Basins and Cultivated Lands Recharge Evaluation on The Basis of Experimental & Mathematical Analysis in Hashimiya-Iraq

Moutaz Al Dabbas

Geology department, College of Sciece, Baghdad University Najah M. Lateef

Arkan Radi Ali

Civil Dept., Technical Instite of Babylon ,AL-Furat AL-Awsat ,Technical University Arkan.radi@vahoo.com

Abstract

A mathematical, field measurements and laboratorial study was adopted to evaluate the validity of many mathematical forms including Horton Model for estimating infiltration rate and infiltration potential, they are; power, polynomial and mixed formulas. The work composed of infiltration field measurements for Hashyimia Region located in the middle of Iraq which for simplicity and accuracy is divided into nine administration agricultural sectors namely as; Jerboevia, Hashvimia, Niwedra, Tebra, Sada, Zineyia, H3, Fayadhiya, and Bazul sectors. The current study proved that the power formula is the best fit to the measuring data than polynomial, mixed and Horton models since the power formula offers 0.996 correlation factor whereas polynomial, mixed and Horton offer 0.947, 0.958 and 0.84 respectively. It is worth to mention that in all cases Horton Model offers less infiltration potential after 2hrs since infiltration process is started

A real infiltration depth that accumulated in a subsurface bearing layer is also estimated depending upon field sampling and laboratorial testing of geologic formation textures to determining the porosities throughout Hashyimia Region.

Keywords: Horton Model, Infiltration Rate, Infiltration Potential, Powre Formula, Polyonemial Formula, Mixeed Formula, Real Infiltration Depth

الخلاصة

القياسات الرياضية والحقلية والدراسة المختبرية المعتمدة لتققيم صلاحية الاشكال الرياضية ضمنها نموذج هورتن في حساب الترشيح وهي معادلة الطاقة والمتعددة والمختلطة . العمل شمل قياسات الترشيح لمنطقة الهاشمية الواقعة وسط العراق والتي قسمت الى تسعة قطاعات اداريةهي الجربوعية ،الهاشمية ،النويدرة ،الطبرة ،السادة ،الزينية ،اج3،الفياضية والتياس.الدراسة الحالية أثبتت أن معادلة الطاقة هي الافضل حيث يصل معامل الار تباط 0.996 بينما للمتعددة والمختلطة و هو رتن 0.958،0.947 و 0.84 على التوالى. من الجدير بالذكر ، ان نموذج هورتن يعرض ترشيح اوطأ بعد الساعتين.عمق الترشيح الحقيقيي في الطبقة الحاملة التحت سطحية حسبت بالاعتماد على النماذج الحقلية والفحوصات المختبرية لنسيج التكوين الجيولوجي لتحديد المسامية لمنطقة الهاشمية.

الكلمات المفتاحية: نموذج هورتن ،معدل الترشيح،الترشيح الاقصى ،معادلة الطاقة ،المعادلة المختلطة وعمق الترشيح الحقيقي.

Introduction:

Infiltration potential estimate depends mainly upon field measurements and mathematical integration for the area under infiltration rate-time curve.

There are several models have been issued representing downward infiltration movements but they constrained with assumptions and limitations (Horton ,1940; Green 1986 ; Mishra et.al., 2003 and Swamee et.al., 2012) developed models predominantly applicable when rainfall intensity exceeding initial soil capability of water absorption. To solve this drawback, many attempts were made to modify the basic model of Horton to be applicable widely, among them (Mein and Larson, 1973; Bauer ,1974; Green ,1986; Akan , 1992; Gavin and Xue ,2007; Ismail , 2013) generalized Horton Eq. to be applicable for low rainfall intensity since it is originally derived to predict infiltration rates that rainfall intensity exceeding the initial infiltration rate of soil.

