A Graham-type Trade Model with Keynesian Unemployment: Simultaneous Determination of International Values, Wage Rates, and Employment Quantities

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Simultaneous Determination of International Values, Wage Rates, and Employment Quantities

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Summary:

This paper provides a Graham-type trade model with Keynesian unemployment. In the real world, the existence of unemployment, the greater part of which is involuntary, is a normal state. Nevertheless, many of the conventional trade models assume full employment. Although there is a literature which focuses on the relations between international trade and unemployment, the unemployment dealt with in the literature is voluntary in the almost cases. In our model, involuntary unemployment or Keynesian unemployment occurs. Though labor endowments are given, full employment is not presupposed. Rather, employment quantities are determined simultaneously with international values and wage rates.

Keyword:

Multi-country multi-commodity; Link commodities; Keynesian unemployment, Quantity adjustment, F. D. Graham

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Contents with partial summary:

1 Introduction

In Sato (2017b), we provided a full employment version of the Graham-type trade model, which is a multi-country multi-commodity Ricardian trade model and attaches great importance to link commodities. In the present paper, we provide an underemployment version of the model.

Many of the conventional trade models assume full employment: e.g. Heckscher-Ohlin-Samuelson model (Samuelson, 1948, 1949), Dornbusch-Fisher-Samuelson model (Dornbusch et al. 1977), Krugman model laying the foundation of the New Trade Theory (Krugman, 1980), Melitz model forming the basis of the New New Trade Theory (Melitz, 2003). In the real world, however, the existence of unemployment, the greater part of which is involuntary, is a normal state. Therefore, there is also a literature which focuses on the relations between international trade and unemployment. The unemployment dealt with in the literature, in the almost cases, occurs due to the existence of imperfect labor markets (higher wage rates than market clearing level, mismatches in employment, divided labor markets, labor market frictions, and so on[1]). On the other hand, there are almost no trade models incorporating unemployment that occurs due to a shortage of effective demand[2].

In our model, involuntary unemployment or Keynesian unemployment occurs[3]. Although labor endowments are given in the model, full employment is not presupposed. Conversely, employment quantities are determined simultaneously with international values and wage rates.

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[1] See Davidson and Matusz (2004, 2010), Dutt et al. (2009), Helpman and Itskhoki (2010), and Belenkiy and Riker (2015). Also see p. 67 of Choi and Harrigan eds. (2003) and p.115 of Feenstra (2016).

[2] Chapter 7 of Dosi et al. (1990) criticized Dornbusch-Fisher-Samuelson model for assuming full employment and extended the model to an underemployment case. This model, however, had only 4 conditional expressions for 7 endogenous variables, and therefore, did not have appropriate construction as a model.

[3] An early attempt at incorporating involuntary unemployment into a trade model was Haberler (1950). However, as indicated by Tabuchi (2017, pp. 274-5), this unemployment was voluntary unemployment according to the definition of J. M. Keynes (Keynes, 1936: see Chapter 2 and 3).

The remainder of this paper is constructed as follows. Section 2 shows the framework of the Graham-type trade model and gives a definition of terms. In Section 3, a Graham-type trade model with unemployment is set and the way to derive equilibrium solutions of the model is presented. Section 4 exemplifies simultaneous equations in a 3-country 4-commodity case. In Section 5, by using 2-country 3-commodity, the probability of multiple equilibria is explained, and in Section 6, by using 2-country 2-commodity, the reason for the probability. In Section 7, we investigate the effect of changes in demand on employment and wage rates. In Section 8, we describe the possibility of excessive demand and unusual shortage of demand. In Section 9, by mitigating the condition of trade equilibrium, we examine the case of trade imbalance. In Section 10, 3-country 4-commodity numerical examples are set and the equilibrium solutions are derived practically. Moreover, a numerical simulation of multiple equilibria is conducted. Section 11 is concluding remarks.

2 Framework of the Graham-type trade model and definition of terms

In the Graham-type trade model[4], there are three given conditions: production techniques expressed by constant labor input coefficients, labor endowments, and demand structure expressed by expenditure coefficients in the full employment version and by physical units in the underemployment version. By these given conditions, (1) patterns of the international division of labor (IDL), (2) international values or world relative prices, (3) wage rates in each country, (4) production volumes, therefore, (5) employment quantities in each country are determined. Of course, in the full employment version, employment quantities are equal labor endowments. We define collectively the four of above (1) to (4) in the full employment version and the five of above (1) to (5) in the underemployment version as *equilibrium solutions*. Other terms are defined as follows.

Given the IDL, some sectors in each country continue their production activities and others cease them. We call the former *active points* and the latter *non-active points*.

[4] The name comes from F. D. Graham's theory of international values (Graham, 1923, 1932, 1948). See Sato (2017a, 2017b). Graham's original theory presupposes full employment.

The IDL patterns have to be *reasonable*. Here, "reasonable" means a situation that next two matters are fulfilled: production costs of active points equal commodity prices, and production costs of non-active points are higher than commodity prices.

In the Graham-type trade model, the most important keyword is *link commodities* that are commodities produced in common in more than one country[5]: e.g. cars produced in Japan, USA, and Germany, IT products in China, Korea, and Japan, and beef in Brazil, Australia, and USA. The link commodities determine the relative wage rates of the countries producing the same link commodities, thereby determining the relative prices of all commodities produced in these countries and the relative production costs of non-active points in these countries. As a same commodity has an identical price, the relative labor productivities (inverses of the labor input coefficients) of the link commodities are precisely the relative wage rates, and the relative prices and the relative production costs are obtained by multiplying the relative wage rates by constant labor input coefficients[6].

When countries have the same link commodities, we say, these countries are directly linked. Countries, however, can also be indirectly linked. Suppose that there are countries A, B, and C, and that A and B produce a link commodity in common, moreover, that B and C produce another link commodity in common. In this case, all the three countries are linked: A and C are not directly but indirectly (via B) linked. The term *link* means not only "link directly" but also "link indirectly" and we use the term *linkage* to express "the state of being linked".

The IDL patterns can be classified into two types. One is when all the countries are linked through link commodities. We refer to this as *the linkage type*. In this type, the relative wage rates of all countries and the relative prices of all commodities can be expressed by labor input coefficients according to each IDL pattern. In other words,

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[5] Graham (1948, p. 254 and p. 332).

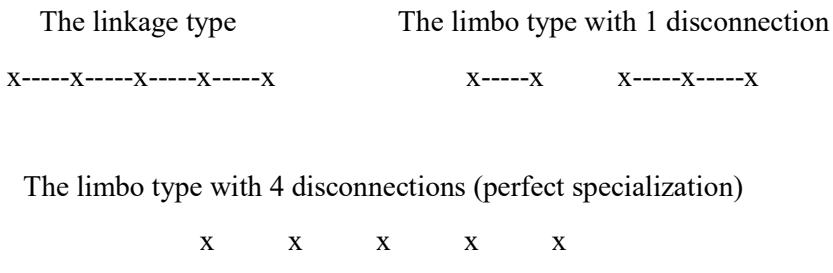
[6] Suppose that countries A and B produce a same link commodity (e.g. commodity 1). Then, the commodity's price (p_1) is expressed as the product of wage rates (w_A, w_B) and labor input coefficients (a_{A1}, a_{B1}), or $p_1 = w_A a_{A1} = w_B a_{B1}$. Therefore, $w_B/w_A = a_{A1}/a_{B1}$. Further, suppose that country A produces commodity 2 and country B commodity 3. Then, because $p_2 = w_A a_{A2}$ and $p_3 = w_B a_{B3}$, the relative price $p_3/p_2 = (w_B/w_A)(a_{B3}/a_{A2}) = (a_{A1}/a_{B1})(a_{B3}/a_{A2})$. Furthermore, for example, because production costs of commodity 4 in country A (C_{A4}) are $w_A a_{A4}$ and those in country B (C_{B4}) are $w_B a_{B4}$, the relative production cost $C_{B4}/C_{A4} = (w_B/w_A)(a_{B4}/a_{A4}) = (a_{A1}/a_{B1})\,(a_{B4}/a_{A4})$.

once the IDL patterns are determined, all the relative wage rates and commodity prices (hereafter, the wage rates/prices) are determined by the patterns themselves, or there is a one-to-one correspondence between the IDL patterns and the wage rates/prices (see **fn. 6**).

The second IDL type is called *the limbo type*. In this type, the linkage of countries is not perfect, and one or more *disconnections* of the linkage occur[7]. Therefore, to determine all the wage rates/prices only by the IDL patterns is not possible. Theoretically, the disconnection can occur in the range from 1 to M–1. When there are M–1 disconnections, perfect specialization patterns are formed, which have no link commodities. We need to pay attention that, except perfect specialization patterns, there are link commodities also in the limbo type and that the link commodities perform the above-mentioned functions.

Figure 1 illustrates these two types in a five-country case. Five countries (expressed by x) are all linked in the linkage type, whereas in the limbo type, the linkage is disconnected in 1 or 4 places and five countries are divided into 2 or 5 groups, within which more than one country is linked unless the groups consist of a single country.

