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PROSPECTIVE MATHEMATICS TEACHERS' ABILITY TO IDENTIFY MISTAKES RELATED TO ANGLE CONCEPT OF SIXTH GRADE STUDENTS

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Abstract:
In the present study we try to highlight prospective mathematics teachers' ability to identify mistakes of sixth grade students related to angle concept. And also we examined prospective mathematics teachers' knowledge of angle concept. Study was carried out with 30 sixth-grade students and 38 prospective mathematics teachers. Sixth grade students required to define the concept of angle with their own statements and describe what it brought to their minds by writing their responses on a paper. Students written responses were examined by the researchers and students' mistakes in their definitions were determined. A data collection form that included students' definitions of angle (correct, partly correct and incorrect) was obtained. Prospective teachers were required to define the concept of angle with their own statements and evaluate whether sixth grade students' responses were correct by explaining the reasons in written form. The data obtained from written forms of prospective teachers were analysed through the descriptive analysis technique. Prospective mathematics teachers' comments they had made in response to each student are coded. The results of the research show that prospective mathematics teachers didn't have problems in determining students' definitions of angle as correct, incorrect or partially correct. But they have problems about expressing the students' failures and to clearly identifying the mistakes in the incorrect definitions of students. Additionally, almost all of the candidates have made only the static definition of the angle concept.

Key words: prospective mathematics teachers, angle concept, students' mistakes, teacher knowledge about students' mistakes

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1. Introduction

Geometry is one of the most important branches of mathematics education and its place in education cannot be discussed. It plays a bridge role in establishing connections between daily life and mathematical concepts. It is thought that the mathematics is cumulative and previous knowledge and concepts constituted a step for the latter in mathematics, so to determine the misconceptions of concepts and lack of knowledge, and to find solutions to eliminate these mistakes and deficiencies is crucial.

Angle concept is one of the basic concepts in geometry. But because of many-sided nature, it is clear from the research literature that school students have great difficulty learning the angle concept and grasping the multifaceted nature of the concept of angle (Biber, Tuna & Korkmaz, 2013; Butuner & Filiz, 2016; Clements & Battista, 1992; Clements & Burns, 2000; Dane & Baskurt, 2012; Devichi & Munier, 2013; Erbay, 2016; Mitchelmore, 1998; Mitchelmore & White, 2000; White & Mitchelmore, 2003, Keiser, 2004). Mayberry (1983) indicated that students mostly learn geometric concepts based on a rote learning approach. The properties, scopes, associations, and meanings contained in geometric expressions cannot be taught satisfactorily. One of the important reasons for this is the experience of teachers in this issue who are going to teach the concept of angle. There are also several studies indicating the difficulties prospective mathematics teachers (PMTs) had experienced with angle concept (Ipek, Atasoy & Okumus 2010; Silfverberg & Joutsenlahti, 2014; Tuluk 2015; Yazgan, Argun & Emre, 2009; Yigit 2014).

Gokkurt, Sahin, Soylu and Dogan (2015) stated that in order to perform meaningful learning in mathematics teaching, teachers should be aware of learning difficulties and mistakes of their students related to geometric concepts. When we look at the studies about angle concept with student teachers; Silfverberg and Joutsenlahti (2014) investigated the comprehension of plane angle concept of 191 Finish prospective teachers. The results of their examination showed that even the adults who have completed their years with mathematics studies still cherish various notions (beliefs) on such basic concepts of elementary mathematics as an angle, and these different notions and beliefs can remain very much alive. Yet, these concepts were used in mutual discussions regularly. Ipek, Atasoy and Okumus (2010) described the perceptions of the pre-service mathematics teachers on the angle concept with a case study. And according to their findings pre-service teachers generally emphasized the static dimension of the angle concept rather than its dynamic dimension. Additionally, their angle perception has also negative effects on their angle types and angle measurement perceptions.