An extensive studies were made by many workers in this aspect among them; (Jinguan et.al., 1996) field lysimeter experiments were used to study the relationships between rainfall and recharge by infiltration at different groundwater depths. They applied 1.5, 3, 4.5, and 5 m lysimeters depths at fixed GW levels to develop a mathematical simulation. Accordingly GW algorithm was classified into shallow, intermediate and deep GW. (Swartzendruber ,1997) derived two terms infiltration equations by using the integral of 1D downward Richarg Eq. subjected to initial and boundary conditions. The solution is intrinsic to two stipulations; 1) linear relation between water content (ω) and hydraulic conductivity (k) 2) the bonded water should be increased as a square root of time (t) since start of water application. Although in case of absence these two stipulations, this simple mathematical form fails experimentally, nonetheless the solution stays a useful pedagogically. (Arnold et.al.,2000) used two methods to estimate GW recharge and discharge (base flow) upper of Mississippi River basin. A water balance analysis is used in the first method whereas the second method composed of two steps for both base flow and recharge estimating of daily stream flow: firstly a separation of base flow from daily flow and secondly a modification of hydrograph recession curve technique to estimate groundwater recharge. The two methods are potentially good to provide a reasonable and realistic estimates of base flow and recharge to be latterly used in regional groundwater models.(Rousta et.al., 2012) an artificial recharge of groundwater has been examined by an appropriate technology in Iran. The effects of raw gypsum <5%on infiltration rate enhancement were tested. The treatments include infiltration 1) without gypsum application. 2) 15cm of top soil mixed with 0.5kg/m of gypsum. 3) 15cm of top soil mixed with 1kg/m of gypsum before adding infiltration water. The results showed that a gypsum addition of 0.5 kg/m to infiltrated water significantly increased. (David et.al., 2012) used a data between 2003 and 2007 to determine the feasibility of recharging Stockton Aquifer, California. It is found that by using a tracer test basin water moves down through a saturated alluvial deposits until reaching a 110m more permeable zone in depth and then moves speedily to nearby wells at rates of about 13m/day which confirmed by flowmeter logging and 2D numerical simulation radial groundwater flow model. Marco et al (2016) focused on temporal evolution infiltration rates in basins overlying highly permeable aquifers. An infiltration basin of 16 hectares north Italy has been monitored via 4 years. Field and laboratorial test data were characterized to construct a saturated-unsaturated numerical model. It is found that a basin infiltration is extremely affected by natural recharge patterns.

In the current paper, a comparative study was made between Horton Model and several suggested mathematical forms to evaluate infiltration potential of an agricultural region of Hashyimia which is located in the middle of Iraq.

Significance of study

It is frequently encountered during infiltration measuring and estimating that Horton Formula is not frequently in the level of ambition to fit so many infiltration field data within a certain basin and cultivated fields as occurred in Hashyimia Region, therefore it is suggested to make a comparative study between Horton formula and many other mathematical forms to evaluate the validity of Horton.

Purpose of study:

The current study is aimed to:

- 1- Compare the infiltration calculus of Horton Model with many selected mathematical formulas.
- 2- Evaluating a water recharge to the unconfined bearing layer of Hashimiya Region

Geography & Topography of the Study Area

Hashimiya Region of 110km² in area located in the southern-east part of Babylon Governorate and bounded between longitudes of 44° 36' and 44° 47' and

latitudes of 32° 15′ and 32° 25′. Nine streams are found to recharge the unconfined bearing layer namely as; Jerboeya Hashimiya, Niwedra, Tebra, Sada, Zineya, H3, Fayadhia, Bazul pass the area and to satisfy the water demand WD and urban water requirement WU successively. Whereas D_1 and D_2 are existing drains passing the area from the north toward the south as indicated in the location map of Fig.(1).In general, the area appears to be flat and reduces gradually in elevation from northern east to southern west. In general the highest elevation is 26.5 m a. s. 1 whereas the lowest is 22.5 m a. s. 1. Since the area is too large to evaluate the infiltration therefore it is divided into a number of divisions they are; Jerboeya Hashimiya, Niwedra, Tebra, Sada, Zineya, H3, Fayadhia, Bazul as shown in Fig. (2).





