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Fuzzy H_2 Guaranteed Cost Sampled-Data Control of Nonlinear Time-Varying Delay Systems

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Abstract: We present and study a delay-dependent fuzzy H_2 guaranteed cost sampled-data control problem for nonlinear time-varying delay systems, which is formed by fuzzy Takagi-Sugeno (T-S) system and a sampled-data fuzzy controller connected in a closed loop. Applying the input delay approach and stability theorem of Lyapunov-Krasovskii functional with Leibniz-Newton formula, the H_2 guaranteed cost control performance is achieved in the sense that the closed-loop system is asymptotically stable. A new sufficient condition for the existence of fuzzy sampled-data controller is given in terms of linear matrix inequalities (LMIs). Truck-trailer system is given to illustrate the effectiveness and feasibility of H_2 guaranteed cost sampled-data control design.
Keywords: fuzzy T-S system; sampled-data; nonlinear systems; time-varying delay; H_2 guaranteed cost control

1 Introduction

Fuzzy Takagi–Sugeno(T-S)models [1] are used to describe nonlinear systems by a set of IF–THEN rules which gives a local linear representation. Since the work of Tanaka and Sugeno [2] on stability analysis and stabilization being published, many efforts have been made in developing systematic theory for such systems.

Because of the fast development of the digital circuit technology, using computers to design controller to reduce the implementation cost and time is more and more popular. The system of control is a sampled-data system. In sampling period,its control signals are constant. The overall control system becomes a sampled-data system, where the control signals are kept constant during the sampling period. It's a popular trend to study the analysis and synthesis of fuzzy sampled-data systems in many papers, see, for instance, [3–12] and the references therein. Of these works, stability analysis [3], stabilization [11], H_∞ control [4,6,7,9], H_2 GC control [8,10], fault-tolerant control [12] and tracking control [5] are researched, respectively.

Stability and robust stability theory was adopted in sampled-data time-delay systems [3, 4, 7, 9]. In industrial systems and information networks, it's popular to use time-delay systems. So, we should study time-delay systems and design some controllers for them. There have two ways for the stability analysis and synthesis of time-delay fuzzy T–S systems, i.e. delay-independent and delay-dependent approaches. With no respective of the size of the delay, we use delay-independent approach to assure stable conditions. The delay-dependent approach, contrast with the delay-independent approach, is complex in design procedure. So, it always have more conservative results. The delay-dependent approach supplies an upper bound of the time-delay.

It deals with the size of the time-delay, as a consequence, it usually provides less conservative results.

Among these works [3, 4, 7, 9], [7] is delay-independent and [3, 4, 9] are delay-dependent, where time delay is assumed to be constant. However, in practical engineering systems, the occurrence of time delay phenomena is often time-varying. Thus, fuzzy sampled-data control for time-varying delay systems is more appealing. In fuzzy sampled-datacontrol, there is no report aboutH_2 guaranteed cost controlproblem for the nonlinear time-varying delay systems.

In this paper, we consider the delay-dependent sampled-data H_2 guaranteed costperformance problem of the nonlinear time-varying delay system represented by a fuzzy T-S model. A Lyapunov-Krasovskii functional with Leibniz-Newton formula is employed to obtain new sufficient conditions in terms of linear matrix inequalities (LMIs) to the fuzzy H_2 guaranteed cost control performance. Based on the stability condition, the guaranteed cost control is minimized for the closed-loop system. We use truck-trailer system to prove the effectiveness and the feasibility of the proposed method.

The main contributions and advantages of the present paper are summarized as follows: (i) The H_2 design via fuzzy sampled-data control for nonlinear systems with time-varying delay is first obtained. (ii) Fuzzy sampled-data control algorithm is less conservative. Comparing with the existing works, the dimension of the LMIs in this paper is simplified, which adds the existence of feedback gains and lowers the implementation time. Experimental results illustrate that the fuzzy sampled-data controller has a larger sampling interval.

Notations: Throughout this paper, if not explicitly stated, we assumed that matrices have compatible dimensions. The notation $P>0(<0)$ is used to denote a positive (negative) definite matrix. The transpose of a matrix $P$is denoted by P^T. The symbol $*$ stands for the transposed element in symmetric positions.

2 Problem formulation

Consider the following nonlinear time-varying delay system:

$$\dot{x}(t) = f(x(t), x(t-d(t)), u(t)), \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, f is a nonlinear function, and $d(t)$ is time-varying delay.

