEKB [55] is a two-ring energy-asymmetric $e^+e^-$ collider and aims to produce huge number of $B$ and anti-$B$ meson pairs. Figure 3-1 shows a schematic layout of KEKB accelerator. A linear accelerator (Linac) accelerates an electron and positron up to the required energy and injects them to the storage rings. THE KEKB accelerator has two different storage rings: the ring for 8 GeV electrons is called the High Energy Ring (HER), and the one for 3.5 GeV positrons is called the Low Energy Ring (LER). The HER and LER were constructed side by side in the tunnel used for TRISTAN experiment. The two rings cross at one point called the interaction point (IP), where electrons and positrons collide with a crossing angle of $\pm 11$ mrad. The crossing angle was one of the novel features of the KEKB design, providing effective beam separation after collision without a high detector background level.
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Figure 3-1: Schematic layout of KEKB accelerator
3.1. KEKB ACCELERATOR

The center-of-mass energy is designed to be

$$\sqrt{s} = 2\sqrt{E_{\text{HER}} \cdot E_{\text{LER}}} = 10.58 \text{ GeV}, \quad (3.1)$$

which corresponds to the mass of the $\Upsilon(4S)$ resonance, just above $B\bar{B}$ production threshold. The cross-section for various processes in $e^+e^-$ collisions at the $\Upsilon(4S)$ resonance are summarized in Table 3.1. The $b\bar{b}$ production cross-section is about 1.1 nb. The $\Upsilon$ resonance stand on top of a large continuum background coming from light-quark pair production ($e^+e^- \rightarrow q\bar{q}$ with $q = u, d, s, c$). The $e^+e^-$ storage rings operating at the $\Upsilon(4S)$ resonance are called $B$-factories. The $\Upsilon(4S)$ dominantly decay

Table 3.1: Cross-section for various processes in $e^+e^-$ collisions at $\sqrt{s} = 10.58$ GeV. QED refers to Bhabha and radiative Bhabha processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sigma$ [nb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>1.1</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>1.3</td>
</tr>
<tr>
<td>$q\bar{q}$ ($q = u, d, s$)</td>
<td>2.1</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>0.93</td>
</tr>
<tr>
<td>QED (25.551° &lt; $\theta$ &lt; 159.94°)</td>
<td>37.8</td>
</tr>
<tr>
<td>$\gamma\gamma$</td>
<td>11.1</td>
</tr>
</tbody>
</table>

to $B^0\bar{B}^0$ and $B^+B^-$ pairs which are created with a Lorentz boost

$$\beta\gamma = \frac{E_{\text{HER}} - E_{\text{LER}}}{2\sqrt{E_{\text{HER}}E_{\text{LER}}}} = 0.425, \quad (3.2)$$

due to the energy asymmetry. For measurement of time dependent asymmetry, the distance of the decay vertices ($\Delta z$) of the $B$ meson pairs is measured instead of the difference of the decay time ($\Delta t$) from the relation $\Delta z \sim c\beta\gamma\Delta t$. The typical $B$-meson decay length is dilated from $\sim 20$ $\mu$m to $\sim 200$ $\mu$m by the Lorentz boost.

The design instantaneous luminosity of KEKB is $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$. However, it
CHAPTER 3. EXPERIMENTAL APPARATUS

exceeded this goal in 2004, and the world’s highest luminosity of

\[ \mathcal{L} = 2.11 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1} \]  

has been achieved in June 2009. Several improvements during that period increased the instantaneous luminosity. In early 2004, a new operation method called continuous injection was successfully introduced, which removes the dead time of the ordinary injection method. In early 2007, a new instrument called a crab cavity was installed. In the original design of KEKB, the two beams do not collide head-on, but with a small crossing angle of \( \pm 11 \text{ mrad} \). The crab cavities kick the beams in the horizontal plane, and make the head-on collisions, while retaining the crossing angle of beams.

Figure 3-2 shows the history of the luminosity. The total integrated luminosity had reached 1000 \( \text{fb}^{-1} \), which is one of the primary targets of the KEKB project, by finishing the data taking in June 2010.

3.2 Belle Detector

Belle detector is a general-purpose \( 4\pi \) detector composed of many sub-detector. The excellent performances of the particle identification and tracking system, and large angular coverage, make it very efficient to reconstruct \( B \) decays. A super-conducting solenoidal magnet producing a 1.5 T field is used for the momentum measurements.

\( B \)-meson decay vertices are measured by a silicon vertex detector (SVD) situated outside of a cylindrical beryllium beam-pipe. Charged particle tracking is provided by a wire drift chamber (CDC) together with the SVD. Particle identification (PID) is provided by \( dE/dx \) measured in CDC, aerogel Cherenkov counters (ACC) and time-of-flight counters (TOF) situated radially outside of the CDC. Electromagnetic particles are detected by an array of CsI(Tl) crystals (ECL) located inside the solenoid
Figure 3-2: The integrated luminosity of $B$-factories: Belle (blue) and BABAR (green).
coil. The outermost detector is the $K_L$ and muon detector (KLM), layers of resistive plate counters instrumented in the iron flux return. A pair of BGO crystal arrays (EFC), which is placed on the surfaces of the QCS (Quadrupole Collision Superconducting magnet) cryostat, covers forward and backward regions uncovered by the other detectors.

A major detector upgrade has been performed in the summer of 2003. A 3-layer SVD with a 2 cm radius beam-pipe was used until the summer of 2003. A data sample corresponding to a integrated luminosity of 140 fb$^{-1}$ was collected with this configuration. In the summer of 2003, a 4-layer SVD, a 1.5 cm radius beam-pipe, and a small-cell inner drift chamber were installed.

The origin of the coordinate system is defined as the position of the nominal IP. The $z$ axis is aligned with the direction opposite to the positron beam and is parallel to the direction of the magnetic field within the solenoid. The $x$ axis is horizontal and points towards the outside of the ring, and the $y$ axis is vertical. The polar angle $\theta$ and azimuthal angle $\phi$ are measured relative to the positive $z$ and $x$ axes, respectively. The radial distance is defined with $r = \sqrt{x^2 + y^2}$.

The following subsections provide a more detailed description of every sub-detector.

### 3.2.1 Silicon Vertex Detector (SVD)

The Silicon Vertex Detector (SVD) provides very precise position measurements and play a crucial role in measuring time-dependent $CP$ violation in the neutral $B$ meson system.

Figure 3-4(a) illustrates the end and side views of SVD1, which is used to the summer of 2003. The SVD1 consists of three concentric cylindrical layers arranged to cover $23^\circ < \theta < 139^\circ$. Its coverage corresponds to 86% of full solid angle. The three layers at 30.0 mm, 45.5 mm and 60.5 mm radii surround the beam pipe that has a double-wall beryllium cylinder of 2.0 cm radius. There are 8/10/14 ladders along $\phi$ in layers 1/2/3, respectively.
3.2. BELLE DETECTOR

In the summer of 2003, a new vertex detector, SVD2, was installed [58]. Figure 3-4(b) shows the configuration of the SVD2. The SVD2 consists of four concentric cylindrical layers and the polar angle acceptance is improved to cover $17^\circ < \theta < 150^\circ$, which is the same as CDC and corresponds to the 92% of the full solid angle. The four layers at 20.0 mm, 43.5 mm, 70.0 mm, and 88.0 mm radii surround the beam pipe whose radii is 1.5 cm. There are 6/12/18/18 ladders in layers 1/2/3/4, respectively.

Both SVD1 and SVD2 used a common double-sided silicon strip detectors (DSSDs) design. A DSSD is essentially a depleted $p n$ junction. A charged particle passing through the junction liberates electrons from the valence band into the conduction band creating electron-hole pairs. These pairs create currents in the $p^+$ and $n^+$ strips located on the surface of the DSSD. The $p^+$ strips are aligned along the beam axis and therefore measure the azimuthal angle $\phi$. The $n^+$ strips are aligned perpendicularly to the beam axis and measure $z$. The readout chain of DSSDs is based on CMOS-integrated circuit placed outside of the tracking volume.

The impact parameter resolution $\sigma_{r\phi}$ and $\sigma_z$ measured using cosmic rays events
are shown in Fig. 3-5. Obtained performance are

\[
\sigma_{r\phi}(\mu m) = 19.2 \oplus 54.0/\tilde{p} \quad \text{for SVD1},
\]

\[
\sigma_{r\phi}(\mu m) = 21.9 \oplus 35.5/\tilde{p} \quad \text{for SVD2},
\]

\[
\sigma_z(\mu m) = 42.2 \oplus 44.3/\tilde{p} \quad \text{for SVD1},
\]

\[
\sigma_z(\mu m) = 27.8 \oplus 31.9/\tilde{p} \quad \text{for SVD2},
\]

where pseudo momentum of \( \tilde{p} \) is defined as \( \tilde{p} = p\beta \sin^{3/2}\theta \) for \( r-\phi \) side and \( \tilde{p} = p\beta \sin^{5/2}\theta \) for \( z \) side. The SVD upgrade significantly improved the impact parameter resolution in both the \( r-\phi \) and \( z \) coordinates.

### 3.2.2 Central Drift Chamber (CDC)

The main role of the Central Drift Chamber (CDC) \[58\] is detection of charged particle tracks and determination of their momenta from their curvature in the magnetic field of 1.5 T provided by the superconducting solenoid. The CDC also provides particle identification information in the form of \( dE/dx \) measurements for charged particles.

Figure 3-6 shows the structure of the CDC. It is asymmetric in the \( z \) direction in order to provide an angular coverage of \( 17^\circ < \theta < 150^\circ \). The CDC has 50 cylindrical layers of anode wires, which consist of 32 axial- and 18 stereo-wire layers, and three cathode strip layers. In summer of 2003, the inner three layers have been replaced by two small-cell layers for making a space of SVD2, maintaining the performance of the trigger. Axial wires are parallel to the \( z \) axis, while stereo wires are slant to the \( z \) axis to provide \( z \) position information. A total number of drift cells is 8400(8464) for SVD1(SVD2) configuration.

A mixture of helium (50%) and ethane (50%) gas fills the chamber. A charged particle passing through CDC ionizes the gas. A charge avalanche is caused by the electrons produced by the gas ionization and drifts to a sensitive wire with a specific
3.2. BELLE DETECTOR

Figure 3-4: Detector configuration of SVD [58].
drift velocity, then the measured signal height and drift time provides information of the energy deposit and distance from the sensitive wire. Even though the gas mixture has a low Z to minimize the multiple-Coulomb scattering, a good $dE/dx$ resolution is provided by the large ethane component. The transverse momentum resolution measured using the cosmic ray events are shown in Fig. 3-7(a). Obtained performance is

$$\frac{\sigma_{p_t}}{p_t}(\%) = 0.19p_t \oplus 0.30/\beta.$$  \hspace{1cm} (3.8)

A scatter plot of measured $<dE/dx>$ and particle momentum is shown in Fig. 3-7(b) together with the expected mean energy losses for different particle species. Populations of pions, kaons, protons, and electrons can be clearly seen. The $<dE/dx>$ resolution was measured to be 7.8% in the momentum range from 0.4 to 0.6 GeV/c. It provides $K/\pi$ separation up to 0.8 GeV/c and also in the region of the relativistic rise (above 2.5 GeV/c).

Figure 3-5: Impact parameter resolutions (left) in $z$ and (right) in $r-\phi$ coordinates for the SVD1 and SVD2 [58].
3.2. BELLE DETECTOR

3.2.3 Aerogel Cherenkov Counter (ACC)

Aerogel Cherenkov Counter (ACC) [58] provides an information to separate $K^\pm$ from $\pi^\pm$ in high momentum range ($1.2 \text{ GeV}/c < p < 3.5 \text{ GeV}/c$), which extend the momentum coverage beyond the reach of CDC and TOF. ACC is silica aerogel threshold Cherenkov counters, which detect if a particle emits Cherenkov light or not and distinguishes particle species. Cherenkov radiation is emitted when a charged particle passes through a material medium at a speed greater than the phase velocity of light in that medium. The condition to emit Cherenkov light is given as

$$n > \frac{1}{\beta} = \sqrt{1 + \left(\frac{m}{p}\right)^2},$$  \hspace{1cm} (3.9)

where $m$ and $p$ are the particle mass and the momentum and $n$ is the refractive index of the matter.

Figure 3-8 illustrates the configuration of the ACC in the Belle detector. ACC consists of 960 counter modules segmented into 60 cells in the $\phi$ direction for the barrel part and 228 modules arranged in five concentric layers for the forward end-cap part.
(a) Transverse momentum resolution measured by CDC with SVD.

(b) $dE/dx$ versus charged track momentum in collision data.

Figure 3-7: CDC performances for transverse momentum resolution ranging from 0.4 to 0.6 GeV/$c$ and $dE/dx$ particle identification capability.
of the detector. All the modules are arranged in a semi-tower geometry, pointing to the IP. In order to obtain good $K/\pi$ separation for the whole kinematical range, the refractive indices of aerogels are selected to be between 1.01 and 1.03, depending on their polar angle region. The choice of the refractive index for the barrel ACC is optimized for separation of high momentum pions and kaons from the two-body $B$ decay, such as $B \to \pi\pi$ and $K\pi$. For the end-cap ACC, $n = 1.030$ has been chosen to cover low momentum region, which is necessary for flavor tagging, to cover lack of TOF in the endcap. A typical single ACC module is shown in Fig. 3-9 for the barrel and the end-cap ACC. Five aerogel tiles are stacked in thin (0.2 mm thick) aluminum box of approximate dimensions $12 \times 12 \times 12$ cm$^2$. To detect the Cherenkov lights, two(one) fine-mesh type photomultiplier tubes (FM-PMTs) are attached to each module in the barrel (end-cap) part. These FM-PMTs are designed to operate in strong magnetic fields of 1.5 T.

