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Dynamic Analysis of Gough Stewart Robot Manipulator by Using Lagrange Formulation in Matlab Software

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Abstract:

In this paper, Gough Stewart parallel manipulator with six-degree of freedom used as a model to derived the inverse kinematics equations and inverse dynamic equations. The inverse kinematic problem is very simple to derive then used to computed the jacobian matrix and the lengths of the linkages to determine the path trajectory of the moving platform. The dynamic equation based on the Lagrange method by calculated the kinetic and potential energies for the model. The dynamic equation inserted in Simulink block as Matlab file lead to computed the forces of the linkages at any time of simulation and can be used to explain if the path contain a singularities where the forces increasing very quickly in singular point compare with other points during the path of the moving platform. Finally, these blocks can be used in the other models with different parameters inputs.

Keywords: Parallel Robot, Gough-Stewart Manipulator, Matlab, Simulink, Inverse Kinematics, dynamic analysis, Lagrange formulation.

1- Introduction:

Robots can be considered as the most advanced automatic systems and robotics, as a technique and scientific discipline can be considered as the evolution of automation with interdisciplinary integration with other technological fields[1]. Robots can be found today in the manufacturing industry, agricultural, military and domestic applications, space exploration, medicine, education, information and communication technologies, entertainment, etc[2]. There are two types of the robots serial and parallel robot. A serial link manipulator consists of a sequence of mechanical links connected together by actuated joints[3], and Parallel robots called parallel kinematic machines (PKM)[4], as shown in Fig (1)[5].

From this definition, we see that the PKM are composed of different elements:

• The (fixed) base, which is the fixed element of the robot

• The (moving) platform on which is usually mounted the end-effector,

• The kinematic chains, linking the base to the platform, and also called the robot Legs[6].

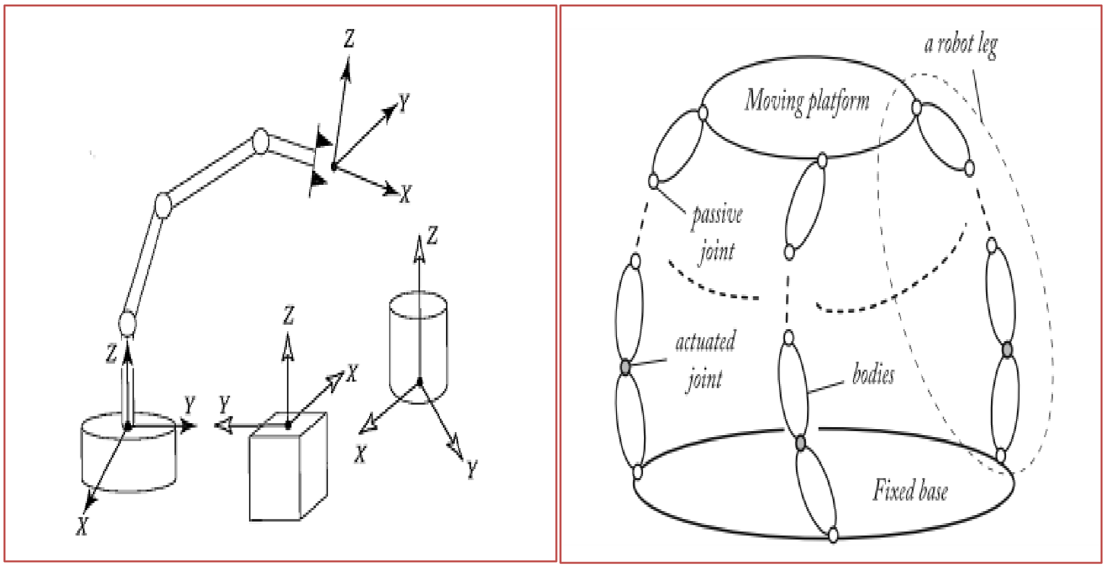

Fig (1): Serial and parallel robot.

In contrast to the open-chain serial manipulators, the dynamic modelling of parallel manipulators presents an inherent complexity due to their closed-loop structure and kinematic constraints. Nevertheless, the dynamic modelling is quite important for their control, particularly because parallel manipulators are preferred in applications where precise positioning and good dynamic performance under a high load are the prime requirements[7].

The inverse dynamic model gives the motorized joint forces as a function of the position, velocity and acceleration of the mobile platform. The direct dynamic model of the robot gives the platform

acceleration as a function of the state of the robot (platform position and velocity) and the input forces of the active joints[8].

In this paper, the Lagrange formulation is employed for solving the inverse dynamics of a Stewart-Gough manipulator. This method used the concept of Jacobian matrices leads to form of the dynamical equations of motion.

2- Gough Stewart Model:

A Gough-Stewart parallel manipulator typically consists of a moving platform and a fixed base that are connected together by six links. Because of the closed-loop architecture, not all of the joints can be independently actuated. In general, the number of actuated joints is equal to the number of degrees of freedom of the manipulator.

In this paper, the model of the manipulator consists of the fixed base and moving platform, six linear actuators which are connected to the base by universal joints (have two degrees of freedom) and connected with the moving platform by a ball joint (have three degrees of freedom) as shown in Fig (2)[9].

