European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.825887
Volume 3 | Issue 8 | 2017

DEVELOPMENT OF RESEARCH AND STUDY PATHS IN THE PRE-SERVICE TEACHER EDUCATION

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Abstract:
In this paper, we present results of an implementation of research and study course carried out into a teacher training course in the University. The framework of the Anthropological Theory of the Didactic (ATD) is adopted, and a co-disciplinary Research and Study Path (RSP) whose generative question requires studying mathematics and physics together is carried out by training teachers of Mathematics at University. Some conclusions concerning on the conditions, restrictions and relevance of introducing the RSP in teachers training courses at the university are presented.

Keywords: co-disciplinary research and study path, pre-service teachers training, ATD, modeling

1. Introduction

The training of mathematics teachers has been the subject of numerous investigations in the field of Mathematical Education (Cirade, 2006; Chevallard y Cirade, 2009, Chevallard, 2005; Gómez, 2007; Llinares; Valls; Roig, 2008; Godino, 2009; Ribeiro; Carrillo; Monteiro, 2010; Font, 2011; Ruiz Olarría, Bosch & Gascón, 2014). These authors emphasize the importance that the training of the mathematics teacher includes knowledge that exceeds the mathematical contents that the teacher should teach. In this line, the notion of Pedagogical Content Knowledge (PCK) developed by Shulman (1987), which specifically in mathematics, originates Mathematical Knowledge for

Teaching (MKT), as an essential mathematical knowledge in teacher training (Ball, 2000, Ball, Lubienski, Mewborn, 2001, Hill, Ball, Schilling, 2008).

According to Chevallard´s Anthropological Theory of the Didactic (ATD), that is the framework of this work, the training of mathematics teachers requires professional knowledge whose construction and development is the responsibility of the community of researchers in didactics of Mathematics, in close collaboration with the teaching profession (Chevallard and Cirade, 2009). The didactic phenomenon called monumentalism is characteristic of the paradigm of the "visit of the works" and has been described by Chevallard (2001, 2012). This paradigm focuses on the teaching of answers rather than questions, ignoring the fact that knowledge always arises as a response to a question, which if hidden, leads to the presentation of scholastic knowledge that lack motives and reasons for being. Knowledge is signaled as if it were an historical monument, which at most is seen and venerated. To overcome the monumentalism prevailing in educational systems, the ATD has proposed the Paradigm of Research and Questioning the World (Chevallard, 2012, 2013a) advocating an epistemological and didactic revolution of the teaching of mathematics and school disciplines (Chevallard, 2012), where knowledge should be taught by its usefulness or potential uses in life.

However, to opt for such paradigm, it is necessary to train future teachers in a different way, to provide them with the necessary equipment to develop a teaching based on questions. It is very difficult to put into practice a teaching based on questioning and inquiry, since it requires systems of teacher training that are appropriate, and are not generally available. In this paper, we describe the results obtained in two courses of mathematics teachers in training at the university, when addressing a question that places future teachers in an investigation and questioning situation.

The results were obtained in two courses of pre-service mathematics teacher education I1 (N=12) and I2 (N=13) during a teaching inspired in the paradigm of questioning the world, by means of a Research and Study Course (RSC). To learn what an RSC is, and which kind of teaching is involved in, the trainee teachers (TT) must deeply experience a genuine RSC. Thus, a physics and mathematics co-disciplinary RSC was designed implemented and analyzed with the students. In this context co-disciplinary means that, physics does not only trigger the study of mathematics, but rather that both disciplines play a central role, being necessary to study both of them. The starting point of the RSP is the question **Q_0: Why did the Movediza stone in Tandil fall?** Which, to be answered – in a provisional and unfinished way- needs the study of Physics and Mathematics jointly.

The rationale of the paper is to describe the trainee teachers' activities and their difficulties when they must experience a genuine RSP and to face a strong question.

2. The research and study courses

The ATD defines the RSP as devices that allow the study of mathematics by means of questions. The RSP establish that the starting points of mathematical knowledge are questions called generative questions, in the framework of the ATD, because its study should generate new, derived, questions. Teaching by means of RSP is complex and demands rootle changes in the roles of the teacher and students.

The study of a question Q as starting point of knowledge supposes the emergence of a didactic system denoted by $S(X;Y;Q)$. In the case of a mathematics classroom, this means that a group of students (X) helped by one or more teachers (Y) will build an answer R to the question Q. The operation of this system responds to a scheme that Chevallard (2013b) calls Herbartian scheme. In its reduced form, this scheme is written as follows: $S(X;Y;Q) \hookrightarrow R^{\heartsuit}$.

The symbol ♥ indicates that the answer to Q was produced under certain constraints, "working" as a response to that question under those limitations (Chevallard, 2009). The elaboration of $R^{\heartsuit}$ from Q needs the "fabrication" of a didactic medium M. This is expressed by the semi-developed Herbartian scheme:

$$[S(X;Y;Q) \rightharpoonup M] \hookrightarrow R^{\heartsuit}$$

That is, the didactic system constructs and organizes ($\hookrightarrow$) the medium M with the aim to generate or produce ($\hookrightarrow$) an answer $R^{\heartsuit}$. This scheme indicates that the elaboration of M is articulated in a complex way with the elaboration of the response. This observation is applied in the developed Herbartian scheme (Chevallard, 2009), which is written as follows:

$$\left[S(X;Y;Q) \rightharpoonup \{R_1^{\diamond}, R_2^{\diamond}, R_3^{\diamond}, \dots, R_n^{\diamond}, Q_{n+1}, \dots, Q_m, O_{m+1}, \dots, O_p\}\right] \hookrightarrow R,$$

where $M = \{R_1^{\diamond}, R_2^{\diamond}, R_3^{\diamond}, \dots, R_n^{\diamond}, Q_{n+1}, \dots, Q_m, O_{m+1}, \dots, O_p\}$ is the didactic medium to study (Q). The available responses $R_i^{\diamond}$, the derived questions Q_j and other Works O_l are potential instruments to study Q. These instruments have to be conveniently studied in "quality" and "quantity", to be used effectively and efficiently in the study of Q, that is in the construction and validation of $R^{\heartsuit}$ (Chevallard, 2009). The objects noted by $R_i^{\diamond}$, with $i = 1, \dots, n$ are "already made" answers available ,for example, a book, a web page, a teacher's course, etc. The entities Q_j with $j = n + 1, \dots, m$ are other works - for instance, theories, experimental setups, praxeologies, etc., considered useful to deconstruct *R* and extract what is necessary there to construct the response $R^{\heartsuit}$ (Ibid). Introducing the developed Herbartian scheme, Chevallard (2012) specifies what can be described as a study and research course (RSP).