Figure 1: An example of the two types of the IDL patterns

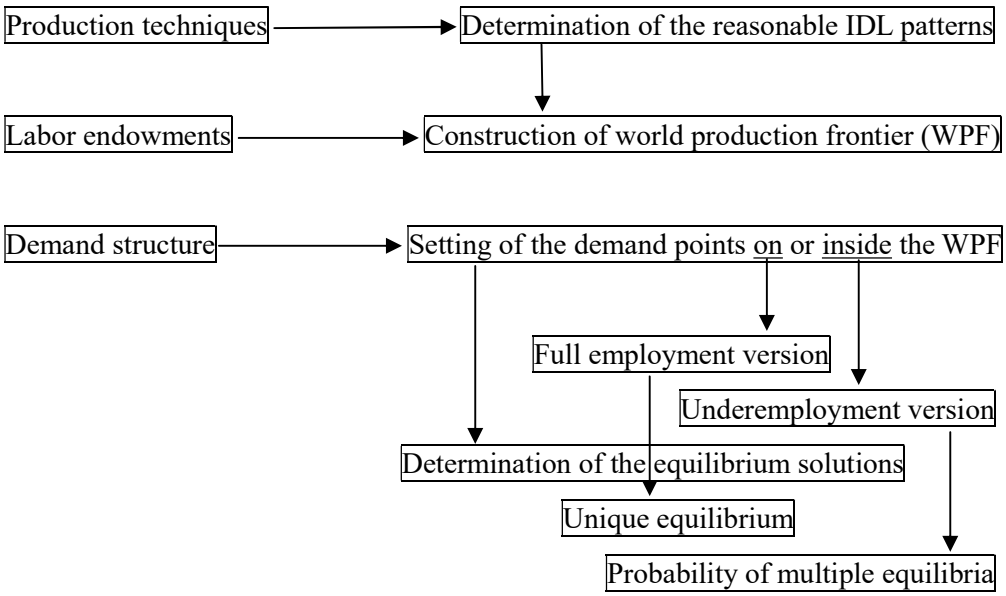


At last of this session, we show the framework of the Graham-type trade model in **Figure 2**. The left three items are the given conditions. Production techniques determine the reasonable IDL patterns by only themselves, or independently of other two conditions. The reasonable IDL patterns and the labor endowments in each country combined to construct the world production frontier (WPF) or the world production

[7] Graham called such a state of disconnection “limbo” (Graham, 1948, p. 35).

possibility set. Demand structure sets the location of demand points on the WPF in the full employment version or inside the WPF in the underemployment version. The production points are determined in accord with the demand points, and thereby equilibrium solutions are determined. Although the equilibrium solution is unique in the full employment version, in the underemployment version, there is a probability of multiple equilibria.

Figure 2: Framework of the Graham-type trade model



3 Setting of the model and steps to derive the equilibrium solutions

At first, we set a Graham-type trade model with unemployment as follows.

1. There are M countries and N commodities. Here, M and N are integers more than 2 and N is larger than M.
2. There are no intermediate goods and no profits. All commodities are for consumption.

3. Trade equilibrium (national expenditure equals national income) in each country is fulfilled[8].
4. There are no transport costs and no trade barriers.
5. There are no international movements of labor and domestic wage rates are equal in all sectors.
6. For each country, production techniques expressed by constant labor input coefficients are given. Although we don't absolutely need the information about the labor input coefficients of some sectors (e.g. the car industries in developing countries or the crude oil extraction industries in non-oil-producing countries) in which a probability of having a comparative advantage is almost zero, we give all sector's data for convenience of explanation and assume that the degrees of comparative advantage between two countries selected arbitrarily differ in every sector.
7. Although labor endowments in each country are given, full employment is not a precondition and unemployment can exist. Too high unemployment rates, however, destabilize societies, and therefore, are not sustainable. Then, we set the upper limit of unemployment rates. Each country has to be subject to the labor quantity constraints (LQCs) and the unemployment rate constraints (URCs).
8. Demand conditions in each country are given in terms of physical units.

Next, we explain how to obtain the equilibrium solutions. There is no way to obtain the solutions at one stroke, so that we have to follow a little complicated procedure, which comprises of four steps. In step 1, we search for and identify reasonable IDL patterns. In step 2, according to the each identified reasonable IDL pattern, we set up simultaneous equations and solve them mathematically. In step 3, we select one or more solution sets that fulfill the conditions. In step 4, we inspect whether the selected solution sets fulfill the LQCs and URCs or not. We describe the details of each step below.

[8] This condition is possible to be mitigated (see **Section 9**).

Step 1: Searching for and identifying reasonable IDL patterns

First, we have to search for and identify reasonable IDL patterns. Whether an IDL pattern is reasonable or not is determined only by the labor input coefficients. The way to identify is common between the full employment version and the underemployment version, but there is a difference between the linkage type and the limbo type. Therefore, we explain it separately.

In the linkage type, as the entire wage rates/prices and production costs of the non-active points are already known according to the IDL patterns, we only have to compare the commodity prices with the production costs (see **fn. 6**).

The number of reasonable patterns of the linkage type is (M+N–2)!/{(M–1)!(N–1)!} in an M-country N-commodity case[9]. If M and N are large, because of a large number of IDL patterns to be judged[10], it is difficult even to identify the patterns. Including the rest of the process, the support of computer program would be needed in order to calculate actually[11].

In the limbo type, because not all the wage rates/prices are determined according to the IDL patterns, we have to adopt a different way from the linkage type. There are two methods. We explain those in the case of the IDL patterns with l disconnections (l: an integer and $0 \leq l \leq$ M-1)[12]. In this case, countries are divided into l+1 groups and the IDL has to be reasonable within each group and also among groups. Reasonableness of the IDL within each group is able to be checked easily because the relative wage rates are determined by linkage. For the IDL among groups to be reasonable, there is a condition. It is that relative wage rates between countries belonging to different groups have to be within a specific range. This wage rates constraint has to be met between all the combinations of two out of the l+1 groups, or there are ${}_{l+1}C_2$ constraints relating

[9] Based on Shiozawa (2012, p. 50), the number of reasonable IDL patterns with l disconnections is (M+N–l–2)!/{(M–l–1)!(N–l–1)!l!}. Since the linkage type has no disconnection, by substituting zero for l in this expression, we obtain the number of linkage type IDL patterns.

[10] The number is {M^ (N−1)} {N^ (M−1)} according to Shiozawa's direct suggestion.

[11] We uploaded the programs to identify the reasonable IDL patterns in the 3-country 4-commodity case to the web site https://www.researchgate.net/profile/Hideo_Sato2. See "grahamprogramm0" and "grahamprogramm1". These are possible to download.

[12] The value of l is greater than or equal to one in the limbo-type and zero in the linkage type.

wage rates. If wage rates satisfying all these conditions can exist under an IDL pattern, the IDL pattern is judged as reasonable. Contrary, if these conditions are contradictory each other, the IDL pattern is judged as not reasonable. For convenience of explanation, we describe this method accompanied by identification of the range of wage rates as *the judging method 1*.

We explain this method by using a numerical example in Jones (1961). **Table 1** is the example: country names, commodity names, and arrangements are changed.

Table 1: Jones' numerical example

	Labor input coefficients		
	Comm.1	Comm.2	Comm.3
Country A	2	**3**	10
Country B	4	5	**10**
Country C	**3**	7	10

We examine which IDL pattern is reasonable among the limbo type with two disconnections or perfect specialization patterns. The number of these patterns is 6. At first, we examine the pattern printed in boldface. Conditions that this pattern is reasonable are as follows (w_i: wage rate of country i).

$3w_C < 2w_A$ and $3w_C < 4w_B$ (commodity 1 in country C)

$3w_A < 5w_B$ and $3w_A < 7w_C$ (commodity 2 in country A)

$10w_B < 10w_A$ and $10w_B < 10w_C$ (commodity 3 in country B)

By deforming and rearranging these, we obtain three (${}_{2+1}C_2 = 3$) wage rates constraints: $3w_A/5 < w_B < w_A$, $3w_A/7 < w_C < 2w_A/3$, and $w_B < w_C < 4w_B/3$. Although the range that fulfills all the three constraints is very narrow, such a range certainly exists. Therefore, this IDL pattern is reasonable.

Next, we examine the pattern that country A produces commodity 1, country B commodity 2, and country C commodity 3. Conditions making this pattern reasonable are as follows.

$2w_A < 4w_B$ and $2w_A < 3w_C$ (commodity 1 in country A)

$5w_B < 3w_A$ and $5w_B < 7w_C$ (commodity 2 in country B)

$10w_C < 10w_A$ and $10w_C < 10w_B$ (commodity 3 in country C)

By deforming and rearranging these, we obtain three constraints: $w_A/2 < w_B < 3w_A/5$, $2w_A/3 < w_C < w_A$, and $5w_B/7 < w_C < w_B$. The range fulfilling all the three inequalities does not exist[13]. Therefore, this pattern is not reasonable. Other four patterns are the same.

This method is very laborious. There is a far easier method, which uses the identified linkage type IDL patterns: we call it *the judging method 2*. If, while holding the condition that all the commodities are produced and all the countries produce at least one commodity, we remove one active point of a linkage type IDL pattern, one disconnection occurs and a limbo type IDL pattern with one disconnection is derived. Further, by adding the same operation to this newly obtained pattern, we can obtain an IDL pattern with two disconnections. By repeating the same operation up to M–1 disconnections, we can identify all the limbo type IDL patterns (we show an example in **Section 8**).

Because all the patterns from with one disconnection to with M–1 disconnections are summed, the number of the limbo type IDL patterns are very large: $\sum(M+N-l-2)!/\{(M-l-1)!(N-l-1)!l!\}$ ($l = 1, 2, \cdots, M-1$).

Step 2: Setting up and solving simultaneous equations

We explain this step in the case that the number of the disconnections is l. When the value of l is zero, the IDL patterns are the linkage type. Otherwise, the IDL patterns are the limbo type. In the second step, for all the reasonable IDL patterns[14], we have to set up simultaneous equations and solve them.