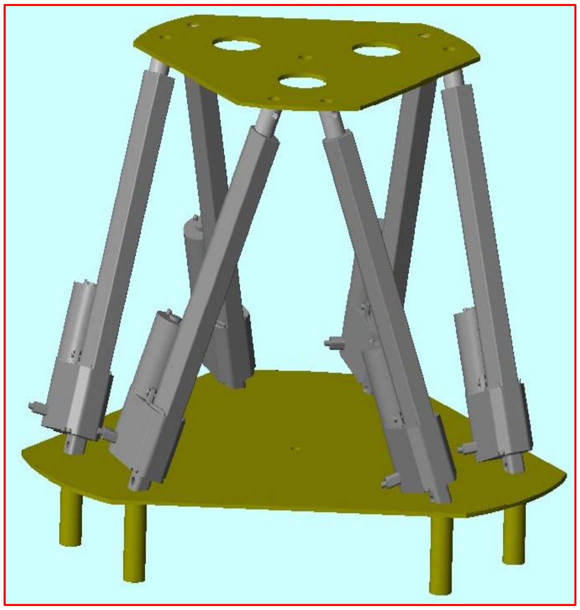
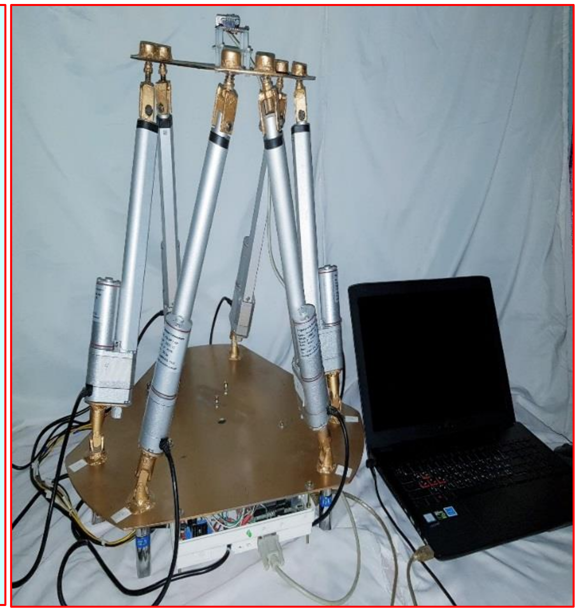

Fig (2): The model of Gough-Stewart manipulator.

3- Inverse Kinematics Problems (IKP) of Gough-Stewart Manipulator:

Solving of (IKP) is used to calculate the length of each linkage and its change when the coordinates and orientation of the moving platform center is known, from Fig (3) below the length of the linkage (Li) can be calculated as a vector as shown in equation (1).

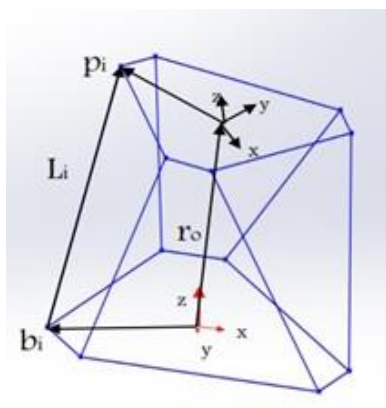

Fig (3): Mechanism of Gough-Stewart manipulator.

$$L_i = {}^{G}r_0 + {}^{G}R_L \, . \, {}^{L}P_i - b_i \; [10] \qquad i = 1 \ldots 6 \qquad \text{……(1)}$$

$${}^{G}r_0 = [X \quad Y \quad Z]^T$$

$${}^{G}R_L = R_z(\gamma).R_y(\beta).R_x(\alpha) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$${}^{L}P_i = [p_{xi} \quad p_{yi} \quad p_{zi}]^T$$

$b_i = [b_{xi} \quad b_{yi} \quad b_{zi}]^T$

r_{11}=cos(γ) cos(β)

r_{12}=cos(γ) sin(β) sin(α)-sin(γ) cos(α)

r_{13}=cos(γ) sin(β) cos(α)+sin(γ) sin(α)

r_{21}=sin(γ) cos(β)

r_{22}=sin(γ) sin(β) sin(α)+cos(γ) cos(α)

r_{23}=sin(γ) sin(β) cos(α)-cos(γ) sin(α)

r_{31}=-sin(β)

r_{32}=cos(β) sin(α)

r_{33}=cos(β) cos(α)

The length of i linkage can be obtained from the equation below which is derived by square equation (1):

$$L_i = \left(\left(X + (r_{11} * p_{xi}) + \left(r_{12} * p_{yi}\right) + (r_{13} * p_{zi}) - b_{xi}\right)^2 + \left(Y + (r_{21} * p_{xi}) + \left(r_{22} * p_{yi}\right) + (r_{23} * p_{zi}) - b_{yi}\right)^2 + \left(Z + (r_{31} * p_{xi}) + (r_{32} * p_{yi}) + (r_{33} * p_{zi}) - b_{zi}\right)^2\right)^{0.5} \quad \text{...... (2)}$$

4- Jacobian Matrix Of The Gough-Stewart Manipulator Robot:

The Jacobian matrix produce from the relationship between the velocities of the moving platform and the movement of the actuators[4].

As shown in the geometry of the manipulator in the Fig (4), the link input denoted by $\dot{L} = [\dot{L}_1 \quad \dot{L}_2 \quad \dot{L}_3 \quad \dot{L}_4 \quad \dot{L}_5 \quad \dot{L}_6]^T$, and the output of the platform denoted by $\dot{x} = \left[\dot{X} \quad \dot{Y} \quad \dot{Z} \quad \dot{\alpha} \quad \dot{\beta} \quad \dot{\gamma}\right]^T$, where the linear velocity of the platform $\dot{v}_p = [\dot{X} \quad \dot{Y} \quad \dot{Z}]^T$and the angular velocity of the platform $\dot{w}_p = [\dot{\alpha} \quad \dot{\beta} \quad \dot{\gamma}]^T$. The jacobian matrix (equation 3) can be obtained by derived the velocity of each link as shown:

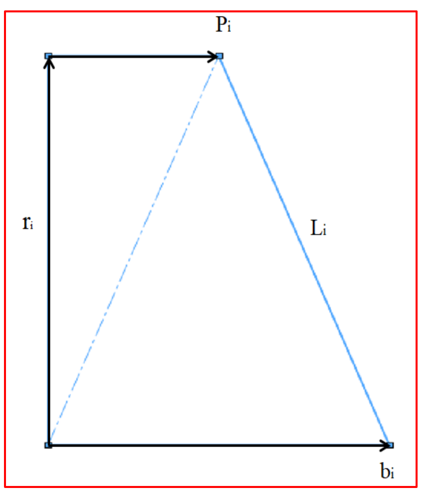

Fig (4): Scheme of the i leg.

$$J^{-1} = \begin{bmatrix} U_{x1} & U_{y1} & U_{z1} & (\left({}^{G}R_{L} * {}^{L}P_{1}\right) \times \vec{U}_1)^T \\ U_{x2} & U_{y2} & U_{z2} & (\left({}^{G}R_{L} * {}^{L}P_{2}\right) \times \vec{U}_2)^T \\ U_{x3} & U_{y3} & U_{z3} & (\left({}^{G}R_{L} * {}^{L}P_{3}\right) \times \vec{U}_3)^T \\ U_{x4} & U_{y4} & U_{z4} & (\left({}^{G}R_{L} * {}^{L}P_{4}\right) \times \vec{U}_4)^T \\ U_{x5} & U_{y5} & U_{z5} & (\left({}^{G}R_{L} * {}^{L}P_{5}\right) \times \vec{U}_5)^T \\ U_{x6} & U_{y6} & U_{z6} & (\left({}^{G}R_{L} * {}^{L}P_{6}\right) \times \vec{U}_6)^T \end{bmatrix} \quad \text{……(3)}$$

5- Dynamic Equations of Gough stewart:

The difference between the kinetic and potential energy referred to the Lagragian function[11]:

$$\mathcal{L} = K.E - P.E$$

The final Lagrange equation (equation 4) include the mass matrix $M(X)$, Coriolis matrix $C(X,\dot{X})$, and gravity matrix $G(X)$

$$M(X)\ddot{X} + C(X,\dot{X})\dot{X} + G(X) = \tau \quad \text{…… (4)}$$

Here the Gough-Stewart manipulator divided into two subsystem the upper moving platform and the six legs. The kinetic and potential energy of these subsystems should be computed to derive the dynamic equations for the Gough-Stewart manipulator.

- **Dynamic analysis of the upper moving platform:**

Here the Lagrange formulation for the dynamic analysis (equation 66) applied on the moving platform for the Gough-Stewart manipulator to obtain its mass matrix (M), Coriolis matrix (V) and gravitational matrix (G), which obtained after calculate the kinetic and potential energy for the moving platform as shown in the equations below:

$$M_{pl} = \begin{bmatrix} m_{pl}[\mathrm{I}]_{3X3} & 0_{3X3} \\ 0_{3X3} & I_{pl} \end{bmatrix} \quad \text{……(5)}$$

$$C(x)_{pl}\dot{x} = \frac{1}{2}\begin{bmatrix} 0_{3X3} & 0_{3X3} \\ 0_{3X3} & w_{pl}\mathrm{x}I_{pl} \end{bmatrix}\dot{x} \quad \text{……(6)}$$

$$G_{pl} = [0 \quad 0 \quad m_{pl}g \quad 0 \quad 0 \quad 0]^T \quad \text{……(7)}$$

- **Dynamic analysis of the linkages:**

The dynamic analysis of the linkages of the Gough-Stewart manipulator robot done by decompose each linkages into two parts (only rotate motion), the lower part and the upper part (rotate and linear motion) as shown in Fig (5). Assuming the center of the masses of each part are considered to be located at S_1 and S_2 from its ends and their masses denoted by m_1 and m_2.

This analysis was done by calculating the kinetic energy and potential energy for the linkages to obtain the mass, Coriolis, and gravity matrices as below:

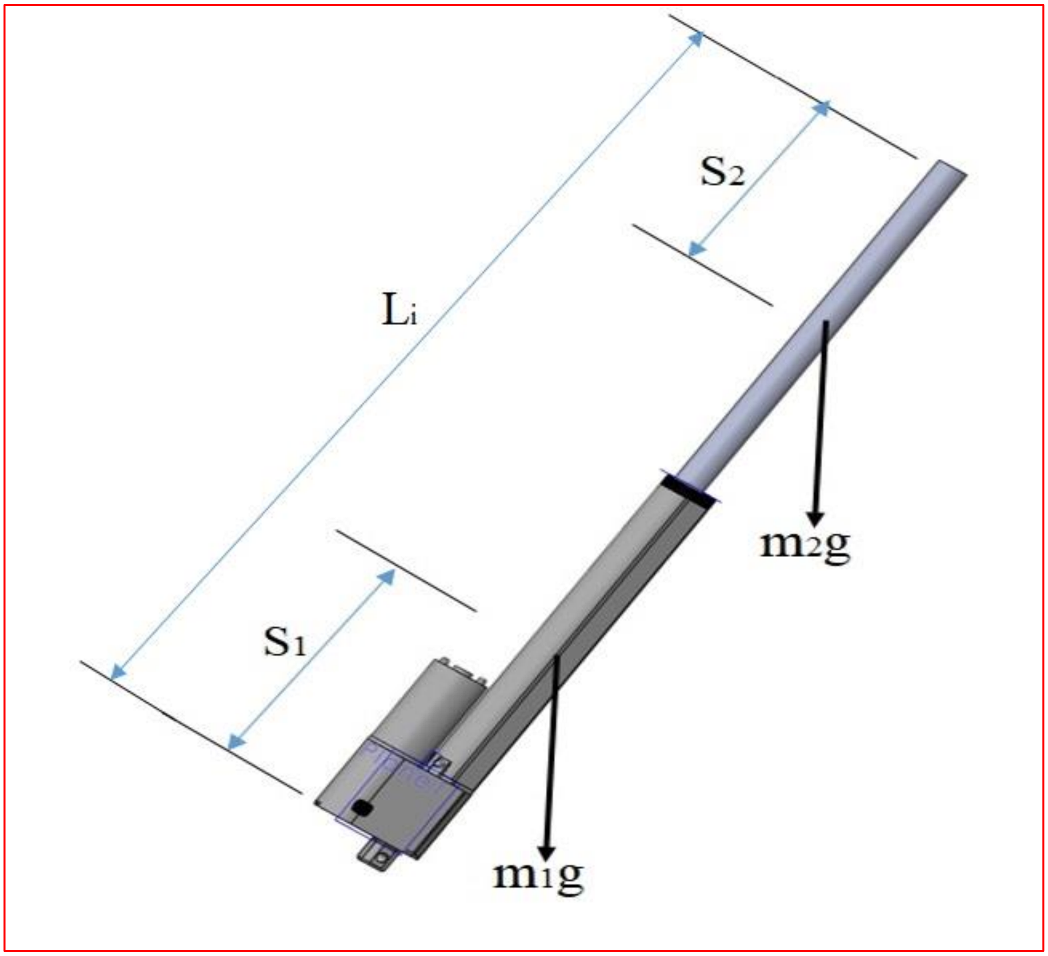

Fig (5): Two parts of i linkages.

Where:

S_1: the distance from the lower joint to the center of the lower part of i leg.

S_2: the distance from the upper joint to the center of the upper part of i leg.

m_1: the mass of the lower part of i leg.

m_2: the mass of the upper part of i leg.

$$M_{Li} = m_2 \vec{U}_i \vec{U}_i^{\,T} - m_{eq} \vec{U}_i^{\,2} \times - \frac{1}{|L_i|^2} I_{eq} \vec{U}_i^{\,2} \times \quad \textbf{......(8)}$$

$$C_{Li}(x_{Li},\, \dot{x}_{Li})\, \dot{x}_{Li} = (-\frac{1}{|L_i|^2} m_2 S_2 \vec{U}_i\, \dot{x}_{Li}^{\,T} \vec{U}_i^{\,2} \times - \frac{2}{|L_i|}\left(\frac{1}{|L_i|} m_2 S_2 - \frac{1}{|L_i|^2}\left(I_{eq} + |L_i|^2 m_{eq}\right)\right) |\dot{L}_i| \vec{U}_i^{\,2} \times)\, \dot{x}_{Li} \quad \textbf{......(10)}$$

$$G_{Li} = \frac{\partial P_{Li}}{\partial\, x_{Li}} = g((m_1 S_1 + m_2(|L_i| - S_2))\vec{U}_i^{\,2} \times - m_2 \vec{U}_i \vec{U}_i^{\,T}) \quad \textbf{......(11)}$$

$$I_{eq} = \begin{bmatrix} I_{xx1} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix} + \begin{bmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz2} \end{bmatrix}$$

$$m_{eq} = \frac{1}{|L_i|^2}(m_1 {S_1}^2 + m_2 {S_2}^2)$$

- **Dynamic analysis of the whole Gough-Stewart manipulator robot:**

To evaluate the final dynamic equations of the manipulator, the coordinates of the linkages x_{Li} in terms of the general coordinates of the moving platform x_{pl} to obtain the general mass, Coriolis, and gravity matrices, which used in Lagrange formulation to find the forces on the linkages.

$$\dot{x}_{Li} = J_i\, \dot{x}_{pl} \qquad \text{……(12)}$$

$$J_i = \begin{bmatrix} \mathrm{I}_{3\times3} & -{}^{L}P_i \times \end{bmatrix} \qquad \text{……(13)}$$

$$\ddot{x}_{Li} = J\ddot{x}_{pl} + \dot{J}\, \dot{x}_{pl} \qquad \text{……(14)}$$

$$\dot{J}_i = \begin{bmatrix} 0_{3\times3} & -(w_{pl} \times {}^{L}P_i) \times \end{bmatrix}$$

$$= \begin{bmatrix} 0_{3\times3} & -w_{pl} \times {}^{L}P_i \times + {}^{L}P_i \times w_{pl} \times \end{bmatrix} \qquad \text{……(15)}$$

The Lagrange formulation for the linkages can be written as:

$$M_{Li}\, \ddot{x}_{Li} + C_{Li}\, \dot{x}_{Li} + G_{Li} = (J^{-1})^T F_{Li} \qquad \text{……(16)}$$

$$M_{Li}(J_i\ddot{x}_{pl} + \dot{J}_i\, \dot{x}_{pl}) + C_{Li}(J_i\, \dot{x}_{pl}) + G_{Li} = (J^{-1})^T F_{Li} \qquad \text{……(17)}$$

Hence the mass, Coriolis, and gravity matrices for whole manipulator can be written as:

$$M = M_{pl} + \sum_{i=1}^{i=6} M_{Lini} \qquad \text{……(18)}$$

$$C = C_{pl} + \sum_{i=1}^{i=6} C_{Lini} \qquad \text{……(19)}$$

$$G = G_{pl} + \sum_{i=1}^{i=6} G_{Lini} \qquad \text{……(20)}$$

$$(J^T)^{-1}(M\ddot{x}_{pl} + C\, \dot{x}_{pl} + G = F$$

Where:

$$M_{Lini} = {J_i}^T M_{Li} J_i \qquad \text{……(21)}$$

$$C_{Lini} = {J_i}^T M_{Li} \dot{J}_i + {J_i}^T C_{Li} J_i \qquad \text{……(22)}$$

$$G_{Lini} = {J_i}^T G_{Li} J_i \qquad \text{……(23)}$$

6- Modeling of the Dynamic equations in Matlab:

In this paper, the inverse dynamic model in the last equation employed to calculate the forces of the linkages, which need to compute the mass, Coriolis, gravity, and jacobian matrices based on the input parameters, the overall modeling of the dynamic analysis have been done in the Matlab software (Simulink blocks), which represented in the Fig (6).