The performance of ACC is confirmed using the decay chain $D^{*-} \to D^{0}\pi^{-}$ followed by $D^{0} \to K^{+}\pi^{-}$. The slow $\pi^{-}$ from $D^{*-}$ allows to identify the daughter $K$ and $\pi$ from the $D^{0}$ directly by their relative charges with respect to the slow pion. Figure 3-10 shows the distribution of the number of photoelectrons, where the $K/\pi$ separation is good and consistent with MC.

### 3.2.4 Time-of-Flight Counter (TOF)

The Time-of-Flight Counter (TOF) gives particle identification to distinguish charged kaon from pions in the momentum region, below 1.2 GeV/c. TOF also provides fast timing signals for the trigger system, together with thin trigger scintillation counters (TSC). The TSC is used for keeping the fast trigger rate below 70 kHz.

The mass of the particle $m$ can be determined from the time-of-flight $T$ measured with the TOF and the momentum $p$ measured with the CDC, according to the
Figure 3-8: Arrangement of the ACC \[58\].

Figure 3-9: Schematic drawing of a typical ACC counter module: (a) barrel and (b) end-cap ACC \[58\].
Figure 3-10: Distribution of photo-electron for $K^\pm$ and $\pi^\pm$ in $D^{*\mp}$ decays. Each plot corresponds to the different set of modules with a different refractive index.
following relation:

\[ m = p \sqrt{\left( \frac{cT}{L} \right)^2 - 1}, \]  

(3.10)

where \( L \) is a length of the flight.

The TOF system consists of 128 TOF counters and 64 TSC counters. Two trapezoidally shaped TOF counters and one TSC counters form one module as shown in Fig. 3-11. In total 64 TOF/TSC modules located at a radius of 1.2 m from the IP cover a polar angle range from 34° to 120°. Each TOF counter is read out by a FM-PMT at each end. Each TSC counter is read out by only one FM-PMT from the backward end.

Figure 3-12(a) shows TOF time resolution for forward and backward PMTs and for the weighted average as a function of \( z \) position. The resolution for the weighted average time is about 100 ps with a small \( z \) dependence. Figure 3-12(b) shows the mass distribution for each track in hadron events. Clear peaks corresponding to pion, kaon and proton are seen. The data points well agree with a MC prediction (histogram) obtained by assuming resolution of \( \sigma_{TOF} = 100 \) ps.

![Figure 3-11: Schematic drawing of a TOF/TSC module](image)

### 3.2.5 Electromagnetic Calorimeter (ECL)

The main purpose of the electromagnetic calorimeter (ECL) is the detection of electrons and photons from \( B \) meson decays with high efficiency and good resolution in
3.2. BELLE DETECTOR

(a) Time resolution for $\mu$-pair events.

(b) Mass distribution from TOF measurements for particle momenta below 1.2 GeV/c. Histogram shows the expectation assuming time resolution of 100 ps. The point with error bars are data.

Figure 3-12: TOF performance [58].
energy and position. The ECL measures energy deposited by electromagnetic showers. High energy electrons and photons entering the calorimeter initiate an electromagnetic shower through subsequent bremsstrahlung and electron pair production processes followed by Coulomb scatterings. As a result, all of the incident energy is absorbed as ionization or excitation (light) in the calorimeter. Other particles only deposit small amounts of energy via $dE/dx$ ionization. The matching of the energy measured by the ECL and the momentum measured by the CDC is used for the electron identification.

The overall configuration of the ECL is shown in Fig. 3-13. ECL contains 8,736 thallium doped CsI crystal counters. The ECL consists of three sections: the forward endcap section consist of 1152 crystals and cover $12.4^\circ < \theta < 31.4^\circ$; the barrel section consist of 6,624 crystals and cover $32.2^\circ < \theta < 128.7^\circ$; and the backward endcap section consist of 960 crystals and cover $130.7^\circ < \theta < 155.1^\circ$; Each crystal has tower shape with about 6 cm $\times$ 6 cm cross section and 30 cm length (16.2 radiation lengths). Total weight of the crystals is about 43 tons. The light of each crystals is read-out by two PIN photodiodes and a preamplifier mounted at the end of each crystal. The energy resolution is measured by a beam test \cite{ref58} to be
\[
\sigma_E(E\%) = \frac{0.066}{E} \oplus \frac{0.81}{\sqrt{E}} \oplus 1.34 \quad (E \text{ in GeV}),
\] (3.11)
where the value is affected by the electronic noise (1st term), the shower leakage fluctuation (2nd and 3rd terms), and the systematic effect such as the uncertainty of calibration (3rd term). The spacial resolution is approximately found to be 0.5 cm/$\sqrt{E}$ ($E$ in GeV).

### 3.2.6 $K_L$ and Muon Detector (KLM)

The purpose of $K_L$ and Muon Detector (KLM) \cite{ref55} is to identify $K_L$’s and muons with high efficiency over a broad momentum range greater than 600 MeV/$c$.

The KLM consists of successive layers of charged particle detector (resistive plate
Figure 3-13: Configuration of ECL, made of thallium-doped CsI crystals, two endcaps and a barrel section that contains most of the crystals [58].

counters, RPC [58] and iron plates (4.7 cm thick). The neutral $K_L$ meson produces a hadronic shower when interacting in the iron, allowing for position detection. However, no useful measurement of its energy is possible because of the fluctuations of this shower. The muons go through all the detector and the hits in the RPC allow for energy and position measurements. Other particles, such as pions and kaons are stopped in the system and can easily be separated from muons.

The KLM contains 15 detector layers and 14 iron layers in the barrel part ($45^\circ < \theta < 125^\circ$), and 14 detector layers in each of the forward and backward endcaps region ($20^\circ < \theta < 155^\circ$). The iron plates provide 3.9 interaction length of material, in addition to 0.8 interaction length of ECL. The iron layers also serve as a return yoke for the magnetic flux provided by the superconducting solenoid.

KLM layers are grouped in superlayers, as shown in Fig. 3-14. A superlayer is made of $\theta$ and $\phi$ cathode strips surrounding two RPCs. Resistive-plate counters have two parallel-plate electrodes separated by a gas-filled gap. An ionizing particle
traveling the gap initiates a streamer in the gas that results in a local discharge. This discharge creates a signal on the external cathode strips which can be used to record the location and time of the ionization. The number of $K_L$ clusters per event is in good agreement with the prediction. Typical muon identification efficiency is 90% with a fake rate around 2%.

![Diagram of a KLM super layer](image)

Figure 3-14: Cross section of a KLM super layer, that are placed surrounding ECL [58].

### 3.2.7 Trigger and Data Acquisition

An important part of the Belle experiment is the trigger and the data acquisition (DAQ) systems. Most of events are not interesting for physics studies, like $e^+e^-$ scattering (Bhabha interaction), beam-gas interaction in the beam pipe, cosmic rays, etc. The purpose of the trigger is to reject uninteresting events as much as possible...
Table 3.2: Total trigger rates with $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$ from various processes at $\Upsilon(4S)$.
† The values is pre-scaled by a factor 1/100.

<table>
<thead>
<tr>
<th>Process</th>
<th>Rate [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(4S) \rightarrow BB$</td>
<td>12</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q}, (q = u, d, s, c)$</td>
<td>28</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \ell^+\ell^-, (\ell = \mu, \tau)$</td>
<td>16</td>
</tr>
<tr>
<td>Bhabha ($\theta_{\text{lab}} &gt; 17^\circ$)</td>
<td>4.4 †</td>
</tr>
<tr>
<td>$\gamma\gamma$ ($\theta_{\text{lab}} &gt; 17^\circ$)</td>
<td>0.24 †</td>
</tr>
<tr>
<td>two-photon process ($\theta_{\text{lab}} &gt; 17^\circ$, $p_t &gt; 0.3$ GeV/c)</td>
<td>35</td>
</tr>
<tr>
<td>Total</td>
<td>86</td>
</tr>
</tbody>
</table>

and to forward interesting ones to the DAQ system. With high efficiency, within a very short decision time. The trigger rates at high luminosity $10^{34}$ cm$^{-2}$s$^{-1}$ for various processes of interest are listed in Table 3.2. The Belle trigger system consists of a hardware trigger and a software trigger.

An overview of the hardware trigger system [58] is shown in Fig. 3-15. It consists of the sub-detector trigger systems and the central trigger system called the Global Decision Logic (GDL). The GDL receives sub-detector triggers within 1.85 $\mu$s after the collision and issues a decision 2.2 $\mu$s after the collision. The sub-detector trigger systems are based on two categories: track triggers and energy triggers. CDC and TOF are used to yield trigger signals for charged particles. The ECL trigger system provides triggers based on total energy deposit and cluster counting of crystal hits. These two categories allow sufficient redundancy. The KLM trigger gives additional information on muons and the ECL triggers are used for tagging two photon events as well as Bhabha events.

When the hardware trigger is issued, the Data Acquisition system (DAQ) collects signal data from sub-detectors and them on the data storage system. Figure 3-16 shows the overview of the DAQ system. The entire system is segmented into 7 subsystems running in parallel, each handling the data from a sub-detector. The signals from most sub-detectors go through a charge-to-time (Q-to-T) converter and
are processed by a time-to-digital converter (TDC). The KLM does not have a Q-to-T converter since the pulse does not provide useful information. For the SVD, DSSDs are read out by on-board chips and passed to flash analog-to-digital converters (FADC). The readout sequence starts when the sequence controller (SEQ) receives a final trigger from the GDL and distributes a common stop signal to the TDCs. The event builder converts detector-by-detector parallel data streams to an event-by-event data river and sends the data to an online computer farm.

The online computer formats an event data into an offline event format and performs a background reduction (a hardware trigger) after a fast event reconstruction. The data are then sent to a mass storage system via optical fibers.
3.2. BELLE DETECTOR

Figure 3-16: Overview of the Belle Data Acquisition (DAQ) system [58].
CHAPTER 3. EXPERIMENTAL APPARATUS
Chapter 4

Analysis Overview

In this chapter an overview of the analysis is presented. In order to avoid possible bias, all analysis procedures are defined by studying Monte Carlo (MC) samples, followed by a verification of the analysis strategy taken by 5% of the real data. Finally the analysis procedures are applied to the full data set without modifying the procedure at that point. Each of the steps taken are described in the same order as they were conducted.

To set the Analysis Strategy, MC samples were produced for the $e^+ e^- \rightarrow Z' \mu^+ \mu^- \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ and its background. These samples were used to perform a $Z'$ signal search.

The search for a $Z'$ candidate is done by scanning through the dimuon invariant mass distribution for a resonance, a bump search. The region searched is the whole Belle mass range starting from the dimuon threshold to the kinematic upper limit, $0.212 \sim 10 \text{ GeV}/c^2$.

As for the background encountered in the $Z'$ search, it is studied by applying the same selection criteria as used for the signal. After the reconstruction, the signal MC is used to find the signal shape for different $Z'$ masses, and then it is used to scan for the signal over the background dimuon invariant mass distribution.

Figure 4-1 shows the flowchart of the analysis. The blue rectangle separates the
steps that were done with the MC samples, the MC study. The steps inside the red rectangle are independent of the MC samples, they are related with real data samples. Description of each step is explained further below.

Figure 4-1: Flowchart of the analysis procedure, divided into SIGNAL and BACKGROUND oriented, on the left and right respectively. The MC study is separated by the blue rectangle and the Real data steps are surrounded by a red rectangle.
0. The $Z'$ model in this study has no analytical, "theoretical", cross section. In its place is the output from the MadGraph generator (explained in deeper detail in chapter 5). Used to calculate the upper limit on the coupling $g'$ in step number 7.

The background channels cross section is also taken from their generators (also explained further in chapter 5) output as the cross section.

1. The MadGraph MC generated signal sample depends on many input parameters: the Lagrangian for $Z'$ interactions including the $Z'$ mass and its coupling $g'$ to muons, as well as decay channels and their branching ratios. The background sample depends on analogous parameters, both cases will be discussed in more detail on the next chapter.

Finally the signal and background samples undergo the Belle detector simulation using gsim, in order to compute Belle’s detector response to the signal and background events.

2. With the MC samples at hand, a series of selection cuts is applied aiming to remove as much background events while keeping as much signal events as possible.

To reconstruct the $Z'$ candidates, 2 oppositely charged muons are paired. However, since in the final state there are four muons, $e^+e^- \rightarrow \mu^+\mu^- Z' \rightarrow \mu^+\mu^-\mu^+\mu^-$, all of the possible combinations are considered. Each event corresponds to 4 possible pairings. The invariant mass distribution of muon pairs, $Z'$ candidates, is scanned looking for a peak.

3. After the selection is completed the detection efficiency can be calculated by using the final number of $Z'$ candidates, the ones that survived the cuts in the previous step, divided by the number of $Z'$ generated in the MC generation step.
In the background case, after the selection criteria are applied it is possible to identify which are the most relevant background channels, the ones that survive the cuts.

4. The **signal shape** is studied by fitting the invariant mass distribution from the reconstructed $Z' \rightarrow \mu^+\mu^-$ candidates. The signal shape parameters, peak width and mean, are obtained and parameterized as a function of the $m_{Z'}$.

5. With the signal shape obtained and parameterized by $Z'$ mass, the **Dimuon Invariant Mass** distribution for the background is fitted with a function made up of the signal shape and the background.

6. From the result of the fit a step before, the **Signal Yield** and its error are obtained, as well as the **90% upper limit** on the signal yield, estimated by assuming a single gaussian distribution.

