

# Study on Accuracy Improvement of Moment Method Analysis for Scattering Problems Including Dielectric Material

著者	Saeedfar Amin
号	51
学位授与番号	3661
URL	<a href="http://hdl.handle.net/10097/37329">http://hdl.handle.net/10097/37329</a>

	サイードファル アミン
氏名	Saeedfar Amin
授与学位	博士(工学)
学位授与年月日	平成18年9月13日
学位授与の根拠法規	学位規則第4条第1項
研究科, 専攻の名称	東北大学大学院工学研究科(博士課程) 電気・通信工学専攻
学位論文題目	Study on Accuracy Improvement of Moment Method Analysis for Scattering Problems Including Dielectric Material (誘電体を含む散乱問題のモーメント法解析の高精度化に関する研究)
指導教員	東北大学教授 澤谷 邦男
論文審査委員	主査 東北大学教授 澤谷邦男 東北大学教授 杉浦 行 東北大学教授 山口 正洋 東北大学助教授 陳 強

## 論文内容要旨

**Abstract:** Analysis of electromagnetic wave scattering by three-dimensional dielectric and magnetic bodies is faced in various systems and design processes due to its wide range of applications such as analysis of dielectric radar targets, specific absorption rate (SAR) calculation for the analysis of electromagnetic wave interaction between antenna and biological body, design of printed antennas on finite dielectric substrates and suppression of undesired wave emission in high-frequency PCB's using magnetic absorbers. Two popular numerical techniques for these kinds of problems are based on either integral equations or partial differential equations. Integral equations are often numerically solved with the method of moments (MoM), which is one of the most generally accepted computational techniques for electromagnetic (EM) problems. The finite difference time domain (FDTD) technique is also one of the most popular numerical method for solution of partial differential equations appeared in EM area. In this thesis a new approach for solution of the Tensor-Volume Integral Equation (TVIE) using Galerkin-based MoM for three-dimensional dielectric bodies is proposed. Three problems of plane wave scattering by a dielectric sphere, a thin-wire antenna in close proximity to a dielectric body, and printed dipole on finite size dielectric substrate are investigated. In all cases, cubic modeling is applied and a combination of entire-domain and sub-domain basis functions, including three-dimensional polynomial functions with fixed or different degrees is utilized for field expansion inside dielectric bodies. Power polynomial and modified Legendre polynomial are adopted for this purpose, and their properties are discussed over the proposed mixed-domain MoM formulation. Moreover, an extreme care is exercised in calculation of the principal value integral for singularity extraction of the dyadic Green's function. Also, an integral degree reduction method is applied to perform more efficient numerical integration. These two tasks cause to obtain more accurate results through the Galerkin's method computations. Numerical examples show that based on the proposed method, a relative fast algorithm and suitable accuracy are achieved compared with conventional MoM. The accuracy of the proposed method is verified by comparing it with the Mie theory, conventional MoM, FDTD method and experimental data.

### Chapter 1: Introduction

Two popular numerical techniques for analysis of electromagnetic wave scattering by dielectric bodies are based on either integral equations or partial differential equations. In the integral equation approach, we usually start from Maxwell equations and apply theory of the Green's function and electromagnetic equivalence theorems to derive some integral equations according to the geometry of the problem. The integral equation method is based on the boundary condition satisfied on the surface of the conducting scatterer or on the condition for the polarization current in dielectric material. Integral equations (IE) are often numerically solved with the method of moments (MoM), which is one of the most generally accepted computational techniques for electromagnetic (EM) problems. The finite difference time domain (FDTD) technique is also one of the most popular numerical method for solution of partial differential equations appeared in EM area by which the Maxwell equations are solved directly in time domain. FDTD method can be used for an open space using the finite number of cells. In this work, we will deal with solution of the corresponding integral equations based on MoM. There are two general methods to analyze scattering problems including dielectric materials using IE method and MoM. The first approach is the exact treatment using the corresponding Green's function for a given topology of a scattering problem. Indeed, the exact Green's function method can be applied for few number of problems and for arbitrary geometries the second method, which is free space Green's function method should be utilized. In this work, we go for the second method dealing with volume integral equations for three-dimensional arbitrary dielectric scatterers and we will use the tensor-volume integral equation and try to remove the strong singularity of the dyadic Green's function numerically with extreme care during the moment method computations. By

adopting the tensor-volume integral equation, the next step is to discretize that integral equation using MoM. The first task in MoM is to model the dielectric scatterer using a suitable geometric modeling. In this study, we keep our attention on the first and the old kind of geometric modeling due to its simple realization for dielectrics with shape that is cubic modeling.

