# Kinematics Simulation of Gough-Stewart Parallel Manipulator by Using Simulink Package in Matlab Software 

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#### Abstract

: The Gough Stewart Robotic manipulator is a parallel manipulator with six-degree of freedom, which has six equations of Kinematics (Inverse and forward), with six variables (Lengths, Position, and Orientation). In this work derived the inverse equations, which used to compute the lengths of the linkages and its changes depended on the position and orientation of the platform's center, then derived the forward equations to calculate the position and orientation of the moving platform in terms of the lengths. This theoretical model of the kinematics analysis of the Gough Stewart has been built into the Simulink package in Matlab to obtain the lengths, position, and orientation for the manipulator at any time of motion. The input parameters (Position and Orientation) in inverse blocks compared with the output parameters (Position and Orientation) in the forward blocks, which show good results.


Keywords: Robot, Gough-Stewart, Inverse Kinematics, Forward Kinematics, Matlab, Simulink.

## 1- Introduction:

With the advancement in the field of robotic technologies, robotic manipulators are widely used in the applications of factory automation, which are required to carry out continuous and delayed work, such as lifting and transporting radioactive substances and executing the work in a hazardous [1]. At the last time, great attention has been shown in the most paper in the field of the parallel robot. Where the parallel robot more popular due to it did specific applications over than serial robot. And has high rigidity, high load capacity, good dynamic performance, and easier solution for the inverse kinematics equations, but the parallel robot not used in most industrial applications because of the limitation of its workspace and the complexity in the solution of the forward kinematics equations [2]. Gough-Stewart manipulator is a fully parallel robot used to control the motion of the flight simulator base. The Stewart platform is a six degree of freedom parallel manipulator, where the moving platform connected to the base by six actuators which treated as links. Each actuator connected to the base and moving platform by joints in form universal-universal, sphericalspherical, or universal-spherical. The geometry of the fixed base and moving platform contain a six points which have coordinates in the global coordinates frame and local coordinates frame respectively, while the coordinates of the points on the moving platform must be transformed to the global frame by using its position and orientation [3]. The first used of the Gough-Stewart platform as a flight simulator in 1965 by stewart [4]. The lengths of the links are calculated in inverse kinematics analysis in terms of the position, orientation, translation and angular velocities, and translation and angular acceleration for the center of the moving platform, also can be computed the rate change of the length with respect to time for each link in inverse kinematics equations [5]. While the forward equations for the Gough-Stewart manipulator used to determine the position, orientation, and its rate change with time in time based on the lengths of the linkages for the manipulator. The solution of the inverse kinematics equation are very easy compare with the solution of the forward kinematics equations [6].

This paper deals with the solution of the kinematics problem analysis for Gough Stewart manipulator, where solved the invers kinematics by using the position and orientation at initial and final position for the moving platform to obtained the lengths of the linkages, then used these lengths to solve the forward kinematics analysis by Newton Raphson method to obtained these position and orientation. All these calculation done by Simulink in Matlab software.

## 2- Gough Stewart Model:

A Gough-Stewart parallel manipulator typically consists of a moving platform and a fixed base that are connected together by six links. Because of the closed-loop architecture, not all of the joints can be independently actuated. In general, the number of actuated joints is equal to the number of degrees of freedom of the manipulator. In this paper, the model of the manipulator consists of the fixed base and moving platform, six linear actuators which are connected to the base by universal joints (have two degrees of freedom) and connected with the moving platform by a ball joint (have three degrees of freedom) as shown in Fig (1).


Fig (1): The mechanism of the Stewart [7].

## 3- Inverse Kinematics Equations:

The tables 1 and 2 show coordinates of the base and moving platform as shown in Fig (2):
Table 1: the coordinates of the base of the Gough-Stewart

| Base Coordinates |  |  |  |
| :---: | :---: | :---: | :---: |
| Point No | X | Y | Z |
| 1 | 48.6989 | 219.6666 | 0 |
| 2 | 165.8874 | 152.0078 | 0 |
| 3 | 165.8874 | -152.0078 | 0 |
| 4 | 48.6989 | -219.6666 | 0 |
| 5 | -214.5863 | -67.6588 | 0 |
| 6 | -214.5863 | 67.6588 | 0 |