The reason of angles being difficult to understand may stem from the fact of multiple definitions used for the concept. An angle can be defined using a static angle representation; as a part of the plane included between two rays meeting at their endpoints or a dynamic representation; as the amount of rotation necessary to bring one of its rays to the other ray without moving out of the plane (Kieran, 1986).

The definitions of angle concept in the modern sense falls into one of three categories according to Keiser's (2004) interpretation are given below.

1. The angle is the difference of direction between two straight lines.
2. The angle is the quantity or amount (or the measure) of the rotation necessary to bring one of its sides from its own position to that of the other side without its moving out of the plane containing both.
3. The angle is the portion of a plane included between two straight lines in the plane that meet in a point.

The grade 6 mathematics textbooks in Turkey provide only one definition of the angle concept and it is a static definition (Aydin & Gundogdu, 2016). The definition of the angle concept given as textbook format can be seen in Figure 1.

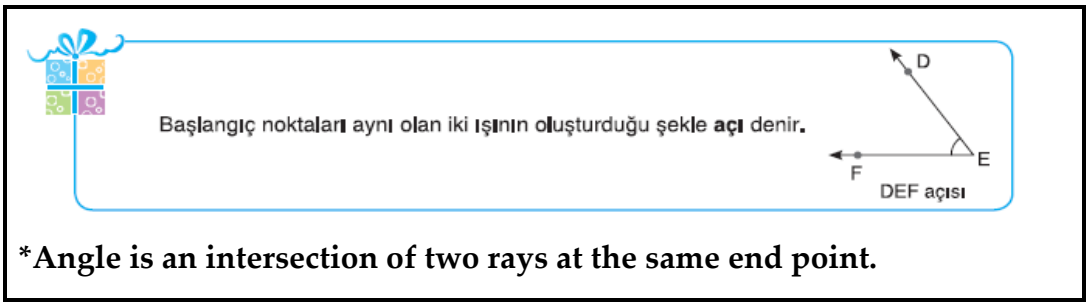


Figure 1: Usual definition of angle concept in 6th grade textbook of Turkey

Because of many different definitions in the field related to the concept of angle, the boundaries of the concept cannot be determined precisely. Developing understanding in mathematics is an important but difficult goal and in order to achieve this goal, teachers must be aware of student difficulties, the sources of the difficulties and design instruction to diminish them (Yetkin, 2003). Therefore, there is a need for new research to be done in this regard about angle concept. In this context the purpose of this study was to investigate the PMTs' ability to identify mistakes held by elementary students in angle concept definitions and their proposed corrections to overcome those mistakes. Another purpose was to determine PMTs' knowledge about angle concept.

2. Methodology

2.1 Research Design

In current research, case study which is one of the qualitative research designs was used because it answered "*how*" and "*why*" questions. Case study is a detailed description and analysis of case or cases (Merriam, 2009). It provides to examine a case, a relation or a process in all its aspects (Cepni, 2012). Because of that the aim of this study was to investigate conceptual knowledge of PMTs regarding the concept of angle and their abilities to determine students' mistakes in their definitions of angle, this approach was preferred for detailed examination.

2.2 Participants

The participants in this study were 38 prospective middle school mathematics teachers who were enrolled in 4th grade of elementary mathematics education program at a public university in Turkey. Senior PMTs were selected since participants were expected to have necessary knowledge about the concept of angle and main features of it and enough ability to link what they learned to students' answers. It can be said that these participants took most of both mathematics and education courses so that their levels were appropriate to define a mathematical concept and determine students' mistakes related to the same concept by using their current knowledge.

