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WASTE OPERATIONS MANAGEMENT: LOCATION AND ROUTING MODELS

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Executive Summary

WASTE OPERATIONS MANAGEMENT:
LOCATION AND ROUTING MODELS

The purpose of our MA dissertation is to demonstrate how evidence-based research can help companies and not-for-profit organizations to engage in Waste Management (WM) and Reverse Logistics (RL) effectively and efficiently. Our focus is in identifying the most suitable routing and location models for end-of-life products and waste.

To achieve this, we draw on both primary sources and secondary sources. In particular, we review the latest studies in the international literature that apply routing and location models on real-life case studies from all over the world. We then use these insights to recommend an appropriate WM and RL strategy to three Greek organizations.

The dissertation is divided in two parts. In the first we present and analyze 20 recent studies, 10 on routing and 10 on location models. In each case we present in detail the problem, the data, the objectives, the constraints and the mathematical model with its variables. In the second part we investigate three different Greek organizations engaged in RL and WM. Based on document analysis and in-depth interviews with the organizations' representatives, we argue that all of them operate on the basis of 'practical experience' instead of formal models, a practice which often leads to sub-optimal results. The dissertation concludes by proposing the most suitable model, for a given set of objectives, for each of the organizations in our Greek case studies.

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1. Introduction

1.1. Definition and Importance of Waste Management and Reverse Logistics

Growing awareness that we live in a world of finite resources and of increased environmental pressures induced changes in the way production and consequently logistics are perceived. Production does not simply deliver finished products that as soon as they enter the market simply disappear, and logistics is no longer preoccupied simply with the unidirectional flow of products between the producer and the consumer. The need to keep costs down and concerns about the environmental implications of large-scale production has forced companies and researchers alike to re-consider the role and usefulness of logistics. In other words, the simple deductions that growing production leads to growing waste which is damaging the environment, and that products which are not absorbed by the market and end up in landfills is a wasteful and polluting practice, can no longer go unnoticed. The proper management of reverse flows of materials and products has a great influence on economic performance and a positive environmental impact as well. Considering that green business ethics and performance may actually affect the image and, therefore, the economic indicators of a company and an industry, the link between environmental and economic performance is closer than ever before.

But what exactly are reverse logistics (RL) and waste management (WM), and, more importantly, how can they improve a company's economic performance and/or minimize environmental damage? The following chapters deal with the latter part of the question, but a definition of the concepts of RL and WM is necessary before we proceed.

Stock (1992) argued that "…the term [reverse logistics] often used to refer to the role of logistics in recycling, waste disposal, and management of hazardous materials; a broader perspective includes all relating to logistics activities carried out in source reduction, recycling, substitution, reuse of materials and disposal". Kopicky (1993) called "Reverse Logistics… a broad term referring to the logistics management and disposing of hazardous or non-hazardous waste from packaging and products. It includes reverse distribution which causes goods and information to flow in the opposite direction of normal logistics activities''. [1]

More recently the term RL is used interchangeably with 'Green Logistics', 'Closed Loop Supply Chain Management' or 'Waste Management'. However, RL need not be confined to waste management or recycling only. The defining feature of RL is the logistics activities that, as Kopicky (1993) argued, concentrates on the flow of goods and information in the opposite than the usual direction. Thus, not only waste but also returned products fall within the scope of RL.

Waste Management can be defined as the collection, transport, processing, recycling or disposal and monitoring of waste materials. The waste flow starts from hazardous waste producers (e.g. hospitals, electricity plants) and non-hazardous waste producers (e.g. households) and continues either to transfer stations, material recovery facilities, incinerators, or sanitary landfills. In some cases unfortunately it may even end up in unsupervised and illegal landfills.

The main driving forces for companies, private and public organizations to accept returns, recycle and manage waste are found in (1) economics, which relates to all recovery actions where the company has direct or indirect economic benefits; (2) legislation, which refers here to any laws indicating that a company should recover its products or accept them back as recycling quotas; (3) extended responsibility, which refers to a set of values or principles that a company or an organization adopts to become responsibly engaged with reverses.

The key issues that affect both reverse logistics and waste management are the planning for material flows, the network structure and the classification and routing of materials. [2] The collection of returned goods and waste is a supply-driven flow rather than demand-driven. This kind of flow of materials is unpredictable and uncertain with respect to quantity, timing and condition of items. [3] Furthermore, most logistic systems are not programmed or equipped to handle reverse product movement. The returned items very often cannot be transported, stored, or handled in the same way as do the outgoing products. Similarly, waste transportation, storing and handling requires a completely different infrastructure and equipment. Reverse distribution costs may be several times higher than the initial distribution costs. Finally, high value products may justify high transportation costs, whereas transporting low value goods may not be economical. [3]. Therefore, vehicle routing and the facility location are fundamental components in WM and RL. The profit or non-profit organizations engaged in WM and RL have to decide where to open new facilities, treatment and disposal centers, and how to route different types of waste or returns, in order to minimize the operating cost in balance with social responsibility and environmental concern.

1.2. Scope of the project

The scope of the dissertation is twofold. Firstly, we analyze recent real-life studies in vehicle routing and facility location problems which use different mathematical models, try to achieve different objectives, and were faced with different constraints. The case-studies refer to the successful application of models in different geographical areas. Secondly, we investigate how different Greek organizations manage urban/residential waste (Municipality of Panorama, Thessaloniki) and industrial waste (ELDIA and company 'T') with an emphasis on their routing and location strategy (The company of our third case-study operating in industrial waste management preferred to stay anonymous. Henceforth it will be referred to as company 'T'). Finally, we propose a reliable and tested routing model for each of our Greek case-study.

1.3. Methodology

As Marianne W. Lewis (1998)[4] mentioned, **"**Existing case studies offer a potentially effective and efficient means for comparing complex and disparate operations settings". Therefore, we identified 20 of the most recent case-studies in the international literature that employ advanced mathematical models on routing and facility location problems and that succeeded in improving performance under a different set of objectives, constraints and environments. The analysis of the cases includes the description of the problem, the data, the objectives, the constraints and the mathematical model. This way practitioners and decision makers can find easily more information about the problems and their solutions in other countries or corporations.

Our own Greek case-studies provide an interesting combination of private and public enterprises that deal with urban and industrial waste. We focused on their management practices, in order to explore the potential application of the models we introduce in the first part of the dissertation. We collected as much information as possible about their routing and location strategy by visiting the companies' premises and conducting a series of in-depth interviews. In particular, we extracted all relevant information (e.g. fleet, capacity, number of customers, distances, types of depots, location criteria etc.) from two interviews with the mayor of Panorama, three meetings with the Cleaning Services Supervisor of Panorama, two with the CEO of ELDIA ("Elliniki Etairia Diaxeirisis Aporimmaton") and two meetings with the CFO of company 'T'.

1.4. Outline

The dissertation is structured as follows. The following chapter explores ten case-studies on location facility models that are classified according to their objectives (multiple and single). Chapter 3 includes ten different case-studies on the vehicle routing problem classified by constraints. In Chapter 4 we present our own case-studies and we argue that the routing and location practice is based on practical experience only leading to sub-optimal results. Finally, we propose a suitable routing model for each of our three case-studies that should lead to the optimization of their routing schedule.

2. Facility Location models in reverse logistics and waste management

The goal of this chapter is to present the latest research advances in the field of RL and WM with regard to facility location questions and problems, and to demonstrate that a systematic and model-based treatment of problems can lead to improved solutions. Identifying the ideal location of a facility is a core aspect of the distribution system design. Nearly all industries receive returned products and a growing number of them are engaged in waste management. RL and WM are important not only because they help minimize costs, but also because they are directly associated to issues of green business, environmental regulations, business ethics and also improving customer services. RL and WM, therefore, are often asked to serve multiple and sometimes conflicting objectives under conditions that can be quite complex. This is evident in the sophistication of the RL and WM we analyze in this chapter. We focus on the ten latest studies on the facility location problem where we investigate the case study problem, the objectives of the model, the constraints and the mathematical model itself. We commence with the single objective model studies and the multiple objective models follow.

2.1. Location models with single objectivee

2.1.1. Study 1 [5]

Description of the study
In this study is presented a two-level location problem with three types of facilities to be located in a specific remanufacturing network (RNM). It is assumed that are four types of participants: producers, intermediate centres, remanufacturing centres and customers. At the customers' level, there are product demands and used products ready to be recovered. Intermediate reprocessing centres are only used in the reverse channel and are responsible for some essential activities, such as cleaning, disassembly, checking and sorting, before the return-products are shipped back to remanufacturing centres. Remanufacturing centres accept the checked returns from intermediate centres and are responsible for the process of remanufacturing. As a member of the "forward" channel, producers are in charge of the "traditional" production in order to meet, together with remanufacturing centres, the product demands of the customers. In such a system there are two kinds of flow. One flow (the "reverse" flow) from customers through intermediate centres to remanufacturing centres is formed by used products, while the other ("forward" flow) from remanufacturing centres or producers directly to customers (without passing through the intermediate centres) consists of "new" products. The objective of the proposed model is the minimization of the total cost included fixed and variable cost.
Data

- Number of location or possible facilities
- Number of customers
- Number of intermediate centres (intermediate reprocessing centres are only used in the reverse channel and are responsible for some essential activities, such as cleaning, disassembly, checking and sorting, before the return-products are shipped back to remanufacturing centres.
- Number of remanufacturing centres
- Number of producers
- Quantity of products demands
- Quantity of used products ready to recover

- Fixed cost of setting up a producer site
- Fixed cost of setting up a intermediate center
- Unit production cost
- Unit reprocessing cost
- Unit remanufacturing cost
- Cost of shipping unit product from both producers or remanufacturing centres to customers
- Cost of sending unit return product from customers to intermediate center and from intermediate to remanufacturing center
- Unit disposal cost at intermediate and remanufacturing center
- Percentage at which return products will be disposed of at intermediate or remanufacturing centres
- the total demand for products in the whole system is greater than the quantity of products that can be obtained by remanufacturing
- The cost of producing a unit product is assumed to be much greater than the cost of obtaining a remanufactured product.

Objectives

- Minimization of the total cost of the system(fixed and variable)

Constraints

- Demand for products and return-items must be fully met
- Location and allocation variables constraints: in which three types of facility are related to different location variables and two flows are linked to corresponding allocation variables. Specifically, the locations and numbers of producers are concretely determined by the quantitative relationship between the "forward" and "reverse" flows at a site
- The amount of products to meet the customer demand is greater than or equal to the amount of remanufactured products from reverse flows
- integrality of the location variables constraint
- Nonnegative constraint

Mathematical Model

Parameter

$j \in J = \{1, 2, \ldots, m\}$, index of potential location sites for both producers and remanufacturing centres,
$k \in K = \{1, 2, \ldots, mn\}$, index of potential location sites for intermediate centres
$i \in I = \{1, 2, \ldots, n\}$, index of customers,
f_i =fixed cost of setting up a producer at site *j*,
fr_j =fixed cost of setting up a remanufacturing center at site *j*,
fc_k =fixed cost of setting up an intermediate center at site *k*,
h_i =product demand at customer site *i*,
hr_j =available quantities of return-products ready for recovery at customer *i*,
cp_j =unit production cost at producer *j*,
ct_k =unit reprocessing cost at intermediate center *k*,
cm_j =unit remanufacturing cost at remanufacturing center *j*,
c_{ij} =cost of shipping unit product (including remanufactured product) from producer *j* or remanu-facturing center *j* to customer *i*,
cc_{ik} =cost of sending unit return-product from customer *i* to intermediate center *k*,
cr_{kj} =cost of sending unit checked return-product from intermediate center *k* to remanufacturing center *j*,
ccd= unit disposal cost at intermediate centres (this cost is assumed to be the same for all intermediate

centres),
crd= unit disposal cost at remanufacturing centres (this cost is assumed to be the same for all remanufacturing centres),
β= percentage at which the return-products will be disposed of at intermediate centres (this percentage is assumed to be the same for all intermediate centres),
γ= percentage at which the checked return-products will be disposed of at remanufacturing centres (this percentage is assumed to be the same for all remanufacturing centres).

Decision variables

Y_j = 1, if a producer is located and set up at potential site *j*, 0, otherwise,

YR_j = 1, if a remanufacturing centre is located and set up at potential site ***j***, 0, otherwise,

YC_k = 1, if an intermediate centre is located and set up at potential site *k*, 0, otherwise,

X_{ij} = fraction of product demand at customer *i* which is met by producer *j* or remanufacturing centre *j* or a combination of them located at *j*,

XR_{ikj} = fraction of quantity of return-products at customer *i* that is taken back through intermediate center *k* to remanufacturing center *j*.

The problem is formulated as an incapacitated facility location model, named RMNU, for the remanufacturing network system (RMN):

$$Min \sum_j f_j Y_j + \sum_j fr_j YR_j + \sum_k fc_k YC_k + \sum_j \sum_i c'_{ij} h_i X_{ij} + \sum_i \sum_k \sum_j cr'_{ikj} hr_i XR_{ikj} \quad (1)$$

s.t.

$$\sum_j X_{ij} = 1 \; \forall i, \quad (2)$$

$$\sum_j \sum_k XR_{ikj} = 1 \forall i, \quad (3)$$

$$\sum_i h_i X_{ij} \geq (1-\gamma)(1-\beta) \sum_k \sum_i hr_i XR_{ikj} \forall j, \quad (4)$$

$$\sum_i h_i X_{ij} - (1-\gamma)(1-\beta) \sum_i \sum_k hr_i XR_{ikj} \leq Y_j M \;\forall j, \quad (5)$$

$$XR_{ikj} \leq YC_k \forall i,k,j \quad (6)$$

$$(1-\beta) hr_i XR_{ikj} \leq YR_j M' \forall i,k,j \quad (7)$$

$$Y_j YR_j YC_k = 0,1 \forall k,j \quad (8)$$

$$X_{ij} XR_{ikj} \geq 0,1 \forall i,k,j \quad (9)$$

Where:

$$cr'_{ij} = c_{ij} + cp_j$$

$$cr'_{ikj} = cc_{ik} + ct_k (1-\beta) + ccd\beta + cr_{kj}(1-\beta) + cm_j (1-\gamma)(1-\beta)$$
$$+ crd\gamma(1-\beta) - (1-\gamma)(1-\beta)cp_j$$

Equation (1) presents the objective of the problem which is the minimization of the total cost (fixed and variables)
Equation (2) and (3) present the constraints that stipulate respectively the demands for products and return-items must be fully met
Equation (4) is the constraint that represents the relationship between the forward flows and return flows at each site. In other words the amount of products to meet the customer demand is greater than or equal

to the amount of remanufactured products from reverse flows. In fact, the difference in quantity between these two amounts represents the sum of products produced by "traditional" production at producer *j*, denoted as $XP_j(\sum_i h_i X_{ij} - (1-\gamma)(1-\beta)\sum_k \sum_i hr_i XR_{ikj}$ thus the constraint means simply $XP_j \geq 0$

Equation (5) and (6) represent the constraints that link the location and allocation variables, in which three types of facility are related to different location variables and two flows are linked to corresponding allocation variables. Specifically, the locations and numbers of producers are concretely determined by the quantitative relationship between the "forward" and "reverse" flows at a site *j*

Equation (7) presents the constraint which is represented as being between flow amount and location rather than between flow fraction and location because the possibility of taking *β* as value 1 is considered.

Equation (8) presents the constraint that specifies the integrality of the location variables.

Equation (9) presents the non-negative constraint

2.1.2. Study 2 [6]

Description of the study

The purpose of this study is to present a multi period multi echelon forward–reverse logistics network model for design purposes under risk. The model is a formulation for the forward–reverse logistics network design problem. The network is a multi-period multi-echelon, where it consists of suppliers, facilities, distributors, and first customers in the forward direction. In the reverse direction it consists of disassembly, disposal, redistribution locations and second customers.

In the forward direction, suppliers are responsible for supplying of raw materials to facilities. Facilities are responsible for manufacturing of virgin products and supplying some of them to the distributors and storing the rest for the next periods; if it is profitable. Distributors are responsible for the distribution of new products to the first customers and/or storing some of them for the next periods, and customers' nodes may represent one customer, a retailer, or a group of customers and retailers. In the reverse direction, the first customers return the used products to the disassembly locations.

In reverse direction, disassembly locations are responsible for disassembling and sorting of the returned products to recyclable, remanufacturable, repairable, and disposable and they are also responsible for supplying the recyclable to the suppliers, the remanufacturable to the facilities, the disposable to the disposal locations, and to repair the repairable products and supplying them directly to the redistribution locations. Suppliers are responsible for recycling of the returned products and supplying of recycled materials to facilities. Facilities are responsible for remanufacturing of used products and supplying them to the redistribution locations. Disposal locations are responsible for disposing of disposable products. Redistribution locations are responsible for the distribution of refurbished products to the second customers. The objective of the model is to maximize the total expected profit.

Models Data and Assumptions

- The model is a multi-period.
- Customers' locations are known and fixed with stochastic demands.
- The returned quantities are stochastic and depend on the first customer demand.
- The quality of remanufactured and repaired products is different from the new ones.
- The potential locations of suppliers, facilities, distributors, disassemblies, and redistributors are known.
- Costs parameters (fixed, material, manufacturing, non-utilized capacity, shortage, transportation, holding, recycling, remanufacturing, disassembly, and disposal costs) are known for each location and time period.

- Capacity of each location is known for each time period.
- The shortage cost depends on the shortage quantity and time.
- The holding cost depends on the residual inventory at the end of each period.
- Integer number of batches is transported.

Objective

- Maximize the total expected profit of the forward–reverse network

Constraints

- Balance constraints
- Capacity constraints
- Linking shipping constraints
- Shipping linking constraints
- Maximum number of activated location constraints

Model

Sets:

S: potential number of suppliers, indexed by s.
F: potential number of facilities, indexed by f.
D: potential number of distributors, indexed by d.
C: potential number of first customers, indexed by c.
A: potential number of disassembly locations, indexed by a.
R: potential number of redistributors, indexed by r.
P: potential number of disposal locations, indexed by p.
K: potential number of second customers, indexed by k.
T: number of periods, indexed by t.

Parameters:

D_{ct} Demand of first customer c in period t,

l_{ct} Demand mean of first customer c in period t,

r_{ct} Demand standard deviation of first customer c in period t,

D_{kt} Demand of the second customer k in period t,

P_{ct} Unit price at the first customer c in period t,

P_{kt} Unit price at second customer k in period t,

F_i Fixed cost of opening location i.

DS_{ij} Distance between any two locations i and j.

$DS_{ij} = \sqrt{(xj - xi)2 + (yj + yi)2}$ .where, xi and yi represent the Cartesian coordinates of location i.

CS_{it} Capacity of supplier s in period t,

CRS_{st} Recycling capacity of supplier s in period t,

FC_{ft} Manufacturing capacity in hours of facility f in period t,

RFC_{ft} Remanufacturing capacity in hours of facility f in period t,

SC_{ft} Storage capacity of facility f in period t,

DC_{dt} Maximum capacity of distributor d in period t,

AC_{at} Capacity of disassembly a in period t,

RC_{rt} Capacity of redistributor r in period t,

PC_{pt} Capacity of disposal p in period t,

MC_{st} Material cost per unit supplied by supplier s in period t,

RC_{st} Recycling cost per unit recycled by supplier s in period t,

FC_{ft} Manufacturing cost per unit manufactured by facility f in period t,

RFc_{ft} Remanufacturing cost per unit remanufactured by facility f in period t,

DAc_{at} Disassembly cost per unit disassembled by disassembly location a in period t,

RPc_{at} Repairing cost per unit repaired by disassembly location a in period t,

Pc_{pt} Disposal cost per unit disposed by disposal location p in period t,

N_{cf} Non utilized manufacturing capacity cost per hour of facility f,

RN_{cf} Non utilized remanufacturing cost per hour of facility f,

S_c : Shortage cost per unit per period,

Fh_f Manufacturing time per unit in hours at facility f,

RF_{hf} Remanufacturing time per unit in hours at facility f,

FH_f Holding cost per unit per period at the store of facility f,

DH_d Holding cost per unit per period at distributor d store,

Bs, Bf, Bd, Ba & Br: batch size from supplier s, facility f, distributor d, disassembly a, and, redistributors r respectively.
Tc: transportation cost per unit per kilometre.
RR: return ratio at the first customers.
Rc: recycling ratio.
Rm: remanufacturing ratio.
Rc: repairing ratio.
Rp: disposal ratio.

Decision variables

L_i Li binary variable equals 1 if location i is open and 0 otherwise.

L_{iij} Binary variable equals 1 if a transportation link is established between any two locations i and j.

Q_{ijt} Flow of batches from location i to location j in period t.

l_{fft} Flow of batches from facility f to its store in period t,

l_{fdt} Flow of batches from store of facility f to distributor d in period t,

R_{ft} The residual inventory of the period t at store of facility f.

R_{dt} The residual inventory of the period t at distributor d.