Table 2: the coordinates of the moving platform of the Gough-Stewart

| Moving Platform Coordinates |  |  |  |
| :---: | :---: | :---: | :---: |
| Point No | X | Y | Z |
| 1 | -51.3030 | 140.9539 | 275 |
| 2 | 147.7212 | 26.0472 | 275 |
| 3 | 147.7212 | -26.0472 | 257 |
| 4 | -51.3030 | -140.9539 | 275 |
| 5 | -96.4181 | -114.9067 | 275 |
| 6 | -96.4181 | 114.9067 | 275 |



Fig (2): The Coordinates of the base and platform.
The lengths of the links computed to satisfy the position and orientation of the moving platform, which used to generate the trajectory of the manipulator at any time to determine the velocities and acceleration [8]. To find the inverse kinematics of the Stewart based on the geometry of the Gough-Stewart manipulator, which shown in Fig (3), the lengths of all links are function of the position and orientation of the moving platform center, the inverse analysis is expressed as in the equations below.


Fig (3): Description of the Stewart.
$L_{i}=r_{o}{ }^{G}+R_{L}^{G} \cdot P_{i}{ }^{L}-b_{i}{ }^{G}[9]$
$\mathrm{i}=1 \ldots 6$
Where:
$r_{o}{ }^{G}=\left[\begin{array}{lll}X & Y & Z\end{array}\right]^{T}$
$R_{L}^{G}=R_{z}(\gamma) \cdot R_{y}(\beta) \cdot R_{x}(\alpha)=\left[\begin{array}{lll}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{array}\right]$
$P_{i}^{L}=\left[\begin{array}{lll}p_{x i} & p_{y i} & p_{z i}\end{array}\right]^{T}$
$b_{i}{ }^{G}=\left[\begin{array}{lll}b_{x i} & b_{y i} & b_{z i}\end{array}\right]^{T}$
$L_{i}$ : is the length of i linkage.
$r_{o}{ }^{G}$ : The coordinates of moving platform in the Global Coordinate system.
$R_{L}^{G}$ : Rotation matrix of the local Coordinate system relative to Global coordinate system.
$P_{i}{ }^{L}$ : The coordinates of i joint of with i link moving platform in the local coordinate system.
$b_{i}{ }^{G}$ : The coordinates of i joint of fixed platform with the i link in global coordinate system.
$r_{11}=\cos (\gamma) \cos (\beta)$
$r_{12}=\cos (\gamma) \sin (\beta) \sin (\alpha)-\sin (\gamma) \cos (\alpha)$
$r_{13}=\cos (\gamma) \sin (\beta) \cos (\alpha)+\sin (\gamma) \sin (\alpha)$
$r_{21}=\sin (\gamma) \cos (\beta)$
$r_{22}=\sin (\gamma) \sin (\beta) \sin (\alpha)+\cos (\gamma) \cos (\alpha)$
$r_{23}=\sin (\gamma) \sin (\beta) \cos (\alpha)-\cos (\gamma) \sin (\alpha)$
$r_{31}=-\sin (\beta)$
$r_{32}=\cos (\beta) \sin (\alpha)$
$r_{33}=\cos (\beta) \cos (\alpha)$
The length of i linkage can be obtained from the equation below which is derived by square equation (1):
$L_{i} \cdot L_{i}{ }^{T}=\left[r_{o}{ }^{G^{T}} \cdot r_{o}{ }^{G}+b_{i}{ }^{G^{T}} \cdot b_{i}{ }^{G}+P_{i}{ }^{L^{T}} \cdot P_{i}{ }^{L}+2 \cdot P_{i}^{L^{T}} \cdot R_{L}^{G} \cdot b_{i}{ }^{G}-2 \cdot b_{i}{ }^{G^{T}} \cdot R_{L}^{G} \cdot P_{i}{ }^{L}-2 . P_{i}^{L^{T}} \cdot b_{i}{ }^{G}\right]$
From above equation the change in length of i linkage present in the following equations:
$q_{i}=\left(\left(X+\left((\cos (\gamma) * \cos (\beta)) * p_{x i}\right)+\left(((\cos (\gamma) * \sin (\beta) * \sin (\alpha))-(\sin (\gamma) * \cos (\alpha))) * p_{y i}\right)+\right.\right.$
$\left.\left(((\cos (\gamma) * \sin (\beta) * \cos (\alpha))+(\sin (\gamma) * \sin (\alpha))) * p_{z i}\right)-b_{x i}\right)^{2}+\left(Y+\left((\sin (\gamma) * \cos (\beta)) * p_{x i}\right)+\right.$ $\left(((\sin (\gamma) * \sin (\beta) * \sin (\alpha))+(\cos (\gamma) * \cos (\alpha))) * p_{y i}\right)+(((\sin (\gamma) * \sin (\beta) * \cos (\alpha))-(\cos (\gamma) *$ $\left.\left.\sin (\alpha))) * p_{z i}\right)-b_{y i}\right)^{2}+\left(Z+\left((-\sin (\beta)) * p_{x i}\right)+\left((\cos (\beta) * \sin (\alpha)) * p_{y i}\right)+((\cos (\beta) * \cos (\alpha)) *\right.$ $\left.\left.\left.p_{z i}\right)-b_{z i}\right)^{2}\right)^{0.5}-L_{i}$