The number of the active points matters. It is known that the number is $M+N-1-l$ (McKenzie, 1954, p. 175). Why $M+N-1-l$? Let us suppose a perfect specialization pattern that all the commodities are produced. In this situation, the number of the active points is N and there is no linkage among countries, or the number of the disconnections is M –1. To link any two countries under this situation, we need to add one active point. Then, the number of active points is N+1, that of the linkage is 1, and that of the disconnections is M–1–1. In this way, active points increase by one every

[13] We obtain $w_C < 3w_A/5$ from the first and third inequalities. However, this contradicts $2w_A/3 < w_C$ in the second inequality.

[14] The total number is $\sum(M+N-l-2)!/\{(M-l-1)!(N-l-1)!l!\}$ ($l = 0, 1, 2, \cdots, M-1$).

time when the linkages increase (or the disconnections decrease) by one. Thus, when all the countries are linked and there is no disconnection, the active points are M+N–1.

If there are more active points than M+N–1–l, one or more multiple linkages occur between some countries, and multiple relative wage rates occur between the same two countries. Therefore, the number of the active points has to be exactly M+N–1–l.

Let us consider the composition of the simultaneous equations. Firstly, equations expressing commodity prices, which take the form of "$p_j = w_i a_{ij}$", exist by the number of active points (namely M+N–1–l), where p_j, w_i, and a_{ij} (> 0) denote price of commodity j, wage rate in country i, and labor input coefficients of commodity j in country i respectively. Secondly, there are N equations expressing supply-demand balance for each commodity, which take the form "$\Sigma x_{ij} = \Sigma d_{ij}$ (i= A, B, ⋯, M)", where x_{ij} and d_{ij} (> 0) denote production volumes and demand volumes of commodity j in country i. However, because one of N equations is not independent owing to Walras' Law, independent equations are N–1. Thirdly, there exist N equations expressing trade equilibrium (national expenditure = national income) in each country, which take the form of "$\Sigma p_j x_{ij} = \Sigma p_j d_{ij}$ (j = 1, 2, ⋯, N)". The total number of the independent equations is 2M+2N–2–l.

Next, let us count the number of unknowns. Commodity prices except numéraire are N–1, wage rate in each country M, and production volumes of active points M+N–1–l. The total unknowns are 2M+2N–2–l. Thus, we can solve all the equation sets mathematically.

Step 3: Selecting solution sets that fulfill the conditions

Not all the obtained mathematical solutions are meaningful economically, leading to the next step. We have to select solution sets fulfilling following conditions from the "$\sum$(M+N–l–2)!/{(M–l–1)!(N–l–1)!l!}(l=0, 1, 2, ⋯, M–1)" solution sets. One of the conditions is that all the production volumes and wage rates are positive. Another condition is that, in the limbo type, the obtained wage rates are within the adequate range in the case of the judging method 1, or that the solution set passes *a competitiveness test* in the case of the judging method 2. This test is to check whether non-active points are competitive or not by comparing the production costs of

non-active points with the commodity prices. As the entire wage rates/prices are already obtained, the test is simple. If at least one non-active point is competitive, the set is disqualified. Of course, verifying the range of wage rates and the competitiveness test are equivalent.

Although the solution set that fulfills the conditions is one only in the full employment version, in the underemployment version, there can be multiple.

Step 4: Inspecting whether selected solution sets fulfill the LQCs and URCs

The solution sets selected in step 3 are not equilibrium solutions but candidates for them. We have to calculate employment quantities in each country from the obtained production volumes and the labor input coefficients to inspect whether the candidates fulfill the LQCs and URCs. The candidates that pass the inspection are the equilibrium solutions, which can be multiple or unique.

4 Simultaneous equations in a 3-country 4-commodity case

We show the simultaneous equations in a 3-country 4-commodity case. There are the three countries of A, B, and C and the four commodities of 1, 2, 3, and 4. In addition to previously mentioned p_j, w_i, a_{ij}, d_{ij}, and x_{ij}, We define L_i (> 0) and α_i ($0 \leq \alpha_i < 1$) as labor endowments and the upper limit of unemployment rate in country i respectively. The numéraire is commodity 1. Consumption volumes equal demand volumes and export-import volumes are differences between production volumes and consumption volumes in each country. We provide each one example of the linkage type and the limbo type. Also, we provide the LQCs and URCs, which are used in the step 4.

At first, as an example of the linkage type, we take the IDL pattern that country A produces commodities 1 and 2, country B commodities 2 and 3, and country C commodities 3 and 4.

Price equations ($p_1 = 1$)

$p_1 = w_A a_{A1}$

$p_2 = w_A a_{A2}$ $\qquad$ $p_2 = w_B a_{B2}$

$p_3 = w_B a_{B3}$ $p_3 = w_C a_{C3}$

$p_4 = w_C a_{C4}$

Conditions of supply-demand balance (only three of the four are independent)

$x_{A1} = d_{A1}+d_{B1}+d_{C1}$

$x_{A2}+x_{B2} = d_{A2}+d_{B2}+d_{C2}$

$x_{B3}+x_{C3} = d_{A3}+d_{B3}+d_{C3}$

$x_{C4} = d_{A4}+d_{B4}+d_{C4}$

Conditions of trade equilibrium

$p_1x_{A1}+p_2x_{A2} = p_1d_{A1}+p_2d_{A2}+p_3d_{A3}+p_4d_{A4}$

$p_2x_{B2}+p_3x_{B3} = p_1d_{B1}+p_2d_{B2}+p_3d_{B3}+p_4d_{B4}$

$p_3x_{C3}+p_4x_{C4} = p_1d_{C1}+p_2d_{C2}+p_3d_{C3}+p_4d_{C4}$

Labor quantity constraints (LQCs) and unemployment rate constraints (URCs)

$(1-\alpha_A)L_A \le a_{A1}x_{A1}+a_{A2}x_{A2} \le L_A$

$(1-\alpha_B)L_B \le a_{B2}x_{B2}+a_{B3}x_{B3} \le L_B$

$(1-\alpha_C)L_C \le a_{C3}x_{C3}+a_{C4}x_{C4} \le L_C$

There are the total 12 independent equations (6 price equations, 3 independent equations of supply-demand balance, and 3 equations of trade equilibrium) and the 12 unknowns (3 commodity prices except numéraire, 3 wage rates, and 6 production volumes). Although we have to rewrite these in the case of other patterns, in any case, we can obtain the mathematical solutions.

As to the limbo type, we suppose that country A produces commodities 1 and 2, country B commodities 3 and 4, and country C commodity 4 only. Equations are as follows.

Price equations ($p_1 = 1$)

$p_1 = w_A a_{A1}$

$p_2 = w_A a_{A2}$

$p_3 = w_B a_{B3}$

$p_4 = w_B a_{B4}$ $p_4 = w_C a_{C4}$

Conditions of supply-demand balance (only three of the four are independent)

$x_{A1} = d_{A1}+d_{B1}+d_{C1}$

$x_{A2} = d_{A2}+d_{B2}+d_{C2}$

$x_{B3} = d_{A3}+d_{B3}+d_{C3}$

$x_{B4}+x_{C4} = d_{A4}+d_{B4}+d_{C4}$

Conditions of trade equilibrium

$p_1x_{A1}+p_2x_{A2} = p_1d_{A1}+p_2d_{A2}+p_3d_{A3}+p_4d_{A4}$

$p_3x_{B3}+p_4x_{B4} = p_1d_{B1}+p_2d_{B2}+p_3d_{B3}+p_4d_{B4}$

$p_4x_{C4} = p_1d_{C1}+p_2d_{C2}+p_3d_{C3}+p_4d_{C4}$

Labor quantity constraints (LQCs) and unemployment rate constraints (URCs)

$(1-\alpha_A)L_A \le a_{A1}x_{A1}+a_{A2}x_{A2} \le L_A$

$(1-\alpha_B)L_B \le a_{B3}x_{B3}+a_{B4}x_{B4} \le L_B$

$(1-\alpha_C)L_C \le a_{C4}x_{C4} \le L_C$

Independent equations decrease by one from 12 to 11. However, the unknowns also decrease by one to be 11. Therefore, as well as the linkage type, we can solve them mathematically.

Between both types, there is one difference, though. In the linkage type, the wage rates/prices are obtained only from the price equations regardless of other conditions and are expressed only by the labor input coefficients like below.

$w_A = 1/a_{A1}$

$w_B = (a_{A2}/a_{B2})w_A = a_{A2}/(a_{B2}a_{A1})$

$w_C = (a_{B3}/a_{C3})w_B = (a_{B3}a_{A2})/(a_{C3}a_{B2}a_{A1})$

$p_2 = a_{A2}/a_{A1}$

$p_3 = (a_{B3}a_{A2})/(a_{B2}a_{A1})$

$p_4 = (a_{C4}a_{B3}a_{A2})/(a_{C3}a_{B2}a_{A1})$

In the limbo type, some of the wage rates/prices are not obtain unless other conditions are taken into consideration. Easiness (in the linkage type) and difficulty (in the limbo type) in judging reasonableness of IDL patterns in the first step stem from this difference.

5 Probability of multiple equilibria: An analytical explanation using a 2-country 3-commodity example

Whether equilibrium is unique or multiple depends on the given conditions. We explain this by using a 2-country 3-commodity (countries A and B: commodities 1, 2, and 3) example. Here, we don't take the LQCs and URCs into consideration because the purpose of this session is to show probability of multiple equilibria. Labor input coefficients (a_{ij}) and demand volumes for each commodity (d_{ij}) are given by using the same notation as the previous section. The numbers of commodities are assigned in order of diminishing country A's comparative advantage: therefore, $a_{B3}/a_{A3} < a_{B2}/a_{A2} < a_{B1}/a_{A1}$. Then, there are five reasonable IDL patterns, namely, (A123; B3), (A12; B3), (A12; B23), (A1; B23), and (A1; B123).

