## Research Note

# Occurrence of Bimodal Classroom Achievement in Ontario 

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## Purpose

Classroom based histograms of student achievement often contain structural characteristics suggesting underlying, or latent, achievement behaviors within the classroom. Using a single achievement variable per student and a frequency of occurrence, the number of classrooms exhibiting a bimodal distribution was studied. Bimodality was interpreted as having two or more significant local maximums in the histogram. The methodology examines the histogram directly to avoid assumptions about latent variables, using techniques from pattern recognition ( Ng , 2006; Rosin, 2001) to establish which classrooms exhibit the characteristics of a bimodal distribution.

The theoretical basis for the study was the development of a technique for identifying classrooms that exhibit characteristics of different distributions of student ability level (Sibbald, 2013). This was an empirical study that differs from other methodological approaches, such as inspection (Humphry \& Heldsinger, 2014) or teacher identification of groups (Hallinan \& Sorensen, 1983), because it is data driven. No existing technique was found and a method was developed using pattern recognition techniques for feature extraction (Zheng \& Xue, 2009), statistical and template matching techniques in images (Mahalakshmi, Muthaiah, \& Swaminathan, 2012). This approach using feature analysis has found utility in other fields (Clark, 1977; Guo, Gallagher, Bland, \& Camm, 2009; Harlow, 2011; Jaskierniak, Lane, Robinson, \& Lucieer, 2011).The research objective of the study was to establish whether the occurrence of bimodality justified fine-tuning the technique and the provision of an objective methodology for future investigations regarding the value of such information within education systems. It was also intended to allow an appraisal of whether the resulting statistics were concordant with teacher experiences.

## Methodology/Techniques

Student achievement on a 24 item standardized instrument, an Ontario Educational Quality and Accountability Office (EQAO, 2014) assessment was used to create a classroom histogram reflecting frequency at each level (o to 24) of achievement. Increases in the histogram that pass through a threshold (Coudray, Buessler, \& Urban, 2010; Rosin, 2001) were interpreted as signifying the existence of a peak that exceeds the threshold in the histogram. When more than one peak was found, the histogram was deemed "potentially bimodal." The threshold was selected based on practical considerations from teaching experience, but different values were
subsequently tested to assess its impact.
Potentially bimodal histograms were compared to a uniform step distribution using a chisquared goodness of fit test. The uniform step had a range from the first to the last value that exceeded the threshold and a value that was the class-size divided by the number of cells. Statistical significance at the $95 \%$ level was interpreted as classifying the potentially bimodal classroom histogram as a uniform distribution, otherwise it was deemed to be bimodal.

## Data Source

Population data from 6,943 classrooms was used. It corresponded to 143,925 grade nine students in Ontario, who wrote a grade nine standardized EQAO math assessment during the 2010/2011 school year. Individual student achievement was determined using 24 multiplechoice items leading to a frequency histogram of student achievement in each classroom. Figures 1 and 2 show two classroom histograms. Both were potentially bimodal with the first being bimodal and the second being a uniform step distribution at the $95 \%$ confidence level.


Figure 1: Example of a bimodal classroom distribution.


Figure 2: Example of a uniform classroom distribution.

## Results/Conclusions

The results found that using a $95 \%$ confidence level to remove uniformly distributed classrooms, 26.1\% (1,809) of all Ontario grade 9 classrooms exhibited bimodal characteristics. A further $36.3 \%(2,520)$ of classrooms were potentially bimodal but determined to be uniformly distributed. The composition of the bimodal classrooms, according to the number of modes is provided in Table 1.

A preliminary examination of the threshold value and the confidence value for the goodness of fit test was done. The impact of the value of the threshold on the number of classrooms potentially having a bimodal distributions appears in Table 2. The value 4329 that appears in the table means that there were 4329 (of the 6943 total) classroom histograms which have 2 or more peaks and that each peak exceeded 3 students. The value is a total count for potentially bimodal classrooms and does not show that subsequent goodness of fit testing determined that, in this particular case, 1,809 were bimodal while 2,520 were uniform distributions.

Table 2 demonstrates that the threshold for potential bimodality in classroom distributions is somewhat unclear. A value of three was used based on the pragmatic argument that smaller groups diminish instructional time that a teacher can have with each group. A minimum cluster size of four seemed tangible from the point of view of teaching and would not create an unwieldy number of groups in a classroom.

Table 1
Number of multi-modal classrooms

|  | Number of modes |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | $>5$ |
| Count (Percent) | $1414(20.4 \%)$ | $321(4.6 \%)$ | $63(0.9 \%)$ | $10(0.1 \%)$ | $1(0.0 \%)$ |

Table 2
Number of modes for different detection thresholds

| Number of peaks | Threshold for peak detection |  |  |  |
| :---: | ---: | ---: | ---: | ---: |
|  | 2 | 3 | 4 | 5 |
| $0+$ | 6943 | 6943 | 6943 | 6943 |
| $1+$ | 6600 | 6027 | 4227 | 2014 |
| $2+$ | 6168 | 4329 | 1495 | 267 |
| $3+$ | 4877 | 2007 | 194 | 12 |
| $4+$ | 2774 | 537 | 7 | 0 |
| $5+$ | 986 | 51 | 0 | 0 |
| $6+$ | 218 | 2 | 0 | 0 |
| $7+$ | 18 | 0 | 0 | 0 |
| $8+$ | 2 | 0 | 0 | 0 |

Table 3
Impact of confidence level on potentially bimodal distributions

|  | Confidence Level |  |  |
| :--- | :---: | :---: | :---: |
|  | $90 \%$ | $95 \%$ | $99 \%$ |
| Bimodal distribution | $1546(22.3 \%)$ | $1809(26.1 \%)$ | $2358(34.0 \%)$ |
| Uniform distribution | $2783(40.1 \%)$ | $2520(36.3 \%)$ | $1971(28.4 \%)$ |

The confidence level of the goodness of fit test was examined for a fixed threshold value of three. This was done with three confidence levels, $90 \%$, $95 \%$, and $99 \%$, with separation of potentially bimodal classrooms into bimodal classrooms and uniform step classrooms. The results, given in Table 3, show that increases in the confidence level reduce the number of uniformly distributed cases and increases the number that are bimodal. Since the designation as potentially bimodal does not depend on the confidence interval, the confidence level is a decision criterion, for potentially bimodal classrooms, between the two specific distributions.

## Educational Importance

The findings demonstrate that data informed achievement groupings are identifiable and indicate a high level of need for achievement-differentiated instruction (Pierce \& Adams, 2004; Tomlinson, Brighton, Herberg, Callahan, Moon, et al., 2003). The results suggest that teachers could potentially utilize modal information to tailor pedagogical strategies. Underlying structures could also inform policy initiatives; for example, Berg (2008) indicates a bimodal distribution was indicative of socio-economic differences.

The methodology has demonstrated that there are sufficient bimodal (and uniform step distributions) histograms of student achievement to warrant fine tuning the technique. Classroom achievement that produces bimodal achievement scores can be investigated further by considering mixtures of distributions that has proved fruitful in other fields of research. This will help to clarify what underlying phenomena led to the distinct clusters in the achievement histogram.

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