Fig(2) Agricultural Sectors of Hashimiya Region

Mathematical Back Ground & Conceptualization

Since Horton model has a drawback of inapplicability when rainfall intensity is less than initial infiltration rate of soil and less ability to fit all kind of data exactly, a curiosity to make a comparative with other mathematical forms for infiltration estimation.

1- Horton Model

By using Horton equation of the form: $f_p(t) = f_c + (f_o - f_c)e^{-kt}$(1) Where:

 $f_n(t)$: Infiltration rate or infiltration capacity at a time (t).

 f_c : Final infiltration capacity. f_o : Initial infiltration capacity.

K: a constant representing the rate of decrease in infiltration capacity

Horton formula may be used to determining the infiltration rate and infiltration potential after the equation has been fitted to a specified recorded infiltrometer data. The difficulty in using Eq. (1) is:

- 1- The determination of accurate constant k which is mainly depended upon the fitted data.
- 2- Achieving an experimental trial to estimate the rainfall intensity and initial infiltration capacity of soil and then knowing the possibility of the model application

Determination of Horton Constant (K) and data fitting

In order to find out the value of the constant K

1- Entering the natural logarithmic for both sides of Eq. (1) as;

 $\ln(f_n(t) - f_c) = \ln(f_0 - f_c) + \ln(e^{-kt})$

By applying the characteristics of natural logarithm one may obtain:

$$\ln \frac{f_p(t) + f_c}{f_o - f_c} = -kt$$

Further simplifications offer:-
 $k = \frac{1}{t} \ln \left(\frac{f_o + f_c}{f_p(t) - f_c} \right)$

Infiltration Potential Calculus

The calculation of the infiltration potential of rainy storm or of irrigation water within a finite time interval may be done by a mathematical approach as:

.....(2)

Integrating both sides of eq. (1)

Where $AF_p(ti)$: The infiltration potential for a time period.

Powre Fitting Formulla

In this technique a power form is assumed to fit the measured field data of an infiltrometer, it may be in the form of:

 $f_n(t) = at^{-k}$ Where $f_p(t)$: is the infiltration rate, t: is a time since the infiltration process is started, and (a and k): are constants dependent upon the nature of a fitted data.

Determination of Constants (ai, ki) and Data Fitting

Take the natural logarithm for both sides of Eq. (4) and a necessarily simplification may be made as:

 $\ln f_p(ti) = \ln at^{-ki}$

 $\ln f_p(ti) = lnai - kln(ti) \tag{5}$

The data then is plotted to semi-log paper to find the intersection distance in the ordinate which represents (lna) whereas (k) represents the slope of the fitted line or by using an alternative computer technology of Axil Software that offers the constants a and k easily.

Infiltration Potential Estimation

The finding out of Infiltration Potential is computed by taking the integral for both sides of Eq. (4) as follows

2- Polyonemial Fitting Formulla

A polynomial of any power degree is also used to fit a specified data in order to obtain the infiltration capacity. The polynomial power degree depends mainly on data accuracy and no. of observations. It is may be of the form

 $f_p(t) = at^{ni} - bt^{ni-1} - ct^{ni-2} + \dots + di$ (7)

The measured data of Table (1) should be used to formulate a set of simultaneous equation, then the constants a, b, c ... can be obtained by solving the resulting matrix by Gauss Elimination or any other traditional method.

Infiltration Potential Estimation

3- Mixeed Fitting Formulla

This method represents a product of Horton and power functions which may be in the form of;

$$f_n(t) = a.i(te^{ti})^{-ki}$$
(10)

Determination of a & k Constants

To facilitate the determination of the constants a & k, a simple transformation is achieved before issuing of estimation that is by assuming $z = te^{t}$ therefore Eq.(10) may be rewritten to be of the form;

$$f_p(t) = a(z.i)^{-ki}$$
(11)

Eq. (11) may be treated as a simple power formula.