First sales = $\sum_{d\in D} \sum_{c\in C}\sum_{t\in T} Q_{dct} B_d P_{ct}$ (1)

Second sales= $\sum_{r\in R}\sum_{k\in K}\sum_{t\in T} Q_{rkt} B_r P_{kt}$ (2)

$$\sum_{s\in S} F_s L_s + \sum_{f\in F} F_f L_f + \sum_{d\in D} F_d L_d + \sum_{a\in A} F_a L_a + \sum_{r\in R} F_r L_r + \sum_{p\in P} F_p L_p \quad (3)$$

$$\sum_{s\in S}\sum_{f\in F}\sum_{t\in T} Q_{sft} B_s MC_{st} - \sum_{a\in A}\sum_{s\in S}\sum_{t\in T} Q_{ast} B_a (MC_{st} - RC_{st}) \quad (4)$$

$$\sum_{f\in F}\sum_{d\in D}\sum_{t\in T} Q_{fdt} B_s MC_{st} - \sum_{a\in A}\sum_{s\in S}\sum_{t\in T} Q_{ast} B_a (MC_{st} - RC_{st}) \quad (5)$$

$$\sum_{f\in F}\left(\sum_{t\in T}\left((FC_{ft} / Fh_t) L_f - \sum_{d\in D}(Q_{dft} B_f) - \sum_{d\in D}(I_{dft} B_f)\right) NC_f\right)$$
$$+ \sum_{f\in F}\left(\sum_{t\in T}\left(RFC_{ft} / RFh_f\right) L_f\right) - \sum_{r\in R}(Q_{fdt} B_f)\ \ ) RNc_f) \quad (6)$$

$$\left(\sum_{c\in C}\left(\sum_{t\in T}\left(\sum_{t=1}^{t} D_{ct} - \sum_{t=1}^{t}\sum_{d\in D} Q_{dct} B_d\right)\right)\right) S_c \quad (7)$$

$$\sum_{c\in C}\sum_{a\in A}\sum_{t\in T} Q_{cat} P_c B_c QL_C \quad (8)$$

$$\sum_{c\in C}\sum_{a\in A}\sum_{t\in T} Q_{cat} B_c DA_{ac} \quad (9)$$

$$\sum_{a\in A}\sum_{s\in S}\sum_{t\in T} Q_{ast} B_a C_{st} \quad (10)$$

$$\sum_{r\in F}\sum_{r\in R}\sum_{t\in T} Q_{frt} B_f RFc_{ft} \quad (11)$$

$$\sum_{a\in A}\sum_{r\in R}\sum_{t\in T} Q_{art} B_a RPc_{at} \quad (12)$$

$$\sum_{a\in A}\sum_{p\in P}\sum_{t\in T} Q_{apt} B_a Pc_{pt} \quad (13)$$

$$\sum_{t\in T}\sum_{s\in S}\sum_{f\in F} Q_{sft} B_s TcDS_{sf} + \sum_{t\in T}\sum_{f\in F}\sum_{d\in D} Q_{fdt} B_f TcDS_{fd} +$$
$$\sum_{t\in T}\sum_{a\in A}\sum_{s\in S} Q_{ast} B_a TcDS_{as} + \sum_{t\in T}\sum_{a\in A}\sum_{p\in P} Q_{aft} B_a TcDS_{af} +$$
$$\sum_{t\in T}\sum_{a\in A}\sum_{p\in P} Q_{apt} B_a TcDS_{ap} + \sum_{t\in T}\sum_{a\in A}\sum_{r\in R} Q_{art} B_a TcDS_{ar} +$$
$$\sum_{t\in T}\sum_{f\in F}\sum_{r\in R} Q_{frt} B_f TcDS_{fr} + \sum_{t\in T}\sum_{r\in R}\sum_{k\in K} Q_{rkt} B_r TcDS_{rk} \quad (14)$$

$$\sum_{f\in F}\sum_{t\in T} R_{ft} FH_f + \sum_{d\in D}\sum_{t\in T} R_{dh} DH_d \quad (15)$$

$$\sum_{s\in S} Q_{sft} B_s = \sum_{d\in D} Q_{fdt} B_f + I_{fft} B_f, \quad \forall\ t \in T, \forall f \in F, \quad (16)$$

$$I_{fft}B_f + R_f(t-1) = R_{ft} + \sum_{d\in D} Q_{fdt}B_f \,, \quad \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{f} \in \mathrm{F}, \tag{17}$$

$$\sum_{d\in D}(Q_{fdt} + I_{fdt})B_f + R_d(t-1) = R_{dt} + \sum_{c\in C} Q_{dct}B_d \,, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{d} \in \mathrm{D}, \tag{18}$$

$$\sum_{d\in D} Q_{dct}B_d \le D_{ct} + \sum_{1}^{t} D_c(t-1) - \sum_{d\in D} Q_{dc(t-1)}B_d \,, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{c} \in \mathrm{C}, \tag{19}$$

$$\sum_{a\in A} Q_{cat}B_c \le (\sum_{d\in D} Q_{dct}B_d)RR \,, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{c} \in \mathrm{C}, \tag{20}$$

$$\sum_{c\in C} Q_{cat}B_c = \sum_{s\in S}(Q_{ast}B_a) + \sum_{f\in F}(Q_{aft}B_a) + \sum_{r\in R}(Q_{art}B_a) + \sum_{p\in P}(Q_{apt}B_a),$$

$$\forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{a} \in \mathrm{A}, \tag{21}$$

$$\sum_{c\in C}(Q_{cat}B_c)Rc = \sum_{s\in S}(Q_{ast}B_a), \quad \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{a} \in \mathrm{A}, \tag{22}$$

$$\sum_{c\in C}(Q_{cat}B_c Rm) = \sum_{f\in F}(Q_{aft}B_a), \quad \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{a} \in \mathrm{A}, \tag{23}$$

$$\sum_{c\in C}(Q_{cat}B_c Rr) = \sum_{r\in R}(Q_{rft}B_a), \quad \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{a} \in \mathrm{A}, \tag{24}$$

$$\sum_{c\in C}(Q_{cat}B_c Rd) = \sum_{p\in P}(Q_{apt}B_a), \quad \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{a} \in \mathrm{A}, \tag{25}$$

$$\sum_{a\in A}(Q_{aft}B_a) = \sum_{r\in R}(Q_{frt}B_f), \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{f} \in \mathrm{F}, \tag{26}$$

$$\sum_{a\in A}(Q_{art}B_a) + \sum_{f\in F}(Q_{frt}B_f) = \sum_{k\in K}(Q_{rkt}B_r), \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{r} \in \mathrm{R}, \tag{27}$$

$$\sum_{r\in R}(Q_{rkt}B_r) \le D_{kt}, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{k} \in \mathrm{K} \tag{28}$$

$$\sum_{f\in F}(Q_{sft}B_s) \le SC_{st}L_s, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{s} \in \mathrm{S}, \tag{29}$$

$$(\sum_{d\in D} Q_{fdt}B_f + \sum_{d\in D} I_{fft}B_f)MH_f \le FC_{ft}L_f, \quad \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{f} \in \mathrm{F}, \tag{30}$$

$$R_{ft} \le SC_{ft}L_f, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{f} \in \mathrm{F}, \tag{31}$$

$$\sum_{f\in F}(Q_{fdt} + I_{fdt})B_f + R_{dt-1} \le DC_{dt}L_d, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{d} \in \mathrm{D}, \tag{32}$$

$$\sum_{s\in S} Q_{ast}B_a + \sum_{f\in F} Q_{aft}B_a + \sum_{r\in R} Q_{art}B_a + \sum_{p\in P} Q_{apt}B_a \le PC_{pt},$$

$$\forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{a} \in \mathrm{A}, \tag{33}$$

$$\sum_{k\in K} Q_{rkt}B_r \le RC_{rt}, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{r} \in \mathrm{R}, \tag{34}$$

$$\sum_{a\in A} Q_{ast}B_a \le RC_{st}, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{s} \in \mathrm{S}, \tag{35}$$

$$\sum_{a\in A} Q_{apt}B_a \le PC_{pt}, \ \forall \ \mathbf{t} \in \mathrm{T}, \forall \mathbf{p} \in \mathrm{P}, \tag{36}$$

$$Li_{sf} \le \sum_{t\in T} Q_{sft}, \ \forall \ \mathbf{s} \in \mathrm{S}, \forall \mathbf{f} \in \mathrm{F}, \tag{37}$$

$$Li_{fd} \le \sum_{t\in T}(Q_{fdt} + I_{fdt}), \ \forall \ \mathbf{f} \in \mathrm{F}, \forall \mathbf{d} \in \mathrm{D}, \tag{38}$$

$$Li_{dc} \le \sum_{t\in T} Q_{dct}, \ \forall \ \mathbf{d} \in \mathrm{D}, \forall \mathbf{c} \in \mathrm{C}, \tag{39}$$

$$Li_{ca} \le \sum_{t\in T} Q_{cat}, \ \forall \ \mathbf{a} \in \mathrm{A}, \forall \mathbf{c} \in \mathrm{C}, \tag{40}$$

$$Li_{as} \le \sum_{t\in T} Q_{ast}, \ \forall \ \mathbf{s} \in \mathrm{S}, \forall \mathbf{a} \in \mathrm{A}, \tag{41}$$

$$Li_{af} \le \sum_{t\in T} Q_{aft} \qquad \forall f \in F, \forall a \in A, \tag{42}$$

$$Li_{ar} \le \sum_{t\in T} Q_{art} \qquad \forall r \in R, \forall a \in A, \tag{43}$$

$$Li_{ap} \le \sum_{t\in T} Q_{apt} \qquad \forall p \in P, \forall a \in A, \tag{44}$$

$$Li_{fr} \le \sum_{t\in T} Q_{apt} \qquad \forall r \in R, \forall f \in F, \tag{45}$$

$$Li_{rk} \le \sum_{t\in T} Q_{rkt} \qquad \forall k \in K, \forall r \in R, \tag{46}$$

$$\sum_{t\in T} Q_{sft} \le MLi_{sf} \qquad \forall f \in F, \forall s \in S, \tag{47}$$

$$\sum_{t\in T} (Q_{dct} + I_{fdt}) \le MLi_{fd} \qquad \forall f \in F, \forall d \in D, \tag{48}$$

$$\sum_{t\in T} Q_{dct} \le MLi_{dc} \qquad \forall d \in D, \forall c \in C, \tag{49}$$

$$\sum_{t\in T} Q_{cat} \le MLi_{ca} \qquad \forall a \in A, \forall c \in C, \tag{50}$$

$$\sum_{t\in T} Q_{ast} \le MLi_{as} \qquad \forall s \in S, \forall a \in A, \tag{51}$$

$$\sum_{t\in T} Q_{aft} \le MLi_{af} \qquad \forall f \in F, \forall a \in A, \tag{52}$$

$$\sum_{t\in T} Q_{art} \le MLi_{ar} \qquad \forall r \in R, \forall a \in A, \tag{53}$$

$$\sum_{t\in T} Q_{apt} \le MLi_{ap} \qquad \forall p \in P, \forall a \in A, \tag{54}$$

$$\sum_{t\in T} Q_{frt} \le MLi_{fr} \qquad \forall r \in R, \forall f \in F, \tag{55}$$

$$\sum_{t\in T} Q_{rkt} \le MLi_{rk} \qquad \forall r \in R, \forall k \in K, \tag{56}$$

$$\sum_{s\in S} L_s \le S \tag{57}$$

$$\sum_{f\in F} L_f \le F \tag{58}$$

$$\sum_{d\in D} L_d \le D \tag{59}$$

$$\sum_{a\in A} L_a \le A \tag{60}$$

$$\sum_{r\in R} L_r \le R \tag{61}$$

$$\sum_{p\in P} L_p \le P \tag{62}$$

The objective of this problem is to maximize the total expected profit which is equal to total expected income minus the total expected cost. The total expected income = first sales + second sales. The total expected cost = fixed costs + material costs + manufacturing costs + non-utilized capacity costs + shortage costs + purchasing costs + disassembly costs + recycling profit + remanufacturing cost + repairing cost + disposal cost + transportation costs + inventory holding costs.

Equation (1) presents the first sale
Equation (2) presents the second sales
Equation (3) presents the fixed cost
Equation (4) presents the material cost
Equation (5) presents the manufacturing cost
Equation (6) presents the non-utilized capacity cost
Equation (7) presents the shortage cost for distributor
Equation (8) presents the purchasing
Equation (9) presents the disassembly cost
Equation (10) presents the recycling cost
Equation (11) presents the remanufacturing cost
Equation (12) presents the repairing cost
Equation (13) presents the disposal cost
Equation (14) presents the transportation cost
Equation (15) presents the inventory holding cost
Equation (16) presents the balance constraint that ensure at each period, the flow entering to each facility from all suppliers is equal to the sum of the exiting from this facility to each facility and distributor stores.
Equation (17) presents the balance constraint that ensure that the sum of the flow entering each facility store and its residual inventory from the previous period is equal to the sum of the exiting to each distributor and the residual inventory of the existing period.
Equation (18) presents the balance constraint that ensure that the sum of the flow entering each distributor store from each facility or facility store and its residual inventory from the previous period is equal to the sum of the exiting to each customer and the residual inventory of the existing period.

Equation (19) presents the balance constraint that ensures that the sum of the flow entering to each customer does not exceed the sum of the existing period demand and the previous accumulated back orders.

Equation (20) presents the balance constraint that ensure at each period, the flow exiting from each customer to all disassembly locations does not exceed the sum of the entering to each customer.

Equation (21) presents the balance constraint that ensure at each period, the flow entering each disassembly location from all customers is equal to the sum of flow exiting to each supplier for recycling, to each facility for remanufacturing, to each redistributor (repaired), and to each disposal location for disposing.

Equation (22) presents the balance constraint that ensures at each period, the flow exiting from disassembly location to all suppliers to be recycled is equal to the entering to each disassembly location from all customers multiplied by the recycling ratio.

Equation (23) presents the balance constraint that ensures that the flow exiting from disassembly location to all facilities to be remanufactured is equal to the entering to each disassembly location from all customers multiplied by the remanufacturing ratio.

Equation (24) presents the balance constraint that ensures that the repaired flow exiting from disassembly location to all redistributors' locations to be redistributed is equal to the entering to each disassembly location from all customers multiplied by the repairing ratio.

Equation (15) presents the balance constraint that ensures that the flow exiting from disassembly location to all disposal locations to be disposed is equal to the entering to each disassembly location from all customers multiplied by the disposing ratio.

Equation (126) presents the balance constraint that ensures that the sum of the remanufactured flow entering to each facility from each disassembly location is equal to the sum of the exiting to each redistributor's location.

Equation (27) presents the balance constraint that ensure that sum the of remanufactured flow entering to each redistributors' location from all facilities and the repaired flow entering to it from all the disassembly locations is equal to the sum of flow exiting to each second customer.

Equation (29) presents the balance constraint that ensure that flow entering to each second customer from all redistributors does not exceed the second customer demand at each period.

Equation (29) presents the capacity constraint that ensures at each period, the sum of the flow exiting from each supplier to all facilities does not exceed the supplier capacity.

Equation (30) presents the capacity constraint that ensures at each period, the sum of the flow exiting from each facility to all facilities' stores and to all distributors does not exceed the facility capacity.

Equation (31) presents the capacity constraint that ensures that the residual inventory at each facility store does not exceed its storing capacity at each period.

Equation (32) presents the capacity constraint that ensures at each period, the sum of the residual inventory at each distributor from the previous period and the flow entering at the existing period from the facilities and facilities stores does not exceed this distributor capacity.

Equation (33) presents the capacity constraint that ensures at each period, the sum of the flow exiting from each disassembly location to all suppliers, facilities, redistributors and disposal locations does not exceed this disassembly location capacity.

Equation (34) presents the capacity constraint that ensures at each period, the flow exiting from each redistributor's to the second customers does not exceed this redistributors' capacity.

Equation (35) presents the capacity constraint that ensures at each period, the flow entering each supplier from all disassembly location does not exceed this supplier recycling capacity.

Equation (36) presents the capacity constraint that ensures at each period, the flow entering to each disposal location from all disassembly location does not exceed this disposing capacity.

Equations (37) to (46) present the linking shipping constraints that ensure that there are no links between any locations without actual shipments during all periods.

Equations (47) to (56) present the shipping- linking constraints that ensure that there is no shipping between any non linked locations.
Equation (57) to (62) present the maximum number of activated locations constraints that limit the number of activated locations, where the sum of binary decision variables which indicate the number of activated locations, is less than the maximum limit of activated locations (taken equal to the potential number of locations).

2.1.3. Study 3 [7]

Description of the study

The goal of this study is to design a closed-loop supply chain logistics system that can minimize the total transportation and the operation costs. The closed-loop logistics comprises two parts: forward logistics and reverse logistics. For the forward logistics, as a conventional logistics, after manufactory, the distributors will deliver the final products to the customers to satisfy their demands and the position of the customers is typically the end of the process. For the reverse logistics, the flow of used products is processed from the customers back to the dismantlers to do the sorting or disassembling for recovery, reuse or disposal. To analyze simultaneously both forward & reverse logistics is more complicated; thus, a GA is developed in order to solve the closed-loop logistic model.

Data

- Number of suppliers
- Number of manufactories
- Number of customers
- Number of distribution centres
- Number of dismantlers
- Fixed operating cost of manufactories, distribution centres, dismantlers
- Unit cost of transportation from each manufacturing to distribution centres, rfom DC to customers, from DC to dismantlers
- Unit cost of recovery from customers to DC
- Recovery percentage of customers
- Amount shipped from manufactory
- Amount shipped from distribution centre to customers
- Amount shipped from distribution centre to dismantler
- Amount shipped from dismantler to manufactory
- Quantity recover to distribution centre from customers

Objectives

- minimization of the total cost(transportation and operations)

Constraints

- capacity of suppliers
- capacity of manufactories
- capacity of forward and reverse logistics in distribution centre

- capacity of dismantler
- flow conservation constraints
- customer demand constraints
- binary variables constraint
- non-negative constraints

<u>Mathematical Model</u>

Indices
I the number of suppliers with *i* = 1, 2,..., *I*
J the number of manufactories with *j* = 1, 2,..., *J*
K the number of DCs with *k* = 1, 2,..., *K*
L the number of customers with *l* = 1, 2,..., *L*
M the number of dismantlers with *m* = 1, 2,..., *M*

Parameters

a_i Capacity of suppliers i

b_j *Capacity of manufactory j*

Sc_k Total capacity of forward and reverse logistics in the DC k

pd_k The percentage of the total capacity for reverse logistics in DC k

pc_I Recovery percentage of customers I

pl_m The land filling rate of dismantler *m*

d_I Demand of the customer *l*

e_m Capacity of dismantler *m*

S_{ij} Unit cost of production in manufactory *j* using materials from supplier *i*

t_{jk} Unit cost of transportation from each manufactory *j* to each DC *k*

u_{kl} Unit cost of transportation from DC *k* to customer *l*

V_{km} Unit cost of transportation from DC *k* to dismantler *m*

w_{mj} Unit cost of transportation from dismantler *m* to manufactory *j*

Ru_{ik} Unit cost of recovery in DC *k* from customer *l*

f_j Fixed cost for operating manufactory j

g_k Fixed cost for operating DC *k*

h_m *Fixed* cost for operating dismantler *m*

ϕ fixed cost for landfill per unit

Variables

x_{ij} Quantity produced at manufactory *j* using raw materials from supply *i*

y_{ik} Amount shipped from manufactory *j* to DC *k*

z_{kI} Amount shipped from DC *k* to customer

o_{km} Amount shipped from DC *k* to dismantler *m*

Rd_{mj} Amount shipped from dismantler *m* to manufactory *j*

Rz_{Ik} Quantity recovered at DC *k* from customer *I*

$$a_j = \begin{cases} 1 \text{ if production take place at manufactory j} \\ 0 \text{ otherwise} \end{cases}$$

$$\beta_\kappa = \begin{cases} 1 \text{ if DC k is open} \\ 0 \text{ otherwise} \end{cases}$$

$$\delta_m = \begin{cases} 1 \text{ if dismantler m is open} \\ 0 \text{ otherwise} \end{cases}$$

$$Min \sum_j \sum_j S_{ij} X_{ij} + \sum_k \sum_I u_{kI} z_{kI} + \sum_k \sum_m v_{km} o_{km} +$$

$$\sum_m \sum_j w_{mj} Rd_{mj} + \sum_i \sum_k Ru_{Ik} Rz_{Ik} + \sum_j f_i a_j \quad (1)$$

$$+ \sum_k g_k \beta_k + \sum_m h_m \delta_m + \varphi \sum_m \left[pl_m \sum_k o_{km} \right]$$

Subject to

$$\sum_j x_{ij} \le a_i, \forall i \quad (2)$$

$$\sum_k y_{jk} \le b_j a_j, \forall j \quad (3)$$

$$\sum_i x_{ij} + \sum_m Rd_{mj} = \sum_k y_{jk} \forall j \quad (4)$$

$$\sum_i z_{kl} + \sum_m o_{km} \le Sc_k \beta_k \forall \kappa \quad (5)$$

$$\sum_I y_{jk} = \sum_I z_{kI} \forall k \quad (6)$$

$$\sum_m o_{km} \le \lfloor pd_k Sc_k \beta_k \rfloor, \forall \kappa \quad (7)$$

Where $\lceil \quad \rceil$ floor for Gauss' symbol

$$\sum_{\mathrm{I}} Rz_{ik} = \sum_m o_{km} \forall k \quad (8)$$

$$\sum_k Rz_{Ik} \geq \left\lceil pc_I \sum_k z_{kI} \right\rceil \forall I \quad (9)$$

Where $\lceil \; \rceil$ ceiling for Gauss' symbol

$$\sum_k z_{kI} \geq d_I, \forall I \quad (10)$$

$$\sum_j Rd_{mj} + \left\lfloor pl_m \sum_k o_{km} \right\rfloor \leq e_m \delta_m, \forall m \quad (11)$$

$$\sum_\kappa o_{km} = \sum_j Rd_{mj} + \left\lfloor pl_m \sum_k o_{km} \right\rfloor \forall m \quad (12)$$

$$\sum_\kappa o_{km} = \sum_j Rd_{mj} + \left\lfloor pl_m \sum_k o_{km} \right\rfloor \forall m \quad (13)$$

$$a_{j,} \beta_{k,} \delta_{m,} \in \{0,1\}, \forall j,k,m \quad (14)$$

$$x_{ij,} y_{jk,} z_{kl,} o_{km,} Rd_{mj,} Rz_{Ik,} \in N \cup \{0\}, \forall i,k,l,m \quad (15)$$

Equation (1) presents the objective functions to minimize the total transportation and operation cost.
Equations (2) and (3) present the capacity constraints that represent the limit of the capacity for suppliers and manufactories in forward logistics.
Equations (4), (6), (8) and (12) present the constraints that satisfy the law of the flow conservation by in-flow equal to out-flow.
Equation (5) presents the capacity constraints that show that the total flows of forward and backward cannot exceed the total capacity of DC.
Equation (7) and (11) present the constraints that mean the reverse limit of the capacity for DCs and dismantlers.
Equation (9) presents the constraint that describes the customer recovery relationship with the recovery rate.
Equation (10) presents the constraint that satisfies the customer demand.
Equation (13) denotes the binary variables constraints
Equation (14) is the non-negative, integral condition constraint.

2.1.4. Study 4 [8]

<u>Description of the study</u>

This case study presents an innovative optimization model to outline an optimal regional coordination of solid waste routing and possible landfill/incinerator construction under an uncertain environment in Lower Rio Grande Valley, in South Texas.

<u>Data</u>

- Number of landfills
- Cost of annual shipping
- Operation cost of existing facilities

- Construction cost of new landfills
- Operating cost of new landfills
- Distance from city to facilities
- Solid waste generation in city
- Landfill tipping fee

Objectives

- Minimization of the total net cost-> as the difference between system cost and benefit over a multi period short term planning horizon.

Constraints

- Mass balance constraints
- Capacity limitation constraints for existing facilities
- Capacity limitation constraints for potential facilities
- Conditionally constraint
- Non-negativity constraint

Mathematical Model

Variables

$C1_t^{\pm}$ = total annual shipping cost ($/Y);

$C2_t^{\pm}$ = the total construction cost of the new landfill or incinerator ($/Y);

$C3_t^{\pm}$ = total annual operating cost for existing landfills ($/Y);

$C4_t^{\pm}$ = operating cost for a new landfill or incinerator ($/Y);

$B1_t^{\pm}$ =landfill space-saving benefit using the landfill tipping fee as a surrogate index ($/Y);

$B2_t^{\pm}$ = electricity generation benefit for incineration ($/Y);

T = time periods in long-term planning: 2004 (t = 1), 2005 (t = 2), 2006 (t = 3), 2007 (t = 4);

$X_{ijt}^{\pm}$ = waste stream shipped from i city to j landfill (tons per year, or TPY);

$DC_{jt}^{\pm}$ = design capacity of facility (either landfill or incinerator) j (TPY);

$I_{jt}^{\pm}$ = a binary integer used for sifting through new facility options.

Parameters

$DS_{ijt}^{\pm}$ = average shipping distance from the city i to facility j (km);

$PS_{ijt}^{\pm}$ = unitary shipping cost for shipping waste from city I to facility j [1.79, 2.50]($/ton/km);

$T_j^{\pm}$ = range of existing landfill tipping fees on a per ton basis at a specific landfill, where $T_4^{\pm}$ _represents a range of LRGV tipping fees;

$LFC_{jt}^{\pm}$ = fixed cost for constructing a new landfill j ($);

$LVC_{jt}^{\pm}$ = variable cost for constructing a new landfill j ($/ TPY);

$LFO_{jt}^{\pm}$ = fixed cost for operating a new landfill j ($/Y);

$LVC_{jt}^{\pm}$ = variable cost for operating a new landfill j ($/Y-TPY);

$IFC_{jt}^{\pm}$ = fixed cost for constructing a new incinerator j ($);

$IVC_{jt}^{\pm}$ = variable cost for constructing a new incinerator j ($/ TPY);

$IFO_{jt}^{\pm}$ = fixed cost for operating a new incinerator j ($/Y);

$IVO_{jt}^{\pm}$ = variable cost for operating new incinerator j ($/YTPY);

$G_{it}^{\pm}$ = solid waste generation rate in city i (TPY).