2.3 Data Collection Tools

The process of data collection consisted of two phases. In first phase, one open ended question related to the concept of angle was asked to 30 6^{th} grade students in a middle school. They were required to define the concept of angle with their own statements and describe what it brought to their minds by writing their responses on a paper. In second phase, in order to prepare data collection form for PMTs, students written responses were examined by three researchers and students' mistakes or deficiencies in their definitions were determined. 6 of these responses, 2 correct, 2 partly correct and 2 incorrect, were chosen with consensus. It was paid attention not to choose similar mistakes or deficiencies and unclear definitions. Therefore, a form that included students' definitions of angle was obtained. This form was given each PMT by the researchers. PMTs were required to define the concept of angle with their own statements and evaluate whether students' responses were correct by explaining the reasons in written form. They were also expected to determine the mistakes in students' responses if there is any. Besides, PMTs were given enough time to examine each student's response and to give their own answers.

2.4 Data Analysis

The data obtained from written forms of PMTs were analysed through the descriptive analysis technique. Descriptive analysis is a technique which provides to analyse, organize and interpret verbal and written data with the help of predetermined categories (Yıldırım & Simsek, 2013). In this direction, PMTs' definitions of angle were analysed with classification of Mitchelmore and White (2000) and PMTs' explanations about students' responses were analysed with through using the coding categories of Gokkurt, Sahin, Soylu and Dogan (2015). These codes and categories were presented below.

2.5 The Codes and Categories for Determining Mistakes

- Not to determine the mistakes: It includes the cases which PMTs are not able to determine the mistakes in students' answers. They make wrong explanation or do not give answers.
- Determine the mistakes partly true: It involves the cases which PMTs are not able to determine students' mistakes in their definitions completely. Their explanations include some ideas which are not true completely or insufficient.
- Determine the mistakes completely true: It reflects the cases which PMTs are able to determine students' mistakes. They offer true explanations and gave expected answers.

2.6 The Codes and Categories for Definitions of Angle

- an amount of turning about a point between two lines
- a pair of rays with a common end-point
- the region formed by the intersection of two half-planes
- inappropriate definitions

Classification of PMTs' angle definitions was made by considering the mathematical concepts such as point, line, ray, end-point, region, intersection, half-planes. The values of percentage and frequency for each category were calculated. PMTs' answers were coded based on the properties of categories. The values of percentage and frequency for each category were calculated. The data were labelled separately by two researchers and the percentage of conformity was found to be %91. Researchers reached a consensus after a meeting and matched the responses and categories.

3. Findings

In this section, the findings of the study and relevant explanations were presented. The data were analysed in two steps. At first, a correction table was designed for incorrect or partly correct expressions in the definition of the angle concept of the sixth grade students on the data collection form. Table 1 shows that the definitions of Ayşe and Figen were incorrect because they did not include the essential concepts which were "*the same origin*" and "*two rays*" instead they talked about incorrect or irrelevant concepts such as "three rays" and "*two line segments*".

According to table, the definitions of Beyza and Deniz were partly correct because although they mentioned "*two rays*" which was one of the essential features for identifying angle, they did not refer to the concept of "*the same origin*" in their definitions. If these deficiencies and mistakes are corrected, these students' definitions regarding angle concept will be true. At second step of this study, the PMTs were expected to determine the mistakes in students' responses and offer similar correction expressions that were indicated on Table 1.

Table 1: Incomplete or wrong expressions in the definition of the angle concept of sixth grade students

6th Grade Students' Nicknames	The correctness of students answers	Corrections
Ayşe	Incorrect	The same origin Two rays instead of two or three rays
Beyza	Partly correct	The same origin
Cenk	Correct	---
Deniz	Partly correct	The same origin
Efe	Correct	---
Figen	Incorrect	The same origin Two rays instead of two line segments