Infiltration Potential Estimation

The easier procedure to achieve this integral is by using a simple transformation of *Tylor Expansion* Thomas (1979), and Erwin Kryziyk (1972). Eq. (12) in the expanded form may be rewritten as:

expanded form may be rewritten as: $\int_{0}^{t} f_{p}(t) dt = \int_{0}^{ti} a(t^{ki} - kt^{ki+1} + \frac{k^{2}t^{ki+2}}{2!} - \frac{k^{3}t^{ki+3}}{3!} + \frac{k^{4}t^{ki+4}}{4!} - \frac{k^{5}t^{ki+5}}{5!} + \cdots) dt \dots \dots (13)$ By achieving the usual integral on both sides of Eq. (13) we obtain the

By achieving the usual integral on both sides of Eq. (13) we obtain the following

3.7.1.2 Infiltrometer Setup & Field Measurements

The testing procedure is initiated by removing the unfavorable soil crust of about $9m^2$ and maintaining soil surface fairly leveled and undisturbed. Then, the two rings are driven consecutively to a depth of 10-12 cm in the surface soil by using a hammer of 5.5 kg. A steel cover is place horizontally on the upper edges of the cylinders for protection purposes of hammer knocking. Then, water is firstly added to the outer ring which is provided with a graduated scale. The two cylinders should be filled with water to equal levels. After the water is set in the infiltrometer, a time-height records are instantaneously taken for a period of about 4:30 hours. The methodology of field measurements carried out by the researcher within Hashyimia Region in (2015)

Estimation of Infiltration Potential

The Infiltration Potential is estimated in each sector individually according to Eqs. (3, 6, 8, and 14, but for abbreviation purpose only Fayadhiya sector is presented in Table (1) whereas Table (2) presents the fitted forms of the four models calculus with their constants and correlation coefficients

			Potential Infiltration, mm						
Accu. Depth of Infiltrometer Readings, mm	Time, hrs	Inf. Rate mm/hr	Horton $R^2 = 0.84$	Power R^2 = 0.996	Polyonemial $R^2 = 0.947$	$Mixeed$ R^{2} $= 0.958$			
2	0.083333	24	1.803899	3.757256	1.765647	0.440196			
3	0.166667	18	3.40493	5.481912	3.404648	1.023715			
4	0.25	16	4.830391	6.837571	4.924912	1.669839			
6	0.5	12	8.27414	9.976155	8.848717	3.802278			
8	1	8	12.7814	14.55541	14.44848	8.39324			
9	1.25	7.2	14.36609	16.43768	16.45282	10.71322			
10	1.5	6.666667	15.70599	18.15492	18.11284	13.00557			
11	1.75	6.285714	16.88722	19.74605	19.5434	15.25158			
12	2	6	17.96561	21.23664	20.8396	17.43811			
14	2.5	5.6	19.94583	23.98291	23.31074	21.5955			
15	3	5	21.80867	26.48839	25.89248	25.41882			
16	3.5	4.571429	23.62219	28.80989	28.63522	28.85981			
16	4	4	25.41498	30.98469	31.2736	31.86801			
16	4.5	3.555556	27.19907	33.03888	33.22647	34.3832			

Table (1) Regional Infiltration Calculus of Fayadhiya Sector

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	Table (2) Models Calculus						
Model	Equation	R^2					
Horton	$f_p(t) = 3.557 + (8 - 3.557)e^{-1.525935t}$						
	$AF(t) = 2.557t + \frac{(8 - 3.557)}{(1 - 2^{-1.525935t})}$	0.84					
	$AI_p(t) = 5.557t + \frac{1.52595}{1.52595}$						
Powre	$f_p(t) = 8.17712t^{-0.45}$						
	$8.17712t^{1-0.5}$	0.996					
	$AF_p(t) = \frac{1 - 0.499}{1 - 0.499}$						
Polyonemial	$f_p(t) = -3.557 - (8 - 3.557)e^{-1.525935t}$						
	$AF_p(t) = 0.210525t^{4i} + 2.383t^{3i} - 9.704t^{2i} - 21.98t$						
	$AF_{n}(t) = \left(\frac{a}{1+t}t^{ki+1} - \frac{aki}{(1+t)}t^{ki+2} + \frac{aki^{2}}{(1+t)^{2}}t^{ki+3}\right)$						
	ki + 1 $(ki + 2)$ $(ki + 3)2!$						
Mixeed	$-\frac{aki^3}{ki-4}-\frac{aki^4}{ki-5}$						
	(ki + 4)3! $(ki + 5)4!$						
	$-\frac{aki^5}{t^{ki-6}}$	0.900					
	$-\frac{1}{(ki+6)5!}$						
	a = 11.749, and $k = 0.243$						