The following fuzzy T-S model with time-varying delay described by IF–THEN rules is used to represent nonlinear time-varying delay system:

$$\text{IF } \xi_1(t) \text{ is } M_{i1} \text{ and } \cdot \quad \cdot \quad \cdot \text{ and } \xi_p(t) \text{ is } M_{ip}, \text{ THEN}$$

$$\dot{x}(t) = A_i x(t) + A_{id} x(t-d(t)) + B_i u(t), i = 1, \cdots, L, \tag{2}$$

where A_i, B_i and A_{id} are constant matrices of appropriate dimensions. L is the number of IF–THEN rules, M_{ij} are fuzzy sets and $\xi_1, \ldots, \xi_p$ are premise variables, $\xi(t) = [\xi_1 \ldots \xi_p]^T$, and $\xi(t)$ is assumed to be given or a measurable function vector. We consider the following two cases for the time-varying delay.

Case 1: $d(t)$ is a differentiable function satisfying for all $t \geq 0$:

$$0 \leq d(t) \leq d_M \text{ and } \dot{d}(t) \leq d_D,$$

where d_M and d_D are constants.

Case 2: $d(t)$ is a continuous function satisfying for all $t \geq 0$:

$$0 \leq d(t) \leq d_M,$$

where d_M is a constant.

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{L} \lambda_i(\xi(t))[A_i x(t) + A_{id} x(t - d(t)) + B_i u(t)], \tag{3}$$

where $\lambda_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{i=1}^{L} \beta_i(\xi(t))}$, $\beta_i(\xi(t)) = \prod_{j=1}^{p} M_{ij}(\xi_j(t))$ and $M_{ij}(.)$ is the grade of the membership function of M_{ij}. $\beta_i(\xi(t)) \geq 0$, $i = 1, 2, \ldots, L$, $\sum_{i=1}^{L} \beta_i(\xi(t)) > 0$ for any $\xi(t)$, $\lambda_i(\xi(t)) \geq 0$, $i = 1, 2, \ldots, L$, $\sum_{i=1}^{L} \lambda_i(\xi(t)) = 1$.

We design the following fuzzy sampled-data controller for (3):

IF $\xi_1(t_k)$ is M_{j1} and $\cdot \quad \cdot \quad \cdot$ and $\xi_p(t_k)$ is M_{jp}, THEN

$$u(t) = K_j x(t_k), \quad j = 1, 2, \ldots, L,$$

where K_j is the sate feedback gain, the time t_k is the sampling instant satisfying $0 < t_1 < t_2 < \cdots < t_k < \cdots$, and sampling interval is a constant, i.e. $t_{k+1} - t_k = h_k = h$. The overall fuzzy sampled-data controller is as follows:

$$u(t) = \sum_{j=1}^{L} \lambda_j(\xi(t_k)) K_j x(t_k). \tag{4}$$

By using input delay approach, (4) is equivalent to (5)

$$u(t) = \sum_{j=1}^{L} \lambda_j(\xi(t_k)) K_j x(t - \tau(t)). \tag{5}$$

The closed-loop system (3) with (5) is given by

$$\dot{x}(t) = \sum_{i=1}^{L} \sum_{j=1}^{L} \lambda_i(\xi(t)) \lambda_j(\xi(t_k))[A_i x(t) + A_{id} x(t - d(t)) + B_i K_j x(t - \tau(t))]. \tag{6}$$

The following H_2 guaranteed costcontrol performance

$$J = \int_0^{\infty} (x^T(t) Q x(t) + u^T(t) R u(t)) dt. \tag{7}$$

must be minimized, where the weighting positive-definite matrices Q and R are specified beforehand according to the design purpose.

Determine a sampled-data state feedback controller such that the closed-loop system (6) is asymptotically stable and the upper bound of H_2 guaranteed cost function is minimized.

