Dualities and generalized gauge structures in string theory and M-theory

著者	HELLER Marc Andre
号	77
学位授与機関	Tohoku University
学位授与番号	理博第3080号
URL	http://hdl.handle.net/10097/00121790

論 文 内 容 要 旨

(NO. 1)

氏　名	HELLER, Marc Andre	提出年	平成 29 年
学位論文の題目	Dualities and generalized gauge structures in string theory and M-theory (弦理論とM理論における双対性と一般化されたゲージ構造)		

論 文 目 次

1. Introduction
2. String theory: A synopsis
3. Dualities in string theory and M-theory
    3.1 Introduction
    3.2 Physical preliminaries
        3.2.1 Toroidal compactification of string theory and T-duality
        3.2.2 T-duality and non-geometric spaces
        3.2.3 Kaluza-Klein reduction and gauged supergravity
        3.2.4 Scherk-Schwarz reduction
        3.2.5 Reduction with duality twists
        3.2.6 Non-geometric fluxes from orbifold CFTs
        3.2.7 Generalized geometry
        3.2.8 Double field theory
        3.2.9 M-theory and U-duality
        3.2.10 On the emergence of U-duality
        3.2.11 Exceptional generalized geometry
    3.3 Mathematical preliminaries
        3.3.1 Poisson geometry, cohomology and connections
        3.3.2 L_∞-algebras
        3.3.3 Courant algebroids
        3.3.4 Graded manifolds and supergeometry
        3.3.5 AKSZ sigma models
        3.3.6 Current algebras
        3.3.7 Deligne cohomology and n-gerbes
    3.4 Twisted Courant algebroids and fluxes
        3.4.1 Graded symplectic manifold
        3.4.2 Courant algebroid with trivial R-flux

3.4.3 Classification of twists

3.4.4 Non-geometric β-twisted Courant algebroid and cohomology

3.4.5 Courant algebroid with frame bundle

3.4.6 Courant algebroid with geometric flux

3.4.7 Fully fluxed Courant algebroid

3.5 Double field theory and T-duality

3.5.1 Pre-QP-manifold

3.5.2 Generalized fluxes

3.5.3 Non-geometric Courant algebroids

3.5.4 T-duality

3.6 Poisson-Courant algebroid

3.6.1 Supergeometry of the Poisson-Courant algebroid

3.6.2 Poisson-Courant algebroid cohomology versus standard Courant algebroid cohomology

3.6.3 Flux duality I: QP-manifolds

3.6.4 Flux duality II: Cohomologies

3.6.5 Embedding in double field theory

3.6.6 Poisson-Courant sigma model

3.6.7 Flux duality III: Courant sigma models

3.6.8 Contravariant Poisson algebra

3.6.9 Contravariant current algebra

3.7 Generalized geometries

3.7.1 Generalized geometry

3.7.2 B_n-generalized geometry

3.7.3 Exceptional generalized geometry with M2-branes

3.7.4 Exceptional generalized geometry with M2- and M5-branes

3.8 Summary

4. Higher gauge theory and multiple M5-branes

4.1 Introduction

4.2 Physical preliminaries: M-branes

4.3 Mathematical preliminaries

4.3.1 Category theory and higher gauge theory

4.3.2 Higher gauge theory from QP-manifolds

4.4 Higher gauge theories of multiple M5-branes

4.4.1 Off-shell covariantization

4.4.2 Graded symplectic manifold setup and classification

4.4.3 Symplectic NQ-manifold of a semistrict Lie 2-algebra

4.4.4 Higher gauge theory of a semistrict Lie 2-algebra

4.4.5 Higher gauge theory of multiple M5-branes

4.5 Summary
5. Discussion and outlook
A. Detail calculations
A.1 Special subset of solutions
A.2 Residual gauge transformations on constraint surface
B. Conventions and formulas
B.1 Graded differential geometry
B.2 Graded symplectic geometry
B.3 Graded functional analysis I: Fields and derivatives
B.4 Graded functional analysis II: Graded symplectic geometry
B.5 Graded functional analysis III: Integration

The present thesis is divided into two parts: Dualities in string theory and M-theory, and Higher gauge theory and multiple M5-branes.