Some important characteristics of the RSP are the following:

a. The RSP are generated by a question Q_0 called generative. A question whose answer is not immediate and it allows the formulation of sub-questions, called derived questions (Chevallard, 2004).
b. The search for answers to the questions refers not only to the construction or reconstruction of knowledge but also, to the information search and their consequent analysis and evaluation, as an indispensable resource that contributes to the construction of responses. This process generates diverse levels of action that are indispensable to developing the RSP: observing existing responses, analysing them, evaluating them, developing a new response, and finally defending the response produced (Chevallard, 2012).
c. During the development of a RSP, the didactic medium *M* is constructed at the same time that the answers to the questions are generated, it is not previously determined, it is constructed throughout the course and any component accepted and validated by the study community could be incorporated.
d. In a teaching by means of a RSP, the study community is not limited to the group composed by the teacher and the students. This study community expands and incorporates, at least momentarily, any person or institution being useful or contributing pertinent work to the construction of the answers.
e. The teacher is considered the director of the study process (Chevallard, 2009), being a source of information as any other media, although responsible for guiding the study process. The teacher does not occupy the central place in the class, and is not considered the sole source of knowledge.
f. Students broaden their possibilities for action; they ask questions, propose resources and sources of information. They construct and respond to the questions, evaluate, disseminate, defend, and receive critically the responses of other students (Chevallard, 2012).
g. Knowledge is considered in an integral way, that is to say, as a set of interrelated knowledge. This articulation between knowledge is generated in the modelling processes. The Modelling is considered like a process of reconstruction and articulation of knowledge of increasing complexity (Barquero, Bosch and Gascón, 2011), where the RSP allows proposing increasingly broad generative issues. Modelling is not to "apply" a mathematical knowledge to a situation in context, on the other hand, modelling involves delimiting a system, for example

economic, physical, biological, etc., describing the variables of the system, the relationships between the variables, formulating a first model and testing it and refining it as much as necessary until it is able to construct a reasonable response that can be validated and accepted by the study community.

Before facing the actual teaching process by means of an RSP, there is a priori analysis stage, where the specific and didactic knowledge which could be involved within an RSP is set up and the Epistemological Model of Reference (EMR) is elaborated. Here the potential set of questions which the study and the research that Q_0 might encompass, together with the knowledge, mathematics and physics in this case, necessary to answer those questions is analyzed (Chevallard 2013a). In summary, the EMR underlies the whole of the teacher, student and researcher´s activity, being always likely and desirable to identify and clarify it, emphasizing the dynamic nature of the EMR.

3. Material and Methods

3.1 Questions

We have structured our work around the following questions that guide our research

1. Which was the role of the students and the teacher during the RSP?
2. Which mathematical and physical contents were studied along the RSP?
3. Which mathematical and physical models were developed by the students during the RSP?
4. Which were the most relevant constraints to develop the RSP in this level?

3.2 Methodology

This work involves a qualitative and exploratory research that aims to carry out a research and study course as it is proposed by the ATD, in a mathematics teacher training course at the University. The RSP was implemented in a state university, in the city of Tandil, Argentina, in a discipline which is part of the didactic studies within the Mathematics Teaching Training Course, in which two of the researchers are also teachers. There were two implementations, where N=12 and N=13 students from the last year (4^{th}), aged 21-33 took part in it.

It is important to notice that these students had had not studied physics before at the university, but had a relatively strong mathematical formation. In addition, the students had studied the ATD in two previous Didactics courses; however, they report difficulties to understand what an RSP is, and how it works? In this respect, we propose to design, implement and analyze a physics and mathematics co-disciplinary RSP, adapted to the institution in which it is developed.

As we have mentioned, the RSP is genuinely co-disciplinary in the sense that the interplay between physics and mathematics plays a central role and calls the study of both disciplines on an equal footing.

The RSP was carried out in a total of 7 weekly hours provided in two lessons per week. In both implementations, which we will identify as I1 and I2, respectively, three work groups were organized with approximately 4 members each.

In a RSP, the generative question Q_0 has to be pointed out by the teacher, and this was made in the first lesson. Then, the students started their research in the library, by selecting some texts, documents etc. as possible $R^{\lozenge}_i$. In every class, each group presented and discussed with the teacher and the other groups their findings and possible ways to face Q_0. In the second class, many derived questions Q_i were made explicit by the students, and the community of study selected the questions Q_i to be studied as well as their related knowledge O_k. The regular dynamic during the RSP was characterized by the roles of the teacher and students described in the previous section of this text.

Recordings of each class were obtained and the student productions were digitalized and returned in the subsequent class. The teacher wrote a class diary writing down the tasks performed by the study group. The remaining researchers of the team performed non-participant observation during classes. The data analysis was performed by using the categories provided by the *Developed Herbartian model* (Chevallard, 2013) summarized before.

3.3 The Epistemological model of reference (EMR) and the RSP

As we mentioned, the starting question Q_0 is: Why did the Movediza Stone in Tandil fall down? This enormous basalt stone has remained the city's landmark, providing it with a distinctive feature. Many local people and national celebrities visited the place to closely observe the natural monument. It was a 248-ton rock, sitting on the top of a 300-meter-high hill (above sea level), which presented very small oscillations when disturbed in a specific spot, (Figure 1). Unexpectedly, on February 28, 1912, the stone fell down the cliff and fractured into three pieces, filling the town with dismay by the loss of their symbol. For over 100 years, the event produced all kinds of conjectures and legends for the causes of the fall. Within the two groups where the RSP was performed, there existed a certain curiosity and interest in finding a scientific answer to this question. Once in contact with the available information, the question evolved into: What are the conjectures about the causes the Movediza Stone fall, and which is the most likely from a scientific viewpoint? Assuming that the fall can be explained by means of the Mechanical Resonance phenomenon, several questions Q_i emerged which are linked to the physical and mathematical knowledge necessary to answer Q_0.

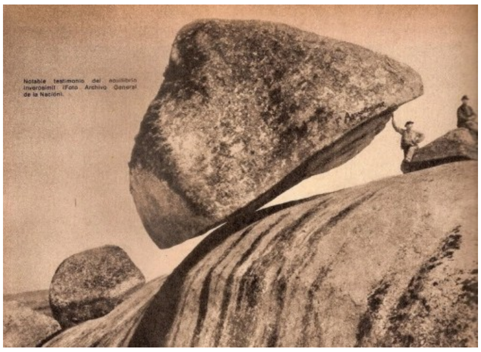


Figure 1: Photography of the Movediza Stone (Photo Archivo General de la Nación Argentina, available in: http://bibliocicop.blogspot.com.ar/2012/02/piedra-movediza-100-anos-de-su-caida.html)

If we consider that the stone was an oscillating system, the study can be carried out within the Mechanic Oscillations topic, starting from the ideal spring or the pendulum. In this case, frictionless systems are used, in which the only force in action is the restoring force depending (for small amplitude oscillations) in a linear way on the deviation respect to the equilibrium position. This model is known as simple harmonic oscillator whose motion, via Newton equations, is described by a second-order linear differential equation.