## 4- Forward Kinematics Equations:

The solving of (FKP) is used to calculate the coordinates and orientation of the moving platform center and its change when the length of each linkage is known.

From equation (3), the change of the length of each linkage obtained in six non-linear equations, which solved numerically by Newton-Raphson method ( $q_{1}, q_{2}, q_{3}, q_{4}, q_{5}$, and $q_{6}$ ), these systems of equations can solve numerically by Newton-Raphson Method as below steps[10]:

## - step 1:

The system can be written in the form:

$$
\begin{align*}
& f_{i}(x)=0 \\
& f_{i}(x)=\left(\left(X+r_{11} \cdot p_{x i}+r_{12} \cdot p_{y i}+r_{13} \cdot p_{z i}-b_{x i}\right)^{2}+\left(Y+r_{21} \cdot p_{x i}+r_{22} \cdot p_{y i}+r_{23} \cdot p_{z i}-b_{y i}\right)^{2}+(z+\right. \\
& \left.\left.r_{31} \cdot p_{x i}+r_{32} \cdot p_{y i}+r_{33} \cdot p_{z i}-b_{z i}\right)^{2}\right)^{0.5}-L_{i}-q_{i}  \tag{4}\\
& f(x)=\left[\begin{array}{lllll}
f_{1}(x) & f_{2}(x) & f_{3}(x) & f_{4}(x) & f_{5}(x) \\
f_{6}(x)
\end{array}\right]^{T}  \tag{5}\\
& x=\left[\begin{array}{llllll}
X & Y & Z & \alpha & \beta & \gamma
\end{array}\right]^{T} \tag{6}
\end{align*}
$$

## - step 2:

## - Derivative $f(x)$ :

The following equations obtained From derivative of the $f(x)$ :

$$
D f^{-1}=\left[\begin{array}{llllll}
\frac{\partial q_{1}}{\partial X} & \frac{\partial q_{1}}{\partial Y} & \frac{\partial q_{1}}{\partial Z} & \frac{\partial q_{1}}{\partial \alpha} & \frac{\partial q_{1}}{\partial \beta} & \frac{\partial q_{1}}{\partial \gamma}  \tag{7}\\
\frac{\partial q_{2}}{\partial X} & \frac{\partial q_{2}}{\partial Y} & \frac{\partial \mathrm{q}_{2}}{\partial Z} & \frac{\partial \mathrm{q}_{2}}{\partial \alpha} & \frac{\partial \mathrm{q}_{2}}{\partial \beta} & \frac{\partial \mathrm{q}_{2}}{\partial \gamma} \\
\frac{\partial q_{3}}{\partial X} & \frac{\partial q_{3}}{\partial Y} & \frac{\partial q_{3}}{\partial Z} & \frac{\partial q_{3}}{\partial \alpha} & \frac{\partial q_{3}}{\partial \beta} & \frac{\partial q_{3}}{\partial \gamma} \\
\frac{\partial q_{4}}{\partial X} & \frac{\partial q_{4}}{\partial Y} & \frac{\partial q_{4}}{\partial Z} & \frac{\partial q_{4}}{\partial \alpha} & \frac{\partial q_{4}}{\partial \beta} & \frac{\partial q_{4}}{\partial \gamma} \\
\frac{\partial q_{5}}{\partial X} & \frac{\partial q_{5}}{\partial Y} & \frac{\partial q_{5}}{\partial Z} & \frac{\partial q_{5}}{\partial \alpha} & \frac{\partial q_{5}}{\partial \beta} & \frac{\partial q_{5}}{\partial \gamma} \\
\frac{\partial q_{6}}{\partial X} & \frac{\partial q_{6}}{\partial Y} & \frac{\partial q_{6}}{\partial Z} & \frac{\partial q_{6}}{\partial \alpha} & \frac{\partial q_{6}}{\partial \beta} & \frac{\partial q_{6}}{\partial \gamma}
\end{array}\right]
$$

## - Step 3:

The equation of the (Newton Raphson) were used to calculate the unknown variable defined below:
$x_{j+1}=x_{j}-D f^{-1} . f\left(x_{j}\right)$
The (FKP) solved by the Newton-Raphson method, while the initial lengths of each linkages, its change, first guess value of position and orientation of the platform sub in equation (4) and (7), then apply the matrix obtained used in equation (8) to get the new coordinates of the moving platform, this procedure repeated for $j$ iteration until the difference between $x_{j+1}$ and $x_{j}$ converge to zero. The above analysis is done by Matlab software.