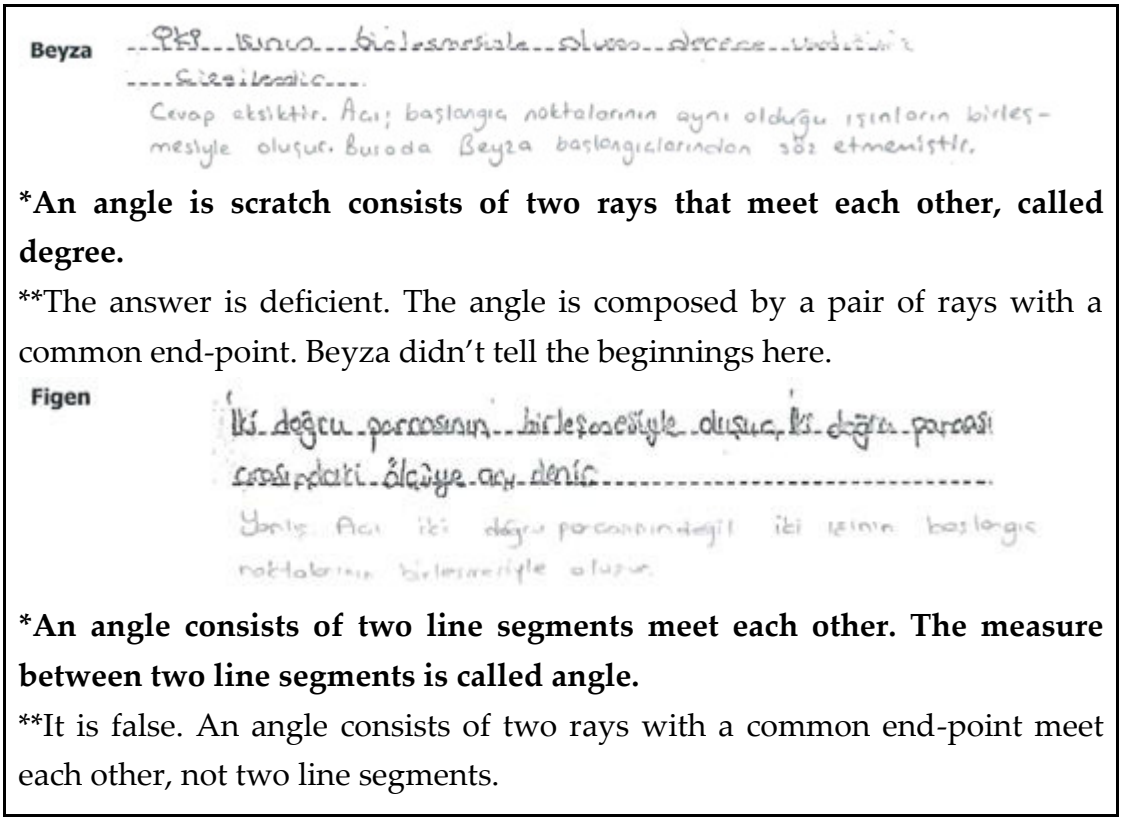
PMTs' answers related to the correctness of the students' definitions of angle were analysed under three categories which were "*Not to determine the mistake*", "*Determine the mistake partly true*" and "*Determine the mistake completely true*". The frequencies of their answers were presented in Table 2.

Table 2: The frequencies of PMTs' answers according to categories

6th Grade Students' Nicknames	Not to determine the mistake		Determine the mistake partly true	Determine the mistake completely true
	Wrong	Empty		
Ayşe	2	1	20	15
Beyza	3	2	21	12
Cenk	2	1	3	32
Deniz	1	1	28	8
Efe	2	1	1	34
Figen	6	0	24	8

It has been seen that PMTs generally have no problems while determining the students who defined the angle concept correctly. Of the 38 prospective teachers, 32 PMTs understood that Cenk's definition was correct whereas 34 of them accepted Efe's definition as correct. PMTs have difficulty in expressing where the mistake is although they can identify it as false. Prospective teachers were aware of that Ayşe and Figen's angle descriptions were wrong. 8 prospective teachers for the definition of Ayşe and 15 prospective teachers for the definition of Figen were able to address the mistakes completely. But a large part of them could not make sufficient explanations about what these mistakes were. Similar findings were also seen for the angle definitions that students did partially correct. Prospective teachers might have felt that there were shortcomings in their responses, but they could not express them fully. 12 prospective teachers determined that Beyza's response was partly correct, while 8 identified the deficiencies in the definition of Deniz.