Progressively, the system becomes more complex. If friction-produced damping is considered, it provides a new term to the differential equation connected to the first derivative of the position (speed). Finally, it is possible to study systems that apart from being damped, are under the influence of an external force, and therefore called driven systems. In the case that the external force is periodic and its frequency is approximately equal (the order of the approximation will be clarified later) to the natural (free of external forces) frequency of the oscillating system, a maximum in the oscillation amplitude is produced, generating the phenomenon known as mechanical resonance.

By increasing the complexity of the model, it is possible to consider a suspended rotating body, instead of a punctual mass. This leads to the study of the torque and the moment of inertia of an oscillating body. Here again, the linear system is for small amplitude oscillations and the damped and driven cases can be also considered, corresponding to the same mathematical model, but in which the parameters have a different physical interpretation.

However, as it refers to a suspended oscillating body, this is not a suitable physical model for the Movediza stone system. Since that, the base of the Stone was not flat, it is necessary to consider more precise models of the real situation. This leads to the mechanics of supported (and not hanging) oscillating rigid solids. In this case, we consider a rocker-like model in which the Movediza stone base is curved and it lies on a flat surface, where the oscillation is related to a combined translational and rotational motion (Otero et al. 2016a 2016b).

The application of Newton laws to the rocker model of the stone leads to a differential equation where the parameters are specific of the Movediza system: mass, geometry, inertia moments, friction at the base, external torque, etc., which is given by the following *effective* Harmonic oscillator mathematical model of the Movediza physical system:

$$\ddot{\varphi}+\gamma\dot{\varphi}+w_0^2\varphi=(M_0/I)\cos(wt) \qquad (1)$$

The stationary solution to equation (1) is

$$\varphi(t)=\varphi_M\cos(wt-\psi)$$

being the amplitude ϕ_M and the phase ψ

$$\varphi_M=\frac{M_0/I}{\sqrt{(w_0^2-w^2)^2+w^2\gamma^2}} \qquad \psi=tg^{-1}\left(\frac{\gamma w}{w_0^2-w^2}\right) \qquad (2)$$

The maximum of φ_M is for $w_m=\sqrt{w_0^2-\frac{\gamma^2}{2}}$. The parameters: M_0 (external torque), I (inertia moment), w_0 (natural oscillation system frequency) and γ (damping coefficient), must be estimated. Detailed data about the shape, dimensions and center of mass position of the Movediza stone are available (Peralta et al. 2008) after a replica construction and its relocation in 2007 on the original place (although fixed to the surface and without possibility to oscillate). These data bring us the possibility to estimate some parameters in our model, as e.g. mass, inertia moment, and the distance of 7.1 m, from which the external torque could be exerted efficiently by up to five people (according to historical chronicles) to start the small oscillation. By using these values, it is possible to study the behavior of the $\varphi_M(w)$ function for w_0 in a range of frequencies between 0,7 Hz and 1 Hz, historically recognized (Rojas, 1912) as the natural oscillation frequencies in the Movediza stone system and calculate for each case the maximum amplitude $\varphi_M(w_m)$.

The Stone would fall if $\varphi_c\le\varphi_M(w_m)$, being $\varphi_M(w_m)=M_0/w_0I\gamma$. Note that if γ is very small (as is expected to be in this case) we can neglect it from $w_m=\sqrt{w_0^2-\frac{\gamma^2}{2}}$

,leading to $w_m \approx w_0$, which is the approximation that we mentioned in the previous section and we will use hereafter. By using this approximation in Eq.(2) (left) the falling condition becomes $\varphi_c \leq (M_0 / w_0 I\gamma)$.

The value of φ_c can be determined by an elementary stability analysis, which per the dimensions of the base of the stone and the center of mass position is estimated to be approximately of 6°.

Note that in the present model γ is a free parameter, for which we set "ad doc" a magnitude order $\gamma \geq 10^{-2}$. This is justified in the frame of a more sophisticated model that we will comment briefly below. With this constraint, we find several situations, comprising different torques within the mentioned frequencies interval, supporting the overcoming of the critical angle, i.e., predicting the fall.

Finally, in search of a more appropriate approximation of the physics model for the damping that is clearly not due to air, we consider a more sophisticated model of the stone as a deformable solid, where the contact in the support is not a point but a finite extension, along which the normal force is distributed, being larger in the motion direction and generating a rolling resistance, manifested through a torque contrary to the motion. The rolling resistance depends on the speed stone, giving a physical interpretation to the damping term. Therefore, the physics behind the damping is the same that makes a tire wheel rolling horizontally on the road come to a stop, but in the case of the stone, the deformation is much smaller. Although the deformable rocker model has extra free parameters, tabulated values of rolling resistance coefficient for stone on stone, which are available in the specialized literature, allowed us to estimate and justify the damping values that we incorporate otherwise ad-hoc in the rigid rocket Movediza model.

4. Description of the RSP developed in each implementation

To describe the RSPs we consider the components in the Herbartian model, analyzing all collected data for each course. We discuss, based on those components, the RSP experienced in each implementation.

During the implementations, the students aim at answering how and why the stone fell down the cliff. The TTs busily searched for an "already-made" mathematical and physical model, which allowed them to solve a differential equation in a specific way. Initially in both implementations, several physical and mathematical questions arose; the main preoccupation of the TTs was to study the oscillations and resonance topics, because it was a thoroughly new knowledge to them. In both groups, the underlying mathematics did not seem to present difficulties at the beginning, given that in parallel they were carrying out a differential equations course.

In both implementations, the study was dominated by the need to find a physical model suitable for the situation. Among all the already made answers $R^{\lozenge}_i$ they found, the TTs decided on the physical pendulum model initially, whose mathematical model might be adequate to the problem, although physically inadequate. However, the path performed in the first implementation was different from the second as we can detail below.

A. The First Implementation I1

In the Table 1 we summarize, class by class, the RSP developed in seven lessons with twelve students,. In the first column, we describe the most relevant actions performed in every lesson by the study group (SG) (teacher and students). In the second column, we detail the derived questions pointed out by the students, and in the third column we describe the "already made answers" founded by de study community and introduced into the medium *M*. Finally, in the last column of the Table 1, we consider the knowledge field area that has been effectively studied.