Although there are different reports of similar studies; nevertheless, some improvements are introduced in the previous researches regarding MoM in this thesis. First, similar to the conventional approaches, cubic block modeling and the tensor-volume integral equation, which includes the free-space dyadic Green's function, are adopted for a scattering problem consisting of a three-dimensional dielectric sphere, illuminated by a plane wave. We have chosen the sphere model to compare the accuracy of the proposed method with the exact solution of Mie scattering. Also, systems of coupled tensor-volume/line and tensor-volume/surface integral equations are applied for a radiation problem including a thin-wire antenna adjacent to a dielectric scatterer and printed dipole antenna on dielectric substrate with finite size, respectively. Finally, Galerkin-based MoM is utilized to solve the integral equations. Symbolically, three main models that we treat in this research are shown in Figure 1.

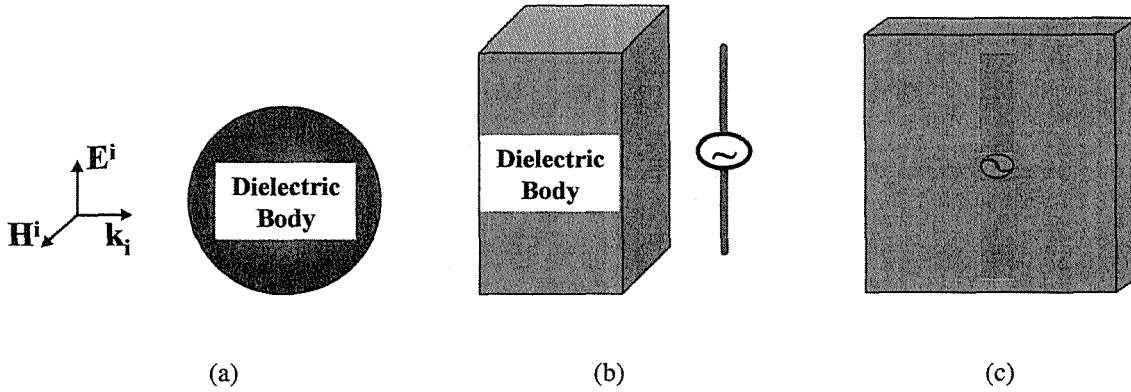


Fig. 1:

- (a) Plane wave scattered by dielectric sphere
- (b) Dipole antenna in the vicinity of the dielectric box
- (c) Printed antenna over the finite dielectric substrate

## Chapter 2: Fundamental Concepts for Subdomain MoM Using Polynomials

In this chapter, basic concepts that are applied in this research will be reviewed. The required volume, surface and line integral equations for this work will be introduced in this part. Also, some notes regarding analytic treatment of the singular volume integral equation are discussed. General MoM formulation for a three-dimensional scatterer, which has both dielectric and magnetic properties, will be introduced and the point-matching formulation with a different notation compared with conventional approaches will be presented. After that, we introduce the power polynomial basis function and apply it in sub-domain Galerkin's method. The exact MoM formulation will be derived and then, numerical results will be presented. The main integral equation that we deal with in this work is as follows

$$\mathbf{E}^i(\mathbf{r}) = \hat{\mathbf{L}}_E(\mathbf{J}_p) + \frac{\mathbf{J}_p(\mathbf{r})}{j\omega\epsilon_0(\epsilon_r^*(\mathbf{r})-1)}, \quad \mathbf{r} \in V$$

$$\hat{\mathbf{L}}_E(\mathbf{J}_p) = j\omega\mu_0 PV \cdot \iint_V \overline{\overline{G_0}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_p(\mathbf{r}') dv' + \frac{1}{j\omega\epsilon_0} \overline{\overline{L}} \cdot \mathbf{J}_p(\mathbf{r}) \quad (1)$$