The examples from PMTs' responses to students' definitions of angle concept were presented below. In the figures, "*" represents the students' definitions of angle concept whereas "**" refers to prospective teachers' responses regarding the correctness of students' answers and their correction expressions for these definitions.



***An angle is scratch consists of two rays that meet each other, called degree.**

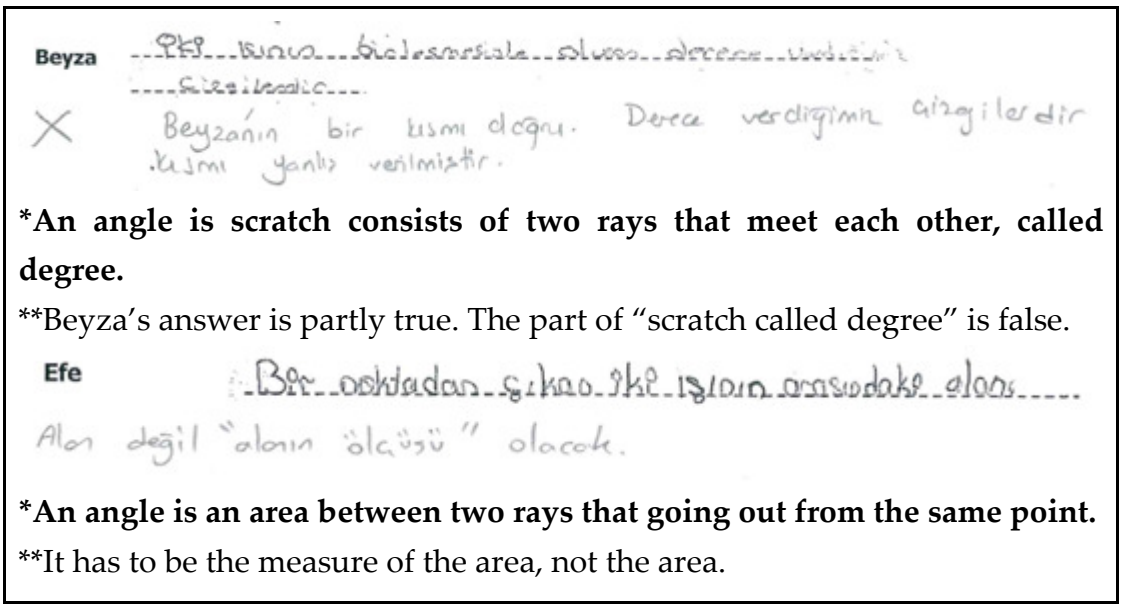
**The answer is deficient. The angle is composed by a pair of rays with a common end-point. Beyza didn't tell the beginnings here.

***An angle consists of two line segments meet each other. The measure between two line segments is called angle.**

**It is false. An angle consists of two rays with a common end-point meet each other, not two line segments.

Figure 2: The examples for the PMTs who determine the mistake completely true

The examples of PMTs' answers which he could determine the students' mistakes completely true and expressed the reasons clearly can be seen in Figure 2.



***An angle is scratch consists of two rays that meet each other, called degree.**

**Beyza's answer is partly true. The part of "scratch called degree" is false.

***An angle is an area between two rays that going out from the same point.**

**It has to be the measure of the area, not the area.

Figure 3: The examples for the PMTs who determine the mistake partly true

Figure 3 presents the examples from the responses of PMTs who could partially identify the students' mistakes, explain some of the corrections or simply indicate that there was a mistake without providing sufficient explanation.