For each IDL pattern, let us set up simultaneous equations and solve them. If obtained solution sets fulfill the above-mentioned conditions (all the production volumes [x_{ij}] of active points being positive in linkage type, and, in addition to that, wage rates being within an adequate range in limbo type), the IDL patterns are reasonable. In the following, we derive conditions making each IDL pattern reasonable as a relational expression between a_{ij} and d_{ij}. The numéraire is commodity 1. Three parentheses after hash marks (#) show the IDL patterns, commodity prices (1; 2; 3), and wage rates (A; B) in order. Among five equations, the former three are the conditions of supply-demand balance and the next two are the conditions of trade balance.

At first, we examine about the pattern (A123; B3).

(A123; B3), (1; a_{A2}/a_{A1}; a_{A3}/a_{A1}), (1/a_{A1}; [a_{A3}/a_{A1}]/a_{B3})

$x_{A1} = d_{A1}+d_{B1}$

$x_{A2} = d_{A2}+d_{B2}$

$x_{A3}+x_{B3} = d_{A3}+d_{B3}$

$x_{A1}+x_{A2}(a_{A2}/a_{A1})+x_{A3}(a_{A3}/a_{A1}) = d_{A1}+d_{A2}(a_{A2}/a_{A1})+d_{A3}(a_{A3}/a_{A1})$

$x_{B3}(a_{A3}/a_{A1}) = d_{B1}+d_{B2}(a_{A2}/a_{A1})+d_{B3}(a_{A3}/a_{A1})$

While it is obvious that production volumes x_{A1}, x_{A2}, and x_{B3} are positive, it is not certain whether x_{A3} is positive. Then, let us solve the third and fifth equations for x_{A3}.

$$x_{A3} = d_{A3}+d_{B3}-x_{B3}$$
$$= d_{A3}+d_{B3}-\{d_{B1}+d_{B2}(a_{A2}/a_{A1})+d_{B3}(a_{A3}/a_{A1})\}/(a_{A3}/a_{A1})$$
$$= d_{A3}-\{d_{B1}+d_{B2}(a_{A2}/a_{A1})\}/(a_{A3}/a_{A1})$$
$$= d_{A3}-(a_{A1}d_{B1}+d_{B2}a_{A2})/a_{A3}$$

Therefore,

$$x_{A3} > 0 \Leftrightarrow\ a_{A3}d_{A3} > a_{A1}d_{B1}+a_{A2}d_{B2}.$$

The latter inequality is the condition that the pattern (A123; B3) is reasonable.

Next, we examine the pattern (A12; B3). As this IDL pattern is limbo type, not all the commodity prices and wage rates are determined. By expressing country B's wage rate as w_B, commodity prices, wage rates, and equations are shown as follows.

(A12; B3), (1; a_{A2}/a_{A1}; $a_{B3}w_B$), (1/a_{A1}; w_B)

$$x_{A1} = d_{A1}+d_{B1}$$
$$x_{A2} = d_{A2}+d_{B2}$$
$$x_{B3} = d_{A3}+d_{B3}$$
$$x_{A1}+x_{A2}(a_{A2}/a_{A1}) = d_{A1}+d_{A2}(a_{A2}/a_{A1})+d_{A3}a_{B3}w_B$$
$$x_{B3}a_{B3}w_B = d_{B1}+d_{B2}(a_{A2}/a_{A1})+d_{B3}a_{B3}w_B$$

It is self-evident that all the production volumes are positive. However, we have to identify the range of w_B that make the pattern (A12; B3) reasonable. Because production costs of commodity 2 in country B have to be higher than commodity 2's price and production costs of commodity 3 in country A have to be higher than commodity 3's price, the following inequalities have to be satisfied.

$$a_{B2}w_B > a_{A2}/a_{A1} \text{ and } a_{A3}/a_{A1} > a_{B3}w_B$$

By simplifying, the following wage rates constraints are obtained.

$$a_{A3}/(a_{A1}a_{B3}) > w_B > a_{A2}/(a_{A1}a_{B2})$$

On the other hand, by solving the above first, second, and forth equations for w_B,

$$d_{A1}+d_{B1}+(d_{A2}+d_{B2})(a_{A2}/a_{A1}) = d_{A1}+d_{A2}(a_{A2}/a_{A1})+d_{A3}a_{B3}w_B$$
$$d_{B1}+d_{B2}(a_{A2}/a_{A1}) = d_{A3}a_{B3}w_B$$
$$w_B = \{d_{B1}+d_{B2}(a_{A2}/a_{A1})\}/(d_{A3}a_{B3})$$

By substituting this expression for w_B in the wage rates constraints,

$$a_{A3}/(a_{A1}a_{B3}) > \{d_{B1}+d_{B2}(a_{A2}/a_{A1})\}/(d_{A3}a_{B3}) > a_{A2}/(a_{A1}a_{B2})$$
$$a_{A3}/a_{B3} > (d_{B1}a_{A1}+d_{B2}a_{A2})/(d_{A3}a_{B3}) > a_{A2}/a_{B2}$$

$a_{A3}d_{A3} > d_{B1}a_{A1}+d_{B2}a_{A2} > (a_{A2}/a_{B2})d_{A3}a_{B3}$

This is the condition making the pattern (A12・B3) reasonable.

In the same way, the conditions making each pattern reasonable are obtained as follows.

(A123; B3): $a_{A1}d_{B1}+a_{A2}d_{B2} < a_{A3}d_{A3}$

(A12; B3): $(a_{A2}/a_{B2})a_{B3}d_{A3} < a_{A1}d_{B1}+a_{A2}d_{B2} < a_{A3}d_{A3}$

(A12; B23): $(a_{A2}/a_{B2})a_{B3}d_{A3} < a_{A1}d_{B1}+a_{A2}d_{B2}$

and $(a_{B2}/a_{A2})a_{A1}d_{B1} < a_{B2}d_{A2}+a_{B3}d_{A3}$

(A1; B23): $(a_{B2}/a_{A2})a_{A1}d_{B1} < a_{B2}d_{A2}+a_{B3}d_{A3} < a_{B1}d_{B1}$

(A1; B123): $a_{B2}d_{A2}+a_{B3}d_{A3} < a_{B1}d_{B1}$

Although the size relations among the former three and among the latter three of six inequalities are clear, the relations among all the six are not clear from the above inequalities themselves. Let us set d_{A3} as a criterion for comparison and rewrite the above inequalities. Then, the following is obtained.

(A123; B3)：$(a_{A1}d_{B1}+a_{A2}d_{B2})/a_{A3} < d_{A3}$

(A12; B3)：$(a_{A1}d_{B1}+a_{A2}d_{B2})/a_{A3} < d_{A3} < a_{A1}a_{B2}d_{B1}/(a_{A2}a_{B3})+(a_{B2}/a_{B3})d_{B2}$

(A12; B23)：$a_{A1}a_{B2}d_{B1}/(a_{A2}a_{B3}) - (a_{B2}/a_{B3})d_{A2} < d_{A3} < a_{A1}a_{B2}d_{B1}/(a_{A2}a_{B3})+(a_{B2}/a_{B3})d_{B2}$

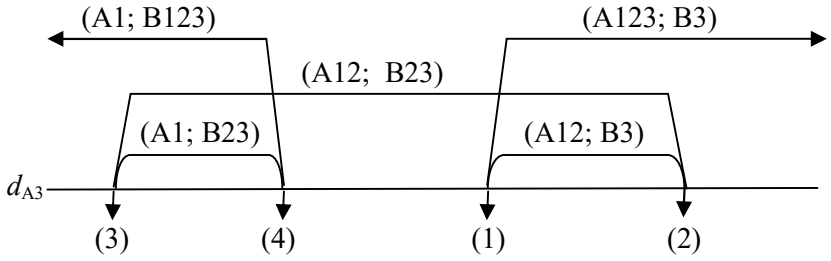
(A1; B23)：$a_{A1}a_{B2}d_{B1}/(a_{A2}a_{B3}) - (a_{B2}/a_{B3})d_{A2} < d_{A3} < (a_{B1}/a_{B3})d_{B1} - (a_{B2}/a_{B3})d_{A2}$

(A1; B123)：$d_{A3} < (a_{B1}/a_{B3})d_{B1} - (a_{B2}/a_{B3})d_{A2}$

Here, if we express $(a_{A1}d_{B1}+a_{A2}d_{B2})/a_{A3}$ as (1), $a_{A1}a_{B2}d_{B1}/(a_{A2}a_{B3})+(a_{B2}/a_{B3})d_{B2}$ as (2), $a_{A1}a_{B2}d_{B1}/(a_{A2}a_{B3}) - (a_{B2}/a_{B3})d_{A2}$ as (3), and $(a_{B1}/a_{B3})d_{B1} - (a_{B2}/a_{B3})d_{A2}$ as (4), the relation of 0 < (1) < (2), (3) < (2), and (3) < (4) are certain, but the relations between (1) and (4), (1) and (3), and (2) and (4) are uncertain, and moreover, (3) and (4) are possible to be negative[15]. According to labor input coefficients and demand quantities, there can be five arrangement patterns: (1) < (3) < (4) < (2), (3) < (1) < (2) < (4), (1) < (3) < (2) < (4), (3) < (1) < (4) < (2), and (3) < (4) < (1) < (2). We will illustrate the relations between parameters and the IDL patterns by taking "(3) < (4) < (1) < (2)" as an example.