Lemma 2.1 (Gu et al. [13]). *For any positive definite symmetric constant matrix $M \in R^{n \times n}$, scalars r_1, r_2 satisfying $r_1 \leq r_2$, if $\varpi : [r_1, r_2] \to R^n$ is a vector function such that the integrations concerned are well defined, then*

$$\left(\int_{r_1}^{r_2} \varpi(s) ds \right)^T M \left(\int_{r_1}^{r_2} \varpi(s) ds \right) \leq (r_2 - r_1) \int_{r_1}^{r_2} \varpi^T(s) M \varpi(s) ds. \tag{8}$$

Remark 1: The premise variables $\xi_1, \ldots, \xi_p$ can be function of measurable state variables $x(t)$ and $x(t-d)$, or combination of measurable state variables. The limitation of design of fuzzy T–S approach is that some state variables must be measurable to construct fuzzy controller. ***Remark 2:*** It should be noted that the control signal $u(t)$ holds constant during the period of $t_k \le t \le t_{k+1}$.

3 Fuzzy H_2 Guaranteed Cost Sampled-Data Control

In this section, we present a H_2 guaranteed cost sampled-data control scheme of the fuzzy system and minimization of the upper bound of (7).

Here, we give some sufficient conditions for the stability of the closed-loop system (6) in terms of LMIs.

Theorem 1. Suppose that, under case 1, for given matrices $Q > 0$, $R > 0$, scalars $h > 0$, $d_M > 0$, $d_D > 0$, $\mu > 0$,there exist matrices $\overline{P} > 0$, $\overline{R_1} > 0$, $\overline{R_2} > 0$, $\overline{R_3} > 0$, such that the following LMIs hold for all $i, j = 1, 2, \cdots, L$,

$$\Sigma_{ij} = \begin{bmatrix} \Sigma_{ij11} & \Sigma_{ij12} & \Sigma_{ij13} & 0 & \Sigma_{ij15} & \Sigma_{ij16} \\ * & \Sigma_{ij22} & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{ij33} & \Sigma_{ij34} & \Sigma_{ij35} & 0 \\ * & * & * & \Sigma_{ij44} & 0 & 0 \\ * & * & * & * & \Sigma_{ij55} & \Sigma_{ij56} \\ * & * & * & * & * & \Sigma_{ij66} \end{bmatrix} < 0, \tag{9}$$

where

$$\begin{aligned} \Sigma_{ij11} &= A_i\overline{P} + \overline{P}A_i^T + \overline{R}_1 - \overline{R}_2 - \overline{R}_3, \Sigma_{ij12} = \overline{P}, \Sigma_{ij13} = B_i\overline{K_j} + \overline{R}_2, \Sigma_{ij15} = \mu\overline{P}A_i^T, \\ \Sigma_{ij16} &= A_{id}\overline{P} + \overline{R}_3, \Sigma_{ij22} = -Q^{-1}, \Sigma_{ij33} = -\overline{R}_2, \Sigma_{ij34} = \overline{K}_j^T, \Sigma_{ij35} = \mu\overline{K}_j^T B_i^T, \\ \Sigma_{ij44} &= -R^{-1}, \Sigma_{ij55} = -2\mu\overline{P} + h^2\overline{R}_2 + d_M^2\overline{R}_3, \Sigma_{ij56} = A_{id}\overline{P}, \\ \Sigma_{ij66} &= -(1-d_D)\overline{R}_1 - \overline{R}_3. \end{aligned}$$

Then there exists a sampled-data controller (4) with $K_j = \overline{K_j P}^{-1} (j = 1, 2, \cdots, L)$ such that H_2 guaranteed cost control performance (7) is minimized in the sense that the closed-loop system (6) is asymptotically stable.

Proof. Choose the Lyapunov-Krasovskii functional:

$$V(x_t) = V_1(x) + V_2(x_t) + V_3(x_t) + V_4(x_t), \tag{10}$$

where

$$V_1(x) = x^T(t)Px(t), \quad V_2(x_t) = \int_{t-d(t)}^{t} x^T(s)R_1x(s)ds,$$

$$V_3(x_t) = h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta, V_4(x_t) = d_M\int_{-d_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta$$

and $P > 0$, $R_1 > 0$, $R_2 > 0$,$R_3 > 0$.