7. From the 90% limit on the signal yield the cross section and $g'$ coupling constant **upper limit** were calculated.

8. Before having access to the full Belle data set, a 5% sample from it is analysed. Then this 5% data sample dimuon invariant mass distribution is compared to the scaled background distribution to **validate** the analysis strategy.

9. Finally after the analysis strategy is defined, validated and without changing any of the selection criteria to guarantee a blind analysis, the full data set from Belle is analyzed, **box opening**, to scan for a $Z'$ signal.

This concludes the analysis overview, in the next chapter each step will be explained fully, followed by the results.
Chapter 5

Monte Carlo Study

In this chapter the Monte Carlo (MC) study mentioned previously in chapter 4 is unfolded completely along with the other analysis steps, starting with the MC generators description.

5.1 Monte Carlo Samples

5.1.1 Signal

As mentioned in the last section of chapter 2.5, 107 different $Z'$ mass candidates signal samples were generated using Madgraph [46]. Actually called MadGraph5aMC@NLO a MC generator aimed at the LHC and at a 1 TeV $e^+e^-$ collider, in which tree-level and next-to-leading order cross sections as well as their matching parton shower simulations are calculated based on a few key physical parameters required, such as collision energy, a theoretical cross-section, branching ratios, decay modes, decay width and the coupling.

As a meta-code written in Python, it writes a (Python, C++, Fortran) code tailored for the desired model. To do that for the present study the following parameters were set:
• the $Z'$ Lagrangian $\mathcal{L} = -g'_\mu \bar{\mu} \gamma^\mu Z'_\mu \mu + g'_\tau \bar{\tau} \gamma^\mu Z'_\mu \tau - g'_\nu \bar{\nu}_{\mu,L} \gamma^\mu Z'_\mu \nu_{\mu,L} + g'_\nu \bar{\nu}_{\tau,L} \gamma^\mu Z'_\mu \nu_{\tau,L}$

• the $Z'$ branching ratios as in Fig.2-5

• number of desired events = 10 runs $\times$ 10000 events = 100000

• desired beam energies = $\Upsilon(1S, 2S, 3S, 4S, 5S, 6S)$

• decay modes, $e^+e^- \rightarrow \mu^+\mu^- Z'$, $Z' \rightarrow \mu^+\mu^-$

• the coupling between $Z'$ and the muons, $g'_{\mu \mu} = 0.1$

• the decay width for $Z'$ is set as prompt

• the $Z'$ desired mass

Two other fundamental programs were used concomitantly with MadGraph, one of them was FeynRules[60] a Mathematica package that allows for the implementation of new particle physics models from their particle contents and its Lagrangian. It derives the Feynman rules and stores them in a generic file, allowing for easy translation into any other Feynman diagram calculation program, such as MadGraph. The other one was FeynArts[61] another Mathematica package used for generation and visualization of Feynman diagrams and amplitudes. The result of the output from FeynRules used in FeynArts is in Fig 5-1.

These Feynman diagrams were generated using Brian Shuve’s Lagrangian 2.2 and also his MadGraph model. The most significant contributions are from the $Z$ and $\gamma$ mediated interactions, the mediation by the Higgs, $h$ is negligible.

A fundamental part of the analysis depends on the output of the Madgraph generated signal output, which is the cross section for each mass candidate.

---

1 mass of the $Z'$ candidates = 0.212, 0.22, 0.23, 0.24, 0.25, 0.3, 0.41, 0.42, 0.43, 0.44, 0.45, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4.0, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 5.0, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 7.0, 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.8, 7.9, 8.0, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 9.0, 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 10.0 GeV/$c^2$
Figure 5-1: Feynman diagrams for the channels involved in the $Z'$ production

In Figure 5-1 it is worth mentioning that even though there is a $Z'$ production channel that could have a mediation of a virtual Higgs Boson, $h$, it is a very small contribution compared to the other channels.

Also in the main MC samples used to define the analysis strategy, corresponding to the diagrams in Figure 5-1, the Initial State Radiation (ISR) effects are not included. Later some ISR samples were produced for the $Z'$ model following [62].

The examples on Figure 5-2 can be extended for all the channels from Figure 5-1. The ISR effects are very relevant because they worsen the resolution of the reconstructed $Z'$ resonance peak width and decrease the signal yield.

In this study there is no analytical cross-section for the $Z'$, the $Z'$ output cross-section from Madgraph will be called “theoretical” cross-section, $\sigma^{th}$ visible in Fig 5-3. Different $Z'$ masses hypotheses were produced at different $E_{cm}$ center of mass energies. Based on the data samples taken by Belle. Starting from 9.46, 10.02, 10.35, 10.58, 10.89 up to 10.98 GeV/c$^2$, corresponding to $\Upsilon(1S, ..., 6S)$ masses.

5.1.2 Background

Before discussing the MC sample generators for the main background channels all of the initial expected contributions will be listed.

Table 5.1 lists all of the background channels and their corresponding cross sec-
tions, besides the number of events. The channels listed as an irrelevant cross section means that after the preselection cuts and the further cuts inspired partially by the BABAR search (they will be explained fully in the following sections) are applied, the background channels disappears, or becomes negligible.

Anticipating the surviving and significant channels as $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ produced by $AAFH(Diag36)$ and $e^+e^- \rightarrow \mu^+\mu^- J/\psi$ or $e^+e^- \rightarrow \pi^+\pi^- J/\psi$, where $J/\psi \rightarrow \mu^+\mu^-$ by $EvtGen$.

**AAFH(Diag36)**

Cross section generator for $e^+e^-$ two photon scattering into four lepton final states, such as $e^+e^-e^+e^-, e^+e^-\mu^+\mu^-, \mu^+\mu^-\mu^+\mu^-$ and $\mu^+\mu^-\tau^+\tau^-$, the calculations are per-

Figure 5-2: Initial State Radiation channels example.
### 5.1. MONTE CARLO SAMPLES

#### Table 5.1: Background Channels

<table>
<thead>
<tr>
<th>generator</th>
<th>channel</th>
<th>cross section (fb)</th>
<th>number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>KKMC</td>
<td>$e^+e^- \rightarrow c\bar{c}$</td>
<td>1330.1262167</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow d\bar{d}$</td>
<td>404.60226383</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow s\bar{s}$</td>
<td>378.85979994</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \tau^+\tau^-$</td>
<td>916.65184150</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \mu^+\mu^-$</td>
<td>1143.54372146</td>
<td>$10^6$</td>
</tr>
<tr>
<td>BBBREM</td>
<td>$e^+e^- \rightarrow e^+e^-\gamma$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td>AAFH(Diag36)</td>
<td>$e^+e^- \rightarrow e^+e^-e^+e^-$</td>
<td>$3.9 \times 10^6$</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$</td>
<td>$0.0214 \times 10^6$</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow e^+e^-\mu^+\mu^-$</td>
<td>$19.1 \times 10^6$</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow e^+e^-\mu^+\mu^-$</td>
<td>$0.000336 \times 10^6$</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$</td>
<td>76.4</td>
<td>$10^6$</td>
</tr>
<tr>
<td>PHOKHARA</td>
<td>$e^+e^- \rightarrow \mu^+\mu^-\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow n\bar{n}\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow p\bar{p}\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \pi^+\pi^-\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \Lambda\bar{\Lambda}\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow K^+K^-\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow K^0K^0\gamma_{\text{ISR}}$</td>
<td>irrelevant</td>
<td>$10^6$</td>
</tr>
<tr>
<td>BABA</td>
<td>$e^+e^- \rightarrow \mu^+\mu^-$</td>
<td>irrelevant</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow \gamma\gamma$</td>
<td>irrelevant</td>
<td>$2.9 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>$e^+e^- \rightarrow e^+e^-$</td>
<td>irrelevant</td>
<td>$2 \times 10^6$</td>
</tr>
</tbody>
</table>
Figure 5-3: Cross sections for $Z'$ production at different center of mass energies.

To evaluate the total cross section for such events, the 36 lowest order Feynman diagrams, $Diag_{36}$, have to be evaluated. This Monte Carlo event generator produces unweighted events for the various channels, allowing for a direct comparison with experimental data. The program initially selects one of the Feynman Diagram Groups: A, B, C, D, E or F in Fig. 5-4, then generates an event. This sets the optimal phase space variables for a peaking structure. The sample of “raw” events can then be modified to obey the exact cross section desired, done by imposing the weight which takes contributions from the different groups of diagrams, together with a rejection algorithm. For each event the cross section is evaluated on the amplitude level.

**EvtGen**

A particle decay simulation package relevant for B meson decays and other B physics related resonances [51]. Decay amplitudes are used for the simulations, the
amplitude for each step in a decay process, including all angular and time-dependent correlations.

The event selection algorithm works considering all decay steps into the decay amplitude, which can then be used to obtain the probability, taking the \( e^+e^- \rightarrow \mu^+\mu^- J/\psi \) background channel as an example.

\[
A = \sum_{\lambda_+\lambda_-} A^{J/\psi \rightarrow \ell^+\ell^-}_{\lambda_+\lambda_-}, \quad P_{J/\psi} = \sum_{\lambda_+\lambda_-} |A^{J/\psi \rightarrow \ell^+\ell^-}_{\lambda_+\lambda_-}|^2 \quad (5.1)
\]

The \( \lambda_\ell \) denotes the states of spin degrees of freedom for \( \ell^\pm \), and \( A^{J/\psi \rightarrow \ell^+\ell^-}_{\lambda_+\lambda_-} \) is the decay amplitude. In this case the production is made by \textit{PYTHIA/JETSET} interactions, and \textit{EvtGen} evaluates the decay amplitude for \( J/\psi \rightarrow \ell^+\ell^- \), so to have the accurate cross section the corresponding \( J/\psi \) Branching Ratio must be considered.
5.2 Selection Criteria

After the initial signal and background MC sample generation is completed they go under the Belle response simulation. The Belle detector simulation and digitalization is done by \textit{gsim}, a standard Belle analysis tool describing the Belle detector response by using \textit{GEANT3} \[59\]. With the MC signal and background samples undergone the Belle detector response, it is possible to move on to the selection criteria used to pick the relevant events to the $Z'$ search, starting with the Particle Identification.

5.2.1 Particle Identification (PID)

PID for $K^\pm/\pi^\pm$

The $K^\pm/\pi^\pm$ identification \[58\] is based on the complementary measurements performed in three sub-detectors:

- $dE/dx$ measurement by CDC
- the Cherenkov light yield in ACC
- the time-of-flight information from TOF

The momentum coverage of kaon over pion separation of each sub-detector is illustrated in Fig. \[5-5\], that is how much can each sub detector contribute in distinguishing kaons from pions depending on their momentum. A separation of more than $3\sigma$ between kaons and pions is realized up to momenta of 3.5 GeV/$c$. The likelihood functions $L_K$ and $L_\pi$ are constructed on the product of the likelihood functions for three discriminants.

$$L_i = L_i^{dE/dx} \cdot L_i^{ACC} \cdot L_i^{TOF} \quad (i = K, \pi). \quad (5.2)$$
The likelihood ratio $\mathcal{P}_{K/\pi}$ is then calculated as

$$\mathcal{P}_{K/\pi} = \frac{\mathcal{L}_K}{\mathcal{L}_K + \mathcal{L}_\pi}$$

(5.3)

The performance of kaon identification is checked by measuring the decay chain $D^{*+} \to D^0 \pi^+$; $D^0 \to K^- \pi^+$. With $\mathcal{P}_{K/\pi} > 0.6$, the average kaon efficiency and $\pi$ fake rate over $0.5 < p < 4.0$ GeV/c are about 88% and 8.5%.

![Figure 5-5: Separation power of kaon over pion identification provided by different sub-detectors in Belle [58].](image)

As for the likelihood ratio $\mathcal{P}_{K/\pi}$ dependency with the momentum of kaons or pions, this correlation is visible in Fig 5-6 for kaons and pions separately.

**PID for $e$**

The electron identification [58] is based on differences in the shape of the electromagnetic shower and the velocity of electrons and hadrons with same momentum. The following five discriminants are used in the electron identification.

1. the ratio of cluster energy and track momentum
2. the value of $dE/dx$ measured by the CDC
3. matching between the track and ECL cluster
4. cluster shape parameter
Figure 5-6: Scatter plot of the track momentum and the likelihood ratio $P_{K/\pi}$ for kaon (closed red circle) and pion (open blue circle) tracks [58].

5. ACC light yields

A likelihood function for the electron

$$\mathcal{L}_e = \frac{\prod_{i=1}^{5} \mathcal{L}_e(i)}{\sum_{i=1}^{5} \mathcal{L}_e(i) + \sum_{i=1}^{5} \mathcal{L}_{\text{non-e}}(i)}.$$  \hspace{1cm} (5.4)

and non-electron $\mathcal{L}_{\text{non-e}}$ which has an analogous likelihood function to (5.4) hypotheses are constructed by combining the probability density functions from the above five variables enumerated in the list 5.2.1. The likelihood ratio $P_e$ is then calculated as

$$P_e = \frac{\mathcal{L}_e}{\mathcal{L}_e + \mathcal{L}_{\text{non-e}}}.$$  \hspace{1cm} (5.5)

The performance of electron identification is estimated using the dedicated hadronic MC samples. With $P_e > 0.5$, the average electron efficiency over $1.0 < p < 3.0$ GeV/c are about 92%. The average pion fake rates are determined using inclusive
$K^0_S \rightarrow \pi^+\pi^-$ decays and is found to be 0.22\% over $0.5 < p < 3.0$ GeV/$c$. In Fig 5-7 the comparison between the likelihood ratios, for the a normalized amount of events, for both electrons and the pions.