$EG_{j}^{\pm}$ = electricity generation capacity of incineration unit [888, 1038] (KW (hr/ton);

$E_{j}^{\pm}$ = electricity price [0.131, 0.142] ($/kW-hr).

$$\text{Minimize } \sum f^{\pm} = \text{upper bound net total cost } (f^{+}) + \text{lower bound net total cost } (f^{-}) \tag{1a}$$

Components of $f^{\pm}$ are :

$$f^{+} = \sum_{t \in t'} \left\{ \begin{array}{l} \left[(1+f)/(1+r)\right]^{t-1} \\ \left(C1_t^{+} + C2_t^{+} + C3_t^{+} - B1_t^{-} - B2_t^{-}\right) \end{array} \right\} \tag{1b}$$

$$f^{-} = \sum_{t \in t'} \left\{ \begin{array}{l} \left[(1+f)/(1+r)\right]^{t-1} \\ \left(C1_t^{-} + C2_t^{-} + C3_t^{-} - B1_t^{+} - B2_t^{+}\right) \end{array} \right\} \tag{1c}$$

Components of $C_t^{\pm}$ are :

$$C1_t^{+} = \sum_{i=1}^{27} \sum_{j=1}^{6} \left[X_{ijt}^{+} DS_{ijt}^{+} PS_{ijt}^{+} \right] \tag{2a}$$

$$C1_t^{-} = \sum_{i=1}^{27} \sum_{j=1}^{6} \left[X_{ijt}^{-} DS_{ijt}^{-} PS_{ijt}^{-} \right] \tag{2b}$$

$$C2_t^{+} = \sum_{i=4}^{5} \left[LFC_{jt}^{+} I_{jt}^{+} + LVC_{jt}^{+} DC_{jt}^{+} \right] + \sum_{i=6}^{6} \left[IFC_{jt}^{+} I_{jt}^{+} + IVC_{jt}^{+} DC_{jt}^{+} \right] \tag{3a}$$

$$C2_t^{-} = \sum_{i=4}^{5} \left[LFC_{jt}^{-} I_{jt}^{-} + LVC_{jt}^{-} DC_{jt}^{-} \right] + \sum_{i=6}^{6} \left[IFC_{jt}^{-} I_{jt}^{-} + IVC_{jt}^{-} DC_{jt}^{-} \right] \tag{3b}$$

$$C3_t^+ = \sum_{i=1}^{3} \left[LFO_{jt}^+ I_{jt}^+ + LVO \sum_{i=1}^{27} x_{ijt}^+ \right]$$

$$+ \sum_{j=4}^{5} \left[LFO_{jt}^+ I_{jt}^+ + LVO_{jt}^+ \sum_{i=1}^{27} x_{ijt}^+ \right] \quad (4a)$$

$$+ \sum_{i=6}^{6} \left[IFO_{jt}^+ I_{jt}^+ + IVO_{jt}^+ \sum_{i=1}^{27} x_{ijt}^+ \right]$$

$$C3_t^- = \sum_{i=1}^{3} \left[LFO_{jt}^- + LVO_{jt}^- \sum_{i=1}^{27} x_{ijt}^- \right]$$

$$+ \sum_{j=4}^{5} \left[LFO_{jt}^- I_{jt}^- + LVO_{jt}^- \sum_{i=1}^{27} x_{ijt}^- \right] \quad (4b)$$

$$+ \sum_{i=6}^{6} \left[IFO_{jt}^- I_{jt}^- + IVO_{jt}^- \sum_{i=1}^{27} x_{ijt}^- \right]$$

Components of $B_t^{\pm}$ are :

$$B1_t^+ = \sum_{i=1}^{27} \sum_{j=1}^{3} \left[X_{ijt}^+ T_j^+ \right] + \sum_{i=1}^{27} \sum_{j=4}^{5} \left[X_{ijt}^+ T_j^+ \right] \quad (5a)$$

$$B1_t^- = \sum_{i=1}^{27} \sum_{j=1}^{3} \left[X_{ijt}^- T_j^- \right] + \sum_{i=1}^{27} \sum_{j=4}^{5} \left[X_{ijt}^- T_j^- \right] \quad (5b)$$

$$B2_t^+ = \sum_{i=1}^{27} \sum_{j=6}^{6} \left[X_{ijt}^+ EG_j^+ E_j^+ \right] \quad (6a)$$

$$B2_t^- = \sum_{i=1}^{27} \sum_{j=6}^{6} \left[X_{ijt}^- EG_j^- E_j^- \right] \quad (6b)$$

$$G_{it}^+ = \sum_j X_{ijt}^+ \qquad \forall i,t \quad (7a)$$

$$G_{it}^- = \sum_j X_{ijt}^- \qquad \forall i,t \quad (7b)$$

$$\sum_{i=1}^{27} X_{ijt}^+ \le DC_{jt}^+ \qquad \forall t \in t' \text{and } j = 1-6 \quad (8a)$$

$$\sum_{i=1}^{27} X_{ijt}^- \le DC_{jt}^- \qquad \forall t \in t' \text{and } j = 1-6 \quad (8b)$$

$$DC_{jt,\min} I_{jt}^+ \le DC_{jt}^+ \qquad \forall t \in t' \text{and } j = 4-6 \quad (9a)$$

$$DC_{jt,\min} I_{jt}^+ \ge DC_{jt}^+ \qquad \forall t \in t' \text{and } j = 4-6 \quad (9b)$$

$$DC_{jt,\min} I_{jt}^- \le DC_{jt}^- \qquad \forall t \in t' \text{and } j = 4-6 \quad (9c)$$

$$DC_{jt,\min} I_{jt}^- \ge DC_{jt}^- \qquad \forall t \in t' \text{and } j = 4-6 \quad (9d)$$

$$\sum_t I_{jt}^+ \le 1 \qquad \forall t \in t' \text{and } j = 4-6 \quad (10a)$$

$$\sum_t I_{jt}^- \le 1 \qquad \forall t \in t' \text{and } j = 4-6 \quad (10b)$$

$$X_{ijt}^{-} \leq X_{ijt}^{+} \qquad \forall i,j,t \qquad (10c)$$

$$X_{ijt}^{\pm} \geq 0 \qquad \forall i,j,t \qquad (11)$$

$$I_{ijt}^{\pm} \in \{0,1\} \qquad \forall j,t \qquad (12)$$

Equation (1a) presents the objective function
Equations (1b) and (1c) present the components of $f^{\pm}$ (upper and lower bound)
Equations (2a) to (4b) present the components of $C_t^{\pm}$ (total annual shipping cost)
Equations (5a) to (6b) present the components of $B1_t^{\pm}$ (landfill space-saving benefit using the landfill tipping fee as a surrogate index)
Equation (7a) and (7b) presents the mass balance constraint for source location: All solid waste generated in a city should be shipped to treatment or disposal components in the network.
Equation (8a) and (8b) presents capacity limitation constraint for existing facilities: This constraint ensures that the waste inflow destined for final disposal should at least equal the permitted design capacity.
Equation (9a) to (9d) presents the capacity limitation constraint for potential facilities
Equation (10a) to (10c) presents the conditionally constraint: This constraint assures that the selection of a new facility at some point in the planning timeline can happen only once (10a–10b) and the upper bound should be greater than or equal to the lower bound for all decision variables (10c).
Equation (11) and (12) presents the non-negativity constraint: This constraint assures that only positive waste streams are considered in the solution, eliminating infeasibilities while calculating the solution.

2.2. Location modes with multi objective

2.2.1. Study 1 [9]

<u>Description of the study</u>

The purpose of this analysis is to present a new multi-criteria mixed-integer linear programming model to solve the location–allocation problem for municipal solid waste management at the regional level in the Central Macedonia, Greece. The solution consists of location and technologies for transfer stations, material recovery facilities, incinerators and sanitary landfills and waste flow between these locations. The objectives are multiple, such as minimization of greenhouse effect, disposals and total cost, and maximization of energy and material recovery, assumed as equal.

<u>Data</u>

- Set of locations of waste producers
- Quantity of waste (in tons/day) produced by a producer
- Set of possible sites for locations of transfer stations
- Quantity of incompact waste
- Number of opened transfer stations
- Set of possible sites for the locations of material recovery facilities
- Set of typology of material recovery facilities
- Quantity of waste that carry to material recovery facilities

- Waste flow(in tons/day) that transfer from different typologies of sites
- Energy recovery coefficient from material recovery facility, incinerator, landfill (in MW h day/ton)
- Efficiency degree(assumed to be 0,4 for that case)
- Set of possible sites for the location of an incinerator
- Set of possible typologies of incinerators
- Quantity of waste that carry to incinerator of specific typology
- Waste flow from incinerator at a site
- Set of possible sites for the locations of a landfill
- Set of possible typologies of landfills
- Quantity of waste that carry to landfills
- Compacted waste that carry to a landfill quantity of waste residue
- Installation cost (in Euro/ton) of facilities (/typology)
- Transportation cost (in Euro/ton) from specific facilities to sites(/typology)
- Treatment cost(in Euro/ton) of the waste/residue of facility at site (/typology)
- Distance between waste producers and transfer station and landfill

Objectives

- Minimization of greenhouse effect
- Minimization of final disposal to the landfill
- Maximization of the energy recovery
- Maximization of material recovery
- Minimization of the total cost (installation, transportation and treatment cost)

Constraints

- Service demand constraints: the amount of waste produced at a waste producer is equal to the sum of waste flow to other possible facilities
- Mass input-output relation constraints: indicate that no transfer station may keep the waste and reduction on the output of incinerator
- Minimum amount requirement constraints: facility is opened, only if the minimum amount of waste processed by that facility is available
- Capacity constraints
- Constraints of the maximum number of opened facilities
- Nonnegativity constraints

Mathematical Model

Variables

0–1 facility location variables u, v, w, x, where the variable has value one if the corresponding new facility is opened and zero otherwise,
Continuous waste flow variables a, e, 1, f, h, g, i, j,d representing the quantity of flow between facilities.
It is denoted x as the vector of variables used in the model, i.e., x =(φ,α,χ,ε,ς,ψ,ζ,θ,ω,η,ι,,κ,δ)

Notations
Waste producer
I set of locations of waste producers
a_i Quantity of waste (in ton/day) produced by a waste producer i $\in$ I
Transfer station

P set of possible sites for the location of a transfer station
M set of possible typologies for a transfer station

φ_{π}^{μ} Binary variable for locating a transfer station at site π ∈ Π with typology μ ∈ M

$\alpha_{i\pi}^{\mu}$ Quantity of incompact waste (in ton/day) generated by a waste producer i ∈ I and carried to a transfer station located at site π ∈ M with typology μ ∈ M

b_{π}^{μ} Waste flow (in ton/day) variable from all waste producers to transfer station π at site μ, i.e. $b_{\pi}^{\mu} = \sum_{i \in I} a_{i\pi}^{\mu}$

$k_{-\pi}^{\mu}$ Lower limit capacity (in ton/day) of local transfer station at site π ∈ Π with typology μ ∈ M

$k_{\pi}^{-\mu}$ Upper limit capacity (in ton/day) of local transfer station at site π ∈ Π with typology μ ∈ M

p^{φ} Maximum number of opened transfer stations

Material Recovery Facility (MRF)

P set of possible sites for the location of aMRF
V Set of possible typologies for a MRF

χ_{ρ}^{v} Binary variable for locating a MRF at site p ∈ P with typology v ∈ V

$\varepsilon_{i\rho}^{v}$ Quantity of waste (in ton/day) variable generated by a waste producer i ∈ I and carried to a MRF located at site p ∈ P with typology v ∈ V

$\varsigma_{\pi\ \pi}^{\mu\ v}$ Waste flow (in ton/day) variable from a transfer station located at site π ∈ Π with typology μ ∈ M to a MRF at site p ∈ P with typology v ∈ V

c_{π}^{n} waste flow (in ton/day) variable to a MRF at site p ∈ P with typology v ∈ V i.e., waste flow from all waste producers and transfer stations to a MRF at site p ∈ P with typology v ∈ V . i.e.,

$c_{p}^{v} = \sum_{i \in I} \varepsilon_{i\ p}^{v} + \sum_{\mu \in M} \sum_{\pi \in \Pi} \varsigma_{\pi\ \rho}^{\mu\ v}$

g_{p}^{v} Lower limit capacity (in ton/day) of a MRF at site p ∈ P with typology v ∈ V

g_{p}^{v} Upper limit capacity (in ton/day) of a MRF at site p ∈ P with typology v ∈ V

s_{p}^{v} Waste and residue (in ton/day) variable from a MRF at site p ∈ P with typology v ∈ V

i.e. $s_{p}^{v} = \sum_{o \in O} \sum_{\tau \in T} \kappa_{p\tau}^{vo}$

$A_{v}^{G\ H\ E}$ Emission coefficients for greenhouse effects (in ton of CO_2-equivalent of CO_2 and CH_4 day/ton of waste year) from an MRF with typology v ∈ V

A_{v}^{E} Energy recovery coefficient (in MW h day/ ton year) from anMRF with typology v ∈ V

A_{v}^{M} Coefficient for calculating the material recovery (ton.day/ton of waste year) from an MRF with typology v ∈ V

f^{o} Efficiency degree of MRF

p^{x} Maximum number of opened MRFs

Incinerator (waste to energy facility)

D set of possible sites for the location of an incinerator
E set of possible typologies for an incinerator

ψ_{σ}^{ξ} Binary variable for locating an incinerator at site σ ∈ D with typology ξ ∈ E

$\zeta_{i\sigma}^{\xi}$ Quantity of waste (in ton/day) variable generated by a waste producer i ∈ I and carried to an incinerator located at site σ ∈ D with typology ξ ∈ E

$\theta_{\pi\ \sigma}^{\mu\ \xi}$ Waste flow (in ton/day) variable from a transfer station located at site π ∈ Π with typology μ ∈ M to an incinerator at site σ ∈ D with typology ξ ∈ E

d_{σ}^{ξ} Waste flow (in ton/day) variable to an incinerator at site σ ∈ D with typology ξ ∈ E
i.e., waste flow from all waste producers and transfer stations to an incinerator at site σ ∈ D with typology ξ ∈ E
i.e. $d_{\sigma}^{\xi} = \sum_{i \in I} \zeta_{i\sigma}^{\xi} + \sum_{\mu \in \mathrm{M}} \sum_{\pi \in \Pi} \theta_{\pi\ \sigma}^{\mu\ \xi}$

$h_{-\sigma}^{\xi}$ Lower limit capacity (in ton/day) of incinerator at site σ ∈ D with typology ξ ∈ E

$h_{\sigma}^{-\xi}$ Upper limit capacity (in ton/day) of incinerator at site σ ∈ D with typology ξ ∈ E

q_{σ}^{ξ} Waste flow (in ton/day) from an incinerator at site σ ∈ D with typology ξ ∈ E

$\Gamma_{\xi}^{G\ H\ E}$ Emission coefficients for greenhouse effects (in ton of CO_2-equivalent of CO_2 and CH_4 day/ton of waste year) from facilities in an incinerator with typology ξ ∈ E

Γ_{ξ}^{E} Energy recovery coefficients (in MW h day/ ton year) from an incinerator with typology ξ ∈ E

$\Gamma_{\xi}^{\mathrm{M}}$ Material recovery coefficients (ton.day/tonof waste.year) from an incinerator with typology ξ ∈ E

f^{ξ} Efficiency degree of an incinerator.

p^{ψ} Maximum number of opened incinerators

Sanitary landfill
T Set of possible sites for the location of a landfill
O Set of possible typologies for a landfill
xos

ω_{τ}^{o} Binary variable for locating a landfill at site τ ∈ T with typology o ∈ O

η_{π}^{o} Quantity of waste (in ton/day) variable generated by a waste producer i ∈ I and carried to a landfill located at site τ ∈ T with typology o ∈ O

$\iota_{\pi\ \tau}^{\mu\ o}$ Compacted waste flow (in ton/day) variable from a transfer station at site π ∈ Π with typology μ ∈ M to a landfill at site τ ∈ T with typology o ∈ O

$\kappa_{\rho\ \tau}^{\nu\ o}$ Waste residue flow (in ton/day) variable from a MRF at site P ∈ P with typology v ∈ V to a landfill at site τ ∈ T with typology o ∈ O

$\delta_{\sigma\ \tau}^{\xi\ o}$ Waste residue flow (in ton/day) variable from an incinerator at site σ ∈ D with typology ξ ∈ E to a landfill at site τ ∈ T with typology o ∈ O

e_{τ}^{o} Waste flow (in ton/day) variable to a landfill at site τ ∈ T with typology o ∈ O
, i.e. waste flow from all waste producers, all transfer stations, all MRFs, and all incinerators to landfill at site τ ∈ T with typology o ∈ O

i.e. $e_{\tau}^{o} = \sum_{i \in I} \eta_{i\tau}^{o} + \sum_{\mu \in \mathrm{M}} \sum_{\pi \in \Pi} \iota_{\pi\ \tau}^{\mu 0} + \sum_{\nu \in V} \sum_{p \in P} k_{\rho\ \tau}^{\nu 0} + \sum_{\xi \in \mathrm{E}} \sum_{\sigma \in D} \delta_{\sigma\ \tau}^{\xi 0}$

$u_{-\tau}^{o}$ Lower limit capacity (in ton/day) of a landfill at site τ ∈ T with typology o ∈ O

u_{τ}^{-o} Upper limit capacity (in ton/day) of a landfill at site τ ∈ T with typology o ∈ O

$E_{o}^{G\ H\ E}$ Emission coefficients for greenhouse effects (in ton of CO_2-equivalent of CO_2 and CH_4. day/ton of waste.year) from facilities in a landfill with typology o ∈ O

E_o^E Energy recovery coefficients (in MW h day/ ton year) from a landfill with typology o ∈ O

p^ω Maximum number of opened sanitary landfills

CFF_{jk}^i Installation cost (in Euro/ton) of facility j at site k with typology i where j ∈ {φ,χ,ψ,ω}, k ∈ {π,ρ,σ,τ}, and i∈ {μ,ν,ξ,o}

$CFVtp_{jj'kk'}^{it}$ Transportation cost (in Euro/ton) from facility j at site k with typology i to facility j' at site k' with typology Ii' where j ≠ j'≠∈ {φ,χ,ψ,ω}, k≠k' ∈ {π,ρ,σ,τ} and i ≠ I i' ∈ {l, m,n,o}. The transportation cost may not dependent on typology

$CFVtr_{jk}^i$ Treatment cost (in Euro/ton) of the waste/residue of facility j at site k with typology i where where j ∈ {φ,χ,ψ,ω}, k ∈ {π,ρ,σ,τ}, and i∈ {μ,ν,ξ,o}

Mathematical Model

$$\min GHE(x) = \sum_{v\in V}\sum_{\rho\in P} A_v^{GHE} c_p^v + \sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D} \Gamma_\xi^{GHE} d_\sigma^\xi + \sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} \mathrm{E}_o^{GHE} e_\tau^o$$

$$\min FIDI(x) = \sum_{i\in I}\sum_{o\in O}\sum_{\tau\in \mathrm{T}} \eta_{i\tau}^o w_\tau^o + \sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi}\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} r_{\pi\tau}^{\mu o} w_\tau^o + \sum_{p\in P}\sum_{o\in O}\sum_{\tau\in \mathrm{T}} k_{p\tau}^{vo} w_\tau^o + \sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D}\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} \delta_{\sigma\pi}^{\xi o} w_\tau^o \quad (1)$$

Or for short

$$\min FIDI(x) = \sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} e_\tau^o w_\tau^o \quad (2)$$

$$\max ER(x) = \sum_{v\in V}\sum_{p\in P} A_p^E c_p^v + \sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D} \Gamma_\xi^{\mathrm{E}} d_\sigma^\xi + \sum_{o\in O}\sum_{\tau\in \mathrm{T}} \mathrm{E}_o^{\mathrm{E}} e_\tau^o \quad (3)$$

$$\max MR(x) = \sum_{v\in V}\sum_{p\in P} A_v^M c_p^v + \sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D} \Gamma_\xi^M d_\sigma^\xi \quad (4)$$

$$\min TC(x) = IC(x) + TransC(x) + TreatC(x) \quad (5)$$

Where

$$IC(x) = \sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi} CFF_{\varphi\pi}^\mu b_\pi^\mu + \sum_{v\in V}\sum_{p\in P} CFF_{xp}^v c_p^v \quad (5.1)$$

$$Trans(x)=\sum_{i\in I}\sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi} CFVtp^{\mu}_{\alpha\varphi\iota\pi}\alpha^{\mu}_{\iota\pi}+\sum_{i\in I}\sum_{v\in V}\sum_{p\in P} CFVtp^{\xi}_{\alpha\chi ip}\varepsilon^{v}_{ip}$$
$$+\sum_{i\in I}\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D} CFVtp^{\xi}_{\alpha\psi i\sigma}\zeta^{\xi}_{i\sigma}+\sum_{i\in I}\sum_{o\in O}\sum_{\tau\in \mathrm{T}} CFVtp^{o}_{\alpha\omega i\tau}\eta^{o}_{i\tau}$$
$$+\sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi}\sum_{v\in V}\sum_{p\in P} CFVtp^{\mu v}_{\varphi\chi\pi\rho}\theta^{\mu\xi}_{\pi\sigma}+\sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi}\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D} CFVtp^{\mu\xi}_{\varphi\psi\pi\sigma}\theta^{\mu\xi}_{\pi\sigma} \tag{5.2}$$
$$+\sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi}\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} CFVtp^{\mu o}_{\varphi\omega\pi\tau}\iota^{vo}_{\rho\tau}+\sum_{v\in V}\sum_{p\in P}\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} CFVtp^{vo}_{x\omega p\tau}k^{vo}_{\rho\tau}$$
$$+\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D}\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} CFVtp^{\xi o}_{\psi\omega\sigma\tau}\delta^{\xi o}_{\sigma\tau}$$

$$TreatC(x)=\sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi} CFVtr^{\mu}_{\varphi\pi}b^{\mu}_{\pi}+\sum_{v\in V}\sum_{p\in P} CFVtr^{\mu}_{xp}c^{v}_{\rho}$$
$$+\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D} CFVtr^{\xi}_{\psi\sigma}b^{\xi}_{\sigma}+\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}} CFVtr^{o}_{\alpha\chi}e^{o}_{\iota} \tag{5.3}$$

$$a_i=\sum_{\mu\in \mathrm{M}}\sum_{\pi\in \Pi} a^{\mu}_{i\pi}+\sum_{v\in V}\sum_{p\in P}\varepsilon^{v}_{i\rho}+\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D}\zeta^{\xi}_{i\sigma}$$
$$+\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}}\eta^{o}_{it}\forall i\in I \tag{6}$$

$$b^{\mu}_{\pi}=\sum_{v\in V}\sum_{p\in P}\varsigma^{\mu v}_{\pi\rho}+\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D}\theta^{\mu\xi}_{\pi\sigma}+\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}}\iota^{\mu o}_{\pi\tau}$$
$$\forall\mu\in \mathrm{M},\pi\in\Pi \tag{7.1}$$

$$f^{v}c^{v}_{p}=\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}}\kappa^{vo}_{\rho\tau}\forall v\in V,\tau\in \mathrm{T}$$
$$f^{\xi}d^{\xi}_{\sigma}=\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}}\delta^{\xi o}_{\sigma\tau}\forall\xi\in \mathrm{E},\sigma\in D \tag{7.2}$$

Transfer Station: $d^{\mu}_{\pi}\geq\kappa^{\mu}_{-\pi}\varphi^{\mu}_{\pi}\forall\mu\in \mathrm{M},\pi\in\Pi$

MRF facilities: $c^{v}_{\rho}\geq g^{v}_{-p}\chi^{v}_{\rho}\forall v\in V,\rho\in \mathrm{P}$ (8)

Incinerators: $d^{\xi}_{\sigma}\geq h^{\xi}_{-\sigma}\psi^{\xi}_{\sigma}\forall\xi\in \mathrm{E},\sigma\in D$

Landfills: $e^{o}_{\tau}\geq u^{o}_{-\tau}\omega^{o}_{\tau}\forall o\in \mathrm{O},\tau\in \mathrm{T}$

Transfer Station: $b^{\mu}_{\pi}\leq\kappa^{-\mu}_{\pi}\varphi^{\mu}_{\pi}\forall\mu\in \mathrm{M},\pi\in\Pi$

MRF facilities: $c^{v}_{\rho}\leq g^{-v}_{p}\chi^{v}_{\rho}\forall v\in V,\rho\in \mathrm{P}$ (9)

Incinerators: $d^{\xi}_{\sigma}\leq h^{-\xi}_{\sigma}\psi^{\xi}_{\sigma}\forall\xi\in \mathrm{E},\sigma\in D$

Landfills: $e^{o}_{\tau}\leq u^{-o}_{\tau}\omega^{o}_{\tau}\forall o\in \mathrm{O},\tau\in \mathrm{T}$

Transfer Station: $\sum_{\mu\in \mathrm{M}}\sum_{\pi\in\Pi}\varphi^{\mu}_{\pi}\leq\rho^{\varphi}$

MRFs: $\sum_{v\in V}\sum_{p\in P}x^{v}_{p}\leq\rho^{x}$ (10)

Incinerators: $\sum_{\xi\in \mathrm{E}}\sum_{\sigma\in D}\psi^{\xi}_{\sigma}\leq\rho^{\psi}$

Landfills: $\sum_{o\in \mathrm{O}}\sum_{\tau\in \mathrm{T}}\omega^{o}_{\tau}\leq\rho^{\omega}$

Equation 1 presents the objective of minimization of the greenhouse effect (GHE). The greenhouse effect describes how greenhouse gases, including carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and chlorofluorocarbons (CFCs) in the earth's atmosphere absorb the amount of heat escaping from the earth into the atmosphere, making the earth's surface warmer. Waste processing in MRF (anaerobic digestion), incinerators and landfills is considered to be the source of greenhouse gases. We define the greenhouse effect as a product of the amount of waste in the facility, and the greenhouse emission coefficient associated with the facility or its typology. It is represented in ton of CO_2-equivalent and CH_4 per year.
Equation 2 presents the objective of minimization of the final disposal to the landfill (FIDI), i.e., the total amount (in tons/year) of waste and/ or residue brought to all landfills from all waste producers and other facilities. It minimized the amount of waste that cannot be recovered or converted further. Such waste occupies valuable landfill space, reducing the site's life
Equation 3 presents the objective of maximization of the energy recovery (ER) (in MW h/ year) from MRFs, incinerators, and sanitary landfills
Equation 4 presents the objective of maximization of the material recovery (MR) (in ton/ year) from MRFs
Equation 5 presents the objective of the minimization of the total cost (TC) (in Euro/day), which includes the installation or opening costs, transportation costs, and treatment costs.
Equation 5.1 presents the Installation cost. As installation cost, it is considered the investment cost per tonne of waste.
Equation 5.2 presents the transportation costs. In defining the transportation cost, the maximum distance between a waste producer and either a transfer station or sanitary landfill (25 km, based on the maximum one-way distance of collection trucks in daily trips) and the 100 km between a transfer station and a landfill are taken into account. Equation 5.3 presents the treatment cost
Equation 6 presents the service demand constraint, the amount of waste produced at a waste producer is equal to the sum of waste flow to other possible facilities.
Equation 7 presents the mass input–output relation constraints, equation 7.1 indicates that no transfer station may keep the waste, Equation 7.2 indicates that Reduction on the output of an MRF and incinerator determined by the mass preservation rate of the MRF and incinerator
Equation 8 presents the minimum amount requirement constraints, which ensure that a facility is opened, only if the minimum amount of waste processed by that facility is available
Equation 9 presents the capacity constraints
Equation 10 presents the constraints on the maximum number of opened facilities

2.2.2. Study 2 [10]

Description of the study

The purpose of this study is to develop three generic facility location models for the integrated distribution and collection of products. These models quantify the value of integrated decision making in the design of forward and reverse logistics networks throughout different stages of a product's life cycle. The formulations extend the incapacitated fixed-charge location model to include the location of used product collection centres and the assignment of product return flows to these centres. The objective of these models is the minimization of the fixed forward and reverses facility costs, of the forward and reverse transportation costs, and minimization of the savings associated with collocation of forward and reverse facilities.