[15] These can be confirmed by taking "$a_{B3}/a_{A3} < a_{B2}/a_{A2} < a_{B1}/a_{A1}$" into account.

Figure 3: Labor input coefficients, demand volumes, and the IDL patterns



In **Figure 3**, on the number line of d_{A3}, four points are arranged according to the assumed order. This figure shows the relationship between values of d_{A3} and the IDL patterns. Only (A123; B3) is possible on the right side of (2) , (A123; B3), (A12; B3), and (A12; B23) between (1) and (2), only (A12; B23) between (4) and (1), (A12; B23), (A1; B23), and (A1; B123) between (3) and (4), and only (A1; B123) on the left side of (3). Although the values on the left side of (1) are possible to be negative, in such a case, some IDL patterns are excluded because the values of d_{A3} are always positive.

Under other arrangement patterns, other results follow. For example, all the five IDL patterns are possible between (3) and (4) in the arrangement pattern (1) < (3) < (4) < (2), and between (1) and (2) in (3) < (1) < (2) < (4).

An important thing to be confirmed is that, as is evident from the inequalities and the **Figure 3**, it is impossible for the limbo type to be the candidate of equilibrium solution independently and the limbo type is always accompanied by one or more linkage type adjoining the limbo type[16]. However, when the LQCs and URCs are taken into consideration, it is possible that only one limbo type IDL pattern is the unique equilibrium solution. About this, we describe later (in **Section 8**).

6 Reason for the probability: An explanation using 2-country 2-commodity numerical examples

What is the reason why multiple equilibria are possible? We explain this by using a 2-country 2-commodity numerical example. Also here, we disregard the LQCs and

[16] In **Section 8**, we conduct a simulation using a 3-county 4-commodity numerical example. These are confirmed also by the simulation.

URCs for the same reason as **Section 5**. Suppose that labor input coefficients and demand volumes are given as **Table 2**.

Table 2: Labor input coefficients and demand volumes

	Labor input coefficients		Demand volumes	
	Comm.1	Comm.2	Comm.1	Comm.2
Country A	1	2	10	10
Country B	2	1	10	10

From the labor input coefficient, three reasonable IDL patterns, or (A12; B2), (A1; B12), and (A1; B2), are derived. By setting commodity 1 as the numéraire and by calculating production volumes, employment quantities, commodity prices, and wage rates, we obtain three sets of solutions, which are shown in **Table 3**. All these solution sets fulfill the conditions, and therefore, there exist multiple equilibria.

Table 3: Equilibrium solutions (case 1)

(A12; B2)	Production volumes		Employment quantities	Wage rates
	Comm.1	Comm.2		
Country A	20	5	30	1
Country B		15	15	2
Commodity prices	1	2		

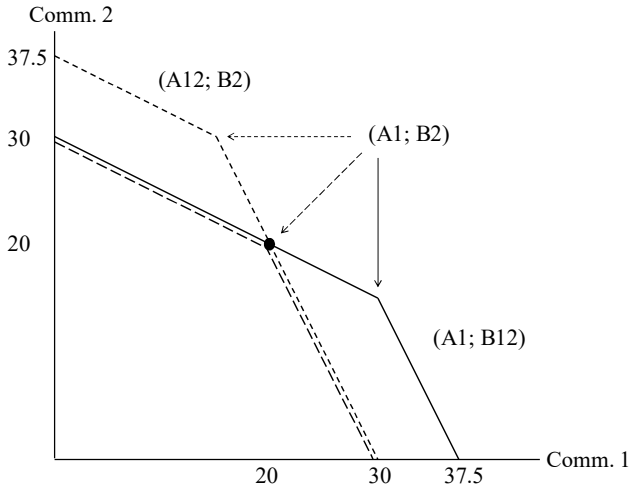
(A1; B12)	Production volumes		Employment quantities	Wage rates
	Comm.1	Comm.2		
Country A	15		15	1
Country B	5	20	30	0.5
Commodity prices	1	0.5		

(A1; B2)	Production volumes		Employment quantities	Wage rates
	Comm.1	Comm.2		
Country A	20		20	1
Country B		20	20	1
Commodity prices	1	1		

If we regard the obtained employment quantities as the labor endowments, the pseudo world production frontiers (PWPFs) are drawn as **Figure 4**: the pattern (A12;

B2) is expressed by the solid line, (A1; B12) by the dotted line, and (A1; B2) by the broken line, which is shifted down a little for identification.

Figure 4: Pseudo world production frontiers (case 1)



Vertices on the three lines are the pattern (A1; B2), the left side lines of each vertex are (A12; B2), and the right side lines are (A1; B12). The black point is the world demand point. The same demand point lies on the three lines, and moreover, on the different IDL patterns. Because, each country's employment quantities are variable in the underemployment version, a world production volume equal to world demand may be attained by various combinations of each country's production volume. **Figure 4** expresses this situation.

However, the equilibrium is not always multiple. Suppose that country A's demand volumes for both commodities are 14 each and country B's those are 6 each. Then, the IDL pattern fulfilling conditions is only one, namely, (A12; B2). In the pattern (A1; B12), commodity 1's production volumes in country B are negative (–1) and, in the pattern (A1; B2), commodity 1 fails to pass the competitiveness test: country B's wage rate is 3/7 and commodity 1's production costs in country B are 6/7, which are lower than the price of commodity 1. Obtained mathematical solutions are summarized in **Table 4**. Country B's employment quantities in the pattern (A1; B12) are 18, which is

obtained by adding commodity 2 sector's employment quantities (= 20) and commodity 1 sector's those (= –2). The PWPFs of Table 4 are drawn as **Figure 5**.

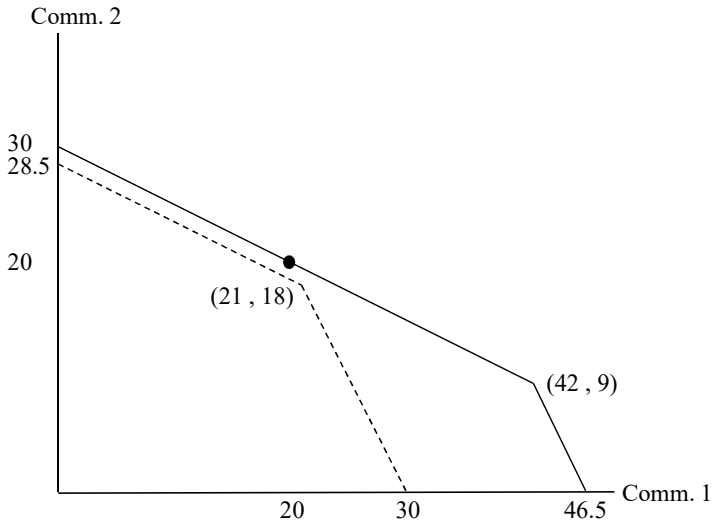
Table 4: Equilibrium solution and mathematical solutions (case 2)

(A12; B2)	Production volumes		Employment quantities	Wage rates
	Comm.1	Comm.2		
Country A	20	11	42	1
Country B		9	9	2
Commodity prices	1	2		

(A1; B12)	Production volumes		Employment quantities	Wage rates
	Comm.1	Comm.2		
Country A	21		21	1
Country B	–1	20	(18)	0.5
Commodity prices	1	0.5		

(A1; B2)	Production volumes		Employment quantities	Wage rates
	Comm.1	Comm.2		
Country A	20		20	1
Country B		20	20	3/7
Commodity prices	1	3/7		

Figure 5: Pseudo world production frontiers (case 2)



The pattern (A12; B2) is expressed by the solid line, (A1; B12) by the dotted line, and the frontier of (A1; B2) is omitted. The values in **Figure 5** are coordinates of vertices. The frontier of the dotted line shows the reason why the pattern (A1; B12) is disqualified under the given demand volumes: in the pattern (A1; B12), country A requires employment quantities of 21 to cover demand volumes 14 each; (commodity 1's price) 1 x 14 + (commodity 2's price) 0.5 x 14 = (wage rate) 1 x 21, while country B requires employment quantities of 18 to cover demand volumes 6 each; 1 x 6 + 0.5 x 6 = 0.5 x 18. The PWPF with these employment quantities does not reach to the demand point. That is to say, in the pattern (A1; B12), there is no combination of each country's production volumes that is able to meet the given demand while keeping trade equilibrium.

Thus, even though the world total demand is the same, if the composition by countries is different, the equilibrium solution is different, which can be multiple (**Figure 4**) or unique (**Figure 5**).

7 Effect of changes in demand on employment and wage rates

Changes in demand volumes cause adjustment processes. If the changes are large enough to bring about changes in the IDL patterns, the equilibrium solutions are re-determined newly. Although this re-determination is also important, in this section, we assume that the IDL patterns do not change. Because the adjustment processes differ between the linkage type and the limbo type, we examine it separately.

In the linkage type, when changes in demand occur, the wage rates/prices never change. This is shown by the fact that the wage rates/prices are expressed only by labor input coefficients (see **Section 4**). In short, the quantity adjustments without price changes are conducted[17].

[17] We call the aspect of the quantity adjustments without price changes the Graham case, and the aspect of the adjustments with price changes the Mill case. To say in this terms, the linkage type is the IDL patterns that the Graham case is exclusive, a large part of the limbo type is the patterns that the Graham case and the Mill case coexist, and a very small part of the limbo type or the perfect specialization patterns, which has M–1 disconnections of the linkage and therefore have no link commodities, is the patterns that the Mill case is exclusive.