![Figure 5-7: Likelihood ratio $P_e$ for electron (red) and pion (blue) tracks.](image)

**PID for $\mu$**

Muons are heavy charged leptons that lose their energy mainly by multiple scattering in the detector material. If a muon’s momentum is greater than 500 MeV, it can penetrate easily to the outermost part of the detector, the KLM. The KLM hits are associated to the reconstructed track, if they are near the extrapolated track from the CDC and SVD to the KLM. The charged track is then refitted with the associated KLM hits, minimizing the $\chi^2$, defined as the deviation of hits from the track, in the units of the corresponding uncertainties. A likelihood function for the muon identification [58] is calculated based on the following two discriminants.

- the difference between the expected and the actual penetration in the KLM
- the distance between KLM hits and the extrapolated track
The likelihood ratio $P_\mu$ is then calculated as

$$P_\mu = \frac{L_\mu}{L_\mu + L_K + L_\pi}.$$  \hspace{1cm} (5.6)$$

The performance of muon identification is checked using two-photon samples $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$. The efficiency is measured to be around 89% for $P_\mu > 0.9$ and 93% for $P_\mu > 0.1$ over $1.0 < p < 3.0$ GeV/$c$. The average pion fake rates are determined using inclusive $K_S^0 \rightarrow \pi^+\pi^-$ decays and is found to be 1.4% for $P_\mu > 0.9$ and 2.8% for $P_\mu > 0.1$ over $1.5 < p < 3.0$ GeV/$c$. Comparing the muon likelihood ratios for muons and pions in Fig 5-8 on the left the likelihood ratio for muons peaks at 1 and on the right it peaks at 0.

Figure 5-8: Likelihood ratio $P_\mu$ for (a) muon and (b) pion tracks, based on double photons events $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ taken with a center of mass energy of the $\Upsilon(4S)$ [58].

**PID Summary**

The particle identification criteria are summarized in Table 5.2 and were taken from Belle Note (BN) 779 [64] and 1256 [65]. However, in this analysis the particle
5.2. SELECTION CRITERIA

Table 5.2: Particle Identification

<table>
<thead>
<tr>
<th>ID</th>
<th>$p/\pi$</th>
<th>$K/\pi$</th>
<th>$p/K$</th>
<th>eid</th>
<th>MUid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon</td>
<td>none</td>
<td>none</td>
<td>none</td>
<td>MUid &gt; eid</td>
<td>MUid &gt; 0.95</td>
</tr>
<tr>
<td>Electron</td>
<td>none</td>
<td>none</td>
<td>none</td>
<td>eid &gt; 0.1</td>
<td>MUid ≤ eid</td>
</tr>
<tr>
<td>Proton</td>
<td>$P(p</td>
<td>\pi) &gt; 0.6$</td>
<td>none</td>
<td>$P(p</td>
<td>K) &gt; 0.6$</td>
</tr>
<tr>
<td>Kaon</td>
<td>none</td>
<td>$P(K</td>
<td>\pi) &gt; 0.6$</td>
<td>$P(p</td>
<td>K) &lt; 0.4$</td>
</tr>
<tr>
<td>Pion</td>
<td>$P(p</td>
<td>\pi) &lt; 0.4$</td>
<td>$P(K</td>
<td>\pi) &lt; 0.4$</td>
<td>none</td>
</tr>
<tr>
<td>not assigned</td>
<td>0.4 ≤ $P(p</td>
<td>\pi)$ ≤ 0.6</td>
<td>0.4 ≤ $P(K</td>
<td>\pi)$ ≤ 0.6</td>
<td>0.4 ≤ $P(p</td>
</tr>
</tbody>
</table>

ID for muons is slightly different.

Initially when performing the MC study of the MUid (denoted $P_{\mu}$ in the previous PID sections, which is the muon id likelihood, it was set as MUid > 0.1, however, when performing a preliminary check of less than 5% of the Belle data (the validation of the analysis strategy), a significant peak around the $\rho$ mass was visible in the invariant dimuon mass distribution. In consideration of not having a reliable Monte Carlo generator for $e^+e^- \rightarrow \rho \mu^+\mu^-$ at hand, to optimize the search for a $Z'$ candidate the MUid requirement effect was studied in less than 5% of Belle data before the process of box opening. The $\rho$ peak had to be reduced to improve the final invariant mass distribution. From the Belle lepton ID and fake ID study, setting the muon id requirement as MUid > 0.95 gave a reasonable result for the signal so it was used to estimate the signal efficiency based on the signal MC Sample, without a $\rho$ sample for the background.

5.2.2 Pre-selection cuts

The pre selection cuts are aimed at the fundamental parameters of each of the collision events. They are caled pre selection because they are made to the samples before the relevant events to the $Z'$ are selected. The $Z'$ relevant events contain 4 charged tracks and the pre selection cuts are made to all charged tracks, events that have less or more than 4 charged tracks included. In Table 5.3 $|dr|$ stands for
Table 5.3: Pre-selection cuts on each track

<table>
<thead>
<tr>
<th>impact parameter</th>
<th>charge assigned</th>
<th>anti-double-count</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>dr</td>
<td>&lt; 0.2 \text{ cm} &amp;</td>
</tr>
</tbody>
</table>

the absolute value of the distance perpendicular to the beam pipe ($z$) between the Interaction Point (IP) and the Vertex position, where the resulting resonance of the $e^+e^-$ collision decayed, its distribution can be seen in Fig 5-9 on the left. In the same table there is $|dz|$ which is the absolute value of the distance parallel to the beam pipe from the IP to the vertex, its distribution can be seen in Fig 5-9 on the right.

![Impact Parameter dr](image1)

![Impact Parameter dz](image2)

Figure 5-9: Impact parameters distributions for $dr$ and $dz$ for the AAFH generated $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ background channel.

The pre-selection cuts were studied on Monte Carlo simulation of signals and backgrounds, specifically signal MC samples generated by MadGraph 5 and the most significant background channel $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ by AAFH or Diag36. Tables 5.3 and 5.4 show the pre-selection cuts which can be divided into two groups.

- applied to each track, from Table 5.3
- applied to a group of tracks, from Table 5.4

To guarantee a prompt decay of a $Z'$ candidate:
5.2. SELECTION CRITERIA

Table 5.4: Pre-selection cuts on a group of tracks

<table>
<thead>
<tr>
<th>ID</th>
<th># charged tracks</th>
<th>vertex fit</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exclusive</td>
<td>4</td>
<td>$\chi^2_{\text{vertex}} &gt; 0.01$</td>
<td>$R2 &lt; 0.9$</td>
</tr>
</tbody>
</table>

- impact parameters: $|dr| < 0.2 \text{ cm}$ and $|dz| < 1.5 \text{ cm}$, defined before table 5.3.

Figure 5-9 shows $dr$ and $dz$ distributions for the main background MC channel, $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$. 

- to guarantee all tracks came from the same vertex a cut was imposed over $\chi^2_{\text{vertex}}$ of the vertex fit. Figure 5-10 shows the $\chi^2_{\text{vertex}}$ distribution for the main background MC channel, it is visible there is a peak around 5 which is the number of degrees of freedom involved between matching the vertex with the 4 outwards charged tracks.

The “anti-double-count” from Table 5.3 refers to a cut on the opening angle between two charged tracks: above $3^\circ$, to avoid multiple counting of a single particle. Figure 5-11 shows the opening angle between two charged particles distribution, with a very big peak for very small angles, the reason to cut them, since it corresponds for particles that were counted multiple times.

In Table 5.4 there is the only channel, which is “Exclusive”, investigated. 4 charged tracks is a requirement that significantly reduces neutral sources of background. The R2 (or fox2, $H_2$ in Eq 5.7) in Table 5.4 is a cut on the Fox-Wolfram 2nd moment.

$$H_l = \sum_{ij} \frac{p_i \cdot p_j}{E_{\text{tot}}^2} P_l(\cos \theta_{ij}) \quad (5.7)$$

Equation 5.7 is the general definition of the Fox Wolfram moments [66], which was originally defined as a way to distinguish jet like events and spherical distributed events in $e^+e^- \rightarrow q\bar{q}$ collisions. Where $p_i$ and $p_j$ are the momenta of two charged particles, $E_{\text{tot}}^2$ is the collision energy squared, $P_l(\cos \theta_{ij})$ are the Legendre Polynomials, and $\theta_{ij}$ the angle between the charged particles. Since the relevant cut is over the
second Fox-Wolfram moment, \( l = 2 \rightarrow P_2(x) = \frac{1}{2}(3 \cos^2 \theta_{ij} - 1) \).

The main purpose of requiring the 2nd Fox-Wolfram moment \( H_2 \) in Eq 5.7 or \( R^2 \) < 0.9 is to guarantee there are no jet like events, in which \( R^2 = 1 \), and that most of the events selected are spherically distributed, where \( R^2 = 0 \).

Figure 5-12 shows the Fox-Wolfram 2nd moment distribution for charged tracks, it is slightly smooth with 2 distinct peaks, that are due to different pairings from the muons in the final state, \( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- \).

![Vertex \( \chi^2 \) distribution](image)

**Figure 5-10: Vertex \( \chi^2 \) distribution**

![Crossing angle between particles](image)

**Figure 5-11: Opening angle between charged particles.**
5.2. SELECTION CRITERIA

5.2.3 Final states selection

The selection of the final state is pretty straightforward since we are considering a $Z'$ candidate decaying into a $\mu$ pair.

All possible combinations of oppositely charged muons from the final state, $e^- e^+ \rightarrow \mu_1^+ \mu_1^- Z' \rightarrow \mu_1^+ \mu_1^- \mu_2^+ \mu_2^-$ are taken into account. Since $Z'$ is assumed to be promptly decayed, not allowing a distinction between which muon pair was a result of its decay or not.

- $Z'_1 \rightarrow \mu_1^+ \mu_1^-$
- $Z'_2 \rightarrow \mu_1^+ \mu_2^-$
- $Z'_3 \rightarrow \mu_2^+ \mu_1^-$
- $Z'_4 \rightarrow \mu_2^+ \mu_2^-$

The final state consists of 4 possible dimuon pairings taken as the sources to reconstruct a $Z'$ candidate, in the case of real data, since there should be only one $Z'$ candidate per event, counting all of the possible combinations end up overestimating the amount of background and signal. However, when doing the Monte Carlo study
CHAPTER 5. MONTE CARLO STUDY

Table 5.5: Energy in Center-of-Mass, HER and LER energies

<table>
<thead>
<tr>
<th>E_{cm} [GeV]</th>
<th>E_{e^-} [GeV]</th>
<th>E_{e^+} [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S</td>
<td>7.130243</td>
<td>3.119481</td>
</tr>
<tr>
<td>2S</td>
<td>7.575786</td>
<td>3.314406</td>
</tr>
<tr>
<td>3S</td>
<td>7.827414</td>
<td>3.42449</td>
</tr>
<tr>
<td>4S</td>
<td>7.998213</td>
<td>3.499218</td>
</tr>
<tr>
<td>5S</td>
<td>8.216371</td>
<td>3.594662</td>
</tr>
<tr>
<td>continuum</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

it is possible to select only the muon pair which decayed from the single $Z'$, this was done to improve the detection efficiency.

5.2.4 Data Set

Data corresponding to two different skims, which are slices of Belle event full data containing only specific physics channels aimed to be analyzed, will be combined by taking care that no double counting is done when reconstructing possible events:

- Hadron BJ Skim, a hadronic data sample containing $e^+e^- \rightarrow B\bar{B}$ and $e^+e^- \rightarrow c\bar{c}(J/\psi)$ \[67\]

- tau pair B Skim, a leptonic data sample containing events not in Hadron BJ and containing $e^+e^- \rightarrow \tau^+\tau^-$ \[68\]

These skims were selected given the detection efficiency calculation for their MC samples, which were compatible to the no skimmed specified samples corresponding to the energies listed in Table \[5.5\] The luminosity of the selected skims add up to 977fb$^{-1}$.

5.2.5 Main Selection Criteria

The analysis strategy consists of looking over the reduced mass distribution of dimuon invariant masses, $m_R = \sqrt{m_{\mu^+\mu^-}^2 - 4m_{\text{PDG}}^2}$ for the different $Z'$ masses signal
Table 5.6: Summary of selection criteria

<table>
<thead>
<tr>
<th>Target Particles</th>
<th>cut value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charged Particles</td>
<td>4 charged tracks requirement, at least 2 ( \mu ) id</td>
</tr>
<tr>
<td>Neutral Particles ( \rightarrow \mu^+\mu^- )</td>
<td>( E^i_{\text{cluster}} &gt; 30\text{MeV} ) &amp; ( \sum_i E^i_{\text{cluster}} &lt; 200 \text{MeV} )</td>
</tr>
<tr>
<td>( \Upsilon(3S,2S) \rightarrow \Upsilon(1S) \rightarrow \mu^+\mu^- )</td>
<td>( \Upsilon(1S) - 100\text{ MeV} &gt; m_{\mu^+\mu^-} &gt; \Upsilon(1S) + 100\text{ MeV} )</td>
</tr>
<tr>
<td>( J/\psi \rightarrow \mu^+\mu^- )</td>
<td>( J/\psi - 30\text{ MeV} &gt; m_{\mu^+\mu^-} &gt; J/\psi + 30\text{ MeV} )</td>
</tr>
<tr>
<td>( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- )</td>
<td>( E_{\text{CMS}} - 500\text{MeV} &lt; m_{4\mu} &lt; E_{\text{CMS}} + 500 \text{ MeV} )</td>
</tr>
</tbody>
</table>

MC samples, seen in Fig 5-16 along with the surviving background channels, seen in Fig 5-13 for the \( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- \) channel.

Figure 5-13: In red there is the reduced mass distribution, which essentially causes a slight leftwards shift of the distribution in blue, which is the invariant dimuon mass for the MC generated \( e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^- \) background sample.