where the dyad  $\overline{\overline{L}}$  is the extracted singularity of the volume integral of dyadic Green's function (source dyad) in a form of a surface integral over an arbitrary shape around the field point,  $\mathbf{J}_p$  is the polarization current inside the dielectric object.  $\overline{\overline{G_0}}$  is the dyadic Green's function,  $\mathbf{E}^i$  is the imposed incident field,  $\epsilon_r^*$  is the complex relative permittivity of the dielectric scatterer and  $V$  is the volume of the dielectric scatterer. It has been proved in the dissertation that  $\overline{\overline{L}}$  has the following form

$$\bar{L} = \lim_{S_\delta \rightarrow 0} \left[ \iint_{S_\delta} \frac{\hat{\mathbf{R}}' \hat{\mathbf{n}}'_\delta}{4\pi |\mathbf{r} - \mathbf{r}'|^2} ds', \quad \hat{\mathbf{R}}' = \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r} - \mathbf{r}'|} \right] \quad (2)$$

and can be reduced to  $\bar{I}/3$  in the case of sphere or square cube modeling for the principal volume. Also, in the case of dielectric scatterer in the presence of a linear antenna, we have to consider the following system of integral equation

$$\begin{pmatrix} \frac{1}{j\omega(\varepsilon(\mathbf{r}) - \varepsilon_0)} + \hat{L}_E & L_A \left( \frac{\hat{\mathbf{z}}}{2\pi a} \right) \\ \hat{\mathbf{z}} \cdot \mu_0 L_E & \hat{L}_A \end{pmatrix} \begin{pmatrix} \mathbf{J}_p \\ I(z) \end{pmatrix} = \begin{pmatrix} 0 \\ E_z^i \end{pmatrix} \begin{matrix} \mathbf{r} \in V \\ \mathbf{r} \in L \end{matrix} \quad (3)$$

where

$$\hat{L}_A^i(*) \square \frac{-1}{j\omega\varepsilon_0} \int \psi(z, z')(*) dz', \quad \psi(z, z') = \frac{e^{-jk\rho}}{4\pi\rho^5} \left( (1 + jk\rho)(2\rho^2 - 3a^2) + (ka\rho)^2 \right) \\ \rho = \left( (z - z')^2 + a^2 \right)^{1/2} \quad (4)$$

and for the printed strip antenna over a finite dielectric substrate, the above line integral is transformed to a surface integral. In this chapter, we will show that the point matching solution of (1) is given by

$$I_{3N \times 1} = (Z_{3N \times 1})^{-1} \times V_{3N \times 1} \quad (5)$$

where

$$V_{3N \times 1} = [e_\beta]_{3N \times 1}, \quad v_\beta = E_{mj}^i, \quad \beta = 3(j-1) + m, \quad I_{3N \times 1} = [x_\alpha]_{3N \times 1}, \quad i_\alpha = J_{ni}, \quad \alpha = 3(i-1) + n \\ Z_{3N \times 1} = [z_{\beta\alpha}]_{3N \times 3N}, \quad z_{\beta\alpha} = \begin{cases} 0, & m \neq n \\ s_j = j\omega\mu_0 s + \frac{1}{j3\omega\varepsilon_0} + \frac{1}{j\omega\varepsilon_0(\varepsilon_r^*(\mathbf{r}_j) - 1)}, & m = n, \quad i = j \\ j\omega\mu_0 \Delta V_i^3 G_{mn}(\mathbf{r}_j, \mathbf{r}_i), & i \neq j \end{cases} \quad (6)$$

For more accurate solution, using Galerkin's based MoM, we apply the following polynomial as the basis and test function

$$\mathbf{J}_p = \sum_{n=1}^N \sum_{c=1}^3 \sum_{i=0}^{N_{nx}} \sum_{j=0}^{N_{ny}} \sum_{k=0}^{N_{nz}} a_{ijk}^{(nc)} \mathbf{g}_{ijk}^{(nc)}, \quad \mathbf{g}_{ijk}^{(nc)} = x^i y^j z^k u_n(\mathbf{r}) \hat{\mathbf{c}}_c, \quad (\hat{\mathbf{c}}_1, \hat{\mathbf{c}}_2, \hat{\mathbf{c}}_3) = (\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$$

$$N = \text{total number of blocks}, \quad n = \text{block index}, \quad u_n(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in V_n \\ 0, & \text{Otherwise} \end{cases} \quad (7)$$