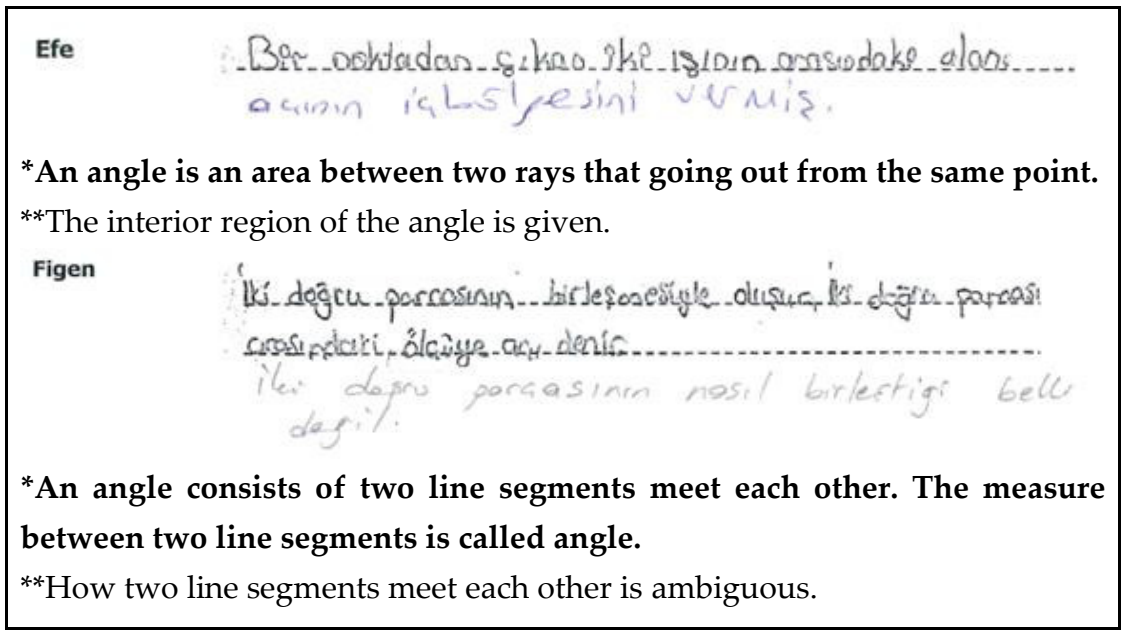


Figure 4: The examples for the PMTs who determine the mistake wrong

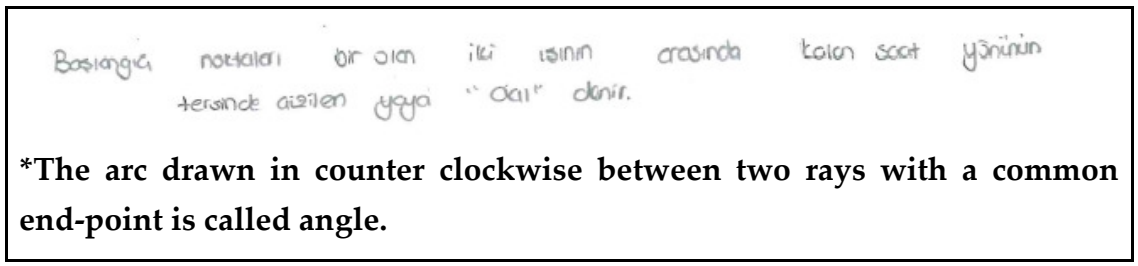
Figure 4 shows the examples from the answers of PMTs who were not able to identify student mistakes and made irrelevant explanations. The frequencies of PMTs' definitions of angle concept according to the categories of Mitchelmore and White (2000) which were indicated in method can be seen in Table 3.