Considering the specific decay channel for \( Z' \rightarrow \mu^+\mu^- \) we will look for 4 charged final states and try to reconstruct possible \( Z' \) candidates using a muon pair. Table 5.6 lists final selection criteria adapted from BABAR previous analysis and tuned for the Belle detector.

Following the same order starting from the first row as in table 5.6 these were
analogous cuts to the BABAR analysis:

1. 4 charged tracks requirement for every event and 2 $\mu$ to have a full $\mu$ likelihood (MUId in table 5.2)

2. Based on the Electromagnetic Calorimeter (ECL), described in chapter 3, a requirement that the sum of energies of electromagnetic clusters, the secondary decays caused by high energy particles interacting with dense matter, not associated with any charged tracks with energy above 30 MeV be less than 200 MeV.

3. Some of the data taken at Belle were during collisions with center of mass energy equal to $\Upsilon(3S,2S)$. Since these particles could decay into $\Upsilon(1S)$ then decay into a muon pair, the $\Upsilon(1S)$ resonance also needs to be vetoed.

4. As mentioned in subsection 5.2.6, some of the data samples planned on being analyzed might contain $J/\psi \rightarrow \mu^+\mu^-$ decays. Therefore, this decay could cause a resonance similar to a $Z'$ signal, so the $J/\psi$ resonance has to be vetoed.

5. The final cut is to guarantee that events which had a Initial State Reaction, where a photon would take some of the collision energy making the initial state and final state energies difference, are included in the analysis and not rejected.

Besides the cuts described above a Four Constraint Fitter shown on Table 5.7, described in detail by BN 1238 [69], was used to improve the resolution of the signal resonance region, this is done essentially by constraining the initial state four momentum, $\mathbf{T} = (T^E, T^x, T^y, T^z)$ to the final state four momentum of the four charged tracks, $\mathbf{P}_{1234} = (E_i, P^x_i, P^y_i, P^z_i)$.

Regarding the Electromagnetic Calorimeter cut, $E_\gamma > 0.03 \text{ GeV} \& \sum E_{\text{cluster}} < 0.2 \text{ GeV}$, on Figure 5-14 there are two ECL cluster deposited energy plots, by non charged particles (photons), energy sums. The left one without the cut and the right one with the cut. The peak close to 0 is due to the clusters with 0 energy, events where there
5.2. SELECTION CRITERIA

Table 5.7: Four Constraint Fitter

\[ P_1 + P_2 + P_3 + P_4 - T = 0 \]
\[ \sqrt{P_{1}^2 + m_1^2} + \sqrt{P_{2}^2 + m_2^2} + \sqrt{P_{3}^2 + m_3^2} + \sqrt{P_{4}^2 + m_4^2} - T_E = 0 \]

are no photons not associated to charged tracks the energy deposited in the ECL crystal clusters would be null.

Figure 5-14: On the left energy deposited on ECL clusters not associated with any charged track without restrictions. On the right, the same but only photons with greater than 30 MeV energies, and ECL clusters with less than 200 MeV deposited energies.

To tune the cut aimed at rejecting $J/\psi \to \mu^+\mu^-$ the $J/\psi$ resonance peak was fitted in Figure 5-15. After discovering its width though the fit, any event containing a dimuon pair with a mass in the vicinity of the $J/\psi$ mass was rejected, $m_{J/\psi} \pm 30$ MeV. Meaning that the four different muon pair combinations are rejected. This completely reduces $J/\psi$ background sources.

A triple gaussian was used to fit the $J/\psi$ resonance peak in Fig 5-15 because of the momentum resolution degradation at the endcaps of the detector. As a result,
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Reduced Di muon mass [GeV/c^2]

Figure 5-15: Triple gaussian fit on the \( J/\psi \) resonance peak to determine its width and then reject any event containing a dimuon pair whose mass is around \( m_{J/\psi} \pm 30\text{MeV} \).

the \( J/\psi \) resonance tail components are wider than a single or a double gaussian could match.

5.2.6 Detection Efficiency

In order to obtain precisely the width of the \( Z' \) resonance from the dimuon reduced mass distributions in Figure 5-16. The MC signal samples generated by Madgraph have an extra requirement. Since from the generator level it is possible to set a tag for muons decaying from the \( Z' \). So that it is possible to tell which muon pair actually decayed from the \( Z' \), allowing for the exclusive selection of the “correct” muon pair combination, instead of considering all of the final state muons pairing, which is done for the real data or background MC samples.

Some MadGraph samples including ISR (Initial State Radiation) interactions were also considered using MadGraph special options, in events corresponding to the diagrams in figure 5-2. The resulting reduced mass distribution are different from the ones without ISR, seen in Figure 5-17, the cross section is also affected by ISR.

Even tough the ISR samples have a different signal shape it is still possible to use
5.2. **SELECTION CRITERIA**

![Graphs showing distribution of dimuon reduced mass for different Z' masses MC samples](image)

**Figure 5-16:** Distribution for the dimuon reduced mass of 3 different Z' masses MC samples, 0.7, 5.0, and 10.0 GeV/c^2. The number of entries is larger for the 5.0 GeV/c^2 case, as this mass range is closer to the KLM (K-Long Muon) acceptance.

![Graphs showing distribution of dimuon reduced mass for different Z' masses MC samples](image)

**Figure 5-17:** Distribution of the dimuon reduced mass of 3 different Z' masses MC samples, 0.712, 5.212, and 10.0 GeV/c^2. The asymmetric tail of the peak gaussian shape prevents us from using an identical fitting procedure.

them to consider the detection efficiency. Since the analysis was performed using non ISR samples for the signal and background a correction will be necessary to increase
### 5.2.7 Background estimation

The background channels from the *PHOKHARA* [70], a $e^+e^-$ annihilation into hadrons, plus an energetic photon from the initial state radiation, next to leading order, cross section MC generator. Tailored for hadronic cross sections produced in the

---

**Table 5.8: Signal samples cut efficiencies**

<table>
<thead>
<tr>
<th>Sample masses (GeV/c$^2$)</th>
<th>0.212</th>
<th>1.0</th>
<th>2.0</th>
<th>3.1</th>
<th>6.0</th>
<th>7.0</th>
<th>8.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial entries</td>
<td>217800</td>
<td>240024</td>
<td>268220</td>
<td>275212</td>
<td>285992</td>
<td>290808</td>
<td>289544</td>
<td>80228</td>
</tr>
<tr>
<td>if $E_\gamma &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>88.8%</td>
<td>90.5%</td>
<td>92.3%</td>
<td>93.3%</td>
<td>93.2%</td>
<td>93.0%</td>
<td>92.4%</td>
<td>86.7%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>98.2%</td>
<td>98.8%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>100%</td>
<td>100%</td>
<td>99.9%</td>
<td>100%</td>
</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>99.2%</td>
<td>99.1%</td>
<td>99.1%</td>
<td>0.73%</td>
<td>98.5%</td>
<td>97.9%</td>
<td>98.7%</td>
<td>99.8%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.8%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
</tr>
<tr>
<td><strong>Total Efficiency</strong></td>
<td>86.3%</td>
<td>88.5%</td>
<td>91.3%</td>
<td>6.8%</td>
<td>91.7%</td>
<td>90.9%</td>
<td>91.1%</td>
<td>86.6%</td>
</tr>
</tbody>
</table>
5.2. SELECTION CRITERIA

Figure 5-18: Summary of the cuts final efficiency as a function of the $Z'$ mass.

Table 5.9: Background cut efficiencies

<table>
<thead>
<tr>
<th>Sample sources</th>
<th>$\mu\mu\mu$</th>
<th>$ee\mu\mu$</th>
<th>$\tau\tau\mu\mu$</th>
<th>$\mu\mu$</th>
<th>$\tau\tau$</th>
<th>$\pi^0\pi^0\pi\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial entries</td>
<td>1.15 x10^6</td>
<td>60</td>
<td>52</td>
<td>396</td>
<td>196</td>
<td>95136</td>
</tr>
<tr>
<td>$E_\gamma &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>90.9%</td>
<td>73.3%</td>
<td>69.2%</td>
<td>64.6%</td>
<td>67.3%</td>
<td>39.1%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>99.2%</td>
<td>100%</td>
<td>100%</td>
<td>98.8%</td>
<td>99.2%</td>
<td>100%</td>
</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>98.9%</td>
<td>100%</td>
<td>97.2%</td>
<td>100%</td>
<td>98.6%</td>
<td>99.2%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.9%</td>
<td>90.9%</td>
<td>0%</td>
<td>55.7%</td>
<td>60.3%</td>
<td>0.04%</td>
</tr>
<tr>
<td><strong>Total Efficiency</strong></td>
<td>89.6%</td>
<td>66.7%</td>
<td>0%</td>
<td>35.6%</td>
<td>40.3%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

1-10 GeV center of mass energy. And BABA [71] a high precision QED calculation, with 0.1% theoretical accuracy of two photon production in $e^+e^-$ annihilation, MC generator for Bhabha scattering events for the range between 1-10 GeV. Channels from both generators amount to 0 events due to the four charged tracks requirement associated with mass conservation and photons unassociated to charged tracks. It is relevant to study these channels to guarantee there is no event in the final mass distribution from unwanted sources to the relevant signal searched.

The surviving background channels cut efficiencies are displayed in Tables 5.9 and 5.10.
Table 5.10: Background cut efficiencies

<table>
<thead>
<tr>
<th>Sample sources</th>
<th>$\pi\pi\pi$</th>
<th>$KK$</th>
<th>$\mu\mu$</th>
<th>$A\bar{A}$</th>
<th>$\pi\pi^0$</th>
<th>$\pi\pi$</th>
<th>$p\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial entries</td>
<td>191084</td>
<td>128</td>
<td>1556</td>
<td>38088</td>
<td>148</td>
<td>1468</td>
<td>32</td>
</tr>
<tr>
<td>if $E_{\gamma} &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>2.2%</td>
<td>37.5%</td>
<td>59.6%</td>
<td>0.6%</td>
<td>5.4%</td>
<td>55.3%</td>
<td>12.5%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $Y(1S) \pm 0.1$</td>
<td>100%</td>
<td>100%</td>
<td>99.0%</td>
<td>100%</td>
<td>100%</td>
<td>99.4%</td>
<td>100%</td>
</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>97.5%</td>
<td>0%</td>
<td>99.2%</td>
<td>87.5%</td>
<td>0%</td>
<td>99.7%</td>
<td>0%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{CMS} \pm 0.5$</td>
<td>0.6%</td>
<td>50%</td>
<td>69.9%</td>
<td>0%</td>
<td>0%</td>
<td>67.5%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Total Efficiency</strong></td>
<td>0.01%</td>
<td>18.8%</td>
<td>41.3%</td>
<td>0%</td>
<td>0%</td>
<td>36.9%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Final entries</td>
<td>24</td>
<td>24</td>
<td>643</td>
<td>0</td>
<td>0</td>
<td>543</td>
<td>4</td>
</tr>
</tbody>
</table>

After applying the cuts and scaling. The surviving major background channels respective to the generated cross section, number of initial events and files, is $AAFH e^+ e^- \rightarrow \mu^+ \mu^- \mu^+ \mu^-$. 

![Surviving Background channels after all cuts](image.png)

Figure 5-19: Surviving background channels plotted together, dominated by $e^+ e^- \rightarrow \mu^+ \mu^- \mu^+ \mu^-$. 

Most of the cuts leave the signal samples untouched, however this is also true for the most relevant background, $AAFH e^+ e^- \rightarrow \mu^+ \mu^- \mu^+ \mu^-$, since $EVTGEN e^+ e^- \rightarrow \mu^+ \mu^- J/\psi, J/\psi \rightarrow \mu^+ \mu^-$ and $e^+ e^- \rightarrow \pi^+ \pi^- J/\psi, J/\psi \rightarrow \mu^+ \mu^-$ are 0 after the $J/\psi$ resonance peak region rejection within $\pm 30$ MeV.
5.3 Reconstruction

5.3.1 Fitting

In order to get the signal shape, initially a fit of a normalized double gaussian is attempted, as in Appendix 3 over histograms such as the ones on Figure 5-16 with the six parameters, heights, means and widths allowed to float in a broad interval. This does not result in a usable fit, but into a way to parameterize the MC $Z'$ signal gaussian width and then proceed to get a better fit. The fit process is performed using the ROOT analysis framework, the minimization algorithm used is Minuit which was translated and adapted from Fortran to C++ in the ROOT framework [72].

$$f(x) = \frac{A}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{B}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} \quad (5.8)$$

Figure 5-20: Evolution of the double gaussian used on the signal MC samples for both widths, with a 9th order polynomial fitted later to be used on the parametrization.