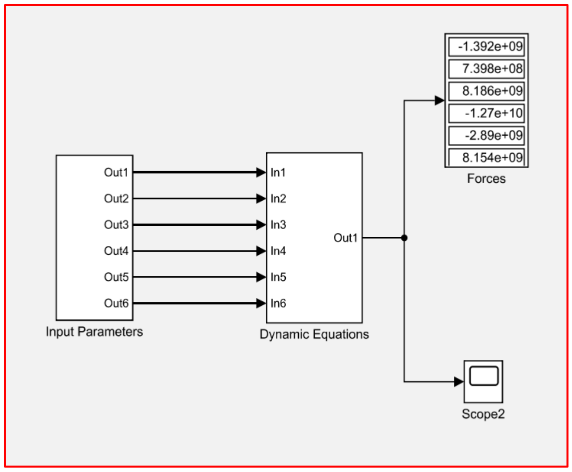

Fig (6): Dynamic model in Simulink.

The subsystems of the inputs parameters include position, orientation, translation and angular velocities, and translation and angular acceleration for the center of the moving platform, as shown in the Fig (7), Fig(8), and Fig (9), lead to commuted the forces in the last block Fig (6).

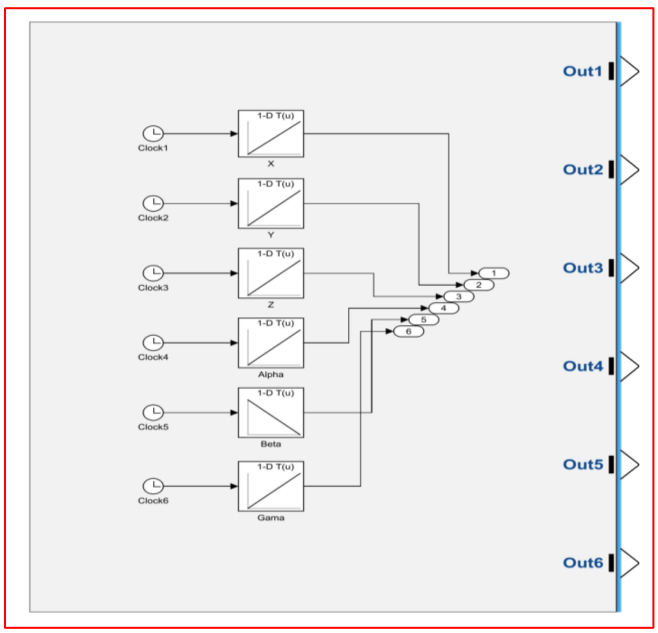

Fig (7): Position and orientation coordinates.

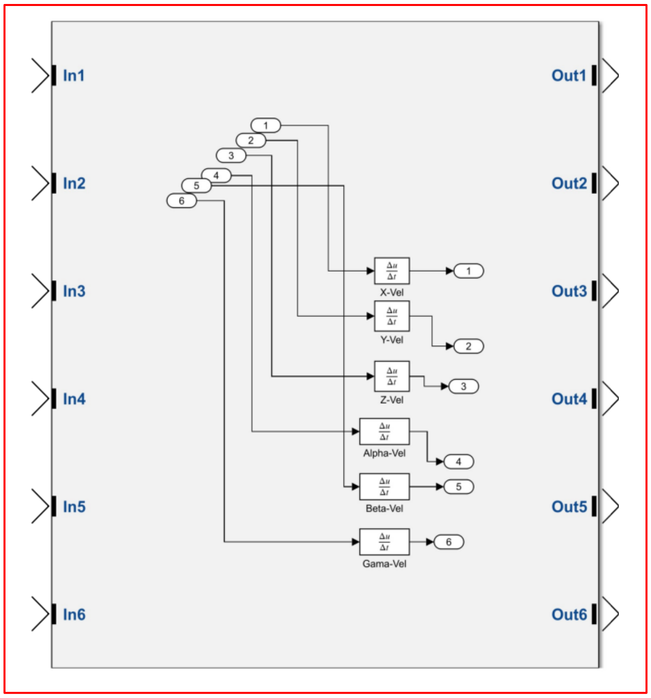

Fig (8): Translation and angular velocities.

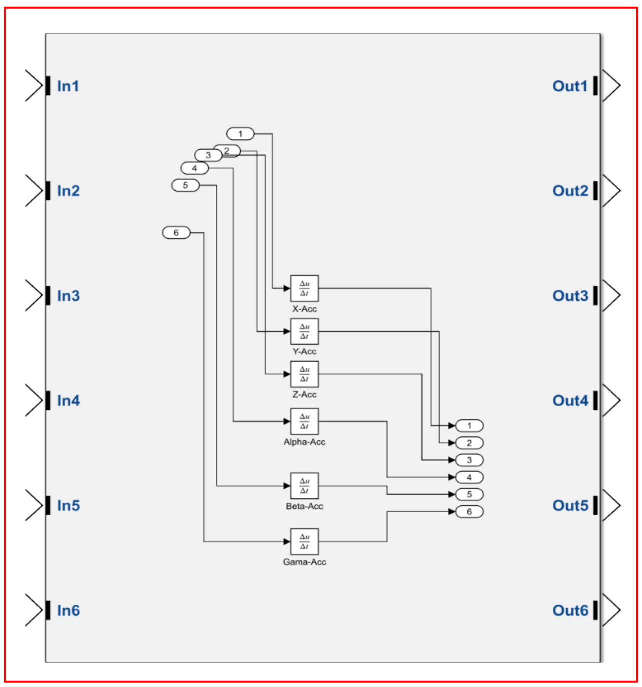

Fig (9): Translation and angular acceleration.