How are the production volumes and the employment quantities adjusted? We explain it by using the linkage pattern in **Section 4**. Define E_{ij} as country i's employment quantities in commodity j sector. Because production volumes are employment quantities divided by labor input coefficients and commodity prices are wage rates multiplied by labor input coefficients, we can deform $p_j x_{ij}$ into $w_i E_{ij}$. In addition, by replace commodity prices with labor input coefficients, we obtain the following expressions from the trade equilibrium expressions in **Section 4**.

$$w_A(E_{A1}+E_{A2}) = d_{A1}+(a_{A2}/a_{A1})d_{A2}+(a_{B3}a_{A2})/(a_{B2}a_{A1})d_{A3}+(a_{C4}a_{B3}a_{A2})/(a_{C3}a_{B2}a_{A1})d_{A4}$$

$$w_B(E_{B2}+E_{B3}) = d_{B1}+(a_{A2}/a_{A1})d_{B2}+(a_{B3}a_{A2})/(a_{B2}a_{A1})d_{B3}+(a_{C4}a_{B3}a_{A2})/(a_{C3}a_{B2}a_{A1})d_{B4}$$

$$w_C(E_{C3}+E_{C4}) = d_{C1}+(a_{A2}/a_{A1})d_{C2}+(a_{B3}a_{A2})/(a_{B2}a_{A1})d_{C3}+(a_{C4}a_{B3}a_{A2})/(a_{C3}a_{B2}a_{A1})d_{C4}$$

By considering $w_A = 1/a_{A1}$, $w_B = a_{A2}/(a_{B2}a_{A1})$, and $w_C = (a_{B3}a_{A2})/(a_{C3}a_{B2}a_{A1})$,

$$E_{A1}+E_{A2} = a_{A1}d_{A1}+a_{A2}d_{A2}+(a_{B3}a_{A2})/(a_{B2})d_{A3}+(a_{C4}a_{B3}a_{A2})/(a_{C3}a_{B2})d_{A4}$$

$$E_{B2}+E_{B3} = (a_{B2}a_{A1}/a_{A2})d_{B1}+ a_{B2}d_{B2}+a_{B3}d_{B3}+(a_{C4}a_{B3}/a_{C3})d_{B4}$$

$$E_{C3}+E_{C4} = \{(a_{C3}a_{B2}a_{A1})/(a_{B3}a_{A2})\}d_{C1}+(a_{C3}a_{B2}/a_{B3})d_{C2}+a_{C3}d_{C3}+a_{C4}d_{C4}$$

These employment expressions show that employment quantities in each country are determined only by domestic demand, and that even increases in domestic demand for commodities produced in foreign countries increases domestic employment and only this. Increases in foreign country's demand never contribute to increases in domestic employment. We explain this mechanism in the case of the increases of country A's demand for commodity 4 (d_{A4}). By partially differentiating the above employment expression of country A by d_{A4}, we obtain

$$\partial (E_{A1}+E_{A2})/\partial d_{A4} = (a_{C4}/a_{C3})(a_{B3}/a_{B2})a_{A2}$$

This shows how much employment is increased by increases of one unit of d_{A4}. The (a_{C4}/a_{C3}) is the opportunity cost of commodity 4 measured with commodity 3 in country C, in other words, it is the reduced production volumes of commodity 3 on the occasion of increasing one unit of commodity 4 by production switching in country C. The (a_{B3}/a_{B2}) is the reduced production volumes of commodity 2 on the occasion of increasing one unit of commodity 3 by production switching in country B. The a_{A2} is a labor quantity required to produce one unit of commodity 2 in country A.

The mechanism that increases of one unit of d_{A4} increase country A's employment by $(a_{C4}/a_{C3})(a_{B3}/a_{B2})a_{A2}$ units is as follows. The demand increases increase country C's

production volumes of commodity 4 by one unit. The other hand, country C has to decrease production volumes of commodity 3 by (a_{C4}/a_{C3}) units in order to maintain trade equilibrium[18]. Because the world demand for commodity 3 doesn't change, country B has to increase production volumes of commodity 3 by (a_{C4}/a_{C3}) units to cover the reduced production volumes. In turn, country B decreases production volumes of commodity 2 by $(a_{C4}/a_{C3})(a_{B3}/a_{B2})$ units. Finally, country A has to cover the volumes, leading to the increase in employment quantities of country A by $(a_{C4}/a_{C3})(a_{B3}/a_{B2})a_{A2}$ units. This is the mechanism that only the Graham-type trade model based on the linkage structure is able to have.

Note that, even though increasing demand volumes are the same, increasing employment quantities may be different, because the combination of opportunity costs, which influence employment quantities, is different according to the IDL patterns. We give a concrete example in **Section 10.**

In the limbo type, changes in demand cause changes in the wage rates/prices in some cases, but not in other cases. We explain this in the case of the limbo type IDL patter in **Section 4**. In this pattern, although country B and country C are linked, the both countries are disconnected from country A. By solving the above-mentioned equations for w_B, we obtain

$$w_B = \{d_{B1}+d_{C1}+(a_{A2}/a_{A1})(d_{B2}+d_{C2})\}/(a_{B3}d_{A3}+a_{B4}d_{A4})$$

This expression shows that country B's or country C's demand increases for commodities that are not produced in these country (increases of d_{B1}, d_{C1}, d_{B2}, and d_{C2}) raises these country's wage rate, and that country A's demand increase for commodities produced countries B and C (increase of d_{A3} and d_{A4}) reduces the wage rates of both countries B and C. This moving of the wage rate may seem to be a little complicated. To make it more understandable, we will apply this moving to the case of a perfect specialization pattern with 2-country 2-commodity. The above moving of the wage rate means that domestic demand increases for the commodity produced in foreign country raise the domestic relative wage rate[19]. This is the exact opposite of conventional trade

[18] Because the right side of the country C's trade equilibrium expression ($p_3x_{C3}+p_4x_{C4} = p_1d_{C1}+p_2d_{C2}+p_3d_{C3}+p_4d_{C4}$) does not change, the increase of x_{C4} has to be accompanied by the decrease of x_{C3}.

[19] The pattern 12) and 22) in **Table 8** are concrete examples of this case.

models (e.g. Mill's theory of reciprocal demand) that presuppose full employment[20]. The mechanism is as follows. The domestic demand increases increase the import volumes from foreign country. The other hand, domestic export volumes don't change. To hold trade equilibrium, the domestic commodity terms of trade have to worsen, and therefore, the domestic relative wage rate has to decline.

Conversely, domestic demand changes for commodities produced in home country or countries linked with home country (changes of d_{A1}, d_{A2}, d_{B3}, d_{B4}, d_{C3}, and d_{C4}) does not change the wage rates/prices, just like the linkage type. The mechanism is also the same with the linkage type.

Thus, the relation between demand and unemployment in the present model is clarified. Unemployment in the model is involuntary unemployment or Keynesian unemployment that occurs due to a shortage of effective demand.

8 Excessive demand and unusual shortage of demand

According to demand conditions, there might be the case that the candidate of the equilibrium solution is only one and the candidate does not fulfill the LQCs or URCs, and therefore, there is no equilibrium solution unless the conditions change. In this case, some kind of demand adjustment is inevitable because an excessive demand or an unusual shortage of demand occurs.

We take country A's LQC and URC in **Section 4** as an example (reproduced below).

$$(1-\alpha_A)L_A \leq a_{A1}x_{A1}+a_{A2}x_{A2} \leq L_A$$

At first, we consider the case not satisfying the URC. Suppose the unemployment rate β. Then, the following equation expressing supply-demand gap holds.

$$w_A L_A - w_A(E_{A1}+E_{A2}) = \beta w_A L_A$$

The first term of the left side is the potential income, the second is the actual income determined by aggregate demand[21], and the right side is the supply-demand gap, or the

[20] This is also contrary to the full employment version of the Graham-type trade model. See Sato (2017b, pp. 15-16).

[21] We can obtain "$w_A(E_{A1}+E_{A2}) = p_1d_{A1}+p_2d_{A2}+p_3d_{A3}+p_4d_{A4}$" from the condition of country A's trade equilibrium.

income lost by unemployment. On the other hand, by multiplying the both side of the URC by w_A and deforming that, we obtain

$$w_A L_A - w_A(E_{A1}+E_{A2}) \leq \alpha_A w_A L_A$$

The right side is the upper limit of acceptable supply-demand gap. Therefore, if the URC is not fulfilled, it means that the actual gap exceeds this upper limit ($\beta w_A L_A > \alpha_A w_A L_A$), that is, there is an unusual shortage of demand.

Next, by multiplying the both side of the LQC by w_A and deforming that, we obtain

$$w_A(E_{A1}+E_{A2}) \leq w_A L_A$$

If the LQC is not fulfilled, it means that country A's demand exceeds the potential income: $w_A(E_{A1}+E_{A2})$ = the aggregate demand > $w_A L_A$, that is, there is an excessive demand which is over the potential income.

In the both cases, demand volumes have to be adjusted sooner or later.

9 Mitigation of trade equilibrium conditions: Trade imbalance case

So far, we have assumed trade equilibrium. This was done to close the model in a most simple way, but we can mitigate the condition to introduce trade imbalance. As an initial state, we use the linkage type IDL pattern in **Section 4**. Suppose that, in next period, country B's demand for commodities 1 and 4 increases by Δd_{B1} and Δd_{B4}, and this country covers the demand increment by trade deficit γ (= $p_1\Delta d_{B1} + p_4\Delta d_{B4}$). Country A's demand volumes are unchanged but this country has trade surplus γ. There is no change in country C. Then, the equations in **Section 4** are rewritten as follows. Here, we change the naming of "Conditions of trade equilibrium" to "Conditions of income-expenditure balance" because there is now the trade imbalance.