After that we perform a fit of the MC signal sample using a triple normalized gaussian in which the widths and the fitting range are initially defined using the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>p0</td>
<td>0.00237 ± 0.002137</td>
<td>0.0005428</td>
</tr>
<tr>
<td>p1</td>
<td>0.001328 ± 0.000755</td>
<td>0.0001999</td>
</tr>
<tr>
<td>p2</td>
<td>-0.000545 ± 0.0001999</td>
<td>0.0002755</td>
</tr>
<tr>
<td>p3</td>
<td>0.0001999 ± 0.0000000</td>
<td>0.0000000</td>
</tr>
<tr>
<td>p4</td>
<td>-2.023e-06 ± 1.304e-08</td>
<td>8.026e-08</td>
</tr>
<tr>
<td>p5</td>
<td>-1.915e-06 ± 1.459e-09</td>
<td>7.105e-09</td>
</tr>
<tr>
<td>p6</td>
<td>7.148e-06 ± 1.672e-10</td>
<td>5.135e-10</td>
</tr>
<tr>
<td>p7</td>
<td>1.054e-07 ± 1.857e-11</td>
<td>9.061e-11</td>
</tr>
<tr>
<td>p8</td>
<td>-1.448e-06 ± 1.803e-12</td>
<td>6.138e-12</td>
</tr>
<tr>
<td>p9</td>
<td>5.643e-10 ± 1.746e-13</td>
<td>9.061e-13</td>
</tr>
</tbody>
</table>

$2\sqrt{M[GeV/c]^2}$
parametrization from Figure 5-20. That is, using the previous fit base gaussian width as an initial guess for the triple gaussian fit, for each $Z'$ candidate mass following:

$$g(x) = \frac{A}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu)^2}{2\sigma_1^2}} + \frac{B}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu)^2}{2\sigma_2^2}} + \frac{C}{\sqrt{2\pi}\sigma_3} e^{-\frac{(x-\mu)^2}{2\sigma_3^2}}$$  \hspace{1cm} (5.9)$$

The fit window is displayed in table 5.11. With the parametrization of the MC $Z'$ signal widths using the double gaussian fits. The width of the “thinner” gaussian is used as a parameter, $\sigma$, to define the fitting interval, this is because its behavior as a function of the $Z'$ is smoother, as seen on the left of figure 5-20. The fit is done centered in the reduced mass of the $Z'$ mass hypothesis since the reduced mass distribution is being used.

$$m_{R_{Z'}} = \sqrt{m_{Z'}^2 - 4m_{\mu+\mu}^2}$$  \hspace{1cm} (5.10)$$

As a way to check the behavior of the triple gaussian fit the weighted width of the three gaussians can be calculated. Firstly the fraction of each of the individual gaussians need to be normalized and calculated using:

$$f_1 = \frac{A}{A+B+C} \quad f_2 = \frac{B}{A+B+C}$$  \hspace{1cm} (5.11)$$

with the fractions above one can quantify the weight of each width,

$$\sigma_{\text{weighted}} = \sqrt{f_1\sigma_1^2 + f_2\sigma_2^2 + (1 - f_1 - f_2)\sigma_3^2}$$  \hspace{1cm} (5.12)$$

the weighted width as a function of the $Z'$ mass is shown in figure 5-21. Its behavior is reasonably smooth with the exception of the higher masses weighted...
widths fluctuations, this is due to the very wide peaks in heavier $Z'$ MC samples, which lead to even wider fitting intervals.

Figure 5-21: Weighted width from the triple gaussian fit as a function of the $Z'$ mass, calculated using Eq 5.12 by different $Z'$ masses

To calculate the amount of background events, a 3rd order polynomial

$$p(x) = a + bx + cx^2 + dx^3,$$  \hspace{1cm} (5.13)

is integrated in the triple gaussian mean vicinity calculated with the weighted width: $m_{R_{Z'}} \pm 3\sigma_{\text{weighted}}$.

After the fit for each of the samples the signal shape parameters are extracted and the 90% confidence level limit on the number of observed events, the signal yield. To obtain the 90% Upper Limit the parameters from the Triple Gaussian fit, Figure 5-22 for each of the $Z'$ different masses are used, and a new triple gaussian with each of its widths fixed plus a 3rd order polynomial is defined.
Figure 5-22: The black continuous line is the triple gaussian fit over signal sample for $m_{Z'} = 0.5\text{GeV/c}^2$, the dashed green line is 3rd order polynomial fitted over the background MC histogram.

$$t(x) = D \left( \frac{A}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu)^2}{2\sigma_1^2}} + \frac{B}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu)^2}{2\sigma_2^2}} + \frac{C}{\sqrt{2\pi}\sigma_3} e^{-\frac{(x-\mu)^2}{2\sigma_3^2}} \right) + (x_0 + ax + bx^2 + cx^3)$$

(5.14)

Using this function we fit the background reduced mass distribution histogram in the same window around the MC signal peak, only allowing the triple gaussian component fraction in Eq 5.14. $D$ and the 3rd order polynomial components, $x0, a, b, c$
in Eq. 5.14 to float, while the triple gaussian signal shape parameters were fixed and parameterized as a function of the $Z'$ mass, they can be seen in detail in appendix C.

Figure 5-23: Single Gaussian with fixed signal width and 3rd order polynomial component fit over background, in pink, in this case displaying a negative fluctuation.

In figure 5-23 the fit of signal MC and the background can be seem closely, this is an example of how the search for a bump is conducted in the real data sample case.

5.3.2 Bias Check

To check the accuracy of the fit its important to check its residuals distribution to look for any unwanted tendencies, besides a Toy MC study was performed to check if the fit procedure had any biases.

The toy Monte Carlo study of the fit was done by generating a random number of entries following a Poisson distribution with mean equal to the number of entries present in the fitted histogram region, Fig. 5-23, the region around the signal peak. After the number of entries is defined, a random number generator is used to fill a new histogram following the fitted 3rd order polynomial distribution. We refit this histogram using Eq. 5.14, and we extract its pull distribution, that is acquired by
repeating the fit 10000 times in the same range. Different entries number for each fit but the same function, allowing the triple gaussian fraction and the third order polynomial parameters to float. Then, the ratio between the resulted fraction, “D”, from the fit results, and its error $\frac{D}{D_{\text{err}}}$ is taken to fill the pull distribution of 10000 entries, each entry corresponding to a different fit. Finally, a single gaussian fit is attempted over the pull distribution, and if its resulting mean is compatible with 0, while its width is compatible with 1, the fit is accepted as non biased.

This process was done for all $Z'$ mass hypotheses, but here three samples fitting process toy MC study is shown, $m_{Z'} = 0.5, 6.0, 9.1$ GeV/$c^2$, in Figs 5-25, 5-26 and 5-27, which shows the overall behavior around the small, intermediate and high mass

By looking at the three different $Z'$ samples one can get an idea about the fit behavior on the whole range studied.

Figure 5-24: Plot of the residuals distribution from triple gaussian plus 3rd order polynomial fit.
Figure 5-25: Toy montecarlo generated background sample for the 3rd order polynomial fit centered at 0.5 GeV/c$^2$, refitted 10000 times with Eq. 5.14, on the left, where the yield and error was extracted as the pull on the right, with a simple gaussian fit overlayed.

Figure 5-26: Toy Monte Carlo generated background sample for the 3rd order polynomial fit centered at 6.0 GeV/c$^2$, refitted 10000 times with Eq. 5.14 on the left, where the yield and error was extracted as the pull on the right, with a simple gaussian fit overlayed.
Figure 5-27: Toy Monte Carlo generated background sample for the 3rd order polynomial fit centered at 9.1GeV/c², refitted 10000 times with Eq. 5.14 on the left, where the yield and error was extracted as the pull on the right, with a simple gaussian fit overlayed.

5.4 Expected Results

5.4.1 90% Confidence Level Estimation

To obtain the upper limit on the cross section for $Z'$ using $\sigma = \frac{N}{\epsilon BR_{\text{Rec}}}$, it is first necessary to set an upper limit on the signal yield. Besides the already calculated detection efficiency and the branching ratio from Figure 2-5 as well as the Belle
expected luminosity of 977fb$^{-1}$.

Using the resulting number of observed events as $D$, from the previous fit 5.14, a single gaussian is defined as the probability density function, $pdf$ with mean $A = D$, the amplitude, and width as $Ds$ error, it can be seen in figure 5-28

![Figure 5-28: Probability Density Function assumed as a single gaussian with mean defined as previous, gaussian + 3rd order poly fit, gaussian height, the black line sets the positive limit from which to start counting the number of observed events.](image)

And then integrate it in the 90% interval. In the displayed case on Figure 5-23 $A$ is negative, the $pdf$ is truncated at 0 and integrated to 90% counting from 0. This procedure is done for all of the 107 $m_{Z'}$ candidates and using the $\sigma = \frac{N}{\sqrt{BR \epsilon}}$ one could obtain the expected cross section upper limit in figure 5-29.

The significance in Fig 5-30 was studied for the different $m_{Z'}$ values using

$$S = \text{sign}(N)_{\text{obs}} \sqrt{\frac{\chi^2_{\text{gaus+pol}}}{\chi^2_{\text{pol}}}}$$

(5.15)

where $\text{sign}(N)_{\text{obs}}$ is the sign of the number of observed events, that could be positive or negative, and $\chi^2_{\text{gaus+pol}}/\chi^2_{\text{pol}}$ is the ratio between the 3rd order polynomial plus the triple gaussian, equation 5.14 fit over the background $\chi^2_{\text{gaus+pol}}$ from the 3rd
Visible Cross Section (from background count) by Mass

Figure 5-29: Expected Cross section upper limit considering model dependent – in red, with the Branching Ratio – and model independent – in black, without the Branching Ratio – cases. This result was considering the Belle biggest luminosity (711 fb\(^{-1}\)) for data taken at \(\Upsilon(4S)\) center of mass energy.

order polynomial, equation 5.13 only. The result for the 107 \(Z'\) mass hypotheses is shown in figure 5-30.

Since there is no signal in the background MC sample for \(e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-\) it was expected that the behavior of the significance in figure 5-30 following equation 5.15, would be mostly flat around 1 or close to 0. This would be caused by \(\chi^2_{\text{gaus+pol}} = \chi^2_{\text{pol}}\) or \(\chi^2_{\text{gaus+pol}} < \chi^2_{\text{pol}}\) respectively. However, that is not what happened, as seen in figure 5-30. The observed behavior can be explained by the fact that each fit range is different, after all, the \(Z'\) resonance width increases with its mass, as shown earlier in figure 5-17. Given the fluctuations in the reduced dimuon mass distribution of the background MC sample, the changes in the fitting range result in fluctuations for \(\chi^2_{\text{gaus+pol}}\) as well.

5.4.2 MC signal sample 90% CL \(g'\) coupling

From the cross section distribution in Figure 5-29, we multiply it by the corresponding branching ratio from Fig 2-5, for each of the \(Z'\) candidate masses, after that
5.4. EXPECTED RESULTS

Figure 5-30: Significance fluctuations for different values of $m_{Z'}$.

we finally extract the $g'$ value from the theoretical coupling, $g'_{th} = 0.1$, times the square root of the ratio between the evaluated cross section and the “theoretical” one from the Madgraph simulation output in Figure 5-3

$$g'_{val} = g'_{th} \times \sqrt{\frac{\sigma}{\sigma_{th}}} \quad (5.16)$$

The result is in Figure 5-31 which is the $g'$ as a function of different $Z'$ masses for the MC background scaled to the full Belle luminosity 977 fb$^{-1}$.

As expected, due to the size of Belle’s achieved luminosity the $g'$ upper limit we could obtain a more competitive region than BABAR previous result.
Figure 5-31: $g'$ 90% UL from the expected $Z'$ cross section in black, for this analysis, and in red for the BaBar previous result.
Chapter 6

Results

6.1 Comparison of the MC and Real Data reduced mass distributions

After the conclusion of the MC study and the definition of the cuts and selection criteria to be applied to the real data samples The Belle collaboration assigned a referee committee to decide if we could open the box, this is to guarantee a blind analysis method, where the real data is only studied when the analysis strategy is already defined.

The final test before looking into the real data sample was a comparison between 5% of the Belle data and the MC background sample for the relevant surviving channel, \(e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-\).

When comparing 5% of the Belle sample with the surviving scaled MC background sample in figure 6-1 it was expected that the the MC reduced mass distribution (red) level would be higher then the real data (black) case since the real data suffers the effects from the ISR (Initial State Radiation) while the MC background sample compared does not. The explanation for this mismatch between expectation and observation is the presence of the \(\rho\) resonance in the real data case, around 0.7 GeV,
CHAPTER 6. RESULTS

Reduced Di muon mass [GeV/c^2]

Figure 6-1: Real data, 5% of Belle sample, comparison with the surviving scaled background channel.

this was not simulated in the MC sample. However, since the $\rho$ is located in a region where the $Z'$ has a narrow width, and its own width is larger than the $Z'$s it is not a problem.

After the 5% data/MC check the same comparison was made between the bulk data analysed, figure 6-2, taken with $E_{\text{cm}} = \Upsilon(4S)$, which corresponds to the biggest luminosity sample, as seen in table 6.1 and the surviving MC background sample, $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$. In figure 6-2 the behavior difference between MC sample and real data gets smaller with the exception of the $\rho$ resonance and its reflection. It is visible that there is a relevant peak at the $\rho$ mass, 0.77 GeV/c^2, and its “reflection” at the remaining mass, 9.8 GeV/c^2 for the real data, these peaks are not present in the MC sample.

To confirm that this “reflection” bump in the real data reduced mass distribution
6.1. COMPARISON OF THE MC AND REAL DATA REDUCED MASS DISTRIBUTIONS

Figure 6-2: Comparison between Belle data biggest luminosity sample and the MC surviving channel.

is due to the $\rho$ peak, a conditioned distribution was developed. As explained before in the final part of chapter 5 for each valid event there are four entries due to the different final state muon pairings. So two new reduced dimuon mass distributions were made:

- **$\rho$ peak and its reflection** consists of events of the four possible combinations that have a reduced mass in $m_R = 9.7 \pm 0.15$ GeV/$c^2$ seen in the distribution on the left of Fig 6-3

- **NOT $\rho$ or its reflection** consists of events outside the first condition above, seen in the distribution on the right of Fig 6-3

A similar study was made for the $J/\psi$ that is not as relevant as the $\rho$ peak for the real data reduced mass distribution in Fig 6-2 but it can be shown to be completely
Figure 6-3: Reduced mass distributions that correlate $\rho$ peak to its reflection, for Belle $\Upsilon(4S)$ data sample. On the left the muon pair combinations that contain the $\rho$ reflection and on the right the muon pair combinations that do not contain it.

rejected by excluding events that contain any reduced mass in $m_{J/\psi} \pm 30$ MeV/c$^2$ as seen in Fig 6-4.