Power polynomials have the nature of a truncated three-variable Taylor series and behave as entire-domain basis functions. However, the number of unknowns will be large if polynomial basis function is used in a subdomain expansion. This drawback is solved in next chapter. When power polynomials even with low degrees are applied for polarization current expansion as the subdomain MoM, more accurate results are achieved compared with conventional subdomain MoM due to the analytic nature of those kinds of basis functions. It means, whatever the describing function of the unknown field is, it can be approximately expanded in terms of power polynomial functions, yet with some truncation error. This property is particularly useful for polarization current expansion procedure in complex dielectric materials and also in the near field problems because of the non-uniform behavior of fields in such cases. Polynomial basis functions would be able to well satisfy the boundary conditions at interfaces between adjacent cells of different permittivities inside the scatterer body and prevent the production of fictitious charge density in a homogeneous dielectric body, whereas they happen in some conventional MoM. In other words, current distribution inside the dielectric is approximated by three-dimensional polynomials that satisfy the boundary condition for the normal component of the electric equivalent displacement vector on surfaces shared by two adjacent blocks. About conventional sub-domain MoM, particularly pulse basis function with cubic modeling. First of all, when using the cubic cells, one can not properly model the physics of the smooth curved material bodies. Also, the charge density that is associated with the current must exist only on the surface of the body as inside a homogeneous dielectric. However, when we use pulse basis function for the current, this introduces additional surface charge layers, at the interface of the

cells, inside a homogeneous region. It has been reported that this additional charge will affect the accuracy of the near-field quantities but the far-field quantities are reasonably accurate.

In this chapter we could achieve some results with suitable accuracy but with long CPU time. This problem is improved in the next chapter.

### **Chapter 3: Polynomials in Mixed-domain Galerkin's Method**

Traditionally, subdomain expansions have been favored because of their geometric flexibility, easier evaluation of the multiple integrals arising in the MoM technique, and ability to handle localized surface (topological, geometrical) features in scattering problems or present aperture and feed-point distributions in antenna problems. On the other hand, the entire-domain representation, leads to multiple integrals that are difficult to evaluate except for some classes of geometries in which generating curve is part of a separable coordinate system. The mixed-domain expansion bridges the limitations of the two approaches outlined. The mixed-domain expansions are overlapped to provide continuity of currents at the transition from the subdomain to the entire-domain representations. In this work, a mixture of three-dimensional polynomials with various degrees is applied for field expansion inside the dielectric body. In mixed-domain expansion method, polynomials with higher degree are used for some macro-blocks located in the internal part of the dielectric object and polynomials with lower degrees (pulse function as a special case), which behave as sub-domain basis function are used for smaller blocks at corners and boundaries of the dielectric body to provide a suitable estimation of dielectric shape. Consequently, by doing this procedure, the number of blocks has decreased and at same time, the high degree polynomials also exist in the corresponding expansion to improve the accuracy. Therefore, we expect that both CPU time and accuracy are improved by this action in MoM procedure. The numerical results confirm this expectation.

### **Chapter 4: Modified Legendre Polynomials in Mixed-domain Galerkin's Method**

In this chapter, the power polynomial basis functions are replaced with another polynomial, so called modified Legendre polynomials. This task is mainly carried out to have an orthogonal basis function improving the ill-conditioned situation of the occurred problems by decreasing the condition number of the impedance matrix. This basis function is derived from orthogonal Legendre polynomial, which is modified to impose the continuity condition of the currents between neighboring elements. This allows the use of high-order basis functions without introducing ill-conditioning of the resulting MoM matrix. Numerical results confirm that the condition number of the MoM matrix obtained with this new basis is much lower than power polynomial function proposed in the previous part with the same accuracy. In fact, complete orthogonality is not possible when the power polynomial is required to satisfy continuity of the normal component across the element boundaries in sub-sectioning procedure of MoM. In other words, the modification that is applied to enforce continuity essentially destroys the orthogonality of the expansion. By this polynomial replacement, we could improve the condition number of the impedance matrix but not very much the level of the accuracy.