And the subsystems of the dynamic equations consist of invers kinematics blocks to compute the lengths, Jacobian block, mass matrix, Coriolis matrix, and gravity matrix, Fig (10) and Fig (11).

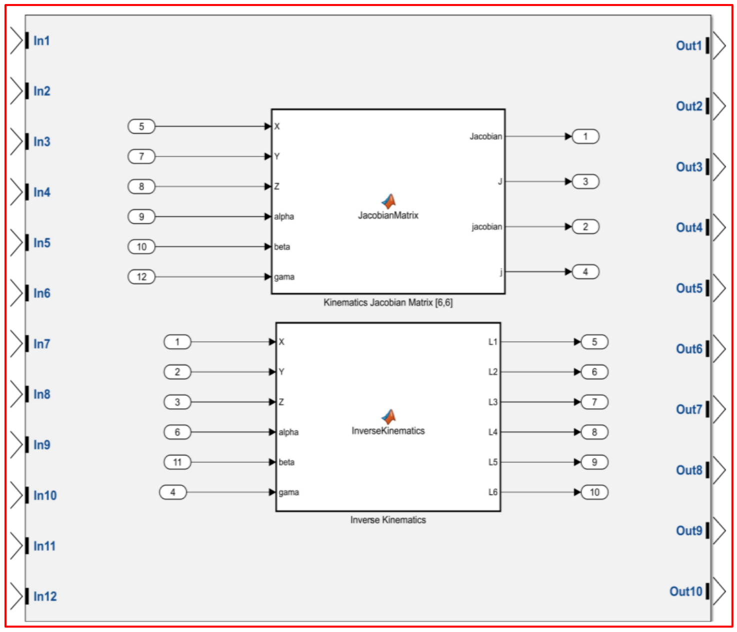

Fig (10): Inverse kinematics and jacobian blocks.

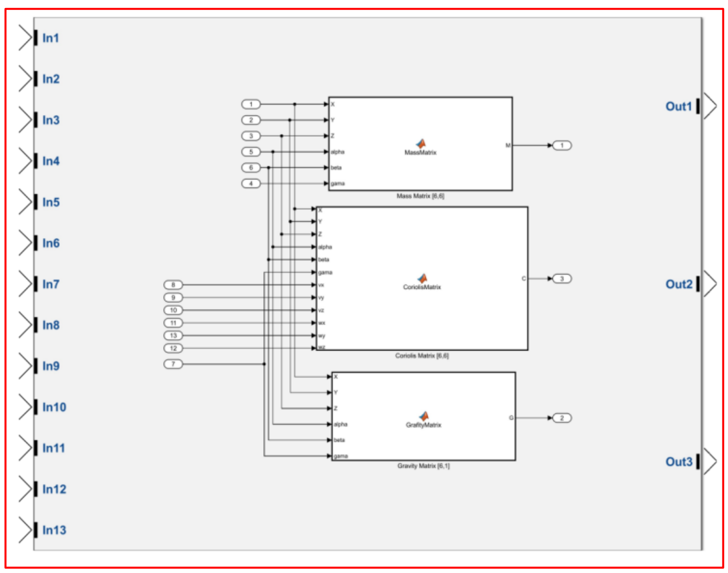

Fig (11): Mass, Coriolis, and Gravity matrices in Simulink.

7- Results and Discussion:

The position and orientation (X, Y, Z, α, β, γ) used in the these analysis represented in Fig (12), while the linear and angular velocities obtained by the first derivative to the input parameters Fig(13) and the linear and angular acceleration from the second derivative for these parameters Fig (14).

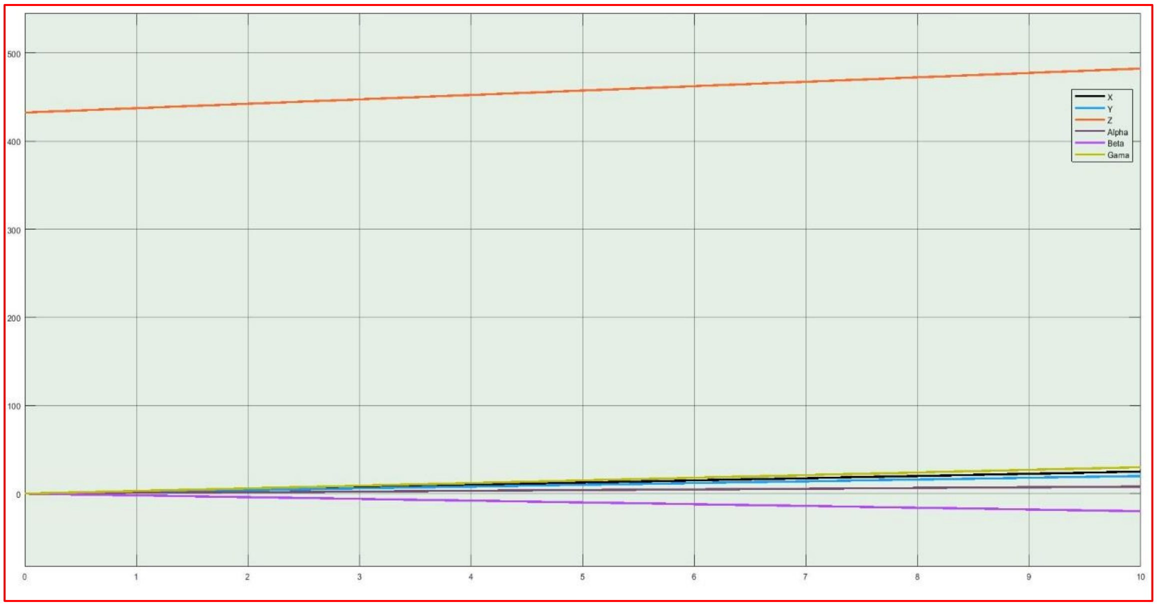

Fig (12): Inputs parameters.