The derivative of Valong the trajectories of the system (6) is computed as follows:

$$\begin{aligned}\dot{V}_1(x) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)\\ &= \sum_{i=1}^{L}\sum_{j=1}^{L}\lambda_i(\xi(t))\lambda_j(\xi(t_k))[x^T(t)A_i^TPx(t) + x^T(t-d(t))A_{id}^TPx(t)\\ &\quad +x^T(t-\tau(t))K_j^TB_i^TPx(t) + x^T(t)PAix(t) + x^T(t)PAidx(t-d(t))\\ &\quad +x^T(t)PB_iK_jx(t-\tau(t)). \end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_2(x_t) &= x^T(t)R_1x(t) - (1-\dot{d}(t))x^T(t-d(t))R_1x(t-d(t))\\ &\leq x^T(t)R_1x(t) - (1-d_D)x^T(t-d(t))R_1x(t-d(t)).\end{aligned} \tag{12}$$

By using Lemma 2.1, we have

$$-h\int_{t-h}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds \leq -\tau(t)\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds \leq -\left(\int_{t-\tau(t)}^{t}\dot{x}(s)ds\right)^T R_2\left(\int_{t-\tau(t)}^{t}\dot{x}(s)ds\right). \tag{13}$$

Leibniz-Newton formula is

$$\int_{t-h}^{t}\dot{x}(s)ds = x(t) - x(t-h). \tag{14}$$

Applying (13) and Leibniz-Newton formula, we have

$$\begin{aligned}\dot{V}_3(x_t) &= h^2\dot{x}^T(t)R_2\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds\\ &\leq h^2\dot{x}(t)^TR_2\dot{x}(t) - (x(t)-x(t-\tau(t)))^TR_2(x(t)-x(t-\tau(t)))^T\\ &= h^2\dot{x}^T(t)R_2\dot{x}(t) - x^T(t)R_2x(t) + x^T(t-\tau(t))R_2x(t) + x^T(t)R_2x(t-\tau(t))\\ &\quad -x^T(t-\tau(t))R_2x(t-\tau(t)).\end{aligned} \tag{15}$$

Similarly, by Lemma 2.1 and Leibniz-Newton formula, we have

$$\begin{aligned}\dot{V}_4(x_t) &\leq d_M^2\dot{x}^T(t)R_3\dot{x}(t) - x^T(t)R_3x(t) + x^T(t-d(t))R_3x(t)\\ &\quad +x^T(t)R_3x(t-d(t)) - x^T(t-d(t))R_3x(t-d(t)).\end{aligned} \tag{16}$$

From (6), for a given $\mu > 0$,

$$\begin{aligned}0 &= -2\mu\dot{x}^T(t)P\dot{x}(t) + \mu\dot{x}^T(t)P\{\sum_{i=1}^{L}\sum_{j=1}^{L}\lambda_i(\xi(t))\lambda_j(\xi(t_k))[A_ix(t) + A_{id}x(t-d(t))\\ &\quad +B_iK_jx(t-\tau(t))]\} + \mu\{\sum_{i=1}^{L}\sum_{j=1}^{L}\lambda_i(\xi(t))\lambda_j(\xi(t_k))[A_ix(t) + A_{id}x(t-d(t))\\ &\quad +B_iK_jx(t-\tau(t))]\}^TP\dot{x}(t)\\ &= -2\mu\dot{x}^T(t)P\dot{x}(t) + \sum_{i=1}^{L}\sum_{j=1}^{L}\lambda_i(\xi(t))\lambda_j(\xi(t_k))[\mu\dot{x}^T(t)PA_ix(t)\\ &\quad +\mu\dot{x}^T(t)PA_{id}x(t-d(t)) + u\dot{x}^T(t)PBiKjx(t-\tau(t)) + ux^T(t)Ai^TP\dot{x}(t)\\ &\quad +\mu x^T(t-d(t))A_{id}^TP\dot{x}(t) + \mu x^T(t-\tau(t))K_j^TB_i^TP\dot{x}(t)].\end{aligned} \tag{17}$$

From (11-12) and (15-17), we obtain

$$\dot{V}(x_t) + x^T(t)Qx(t) + u^T(t)Ru(t) \leq \sum_{i=1}^{L}\sum_{j=1}^{L} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\tilde{x}^T(t)S\prime_{ij}\tilde{x}(t) \tag{18}$$

where

$$\tilde{x}(t) = [\begin{array}{cccc} x^T(t) & x^T(t-\tau(t)) & \dot{x}^T(t) & x^T(t-d(t) \end{array}]^T,$$