Lesson 1: Introducing Q_0 and analyzing derived questions			
Main activities of the SG	Qi (Derived questions)	$R^{\lozenge}i$	Om
The teacher pointed out Q_0 Students searched in the library and internet possible R◊i. They analyzed several hypotheses and pointed out new questions. The teacher asked which conjectures could have scientific support.	How did the Stone arrive to the top of the hill? Did it fell by itself or was it thrown? In this case, how could this have happened? Could the recurring quarry explosions have caused the fall? Were nature phenomena like lightning strikes, the wind or the erosion, the causes of the fall? Could have fallen by the wear of the base? Or for an attack, a blast? Why is there no evidence of blasting? How much force would it take to make the stone fall? Is man able to do it?	Holmberg (1892) El Hage, Levy (2012)	

	Which was the weight of the stone? How was the base of the Movediza Stone (MS)?		
Lesson 2: Analyzing the Equilibrium of the Stone			
The SG analyzed the Holmberg's article proposing "the stone fell by successive accumulation of impulses". G1proposed to study oscillations, G2 proposed to consider the harmonic oscillator because it involves trigonometric functions, and G3 proposed to study forced oscillating systems, supposing that the man threw the stone.	How is an oscillation described? Was the oscillation constant? If it was not constant, which factors influenced the variation? Which kind of oscillations are there? Why are there no sliding marks on the base? Is it because the stone "jumped"? How to explain this jump physically?	Holmberg (1892) Internet (on line physics)	Stone's morphology (Center of mass (CM), base, hill surface, weight) Equilibrium analysis. Free, damped and forced oscillations
Lesson 3: Studying the oscillating systems as possible models of the Stone			
The stone is considered a forced and damped oscillating system. Springs, single pendulum, and physical pendulum models are considered inappropriate. The physical pendulum (PP). Resonance is only mentioned as a particular case. The teacher asked about the differences between the different models.	Which oscillation model would be the most appropriate to describe the stone? Is the oscillation model free, damped or forced? Which mathematical model should be used?	Alonso, Finn (1992) Differential equations course. Boyce (2005)	Oscillating systems (spring, pendulum) PP Differential equations (DE)
Lesson 4: Exploring the physical Pendulum as a model of the Stone			
The teacher asked about the differences between the answers of G2 and G3. The SG decides on the physical pendulum as the most appropriate model for MS. The characteristics of the differential equation are established, the students add other questions.	Which are the physical pendulum motion equation and its solution? What are the parameters? Which was the natural (characteristic) oscillation frequency of the stone? Which is the critical angle of oscillation to fall? How should the force applied to the stone be to cause resonance?	Alonso, Finn (1992). DE course	PP DE

Lesson 5: Damped and forced Physical Pendulum: focusing on Resonance and Moment of Inertia			
The SG studied the resonance phenomenon in the case of the damped and forced physical pendulum, according to the physics textbooks. The main problem was how to calculate the moment of inertia (I)? The SG looked for regular solids as a good approximation of MS. The I was calculated for diverse regular solids, using the Steiner theorem to change the axis of rotation	How would the system resonate? What is the moment of inertia? How is the moment of inertia calculated? What information do we need to calculate I? How to calculate I for irregular solids?	Resnick, Halliday, Krane (2001) Physical Pendulum. Torque, I. Steiner's Theorem Resonance.	Calculation of I for regular solids Resonance phenomenon
Lesson 6: Damped and forced Physical Pendulum: Resonance and Differential Equations			
Discussion about the calculations of I performed by the students for regular solids. Approximation of MS as a truncated cone. Determination of the natural frequency, w0, using calculated I. The damping coefficient γ and torque were not determined, but the students spent some time estimating them.	Which regular solid could approach the stone shape? Is it possible to calculate the damping coefficientγ? Which is the maximum amplitude of oscillation of the stone? Which are the stone dimensions?	I for regular solids. Torque	Linear differential equations of the second order. Parameters
Lesson 7: Analysis of the solutions of the Differential Equation: parameters			
The teacher proposed to verify the solution of the differential equation and estimates the parameters.	Which could be a reasonable γvalue? How large was the external force that caused the resonance?	Stone's morphology (Peralta, et. Al., 2008).	ED parameters. I for a Cone

Table 1: Summary of the RSP developed in the first implementation

As can be observed in the Table 1, during the first two Lessons different alternatives were explored about the causes of stone fall. Among them, the most accepted one, proposed a fall generated by resonance, due to the repetitive action of an external agent, possibly several people. This gave rise to the study of oscillating systems. Note that in Lesson 3 students already mentioned the physical pendulum, which was accepted without question.. On the other hand, in Lessons 5 and 6, the SG spent most of the time in the study of inertia moment concept and their calculation for regular solids, which would result in an appropriate model for the irregular shape of the stone. Several questions were therefore generated, for which the answers were elaborated. The students who had understood the resonance phenomenon for springs, ideal and physical pendulums, calculated the characteristic frequency of the system, making use of the moment of inertia previously obtained. Thus, only a parameter resulted undetermined: the damping. However, in the end as the Table 1 and Table 2 show, the physical pendulum model became an obstacle, because the stone was a supported body, and not hanged. On the other hand, the damping they considered was due to air-friction, whereas in the case of the Movediza stone the main source of friction is the contact with the support surface.

At this point, the external torque (there were different trials to analyze and estimate it) and the solution of the equation remained unstudied. Until this moment, the solution for the differential equation did not seem to present any obstacles. They considered they were facing an initial value problem. Once they had obtained the parameters, which they considered fixed or "given by somebody", the solution seemed simple. However, they had problems to arrive at a final solution, even though this can be found in textbooks of Physics (usually without the deduction). For this reason, it was discarded and they decided to do the calculation on their own. This event complicated the quantification they aimed to obtain, as well as the physical interpretation.

Some groups in this cohort removed the term of damping, to reduce degrees of freedom; although the MS would have been in perpetual motion, this did not create any contradiction to them. Others adopted a damping value due to air-friction, which also led to wrong results.

Finally, he TT's dismissed the solution that was presented in physics textbooks, and they did not interpret the answer in the texts concerning the Stone. Instead, they got complicated with the solution of the differential equation and had to resort to the teachers of the differential equations topic, obtaining thus the notion that it was "a particular case" and "beyond the syllabus". They also drew upon some physicists from the institution, and got hints indicating that the solution was not immediate and needed some elaboration to be found. Considering that the context of the study is a faculty of Sciences, the surprise and discomfort produced by the questioning on the part of some of the students, indicates the magnitude of the drawbacks that the interdisciplinary research teaching must face in the University, as it is opposed to the mainstream pedagogy in this institution.

In summary, in this implementation two fundamental problems were identified that hampered the development of RSP. Firstly, the inadequate decision of the teachers about not intervening to object the physical pendulum model as an inappropriate model to describe the Stone, and secondly, when students strayed in the search for the ED solution, unlike what they did in the second implementation, as we will see below. These teacher's decisions could be justified by certain misunderstanding about the role of the teacher as director of the study in an RSP, and time constraints imposed by the class schedule, which were close to finalizing. The teachers decided on the idea that all physical models conceived in the EMR leads to the same mathematical model.