Conditions of supply-demand balance (only three of the four are independent)

$$x_{A1} = d_{A1}+d_{B1}+d_{C1}+\Delta d_{B1}$$

$$x_{A2}+x_{B2} = d_{A2}+d_{B2}+d_{C2}$$

$$x_{B3}+x_{C3} = d_{A3}+d_{B3}+d_{C3}$$

$$x_{C4} = d_{A4}+d_{B4}+d_{C4}+\Delta d_{B4}$$

Conditions of income-expenditure balance

$$p_1*x_{A1}+p_2*x_{A2} = p_1*d_{A1}+p_2*d_{A2}+p_3*d_{A3}+p_4*d_{A4}+\gamma$$
$$= p_1*d_{A1}+p_2*d_{A2}+p_3*d_{A3}+p_4*d_{A4}+p_1*\Delta d_{B1}+p_4*\Delta d_{B4}$$
$$p_2*x_{B2}+p_3*x_{B3} = p_1*d_{B1}+p_2*d_{B2}+p_3*d_{B3}+p_4*d_{B4}+p_1*\Delta d_{B1}+p_4*\Delta d_{B4}-\gamma$$
$$= p_1*d_{B1}+p_2*d_{B2}+p_3*d_{B3}+p_4*d_{B4}$$
$$p_3*x_{C3}+p_4*x_{C4} = p_1*d_{C1}+p_2*d_{C2}+p_3*d_{C3}+p_4*d_{C4}$$

From the conditions of supply-demand balance, we can know that the production volumes of commodities 1 and 4 increase, and from the conditions of income-expenditure balance, we can also know that employment of countries B and C doesn't change[22] and only country A increases employment. Country A, despite unchanged domestic demand, increases employment and income by absorbing country B's demand increment through the trade surplus. However, this is effective only once. If next period's trade surplus remains γ, employment no longer increases. In order to continue increasing employment through trade surplus, any country has to continue increasing trade surplus: so to speak, an economic growth depending on foreign demand. In the real world, this is possible only in a short time. In a long time, domestic employment increases are realized only by domestic demand increases.

10 Description using 3-country 4-commodity numerical examples

10.1 Identification of the IDL patterns

In this section, we set a 3-country 4-commodity numerical example and derive equilibrium solutions practically. Labor input coefficients are given as **Table 5**. Here, units of the commodities are chosen in such a manner that all the labor input coefficients of country A are one, and the commodities are numbered in order of diminishing country A's comparative advantage between countries A and B[23]. Labor

[22] See the employment expressions in **Section 7** (p. 25).

[23] Although, without this treatment, the number of the IDL patterns to be investigated reaches 432, the number diminishes to 112 owing to this.

input coefficients of country C are given arbitrarily. The numéraire is commodity 1, so that country A's wage rate is always one.

Table 5: Labor input coefficients

	Comm.1	Comm.2	Comm.3	Comm.4
Country A	1	1	1	1
Country B	5	4	3	2
Country C	60	25	30	7

From this table, 10 linkage type IDL patterns, 12 limbo type IDL patterns with one disconnection, and 3 limbo type IDL patterns with two disconnections (perfect specialization patterns) are identified as bellow. In the linkage type, first parentheses show the IDL patterns and e.g. A123 means that country A produces commodities 1, 2, and 3. Second parentheses show commodity prices in order from commodity 1 to 4, and third wage rates from country A to C. In the limbo type, commodity prices are omitted, and w_B means country B's wage rate. In the case of two disconnections, country C needs to fulfill two wage rates constraints.

Linkage type IDL patterns

1) (A1234; B4; C4) (1; 1; 1; 1) (1; 1/2; 1/7)
2) (A1; B1234; C4) (1; 4/5; 3/5; 2/5) (1; 1/5; 2/35)
3) (A1; B1; C1234) (1; 25/60; 1/2; 7/60) (1; 1/5; 1/60)
4) (A123; B34; C4) (1; 1; 1; 2/3) (1; 1/3; 2/21)
5) (A123; B3; C24) (1; 1; 1; 7/25) (1; 1/3; 1/25)
6) (A1; B123; C24) (1; 4/5; 3/5; 28/125) (1; 1/5; 4/125)
7) (A12; B234; C4) (1; 1; 3/4; 1/2) (1; 1/4; 1/14)
8) (A13; B3; C234) (1; 5/6; 1; 7/30) (1; 1/3; 1/30)
9) (A1; B13; C234) (1; 1/2; 3/5; 7/50) (1; 1/5; 1/50)
10) (A12; B23; C24) (1; 1; 3/4; 7/25) (1; 1/4; 1/25)

Limbo type IDL patterns with one disconnection

11) (A123; B4; C4) (1; 1/3-1/2; $2w_B/7$) 12) (A12; B34; C4) (1; 1/4-1/3; $2w_B/7$)
13) (A1; B234; C4) (1;1/5-1/4; $2w_B/7$) 14) (A1; B23; C24) (1; 1/5-1/4; $4w_B/25$)

15) (A1; B3; C234) (1; 1/5-1/3; w_B/10)　16) (A12; B3; C24) (1; 1/4-1/3; 1/25)

17) (A13; B3; C24) (1; 1/3; 1/30-1/25)　18) (A1; B123; C4) (1; 1/5; 4/125-2/35)

19) (A1; B1; C234) (1; 1/5; 1/60-1/50)　20) (A123; B3; C4) (1; 1/3; 1/25-2/21)

21) (A1; B13; C24) (1; 1/5; 1/50-4/125)　22) (A12; B23; C4) (1; 1/4; 1/25-1/14)

Limbo type IDL patterns with two disconnections

23) (A12; B3; C4) (1; 1/4-1/3; 1/25-1/7 and w_B/10-2w_B/7)

24) (A1; B23; C4) (1; 1/5-1/4; 1/60-1/7 and 4w_B/25-2w_B/7)

25) (A1; B3; C24) (1; 1/5-1/3; 1/60-1/25 and w_B/10-4w_B/25)

From this list, we can confirm that there is one (${}_{1+1}C_2$) wage rates constraint in the limbo type patterns with one disconnection and three (${}_{2+1}C_2$) with two disconnections. We will also give concrete examples of the judging method 2. For example, we can derive the pattern 11) by removing A4 from the pattern 1). In the same way, 11) or 12) is derived by removing B3 or A3 from 4), 23) by removing B4 from 12), and so on. As it were, the limbo type is the derivative from the linkage type and the latter is the origin of the former. Such *a relation of derivation* suggests that e.g. 1) and 4) adjoin each other and 11) forms the boundary between 1) and 4) on the world production frontier (hereafter, WPF). According to Shiozawa (2017), the WPF of multi-country multi-commodity has a shape of convex polytope which is covered by facets, the number of which is 10 in the case of 3-country 4-commodity, and each facet represents each IDL pattern of the linkage type, and joints of the facets represent the IDL patterns of the limbo type. In two-dimensional graphs of 2-country (or multi-country) 2-commodity, line segments correspond to the linkage type patterns and vertexes the limbo type patterns, and in three-dimensional graphs of 2-country 3-commodity, surfaces correspond to the linkage type patterns and ridgelines the limbo type patterns[24].

We can also guess such an adjoining relation from wage rates. For example, if we pay attention to the wage rate of 11) which is derived from 1) and 4), we can see that it lies between those of 1) and 4). The IDL patterns adjoining each other on the WPF

[24] See e.g. Shiozawa (2017, p. 6) about the three-dimensional graphs.

resemble each other only in the IDL patterns but also in the wage rates/prices (moreover, as stated in the next subsubsection, in employment quantities).

In the above list, another thing should be noted. There are very large wage rates differentials according to the IDL patterns. The range of the wage rates differentials reaches out from the minimum to maximum of the productivity differentials of individual sectors, namely from 1/2 to 1/5 between countries A and B, from 1/7 to 1/60 between countries A and C, and from 2/7 to 5/60 between countries B and C, which can be calculated from the above list. From a viewpoint of wage rates differentials, not only the production technique but also the IDL patterns are important.

10.2 Derivation of the equilibrium solutions

We give demand volumes as **Table 6**. The LQCs and URCs are introduced later.

Table 6: Demand volumes (case1)

	Demand volumes			
	Comm.1	Comm.2	Comm.3	Comm.4
Country A	80	90	100	110
Country B	40	60	70	110
Country C	20	30	40	50
World total	140	180	210	270

By calculating in accordance with the steps 2 and 3 mentioned in **Section 4**, we have seven candidates of the equilibrium solutions. The IDL patterns, employment quantities, and wage rates of countries B and C are compiled into **Table 7**. Employment quantities are rounded: the same applies hereafter.

Table 7: The IDL patterns, employment quantities, wage rates (case 1)

IDL patterns	4)	5)	7)	10)	12)	16)	22)
A's employment	343	301	300	276	320	294	285
B's employment	730	602	830	733	777	630	770
C's employment	1295	2600	1470	2350	1377	2534	1890
B's wage rates	1/3	1/3	1/4	1/4	15/52	109/350	1/4
C's wage rates	2/21	1/25	1/14	1/25	15/182	1/25	4/77

The numbers of IDL patterns correspond to the numbers of IDL patterns in the previous subsubsection: the four of 4), 5), 7), and 10) are the linkage type and the three of 12), 16), and 22) the limbo type. As described in **Section 5**, it is impossible that the limbo type is independently the candidate of equilibrium solution and the limbo type is necessarily accompanied by one or more linkage type adjoining itself on the WPF: 12) is accompanied by 4) and 7), 16) by 5) and 10), and 22) by 7) and 10). Let us pay attention to each value of the limbo type in **Table 7**. Then, we realize that all the values of the limbo type, in employment quantities as well as wage rates, lie between the values of two adjoining linkage type patterns.

