Automata-theoretic study on infinite games and fragments of modal μ-calculus

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論　文　内　容　要　旨

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学位論文の題　　目	Automata-theoretic study on infinite games and fragments of modal μ-calculus (無限ゲームと様相 μ 計算の部分体系についてのオートマトン理論的研究)		

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Abstract

This thesis is a contribution to the automata-theoretic study on the infinite games and modal μ-calculus. It mainly consists of two parts, preceded by an introduction to the fundamental concepts and results on infinite games, automata and logic (in Chapter 2). The first part is dedicated to determinacy of infinite game recognized by some variants of pushdown automata, and the other to the alternation hierarchy and fragments of modal μ-calculus.

Part I: Determinacy of pushdown ω-languages (Chapter 3)

The determinacy of Gale-Stewart games has been intensively studied in descriptive set theory. Büchi and Landweber [2] first paid attention to the computational aspect of winning sets and winning strategies. Given a Gale-Stewart game $G(A)$, where A is an ω-regular language accepted by a finite Büchi automaton, they showed that one can effectively decide the winner of such $G(A)$ and a winning strategy in $G(A)$ can be constructed by a finite state transducer.

On the other hand, Finkel [4] proved that the determinacy of context-free ω-languages, accepted by nondeterministic Büchi (or Muller) pushdown automata, is equivalent to the determinacy of effective analytic games, even unprovable in the set theory ZFC.

In Part I, we downscale Finkel's results to lower levels of Borel hierarchy. We investigate the determinacy strength of infinite games whose winning sets are recognized by variants of pushdown automata with various acceptance conditions, e.g., safety, reachability and co-Büchi conditions. In terms of the foundational program “Reverse Mathematics”, the determinacy strength of such games is measured by the complexity of a winning strategy required by the determinacy.

Notice that the infinite games recognized by nondeterministic pushdown automata bear some resemblance to those recognized by deterministic 2-stack visibly pushdown automata with the same acceptance conditions. The 2-stack visibly pushdown automata is a kind of input-driven pushdown automata with two stacks. The input alphabet is partitioned into push, pop alphabet for each stack separately, and internal alphabet, which decide its visible actions on the stacks.

So, we start with the determinacy of games recognized by deterministic 2-stack visibly pushdown automata, together with those recognized by nondeterministic ones. Then, for instance, we prove that the determinacy of games recognized by pushdown automata with a reachability condition is equivalent to the weak König lemma, stating that every infinite binary tree has an infinite path. While the determinacy for pushdown ω-languages with a Büchi condition is known to be independent of ZFC, we here show that for the co-Büchi condition, the determinacy is exactly captured by ATR_0, another popular system of reverse mathematics asserting the existence of a transfinite hierarchy produced by iterating arithmetical comprehension along a given well-order.

Finally, we conclude that all the arguments about pushdown automata in Part I are, in fact, replaced by (nondeterministic) 1-counter automata, namely pushdown automata that can check whether the counter is zero or not with only one stack symbol.

The results of Part I have been published in [6] and [7].

Part II: Alternation hierarchy and fragments of modal μ-calculus (Chapter 4)

Modal μ-calculus, first introduced by Kozen [5], is the logic obtained by adding least (μ) and greatest (ν) fixpoint operators to the modal logic. By least and greatest fixpoint, we mean that of the equation $X = \Gamma(X)$, where Γ is a monotone function with X a set variable. Modal logic is just the propositional logic with universal and existential modality.

From an automata-theoretic view, modal μ-calculus is closely related with tree automata. The equivalence between modal μ-calculus and (alternating) parity tree automata over binary trees is established by Emerson and Jutla [3]. Study along this line is motivated by Rabin’s investigations on the decidability of monadic second order logic with two successors [8], and highly concerned with the positional determinacy of parity games [3].

A fundamental issue on modal μ-calculus is that whether the fixpoint operators are necessary, namely, the strictness of alternation hierarchy of modal μ-calculus. The alternation hierarchy classifies the formulas by their alternation depth, that is, the number of alternating blocks of least and greatest fixpoint operators. Note that the alternation depth, in a game-theoretic view, is related with the number of priorities in parity games, and from an automata-theoretic perspective, is concerned with the Rabin index of Rabin tree automata. The strictness of alternation hierarchy of modal μ-calculus was first established by Bradfield [1].

In Part II, we start with comparing three kinds of alternation hierarchy for modal μ-calculus, namely, Niwinski, Emerson-Lei, and simple alternation hierarchy. Next we introduce the descriptive-theoretic and automata-theoretic arguments on strictness of such hierarchies. Moreover, the relations with arithmetic μ-calculus, variable hierarchy, (alternating) parity tree automata and parity games are explained. We also introduce the transfinite extension of modal μ-calculus.

Then we concentrate on the one-variable fragment of modal μ-calculus. Note that apart from the alternation depths, the number of variables contained in a formula also serves as an important measure of complexity for formulas of modal μ-calculus. Thus in Part II, we define the simple alternation hierarchy of one-variable fragment of modal μ-calculus and weak alternation hierarchy. We prove that simple alternation hierarchy of one-variable fragment of modal μ-calculus is strict, which is obtained by analyzing the correspondence with the counterparts of weak alternating tree automata and weak parity games. Moreover, the simple alternation hierarchy of one-variable fragment exhausts the weak alternation hierarchy.

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論文審査の結果の要旨

本博士論文は３つのテーマを扱っている．なかでも主要なトピックは無限ゲームのオートマトン理論的研究である．これは修士論文で行ったプッシュダウン・オートマトン(pda)の記述的階層の研究をさらに進め，種々の pda が定義するゲームの決定性の強さを逆数学的に特定したものである．Finkel の先行結果(pda が Büchi 条件で受理するゲームの決定性は ZFC 集合論から独立である)に対して，本論文では pda が co-Büchi 条件の下で受理するゲームの決定性は逆数学の主要公理の一つ ATR と同値であることを示している．この研究成果は一流専門誌にも受理されており，論文提出者が多方面の専門知識を正確に身につけていることを示している．

２つめのテーマは，様相μ計算，つまり不動点演算をもつ様相論理に関する研究である．変数を１つに制限した式による階層構造について，オートマトンとゲームの視点で新しい知見を得た．これについては，投稿論文を準備中である．

３つめのテーマは，ゲーム木に対するランダム・アルゴリズムについての研究である．研究室の先輩 Peng と沖坂との共著として，すでに 3 編の論文が発表されているが，共同研究であるため，本博士論文には概容のみ述べられている．なお，2016 年 3 月にそれら論文の１つにより International Association of Engineers (Hong Kong)の Best Student Award を共同受賞した．

以上のことは，論文提出者が自立して研究活動を行うに必要な高度の研究能力と学識を有することを示している．したがって，李文娟提出の博士論文は，博士（理学）の学位論文として合格と認める．