Therefore, the answer constructed in the first implementation, I1, is the possible and not definitive answer, according to the conditions of this study community. This also evidence the difficulties faced by any teacher developing and RSP, considering that here the teacher is also an experienced researcher in the ATD. Finally, we would like to mention that there were also difficulties for the TTs to understand the utility and necessity of mathematical models, an aspect that could be related to an epistemological conception close to pure or formal mathematics. All these mentioned difficulties were considered for the second implementation of RSP, that we consider in the following section.

B. The second Implementation I2

In the second cohort (I2), the researchers had already perceived that the fundamental problems seemed to be in the physical models and in the functional modelization understanding. The TTs of the I1did not understand the utility of the mathematical model, neither the role that the parameters could play, which were considered as given, fixed and universal. In consequence, they failed to establish different sets of parameters and did not generate the feasible families of functions and values, whose compatibility with the physical situation could have been analyzed. For this reason, it was decided to devote eight sessions to the development of two intra-mathematical RSPs (Chappaz and Michon 2003; Ruiz et al. 2007), that the TTs could experience in their own flesh, therefore emphasizing the role of the modelization and the use of devices as spreadsheets and graphics software. In the Table 2 we summarize the RSP developed in eleven lessons with thirteen students, class by class, where the columns have the same meaning described in the Table 1.

Lesson 1: Introduction of Q_0 and analysis of derived questions			
Main activities of the study group (SG)	Q_i	$R^\lozenge_i$	O_m
The teacher pointed out Q0. Students searched in the library and internet possible R◊i. There was agreement about that the Movediza stone was in balance and it moved only if perturbed. The students conjectured about the support and the shape of the stone and the hill. They began to study the types of equilibrium; others wanted to know what an oscillation is.	Which hypotheses would be scientifically treatable? Did it fall by the explosions of the quarries, by blasting, because a lightning stroke? Did the stone always move or only if someone "touched it"?	Holmberg (1892) El Hage, Levy (2012). Rojas (1912)	Equilibrium analysis (MS was in balance, it moved if disturbed)

The students pointed out new questions.	What kind of equilibrium did it have; stable, unstable neutral? What caused the imbalance? The stone fell alone or was shot? Is it possible that men have thrown the stone?		
Lesson 2: Equilibrium analysis of the Stone			
Stone's morphology. Hill contact surface. Characteristics of the support where the stone fits MS had a kind of stump which was embedded in the hill. Among the different types of equilibrium, the stone was in stable equilibrium The main topics are oscillations and resonance.	How is the MS morphology? How was the stable equilibrium of the MS disturbed? Why did the stone fall and did not return to the steady state of equilibrium? What kind of motion did the MS have?	Peralta, et. Al., (2008) Ercoli (2015) Tipler (1994)	MS morphology and (reconstruction)
Lesson 3: Study of oscillating systems as possible models of the Stone			
The SG divided the study, G1 studied the harmonic oscillator (HO) (spring and simple pendulum), analyzing the solutions of the motion equations with geogebra. G2 (analyzed the differences between the HO, forced and damped oscillations in the spring case considering amplitude-time graphical representations in each case G3 considered the HO in the context of the physical pendulum. They had difficulties to return to the original problem.	What an oscillation is? How many kinds of oscillations are there? What kind of oscillation had the stone?	Alonso, Finn (1992) Resnick, Halliday, Krane (2001) Tipler (1994) Elmer (2011).	Oscillating systems PP
Lesson 4: Exploration of the physical Pendulum as a model of the Stone			
The teacher proposed to fill the table showed in the figure 3. The teacher and students analyzed the models and the meaning of the angular frequency, w, the equation of motion	How many models are there? How do they differ or not? Which is the closest to	Alonso, Finn (1992) Resnick, Halliday, Krane (2001)	Models, Equations of motion and parameters

and its solution for each case. Regarding damped and forced oscillations for the spring, the corresponding terms of the equation and the meanings of the parameters were also considered. The SG adopted the PP as the most appropriate model to MS.	PM?	Tipler (1994) Elmer (2011).	
Lesson 5: Damped and forced Physical Pendulum: Moment of Inertia			
Students noted that the table (Fig. 3) shows that the mathematical model is the same, but not the physical model. The parameters represent different properties of the system. The SG decided to study only the PP. The teacher demanded to the students to verify the solution of the motion equation presented in the textbooks, since they know differential equations	What the moment of inertia is? What is the maximum amplitude of oscillation in this model? What damping does the stone have?	DE course. Boyce (2005) Zill (2005) Elmer (2011) Landau, Lifschitz (1991)	Moment of Inertia. Rotation axes. Torque PP, DE
Lesson 6: Damped and forced Physical Pendulum: Resonance and Differential Equations			
The students verified the solutions of the DE and they arrived at the solution provided in the textbooks, helped by a text wrote by the teacher. They studied the resonance condition and analyzed the amplitude function to determine the maximum.	Which are the conditions for resonance? What happens with the resonant system? Which is the maximum amplitude of the physical pendulum model of the MS? What is the role of damping?	The function amplitude has a maximum (for small γ) at $\varphi_M = \frac{M_0}{w_0 \cdot I \cdot \gamma}$.	Resonance Solutions of DE
Lesson 7: Discarding the Physical pendulum			
Some students presented strong objections to the PP model. MS is a supported body, not hanged like a PP. Then if we "turn down the PP", the stone would have unstable equilibrium. The GS returned to the supported body model considering that it is more appropriate for PM. The critical angle of oscillation $\phi c = 6°$ was obtained from geometric calculation considering a supported base plane.	Is the PP a good model for the MS? Would there be a model? Which is that model?	Holmberg (1892)	Stability analysis.

Lesson 8: The rocker model			
The model of the rocker, a rigid solid with a combined, oscillating and rolling, motion is introduced by the teacher. A website with physlets was used. The equation of motion is analyzed and the students add the terms of damping and external force. Conclusion: The DE is the same, only the parameters change.	What is rolling motion? Rolling motion is with or without sliding? Which corresponds to the stone? What is the kinetic energy? What is rotational kinetic energy? And mechanical energy?	Teacher's text García (2010)	Rolling motion of rigid solids. Rocker model , DE
Lesson 9: Analysis of the solutions of the Differential Equation: parameters			
Analysis of known and unknown parameters. The moment of inertia was calculated using the data provided by Ercoli (2007). The critical angle had already been calculated. The value w0 = 6,28hz was taken from Rojas (1912). Students failed using various values for the parameters, because they conceive them as fixed and unique, "the" value of γ.	Which are the parameters? Why the parameters are no unique and fixed?	Teacher's text Peralta, et. Al., (2008) Ercoli (2015) García (2010)	ED solutions.Families of functions. Variables and parameters
Lesson 10: How the model works? Different scenarios by means of the families of functions			
The main problem of the students was to recognize the solution as a family of functions. Based on their questions, the teacher proposed to analyze this family of functions by means of spreadsheets and graphics software, varying the different parameters. The students mainly proposed the use of GeoGebra, using sliders for M0, γ, w0. They emphasized the relevance of γ, determining the variation of the maximum amplitude for different values of γ. The students estimate γ to be the order of 10-2, specifically between 0.01 and 0.02, depending on the students (2 to 5) considered. Finally, the students conclude that there is no single set of parameters that support the fall of the stone, as originally they thought.	How does the function $\varphi_M(w)$ behave by varying the parameters? What parameter values should be taken to make the stone fall? So, why did the Stone fall?	Teacher's text	ED solutions Families of functions. Variable and parameters

Lesson 11: Communicating R♥			
Finally, each group made a presentation, justifying the fall due to resonance hypothesis initially considered.			