The comparative graphical presentation of the four models in Fayadhiya sector is shown in Fig. (3)



Fig. (3) Infiltration Potential of Fayadhiya

Comparative Consideration

It is worth immediately to answer the question (what is the best fit method?). The mathematical analysis reveals that Horton Model:

- 1- It is applied when rainfall intensity exceeding the initial infiltration rates
- 2- Offers less correlation coefficient of 0.84.
- 3- Offers less potential infiltration after 2.75 hrs.
- 4- Additional work to discover if the model is applicable or not.

However, the power formula offers a maximum correlation factor of 0.996, therefore it is selected to be compared with Horton Model rather than other comparative forms.

Figs (4 & 5) presents comparisons between the measured and fitted inf. Rate for both Horton and power models respectively.



Fig.(4) Inf. Rate Comparative of Horton Model of Fayadhiya Sector



Fig.(5) Inf. Rate Comparative of Power Model for Fayadhiya Sector

Comments:

- 1- The preceding analysis reveals that Horton Model produces less infiltration potential with correlation coefficient of 0.84, moreover Fig.(4) reflect not a good coincidence between the measured and fitted observations of infiltration rate..
- 2- Power Model shows a good coincidence between the measured and fitted infiltration rates.

Real Bearing Layer Recharge

In this situation, it is an evitable question should be answered is that (How much is the real recharge reaching the unconfined bearing layer within all sectors of Hashyimia Region?) In order to answer the question is first important to investigate the following points:

1- What amount is cultivated and bore lands in each sectors?

2- How long is the time interval for each individual irrigation?

The answers are that it is customized in Hashyimia Region that irrigation time interval of single irrigation is about (2hrs per week in summer) and (2hrs per 3weeks in winter) and 48% of the total area is cultivated. Therefore the infiltration potential at 2hrs of Table (1) is considered.

Correspondingly the Infiltration Potential for dry and summer seasons after 2hrs flooded irrigation period are presented in Table (3) for all sector and shown in the contour maps of Figs. (6 and 7) in summer and winter.

Sector	Ordinates corresponding to		Dry Period Infl. Potential		Wet Period Infl. Potential		Actual Cultivated	Annual Infl. Volume m ³ /year			
	the Or	igin (o)	mm/	day	mm/c	lay	Area, m ²	Horton		power	
	X	у	Horton	power	Horton	power		Summer	Winter	Summer	Winter
Jerboeya	5.35	7.14	4.11	4.71	1.37	1.57	6289510.627	4652980	1550993	5332247	1777416
Hashimiya	8.92	7.85	0.91	1.11	0.330	0.37	5951502.045	974856	353519.2	1189110	396370
Niwedra	7.85	10.35	3.11	3.82	1.037	1.27	7511443.537	4204906	1402086	5164869	1717116
Tebra	8.92	11.42	2.14	2.63	0.71	0.88	4054827.476	1561920	518207	1919555	642284.7
Sada	11.0	9.64	3.03	3.93	1.01	1.31	1817593.316	991315.4	330438.5	1285766	428588.5
Zineya	11.0	8.57	3.9	3.767	1.3	1.318	4512733.441	3167939	1055980	3059904	1070601
Н3	12.85	6.78	2.62	2.87	0.87	0.96	5346913.11	2521604	837326.6	2762215	923946.6
Fayadhiya	14.28	4.99	2.56	3.03	0.86	1.01	6633896.729	3056900	1910562	3618127	1206042
Bazul	13.21	2.85	4.81	6.14	1.6	2.04	4357121.943	3772396	674482.5	4815491	1599935
Total	-	-	27.19	32.007	9.087	10.728	46475542	3353	8411	3890	9584