Data

- Number of forward distribution centres
- Number of reverse distribution centres
- Number of sites

- Quantity of forward demand
- Unit cost of (forward) shipping from candidate site
- Fixed facility cost of a forward distribution center
- Fixed facility cost of a reverse site
- Number of returns per unit
- Unit cost if shipping a return item

Objectives

- Minimization the sum of the fixed forward and reverse facility costs,
- Minimization of the forward and reverse transportation costs,
- Minimization of the savings associated with colocation of forward and reverse facilities.

Constraints

- Forward and reverse demand constraints : requires that all forward and reverse demand nodes be assigned to a facility
- Constraint that ensure that a demand node is not assigned to a facility that has not been opened.
- binary constraints for locating facilities
- nonnegativity constraints
- Constraint to link the forward and reverse sub problems by allowing the savings for colocation to be realized only if a site has both a forward and a reverse facility.
- Constraint of the forward dominant that ensures that the reverse facilities are only located at sites that have an open forward facility.
- Constraint of reverse dominant that ensures that forward facilities are located only at sites that house a reverse facility.

Mathematical Model

This model is developed by the formulation of three extensions to the classical un-capacitated fixed charge location problem that forward and reverse distribution activities through the location of bidirectional distribution centres in addition to dedicated unidirectional facilities.

It is considered two methods to resolve the problem:

a. Using financial incentives to induce the location of bidirectional facilities that is used to formulate the colocation model (CL)
b. Imposing constraints that require one type of network to fit within the other, which is used to formulate the forward dominant (FD) and reverse dominant (RD) models

The FD model is more applicable to products in the early part of their life cycle when few returns are likely and therefore a firm requires that reverse facilities will be colocated with forward facilities.

The RD model is applicable to products nearing the end of their useful life, when the number of returns is likely to be on the same order of magnitude, or even greater than, the number of primary sales (forward flow) of the product.

Decision variables

X_J^F =1 if a forward distribution center is located at site *j*, 0 if not

X_J^R = 1 if a reverse distribution center is located at site *j*, 0 if not

X_J^C =1 if both a forward and a reverse distribution center are located at site *j*, 0 if not

Y_{ij}^F =fraction of forward demand at node *i* that is served by site *j*

Y_{ij}^R = fraction of returns at node *i* that is served by site *j*

$$Min\sum_{j\in J} f_j X_j^F + \sum_{i\in I}\sum_{j\in J} h_j c_{ij} Y_{ij}^F + \sum_{j\in J}\beta_j f_j X_j^R$$
$$+\sum_{i\in I}\sum_{j\in J} a_i h_i \gamma_{ij} c_{ij} Y_{ij}^R - \sum_{i\in I} s_j X_j^C \quad (1)$$

Subject to

$$\sum_{j\in J} Y_{ij}^F = 1 \sum_{j\in J} Y_{ij}^R = 1 \,\forall\, i \in I \quad (2)$$

$$Y_{ij}^F \le X_j^F, Y_{ij}^R \le X_j^R \forall i \in I, j \in j \quad (3)$$

$$X_j^F = \{0,1\}, X_j^R = \{0,1\} \forall j \in J \quad (4)$$

$$Y_{ij}^F \ge 0, Y_{ij}^R \ge 0 \forall\, i \in I,\, j \in j \quad (5)$$

$$X_j^F \ge X_j^C, X_j^R \ge X_j^C \forall j \in J \quad (6)$$

$$X_j^C = \{0,1\} \forall j \in J \quad (7)$$

$$Min\sum_{j\in J} f_j X_j^F + \sum_{i\in I}\sum_{j\in J} h_j c_{ij} Y_{ij}^F + \sum_{j\in J}\beta_j f_j X_j^R$$
$$+\sum_{i\in I}\sum_{j\in J} a_i h_i \gamma_{ij} c_{ij} Y_{ij}^R \quad (8)$$

$$X_j^F \ge X_j^R \forall j \in J \quad (9)$$

$$X_j^F \le X_j^R \forall j \in J \quad (10)$$

Equation (1) presents the objectives of the collocation (CL) models that minimizes the sum of fixed forward and reverse facility costs, the forward and reverse transportation costs, and the savings associated with colocation of forward and reverse facilities.

Equation (2) presents the constraint that requires that all forward and reverse demand nodes be assigned to a facility

Equation (3) presents the constraint that ensures that a demand node is not assigned to a facility that has not been opened.

Equation (4) and (7) presents all the necessary binary constraints for locating facilities

Equation (5) presents the standard Nonnegativity constraints.

Equation (6) presents the constraints that link the forward and reverse sub problems by allowing the savings forCL to be realized only if a site has both a forward and a reverse facility. Note that in the absence of constraint (6) and the CL variables, X_j^C the model decomposes into two incapacitated fixed charge problems: one for forward sites and one for reverse sites.

Equation (8) presents the objective functions of the forward dominant models that minimizes the sum of fixed forward and reverse facility costs, the forward and reverse transportation costs eliminating the term related to colocation model cost savings.

Equation (9) presents the constraint of the forward dominant model that ensure that reverse facilities are only located at sites that have an open forward facility. (For forward dominant models the constraints (6) and (7) from the collocation model removed and replaced from the previous constraint)
Equation (10) presents the constraint of reverse dominant (RD) model that ensures that forward facilities are located only at sites that house a reverse facility.

2.2.3. Study 3 [11]

Description of the study

The purpose of this study is to present a mathematical model which aims to provide a minimum cost solution for the reverse logistic network design problem involving product returns. The aim is to determine the numbers of location of centralised returns centres where returned products from retailers or end-customers can be collected, sorted and consolidated into shipment towards manufactures or distribution repair facilities. This model is applicable in retail industries, which search to manage product returns in a more cost-efficient way. Hence, customers' satisfaction regarding their return products and their convenience can increase, centralized return centres can be fully utilized and the overall R.L. costs (transportation & inventory costs) can be minimized.

Data

- Numbers of customers
- Numbers of collection points
- Number of centralized centres
- Volume of returned products
- Handling cost of unit product per day
- Cost of opened centralized return center
- Annual cost of renting initial collection points
- Daily inventory carrying cost per unit (annual working days)
- Handling cost of unit per day
- Distance from customers to initial collection point and from collection point to centralized centre
- Unit freight rate
- Volume of returned products from initial collection point to centralized center
- Length of collection period (in days) at initial collection point

Objective

- Minimization of the total reverse logistics cost: renting, inventory carrying, material handling, setup and shipping cost.
-

Constraints

- Capacity constraints : the total volume of products returned from initial collection points does not exceed the maximum capacity of a centralized return center
- Each customer is assigned to a single initial collection point
- Each initial collection point should be located within a certain allowable proximity of customers
- Constraint that prevents any return flows from the unopened initial collection point.
- Constraint that makes the incoming flow equal to the outgoing flow at an initial collection point.
- Constraint that maintain a minimum number of initial collection points and centralized return centres for product return

- Non negative constraint
- Binary constraints
- Integrality of decision variables constraint

<u>Model</u>

Indices
i index for customers; $i \in I$
j index for initial collection points; $j \in J$
k index for centralized return centres; $k \in K$

Model parameters
a annual cost of renting initial collection point *j*
b daily inventory carrying cost per unit
w annual working days
r_i Daily volume of products returned by customer *i*
h handling cost of unit product per day
q_k Cost of establishing centralized return center *k*
m_k Maximum capacity of centralized return center *k*
d_{ij} Distance from customer *i* to initial collection point *j*
d_{jk} Distance from collection point *j* to centralized return center *k*
l maximum allowable distance from a given customer to an initial collection point
f (X_{jo}, d_{jo}) *, dj*0*)* $E_{\alpha\beta}$ (= function for freight rate)
Where *α is* a discount rate according to the volume of shipment between initial collection point *j* and centralized return center *k*; *β is* a penalty rate applied for the distance between collection point *j* and centralized return center *k*

$$a = \begin{cases} 1\, for\ x_{jo} \leq \mathrm{p}_1 \\ a_1 for\ p_1 < \mathrm{x}_{jo} \leq p_2 \\ a_2 for\ x_{jo} > \mathrm{p}_2 \end{cases}$$

$$\beta = \begin{cases} 1\, for\ d_{jo} \leq \mathrm{q}_1 \\ \beta_1 for\ \mathrm{q}_1 < \mathrm{d}_{jo} \leq q_2 \\ \beta_2 for\ \mathrm{d}_{jo} > \mathrm{q}_2 \end{cases}$$

E unit freight rate
*p*1*, p*2 volume of returned products for a discount
*q*1*, q*2 distance between collection point *j* and centralized return center *k* for penalties
z minimum number of established initial collection points
g minimum number of established centralized return centres
M arbitrarily set large number.

Decision variables
X_{jk} = volume of products returned from initial collection point *j* to centralized return center *k*,
T_j = length of a collection period (in days) at initial collection point j,

$$Y_{ij} = \begin{cases} 1\ If\ \ customer\ i\ is\ allocated\ to\ initial\ collection\ j(i \in I, i \neq j) \\ 0 othewise \end{cases}$$

$$Z_j = \begin{cases} 1\ If\ \ initial\ collection\ \text{point is established at a site}\ j(j \in J) \\ 0 othewise \end{cases}$$

$$G_k = \begin{cases} 1\ If\ \ a\ centralized\ \text{return}\ centeris\ established\ at\ site\ k\left(k \in K\right) \\ 0 othewise \end{cases}$$

Mathematical model

$$a\sum_j Z_j + bw\sum_j \left\{\sum_i r_i Y_{ij}\frac{(T_{j+1}+1)}{2}\right\} + hw\sum_i r_i + \sum_k q_k G_k + \sum_k \left\{G_k \sum_J (x_{ij}\frac{w}{T_j}) xf(X_{jk} d_{jk})\right\} \tag{1}$$

Subject to

$$\sum_j Y_{ij} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_i Y_{ij} \leq M\ .Z_j \qquad \forall j \in J \tag{3}$$

$$\sum_i r_i Y_{ij} T_j = \sum_k x_{jk} \qquad \forall j \in J \tag{4}$$

$$\sum_j X_{jk} \leq m_k G_k \qquad \forall k \in K \tag{5}$$

$$d_{ij} Y_{ij} \leq 1 \qquad \forall j \in J\ , \forall i \in I \tag{6}$$

$$z \leq \sum_j Z_j \tag{7}$$

$$g \leq \sum_k G_k \tag{8}$$

$$X_{jk} \geq o \qquad \forall j \in J\ , \forall k \in K \tag{9}$$

$$T_j \in (0,1,2,3,4,5,6,7) \qquad \forall j \in J \tag{10}$$

$$Y_{ij}, z_j, G_k \in (0,1) \qquad \forall i \in I\ , \forall j \in J\ , \forall k \in K \tag{11}$$

Equation (1) presents the objective function that minimizes the total reverse logistics cost comprised or renting, inventory carrying, material handling, and setup and shipping cost.
Equation (2) presents the constraint that assures that a customer is assigned to a single initial collection point.
Equation (3) presents the constraint that prevents any return flows from the unopened initial collection point.
Equation (4) presents the constraint that makes the incoming flow equal to the outgoing flow at an initial collection point.
Equation (5) presents the constraint that ensures that the total volume of products returned from initial collection points does not exceed the maximum capacity of a centralized return center.
Equation (6) presents the constraint that ensures assures that each initial collection point should be located within a certain allowable proximity of customers.
Equations (7) and (8) present the constraint that maintain a minimum number of initial collection points and centralized return centres for product return.
Equation (9) presents the constraint that preserves the non-negativity of decision variables X_{jk}.
Equation (10) presents the constraint that limits a range of integrality of decision variables T_j.
Equation (11) presents the constraint that assures the binary integrality of decision variables Y_{ij}, Z_j and G_k

2.2.4. Study 4 [12]

Description of the study

This study focuses on the logistic network design for end-of-lease computer products recovery by developing a deterministic programming model for managing forward and reverse flows. A company delivers computer products to a number of geographically dispersed customers from the original equipment manufacturer according to their demand. A part of end-of-lease computer products associated with the delivered products and having the same dimensions as the original products are collected from the customers and returned to the original equipment manufacturer for the purpose of recovery or safe disposal. In such integrated logistics network, instead of dealing with separate warehouse or collection centres, a type of hybrid processing facility is considered. Both forward products and end of lease returned products are transferred via hybrid processing facilities; both forward flows of products and returned end-of-lease items (EOL reverse flows) will be directed in hybrid processing locations.

Data

- Number of hybrid processing facilities
- One original equipment manufacturer (OEM)
- Set of potential OEMs
- Set of potential hybrid processing facilities
- Set of customers
- Quantity of the supply of forward products
- Quantity of the demand of forward products
- Quantity of the supply of returned products
- Quantity of the demand of returned products
- Shipping cost per unit of forward and return products
- Fixed cost of building hybrid processing facility

Objective

- Minimization of the total cost which includes the cost associated with building up hybrid processing facilities, with shipping forward products from the original equipment manufacturer to customers via hybrid processing facilities, and with collecting end – of – lease returned products from customers to original equipment manufacturer via hybrid processing facilities.

Constraints

- Network flow conservation constraints that ensures that during the shipment of forward products, the quantity of supply of forward products at each depot is equal to the difference between total output and total input of forward products
- Network flow conservation constraint that ensures that the quantity of demand of forward products at each customer is equal to the difference between total input and total output of forward products
- Network flow conservation constraint that ensures that during the collection of returned end- of- lease products, the quantity of supply at each customer is equal to the difference between total output and total input of the returned end- of- lease products
- Shipping capacity constraints
- Depot capacity
- Handling capacity of returned products
- Binary restriction on the location decision variables constraints
- Constraint that guarantee the quantities of forward and end-of-lease returned products shipped along arcs are not less than zero

Mathematical Model

Parameters

M a sufficiently large constant

q Number of hybrid processing facilities to be built up

CD = {1, . . .,a} set of potential OEMs

RD = {1, . . .,b} set of potential hybrid processing facilities

D = CD [RD set of potential depots in the logistics network

C = {1. . . c} set of customers

E = RD [C set of potential hybrid processing facilities and customers

N = D [C set of nodes in the logistics network

A = a(i, j) set of arcs connecting node i and node j in the logistics network, $\forall i, j \in N$

S_k^F Quantity of the supply of forward products at node k, $\forall k \in D$

D_n^F Quantity of the demand of forward products at node n, $\forall n \in C$

S_n^R Quantity of the supply of returned products at node n, $\forall n \in C$

D_k^R Quantity of the demand of returned products at node k, $\forall k \in D$

U_{ij}^F Shipping capacity of arc a(i, j) in the unit of forward products, $\forall i, j \in N$

U_{ji}^R Shipping capacity of arc a(j, i) in the unit of returned products, $\forall i, j \in N$

U_I Capacity for handling returned products at hybrid processing facility l, $\forall I \in RD$

c_{ij}^F Shipping cost per unit of forward products shipped along arc a(j, i), $\forall i, j \in N$

C_{ji}^{R} Shipping cost per unit of returned products shipped along arc a(j, i), $\forall i, j \in N$

C_I Fixed cost associated with building up hybrid processing facility l, $\forall I \in RD$

Variables

x_{ij} Quantity of forward product shipped along arc a (i,j), $\forall i, j \in N$

y_{ij} Quantity of returned products shipped along arc a (i,j), $\forall i, j \in N$

z_k = 1 if the potential depot i is to be chosen, $\forall k \in D$
0 otherwise

$$Min \sum_{i \in RD} C_1 z_1 + \sum_{i \in N} \sum_{j \in N} C_{ij}^F x_{ij} + \sum_{j \in N} \sum_{i \in N} C_{ji}^R y_{ji} \quad (1)$$

$$\sum_{m \in E} x_{km} - \sum_{i \in N} x_{ik} = S_k^F z_k \qquad \forall k \in D \quad (2)$$

$$\sum_{i \in N} x_{in} - \sum_{m \in E} x_{nm} = D_n^F \qquad \forall n \in C \quad (3)$$

$$\sum_{i \in N} y_{ni} - \sum_{m \in E} y_{nm} = S_n^R \qquad \forall n \in C \quad (4)$$

$$\sum_{m \in E} y_{mk} - \sum_{i \in N} y_{ki} = D_k^R z_k \qquad \forall n \in D \quad (5)$$

$$x_{ij} \le U_{ij}^F \qquad \forall i \in N, j \in N \quad (6)$$

$$y_{ji} \le U_{ji}^R \qquad \forall j \in N, i \in N \quad (7)$$

$$\sum_{m \in C} y_{nl} \le U_I \qquad \forall I \in RD \quad (8)$$

$$\sum_{m \in E} x_{km} \le M z_k \qquad \forall k \in D \quad (9)$$

$$\sum_{m \in E} y_{km} \le M z_k \qquad \forall k \in D \quad (10)$$

$$x_{ij} \ge 0 \qquad \forall i \in N, j \in N \quad (11)$$

$$y_{ji} \ge 0 \qquad \forall j \in N, i \in N \quad (12)$$

$$\sum_{p \in CD} z_p = 1 \quad (13)$$

$$\sum_{p \in RD} z_I = q \qquad (14)$$

$$z_k \in \{0,1\} \qquad \forall k \in D \qquad (15)$$

Equation (1) presents the objective function that minimizes the total cost which includes the cost associated with building up hybrid processing facilities, with shipping forward products from the original equipment manufacturer to customers via hybrid processing facilities, and with collecting end – of – lease returned products from customers to original equipment manufacturer via hybrid processing facilities.
Equation (2) present the network flow conservation constraint that ensures that during the shipment of forward products, the quantity of supply of forward products at each depot k, k $\in$D is equal to the difference between total output and total input of forward products
Equation (3) present the network flow conservation constraint that ensures that the quantity of demand of forward products at each customer n, n $\in$ C is equal to the difference between total input and total output of forward products
Equation (4) present the network flow conservation constraint that ensures that during the collection of returned end- of- lease products, the quantity of supply at each customer n, n $\in$ C is equal to the difference between total output and total input of the returned end- of- lease products
Equation (5) present the network flow conservation constraint that ensures that the quantity of demand of returned end- of- lease products at each depot k, k $\in$ D is equal to the difference between total input and total output P of the returned end- of- lease products.
Equations (6) and (7) present the capacity constraint that limit the units of forward and end- of- lease returned products shipped along an arc to its shipping capacity in the network. The realistic meaning of such constraints is that not all products can be shipped via the shortest path in a network due to the limited shipping capacity.
Equation (8) presents the capacity constraint that limits the units of end- of- lease returned products transferred through a collection depot to its capacity for dealing with end- of- lease returned products which is due to the limitation of the certain equipments for the operations of repackaging before the end- of- lease returned products are shipped back to original manufacturer equipment.
Equations (9) and (10) present the capacity constraint that prohibit the units of forward and end- of- lease returned products from being transferred through depots unless the depots are to be built up.
Equations (11) and (12) present the constraint that guarantee the quantities of forward and end-of-lease returned products shipped along arcs are not less than zero. If the decision variable x_{ij} is determined to zero, there is no forward product shipped from i, i $\in$N node to node j, j $\in$ N. If the decision variable y_{ij} is determined to zero, there is no returned end-of lease product transferred from i, i $\in$ N node to node j, j $\in$ N.
Equations (13) and (14) present the constraint that ensure the numbers of original manufacturer equipment and hybrid processing facilities to be built up to specific values, while only one original manufacturer equipment is to be chosen in this paper.
Equation (15) presents the constraint that enforces the binary restriction on the location decision variables.

2.2.5. Study 5 [13]

Description of the study

The purpose of this case study is to develop a novel comprehensive reverse logistic model for the recovery of waste by-products streams in an exchange networks ((Business-to-Business, B2B) among industries in order to design a high degree of sustainability. Every country produce millions tons of by -product /waste materials in the manufacturing sector. The waste and by-products generated by a firm are transferred to the collection centers or if their quality is acceptable, they will be consumed directly by another firm. In the collection centers, after quality inspection, the materials are passed to other facilities such as the VAP centers or the disposal centers. Based on the quality of the generated materials, they may be sent to plants to be used as a substitute for raw materials. After performing value added processes, the materials are sent to the downstream firms. A portion of the material flow deemed "unusable" is sent to a disposal center. Environmental issues & operational costs are integrated.

Data

- Fixed number of locations of plants, collection centers and value-added process centers
- Number of locations of virgin material markets and disposal centers
- Set of virgin material markets
- Set of plants
- Set of collection centers
- Set of value added process centers(e.g. recycling or remanufacturing , disassembly)
- Set of disposal centers
- Set of material types
- Transportation cost per mile per unit of material from virgin material market to plant, from plant to collection center , from collection center to plant, from one plant to other, from collection center to value added process, from collection center to disposal
- Environmental cost of transportation per mile per unit of material from virgin material market to plant, from plant to collection center , from collection center to plant, from one plant to other, from collection center to value added process, from collection center to disposal
- Unit processing cost of materials at collection center, at value added process, at disposal center
- Inventory cost per unit per period for material at plant, at collection center, at value added center
- Cost of opening new collection center
- Disposing cost
- Distance between virgin material market and plan, between plant and collection center, between collection center and value added process centre, between value added process and disposal and plant.
- Demand of plant for materials in a period
- Number of planning periods

Objective

- Minimization of production costs (including facility opening, transportation, processing, and inventory costs)
- environmental costs (including energy, water, and air pollution costs, external environmental costs of producing from virgin materials, disposal costs including tipping fees and effects on local communities).

Constraints

- Constraint that ensures that the total supply of materials from each plant must be equal to the output flows
- Material balance constraints (input flows in the current time period plus the available inventory up to this period in one side and demand or output flows plus inventory to be kept in the current period from the other side)
- Demand constraints
- Shipping from open facilities
- Capacity constraints of plants, collection and disposal centers

- Domain constraints
- Non- negativity constraints.