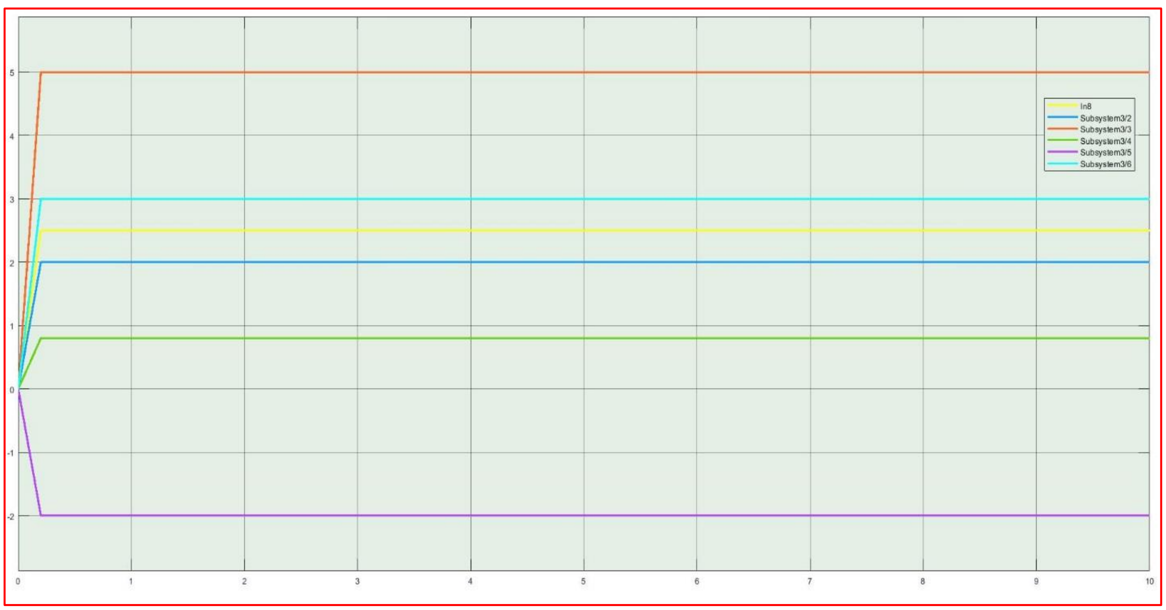

Fig (13): Results of velocities.

Fig (14): Results of acceleration.

From the inverse kinematics, equation can be obtained the lengths of the links from initial to final position Fig (15) and the position and orientation of the moving platform shown in Fig (16).

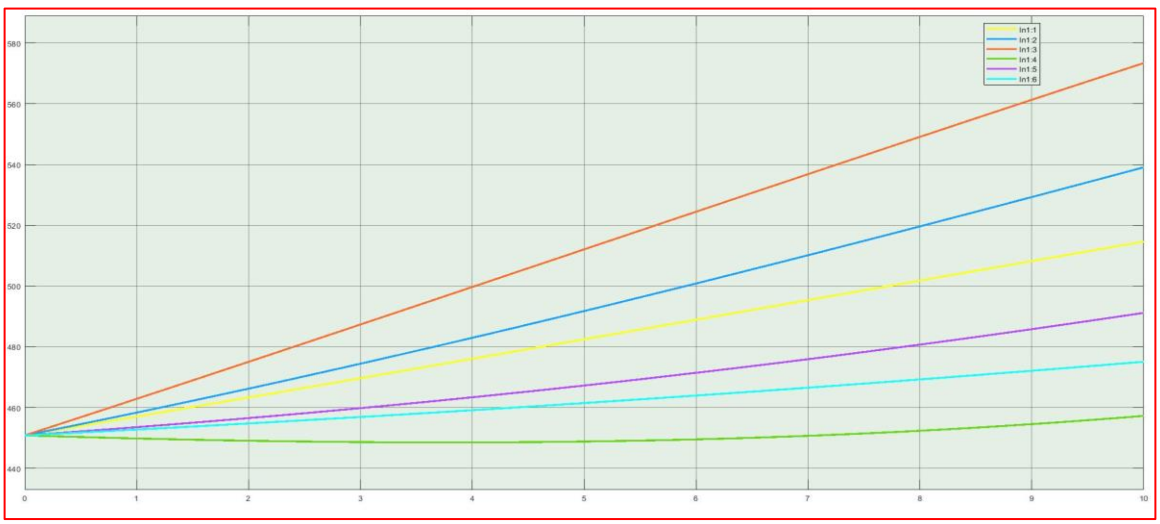

Fig (15): Results of lengths of the linkages.