 Table (3) Infiltration Potential Calculus

x & y are horizontal and vertical distances from the origin respectively, km Comment: The infiltration potential of Table (3) are obtained by dividing the values of table (3) after 2hrs by 7days (single irrigation interval) or summer and by 21 days (single irrigation interval) on winter.



According to the results of a real infiltration which recharging the unconfined bearing layer and the real cultivated lands areas within the agricultural sectors. The amount of water recharging the bearing layer is estimated and listed in Table (3).

Real Infiltration Depth (d_R)



Fig. (8) Real Infiltration Depths

Referring to Fig. (8), one concludes that:

$$d_i = d_v$$
(15)
Where

References: d_i is applied water depth, d_v : depth of solid. But $d_v = nd_R$ therefore Eq. (15) after substitution becomes; $d_R = \frac{d_i}{n}$ (16) Where d_R : is a real Infiltration depth reaching the water bearing stratum. But;

Where d_R : is a real Infiltration depth reaching the water bearing stratum. But; $n = 1 - \frac{\gamma_d}{G_s \gamma_W}$ (17)

Where n is a porosity of soil texture of the geologic formation, G_s : specific gravity, γ_w : water density is taken in (gm/cm³)

Experimental Work

- 1- A core test according to ASTM D 854-00 was carried out for undisturbed samples to find out the average dry density (γ_d) of the bearing layer within the nine sectors and the results are listed in Table (4).
- 2-Standard test methods for an average specific gravity (G_s) determination according to (ASTM D 891 09) is carried out and also shown in Table (4). Fig. (11) presents Porosity distribution in the area.

Referring to Eqs.(16 &17) The real infiltration potential and porosities are estimated respectively throughout Hashyimia Region and included in Table (4) and Figs.(9 & 10)[°]

Table (4) Dry unit weight, specific Gravity, Real Infiltration Depths

Sector	Ordinates from the Origin (o), km		Ave. Dry Density (γ_d)	Ave. Specific Gravity (G _s)	Ave. Poros ity (n)	Ave. oros ity (n) Dry Period Potentia mm/day		l Infl. Wet Perio al Infl. Potent y mm/day	
	X	у	gm/cm ³			Horton	power	Horto n	power
Jerboeya	5.35	7.14	1.7145	2.650	0.353	11.64	13.34	3.88	4.45
Hashimiya	8.92	7.85	1.7621	2.63	0.330	2.75	3.36	1	1.12
Niwedra	7.85	10.3 5	1.6709	2.695	0.380	8.18	10.05	2.72	3.34

Tebra	8.92	11.4 2	1.6944	2.660	0.363	5.897	7.24	1.95	2.42
Sada	11.0	9.64	1.6821	2.670	0.370	8.18	10.62	2.72	3.54
Zineya	11.0	8.57	1.7870	2.688	0.335	11.64	11.24	3.88	3.93
НЗ	12.8 5	6.78	1.7304	2.650	0.347	7.55	8.27	2.50	2.76
Fayadhiya	14.2 8	4.99	1.6838	2.660	0.367	6.97	8.25	2.34	2.75
Bazul	13.2 1	2.85	1.6821	2.67	0.370	13	16.59	4.32	5.51

Conclusions:

It is concluded that:

- 1- The power formula is more capable to fit a recorded data than Horton Model since it offers 0.996 correlation factor whereas Horton Formula offers 0.84.
- 2- Horton Formula offers less Infiltration Potential.
- 3- Power Formula has no limitations and drawbacks.



Fig. (9) Real Infiltration Potential for Dry Season, mm/day



Fig. (10) Real Infiltration Potential for Dry Season, mm/day



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