<u>Mathematical Model</u>

Sets
V: Set of virgin material markets
P: Set of plants
I: Set of collection centers
J: Set of value added process centers
D: Set of disposal centers
K: Set of material types
TP: Set of time periods
Indexes:
v: Index for virgin material markets
p, r: Index for plants
i: Index for collection centers
j: Index for value added process centers
d: Index for disposal centers
k: Index for material types
t: Index for time periods
Model Parameters:
CT_{vpk}: Transportation cost per mile per unit of material *k* from virgin material market *v* to plant *p.*

CT_{pik}: Transportation cost per mile per unit of material *k* from plant *p* to collection center *i*. CT_{ipk} :

Transportation cost per mile per unit of material *k* from collection center *i* to plant *p*. CT_{prk}:

Transportation cost per mile per unit of material *k* from plant *p* to plant *r*.

CT_{ijk} : Transportation cost per mile per unit of material *k* from collection center *i* to value-added process center *j*.

CT_{idk} : Transportation cost per mile per unit of material *k* from collection center *i* to disposal center *d*.

CT_{jpk} : Transportation cost per mile per unit of material *k* from value-added process center *j* to plan *p*.

CT_{jdk} : Transportation cost per mile per unit of material *k* from value-added process center *j* to disposal center *d*.

CN_{vpk}: Environmental cost of transportation per mile per unit of material *k* from virgin material market *v* to plant *p*.

CN_{pik} : Environmental cost of transportation per mile per unit of material *k* from plant *p* to collection center *i*.

CN_{ipk} : Environmental cost of transportation per mile per unit of material *k* from collection center

i to plant *p*.

CN_{prk}: Environmental cost of transportation per mile per unit of material *k* from plant *p* to plant *r*.

CN_{ijk} : Environmental cost of transportation per mile per unit of material *k* from collection center

i to VAP center *j.*

CN_{idk} : Environmental cost of transportation per mile per unit of material *k* from collection center

i to disposal center *d.*

CN_{jpk} : Environmental cost of transportation per mile per unit of material *k* from VAP center *j* to plant *p*.

CN_{jdk} : Environmental cost of transportation per mile per unit of material *k* from VAP center *j* to disposal

center *d.*

CP_{ik} : Unit processing cost of material type *k* at collection center *i*.

CP_{jk} : Unit processing cost of material type *k* at value-added process center *j.* CP_{dk} :

Unit processing cost of material type *k* at disposal center *d.*

h_{pk} : Inventory cost per unit per period for material type *k* at plant *p*.

h_{ik} : Inventory cost per unit per period for material type *k* at collection center *i.*

h_{jk} : Inventory cost per unit per period for material type *k* at value-added process center *j*.

π_{pk} : Backorder cost per unit per period for material type *k* at plant *p*.

π_{ik} : Backorder cost per unit per period for material type *k* at collection center *i.*

π_{jk} : Backorder cost per unit per period for material type *k* at value-added process center *j*.

F_i : Cost of opening collection center *i*.

F_j : Cost of opening VAP center *j*.

CD_{dk} : Unit disposal cost (tipping fee) at disposal center *d* for material type *k*.

CV_{vk} : External environmental cost of producing *a* unit of material type *k* by virgin material market *v.*

CE_{jk} : Energy consumption cost at value-added process center *j* for a unit of material type *k.*

CW_{jk}: Environmental cost of disposing a unit of material *k* into water at value-added process cente*j*.

CW_{dk}: Environmental cost of disposing a unit of material *k* into water at disposal center *d.*

CA_{jk}: Environmental cost of disposing a unit of material *k* in air at value-added process center *j.*

CA_{dk}: Environmental cost of disposing a unit of material *k* into air at disposal center *d*.
T_{vp}: The distance between virgin material market *v* and plant *p.* T_{pi} :

The distance between plant *p* and collection center *i.*

T_{pr}: The distance between plant *p* and plant *r.*

T_{ij} : The distance between collection center *i* and value-added process center *j.* T_{id} :

The distance between collection center *i* and disposal center *d*.

T_{jp} : The distance between value-added process center *j* and plant *p.*

T_{jd} : The distance between value-added process center *j* and disposal center *d.* CAP_{pk}

: The capacity of plant *p* for material type *k*.

CAP_{ik} : The capacity of collection center *i* for material type *k.*

CAP_{jk} : The capacity of value-added process center *j* for material type *k.* CAP_{dk}:

The capacity of disposal center *d* for material type *k.*

S_{pk}^{t}: Total supply of material type *k* at plant *p* in time period *t*.

R_{pk}^{t}: Demand of plant *p* for material type *k* in time period *t.*

T: Number of planning periods.

B : A large number.

w_{jk} : Fraction of material *k* disposed to water at value-added process center *j.*

w_{dk} : Fraction of material *k* disposed to water at disposal center *d.*

a_{jk} : Fraction of material *k* disposed to air at value-added process center *j.*

a_{dk} : Fraction of material *k* disposed to air at the disposal center *d.*

α : A multiplier to adjust material type *k* balance in the constraints.

β: A multiplier to adjust material type *k* balance in the constraints.

δ: Minimum fraction of input material to the collection centers that can be disposed.

γ: Maximum fraction of material that enters the collection center that can be used by the plants.

η: Minimum fraction of material in VAP centers that can be disposed.

τ: Maximum fraction of materials in the plants that can directly be used by the other plants.

Decision Variables:

x_{vpk}^{t} : The flow of material type *k* from virginal material market *v* to plant *p* in period *t*.

x_{prk}^{t} : The flow of material type *k* from plant *p* to plant *r* in period *t*.

x_{pik}^{t} : The flow of material type *k* from plant *p* to collection center *i* in period *t*.

x_{ijk}^{t} : The flow of material type *k* from collection center *i* to VAP center *j* in period *t*.

x_{idk}^{t} : The flow of material type *k* from collection center *i* to disposal center *j* in period *t*.

x_{ipk}^{t} : The flow of material type *k* from collection center *i* to plant *p* in period *t*.

x_{jpk}^{t} : The flow of material type *k* from VAP center *j* to plant *p* in period *t*.

x_{jdk}^{t} : The flow of material type *k* from VAP center *j* to disposal center *d* in period *t*.

Y_i : The indicator of opening collection center *i.*

Y_j : The indicator of opening value-added processing center *j.*

INV_{pk}^{t}: Inventory level of material type *k* at plant *p* at the end of period *t*.

INV_{ik}^{t}: Inventory level of material type *k* at collection center *i* at the end of period *t*.

INV_{jk}^{t}: Inventory level of material type *k* at value added process center *j* at the end of period *t*.

BOR_{pk}^{t} : Backorder of material type *k* at plant *p* at the end of period *t.*

BOR_{ik}^{t} : Backorder of material type *k* at collection center *i* at the end of period

BOR_{jk}^{t}: Backorder of material type *k* at value-added process center *j* at the end of period *t.*

The objective function is as follow:

$$MinZ = \lambda Z_1 + (1-\Lambda) Z_2$$

$$\mathbf{Z}_1 = \sum_{i\in I} F_1\, Y_1 + \sum_{i\in J} F_J Y_J \quad (1)$$

$$\sum_{r\in T}[\sum_{p\in P}\sum_{i\in I}\sum_{k\in K} CT_{pik} x^t{}_{pik} TD_{pi} + \sum_{p\in P}\sum_{r\in R}\sum_{k\in K} CT_{prk} x^t{}_{prk} TD_{pr} \quad (2)$$

$$+\sum_{i\in I}\sum_{p\in P}\sum_{k\in K} CT_{ipk} x^t{}_{ipk} TD_{pi} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} CT_{ijk} x^t{}_{ijk} TD_{ij}$$

$$+\sum_{i\in I}\sum_{d\in D}\sum_{k\in K} CT_{jdk} x^t{}_{jdk} TD_{id} + \sum_{v\in V}\sum_{p\in P}\sum_{k\in K} CT_{vpk} x^t{}_{vpk} TD_{vp}$$

$$+\sum_{i\in J}\sum_{p\in P}\sum_{k\in K} CT_{jpk} x^t{}_{jpk} TD_{jp} + \sum_{i\in J}\sum_{d\in D}\sum_{k\in K} CT_{jdk} x^t{}_{jdk} TD_{jd} \quad (3)$$

$$+\sum_{i\in I}\sum_{k\in K} CP_{ik}(\sum_{p\in P} x^t_{pik}) + \sum_{i\in I}\sum_{k\in K} CP_{jk}(\sum_{i\in I} x^t_{ijk}) + \sum_{d\in D}\sum_{k\in K} CP_{dk}(\sum_{i\in I} x^t_{idk} + \sum_{j\in J} x^t_{idk})$$

$$+\sum_{keK}\sum_{jeI}\left(h_{ik} INV^t_{ik} + \pi_{ik} BOR^t_{ik}\right) + \sum_{keK}\sum_{peP}\left(h_{pk} INV^t_{pk} + \pi_{pk} BOR^t_{pk}\right)$$

$$+\sum_{keK}\sum_{jeJ}\left(h_{jk} INV^t_{jk} + \pi_{jk} BOR^t_{jk}\right) \quad (4)$$

$$\mathbf{Z2}_2 = \sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{k\in K} CN_{pik} x^t_{pik} TD_{pi} + \sum_{p\in P}\sum_{r\in R}\sum_{k\in K} CN_{prk} x^t_{prk} TD_{pr} \quad (5)$$

$$[\ +\sum_{i\in I}\sum_{p\in P}\sum_{k\in K} CN_{ipk} x^t_{ipk} TD_{pi} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} CN_{ijk} x^t_{ijk} TD_{ij}$$

$$+\sum_{i\in I}\sum_{d\in D}\sum_{k\in K} CN_{jdk} x^t{}_{jdk} TD_{id} + \sum_{v\in V}\sum_{p\in P}\sum_{k\in K} CN_{vpk} x^t{}_{vpk} TD_{vp}$$

$$+\sum_{i\in J}\sum_{p\in P}\sum_{k\in K} CN_{jpk} x^t{}_{jpk} TD_{jp} + \sum_{i\in J}\sum_{d\in D}\sum_{k\in K} CN_{jdk} x^t{}_{jdk} TD_{jd}$$

$$+\sum_{j\in J}\sum_{k\in K} CE_{jk}\left(\sum_{i\in I} x^t_{ijk}\right) \quad (6)$$

$$+\sum_{i\in J}\sum_{k\in K} CW_{jk}\left(w_{jk}\sum_{i\in I} x^t_{ijk}\right) + \sum_{d\in D}\sum_{k\in K} CW_{dk} w_{dk}\left(\sum_{i\in J} x^t_{idk} + \sum_{j\in J} x^t_{jdk}\right) \quad (7)$$

$$+\sum_{i\in I}\sum_{k\in K} CA_{jk}\left(a_{jk}\sum_{i\in I} x^t_{ijk}\right) + \sum_{d\in D}\sum_{k\in K} CA_{dk} a_{dk}\left(\sum_{i\in I} x^t_{idk} + \sum_{j\in J} x^t_{jdk}\right) \quad (8)$$

$$+\sum_{v\in V}\ \sum_{p\in P}\ \sum_{k\in K} CV_{vk} x^t_{ipk} \tag{9}$$

$$+\sum_{d\in D}\ \sum_{k\in K}\ CD_{dk}\left(\sum_{i\in I}\ x^t_{idk} + \sum_{j\in J}\ x^t_{jdk}\right)\quad] \tag{10}$$

Constraints Functions

$$\sum_{i\in I} x^t_{pik} + \sum_{r\in R} x^t_{prk} = S^t_{pk}, \qquad \forall k\in K, p\in P, t=1...T \tag{1}$$

$$\sum_{p\in P1} x^t_{pik} + INV^{t-1}_{ik} - BOR^{t-1}_{ik} = \sum_{p\in P} x^t_{ipk} + \sum_{j\in J} x^t_{ijk} + \sum_{d\in D} x^t_{idk} + INV^t_{ik} - BOR^t_{ik}\ ,\ \forall k\in K, i\in I, t=1...T \tag{2}$$

$$\sum_{i\in I} x^t_{ijk} + INV^{t-1}_{ik} - BOR^{t-1}_{jk} = \sum_{k\in K} a_{k'} \sum_{p\in P} x^t_{ipk} + \sum_{k'\in K} \beta_{k'} \sum_{d\in D} x^t_{idk} + INV^t_{jk} - BOR^t_{ik}\ ,\ \forall k\in K, j\in J, t=1...T \tag{3}$$

$$\sum_{i\in I} x^t_{ipk} + \sum_{j\in J} x^t_{jpk} + \sum_{r\in R} x^t_{rpk} + \sum_{v\in V} x^t_{vpK} + INV^t_{pk} - BOR^{t-1}_{pk} = R_{pk} + INV^t_{pk} - BOR^t_{pk},\ \forall k\in K, p\in P, t=1...T \tag{4}$$

$$\sum_{p\in P}\ x^t_{ipk} + \sum_{j\in J}\ x^t_{ijk} + \sum_{d\in D}\ x^t_{idk} \le Y_i\ B\ ,\ \forall k\in K, i\in I, t=1...T \tag{5}$$

$$\sum_{p\in P}\ x^t_{ipk} + \sum_{d\in D}\ x^t_{idk} \le Y_i\ B\ \ ,\ \forall k\in K, j\in J, t=1...T \tag{6}$$

$$\sum_{i\in I} x^t_{ipk} + \sum_{j\in J} x^t_{jpk} + \sum_{r\in R} x^t_{rpk} + \sum_{v\in V} x^t_{vpK} + INV^{t-1}_{pk} - BOR^{t-1}_{pk} \le CAP_{pk}\ ,\ \forall k\in K, p\in P, t=1...T \tag{7}$$

$$\sum_{p\in P} x^t_{pik} + INV^{t-1}_{ik} - BOR^{t-1}_{ik} \le CAP_{ik},\ \forall k\in K, i\in I, t=1...T \tag{8}$$

$$\sum_{i\in I} x^t_{ijk} + INV^{t-1}_{ik} - BOR^{t-1}_{jk} \le CAP_{jk},\ \forall k\in K, j\in J, t=1...T \tag{9}$$

$$\sum_{i\in I} x^t_{idk} + \sum_{j\in J} x^t_{jdk} \le CAP_{dk} \quad ,\forall k\in K, d\in D, t=1...T \tag{10}$$

$$\sum_{d\in D} x^t_{idk} \ge \delta * \sum_{p} x^t_{pik}\ ,\ \forall i\in I, k\in K, t=1...T \tag{11}$$

$$\sum_{p\in P} x^t_{ipk} \le \gamma * \sum_{p\in P} x^t_{pik}\ , \forall i\in I, k\in K, t=1...T \tag{12}$$

$$\sum_{d\in D} x^t_{jdk} \ge \eta * \sum_{i\in I} x^t_{ijk}\ , \forall j\in J, k\in K, t=1...T \tag{13}$$

$$\sum_{r\in P} x^t_{prk} \le \tau * \left(\sum_{r\in P} x^t_{prk} + \sum_{i\in I} x^t_{ipk} + \sum_{j\in J} x^t_{jpk}\right)\ ,\ \forall p\in P, k\in K, t=1...T \tag{14}$$

$Y_{i,}Y_j \in \{0.1\}$ $\forall i \in I, j \in J,$ (15)

$$x^{t}_{pik^3}\, x^{t}_{ipk^3}\, x^{t}_{ijk^3}\, x^{t}_{idk^3}\, x^{t}_{jpk^3}\, x^{t}_{prk^3}\, x^{t}_{jdk^3}\, x^{t}_{vpk^3} \geq 0$$

$\forall$ p $\in$ P, $\forall$ i $\in$ I, $\forall$ j $\in$ J, , $\forall$ d $\in$ D , $\forall$ v $\in$ V, $\forall$ k $\in$ K, t= 1...T (16)

The objective function minimizes two categories of costs: production costs (*Z1*) and environmental costs (*Z2*). A weight, λ and 1-λ, is assigned to each part of the objective function, to differentiate the degree of sensitivity.

In the production costs (Z1) functions include:
Equation (1) facility opening
Equation (2) transportation
Equation (3) processing
Equation (4) inventory/backorder costs
In the environmental costs (Z2) functions include
Equation (5) environmental transportation
Equation (6) energy
Equation (7) water
Equation (8) air pollution cost
Equation (9) external environmental costs of producing from virgin materials (External environmental cost is the extra money that a firm is charged when it refuses to substitute the virgin material market by an acceptable recycled material)
Equation (10) disposal costs including tipping fees

The constraint functions include:

Equation (1) indicates that the total supply of materials from each plant must be equal to the output flows.
Equation (2) to (4) guarantee the balance of material (input flows in the current time period plus the available inventory up to this period in one side and demand or output flows plus inventory to be kept in the current period from the other side) in collection centres, VAP centres, and plants accordingly. Maintaining balance of material in VAP centres is more complex than the other facilities due to the possible chemical reactions. Therefore, by introducing multipliers (α ,β),
Equation (3) can be modified based on different scenarios. Equation (5) and (6) ensure that materials flow through the active facilities.
Equations (7) to (10) are capacity constraints for plants, collection, VAP, and disposal centres.
The next four sets of constraints are added to the model in order to provide flexibility in different real word scenarios.
Equations (11) and (13) assign the least disposal rate for each collection center and VAP center accordingly, based on historical data.
Equations (12) and (14) limits the amount of reused material provided by a collection center and other plants accordingly.
Equations (15) and (16) identify the domain of the decision variables.

2.2.6. Study 6 [14]

Description of the study

The purpose of this problem is to present a heuristic solution methodology for the reverse distribution induced by various forms of reuse of products and materials that can be adapted for end of life, commercial returns and other reverse functions as recycling, remanufacturing, reuses and refurbishing (4R).

Data

- quantity of product that have been recalled , are to be recycled, are to be disposed or hazardous products
- Number of store, retail outlet or customer collection station.
- Number of collection site
- Number of refurbishing site, recycling plant or original manufacturing site
- Total variable cost or transportation a unit recall product from origination site through collection site into of refurbishing site
- Cost of opening a collection site, refurbishing site

Objective

- Minimization the sum of costs to transfer products from origination sites through collection sites to the destination facilities
- Minimization of the fixed cost of opening the collection and destination sites.

Constraints

- constraint set that ensures that all the supply of products available at the origination sites are transported to destination facilities either directly or via collection sites in the network
- Capacity constraint of the collection site and refurbishing site
- Binary constraint
- Constraint that ensure that only an open site can received returns
- Constraint that ensure the minimum number of collection sites remain open and the maximum number that can be opened
- Constraints that limit the minimum number of destination sites remain open and maximum number that can be opened
- Binary restriction constraints

Mathematical Model

Notations

Types of products:

- have been recalled,
- are to be recycled,

- are to be disposed, or
- are hazardous.

- I—{i/i is an origination site}. This is a store, a retail outlet, or a customer collection station. All products are received from customers at origination sites and are passed to collection sites
- J—{j/j is a collection site}. Collection sites are synonymous with intermediate transhipment sites. A collection site receives the collected products from the origination sites. The last collection site denotes a direct shipment from the origination site to the refurbishing facility site at a premium variable cost, thus preventing infeasibility. Note that no product originates at any collection site.
- K—{k/k is a refurbishing facility site}. This site is:
- a refurbishing site,
- a recycling plant,
- a decontamination plant, or the original manufacturing site. The last refurbishing facility site is a dummy site with infinite cost and infinite capacity, and prevents infeasibility in the solution procedure due to insufficient capacity.
- $c_{ijk,}$ Total variable cost of transporting a single unit of recalled product from origination site i through collection site j and onto refurbishing site k. This include the per unit costs for: Processing the recalled product at the origination site.

- The inbound and outbound transportation costs for sending the recalled products from the origination sites to refurbishing sites via the collection sites.
- F_i Cost of opening a collection site j.
- G_k Cost of opening a refurbishing site k.
- a_i Number of hazardous products residing at origination site i.
- B_j Maximum capacity of collection site j.
- D_k Maximum capacity of refurbishing facility k.
- $P_{\min}$ Minimum number of collection sites to open and operate.
- $P_{\max}$ Maximum number of collection sites to open and operate.
- $Q_{\min}$ Minimum number of refurbishing facilities to operate.
- $Q_{\max}$ Maximum number of refurbishing facilities to operate. Note that B0 is set to some arbitrarily high value (999,999).

Variables

X_{ijk} Fraction of units at origination site i that is transported through collection site j and onto refurbishing site k. Use of the index j ̕O 0 indicates that the fractional demand is assigned directly from i to k. The index value zero (0) is not used for subscripts i and k.

$$P_j = \begin{cases} 1 \text{ if collection site j is open} \\ 0\text{otherwise} \end{cases}$$

$$Q_k = \begin{cases} 1 \text{ if refurbishing facility k is open} \\ 0 otherwise \end{cases}$$

$$MinZ = \sum_{j} \sum_{j} \sum_{k} c_{ijk} a_i X_{ijk} + \sum_{j} F_j P_j + \sum_{k} G_k Q_k$$

Subject to

$$\sum_{j} \sum_{k} X_{ijk} = 1 \text{ For all i} \quad (1)$$

$$\sum_{i} \sum_{k} a_i X_{ijk} \leq B_j \text{ For all j} \quad (2)$$

$$\sum_{i} \sum_{j} a_i X_{ijk} \leq D_k \text{ For all k} \quad (3)$$

$$X_{ijk} \leq P_j \text{ For all I,j,k} \quad (4)$$

$$X_{ijk} \leq Q_k \text{ For all I,j,k} \quad (5)$$

$$P_{\min} \leq \sum_{j} P_j \leq P_{\max} \quad (6)$$

$j \neq$ Direct shipment

$$Q_{\min} \leq \sum_{k} Q_k \leq Q_{\max} \quad (7)$$

$k \neq$ Infeasible site

$$0 \leq X_{ijk} \leq 1 \quad (8)$$

$$P_j \in \{1,0\} \quad (9)$$

$$Q_k \in \{1,0\} \quad (10)$$

Equation (1) presents the constraint set that ensures that all the supply of products available at the origination sites are transported to destination facilities either directly or via collection sites in the network

Equation (2) presents the constraint set that limits the units sent through collection site j to the capacity of site j,

Equation (3) presents the constraint set that limits the units ending up at destination site k to the capacity of site k.

Equation (4) presents the constraint set that prohibits units from being routed through collection site j unless the site is opened, and

Equation (5) presents the constraint set that prohibits units from ending up at destination site k unless this site is opened.

Equation (6) presents the constraint that ensures that a minimum number of collection sites remain open and the maximum number of collection sites that can be opened, and

Equation (7) presents the constraint that limits the minimum number of destination sites remain open and the maximum number of destination sites that can be opened.

Equation (8) presents the constraint that requires the decision variable X to be continuous between zero and one,

Equations (9) and (10) present the constraint sets that enforce the binary restriction on the P and Q decision variables.

Conclusion
In this chapter we showed how formal modelling can be applied effectively in real-life RL and WM problems. Despite the complexity of modern demands and the variety of conditions, the results of the ten studies we presented here demonstrate that gains are to be made if the facility location problem is studied systematically. The gains can be translated in terms of financial cost reduction –as is the case in all single objective models ([5],[6],[7],[8]) but also in other terms such as greenhouse gas emissions, solid waste volume, material recovery [9],[10],[11],[12],[13],[14].

3. Vehicle Routing Models

In this chapter we present vehicle routing models in waste collection and reverse logistics that address real life problems in many industries. Real-life vehicle routing problems encounter a number of complexities that have been expressed through the nature of constraints. The constraints include: time window restrictions, heterogeneous vehicle fleet with different travel times, travel costs and capacity, multi-dimensional capacity constraints, order/vehicle compatibility constraints, orders with multiple pickup, delivery and service locations, different start and end locations for vehicles, route restrictions associated to orders and vehicles, and drivers' working hours. We have grouped and categorized the studies according to the time windows constraints, the traffic regulations constraints and the compatibility constraints.

3.1. VRP with time windows constraints (VRPTW)

3.1.1. Study 1 [15]

Description of the Study

This study presents a simplified version of the vehicle routing problem with time windows for the Waste Management Inc., established in Houston, Texas, one of the leading providers of WM comprehensive services in North America. Although, the objective of the simplified version of the problem is to minimize the travel time, Waste management Inc in Texas considers as objectives also the visual attractiveness, the route compactness, the balance workload among vehicles and the minimization of the vehicle numbers, in order to reduce its operation cost and to provide better customer services and determine appropriate prices for its services.