Table 3: Descriptive analysis for the definitions of angle concept made by PMTs

Definition	**f**
An amount of turning about a point between two lines	3
A pair of rays with a common end-point	25
The region formed by the intersection of two half-planes	9
Inappropriate definition	1
TOTAL	**38**

When the angle definitions of the prospective teachers were examined, all PMTs except one of them correctly defined this concept. 34 of these definitions were static whereas only 3 of them were in the definition of dynamic angle. As seen in Table 3, most of the

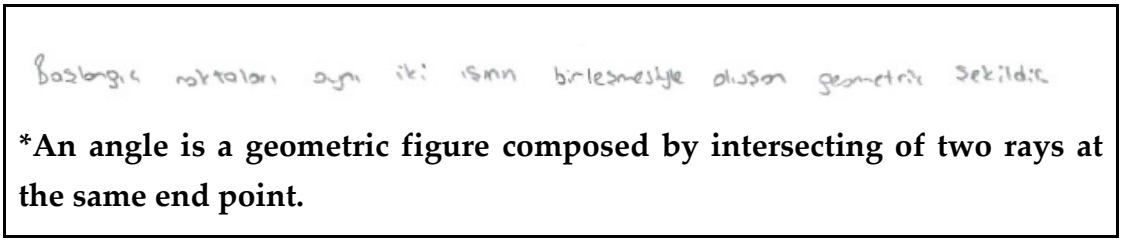
PMTs defined the angle as a pair of rays with a common end-point. Three PMTs defined the angle concept as an amount of turning about a point between two lines. The examples of PMTs' definitions of angle concepts are as the following.



***The arc drawn in counter clockwise between two rays with a common end-point is called angle.**

Figure 5: The example for the PMTs who define angle as
"*an amount of turning about a point between two lines*"

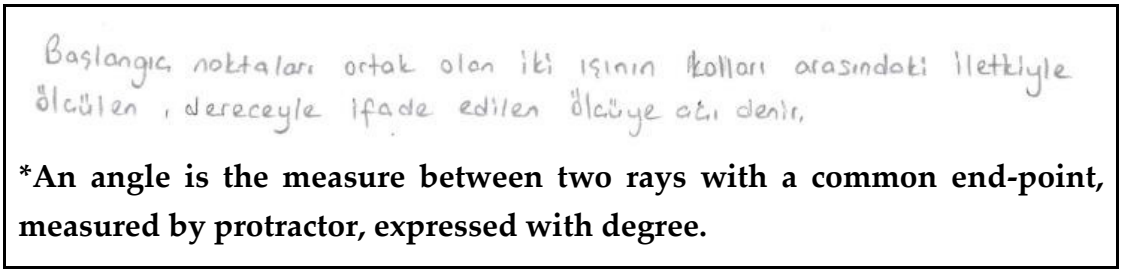
For Figure 5, it can be said that PMTs' these kinds of angle definitions were correct. As mentioned before, only 3 prospective teachers could make the definition of angle concept by referring to "*an amount of turning about a point between two lines*" and in terms of being dynamic.



***An angle is a geometric figure composed by intersecting of two rays at the same end point.**

Figure 6: The example for the PMT who define angle as
"*a pair of rays with a common end-point*"

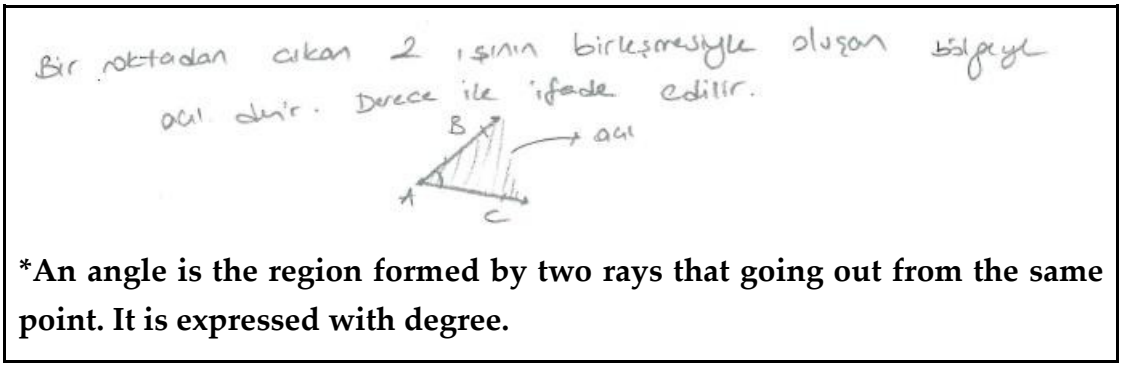
Figure 6 included an example of angle definition of PMTs which was in the category of "*a pair of rays with a common end-point*". It is seen that this definition addressed the essential concepts which were "*two rays*" and "*same point*" and represented an example of static angle definition. Totally 25 PMTs defined the angle concept as a pair of rays with a common end-point. 15 of prospective teachers defined the concept of angle correctly; however, 10 of them indicated the concept of angle in a way that had the same meaning as the measure of angle.