$$S\prime_{ij} = \begin{bmatrix} S\prime_{ij11} & S\prime_{ij12} & S\prime_{ij13} & S\prime_{ij14} \\ * & S\prime_{ij22} & S\prime_{ij23} & 0 \\ * & * & S\prime_{ij33} & S\prime_{ij34} \\ * & * & * & S\prime_{ij44} \end{bmatrix} \tag{19}$$

with

$$\begin{aligned} S\prime_{ij11} &= A_i^T P + PA_i + R_1 - R_2 - R_3 + Q, S\prime_{ij12} = PB_iK_j + R_2, S\prime_{ij13} = \mu A_i^T P, \\ S\prime_{ij14} &= PA_{id} + R_3, S\prime_{ij22} = -R_2 + K_j^T RK_j, S\prime_{ij23} = \mu K_j^T B_i^T P, \\ S\prime_{ij33} &= -2\mu P + h^2 R_2 + d_M^2 R_3, S\prime_{ij34} = PA_{id}, \quad S\prime_{ij44} = -(1-d_D)R_1 - R_3. \end{aligned}$$

Pre- and post-multiplying the matrix $S\prime_{ij}$ in (19) by $diag\left[\begin{array}{cccc} P^{-1} & P^{-1} & P^{-1} & P^{-1} \end{array}\right]$ with$\overline{P} = P^{-1}$,$\overline{R_1} = P^{-1}R_1P^{-1}$,$\overline{R_2} = P^{-1}R_2P^{-1}$,$\overline{R_3} = P^{-1}R_3P^{-1}$,$\bar{Q} = P^{-1}QP^{-1}$,$\bar{K}_j = K_jP^{-1}$ $(j = 1, 2, \cdots, L)$, we have

$$S_{ij} = \begin{bmatrix} \sum_{ij11} + \bar{Q} & \sum_{ij13} & \sum_{ij15} & \sum_{ij16} \\ * & \sum_{ij33} + \bar{K}_j^T R\bar{K}_j & \sum_{ij35} & 0 \\ * & * & \sum_{ij55} & \sum_{ij56} \\ * & * & * & \sum_{ij66} \end{bmatrix}. \tag{20}$$

If (9) is satisfied, then $\Sigma_{ij} < 0$ is equivalent to $S_{ij} < 0$ in (20) by using the Schur complement. And, $S_{ij} < 0$ in (20) is equivalent to $S\prime_{ij} < 0$ in (19). Thus,

$$\dot{V}(x_t) + x^T(t)Qx(t) + u^T(t)Ru(t) < 0. \tag{21}$$

Integrating both sides of (21) from t= 0 to$t = \infty$, we obtain

$$V(x_t(\infty)) - V(x_t(0)) + \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt < 0. \tag{22}$$

Thus, we have

$$J < V(x_t(0)) = x^T(0)Px(0). \tag{23}$$

Now, we provide a stability condition for the fuzzy T–S system (6) under case 2.

Theorem 2. Suppose that, under case 2, for given matrices $Q > 0$, $R > 0$, scalars $h > 0$, $d_M > 0$, $\mu > 0$, there exist matrices $\overline{P} > 0$, $\overline{R_1} > 0$, $\overline{R_2} > 0$such that the following LMIs hold for all $i, j = 1, 2, \cdots, L$

$$\bar{\Sigma}_{ij} = \begin{bmatrix} \bar{\Sigma}_{ij11} & \bar{\Sigma}_{ij12} & \bar{\Sigma}_{ij13} & 0 & \bar{\Sigma}_{ij15} & \bar{\Sigma}_{ij16} \\ * & \bar{\Sigma}_{ij22} & 0 & 0 & 0 & 0 \\ * & * & \bar{\Sigma}_{ij33} & \bar{\Sigma}_{ij34} & \bar{\Sigma}_{ij35} & 0 \\ * & * & * & \bar{\Sigma}_{ij44} & 0 & 0 \\ * & * & * & * & \bar{\Sigma}_{ij55} & \bar{\Sigma}_{ij56} \\ * & * & * & * & * & \bar{\Sigma}_{ij66} \end{bmatrix} < 0, \tag{24}$$