Data

- number of vehicles
- number of landfills
- number of depots
- number of routes
- number of regular stops

Objective

- Minimization of travel time

Constraints

- Each regular stop should be served by exactly one vehicle.
- Each route starts from the depot and returns to a depot
- Vehicle capacity constraint.
- Route capacity constraint
- The number of trips from the regular stops to disposal facilities for a vehicle is equal to the number of actual disposal trips
- The collected garbage volume is reset to zero once the vehicle has visited a landfill
- The number of trips from the disposal facilities to the regular stops should be one less than the number of disposal trips
- The last trip from a disposal facility in that route should be to the depot.
- Lunch break for each route (hour)
- Travel time before and after lunch time
- When a vehicle arrives at a stop, it must leave the stop.
- Nonnegative constraints

Mathematical Model

Notations

K Available vehicle set

N_k Actual number of disposal trips for vehicle k

C vehicle capacity

$$x_{i,j,k} = \begin{cases} 1 \text{ if arc } (\text{i, j}) \text{ is used by vehicle k} \\ 0 \text{ otherwise} \end{cases}$$ Where (i, j)$\in A$, $k \in K$

$w_{i,k}$ Time variable where $i \in V$, $k \in K$ be the start time at node I when it is serviced by vehicle k.

$D_{i,k}$ Where $i \in V$, $k \in K$ be the cumulative demand at node I for vehicle k.

A route represents the travel path of a vehicle, and a subroute represents a sub-travel path that starts from the depot or a disposal facility and ends at a disposal location or the depot.

$$\min \sum_{k \in K} \sum_{(i,j) \in A} t_{i,j} x_{i,j,k}$$

Subject to

$$\sum_{k \in K} \sum_{(i,j) \in A} x_{i,j,k} = 1 \quad \forall \mathbf{i} \in 1,2,\ldots,N, \tag{1}$$

$$\sum_{j=1}^{N} x_{0,j,k} = 1 \quad \forall \mathbf{k} \in K \tag{2}$$

$$D_{i,k} \leq C \quad \forall \mathbf{k} \in K, \ \mathbf{i} \in 1,2,\ldots,N,N+M+1, \tag{3}$$

$D_{m,k}$ = 0 $\forall$ **k** $\in$ K, **m** $\in$ 0,N+1,N+2,…,N+M,

$D_{i,k} + d_j - D_{j,k} \leq (1 - x_{i,j,k})$ Big M, $\forall$ **k** $\in$ K, (4)

i $\in$ 1,2,…,N,N+M+1, **j** $\in$ 0,1,2,…,N+M+1, (5)

$$\sum_{J=1}^{N} d_j \sum_{i=1}^{N+M+1} x_{i,j,k} \leq C * N_k \quad \forall \mathbf{k} \in K, \qquad (6)$$

$$\sum_{i \in 1,..,N,N+M+1,} \sum_{m=N+1}^{N+M} x_{i,m,k} = N_k \quad \forall \mathbf{k} \in K, \qquad (7)$$

$$\sum_{m=N+1}^{N+M} \sum_{j \in 1,..,N,N+M+1,} x_{m,j,k} = N_k - 1 \quad \forall \mathbf{k} \in K, \qquad (8)$$

$$\sum_{m=N+1}^{N+M} x_{m,0,k} = 1 \quad \forall \mathbf{k} \in K \qquad (9)$$

$$\sum_{i=1}^{N+M} x_{i,N+M+1,k} = 1 \quad \forall \mathbf{k} \in K \qquad (10)$$

$$\sum_{j=0}^{N+M} x_{N+M+1,j,k} = 1 \quad \forall \mathbf{k} \in K \qquad (11)$$

$$\sum_{i=0}^{N+M+1} x_{i,j,k} = \sum_{i=0}^{N+M+1} x_{j,i,k} \quad \forall \mathbf{k} \in K, \ \mathbf{j} \in 0,1,\ldots,N+M+1, \qquad (12)$$

$w_{i,k} + s_i + t_{i,j} - w_{j,k} \leq (1 - x_{i,j,k})$ Big M $\forall$ **k** $\in$ K, (i,j) $\in$ A (13)

$w_{i,k} + s_i + s_{N+M+1} + t_{i,j} - w_{j,k} \leq (2 - x_{i,N+M+1,k} - x_{N+M+1,j,k})$ Big M

$\forall$ **k** $\in$ K, (i,j) $\in$ A (14)

$$a_i \sum_{J=1}^{N+M+1} x_{i,j,k} \leq w_{i,k} \leq b_i \sum_{J=1}^{N+M+1} x_{i,j,k} \quad \forall \mathbf{k} \in K, \ \mathbf{i} \in 0,1,\ldots,N+M+1, \qquad (15)$$

$E \leq w_{i,k} \leq L$ $\forall$ **k** $\in$ K, **i** $\in$ 0,1,…,N+M+1, (16)

$x_{i,j,k} \in \{0,1\}$ $\forall$ **k** $\in$ K, (i,j) $\in$ A (17)

$N_k \geq$ 0, Integer $\forall$ **k** $\in$ K, (18)

$D_{i,k} \geq$ 0 $\forall$ **k** $\in$ K, **i** $\in$ 0,1,…,N+M+1, (19)

Equation (1) presents the constraints that impose the rule that each regular stop should be served by exactly one vehicle.

Equation (2) presents the constraints that ensure that each route starts from the depot.

Equation (3) presents the constraints that keep the amount of garbage collected at each stop within the vehicle capacity constraint.

Equation (4) presents the constraints that ensure that the collected garbage volume is reset to zero once the vehicle has visited a landfill, as well as in the particular instance when the vehicle leaves the depot.

Equation (5) presents the constraints that are in place to make sure that the collected garbage reflects the correct incremental volume of a particular stop when a truck visits that stop.

Equation (6) presents the constraints that the actual number of disposal trips is calculated.

Equation (7) presents the constraints that impose the condition that the number of trips from the regular stops to disposal facilities for a vehicle is equal to the number of actual disposal trips.

Equation (8) presents the constraints that impose the condition that the number of trips from the disposal facilities to the regular stops should be one less than the number of disposal trips.

Equation (9) presents the constraints that ensure that the last trip from a disposal facility in that route should be to the depot.
Equation (10) and (11) present the constraints that are introduced to add the lunch break for each route
Equation (12) presents the constraints that ensure that if the vehicle arrives at a stop, it must leave the stop.
Equations (13) through (16) present the constraints that make sure that the time constraints on both the route and vehicle are satisfied.
Equations (14) present the constraints that make sure that the travel time between before-lunch-break stop and after-lunch-break stop is included.
Equations (17) through (19) present the constraints that impose binary conditions, nonnegative integer constraints, and nonnegative conditions on the variable set.

3.1.2. Study 2 [16]

Description of the study

This study considers a truck scheduling problem in the context of solid waste collection in the City of Porto Alegre, Brazil. The problem consists of designing “good” daily truck schedules over a set of previously defined collection trips, on which the trucks collect solid waste in fixed routes and empty loads in one of several operational recycling facilities in the system. The main objective is to minimize the total operating and fixed truck costs, modelling the problem with and without considering balanced unloading.

Data

- Number of trucks (24)
- Number of recycling facilities(8)
- Amount of waste(more than 60 tons /day)
- Maximum amount of solid waste collected in a trip is 1000 kg
- Days of collection (Monday to Friday)
- Workers (one driver and three collectors)
- Travel distance (depots to collection and back)
- Number of collection trips

Objective

- Minimization of the total operating and fixed trucks cost

Constraints

- Trip constraints (each trip must serve only once)
- Sufficiency trucks constraints
- Conservative flow conditions constraints
- Time windows constraint (between 1 and 3 in the morning)
- Every trip has to be assigned to exactly one vehicle
- Each vehicle performs a feasible sequence of trips
- Only one depot exists in the system.

Mathematical models

Modelling the problem without considering balanced unloading

$$\min \sum_{(i,j)\in Z} c_{ij} y_{ij} \quad (2a)$$

s.t.

$$\sum_{j:(i,j)\in Z} y_{ij} - \sum_{j:(j,i)\in Z} y_{ji} = 1, \ \forall \ \mathbf{i} \in N_1, \quad (2b)$$

$$\sum_{j:(i,j)\in Z} y_{ij} - \sum_{j:(j,i)\in Z} y_{ji} = -1, \ \forall \ \mathbf{i} \in N_2, \quad (2c)$$

$$\sum_{j:(i,j)\in Z} y_{ij} - \sum_{j:(j,i)\in Z} y_{ji} = 0, \ \forall \ \mathbf{i} \in R, \quad (2d)$$

$$\sum_{j:(i,j)\in Z} y_{ij} - \sum_{j:(j,i)\in Z} y_{ji} = |N|, \ \mathbf{i} = s, \quad (2e)$$

$$\sum_{j:(i,j)\in Z} y_{ij} - \sum_{j:(j,i)\in Z} y_{ji} = -|N|, \ \mathbf{i} = t, \quad (2f)$$

$$0 \le y_{ij} \le 1, \ \forall (\mathbf{i}, \mathbf{j}) \in Z \setminus (s,t) \setminus D \quad (2g)$$

$$0 \le y_{st}, \quad (2h)$$

$$y_{ij} = 0, \ \forall (\mathbf{i}, \mathbf{j}) \in D, \quad (2i)$$

Modelling the problem considering balanced unloading

$$\min \sum_{(i,j)\in Z} c_{ij} y_{ij} + \sum_{k\in K} P x_k \quad (3a)$$

s.t.

$$\sum_{j:(i,j)\in Z} y_{ij} = 1, \ \forall \ \mathbf{i} \in N, \quad (3b)$$

$$\sum_{j:(i,j)\in Z} y_{ij} = 1, \ \forall \ \mathbf{j} \in N, \quad (3c)$$

$$\sum_{j:(i,j)\in Z} y_{ij} = \sum_{k:(j,k)\in Z} y_{jk}, \ \forall \ \mathbf{j} \in R, \quad (3d)$$

$$\sum_{j\in F^k} \sum_{i:(i,j)\in Z} y_{ij} - u_k \le M x_k, \ \forall \ \mathbf{k} \in K, \quad (3e)$$

Equation (2a) presents the objective function that minimizes the total operating and fixed vehicle costs. Constraints

Equations (2b), (2c) and (2g) present the constraints that assure that each trip is served only once.

Equations (2e), (2f) and (2f) present the constraints that guarantee the sufficiency of vehicles.

Equation (2d) presents the constraints that are conservative flow conditions.

Equation (3 a) presents the objective function minimizes the combination of operating and penalty costs, incurred due to the overloading of recycling facilities.
Equations (3b) and (3c) present the constraints that guarantee that only one truck serves each collection trip.
Equation (3d) presents the constraint that guarantee a conservative flow of trucks arriving at each trip.
Equation (3e) presents the constraint that assures that x_k = 1 if the number of trips assigned into facility *k* exceeds a given limit U_k .

3.1.3. Study 3 [17]

Description of the study

This study addresses an application of operations research technique in waste collection activities in Hanoi, Vietnam. The company is in charge of collecting household and street solid waste in the inner city, and is funded by the city based on the total waste volume collected. The company organization includes two fleets of motorized vehicles and five divisions of manually pushed handcarts. Each division covers each of five districts, and each fleet has a working area covering two or three districts. Since the fleets operate independently and separately but in the same manner, the study is concentrated on the operations of only one fleet. The solid waste collection consists of three stages: manual gathering, picking by vehicles, and transporting and dumping at the landfill.

Data

- Set of depot and landfill
- Set of vehicles
- Set of gather points
- traveling time between points
- the minimum time allowed between two consecutive pickups
- the maximum working time of a vehicle in a shift
- the fixed operating cost per vehicle
- the variable operating cost per travelling time unit
- waste volume (number of handcarts)
- distance between
- number of routes by vehicle

Objective

- minimization of operating costs(include vehicle deployment traveling cost)

Constraints

- time windows constraints
- constraints that ensure that each point in each time window be serviced by at most one vehicle
- Constraints that assure that each pair of connected points be visited by the same vehicle.

- Demand constraints : when a point doesn't have positing demand needs no service
- vehicle visiting the landfill must start from the depot,
- the number of routes made by each vehicle must not exceed the maximum
- number set by the company
- Vehicle capacity constraint : when the total load reaches the truck capacity, the truck must go to the landfill to empty itself
- Binary and nonnegativity constraints

<u>Mathematical Model</u>

Sets

N_o {i\i =1,2) the sets of the depot (i =1) and landfill (i = 2)

N^* The set of gather points numbered consecutively

N $N_o \cup N^*$ the sets of all notes

K the sets of vehicles

P the set of time windows at a gather point; in case of depot or landfill , it represents the set of times a vehicle leaves the depot or visits the landfill

Decision variables

$$x_{ipjq} = \begin{cases} 1 \text{ if point i in time window p is connected to point j in time window q} \\ 0 \text{ otherwise, for all i,j} \in \text{N; p,q} \in \text{P} \end{cases}$$

$$y_{ipk} = \begin{cases} 1 \text{ if point i in time window p is serviced by vehicle k} \\ 0 \text{ otherwise, for all i} \in \text{N; p} \in \text{P; k} \in \text{K} \end{cases}$$

w_{ip} Total load when a vehicle leaves point i in tome window p, for all $i \in N; p \in P$

a_{ip} Arrival time of a vehicle at point i in time window p, for all $i \in N; p \in P$

Parameters

$L_{ip}(U_{ip})^-$ Lower (upper) bound of time window p at point i, for all $i \in N^*; p \in P$

D_{ip} Waste volume at point i in time window p, for all $i \in N^*; p \in P$

S_{ip} Loading/unloading time at point i in time window p, for all $i \in N \setminus \{1\}; p \in P$

T_{ij} Traveling time between points i and j, for all i, j $\in$ N

T_O The minimum time allowed between two consecutive

T_m The maximum working time of a vehicle in a shift

V_k Total capacity of vehicle k, for all $k \in K$

V_m a big number, twice the maximum capacity of vehicles

B the maximum number of routes a vehicle can make in a shift

F the fixed operating cost per vehicle

C the variable operating cost per travelling time unit

Notes

$x_{ipIq} = 0 \forall i \in N^*$ and p,q $\in$ P, since a vehicle must return to the depot from landfill only; there is no waste brought from the landfill only;

$x_{Ip2q} = 0 \forall p, q \in P$ since an empty vehicle from the depot should not go directly to the landfill

$x_{ipjq} = 0 \forall i, j \in N^*$, p>q, p,q $\in$ P since time windows are numbered chronologically; for two consecutive visits, the pth time window should be less than or equal to the qth one.

$x_{ipIq} = x_{2q2q} = 0 \forall \in P$ and $x_{ipip} = 0 \forall i \in N^*$, p $\in$ P, since a node must not be revisited at the same time window. However, a vehicle can visit a node for two consecutive time windows, but it must wait for handcart collection taking T0 time to start loading.

$$\min F \sum_{k \in K} \sum_{q \in P} y_{1qk} + \sum_{q \in P} a_{1q} \tag{1}$$

Subject to

$$\sum_{i \in N} \sum_{p \in P} x_{ipjq} = \sum_{k \in K} y_{jqk} \qquad \forall j \in N; q \in P \tag{2}$$

$$\sum_{j \in N} \sum_{q \in P} x_{ipjq} = \sum_{k \in K} y_{jqk} \qquad \forall j \in N; q \in P \tag{3}$$

$$\sum_{k \in K} y_{jqk} \le 1 \qquad \forall j \in N; p \in P \tag{4}$$

$$y_{ipk} - y_{jpk} \leq 1 - x_{ipjq} \qquad \forall i, j \in N; p, q \in P, k \in K \qquad (5)$$

$$y_{ipk} - y_{jpk} \leq x_{ipjq} - 1 \qquad \forall i, j \in N; p, q \in P, k \in K \qquad (6)$$

$$D_{ip} \sum_{k \in K} y_{ipk} \geq D_{ip} \qquad \forall i \in N^*; p \in P \qquad (7)$$

$$\sum_{k \in K} y_{ipk} \leq D_{ip} \qquad \forall i \in N^*; p \in P \qquad (8)$$

$$\sum_{p \in P} y_{1pk} \leq 1 \qquad \forall k \in K \qquad (9)$$

$$\sum_{p \in P} y_{2pk} \leq B \sum_{p \in P} y_{1pk} \qquad \forall k \in K \qquad (10)$$

$$w_{ip} - V_m (1 - y_{ipk}) \leq V_k \qquad \forall i \in N^*; p \in P, k \in K \qquad (11)$$

$$w_{jq} - D_{jq} - w_{iq} \geq V_m (1 - x_{ipjq}) \qquad \forall i \in N \ ; j \in N^*; p, q \in P \qquad (12)$$

$$w_{jq} - D_{jq} - w_{iq} \leq V_m (1 - x_{ipjq}) \qquad \forall i \in N \ ; j \in N^*; p, q \in P \qquad (13)$$

$$w_{iq} = 0 \qquad \forall i \in N_o; p \in P \qquad (14)$$

$$a_{jq} \geq T_{1j} - T_m (1 - x_{1pjq}) \qquad \forall j \in N^*; p, q \in P \qquad (15)$$

$$a_{jq} \geq a_{iq} + S_{ip} + T_{ij} - T_m (1 - x_{ipjq}) \qquad \forall i \in N \ \setminus \{1\}; j \in N; p, q \in P \qquad (16)$$

$$a_{ip} \leq U_{ip} \qquad \forall i \in N^*; p \in P \qquad (17)$$

$$a_{ip} \leq L_{ip} \qquad \forall i \in N^*; p \in P \qquad (18)$$

$$a_{ip} - a_{i(p-1)} \geq T_o \qquad \forall i \in N^*; p > 1; p \in P \qquad (19)$$

$$a_{1p} \leq T_m \qquad \forall q \in P \qquad (20)$$

$$x_{ipjq}, \ y_{ipk} \ \text{Binary} \qquad \forall i, j \in N \ ; p, q \in P; k \in K \qquad (21)$$

Or other variables are non-negative (22)

Equation (1) presents the objective function that minimizes the total operating costs which include vehicle deployment traveling cost.
Equations (2)- (3)- (4) present the constraints that require that each point in each time window be serviced (either entered or exited) by at most one vehicle.
Equations (5) and (6) present the constraints that assure that each pair of connected points be visited by the same vehicle.
Equations (7) and (8) present the constraints that require each point having positive demand (waste) must be visited, and if there is no demand, the point needs no service.
Equation (9) presents the constraint that requires each vehicle to be utilized for each tour only, while equation (10) presents the constraint that indicates that a vehicle visiting the landfill must start from the depot, and that the number of routes made by each vehicle must not exceed the maximum number set by the company.
Equations (11) (13) (14) present the waste constraints that relate waste load variables w_{ip}, such that when the total load reaches the truck capacity, the truck must go to the landfill to empty itself. Constraint (11) assures that the present waste load on a leaving vehicle should not exceed its capacity at any time; whilst (12) and (13) calculate the cumulative waste volume on a vehicle leaving a node (equal its volume when entering the node plus the demand at the node). Constraint (14) initializes the zero waste volume of each vehicle leaving the depot and landfill
Equations (15) and (16) present the constraints that calculate the arrival times a_{ip} that relates the arrival times at two points that are directly connected to each other.
Equations (17) and (18) present the tome constraints that check for upper and lower bounds of a time window, respectively.
Equation (19) presents the constraint that enforces the minimum inter-arrival time between two consecutive visits at a point,.
Equation (20) presents the constraints that requires that for each vehicle the total working time, equivalently the returning time to the depot, be less than the maximum time allowed. Equations (21) and (22) present the constraints that are necessary binary and nonnegativity constraints

3.2. VRP with traffic regulations constraints

3.2.1. Study 1 [18]

Description of Study

This case study presents the solution of an urban waste collection problem in the municipality of Sant Boi de Llobregat, within the metropolitan area of Barcelona (Spain). This is a typical mixed Capacitated Arc Routing Problem (CARP) which takes account of the traffic regulations since some streets of the city can be traversed in both direction whereas other have only one direction, or some turns are forbidden while others are allowed. The objective of this study is the minimization of the total operating costs (such associated with fuel consumption and the working time of personnel). Additional benefits derived e.g. noise contamination has been reduced.

Data

- set of routs
- number of vehicles
- numbers of single way streets
- numbers of two way streets
- one single depot
- quantity of waste that must be collected

Objective

- Minimization of the total operation costs.

Constraints

- Each route must start and end at the depot,
- Each required link must serviced by one route,
- Vehicle capacity
- Traffic regulation constraint since some streets of the city can be traversed in both direction whereas other have only one direction, or some turns are forbidden whereas others are allowed
- Binary constraints

Mathematical Model

Decision variables

$$x_{pqk} = \begin{cases} 1 \text{ if arc } (\text{p},\text{q}) \in A \text{ belongs to route k} \\ 0 \text{ otherwise} \end{cases}$$

Notations

$\forall$ S$\subset$V,*A(S)* = {*(p, q)* $\in$*A* : *p, q* $\in$ *S*}; $\forall$ *p* $\in$ *V,* δ^+ *(p)* = {*q* $\in$ *V* :*(p, q)* $\in$*A*}, δ^- *(p)* = {*q* $\in$ *V* : *(q, p)* $\in$*A*}.

Note that if *Cp* = *Cq* , then *(p, q)* $\notin$ *A* and *(q, p) /* $\notin$ *A*. Thus, for *p*$\in$*Tt* , for some *t* = 1*, . . . , r*, then δ^+ *(p)* $\notin$*Tt* and δ^- *(p)* $\notin$*Tt* .

(GVRP) min $\sum_{k \in K} \sum_{(p,q) \in A} c_{pq} x_{pqk}$ (1)

s.t.

$$\sum_{k \in K} \sum_{p \in T_1} \sum_{q \in \delta^-(p)} x_{pqk} = 1, \ \forall \ \mathbf{t} = 1,\ldots,r, \quad (2)$$

$$\sum_{k \in K} \sum_{p \in T_1} \sum_{q \in \delta^+(p)} x_{pqk} = 1, \ \forall \ \mathbf{t} = 1,\ldots,r, \quad (3)$$

$$\sum_{k \in K} \sum_{q \in \delta^-(0)} x_{q0k} = |K|, \quad (4)$$

$$\sum_{k \in K} \sum_{q \in \delta^+(0)} x_{q0k} = |K|, \quad (5)$$

$$\sum_{q \in \delta^-(p)} x_{qpk} = \sum_{q \in \delta^+(p)} x_{qpk}, \ \mathbf{t} = 1,\ldots,r, \ \ p \in T_1, \ \forall \ \mathbf{k} \in K, \quad (6)$$

$$\sum_{p \in V} d_p \left[\sum_{q \in \delta^+(p)} x_{pqt} \right] \leq D, \ \forall \ \mathbf{k} \in K, \quad (7)$$

$$\sum_{(p,q) \in A(S)} x_{pqk} \leq |S| - |K|, \ \forall \ \mathbf{s} \in V, \mathbf{k} \in K, \quad (8)$$

$$x_{pqk} \in \{0,1\}, \ \forall \ (p,q) \in A, \ \mathbf{k} \in K. \quad (9)$$

$$u_{pk} - u_{qk} + D \ x_{qpk} \leq D - d_q, \ \forall \ \mathbf{k} \in K, \ \forall \ (p,q) \in A, p \neq 0 \text{ s.t. } d_p + d_q \leq D \quad (10)$$

$$d_p \leq u_{pk} \leq D, \ \forall \ p \neq 0, \mathbf{k} \in K, \quad (11)$$

Equation (1) presents the objective function of the minimization of the total cost.
Equation (2) and (3) present the constraints that ensure that for each cluster of vertices *Tt* , exactly one route "enters" the cluster and exactly one route "leaves" the cluster
Equation (4) and (5) present the constraints that ensure that routes start and end at the depot.
Equation (6) present the constraint that are needed to guarantee that, the route that "enters" the cluster is the same than the route that "leaves" the cluster, and also, that when a route visits a cluster, the same vertex is visited when entering and when leaving the cluster.
Equation (7) present the capacity constraint that ensure that the capacities of the vehicles are not violated,
Equation (8) presents the subtour elimination constraint (The model has |*A*| × |*K*| binary variables and a number of constraints that is exponential in |*V* | due to constraints).
Equations (10) and (11) present the constraints that can be reduced if the subtour elimination constraints expressed according Desrochers and Laporte theory.

3.2.2. Study 2 [19]

Description of the study

This case is focused in the solid waste collection problem that is faced by the municipalities of towns with about 100.000 inhabitants. In this case, the primary objective is to design vehicle collection routes that minimize the total travel cost. This work is concentrated on tactical planning, where a vehicle fleet and the service demand are given and the objective is to design the vehicle trips in order to minimize operational costs taking into consideration service and traffic regulations constraints. The problem is modeled as a Directed Capacitated Arc Routing Problem (DCARP) on a directed graph and it is solved accordingly.