Table 2: Summary of the RSP developed in the second implementation

As the Table 1 and Table 2 show, in both implementations, as a fixed route that is inevitably set by the textbooks, the TTs came across the physical pendulum (see lessons 1-6). However, in the I2, in the lesson 7 of the Table 2, some students presented strong objections to the possibility of using the physical pendulum model in the case of the stone, not so much in relation to a body that is supported but as an "inverted" pendulum. The figure 2 shows some pictures illustrating this questioning. This caused the discussion to be directed once more towards the real system and the point of support, so that the RSP went through the models which refer specifically to the system and that are not, usually present in elementary textbooks, like the rocker.

Furthermore, within the RSP developed in I2, the teacher acted in different way, as soon as the pendulum and spring models emerged. One group studied the AMS for the simple pendulum, the spring, and the physical pendulum, another group studied the spring model in all its possibilities and the third one did not develop further than the AMS in simple pendulum and spring. The synthesis stage corresponding to that class (Lesson 5) allowed the production of a complete answer for the three models and their possibilities, from which the TTs of the I2 arrived at the conclusion that the same mathematical model represented nine different physical systems (Figure 3).

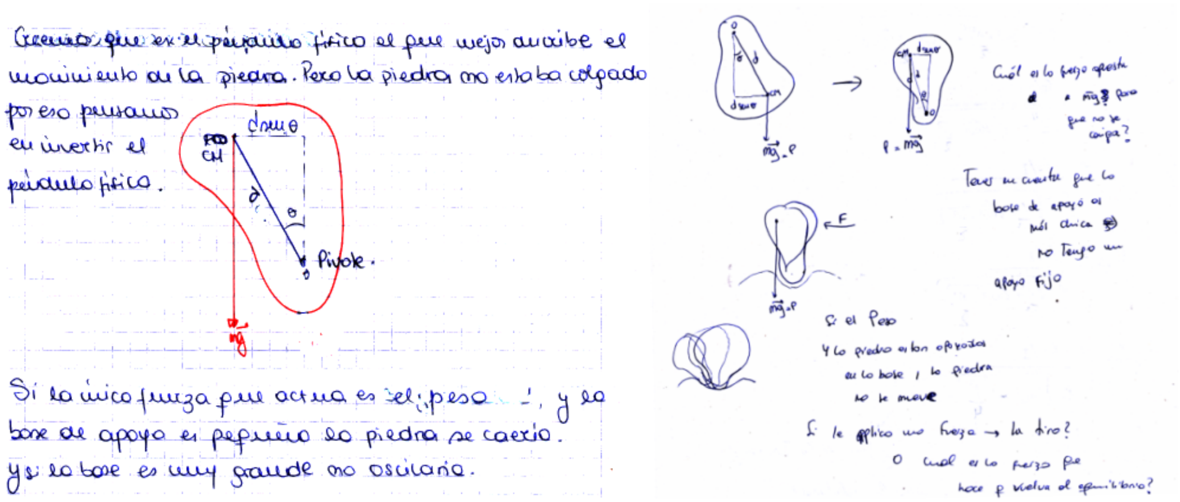
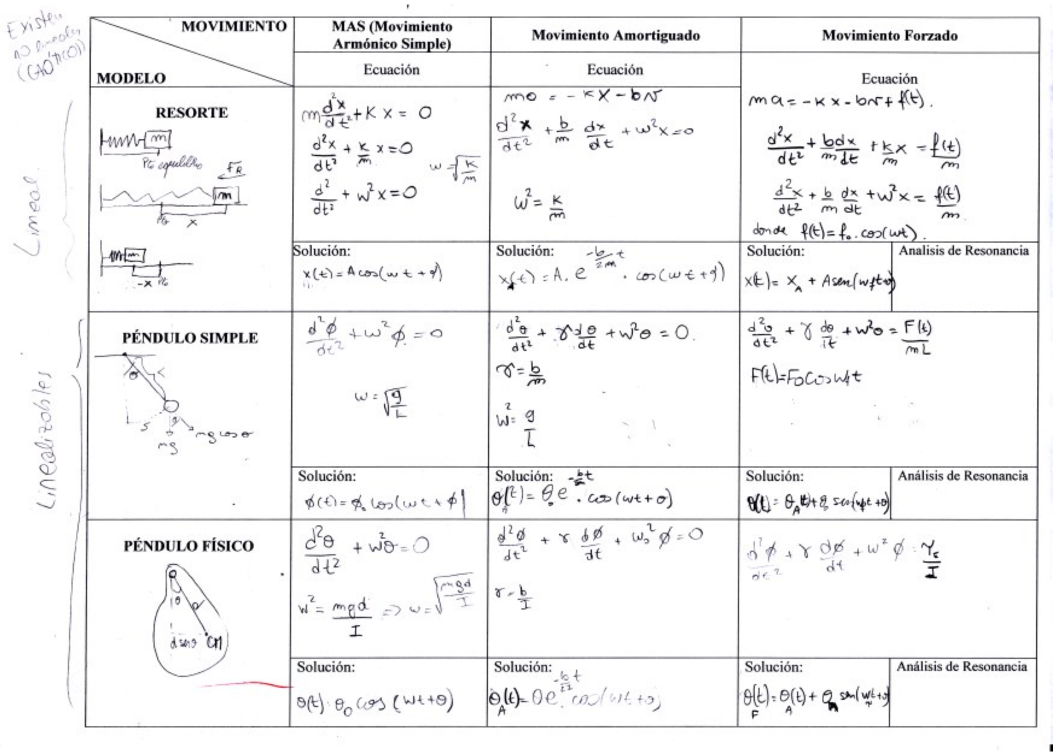


Figure 2: Protocols of the students TT21 and TT24 of implementation 2

The lessons 4 and 5 were devoted to pondering on the differences and similarities between the mathematical and physical models and their connection with the real system to be modelled. After that, the answers to the equations presented in the textbooks were checked out.