where

$$\begin{aligned} \bar{\Sigma}_{ij11} &= A_i\overline{P} + \overline{P}A_i^T - \overline{R}_1 - \overline{R}_2, \bar{\Sigma}_{ij12} = \overline{P}, \bar{\Sigma}_{ij13} = B_i\overline{K_j} + \overline{R}_1, \bar{\Sigma}_{ij15} = \mu\overline{P}A_i^T, \\ \bar{\Sigma}_{ij16} &= A_{id}\overline{P} + \overline{R}_2, \bar{\Sigma}_{ij22} = -Q^{-1}, \quad \bar{\Sigma}_{ij33} = -\overline{R}_1, \quad \bar{\Sigma}_{ij34} = \overline{K}_j^T, \quad \bar{\Sigma}_{ij35} = \mu\overline{K}_j^T B_i^T, \\ \bar{\Sigma}_{ij44} &= -R^{-1}, \quad \bar{\Sigma}_{ij55} = -2\mu\overline{P} + h^2\overline{R}_1 + d_M^2\overline{R}_2, \bar{\Sigma}_{ij56} = A_{id}\overline{P}, \bar{\Sigma}_{ij66} = -\overline{R}_2. \end{aligned}$$

Then there exists a sampled-data controller (4) with $K_j = \overline{K_j P}^{-1} (j = 1, 2, \cdots, L)$ such that H_2 guaranteed cost control performance (7) is minimized in the sense that the closed-loop system (6) is asymptotically stable.

Proof. Choose the following Lyapunov-Krasovskii functional:

$$V(x_t) = V_1(x) + V_2(x_t) + V_3(x_t), \tag{25}$$

where

$$V_1(x) = x^T(t)Px(t), V_2(x_t) = h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta,$$

$$V_3(x_t) = d_M\int_{-d_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta$$

and $P > 0$, $R_1 > 0$, $R_2 > 0$ are to be determined. Then following the similar line in Theorem 1, we can obtain Theorem 2.

If there is no time delay, then we have the following Corollary 1.

Corollary 1. Suppose that, for given matrices $Q > 0$, $R > 0$, scalars $h > 0$, $\mu > 0$, there exist matrices $\overline{P} > 0$, $\overline{R_1} > 0$, such that the following LMIs hold for all $i, j = 1, 2, \cdots, L$

$$\bar{\bar{\Sigma}}_{ij} = \begin{bmatrix} \bar{\bar{\Sigma}}_{ij11} & \bar{\bar{\Sigma}}_{ij12} & \bar{\bar{\Sigma}}_{ij13} & 0 & \bar{\bar{\Sigma}}_{ij15} \\ * & \bar{\bar{\Sigma}}_{ij22} & 0 & 0 & 0 \\ * & * & \bar{\bar{\Sigma}}_{ij33} & \bar{\bar{\Sigma}}_{ij34} & \bar{\bar{\Sigma}}_{ij35} \\ * & * & * & \bar{\bar{\Sigma}}_{ij44} & 0 \\ * & * & * & * & \bar{\bar{\Sigma}}_{ij55} \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \bar{\bar{\Sigma}}_{ij11} &= A_i\overline{P} + \overline{P}A_i^T - \overline{R}_1, \bar{\bar{\Sigma}}_{ij12} = \overline{P}, \bar{\bar{\Sigma}}_{ij13} = B_i\overline{K_j} + \overline{R}_1, \bar{\bar{\Sigma}}_{ij15} = \mu\overline{P}A_i^T, \bar{\bar{\Sigma}}_{ij22} = -Q^{-1}, \\ \bar{\bar{\Sigma}}_{ij33} &= -\overline{R}_1, \quad \bar{\bar{\Sigma}}_{ij34} = \overline{K}_j^T, \quad \bar{\bar{\Sigma}}_{ij35} = \mu\overline{K}_j^T B_i^T, \bar{\bar{\Sigma}}_{ij44} = -R^{-1}, \quad \bar{\bar{\Sigma}}_{ij55} = -2\mu\overline{P} + h^2\overline{R}_1. \end{aligned}$$

Then there exists a sampled-data controller (4) with $K_j = \overline{K_j P}^{-1} (j = 1, 2, \cdots, L)$ such that H_2 guaranteed cost control performance (7) is minimized in the sense that the closed-loop system (6) is asymptotically stable.

In the following, we give the design procedure of fuzzy sampled-data controller.

The H_2 guaranteed cost sampled-data fuzzy control problem can be formulated as the following optimization problem:

$$\min_{\bar{P}} Trace(J)$$

$$s.t.(9)and \begin{bmatrix} J & x^T(0) \\ * & \bar{P} \end{bmatrix} > 0. \tag{27}$$

Design Procedure: The delay-dependentH_2 guaranteed cost sampled-datacontrol for fuzzy time-varying delay system is summarized as follows.