Data

- One or more depots, where vehicles are stationed
- Number of landfills and disposal plants
- Waste types:
  1. Undifferentiated
  2. Plastics
  3. Glass
  4. Paper
  5. Biologic or organic
- Number and types of waste bins
- Number of vehicles

Objectives

- Minimization the total travel cost
- Minimization the total travel distance

Constraints

- Each vehicle can load only bins of given types
- Each vehicle has a capacity specified as a maximum number of bins (the capacity of each vehicle is not exceeded).
- Each bin is set on one specific side of the street
- Bins can be collected only within given time windows
- Bin compatibility constraint
- Continuity constraints
- Integrality constraints
- A vehicle driving along a street can usually collect all bins of a compatible type on the right side of that street. In case of one-way streets, the vehicle can instead collect both bins on the right and on the left side of the street, if adequately equipped.

Model

Parameters

- A set of arcs
- A set of vertices (nodes), where node 0 corresponds to the depot
- G=(V, A) the weighted digraph of interest
- $R \in A$ set of mandatory arcs(clients to service, arcs with bins)

- $V_r \in V$ set of vertices containing the end points of the arcs in and the depot
- K={1,…..,M} Set of available vehicles, each of capacity
- c_{ij} cost of arc (I,j) $\in A$
- q_{ij} request associated with arc (I j) $\in A$

$$q_{ij} \begin{cases} >0 \text{ if } (\text{i, j}) \in \text{R} \\ = 0 \text{ if } (\text{i, j}) \in A \setminus R \end{cases}$$

Assuming G to be connected, it is possible to transform it into a complete graph g=(Vr,A) where each arc (i,j) has a cost c_{ij} defined as follows:

$$c_{ij} \begin{cases} =c_{ij} \text{ if } (\text{i, j}) \in \text{R} \\ = dist(i,j)\ otherwise \end{cases}$$

Dist(ij) being the cost of the shortest in G from I to j. Note that the cost of a mandatory arc (h k) can be consider both explicitly and implicitly, if the arcs is path of a minimum cost path. The use such a path would imply deadheading over (h k)

The decision variables are defined as:

$$x_{ijk} \begin{cases} 1 \text{ if } (\text{i, j}) \text{ of G is traversed by vehicle k} \\ 0\ otherwise \end{cases}$$

A thee indices formulation is as follow:

$$z_{DCARP} = \min \sum_{(ij)\in A} c_{ij} \sum_{k \in k} x_{ijk} \qquad (1)$$

$$s.t \sum_{k \in k} x_{ijk} = 1 \qquad (ij) \in R \qquad (2)$$

$$\sum_{J \in V_R} \sum_{k \in k} x_{ojk} = |K| \qquad (3)$$

$$\sum_{(ij)\in R} q_{ij} T_{ijk} \leq Q \qquad k \in K \qquad (4)$$

$$\sum_{j \in V_R} x_{ijk} = \sum_{j \in V_R} x_{ijk} \qquad i \in V_R k \in K \qquad (5)$$

$$\sum_{i \in S} \sum_{j \notin S} x_{Ijk} >= \sum_{j \in V_R} x_{ijk} \qquad S \subseteq V_R \setminus (0), k \in K, h \in S \qquad (6)$$

$$x_{ijk} \in (0,1) \qquad (ij) \in R, k \in K \qquad (7)$$

The equation (1) is the objective function that minimizes the total travel cost.

The equation (2) is the constraint that each mandatory arc is serviced by exactly one vehicle note however that each mandatory arc can be utilized by more than vehicles if it happens to be on the shortest path from an endpoint of a mandatory arc to that of another mandatory arc: only one vehicle will service it, though)

The equation (3) is presents that the number of vehicle used is the number of available vehicles

The equation (4) is the vehicle capacity constraint that is not exceeded.

The equation (5) is the continuity constraints
The equation (6) is the sub tour elimination
The equation (7) is the integrality constraint

3.3. Vehicle routing problem with pick ups and deliveries

3.3.1. Study 1 [20]

Description of the study

The routing models with time windows can be applied also in cases with pick ups and deliveries (VRP-SPDTW). The VRP-SPDTW is the problem of optimally integrating forward (good distribution) and reverse logistics (returning materials) for cost saving and environmental protection. The vehicle routing problem with simultaneous pickups/ deliveries and time windows can be described as follows: a set of customers is located on a transportation network; each customer requires both a delivery and a pickup operation of a certain amount of goods and returning materials and must be visited once for both operations. The customers should be served by a given fleet of vehicles of limited capacities which are usually assumed to be identical; each vehicle leaves the depot carrying an amount of goods equal to the total amount it must deliver and returns to the depot carrying an amount of returning materials equal to the total amount it has picked up. This case is a mix of delivery and pick-up loading for each vehicle in a given route. Moreover, each customer must be served within a specified time window. The VRP-SPDTW can be in practice applied in many cases, such the soft drink industry and the associated returned goods.

Data

- set of customers
- set of depots
- number of vehicles
- number of customers
- distance between customers
- service time at customers
- travel time between customers
- delivery demand of customers
- pick up demand of customers

Objective

- Minimization of the total distance traveled

Constraints

- each customer is visited by exactly one vehicle for both operation, deliveries and pick ups
- the same vehicle arrives and departs from each customer it serves
- distance constraints : between customers, facility and

- time windows constraints
- pick-up and delivery demands
- vehicle capacity

Mathematical Model

Sets

V set of customers
V0 set of customers plus depot (customer 0):V0 =V U {0}

Parameters

k maximum number of vehicles
Q vehicle capacity
N total number of customers: n=lVl
C_{ij} Distance between customer i and j

S_{ij} Time of beginning of service at customer i by, it is inanition if vehicle k does not serve customer i, where i =1,2,...,n

t_i Service time at customer i, where i =1,2,...,n

t_{ij} travel time (proportional to the Euclidean distance) between customer i and j, where ij=0,1,2;...,n (0 is the depot)

d_i Delivery demand of customer i, where i =1,2,...,n

p_i Pick-up demand of customer i, where i =1,2,...,n

Decision variables

$$x_{ijk} = \begin{cases} 1 \text{ if arc } (\mathrm{i,j}) \text{ belongs to route operated by } \mathrm{k} \\ 0 \text{ otherwise} \end{cases}$$

y_{ij} The demand picked up from customers up to node i and transported in arc (i,j)

z_{ij} The demand to be delivered to customers routed after node i and transported in arc (i,j)

The corresponding mixed integer programming mathematical formulation of VRP -SPDTW is given:

Minimize $\sum_{k=1}^{\bar{k}} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk}$ (1)

s.t. $\sum_{i=0}^{n} \sum_{k=1}^{\bar{k}} x_{ijk}$ = 1, j = 1,...,n (2)

$\sum_{i=0}^{n} x_{ijk} - \sum_{i=0}^{n} x_{jik} = 0$, j = 0,1,...,n; k = 1,..., $\bar{k}$; (3)

$\sum_{j=1}^{n} x_{0jk} \le 1$, k = 1,..., $\bar{k}$, (4)

$$\sum_{i=0}^{n} y_{ji} - \sum_{i=0}^{n} y_{ij} = p_j, \quad \forall \ j \neq 0, \qquad (5)$$

$$\sum_{i=0}^{n} z_{ij} - \sum_{i=0}^{n} z_{ji} = d_j, \quad \forall \ j \neq 0, \qquad (6)$$

$$y_{ij} + z_{ij} \leq \sum_{k=1}^{\bar{k}} x_{ijk}, \ \text{I,j = 0,1,…,n;} \qquad (7)$$

$$s_{ik} + t_i + t_{ij} - M(1 - x_{ijk}) \leq s_{jk}, \ \text{i,j = 0,1,…,n; k = 1,…,} \bar{k}; \qquad (8)$$

$$a_i \leq s_{ik} \leq b_j, \ \text{i,j = 0,1,…,n; k = 1,…,} \bar{k}; \qquad (9)$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk} \leq L, \ \text{k = 1,…,} \bar{k} \qquad (10)$$

$$x_{ijk} \in \{0,1\}, \ y_{ij} \geq 0, \ z_{ij} \geq 0, \ \text{i,j = 0,1,…,n; k = 1,…,} \bar{k} \qquad (11)$$

Equation (1) is the objective function that seeks to minimize the total distance travel.
Equation (2) is the constraint that ensure that each customer is visited by exactly one vehicle
Equation (3) is the constraints that guarantee that the same vehicle arrives and departs from each customer it serves
Equation (4) is the constraint that defines that at most k vehicles are used
Equations (5) and (6) are flow equations for pick-up and delivery demands, respectively
Equation (7) is the constraint that establishes that pick-up and delivery demands will only be transported using arcs included in the solution
Equations (8) and (9) are time windows constraints
Equation (10) is the maximum distance constraints; L is the upper limit on the total load transported by a vehicle in any given section of the route.
Equation (11) is the constraint that defines the nature of the decision variables.

3.3.2. Study 2 [21]

Description of the study

The aim of this study is to quantify the expected benefits of the new advanced planning for the logistic network of Auto Recycling containers. This is a real-life project in optimizing the logistic network for containers with materials from end-of-life vehicles in Nederland. The vehicle routing model is a unique multi-depot pickup and delivery model with alternative delivery locations. The employed heuristic is based on generating a set of promising routes and selecting the optimal combination of routes by solving a set partitioning problem.

Data of the study

- Set of end-of-life vehicle (ELV) dismantlers
- Set of depots
- Set of recyclers

- Number of containers
- Distance and travel times
- Depot locations assumed to have sufficient storage of all containers types to exchange
- Orders may be either one or two containers
- Containers can be delivered either to a depot or to a recycling facility

Objectives

- Optimal combination of routes such that all orders are performed at minimal cost.

Constraints

- Full containers coming from a depot can only delivered to a recycling facility
- A vehicle's route starts and ends at the depot
- Vehicle capacity is limited to two containers
- Every time a full container is picked up from an ELV dismantler, it must be exchanged from an empty one of the same type.

Mathematical Model

Parameters

$\delta_{so,ro}$ =1 if sub-order so belongs to root-order ro, 0 otherwise.

$a_{so,r}$ =1 if sub-order so is contained in route r, 0 otherwise.

c_r =denotes the costs of driving route r in euro.

p_r =denotes the profit or costs (negative p r) of route r as a result of the chosen delivery locations for the orders in route r in euro.

Variables

X_r =1 if route r is selected, 0 otherwise.

The route selection problem

$$CostCor_{so} = a\ LHC_{so} Load_{so} \tag{1}$$

$$\begin{aligned} dist_{so\_A,so\_B} = \min\{ & d(p_{so\_A}, d_{so\_A}) + d(d_{so\_A}, p_{so\_B}) + d(p_{so\_B}, d_{so\_B}), \\ & d(p_{so\_A}, p_{so\_B}) + d(p_{so\_B}, d_{so\_A}) + d(d_{so\_A}, d_{so\_B}), \\ & d(p_{so\_A}, p_{so\_B}) + d(p_{so\_B}, d_{so\_B}) + d(p_{so\_B}, d_{so\_A}), \\ & d(p_{so\_B}, d_{so\_B}) + d(d_{so\_B}, p_{so\_A}) + d(p_{so\_A}, d_{so\_A}), \\ & d(p_{so\_B}, p_{so\_A}) + d(p_{so\_A}, d_{so\_B}) + d(d_{so\_B}, d_{so\_A}), \\ & d(p_{so\_B}, p_{so\_A}) + d(p_{so\_A}, d_{so\_A}) + d(d_{so\_A}, d_{so\_B}) \ \} \\ & - d(p_{so\_A}, d_{so\_A}) - d(p_{so\_B}, d_{so\_B}) \end{aligned} \tag{2}$$

$$\min \sum_r (c_r - p_r) \bullet X_r \tag{3}$$

s.t. $\sum_{r} \sum_{so} \left(\delta_{so,ro} \bullet a_{so,r}\right) \bullet X_{r} = 1 \quad \forall$ ro (4)

$X_{r} \in \{0,1\} \quad \forall$ **r** (5)

The equation (1) presents the route cost from a depot to the recycler.
Where α=Correction factor between 1/4 and 1
LSC_{so} =Line haul costs to deliver a container from the depot of sub-order so to the cheapest recycler in transportation costs and gate fee.
$Load_{so}$ =Number of containers in sub-order
The equation (2) presents the distance measure that based on the best combination of two orders.
The equations (3) to (5) present the optimal combination of routes by the minimum order cost. Note that $\sum_{so} \delta_{so,ro} \alpha_{so,r}$ is either 0 or 1 by construction of the route generator

3.3.3. Study 3 [22]

Description of the study

This study presents the vehicle routing problem with simultaneous distribution and collection (VRPSDC) that is a variation of the capacitated vehicle routing problem, which arises when the distribution of goods from a depot to a set of customers and the collection of the associated waste from them must be performed by the same limited capacity vehicles, while the customers can be visited in any order.

Data

- Set of customers to be visited
- A depot
- Number of vehicles
- demand of pick ups
- demand of delivery

Objective

- Minimization of the overall length of the vehicle route

Constraints

- Every customer must visit once
- Conservation constraints: maximum number of vehicles to be used
- Conservation constraints on the amount of pick up and delivery load
- Vehicle capacity constraints

Mathematical Model

Variables

x_{ij} takes the value 1 if and only if arc(i,j) $\in$ A belongs to the solution

Continuous non negative viarables P_{ij} and D_{ij} indicate respectively the amount of collected load and delivery load carried along arc(I,j)

N set of customers

K maximum number of available vehicle

p_i an integer non negative collection demand

d_i an integer non negative delivery demand

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} \qquad (1)$$

$$\sum_{j\in N^+} x_{ij} = 1 \qquad \forall i \in N, \qquad (2)$$

$$\sum_{j\in N^+} x_{0j} \le K, \qquad (3)$$

$$\sum_{j\in N^+} x_{ij} = \sum_{j\in N^+} x_{ij} \qquad \forall i \in N^+ \qquad (4)$$

$$\sum_{j\in N^+} P_{ij} - \sum_{j\in N^+} P_{ji} = p_i \qquad \forall i \in N \qquad (5)$$

$$\sum_{j\in N^+} D_{ji} - \sum_{j\in N^+} D_{ij} = d_i \qquad \forall i \in N \qquad (6)$$

$$P_{ij} + D_{ij} \le Q x_{ij} \qquad \forall (i,j) \in A \qquad (7)$$

$$P_{ij}, D_{ij} \le Q x_{ij} \qquad \forall (i,j) \in A \qquad (8)$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \qquad (9)$$

Equation (1) presents the objective function that minimizes the overall length of the vehicle route.
Equation (2) presents the constraint that forces every customer to be visited once.
Equation (3) presents the constraint that implies that no more K vehicles can be used
Equations (4) (5) and (6) present the flow conservation constraints on the number of vehicles and on the amount of pick up and delivery load
Equations (7) presents the constraint that ensure that the vehicle capacity is not exceeded

3.4. VRP with compatibility constraints

3.4.1. Study 1 [23]

Description of the study

This case study presents a new multiobjective model for the hazardous waste location and routing problem, which is directly related to population exposure and societal risk. Hazardous waste management involves the collection, transportation, treatment and disposal of hazardous wastes. The hazardous waste location routing problem can be stated as follows: given a transportation network and the set of potential nodes for treatment and disposal facilities, find the location of treatment and disposal centers and the amount of shipped hazardous waste and waste residue, so as to minimize the total cost and the transportation risk. The aim of the models is to answer the following questions: where to open treatment centres and with which technologies, where to open disposal centres, how to route different types of hazardous waste to which of the compatible treatment technologies, and how to route waste residues to disposal centres. The objective is the minimization of the operation cost and transportation risk. There are many types of businesses that generate hazardous waste like dry cleaners, hospitals, and chemical manufactories, as well as residential sources of hazardous waste like batteries etc. Compatibility is an important parameter, since this model manages different types of hazardous waste and different treatment technologies.

Data

- Transportation network
- Number of trucks
- Cost of transportation of unit hazardous and unit of waste residue
- Fixed annual cost of opening a treatment technology
- Number of people in the bandwidth for hazardous waste type
- Amount of hazardous waste type
- Amount of waste residue
- Number of treatment technologies
- Recycle percent of hazardous waste type
- Fixed cost in locating treatment and disposal facilities

Objectives

- minimizing total costs
- minimizing transportation risk

Constraints

- Flow constraints for both hazardous waste and waste residue
- Mass balance constraint for hazardous: all generated non-recycled hazardous waste is transported to and treated at a treatment facility
- Mass and flow balance constraint for waste residue. The treated and non recycled hazardous waste is transformed into waste residue
- Capacity constraints of disposal and treatment technology
- Compatibility constraints:all hazardous waste type is treated only with a compatible treatment technology
- Minimum amount of waste required for the treatment technology

Mathematical Model

Indices
N=(V,A) transportation network
G={1,...,g} generation nodes
T={1,...,t} potential treatment nodes
D={1,...,d} potential disposal nodes
Tr ={1,...,tr} transshipment nodes

W={1,...,w} hazardous waste types
Q={1,...,q} treatment technologies

Parameters

c_{ij} Cost of transporting one unit of hazardous waste on link (i,j) $\in A$

cz_{ij} Cost of transporting one unit of waste residue on link (i,j) $\in A$

fc_{qi} Fixed annual cost of opening a treatment technology q $\in$ Q at treatment node i $\in$ T

fd_i Fixed annual cost of opening a disposal facility at disposal node i $\in$ D

POP_{wij} Number of people in the bandwidth for hazardous waste type w $\in$ W along link (i,j) $\in A$

g_{wi} Amount of hazardous waste type w εW generated at generation node i $\in$ G

α_{wi} Recycle percent of hazardous waste type w $\in$ W generated at generation node i $\in$ G

β_{wq} Recycle percent of hazardous waste type w $\in$ W treated with technology g $\in$ Q

r_{wq} Percent mass reduction of hazardous waste type w $\in$ W treated with technology g $\in$ Q

t_{qi} Capacity of treatment technology q $\in$ Q at treatment node i $\in$ T

t_{qi}^{m} Minimum amount of hazardous waste required for treatment technology g $\in$ Q at treatment center i$\in$T

dc_i Disposal capacity of disposal site i $\in$ D

$com_{w,q}$ 1if waste type w $\in$ W is compatible with technology q $\in$ Q; 0 otherwise

Decision variables

$x_{w,i,j}$ Amount of hazardous waste type w transported through link(i,j)

$z_{i,j}$ Amount of waste residue transported through link(i,j)

$y_{w,i,j}$ Amount of hazardous waste type w to be treated at treatment node i with technology q

d_i Amount of waste residue to be disposed of at disposal node i

$f_{q,i}$ 1 if treatment technology q is established at treatment node i; 0 otherwise

$d_{z,i}$ 1 if disposal site is established at disposal node i; 0otherwise

In the model, the non-recycled amount of generated hazardous wastes ((1 $-\alpha_{w,i}$) $q_{w,i}$) are to be routed ($x_{w,i,j}$) to the compatible treatment technology in the treatment facility

($y_{w,i,j}$) to be located ($f_{q,i}$). After the treatment process, the non-recycled amount of waste residues are to be routed ($z_{i,j}$) to the ultimate disposal facility, which is also to be located (d_i).

Minimize $\sum_{(i,j)\in A}\sum_{w} c_{i,j}x_{w,i,j} + \sum_{(i,j)\in A} cz_{i,j}z_{i,j} + \sum_{i}\sum_{q} fc_{q,i}f_{q,i} + \sum_{i} fd_i dz_i$

AND

Minimize $\sum_{(i,j)\in A}\sum_{w} POP_{w,i,j}x_{w,i,j}$

Subject to

$$(1-a_{w,i})g_{w,i} = \sum_{j:(i,j)\in A} x_{w,i,j} - \sum_{j:(j,i)\in A} x_{w,j,i} + \sum_{q} y_{w,q,i}, \quad w \in W,\ i \in V, \qquad (1)$$

$$\sum_{q}\sum_{w} y_{w,q,i}(1-r_{w,q})(1-\beta_{w,q}) - d_i = \sum_{j:(i,j)\in A} z_{i,j} - \sum_{j:(j,i)\in A} z_{j,i}, \quad i \in V, \qquad (2)$$

$$\sum_{w} y_{w,q,i} \le t_{q,i}f_{q,i}, \quad q \in Q,\ i \in T, \qquad (3)$$

$$d_i \le dc_i dz_i, \quad i \in D \qquad (4)$$

$$\sum_{w} y_{w,q,i} \ge t_{q,i}^{m}f_{q,i}, \quad q \in Q,\ i \in T, \qquad (5)$$

$$y_{w,q,i} \le t_{q,i}com_{w,q}, \quad w \in W,\ q \in Q,\ i \in T, \qquad (6)$$

$$\sum_{q}\sum_{w} y_{w,q,i} = 0, \quad i \in (V-T), \qquad (7)$$

$$d_i = 0, \quad i \in D, \qquad (8)$$

$$f_{q,i} \in \{0,1\}, \quad q \in Q,\ i \in T,$$

$$dz_i \in \{0,1\}, \quad i \in D$$

The cost objective minimizes the total cost of transporting hazardous wastes and waste residues and the fixed annual cost of opening a treatment technology and a disposal facility. The risk objective minimizes the transportation risk, which is measured with population exposure. The amount of shipped hazardous wastes on a given link times the amount of people living along a given bandwidth on that link is to be minimized. As the given bandwidth may differ for each hazardous waste type, the equation is summed for all hazardous waste types.

Equation (1) is the flow balance constraint for hazardous wastes. This constraint ensures that all generated non-recycled hazardous waste is transported to and treated at a treatment facility. The model allows opening a treatment facility at a generation node if that generation node is a potential site. Therefore, part of the generated and non-recycled hazardous waste is either treated at that generation node, if a treatment facility is located at that node, or transported to a node on which a treatment facility is located.

Equation (2) is the mass and flow balance constraint for waste residue. The treated and non recycled hazardous waste is transformed into waste residue by this constraint, which also ensures that the entire generated and non-recycled waste residue is transported to a disposal site and disposed of. The model allows opening a treatment and a disposal facility at the same node, which may be a generation node. So, if a treatment and a disposal facility are located at the same node, some part of the generated waste residues can be disposed of at the same node where they are generated. Otherwise, the generated waste residues are to be transported to a node where a disposal facility is located.

Equation (3) and (4) are capacity constraints. That is, the amount of hazardous wastes treated at a treatment technology should not exceed the given capacity of that treatment technology, and the amount of waste residue disposed of in a disposal facility should not exceed the capacity of that disposal facility.

Equation (5) is the minimum amount of requirement constraint. A treatment technology is not established if the minimum amount of waste required for that technology is not exceeded.

Equation (6) is the compatibility constraint, which ensures that a hazardous waste type is treated only with a compatible treatment technology. The first and second constraints are written for all nodes, which necessitates the seventh and eighth constraints (equation (7) and (8)).It should restrict the model so that no waste is treated and no waste residue is disposed of at nodes that are not among the candidate nodes for treatment and disposal centres.

3.4.2. Study 2 [24]

<u>Description of the study</u>

This case study is concerned with the problem of optimally routing and scheduling the collection of medical waste from a disperse group of facilities in Taiwan. The problem is addressed in two phases. The first phase solves a standard vehicle routing problem to determine a set of individual routes for the collection vehicles. The second phase uses a mixed integer programming method to assign routes to particular days of the week. The objective of the dynamic programming problem faced in the first phase is to minimize the cost and find the location at which the vehicle terminates its route to return to the depot at stage, whereas the objective of the second phase is to minimize the maximal daily travel.

<u>Data</u>

- Number of medical institutions
- Amount of infectious waste
- Number of depots

- Number of locations
- Number of vehicles
- Travel time from locations to depots
- Loading time at locations
- Working time of vehicles
- Cost of routes at the locations

Objective

- Minimize the cost (travel mileage) and find the location at which the vehicle terminates its route to return to the depot at stage
- Minimizes the maximal daily travel.