MODELO \ MOVIMIENTO	MAS (Movimiento Armónico Simple)	Movimiento Amortiguado	Movimiento Forzado	
	Ecuación	Ecuación	Ecuación	
RESORTE	$m\frac{d^2x}{dt^2}+Kx=0$; $\frac{d^2x}{dt^2}+\frac{k}{m}x=0$; $\omega=\sqrt{\frac{k}{m}}$; $\frac{d^2}{dt^2}+w^2x=0$	$ma=-kx-bv$; $\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+w^2x=0$; $w^2=\frac{k}{m}$	$ma=-kx-bv+f(t)$; $\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{f(t)}{m}$; $\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+w^2x=\frac{f(t)}{m}$; donde $f(t)=f_0 . \cos(wt)$	
	Solución: $x(t)=A\cos(wt+\phi)$	Solución: $x(t)=A.e^{-\frac{b}{2m}t}.\cos(wt+\phi)$	Solución: $x(t)=x_A+A\operatorname{sen}(w_f t+\phi)$	Analisis de Resonancia
PÉNDULO SIMPLE	$\frac{d^2\phi}{dt^2}+w^2\phi=0$; $w=\sqrt{\frac{g}{L}}$	$\frac{d^2\theta}{dt^2}+\gamma\frac{d\theta}{dt}+w^2\theta=0$; $\gamma=\frac{b}{m}$; $w^2=\frac{g}{L}$	$\frac{d^2\theta}{dt^2}+\gamma\frac{d\theta}{dt}+w^2\theta=\frac{F(t)}{mL}$; $F(t)=F_0\cos w_f t$	
	Solución: $\phi(t)=\phi_0\cos(wt+\phi)$	Solución: $\theta(t)=\theta_0 e^{-\frac{b}{2}t}.\cos(wt+\sigma)$	Solución: $\theta(t)=\theta_A(t)+\theta_0\operatorname{sen}(w_f t+\theta)$	Análisis de Resonancia
PÉNDULO FÍSICO	$\frac{d^2\theta}{dt^2}+w^2\theta=0$; $w^2=\frac{mgd}{I}\Rightarrow w=\sqrt{\frac{mgd}{I}}$	$\frac{d^2\phi}{dt^2}+\gamma\frac{d\phi}{dt}+w_0^2\phi=0$; $\gamma=\frac{b}{I}$	$\frac{d^2\phi}{dt^2}+\gamma\frac{d\phi}{dt}+w^2\phi=\frac{\tau_e}{I}$	
	Solución: $\theta(t)=\theta_0\cos(wt+\theta)$	Solución: $\theta_A(t)=\theta_0 e^{-\frac{b}{2I}t}\cos(wt+\sigma)$	Solución: $\theta_F(t)=\theta_A(t)+\theta_0\operatorname{sen}(w_f t+\phi)$	Análisis de Resonancia

Figure 3: Protocol of the student TT17. Implementation 2

Firstly, the critical angle was estimated based on the available data (Ercoli, 2015) and an elementary stability analysis (Holmberg, 1982). Later on the model of the base of the movediza stone was sophisticated, as described in the Table 2 (Lesson 8).

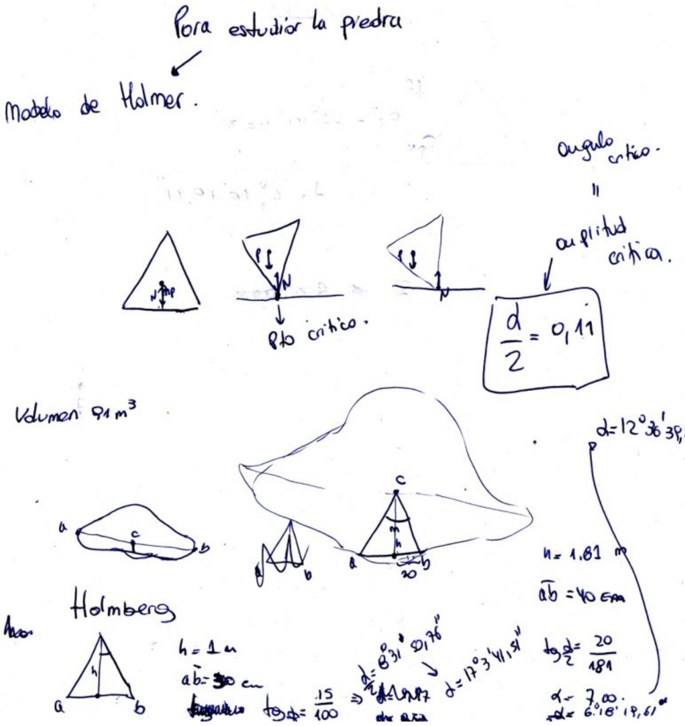


Figure 4: Protocol of the student TT18. Implementation 2

For the study of the rigid solid physical model, a little text was proposed to the students, as a new O_k that could be introduced into the didactic medium *M* by the teacher. In addition, in the Lessons 9 and 10, the students were able to calculate and estimate the parameters of the DE solution, using Geogebra, as illustrated in Fig. 5.

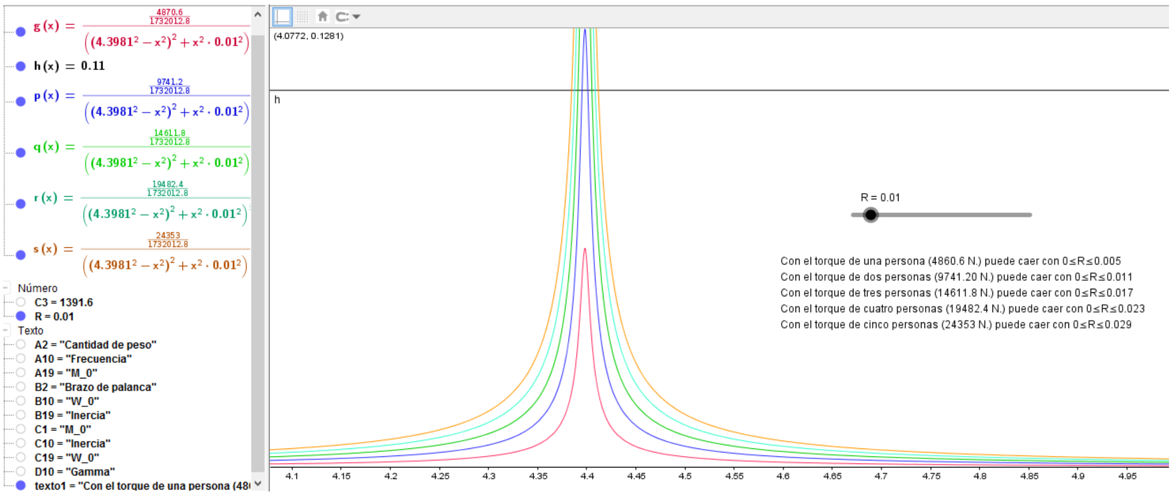


Figure 5: Analysis of the parameters using Geogebra

Later, following a suggestion by the teacher, the students performed several simulations of the maximum amplitude function with the spreadsheets (Fig. 6), which compared with the critical angle, using different values for the parameters. In this way, the students supported, by means of the rocker model of the plausibility, of the Movediza stone fall.