The first part of the present thesis concerns dualities in string theory and M-theory. String theory is presently the most promising candidate for a theory of quantum gravity. In contrast to ordinary particle physics, the fundamental objects of string theory are 1-dimensional open and closed strings. Anomaly-freeness requires supersymmetric string theory to be formulated in 10 spacetime dimensions. To make contact to our 4-dimensional observable world, 6 spatial dimensions are considered to be closed and very small. This is called compactification.

The simplest example of a compactification is a circle, S^1. Since strings are 1-dimensional objects, their closed versions can wind around compact 1-cycles of the compactification. In this case, the winding number w counts how often a closed string winds around the circle. Furthermore, its momentum in the compact S^1-direction is quantized, leading to a momentum mode n. It turns out that string theory is invariant under the exchange $n \leftrightarrow w$ while inverting the radius of the circle, $R \rightarrow 1/R$. This target space symmetry is called T-duality and emerges through the fact that 1-dimensional objects probe geometry differently compared to ordinary point-objects.

The string couples to an antisymmetric (0,2)-tensor $B_{\mu\nu}$, the Kalb-Ramond B-field, whose field strength is given by the so-called H-flux, $dB = H$. The H-flux is a 3-form and can wrap closed 3-cycles of the compactification. In the case of non-zero H-flux, T-duality in isometry directions mixes target space metric and B-field.

In the case of a flat 3-torus compactification, $T^3 = S^1 \times S^1 \times S^1$, with constant H-flux, successive T-duality transformations are possible and lead to new mysterious compactification spaces. Let the T^3 be parameterized by coordinates x^1, x^2 and x^3. It turns out, that the first T-duality transformation in

x^1-direction transforms the H-flux to the so-called geometric f-flux, which measures the torsion of the compactification. The resulting space is a twisted torus. A further T-duality transformation along x^2 leads to a 3-torus with monodromy, which has to be patched by a T-duality transformation. This space ceases to be an ordinary manifold and hence is called non-geometric space. The former f-flux is transformed to a globally non-geometric Q-flux encoding the monodromy. Due to the lack of an isometry direction, the last T-duality along x^3 cannot be computed. However, it is conjectured to induce a further locally non-geometric R-flux, which is believed to be related to non-associative backgrounds. The T-duality chain can be summarized by $H_{123} \leftrightarrow f^1{}_{23} \leftrightarrow Q^{12}{}_3 \leftrightarrow R^{123}$.

In the case of a T^D-compactification, the T-duality group is O(D,D;Z). Generalized geometry treats the metric and B-field on an equal footing by extending the tangent bundle TM to the generalized tangent bundle $TM \oplus T^*M$, which has manifest O(D,D)-symmetry. Here, M is the compactification manifold. It turns out to be the appropriate framework to analyse T-duality on H-flux backgrounds, since the additional cotangent fiber accounts for the string wrapping modes. The local symmetries of generalized geometry are encoded in a so-called Courant algebroid on $TM \oplus T^*M$. However, for the description of arbitrary non-geometric backgrounds, this Courant algebroid turns out to be insufficient.

Double field theory is a field theory, which makes the toroidal T-duality group manifest by doubling the degrees of freedom. To the compactification torus T^D, a double torus is introduced, which parametrizes the winding sector. All fields are taken to depend on the doubled set of coordinates. In this formulation, also T-duality in non-isometry directions is possible and some understanding of the mysterious non-geometric fluxes can be provided. The R-flux arises as $R = d_\beta \beta$, where d_β is the Lichnerowicz-Poisson differential and β is a bivector potential. To restrict the theory to a physical subspace, a strong constraint is imposed.