Constraints

- Vehicle capacity constraints
- Demand constraints
- Working time constraints
- Routes sensibility constraint
- Daily routes assigned constraints
- Multiple visits constraints
- Maximal daily travel mileage constraint

Mathematical Model

Notation

$f_k(s_k, d_k)$ Represents the cost of a terminating route *k* at location *sk*, with servicing *dk* locations;

s_k =state *k*, also denotes the last location of route *k*;

d_k = the number of locations in the route *k*;

e_{s_k} = the cost caused by setting a breakpoint at location *sk* ;

s_{k-1} = s_k - d_k

l_k = lower limit of the accumulated load for stage *k*;

u_k = upper limit of the accumulated load for stage *k*;

q_j = weekly demand (mass quantity) of location *j* ;

t_j = time including the travel time from locations j-1 to *j* and the loading time at location *j* ;

tt_{s_k} Depot = travel time from location s_k to depot;

W = the vehicle capacity;

T = daily working time limit of a vehicle.

B: maximal daily travel mileage (e.g., km) in a week;

j : index for each vehicle route obtained after solving a standard VRP, *j* = 1, 2, . . . ,*m*;

k: index for working days, *k* = 1, 2, . . . , 6;

D_j : travel mileage of route *j* ;

Y_{jk} : = 1 when route *j* is assigned to the *k*th working day; = 0 when route *j* is not assigned to the *k*th working day;
o, p: indices for pairs of routes that visit a medical institution *u*;
v: index for an institution that requires multiple visits;
U^u : One of the allowable day combinations for institution *u*;
W^u : set of routes visiting institution *u*
Z_j : set of indices representing all allowable days for the *j*th route.

Minimize $f_k(s_k, d_k) = e_{s_k} + f^*_{k-1}(s_{k-1})$, (1)

where $f^*_k(s_k) = \min d_k f_k(s_k, d_k)$,

Subject to

$d_k \geq 0$, (2)

$l_k \leq \sum_{j=1}^{s_k} q_j \leq u_k$, (3)

$\sum_{j=s_{k-1}+1}^{s_k} q_j \leq W$, (4)

$\sum_{j=s_{k-1}+1}^{s_k} t_j + tt_{s_{k-1}+1,depot} + tt_{s_{k-1},depot} \leq T$, (5)

Where $S_0 = 0$, and $f^*_0(s_0) = 0$.

Minimize B (6)

Subject to

$B \geq \sum_{j=1}^{m} D_j Y_{jk}$, k = 1,2,…,6, (7)

$\sum_{k=1}^{6} Y_{jk} = 1$, j = 1,2,…,m (8)

$\sum_{k \in U^u} Y_{ok} - \sum_{k \in U^u} Y_{pk} = 0$,

$\forall$ pairs of routes: o and p that visit u and $\forall$ U^u, $\forall$ u**,** (9)

$\sum_{j \in W^u} Y_{jk} \leq 1$, k = 1,2,…,6, and $\forall$ u, (10)

$Y_{jk} = 0$, j = 1,2,…,m; k $\notin Z_j$, (11)

$$Y_{jk} \in (0,1)\ ,\ \forall \text{ j,k.} \quad (12)$$

$$Y_{o1} + Y_{o5} - Y_{p1} - Y_{p5} = 0;$$
$$Y_{o2} + Y_{o6} - Y_{p2} - Y_{p6} = 0. \quad (13)$$

Equation (1) presents the objective function of the periodic routing model that minimizes the cost (travel mileage) and finds the location s_k at which the *k*th vehicle terminates its route to return to the depot at stage *k*.
Equation (2) presents the constraint that allows only the formation of sensible routes. Equation (3) presents the constraint ensures that the state is chosen so that the fulfilled demand is within feasible limits while expressions
Equations (4) and (5) present the constraints that ensure that the vehicle capacity and working time are not exceeded.
Equation (6) presents the objective function of the scheduling routes model that minimizes the maximal daily travel.
Equation (7) presents the constraint that denotes the maximal daily travel mileage.
Equation (8) presents the constraint that denotes that each route can only be assigned to one day.
Equation (9) and (13) presents the constraint that ensures that only one of the allowable day combinations is selected for a medical institution that requires multiple visits. For example, assuming two routes, *o* and *p*, pass the institution *v* that requires two visits while the allowable day combinations are either {Monday, Friday} or {Tuesday, Saturday}
Equation (10) presents the constraint that ensures that when route *o* is assigned to Monday, route *p* will be assigned to Friday. On the other hand, when route *o* is assigned to Tuesday, route *p* is assigned to Saturday.
Equation (11) presents the constraint that denotes that each institution can be visited, at most, once a day.
Equation (12) presents the constraint that ensures that the only allowable day combination can be selected for the route assignment.

<u>Conclusions</u>

Companies and municipalities increasingly face the challenge of managing reverse flows; companies' tasks involve the management of finished goods or raw materials whereas municipalities' tasks involve waste collection and public sanitary services. According to De Brito and Dekker [25] one can affirm that companies get involved with reverse logistics either because they can profit of it, or because they are obliged due to legislation and social pressure. In this chapter we presented some of the proposed vehicle routing models, as characterized by the main constraints that occur depending on the specific cases and circumstances. The usual objective of the routing models is to minimize the overall cost which depends mostly on the overall travel distance. There are also some cases where others than economic objectives prevail, depending on the company's mission. For example in the waste collection industry the corresponding company objective is the minimization of the operating cost, whereas an important part of a municipality's mission is to offer high value public sanitation services. All the above models have been successfully applied to real life problems of municipalities and companies which thereby managed to reduce their cost about 20%.

4. Greek Case Studies

In this chapter we present the waste management operations of three Greek case-studies: one for the Municipality of Panorama in Thessaloniki, northern Greece, in the residential solid waste collection, one for the company 'T', which specializes in industrial solid waste management, and one for ELDIA, specializing in industrial and commercial solid waste management. We explore the way the three organizations work and we propose the most suitable routing model from the pool of models we analyzed in Chapter 3. No facility location models are proposed here for the simple reason that, on the one hand, all three organizations rely on the same landfill and, on the other hand, another study [9] has already proposed an alternative transfer station which will be used in the near future.

4.1. Residential waste management

Residential waste involves municipal solid waste and consists of everyday items such as food scraps, newspapers, product packaging, furniture, clothing, bottles, paint, batteries, small or oversized house appliances. During the municipal planning of waste management the authorities take into account economic, social, political and environmental criteria.

4.1.1. Municipality of Panorama, Thessaloniki

The town of Panorama is a suburb of the city of Thessaloniki, in Northern Greece. It has 30.000 residents, occupying an area of 33.000.000 sq. meters. The Department of Sanitation and Cleaning (DSC) in the Municipality of Panorama has a total annual budget of 2 million Euros, and deploys in total 8 vehicles. More specifically, it deploys 5 garbage trucks for solid waste collection, 1 special truck for recycling items and 2 trucks for the daily collection of oversized items. Additionally, there are 2 open containers for the fragmented massive items to be carried to the Maurorahi landfill facility. The DSC has divided the area in 5 sections. The garbage truck scheduling for each of the 5 trucks is the same each day from Monday to Friday. Each garbage truck runs 1 of the 5 sections daily, except Sunday (none) and Monday (double) with maximum 6.5 h of working time (collecting time and a return trip to the landfill of Mavrorahi).

Data

- 3 sets of waste products: garbage and solid waste in 1.100 green bins, material for recycling in 370 blue bins and massive items and construction materials.
- Single depot
- Single transfer station at 1, 5 Km for massive items.
- Sanitary landfills: Maurorahi single landfill at a distance of 52, 5 Km, so total daily distance (return trip) is 105 Km.
- Location typology of waste facilities: one transfer station only for massive-bulk items in 1, 5 Km distance where they are submitted to fragmentation process.
- Waste volume(100 tons by week)
- Fixed disposal cost 27€/ton

Objective

- Minimization of the total operating cost, taking into account environmental and social responsibility

Constraints

- Maximum of 6.5 working hours for each member of the crew, by Greek Law.
- Working days from Monday - Friday for the green carts.
- Saturday only one garbage truck for the central and market areas.
- Working hours 05.00 – 11.30
- Mass input–output relation constraints for solid urban waste of green carts. No transfer station may keep any solid waste. Contrary to the oversized items that can be kept for fragmentation in the transfer station in a distance of 1.5 Km.
- Each unload to the landfill is weighed and charged accordingly.

Model proposal

The municipality of Panorama organizes the collection of the waste in a heuristic manner. They use no routing software and they don't follow any vehicle routing models. This is bound to lead to sub-optimal results. By cross-examining all the vehicle routing models we analyzed in the previous chapters, we propose for the Panorama municipality the model employed by Jing-Quan Li et al. (2008)[16], analyzed at chapter 3. Despite the obvious differences in terms of population, waste volume, budget and number of staff employed between the municipalities of Panorama and Porto Alegre, there are key structural similarities. In particular, in both cases:

- Every trip is assigned to only one vehicle.
- Each one of vehicles performs a sequence of trips.
- There are no capacity constraints.
- One depot exists in the operating system.
- Each vehicle and its crew (consisting of 1 driver and 2 operators) are directed to pick up waste and transport it for a certain time period before going to the landfill or recycling facility.
- All the trucks start from the same depot and collect waste during a time period of 3-6 hours.
- The trucks, after collection, empty their loads at the landfill or other recycling facilities and return to the depot.
- Town administrators are deeply concerned about productivity of waste collection and transfer operations. They aim at designing ‘’ good’’ vehicle schedules over a set of previously defined collection trips. They are additionally sensitive in the social benefits involved.
- They are constantly growing regions in terms of the residents-waste producers with increasing demand.
- Nevertheless budgets are limited and sanitary activities become more expensive; so do crew salaries and truck maintenance costs.

It is noteworthy that Jing-Quan Li et al. (2008) predicted in the case of Porto Alegre an operating cost reduction of 25.7% if the municipality followed their model. It follows that the municipality of Panorama will also experience a reduction in its operation cost, although the exact amount can be determined only after the model has been applied in practice.

4.2. Industrial and commercial waste management

Industries need to ensure that they dispose of their waste properly, comply with the legislation, and in general that they are responsible about their industrial waste management and specifically their hazardous waste. Commercial waste producers include shopping malls, large retail businesses, restaurants and office buildings. Furthermore, a considerable volume of massive solid waste includes construction scrap for recycling and reuse (e.g. aluminum, steel, rubber, wood, copper). This material often comes from construction or demolition projects which are usually located in residential, commercial or industrial areas. The main difference between commercial and industrial waste collection is the size of containers (buckets or dumpsters). We present two case studies that manage industrial and commercial waste in in the area of Northern Greece.

4.2.1. EL.DI.A. Co. “Elliniki Etairia Diaxeirisis Aporimmaton”, Thessaloniki

Description of the case study

ELDIA is located in the industrial zone of Thessaloniki in a facility of 50.000 m^2 in Neohorouda. ELDIA focuses on waste management and the environmental technology sector. The framework of company's operations includes: the design and management of waste collection and transport systems for the commercial and industrial sector; the design, development and management of waste processing and sorting plants; the design and management of recycling and thermal processing plants.

ELDIA has developed an advanced fleet of 12 specialized vehicles with trailers and 3 cranes for paper and mass materials. It is engaged in the collection and transport of waste in the area of Northern Greece, and it serves about 150 customers at a regular basis including both private companies and the public sector (IKEA, Mediterranean Cosmos, Carrefour, Jumbo, Macro, Lidl, many plants in the industrial area of Sindos Thessaloniki and service customers also in Kozani, Kastoria and Kavala). Besides regular customers-industries, there are casual customers who place orders according to their needs and sometimes in emergency situations.

The orders are received daily by fax. The average number of orders reaches 55 orders daily. Every afternoon the daily orders are gathered and the logistics department conducts the scheduling of pickups and deliveries. Pickups involve the removal of full buckets which may or may not need replacement with empty ones. Deliveries involve the placement of empty buckets to the points of the ordering.

According to Mr. D. Gortzis, ELDIA's CEO: *"… the conditions in the department when scheduling resembles to war conditions, because of the emergency orders and contingencies that occur. That is why we cannot use software for routing, we failed every time we installed one, therefore our routing scheduling is based on experience and practice".*

Routing is scheduled primarily according to the needs of the large-volume regular customers from ELDIA's depot to the regular points of collection. When casual customers occur, they are serviced with in-between stops of the vehicles tripping in the respective area with pickups and deliveries in between the two basic points (depot and regular points of collection). Following collection, the waste is assembled for sorting in ELDIA's facilities for recycling and packaging. What cannot be reused or recycled is transported to a licensed facility for disposal. The Mavrorahi landfill is located at a distance of 48 Km from ELDIA facilities; Mavrorahi is open from 08.00 to 14.00 and it charges private companies at 35€/ton of waste).

All ELDIA's vehicles are self-loading, that means that only one driver-operator is involved. So labor costs cover one person per vehicle. The company recently has introduced a new service which comprises the rental of press containers and mobile waste compactors installed at the client's premises. For example, in the area of Mediterranean Cosmos Mall there are 8 points of collection and 1 press container for organic waste from restaurants. With the use of special vehicles (press containers - compressors, trailers with double buckets and cranes with grab buckets) the company undertakes the collection and final disposal of the waste material in question. Buckets are of two types: a) city buckets of 7 m^3 capacity and, b) Industrial containers of 35 m^3 capacity. Twofold loading is available for every vehicle using the trailer; with one bucket of 35 m^3 and one more on the trailer (of 35 m^3) or a press container. Also manifold loading is enabled when using the cranes. For example, the crane-vehicle when visiting JUMBO or IKEA compresses the paper and then loads it. That means that during the same trip it can service multiple clients because of the capability to compress materials.

Disassembling, checking and sorting materials involve both mechanical and manual processes. Planning and scheduling is more predictable regarding disassembling, checking and sorting for recycling materials, because there is no time pressure or emergencies or contingencies for such materials and they are stored in the ELDIA facilities. Furthermore, glass collection involves the use of grab buckets and a trip is scheduled for whenever there is vehicle availability. Following collection, glass is sold to YIOULA Co., wood is sold to SHELMAN, iron is sold in SIDENOR, and paper (1500 tons/month) is sold either to MEL Co. or to large recycling units abroad.

Data

- Facility location of 50.000 m^2 adjacent to single depot.
- 12 specialized vehicles with trails
- 3 cranes
- Press containers
- Number Regular customers (150)
- Sets of customers (50 industrial & 100 commercial)
- Sets of customers (on a regular basis 45 & casual customers 10)
- Casual customers 10 per day
- 55 average of daily orders
- A disposal facility (Mavrorahi) at 48 Km from ELDIA's headquarters.
- One drive by vehicle, in the case of engine lubricants when collecting from petrol stations and/or auto repair shops (Katerini-Kilkis-Pella), two operators are needed instead of one driver
- Multi points of collection (55 daily)
- Transportation cost 0.8€/Km to 1€/Km.
- For some long distances(Alexandroupoli) ELDIA outsources the fleet of local companies
- Waste volume
- Recycling volume
- 400 City Buckets
- 400 Industrial containers.
- Fixed disposal cost (35€/ton)
- Travel distance from customers to facility and from facility to disposal

Objectives

- Company's objective is the minimization of the operating cost balanced with contingencies orders and high customer level service for big clients.

Constraints

- Pickup and delivery demands
- Each customer (industrial, commercial) is visited by exactly one vehicle for both operations: delivery of empty container and pickup of return materials and solid waste.
- Same vehicle arrives and departs from each customer
- Priority and emergency orders must be serviced immediately
- Time windows constraints, daily time tables and working days Monday to Saturday
- Time windows constraints of the disposal facility (08:00-14:00)
- Vehicle Capacity constraints (containers and buckets) on the amount of pickup and delivery load.
- Streets size constraints: in the narrow streets the trailer and the second bucket from the trucks are removed and cannot be used
- Distance constraints: between customers and between facility and customers
- Two vehicles are booked for long distances services (for remote cities like Florina or Kastoria the company uses only the newly acquired vehicles and avoids using the older ones, so as to avoid engine troubles).

Models Proposal

ELDIA deploys two types of vehicles: the trucks of limited capacity that visit all commercial customers for pickups and deliveries and high capacity vehicles that are used for heavy loads serving regular industrial customers. One of ELDIA's great problems in the scheduling of routing is that every day it has to service first emergency orders and unexpected contingencies. In addition, for some big customers ELDIA does a route more than once daily and sometimes with different vehicles. This increases its cost by vehicle, since it has either to disrupt scheduled routing or to travel more than once from its facility to customer and then back to disposal.

The ELDIA case is a vehicle routing problem with simultaneous pick ups and deliveries with time window constraints. Thus, we propose the application of two studies that also develop a vehicle routing model with simultaneous pick us and deliveries: a) the models employed by Mingyong, Lai (2009), [20] and the model employees by Dell'Amico Mauro (2006), analyzed at chapter 3.[22]The reason why we propose these models is that the also are problems with pickups and delivery problems in the case of a single depot. Moreover, like ELDIA the two case-studies deal with a set of customers and they tackle vehicle capacity and time windows constraints. Nevertheless, operating and managing emergency cases and/or priorities and/or unpredictable demand (dynamic situations) remains an issue for further analysis and has to be solved with dynamic models. Dynamic models will be useful tools in the truck scheduling problems solving, such as a truck breakdown and any severe disruptions in the midst of operations.

4.2.2. "T" Co, Thessaloniki

Description of the case study

This case study attempts to describe the operational part of company 'T' which is established in the industrial zone of Thessaloniki and collects industrial waste from waste producers-industries in the broader area. 'T' is active in the environmental technology sector and aims to offer complete and professional solutions. 'T' owns a fleet of 30 specialized vehicles; it is engaged in the collection and transport of waste in the area of Thessaloniki; it serves more than 100 customers, including plants and warehouses that produce commercial and industrial waste. 'T' currently employs approximately 110 employees for its operations. Recently, the company has introduced a new service which comprises the rental of mobile waste compactors (press containers which can load 4 bucket loaders on 1 press container). These are usually installed either at the client's premises or, when many clients are gathered in the area, at transfer stations in key areas (e.g. the industrial zone of Thermi). Moreover, the 'T' services a large clientele of construction companies at construction and demolition sites. After the collection and sorting phase (1^{st} phase) the trucks go to the landfill (2^{nd} phase). With the use of special vehicles 'T' undertakes the final disposal of the industrial waste material.

Client orders are submitted by phone and they are transmitted to the logistics department. Usually daily orders are carried through the next day. All daily orders from clients are getting categorized according the area of the collection point and the truck features. The objective of the categorization is to minimize the costs of transport. For example, an average fuel cost is 40 lt/100 Km and labor costs reach 10 €/hour. The area in which 'T' operates includes a single depot facility and a dump site for waste storage. When occasional orders are placed there is a request for a specific time delivery. Transport costs are considered to be the highest operational cost. The call center can contact truck drivers over a CB system and the clients over the telephone. The goal is to obtain the maximum load and the largest number of bucket-loaders placements; truck scheduling and routing should decide and design either the shortest distance (thus minimizing the time needed) or the minimum distance in time (and therefore the best operational ability to serve more clients).

At the end of the truck trip of the 1^{st} phase trucks returns to the sorting and recycling facility which is adjacent to the depot. Each truck waste-load is getting weighed when entering the waste facility. Drivers have a half -hour break, before the second phase of the second trip to the Maurorahi landfill and at about 15.00 the empty trucks return at the depot. At the same time the trucks are being examined through cameras placed above the assay-balance. The same truck is getting reexamined and its load is getting assessed at the stage of unloading. This second waste assessment is used, in order to decide what parts of waste will go for recycling.

In the same waste facility there is a large unit called Separation Center for Recycling Material. It is an investment of 2m Euros with 20 employees working there. It is the place where material for reuse is being extracted to be sold for new uses. Iron is then resold to iron companies like SIDENOR, plastic can be exported abroad, and wood to several companies for reuse and glass is sold to glass companies in Athens. If there is waste that can be used as construction material, it is transferred to a facility nearby at a distance of 1 Km so that can be reused in road works.

Clients are charged by the truck trip according the area of servicing. Clients are charged with waste management fees divided in dumpsite fees and processing fees. Anything that cannot be recycled further or reused is transferred to the Mavrorahi landfill. The charges before the landfill of Mavrorahi is 25 €/ton for solid waste.

Data

- Single depot waste facility at a distance of 48 Km from landfill facility.
- A dump site
- Recycling material center: trucks return at the facility for unloading and containers follow the checking and sorting process. Drivers take a break and then they change truck and start a second trip to Maurorahi landfill for disposal of scrap.
- 7 trucks of 8 cubic meters (skip-type) for heavy loading of constructive materials, iron, metals etc.
- 10 trucks of 35 cubic meters (hook-fit-type) for lighter load, such as plastic and wood.
- 800 bucket-loaders are always either in use or available to the clients.
- 100 customers
- Press containers
- Customers demand

Objective

- Minimization of the transportation cost and travel time.
- Balance workload among the trucks.

Constraints

- Vehicle Constraints: skip-type trucks cannot use the type of waste of the hook-fit; a hook-fit can easily load the waste type of skip-type.
- A hook-fit truck can load 9 empty bucket loaders of 8 m^3 and 3 full loaded bucket loaders of 8 m^3 .
- A hook-fit truck can load only 2 empty or full containers (dumpsters) of $35m^3$.
- The company should always have available containers or bucket loaders for emergency cases.
- Time windows constraints of stops and the depot: according to the trade union demands and labor regulations collection and disposal process must take place within 6.5 hours.
- A 30 min break for drivers and unloading for checking and sorting materials.

Model Proposal

Company 'T' used to design routes manually based on experience and adapted to its clients needs. 'T' schedules routing in a practical manner regarding the availability of its trucks, the location of their industrial and commercial customers, the construction areas, and they don't use any specific model for scheduling. Given the conditions under which 'T' operates, the date, the objectives and constraints we propose that 'T' should follow the model developed by Sahoo et al (2005) – presented at chapter 3[15].

Both cases characterized by vehicle capacity constraints; each vehicle can load only waste of given types with the objective to minimize the total travel distance. Other common features between the two case-studies are:

- Both companies need to perform different types of services on the same day with different container sizes, vehicle types, material types, industrial and commercial routes and different service requirements.
- Both companies face difficulties in determining the optimal routing to ensure the best customer service at the minimum cost.
- Serious capacity constraints: A typical commercial WM container is $6m^3$ (8 loose yards), while an industrial one ranges between $15.2m^3$ and $30.4m^3$ (20-40 loose yards).
- 'Both companies must take into account that each truck has a single capacity constraint, such as maximum volume or maximum travel time.
- Both companies possess heterogeneous type of vehicles and need to optimize their routes.
- Trucks first deliver the empty containers to the customers and they pick up the full container, travel to the disposal facility and dispose the contents.
- Trucks of both companies can handle only one industrial container at a time; they compress and transfer these containers to the either to the landfill, empty them and have them back to the customers.
- Intervals are needed for checking waste and sorting materials for recycling or reuse and for drivers' lunch breaks.

5. Conclusions

In the dissertation we reviewed and analyzed the recent literature in reverse and waste operation management focusing on the vehicle routing and facility location problems. We then analyzed three case-studies of Greek companies (two of the private sector and one municipal company) which did not use any routing or facility location models. Drawing on the findings of the literature we analyzed we propose a model for each case study that should help them reduce their costs and meet their objectives.

Environmental legislation and consumers' expectations are encouraging manufacturers and waste producers to be more environmentally conscious. In addition, the benefit of possible cost reduction has played a substantial role in the growing focus on reverse logistics and waste management.

The majority of the academic articles we analyzed here addressed the scheduling and cost issues in the context of remanufacturing, recycling and waste management. The detailed overview of the recent scientific literature gave us an in-depth idea about the problems and the proposed solution in this sector. More importantly, however, it enables one to recognize and analyze the real-life waste management problems in a scientifically informed and systematic fashion. It could also be of some use to practitioners and policy-makers who wish to be informed about waste management developments in other countries and companies.

The analysis of our three Greek case-studies was a good opportunity to show how waste management is organized in a part of Greece and what kind of problems it faces. Perhaps unsurprisingly none of the companies in our case-studies relied on a routing model. We hope that our model proposals are a useful, albeit modest, contribution which can be put to some practical use.

Proposing a model is not as easy task, because identifying the right model depends on many factors. Of course, a simulation of the models we proposed would be necessary before they could be adopted in real-life situations. In any case, we believe that if private or public organizations in Greece started seriously contemplating the prospects of adopting formal routing and facility location models there would be obvious cost and environmental benefits. Thus, it is encouraging to see that a published academic study by Erkut and Karagiannidis [9] has served as the scientific foundation for the creation of the new transfer station in Eykarpia, Thessaloniki.

6. References

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