Using the fixed parameters from signal shape, the triple gaussian widths, and fractions, from the previous chapter 5. The function 5.14 a triple gaussian with a 3rd order polynomial, 10000 mass hypothesis were scanned in the real data from the dimuon threshold 0.212 to 10 GeV/c$^2$ in steps of 0.001 GeV/c$^2$ while the smallest resolution for the MC $Z'$ signal shape is 0.004 GeV/c$^2$. The $J/\psi \pm 30$ MeV was excluded from the fit.

The total samples that were considered by energy range and luminosity are listed in table 6.1.

From 6-5 it is visible that the fit corresponds to slight fluctuations of the data and no significant $Z'$ was observed throughout the whole range analyzed, since every fit has an identical result to this one.

In figure 6-6 the significance was calculated for each of the 10000 fits done for the
6.1. **COMPARISON OF THE MC AND REAL DATA REDUCED MASS DISTRIBUTIONS**

Figure 6-4: Reduced mass distributions showing \( J/\psi \) rejection, for Belle \( \Upsilon(4S) \) data sample. On the left the distribution without the \( J/\psi \) veto and on the right the distribution with \( J/\psi \) veto.

The observed peaks in the cross section as a function of the \( Z' \) mass in figure 6-7 are due to the dimuon threshold, 0.212 GeV/c\(^2\), the \( \rho \) resonance around 0.77549 GeV/c\(^2\) which could not be completely rejected by the cuts employed, and the \( J/\psi \) peak around 3.096916 GeV/c\(^2\) [20] that was rejected. Since the four possible muon

<table>
<thead>
<tr>
<th>( E_{cm} ) [GeV]</th>
<th>name</th>
<th>tauskim A/B ( \mathcal{L} ) [fb(^{-1})]</th>
<th>HadronBJ ( \mathcal{L} ) [fb(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S</td>
<td>1S_scan</td>
<td>8.85338</td>
<td>0.70334</td>
</tr>
<tr>
<td>2S</td>
<td>2S_scan</td>
<td>0</td>
<td>3.5135</td>
</tr>
<tr>
<td>3S</td>
<td>3S_scan</td>
<td>30.8818</td>
<td>2.90674</td>
</tr>
<tr>
<td>4S</td>
<td>on_resonance</td>
<td>696.612</td>
<td>684.385</td>
</tr>
<tr>
<td>5S</td>
<td>5S_scan</td>
<td>151.504</td>
<td>96.1291</td>
</tr>
<tr>
<td>continuum</td>
<td>continuum</td>
<td>85.0875</td>
<td>86.3766</td>
</tr>
</tbody>
</table>
Figure 6-5: Fit example for the real data for the $m'_{Z} = 4.804$ GeV/c$^2$. The dotted blue line is the 3rd order polynomial fit, and the magenta continuous line the 3rd order poly with a triple gaussian.

pair combinations are all taken into account, an effect of the $\rho$ resonance peak is its reflection around $m_{\Upsilon(4S)} - m_{\rho} = 9.8$ GeV, a bump that was not in the simulated reduced mass distribution and which is not rejected by the cuts. As a result, it causes a bump in the cross section result in Fig 6-7. The relationship between $\rho$ and its “reflection”was shown in figure 6-3.

6.2 $g'$ Upper Limit for the Belle data

With the cross section upper limit, the $g'$ coupling between $Z'$ and the muons was calculated using the full Belle data sample using equation 5.16.

As expected from the detection efficiency behavior in figure 5-18, the $g'$ is inversely proportional to the number of observed events, equation 5.16, so with the increase in $Z'$ mass the detection efficiency worsens, the upper limit on the number of observed
events decreases and the $g'$ coupling increases.

### 6.3 Systematics

The different sources of systematic errors are:

- **integrated luminosity**: 1% (BN 982) The luminosity systematic uncertainty was measured using Bhabha and double photon events, $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \gamma \gamma$, very high cross section channels

- **track ID**: 1% (BN 1165) The track ID systematic error is calculated by comparing the track finding efficiency of partially and fully reconstructed $D^*$ decays, $D^* \rightarrow \pi D^0$, $D^0 \rightarrow \pi \pi K^0_s$ and $K^0_s \rightarrow \pi^+\pi^-$ in data and Monte Carlo so 0.5% for each charged track (in this analysis, 4 charged track were required)
CHAPTER 6. RESULTS

Table 6.2: Error associated with different steps of the analysis

<table>
<thead>
<tr>
<th>Det Eff</th>
<th>Det eff error</th>
<th>BR</th>
<th>90% limit on $N_{\text{Obs}}$</th>
<th>90% limit error</th>
<th>90%$\sigma$ (ab)</th>
<th>90%$\sigma$ er (ab)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>0.0014</td>
<td>0.495</td>
<td>47.29</td>
<td>6.9</td>
<td>300.4</td>
<td>43.7</td>
</tr>
<tr>
<td>0.54</td>
<td>0.0016</td>
<td>0.347</td>
<td>145.03</td>
<td>12.04</td>
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<td>65.4</td>
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<tr>
<td>0.02</td>
<td>0.0004</td>
<td>0.334</td>
<td>8.92</td>
<td>2.99</td>
<td>1699.62</td>
<td>568.9</td>
</tr>
</tbody>
</table>

- identification efficiency 2% (determined by varying MUid) First MUid requirement was that it be greater than 0.1, it was then changed to being greater than 0.2, 1% for each muon id

Apart from the standard previous sources of systematic errors, from the detection efficiency calculation error, as well as for the 90% limit on the number of observed events we can estimate the final error on the cross section with:

$$\frac{\delta\sigma}{\sigma} = \sqrt{\left(\frac{\delta N_{\text{obs}}}{N_{\text{obs}}}\right)^2 + \left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta \epsilon}{\epsilon}\right)^2}$$  \hspace{1cm} (6.1)

Detection efficiency systematic error is obtained analogously to (6.1) with a quadratic sum of the systematic errors from track ID and identification.

As examples $m_{Z'} = 0.5, 5.0, 10.0 \text{GeV}$ are displayed in Table 6.2.

The systematic errors from the fitting process were studied for the signal and background shape separately.

For the signal case fixing the width at bigger and smaller values than the best fit allowed to evaluate the difference between the fit results, and estimate the systematic error. This was done by integrating the different peak ranges in Figures 6.3-1, 6.3-2 and 6.3-3, taking the difference between the integration results of the big width gaussian fit in Fig 6.3-1 and the well adjusted width gaussian fit in Fig 6.3-2.

A triple gaussian fit was performed for the MC sample containing the $J/\psi$, a well known and well defined resonance peak.

As for the background shape the usual fit of the background using a 3rd order polynomial is compared with one using a 4th order one. Analogously to the signal shape the difference between the integration results of the 4th order polynomial and
the 3rd order polynomial were considered.
Figure 6-7: Cross section as a function of $Z'$ mass.
6.3. SYSTEMATICS

Figure 6-8: The $g'$ upper limit as a function of $Z'$ masses for BaBar’s previous search in red and this search with Belle luminosity in black.

Figure 6-9: A fit for the $J/\psi$ peak region fixing the width of the gaussian bigger than necessary.
CHAPTER 6. RESULTS

Figure 6-10: The best fit for the $J/\psi$ peak region.

Figure 6-11: A fit for the $J/\psi$ peak region fixing the width of the gaussian smaller than necessary.
Figure 6-12: Comparison of the same background region fit with 4th order polynomial on the left, and 3rd order polynomial on the right.
Chapter 7

Conclusion, Discussion and Future Prospects

Driven by the absence of WIMP dark matter observations, alternative models were developed for low mass regions (below 10 GeV), a region not probed by WIMP searches previously. Among the alternative models, was the dark photon, $A'$, that could mediate WIMP dark matter annihilation into SM particles, and the muonic dark gauge boson, $Z'$. Both are extra $U(1)$ gauge bosons, the $Z'$ was initially proposed as a portal to sterile neutrino dark matter, but it shares another motivation with the dark photon, $A'$, it could be a possible contribution source to account for the $g - 2$ anomaly.

Considering that many dark photon searches, $A' \rightarrow e^+e^-, \mu^+\mu^-$, have already been made. The dark photon parameter space – kinetic mixing factor $\epsilon$ and the dark photon mass $m_{A'}$ – has been substantially constrained, almost completely rejecting the dark photon as a viable contribution for the $g - 2$ anomaly. Therefore it would be relevant to study $Z'$ not only from the perspective of a possible contribution to the $g - 2$ anomaly but also as a channel to sterile neutrino dark matter, a that assumes specific mass and mixing angle for a new neutrino that has no flavor but mixes with the active neutrino types through the mixing angle factor.
A search for the new gauge boson, $Z'$, was done through its decay into muons, $Z' \rightarrow \mu^+\mu^-$. An initial study using Monte Carlo samples produced by MadGraph was done to set the detection efficiency and signal shapes, a similar study was conducted for the main background channel $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$.

After the conclusion of the MC study, calculation of the expected cross section and $g'$ coupling, the defined analysis strategy was done using Belle data taken at the $\Upsilon(4S)$ center of mass (CM) energy, corresponding to about 711 fb$^{-1}$.

The result was that there is no significant $Z'$ signal, visible by the significance evolution in Figure 6-6. Yet, the most stringent Upper Limit was set for the $g'$ coupling between $Z'$ and the muons, in the range between (the dimuon mass threshold) $0.212 \sim 10 \text{ GeV}/c^2$ except around the $\rho$ resonance peak, in which BABAR still has a more stringent result.

## 7.1 Interpretation of the new Upper Limit on $g'$

In Fig 7-1 Belle’s result is compared with other $Z'$ searches, including BABAR’s previous search in red, $Z'$ through a neutrino search by Trident in blue and the $g−2$ favored region, the green band. As expected the full luminosity from Belle gives a little boost in some regions, considering the Belle luminosity the double of BABAR’s, yet in the $\rho$ resonance peak region, it is visible that the Belle result is not so different than BABAR’s. Apparently for their analysis the $\rho$ region was not clearly distinguishable, yet they did observe the same “reflection” from it. Given that the final result rejects any contribution from $Z'$ interactions for the $g−2$ problem, since no $Z'$ signal was found, at least in the $0.212$ (dimuon mass) $\sim 7.0 \text{ GeV}/c^2$ region by checking Fig 7-1.

The Belle result covered completely the $g−2$ favored region, therefore the coupling between $Z'$ and the muons, $g'$, does not contribute to the discrepancy between measurement and theory on the magnetic moment of the muon $g$ factor.

It is possible to conclude that another search using a bigger data sample, luminos-
Figure 7-1: The $g'$ coupling extracted from the $Z'$ cross section Upper Limit from the Belle data (black). In comparison with the BaBar previous search (red), the $g - 2$ favored region (green) and another search by the neutrino search (blue).

...would not be encouraged if aiming only to investigate $Z'$ contributions into the magnetic moment of the muon anomaly and the proton radius anomaly. Yet, studies looking for lower masses than the dimuon threshold, 0.212 GeV, or higher than 7 GeV, are still a possible way to investigate $Z'$, both motivated by the $g - 2$ anomaly and as a channel for sterile neutrinos.

Belle II would be ideal to look for the $Z'$ sterile neutrino connection, as it covers roughly the same energy range as Belle but with almost 50 times its luminosity, allowing it to probe feebler $g'$ values around $10^{-5}$ corresponding to other sterile neutrino masses and mixing angles.
7.2 Lack of \( Z' \) significant signatures with \( g' \sim 10^{-3} \)

In the range of \( g' \sim 10^{-3} \) in Fig 6-8 which is around \( 0.212 < m_{Z'} < 2 \) GeV/\( c^2 \), satisfying the requirements described in chapter 2 for the \( Z' \) decay into sterile neutrinos, through active neutrino mixing. Sterile neutrinos are a good dark matter candidate, after all, in this \( Z' \) mass range there are sterile neutrino candidates that would correspond to the correct thermal relic density of dark matter (similarly to the ‘WIMP miracle’) depending on their masses and mixing angles. The absence of a significant \( Z' \) signature constrains the \( Z' \to \nu_s \bar{\nu}_s \) in the range mentioned earlier for sterile neutrinos with mass \( m_{\nu_s} = 50, 100 \) keV and mixing angle \( \sin 2\theta_0 = 3.5 \times 10^{-8}, 5 \times 10^{-9} \) respectively, this can be confirmed by looking at the figure 2-3.

However, since the \( Z' \) decay into sterile neutrinos conditioned to smaller \( g' \) couplings has not been well constrained yet, \( Z' \to \text{invisible} \) is currently being investigated in Belle and could still be investigated by new experiments, such as Belle 2 scheduled to reach greater luminosity and better sensitivity for very low masses.

It is pertinent to emphasize that differently from WIMPs the sterile neutrino, as a dark matter candidate, due to its SM gauge singlet character cannot interact with any SM particle directly, therefore it can only be searched by indirect channels such as \( Z' \to \nu_s \bar{\nu}_s \).

7.3 \( Z' \) and \( Z' \) alternate models searches

Currently in Belle there the other possible channels for the \( Z' \) decay, \( Z' \to \nu \nu, \tau \tau \), being investigated. For masses lower than the dimuon threshold \( (0.212 \) GeV/\( c^2 \)) there is the possibility of completely covering \( Z' \) contribution to the \( g - 2 \) problem.

Apart from the usual \( Z' \) model, that was the main subject of this study, a recently a modified version has been proposed. In the modified version besides coupling directly to neutrinos and heavy leptons it would also contain a kinetic mixing, identical to the dark photon case, where the SM hypercharge mixes with the Dark sector charge [75].
Appendix A

In chapter 5 the efficiency for each cut for all $Z'$ masses was mentioned but not shown.