***An angle is the measure between two rays with a common end-point, measured by protractor, expressed with degree.**

Figure 7: The example for the PMT who define angle as
"*a pair of rays with a common end-point*"
(confused the angle and angle's measure concepts with each other)

Figure 7 shows the example of PMTs who confused the definition of angle with the measure of angle concept. It is seen that prospective teachers identified angle base on the concepts of "*two rays*" and "*same point*" but they also used the concept of "*measure*". Therefore, these definitions were accepted under the category which was named as "*a pair of rays with a common end-point*".



***An angle is the region formed by two rays that going out from the same point. It is expressed with degree.**

Figure 8: The example for the PMT who define angle as
"*the region formed by the intersection of two half-planes*"

In Figure 8, the example of PMTs' definitions of angle which were considered in the category of "*the region formed by the intersection of two half-planes*" was presented. It is seen that prospective teachers used the concepts such as "*two rays*", "*same point*", "*degree*" and "*region*" in order to identify angle concept. These definitions further exhibited the features of static angle definition because these concepts did not refer to any movement.

4. Discussion and Conclusion

The purpose of this study was to investigate the PMTs' ability to identify mistakes held by elementary students in angle concept definitions and their proposed corrections to overcome those mistakes. The findings show that PMTs didn't have problems in

determining students' definitions of angle as correct, incorrect or partially correct. Although they notice the correct angle definitions, they have problems about expressing the students' failures and to clearly identifying the mistakes in the incorrect definitions of students. The findings of Kilic (2010) also revealed that preservice teachers had difficulty in both identifying the source of students' misconceptions, and errors and generating effective ways to eliminate such misconceptions.

To correct the students' mistakes PMTs should expand their understanding of angle concept, to help them to think of the concept in a flexible way. A student can use her mistakes/errors to develop a deeper understanding of a concept as long as the error can be recognized and appropriate, informative feedback can be obtained. Pedagogical methods that systematically address common student errors produce significant gains in student learning (Fisher & Lipson, 1986). Therefore, students need teachers competent to identify their mistakes and can translate into their own benefits.

When we looked to the findings related to PMTs' angle concept definitions almost all of them define the angle as a pair of rays with a common end-point, which is a static definition. They could not be able to inform their students about the different definitions of angle concept because they do not have them. This may be due from the inclusion of static definitions in the textbooks in Turkey. Angles need to be represented as both static images of 'pointedness' and as dynamic examples of 'turns' in order to develop understanding of the topic (Barmby, Bilsborough, Harries & Higgins, 2009). If students cannot adequately comprehend basic geometric concepts they will not understand and succeed in the subsequent subjects of geometry and this may reduce the achievement of an individual in both school life and daily life (Alkan & Altun, 1998).

Results of this research suggest that; textbooks can be rearranged to include multidimensional definition of the concept of angle in Turkey. PMTs could receive more detailed training on how to teach the concept of angle during teacher education. PMTs must gain experience in assessing student responses, it may be included more intensively in the course of "*teaching practice*".

The conclusions of the present study are limited by the small sample size and the small number of contexts investigated. A more comprehensive study is currently underway and the results will be shared with another study.

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