### **Chapter 5: Conclusions**

In this research we performed a fundamental methodology research regarding MoM for numerical solution of the tensor-volume integral equation. This kind of integral equation appears in the electromagnetic scattering problems, which includes finite dielectric materials without closed form Green's function. More specifically, in this thesis an integral equation approach with a new solution method was used to obtain polarization current inside dielectric bodies which is required for the computation of some field quantities like radar cross-section of dielectric targets, SAR and antenna characteristics in the vicinity of a dielectric scatterer or finite size dielectric substrate. Some improvements were introduced in the previous researches regarding MoM. The Galerkin-based MoM was utilized to solve the integral equations. The first difference of this work with the previous ones was the extreme care, which had been exercised in calculation of the principal value integral for singularity extraction of the dyadic Green's function. Moreover, an integral degree reduction method was applied to perform more efficient numerical integration. These two tasks caused to obtain more accurate results through the Galerkin's method computations. The second difference of this article with the previous ones was use of three-dimensional polynomials of two different kinds for field estimation inside the dielectric body applying a combined subdomain and entire-domain expansion method called mixed-domain moment method. It was demonstrated that a compromise between accuracy and efficiency was achieved by combining the entire-domain and subdomain expansion using this kind of polynomial functions with various degrees. Accuracy of the proposed method was compared with the numerical solutions of different methods such as conventional MoM, point matching and FDTD. Furthermore, in the case of dielectric sphere, numerical results of the proposed technique were compared with exact solutions of the Mie theory.

# 論文審査結果の要旨

電磁界の数値解析法の発展は目覚しく、アンテナや電波伝搬、環境電磁工学などの分野に広く利用されている。モーメント法は積分方程式に基づいた電磁界の数値解析法であり、偏微分方程式に基づいた時間領域差分法（FDTD法）などに比べてメモリサイズが小さく、数値計算時間も短いという大きな利点を有している。しかしながら、誘電体や磁性体が含まれる問題については解析精度が低いために、これまで主に導体からの放射・散乱の解析に用いられてきた。本論文は、誘電体を含む散乱問題に対するモーメント法の解析精度を大幅に改善し、その適用範囲を広げることを目的としたものであり、全編5章よりなる。

第1章は緒言である。

第2章では、誘電体を含む散乱問題にモーメント法を適用するために、誘電体を多数のブロックに分割し、各ブロック中の分極電流をべき級数で展開し、ガラーキン法を用いる手法を導入している。また、基底関数同士の相互インピーダンスの表示式に含まれる6重積分を3重積分に変形し、数値計算時間の短縮を図っている。この手法を誘電体球の散乱断面積、並びに直方誘電体近傍のダイポールアンテナの入力インピーダンスの計算に適用し、それぞれ厳密解及びFDTD法による数値解と良く一致する結果が得られることを示している。また、誘電体球については3重積分化により数値計算時間を約1/5に短縮できることを明らかにしている。これらの結果は、モーメント法の精度を向上させたもので、高く評価できる。

第3章では、第2章の数値解析法の更なる高精度化を図るために、部分領域基底関数と全領域基底関数を併用する手法を開発している。即ち、対象とする誘電体の中心部分に大きなブロックを1つ設け、その分極電流を高次のべき級数で展開し、周辺部分は小さなブロックに分解して低次のべき級数で展開すると共に、分極電流の連続性を保つために、両者のブロック間に重複するブロックを設ける方法を提案している。誘電体球の散乱断面積をこの手法を用いて計算した結果、第2章の手法に比べてさらに高精度化が達成され、また数値計算時間が約半分となることを明らかにしている。さらに、直方誘電体近傍ダイポールアンテナの入力インピーダンスを計算して実験値と比較した結果、本手法が誘電体を含む散乱体の数値解析法として極めて優れていることを確認している。

第4章では、安定した数値計算を可能とするために、第3章で導入した高次べき級数の全領域基底関数をルジャンドルの多項式で置き換えて解析を行っている。相互インピーダンス行列の条件数の比較により、従来のブロックモデルや第3章のべき級数に比べて安定性が改善されることを述べている。また、本手法を誘電体基板上のプリントダイポールアンテナの解析に適用し、FDTD法に比べて精度、メモリサイズ及び数値計算時間の点で優れていることを示している。

第5章は結言である。

以上要するに本論文は、誘電体を含む散乱問題に対するモーメント法について新たな手法を提案し、解析精度の改善と数値計算時間の短縮及び安定性の向上を図り、その適用範囲を大幅に拡大したもので、無線通信工学並びに電気通信工学の発展に寄与するところが少なくない。

よって、本論文は博士(工学)の学位論文として合格と認める。