A further proposal for R-flux backgrounds is provided by the Poisson-Courant algebroid, a Courant algebroid on a Poisson manifold with Poisson tensor Π. It exchanges the roles of the tangent and cotangent bundles of generalized geometry leading to the so-called Poisson-generalized geometry. In this case, the R-flux arises by $R = d_\Pi \beta$.

It is well-known that there are 5 consistent string theory formulations in 10 dimensions, which are limits of an underlying 11-dimensional theory, called M-theory. Its low-energy limit is 11-dimensional supergravity. Compactified on a D-torus, it exhibits a symmetry under split real forms of exceptional Lie groups $E_{D(D)}$. This symmetry is called U-duality and is the fundamental symmetry of M-theory. There are no strings present in M-theory. The dynamical objects are M2- and M5-branes, which couple to the 3-form gauge potential C_3 and its Hodge dual.

Exceptional generalized geometry attempts a geometrization of toroidal U-duality along the lines of generalized geometry. This encodes the wrapping modes of M-branes and symmetries of KK-monopoles.

In this thesis, we construct a twisted Courant algebroid, which naturally encodes the local expressions of all geometric and non-geometric fluxes as well as their Bianchi identities in a unified way. This is done by making use of graded symplectic manifolds and provides the underlying generalized gauge structure associated with T-duality geometry. Furthermore, we provide an analysis of the cohomological structures associated with non-geometric spaces.

We lift our construction up to a O(D,D)-manifest reformulation and recover all generalized fluxes that appear in double field theory. Motivated by this result, we provide a consistent definition of T-duality in the graded symplectic manifold setup.

We provide a thorough investigation of a Courant algebroid structure on a Poisson manifold. We reconstruct it in the supergeometric setting and analyse its cohomology and relation to double field theory and T-duality. We construct a topological membrane with R-flux along the lines of AKSZ sigma models. The boundary theory of the membrane turns out to be a string sigma model with R-flux WZW term. On the loop space of string embeddings, we construct current algebras with R-flux and Poisson-Courant algebroid structure. The resulting current algebras are of Alekseev-Strobl type. Furthermore, as a result of our analysis we formulate a novel duality transformation between the standard Courant algebroid with H-flux and Poisson-Courant algebroid with R-flux, which we call flux duality. We analyse the flux duality on 3 levels: as symplectomorphism between graded symplectic manifolds, as isomorphism between cohomologies and as relation between induced membrane sigma models.

The second part of the present thesis concerns higher gauge theory and multiple M5-branes. The dynamical objects of M-theory are M2- and M5-branes. In contrast to the system of multiple M2-branes, the analysis of systems of multiple M5-branes is still a field which is not very well understood. What is known is that M5-branes interact by M2-branes, which extend between them. Then, the 1-dimensional boundaries of the M2-branes turn out to be soliton solutions of a 6-dimensional superconformal N = (2,0) theory. Since these solutions are charged under a self-dual 2-form gauge field, they are also called self-dual strings. For the case of a single M5-brane, the dynamics of the self-dual strings is governed by a so-called principal 2-bundle with abelian gerbe structure. The dynamics for multiple M5-branes is still unclear. However, it is believed that the governing structure is a non-abelianization of the principal 2-bundle, or a non-abelian gerbe.

The governing structure of the self-dual strings should incorporate a notion of parallel transport of 1-dimensional objects. In ordinary Yang-Mills theory there is a notion of Wilson line, or holonomy of point-objects, or ordinary parallel transport. Exactly the generalization of ordinary parallel transport and the consistent definition of associated higher principal bundles is provided by the young mathematical field of higher gauge theory. Using the viewpoint of higher categorification, a notion of parallel transport of 1-dimensional objects could be defined. It turned out that the underlying higher gauge structure is given by a so-called differential crossed module. This defines a non-abelian gerbe with 3-form curvature H, which is believed to be the appropriate framework for the non-abelianization of the N = (2,0) theory governing the dynamics of multiple M5-branes.