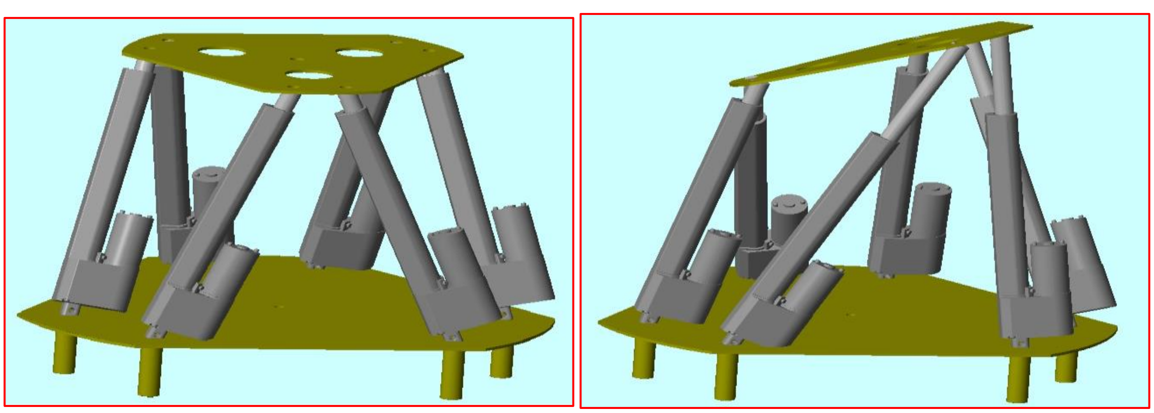

Fig (16): Platform at initial and final position.

In the Fig (17) represents the determinant of the Jacobian matrix at any time, which is very necessary to explain if the trajectory of the moving platform from initial to final position contain a singular point (at det (Jacobian)=zero)

Fig (17): Results of the determinant of the Jacobian matrix.

The links forces result from the inverse dynamic analysis by Lagrange formulation explain in the Fig (18).

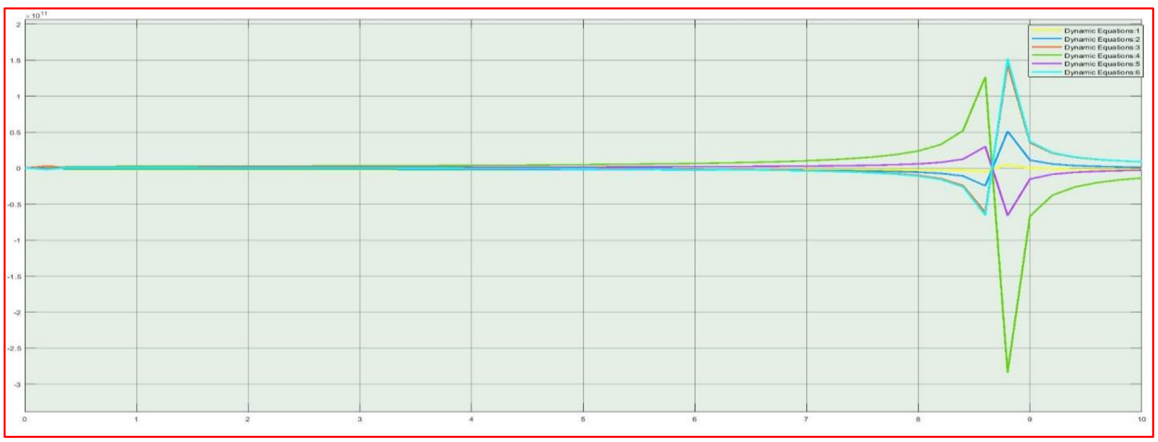

Fig (18): Results of the Linkages forces.

From the above results, there are a high increase in the forces between time 8 and 9 due to the determinant of the Jacobian matrix equal to zero, this mean a singular point appear the trajectory of the moving platform.

8- Conclusion:

1- The invers dynamic analysis used to calculate forces of the links during the motion of the moving platform.

2- The dynamic equations derived by using Lagrange formulation, which required determining the kinetic and potential energy of each moving part.

3- In addition, the Jacobian matrix derived from the kinetic analysis where the equations of the dynamic multiply by the jacobian matrix.

4- After derived the mass, Coriolis, gravity, and jacobian matrices inserted in the Simulink blocks in Matlab software to get the actuators forces.

5- These forces may be increased suddenly due to the singular point during the path of the trajectory when the determinant of the jacobian matrix equal to zero as shown in the above figures.

CONFLICT OF INTERESTS.

- There are no conflicts of interest.

9- References:

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التحليل الديناميكي للروبوت المتوازي كووف ستيوورت باستخدام طريقة لاكرانج ببرنامج الماتلاب

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الخلاصة:

في هذا البحث، استخدم الروبوت المتوازي كووف ستيوورت له ست درجات من الحرية كنموذج لاشتقاق معادلات الكينماتك المعكوسة والمعادلات الديناميكية العكسية. إن اشتقاق التحليل الكينماتيك العكسي بسيط جداً الذي يستخدم لحساب مصفوفة جاكوبيان وأطوال الارجل لتحديد مسار المنصة المتحركة. المعادلة الديناميكية اشتقت بطريقة لاكرانج من خلال حساب الطاقة الحركية والكامنة للنموذج. المعادلة الديناميكية تم ادخالها في قالب السميولنك كملف ماتلاب تؤدي إلى حساب قوى الارجل في أي وقت من المحاكاة ويمكن استخدامها لبيان ما إذا كان المسار يحتوي على شذوذ حيث تتزايد القوى بسرعة كبيرة في نقطة الشذوذ مقارنة بالنقاط الأخرى أثناء مسار المنصة المتحركة. أخيرا يمكن استخدام هذه القالب في النماذج الأخرى مع مدخلات مختلفة.

الكلمات الداله: انسان الي متوازي، كووف ستيوورت، ماتلاب، سيمولنك، الكينماتك العكسي، التحليل الديناميكي، صيغة لاكرانج.