Although we have to proceed to the step 4, before that (while examples are many), we examine the effect that changes in demand have on employment and wage rates. Changes in demand volumes bring about changes in employment quantities. If the changes are large, some of the candidates may disappear or new candidates may appear. Here, let us suppose that, in **Table 6**, country A's demand for commodity 4, which is not produced in country A, increases from 110 to 130 and the others are unchanged. Then, the candidates are unchanged and employment quantities and wage rates have values shown in **Table 8**. For convenience of comparing with **Table 7**, we put an up arrow (↑) in cells with increasing values and a down arrow (↓) in cells with decreasing values.

Table 8: The IDL patterns, employment quantities, wage rates (case 2)

IDL patterns	4)	5)	7)	10)	12)	16)	22)
A's employment	↑ 357	↑ 306	↑ 310	↑ 281	320	↑ 300	↑ 288
B's employment	730	602	830	733	↑ 803	630	↓ 757
C's employment	1295	2600	1470	2350	↑ 1423	2534	↑ 2030
B's wage rates	1/3	1/3	1/4	1/4	↓ 15/56	109/350	1/4
C's wage rates	2/21	1/25	1/14	1/25	↓15/196	1/25	↓ 1/21

In the four linkage type patterns and one limbo type pattern of 16) in which countries A and C are linked, the sole change is increases in employment of country A[25] and there is no change in the wage rates/prices. In the every five cases, the country

[25] About the reason why, despite the same increases in demand, increases in employment quantities differ, see **Section 7**.

increasing production of commodity 4 decreases production of another commodity, and in turn, country A increases the commodity to cover the decreasing production, consequently leading to the increases in country A's employment. To sum up, only the Graham case (quantity adjustments without price changes) exists and the Mill case (adjustments with price changes) does not exist.

In the two limbo type patterns of 12) and 22), the wage rates of countries increasing production of commodity 4 (countries B and C in 12) and country C in 22)), therefore prices of commodities produced by these countries (commodities 3 and 4 in 12) and commodity 4 in 22)) decline. These price changes are accompanied by changes in export-import volumes, therefore production volumes and employment quantities, and consequently trade equilibrium is kept. Anyway, the effect of demand changes on the employment and wage rates is as described in **Section 7**.

Let us introduce the LQCs and URCs. Suppose that country A has 360 labor, country B 800, and country C 2800 and the upper limit of unemployment rates is 25% in common. Each country has to satisfy the following LQCs and URCs.

$$270 \leq \text{Employment quantities of country A} \leq 360$$
$$600 \leq \text{Employment quantities of country B} \leq 800$$
$$2100 \leq \text{Employment quantities of country C} \leq 2800$$

Under these conditions, the seven candidates are narrowed down to the elected three patterns of 5), 10), and 16). However, if we assume the distribution of labor that country A has 360 labor, country B 880, and country C 1600, the elected patterns change to the three of 4), 7), and 12). We can make intently a supposition that the equilibrium solution is unique. For example, under the assumption that country A has 340 labor, country B 810, and country C 1600, only the pattern 12) passes through the LQCs and URCs. Which patterns survive depends on the assumption.

It should be kept in mind that, even though there are many candidates, equilibrium solutions are narrowed down to a few of them by the LQCs and URCs. The reason is as follows. Let us take two patterns arbitrarily from e.g. **Tables 7** and compare each country's employment quantities between both patterns. Then, we can see the fact that, in the pattern which one country's employment is more, one or two of the other countries necessarily have less employment than the other pattern. Under the same

world production (equal to demand) volumes, if one country's production volumes are more, other countries' production volumes are naturally less. The above fact is no more than a reflection of this natural thing. This means that, if an unemployment rate declines in one country, it rises in other countries. Therefore, the number of equilibrium solutions is not so many unless the upper limit of unemployment rates is very high.

When there still remain multiple equilibria after screening by the LQCs and URCs, the model itself is not able to determine which solution is realized finally. Various reasons outside the reach of the model, e.g. path dependency, accident, and so on, determine this. We have to note the following point, however. The equilibrium solutions of the limbo type (limbo solutions) are almost always accompanied by changes in the wage rates/prices in the face of changes in demand. Therefore, especially when the limbo solutions are near the equilibrium solutions of the linkage type (linkage solutions), switching from the limbo solutions to the linkage solutions is probable. Contrary, switching from the linkage solutions to the limbo solutions is almost improbable, because the linkage solutions don't change the wage rates/prices in the face of changes in demand. Considering these, probability that the linkage solutions are realized finally would be high.

If the world demand differs, and if the composition by countries differs under the same world demand, the equilibrium solutions may differ, which can be singular, not multiple. For example, under the demand composition in **Table 9**, of which total is the same as **Table 6**, the candidate is only one of the pattern 5).

Table 9: Demand volumes (case 2)

	Demand volumes			
	Comm.1	Comm.2	Comm.3	Comm.4
Country A	50	60	100	110
Country B	30	40	50	110
Country C	60	80	60	50
World total	140	180	210	270

Employment quantities are 241, 452, and 5350 in order from country A to C. In the case that these employment quantities don't fulfill the LQCs and URCs, as mentioned

in **Section 8**, an excessive demand or an unusual shortage of demand occurs in the countries not fulfilling the constraints.

10.3 Numerical simulation of multiple equilibria

Here, we conduct a numerical simulation using a 3-county 4-commodity example about a probability of multiple equilibria. A starting point is as follows: each country's demand volumes for each commodity are as **Table 5** except country A's demand for commodity 4 (d_{A4}), which increases one by one from one. Then, the relations between values of d_{A4} and the candidates of equilibrium solutions are as the list below. As to the limbo type, we show the origins of the limbo type patterns in { } at the first appearance: the pattern 23) is the limbo type with two disconnections and is derived from the pattern with one disconnection.

Values of d_{A4}:	the candidates of equilibrium solutions
1 – 50:	5), 10), 16) {← 5) and 10)}
51 – 75:	5), 7), 10), 16), 22) {← 7) and 10)}
76–137:	4), 5), 7), 10), 12) {← 4) and 7)}, 16), 22)
138–149:	4), 5), 7), 10), 12), 16), 20) {←4) and 5)}, 22), 23) {←12), 16), 20) and 22)}
150 – 175:	4), 5), 10), 16), 20), 22), 23)
176 – 202:	4), 5), 16), 20), 23)
203 – 211:	4), 5), 20)
212 – 260:	4)
261 – 389:	1), 4), 11) {← 1) and 4)}
390 – :	1)

The above transition of the candidates show concretely the content described in **Section 5**. First, the candidates change with changes in d_{A4}. Second, although the candidates are usually multiple, sometimes singular. Third, there is no case that the limbo type is a single candidate: the limbo type is always accompanied by one or more its origins. Moreover, the origins (the linkage type) and the derivatives (the limbo type)

simultaneously appear or disappear: for example, 7) and 22) appear together when the value of d_{A4} change from 50 to 51; 7) and 12) disappear together when the value of d_{A4} change from 149 to 150. Thus, the analytical results obtained in **Section 5** would be also valid in the multi-country multi commodity case.

It should be noted that the transition of combinations of multiple equilibria and the change of the realized IDL pattern are very different things. Suppose that, when d_{A4} is 130, for some reason or other, 4) out of seven candidates is realized and that only d_{A4} changes. While d_{A4} changing from 130 to 140 and to 160, combinations of the candidates or multiple equilibria change. The IDL pattern would remain 4), however, as long as the LQCs and URCs of country A are satisfied. Even though the combinations of the multiple equilibria change, the IDL pattern once realized stabilize owing to path dependency.

11 Concluding remarks

The present trade model determines not only relative commodity prices and relative wage rates but also production volumes and employment quantities in each country. Not being premised on full employment, the model tells us some important information about the relation between demand and employment.

Changes in demand cause two kinds of adjustment process. Domestic demand increases for commodities produced in home country or countries linked with home country increase domestic employment. The wage rates/prices and foreign employment don't change at all. On the other hand, domestic demand increases for commodities produced in countries not linked with home country raise the domestic relative wage rates to the countries exporting the commodities. This is contrary to the conventional trade models presupposing full employment. As for employment, according to the situation of linkage, various cases can occur. Domestic employment may increase or may not change, and foreign employment may increase or decrease.

In the trade imbalance case, countries which continue to increase a trade surplus can continue to increase domestic employment without increases in domestic demand. However, this is possible only in a short time. In a long time, any country must

increase domestic demand to increase domestic employment. In the trade models presupposing full employment, progress in trade liberalization never causes an employment problem. In our model, this may cause a rise in unemployment rates if domestic demand does not increase sufficiently. Domestic demand matters. We should not rely on foreign demand. This is the most important message from our model.

Thus we have provided a new trade model. Of course, much remains to be done. In order to closer to reality, we should introduce intermediate goods[26] and non-tradable goods which account for substantial share in the real world. Moreover, If we can set any appropriate demand function, our model will be expand more and give us more useful and suggestive information.

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[26] According to WTO (2013), share of intermediate goods accounted for 51-55% of world non-fuel export in the years 2000-2011. In recent years, research into the trade of intermediate goods has started earnestly. See Shiozawa (2017) and Oka (2017).

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