## 5- Simulink Package:

Developed by MathWorks, is a graphical programming environment for modeling, simulating and analyzing multidomain dynamical systems. Its primary interface a graphical block diagramming tool and a customizable set of block libraries. It offers tight integration with the rest of the MATLAB environment and can either drive MATLAB or be scripted from it. Simulink is widely used in automatic control and digital signal processing for multidomain simulation and Model-Based Design[11]. In this work used Simulink to simulation the Inverse and forward Kinematics problems for the Gough-Stewart Robot Manipulator as follow:

- Clock Blocks: to determine the time of the simulation.
- Lockup Blocks: to insert the input parameters (X, Y, Z, Alpha, Beta, Game), which represented the coordinates of the moving platform.
- Matlab Function Blocks: to create the Kinematics function blocks (Inverse and Forward).
- Scope Blocks: to view the results of the Kinematics analysis with respect to time.
- The block diagram of the model in Simulink as shown in the Fig (4):


Fig (4): The block diagram of the model in Simulink.

## 6- Results and discussion:

After building the blocks of the model in the Simulink package, insert the position and orientation of the platform's center as shown in table 3:

Table 3: Position and orientation of the platform.

| Position and Orientation | Start Point | Step | End Point |
| :---: | :---: | :---: | :---: |
| X | 0 | 10 | 100 |
| Y | 0 | 15 | 150 |
| Z | 432.5 | 10 | 532.5 |
| Alpha | 0 | 1.5 | 15 |
| Beta | 0 | -1.5 | -15 |
| Gama | 0 | 2 | 20 |

By substituting the position and orientation in the equation (1) (Kinematics inverse) the lengths of each linkages are obtained at any step, where the input parameters (position and orientation) to the inverse kinematics in Simulink block represented in the Fig (5).


Fig(5): Input parameters (position and orientation) to inverse block.
And the upper platform moved from initial position and orientation ( $0,0,432.5,0,0,0$ ) to final position and orientation ( $100,150,532.5,15,-15,20$ ), as shown in the $\operatorname{Fig}(6)$.


Fig (6): Moving platform after inverse block.

And the solution of the inverse equations gives the lengths of the linkages and its changes at any step of motion, where the lengths obtained by inverse equations in Simulink blocks explained in Fig (7), which are represented the outputs of the inverse equations and inputs for the forward equations.


Fig (7): Output parameters (lengths) from inverse block and input for forward block.
When the lengths computed by inverse equation (1) in $\operatorname{Fig}(7)$, these lengths at each step used to calculate the position and orientation by using the forward equation, which solved numerically in NewtonRaphson method because the forward equation derived from the system of non-linear equations.

The movement of the platform from initial lengths to final lengths according to the forward equations represented in Fig (8).

And the outputs of the forward blocks in the Simulink include the position and orientation of the center for the moving platform, as Fig (9)


Fig (8): Moving platform after forward block.


Fig (9): Output parameters (position and orientation) from forward block.

The forward outputs include the position ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) and orientation $(\alpha, \beta, \gamma)$ compared with the inputs used in inverse kinematics, the forward results less than the inputs in inverse blocks due to the forward equation solved by numerical solution (Newton-Raphson) based on accuracy ( $10^{\wedge}-4$ ), but this deference was very little and can be neglected.

## 7- Conclusion:

1- In this paper derived the kinematics equation (forward and inverse) for the 6-6 Gough-Stewart parallel robot manipulator.

2- These equations are used to calculate the position, orientation, and the lengths of the linkages.
3- Where the accuracy used in numerical solution gives a good result to solve the forward equation.
4- The Matlab software used to simulate the kinematic equation by using Simulink package.
5- The results obtained from inverse kinematics were compatible with the forward results.

## CONFLICT OF INTERESTS.

- There are no conflicts of interest.


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# محاكاة حسابـات الكينماتيك لرويوت كووفسستيوورت المتوازي باستخدام حزمة السميولنك 

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