Remembering that the Initial amount of entries listed in the following tables are the amount of entries after the pre selection cuts and the 4 charged track requirement, besides the 2 positive or 2 negative muon id requirement.

They cut efficiencies are :

<table>
<thead>
<tr>
<th>Sample masses (GeV/$c^2$)</th>
<th>0.212</th>
<th>0.22</th>
<th>0.23</th>
<th>0.24</th>
<th>0.25</th>
<th>0.3</th>
<th>0.41</th>
<th>0.42</th>
<th>0.43</th>
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<td>Initial entries</td>
<td>217800</td>
<td>214636</td>
<td>214776</td>
<td>210692</td>
<td>210404</td>
<td>203092</td>
<td>200888</td>
<td>206012</td>
<td>203720</td>
</tr>
<tr>
<td>$E_\gamma &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>88.8%</td>
<td>89.3%</td>
<td>89.2%</td>
<td>89.5%</td>
<td>89.6%</td>
<td>89.5%</td>
<td>89.2%</td>
<td>89.7%</td>
<td>89.3%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>98.2%</td>
<td>98.3%</td>
<td>98.1%</td>
<td>98.1%</td>
<td>98.1%</td>
<td>98.3%</td>
<td>98.5%</td>
<td>98.6%</td>
<td>98.6%</td>
</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>99.2%</td>
<td>99.3%</td>
<td>99.2%</td>
<td>99.2%</td>
<td>99.3%</td>
<td>99.2%</td>
<td>99.2%</td>
<td>99.2%</td>
<td>99.1%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.9%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
</tr>
<tr>
<td>Final entries</td>
<td>188067</td>
<td>186868</td>
<td>186146</td>
<td>183103</td>
<td>183779</td>
<td>177341</td>
<td>174989</td>
<td>180442</td>
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### Table A.2: Signal samples cut efficiencies

<table>
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<tr>
<th>Sample masses (GeV/c²)</th>
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<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
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</thead>
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<tr>
<td>Initial entries</td>
<td>204412</td>
<td>202492</td>
<td>207728</td>
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<td>226808</td>
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<td>240024</td>
<td>242528</td>
</tr>
<tr>
<td>if $E_γ &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>89.4%</td>
<td>89.5%</td>
<td>89.6%</td>
<td>89.5%</td>
<td>90.1%</td>
<td>90.3%</td>
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<td>90.5%</td>
<td>90.8%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>98.5%</td>
<td>98.6%</td>
<td>98.5%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.7%</td>
<td>98.8%</td>
<td>99.5%</td>
</tr>
<tr>
<td>$J/ψ \pm 0.030$ rejection</td>
<td>99.1%</td>
<td>99.2%</td>
<td>99.2%</td>
<td>99.1%</td>
<td>99.1%</td>
<td>99.1%</td>
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<td>99.1%</td>
<td>99.1%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.8%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
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### Table A.3: Signal samples cut efficiencies

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<td>251516</td>
<td>255764</td>
<td>255524</td>
<td>261892</td>
<td>260032</td>
<td>261700</td>
<td>265636</td>
<td>268220</td>
</tr>
<tr>
<td>if $E_γ &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>91.0%</td>
<td>90.9%</td>
<td>91.7%</td>
<td>91.5%</td>
<td>91.5%</td>
<td>92.0%</td>
<td>92.0%</td>
<td>92.0%</td>
<td>92.3%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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<tr>
<td>$J/ψ \pm 0.030$ rejection</td>
<td>99.1%</td>
<td>99.0%</td>
<td>98.9%</td>
<td>99.1%</td>
<td>99.1%</td>
<td>99.0%</td>
<td>99.0%</td>
<td>99.0%</td>
<td>99.1%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.8%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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### Table A.4: Signal samples cut efficiencies

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<td>268700</td>
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<td>273492</td>
<td>272104</td>
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</tr>
<tr>
<td>if $E_γ &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>92.1%</td>
<td>92.6%</td>
<td>92.7%</td>
<td>93.0%</td>
<td>93.0%</td>
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<td>93.3%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
</tr>
<tr>
<td>$J/ψ \pm 0.030$ rejection</td>
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<td>99.0%</td>
<td>99.0%</td>
<td>99.0%</td>
<td>99.0%</td>
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</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
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<td>99.9%</td>
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<td>99.9%</td>
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<td>99.9%</td>
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### Table A.5: Signal samples cut efficiencies

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<td>277048</td>
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<td>93.3%</td>
<td>93.3%</td>
<td>93.5%</td>
<td>93.3%</td>
<td>93.5%</td>
<td>93.3%</td>
<td>93.3%</td>
<td>93.4%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>99.9%</td>
</tr>
<tr>
<td>$J/ψ \pm 0.030$ rejection</td>
<td>97.3%</td>
<td>0.73%</td>
<td>96.8%</td>
<td>96.9%</td>
<td>98.9%</td>
<td>99.0%</td>
<td>99.0%</td>
<td>99.0%</td>
<td>99.0%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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### Table A.6: Signal samples cut efficiencies

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<td>278036</td>
<td>280444</td>
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<td>if $E_γ &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>93.5%</td>
<td>93.5%</td>
<td>93.5%</td>
<td>93.5%</td>
<td>93.5%</td>
<td>93.2%</td>
<td>93.4%</td>
<td>93.4%</td>
<td>93.5%</td>
</tr>
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<td>$m_2µ$ not in $Σ(1S) ± 0.1$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
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<td>100%</td>
</tr>
<tr>
<td>$J/ψ ± 0.030$ rejection</td>
<td>98.9%</td>
<td>98.8%</td>
<td>98.8%</td>
<td>98.9%</td>
<td>98.8%</td>
<td>98.8%</td>
<td>98.7%</td>
<td>98.7%</td>
<td>98.7%</td>
</tr>
<tr>
<td>$m_4µ$ in $M_{\text{CMS}} ± 0.5$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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</tr>
<tr>
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### Table A.7: Signal samples cut efficiencies

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<td>278456</td>
<td>280308</td>
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<td>282968</td>
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<td>93.3%</td>
<td>93.5%</td>
<td>93.5%</td>
<td>93.3%</td>
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<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$J/ψ ± 0.030$ rejection</td>
<td>98.7%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.6%</td>
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</tr>
<tr>
<td>$m_4µ$ in $M_{\text{CMS}} ± 0.5$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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<td>256505</td>
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### Table A.8: Signal samples cut efficiencies

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<td>284236</td>
<td>284928</td>
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<td>283540</td>
<td>283000</td>
<td>284392</td>
<td>284529</td>
<td>284836</td>
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<tr>
<td>if $E_γ &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>93.3%</td>
<td>93.3%</td>
<td>93.3%</td>
<td>93.2%</td>
<td>93.3%</td>
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</tr>
<tr>
<td>$m_2µ$ not in $Σ(1S) ± 0.1$</td>
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<td>100%</td>
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<td>100%</td>
<td>100%</td>
<td>100%</td>
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</tr>
<tr>
<td>$J/ψ ± 0.030$ rejection</td>
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<td>98.5%</td>
<td>98.4%</td>
<td>98.5%</td>
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</tr>
<tr>
<td>$m_4µ$ in $M_{\text{CMS}} ± 0.5$</td>
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<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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### Table A.9: Signal samples cut efficiencies

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<td>288596</td>
<td>288416</td>
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<td>289200</td>
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<td>289184</td>
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<td>93.1%</td>
<td>93.0%</td>
<td>93.0%</td>
<td>93.0%</td>
<td>92.9%</td>
<td>93.1%</td>
<td>93.1%</td>
<td>92.8%</td>
<td>92.8%</td>
</tr>
<tr>
<td>$m_2µ$ not in $Σ(1S) ± 0.1$</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
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</tr>
<tr>
<td>$J/ψ ± 0.030$ rejection</td>
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<td>98.1%</td>
<td>98.0%</td>
<td>97.9%</td>
<td>97.9%</td>
<td>97.7%</td>
<td>97.9%</td>
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</tr>
<tr>
<td>$m_4µ$ in $M_{\text{CMS}} ± 0.5$</td>
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<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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## Table A.10: Signal samples cut efficiencies

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<th>8.2</th>
<th>8.3</th>
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<tr>
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<td>292228</td>
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<td>286940</td>
<td>288584</td>
<td>287004</td>
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<tr>
<td>if $E_\gamma &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>92.8%</td>
<td>92.6%</td>
<td>92.8%</td>
<td>92.8%</td>
<td>92.6%</td>
<td>92.4%</td>
<td>92.3%</td>
<td>92.5%</td>
<td>92.2%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>98.8%</td>
<td>98.9%</td>
<td>98.9%</td>
<td>99.0%</td>
<td>98.7%</td>
<td>98.7%</td>
<td>98.8%</td>
<td>98.8%</td>
<td>98.8%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
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<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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## Table A.11: Signal samples cut efficiencies

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<th>8.8</th>
<th>8.9</th>
<th>9.0</th>
<th>9.1</th>
<th>9.2</th>
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</thead>
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<td>277108</td>
<td>276420</td>
<td>266580</td>
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<tr>
<td>if $E_\gamma &gt; 0.03 \sum E_{\text{cluster}} &lt; 0.2$</td>
<td>91.8%</td>
<td>91.8%</td>
<td>91.6%</td>
<td>91.0%</td>
<td>91.0%</td>
<td>90.2%</td>
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<td>89.2%</td>
<td>88.4%</td>
</tr>
<tr>
<td>$m_{2\mu}$ not in $\Upsilon(1S) \pm 0.1$</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
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</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>98.6%</td>
<td>98.8%</td>
<td>98.7%</td>
<td>98.6%</td>
<td>98.6%</td>
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<td>98.6%</td>
<td>98.6%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.8%</td>
</tr>
<tr>
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<td>234247</td>
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## Table A.12: Signal samples cut efficiencies

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<th>9.8</th>
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<tbody>
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<td>156036</td>
<td>119068</td>
<td>80228</td>
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<td>86.0%</td>
<td>84.5%</td>
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<td>75.8%</td>
<td>75.1%</td>
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<td>99.9%</td>
<td>100%</td>
<td>100%</td>
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</tr>
<tr>
<td>$J/\psi \pm 0.030$ rejection</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.2%</td>
<td>98.3%</td>
<td>98.9%</td>
<td>99.1%</td>
<td>99.5%</td>
<td>99.8%</td>
</tr>
<tr>
<td>$m_{4\mu}$ in $M_{\text{CMS}} \pm 0.5$</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.9%</td>
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<td>123009</td>
<td>97497</td>
<td>69486</td>
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Appendix B

In chapter 5.2.6 a double gaussian fit was mentioned for the parametrization of the $Z'$ peak here an example of that fit is shown for $m_{Z'} = 0.5, 5.0, 10.0$ GeV.

From these Figures, B1, B2 and B3 the width of the $Z'$ is visible. Also, the bad quality of the double gaussian fit is seem with increasing mass.
Figure B-1: This is the best fit using the double gaussian where the width of the $Z'$ is still not so big.

Figure B-2: This is the fit using the double gaussian for $m_{Z'} = 5.0$ GeV where the width of the $Z'$ is bigger, but the tails are still relatively covered.
Figure B-3: This is the fit using the double gaussian for $m_{Z'} = 10.0$ GeV where the width of the $Z'$ is the biggest, but the tails are not well covered, and neither is the peak.
After defining the signal shape as a triple gaussian in chapter 5, in chapter 6 the next step is taken when the number of observed events has to evaluated. This is done by initially performing of the fixed signal shape with the background.

The signal shape plus background is

\[ t(x) = D \left( \frac{A}{\sqrt{2\pi}\sigma_1} e^{\frac{(x-\mu)^2}{2\sigma_1^2}} + \frac{B}{\sqrt{2\pi}\sigma_2} e^{\frac{(x-\mu)^2}{2\sigma_2^2}} + \frac{C}{\sqrt{2\pi}\sigma_3} e^{\frac{(x-\mu)^2}{2\sigma_3^2}} \right) + (x_0 + ax + bx^2 + cx^3), \]

the initial fractions of each of the gaussian components in eq (C.1) is normalized using

\[ f_1 = \frac{A}{A+B+C}, \quad f_2 = \frac{B}{A+B+C}, \quad f_3 = \frac{C}{A+B+C}. \]  

To explicitly list the correspondence between the triple gaussian fraction and its width:

1. fraction \( f_1 \) that is essentially \( A \) normalized, the biggest fraction corresponds to the thinnest width \( \sigma_1 \)

2. fraction \( f_2 \) that is \( B \) normalized, the intermediate fraction corresponds to the intermediate width \( \sigma_2 \)
3. and finally fraction $f_3$ that is $C$ normalized, the smallest fraction corresponds to the broadest width $\sigma_3$.

Figure C-1: All three triple gaussian component fractions with their fitted functions as a function of the $Z'$ mass.
Figure C-2: Biggest fraction of the triple gaussian signal shape as a function of the $Z'$ mass.

Figure C-3: Thinnest width gaussian with its fitted function for all $Z'$ masses.
Normalized Fraction of the triple gaussian between second height with all of them as a function of mass

Figure C-4: Intermediate fraction of the triple gaussian signal shape as a function of the $Z'$ mass.

Second Width of the Triple Gaussian

Figure C-5: Intermediate width gaussian with its fitted function for all $Z'$ masses.
Figure C-6: Smallest fraction of the triple gaussian signal shape as a function of the $Z'$ mass.

Figure C-7: Broadest width gaussian with its fitted function for all $Z'$ masses.
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Note: “Belle Note”, the Belle Collaboration internal report naming scheme.


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