The resulting 2-form higher gauge theory associated with the differential crossed module incorporates the usual Yang-Mills 1-form gauge field A with 2-form curvature F and a 2-form gauge field B with 3-form curvature H. However, it turns out that it is a generic feature of higher gauge theories, that the consistent definition of a higher Wilson volume, or higher parallel transport, requires all lower form curvatures but the highest form one to vanish. For the 2-form higher gauge theory, it turns out that the covariant gauge transformation of H requires F = 0. This is called fake curvature condition. This requirement highly restricts the dynamics of possible Lagrangian theories. If we want to construct a Lagrangian with non-abelian gauge symmetry of 2-form gauge fields, then the fake curvature condition restricts the construction to a BF-type topological field theory leading to an essentially free theory.

We propose a method, called off-shell covariantization, to circumvent the fake curvature condition towards a non-topologically interacting theory with non-abelian gerbe structure. The method makes use of the prescription to construct higher gauge theories from QP-manifold structures. Off-shell covariantization introduces auxiliary gauge freedom to the initial higher gauge theory of the non-abelian gerbe and constrains the resulting system such that the residual gauge transformations lead to a non-abelian gerbe structure, which does not inherit the fake curvature issue. We apply the off-shell covariantization procedure to the theory, which is a candidate of a theory of M5-branes compactified on a circle.

様式３の(3)

論　文　審　査　等　報　告　書

博士の 専攻分野	博士（理学）	ふりがな 氏　　名	ヘ　ラ　マルク　アンドレ HELLER, Marc Andre
論文審査の 結果の要旨 及びその 担当者氏名	別紙のとおり 論文審査担当者氏名　（主査）綿村　哲 日笠　健一 山口　昌弘 石川　洋 長谷川　浩司		
最終試験の 結果の要旨 及びその 担当者氏名	本学大学院理学研究科の選定した 下記担当者が行った試験に合格した。 試験担当者氏名　（主査）　同上		
博士論文審 査機関の名 称及び組織	名称　審査会 組織　委員５名		
修了の要件	本研究科規定の定める修了要件を満たしている。		
判定の方法	理学研究科委員会の議決による。		

備考　この報告書は、本人に記入させないこと。

別　紙

論文審査の結果の要旨

Heller 氏の博士論文の内容はすでに発表された３篇の論文の内容を踏まえて，研究の背景と動機を含めて，研究の結果を統一的視点からまとめたものである。論文の目的は弦理論およびＭ理論に現れる双対性と呼ばれる非常に高い対称性の構造，特に様々なフラックスが存在する場合の理論の構造を解明することにある。論文は大きく分けて，弦理論およびＭ理論に現れるフラックスに関する研究と，高階ゲージ理論の非アーベル化に関する研究における２つの結果を含む。これらの研究の道具として用いられているのが，次数付シンプレクティック多様体または QP 多様体と呼ばれる超空間上の微分演算であり，論文にはそれらの導入も含まれている。

第１の結果は，超空間上の形式を用いることにより弦理論のコンパクト化によって現れる様々なフラックスを統一的に記述し，結果として得られるビアンキ恒等式など，それまでに知られているフラックスに関する関係式を含むより一般的な式を構成することに成功している。第２の結果は高階ゲージ対称性を非アーベル化したときに現れる虚曲率の問題の一つの解放を与えている。いずれも，それまでに無いアプローチでそれぞれの問題に関して新しい知見を与えるものであり博士論文の内容として十分な結果である。また，論文はレビュー部分も含めて必要十分な内容が研究の背景と議論する問題点と研究の結果が簡潔に書かれている。最終試験による審査および予備審査会における発表も非常に明快かつ簡潔であり，質疑応答も含め十分に合格に値する内容であった。以上の結果は，Heller 君が自立して研究活動を行うに必要な高度の研究能力と学識を有することを示している。

従って，最終試験に関して，審査員全員の一致で，Heller, Marc Andre 提出の博士論文は，博士（理学）の学位論文として合格と認める。