Con torque de $M_0 = 20000\ Nm$ y $\gamma = 0,02$ la piedra se hubiese caído con frecuencias $f \in [0,2\ Hz; 0,8\ Hz]$, lo que equivale a frecuencias angulares $\omega_0 \in [1,257\ Hz;\ 5,027 Hz]$

I (kgm²)	d (m)	F (Kg)	F (N)	M_0 (Nm)	γ
1732012,8	7,1	287,44	2816,90	20000	0,02

f (Hz)	ω_0 (Hz)	ω (Hz)	φ (ω) (rad)
0,8	5,027	5,007	0,052
		5,011	0,062
		5,015	0,075
		5,019	0,092
		5,023	0,108
		5,027	0,115
		5,031	0,105
		5,035	0,088
		5,039	0,072
		5,043	0,060

f (Hz)	ω_0 (Hz)	ω (Hz)	φ (ω) (rad)
0,7	4,398	4,383	0,072
		4,386	0,083
		4,389	0,097
		4,392	0,112
		4,395	0,125
		4,398	0,131
		4,401	0,126
		4,404	0,114
		4,407	0,099
		4,410	0,085

f (Hz)	ω_0 (Hz)	ω (Hz)	φ (ω) (rad)
0,4	2,513	2,483	0,073
		2,489	0,088
		2,495	0,111
		2,501	0,146
		2,507	0,195
		2,513	0,230
		2,519	0,199
		2,525	0,149
		2,531	0,112
		2,537	0,089

f (Hz)	ω_0 (Hz)	ω (Hz)	φ (ω) (rad)
0,3	1,885	1,850	0,085
		1,857	0,104
		1,864	0,133
		1,871	0,179
		1,878	0,252
		1,885	0,306
		1,892	0,250
		1,899	0,177
		1,906	0,131
		1,913	0,102

Figure 6: Numerical analysis of the parameters using a spreadsheet

5. Conclusion

The paper describes the distinctive characteristics of the components of an SRC developed from a genuinely interdisciplinary question in two teacher training courses. Despite the various limitations that have arisen, both courses underwent an interdisciplinary SRC according to their means.

In the following, the questions pointed out in section 3.1 are responded.

1. Which was the role of the students and the teacher during the RSP?

Independently of the difficulties presented, as mentioned before, the TTs experienced a genuine RSP within its means. However, at the beginning there was a visible initial reluctant attitude on the part of the TTs: Why physics should be studied if we are teachers of mathematics? Later, it was gradually understood that the idea was to experience a genuinely co-disciplinary RSP, analyze it and comprehend the teaching model supporting an RSP. In an RSP, the students and the teacher integrate the study community facing together situations of study and research. In both implementations, the TT's studied physics and mathematics thoroughly and showed a good disposition to deal with questions they had never considered before. It is important to highlight the role of the teacher in the RSP. For the teacher, the question Q_0 was also an open question, for which, especially in the first implementation, did not have any a priori closed answer. In this sense both, the students and the teacher took a genuinely active part in the RSP.

2. Which mathematical and physical contents were studied along the RSP?

In both groups, interdisciplinary education is alien to the students, due in part to the imperative of traditional pedagogy. In this sense, the RSP device is very appropriate to foster interdisciplinary study, because it allows studying only the necessary mathematics or physics to answer a question, returning to the original problem. However, it is not only important to decide what content to study, but how to use them, and so the physical and mathematical models and their rationale emerge.

In this work, the question Q_0 triggers, on the side of the physics, the study of oscillating systems, which leads to the study of resonance, motivated by the most plausible conjecture about the fall of the stone. In turn, this physics calls for the study of the equations of motion of these systems, which through Newton's laws give rise to second order differential equations. This

Interplay between both disciplines is clearly reflected in Tables 1 and 2, where we have summarized the mathematical and physical contents studied along the path.

3. Which mathematical and physical models were developed by the students during the RSP?

The construction of a possible answer to the question Q_0, driven the study and the analysis of several physical models related to oscillating systems like springs, single pendulum and physical pendulum, including damped and driven oscillators. However, none of these physical models are adequate to the stone. However, by reanalyzing the real system in more detail, students realized that previous models do not describe some essential aspects of the stone, the most important being the fact that the real system is an object supported on a surface and that is not hanging, like the previous physical models. Then, in the search for a reason to make a supported physical stone model oscillate, the hypothesis that the contact surface between the stone and the base is not flat, but some of the two or both have a certain curvature, emerge, something that in fact had some historical evidence. In this way, the physical model of the rocker arises as the most appropriate to describe the oscillations of a supported object.

Although the students understood that the physical model of the rocker is able to oscillate and could somehow describe the oscillations of the stone, the dynamic analysis of its motion is not within reach of the students, so the mathematical model of the rocker was introduced by the teacher. At this point the students recognized the mathematical similarity with the equations of motion of the pendulum, although the physics is completely different.

Here, is important to notice that the most relevant obstacles there were not only in the physics knowledge, insofar as the physical model was sophisticated and the physical knowledge necessary to treat it was expanded, but in the difficulties of the TT's to use functional modeling involved in the solution of the differential equations. We can say that some aspects of the modeling processes in the sense of the ATD (Barquero, Bosch and Gascón, 2011), were accomplished, and because the RSP evidence the inadequate of the available already made answers to treat the motion of the stone, and the increasing complexity proposed by the RSP.

4. Which are the most relevant constraints to develop the RSP in this level?

Even though the TTs had studied the ATD and other didactic theories, they did it in a traditional way comparable to the traditional training they got. This is reflected in the difficulties they had to understand and to use both physical and mathematical models. It was not expected that the TTs developed the models by themselves, but it was expected that they used the mathematical results presented in the physics textbooks in a pertinent and exoteric manner. This fact did not occur in the first group and improved in the second one from the didactic decision to make a previous incursion into mono-disciplinary RSP, particularly suitable for evidencing the role of the functional modeling. Moreover, this allowed teachers to discuss the relationship between the mathematical model and the physical model and the meaning and role of the parameters.

The TT's behavior is interpreted from the fact that although they have experienced four years of "hard" university studies, the utility of the science they aim at teaching had never been visible. The epistemological conception about the mathematics produced by the traditional paradigm is so ingrained, that it is complex to reverse it. This would be, in our view, the most relevant drawback to permit the TT's at least understand what an RSP is and how the modeling activity works. However, it is important to notice that the sporadic incursions in the modeling activity do not seem enough to allow the TTs to develop such school practices. Although the predominant teaching is mainly traditional, the TTs will face increasing demands for a change to a mathematics teaching based on research, questioning and modeling. It is unlikely that a teacher whose training has been answers-based teaching can teach by means of questions. Therefore, our final message is that the training of teachers must change profoundly.

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