Step 1: Select membership functions and fuzzy rules in (1).

Step 2: Give the upper bound of sampling interval $h > 0$ and a scalar $\mu > 0$.

Step 3: Solve the LMIs (27) to obtain $\overline{K_j}(j = 1, 2, \cdots, L)$ and $\overline{P}$.Thus,$K_j = \overline{K_j P}^{-1}$ $(j = 1, 2, \cdots, L)$ can also be obtained.

Step 4: Increase$h$, and repeat Step 3 until $\overline{K_j}(j = 1, 2, \cdots, L)$ and $\overline{P}$ can not be found.

Step 5: Confirm fuzzy H_2 guaranteed costsampled-datacontrol and stability of the closed-loop system, substitute P,$K_j(j = 1, 2, \cdots, L)$, μ and h into (19) and verify $S_{ij} < 0$.

Step 6: Construct the fuzzy sampled-data controller (4).

4 Simulation example

To test the effectiveness and feasibility of the proposed method, we consider the following truck-trailer system [14]

$$\begin{aligned} \dot{x}_1(t) &= -a\tfrac{v\bar{t}}{Lt_0}x_1(t) - (1-a)\tfrac{v\bar{t}}{Lt_0}x_1(t-t_d) + \tfrac{v\bar{t}}{lt_0}u(t) \\ \dot{x}_2(t) &= a\tfrac{v\bar{t}}{Lt_0}x_1(t) + (1-a)\tfrac{v\bar{t}}{Lt_0}x_1(t-t_d) \\ \dot{x}_3(t) &= \tfrac{v\bar{t}}{Lt_0}\sin(x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t-t_d)), \end{aligned} \tag{28}$$

where $l = 2.8 \quad L = 5.5$, $v = -1.0$, $a = 0.7$, $\bar{t} = 2.0$, $t_0 = 0.5$. $x_1(t) \in [-\pi/2, \quad \pi/2]$,

$\dot{x}_1(t) \in [-3, \quad 3], x_2(t) \in [-\pi/2, \quad \pi/2], \dot{x}_2(t) \in [-2, \quad 2].x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T$,

$[x_1(0) \quad x_2(0) \quad x_3(0)] = \begin{bmatrix} 1.5 & -2 & 5 \end{bmatrix}$.

The nonlinear truck-trailer system is modeled by two-rule fuzzy T-S system.

Rule 1: IF $\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t-t_d)$ is about 0,

$$Then \dot{x}(t) = A_1x(t) + A_{d1}x(t-\tau_d) + B_1u(t). \tag{29}$$

Rule 2: IF $\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t-t_d)$ is about π or $-\pi$,

$$Then \dot{x}(t) = A_2x(t) + A_{d2}x(t-\tau_d) + B_2u(t). \tag{30}$$

where

$$A_1 = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, A_{d1} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{dv^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix}, A_{d2} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix},$$

and $d = 10t_0/\pi$.

The membership functions are defined as

$$\begin{aligned} \lambda_1(\theta(t)) &= \left(1 - \frac{1}{1+\exp(-3(\theta(t)-0.5\pi))}\right) \times \left(\frac{1}{1+\exp(-3(\theta(t)+0.5\pi))}\right), \\ \lambda_2(\theta(t)) &= 1 - \lambda_1(\theta(t)). \end{aligned}$$

A two-rule sampled-data fuzzy controller is employed to stabilize the truck trailer system. The sampled-data fuzzy controller is designed as follows:

$$u(t) = \sum_{j=1}^{2} \lambda_j(\theta(t_k))K_j x(t_k).$$

First, we assume that time delay $d(t) = 0$. Applying various methods of [8] (H_2 control), [10] (H_2 control) and Corollary 1, the dimensions of the LMIs are given in Table 1. It is seen from Table 1 that the dimension of the LMIs is greatly simplified in the proposed method of this paper.

Table 1: The comparison for the dimensions of LMIs (Corolarry 1)

Method	[8]	[10]	Corollary 1
Dimension	25	28	13

Next, we assume that the delay is time-invariant, i.e. $d_D = 0$. By using various the methods of [3](H_2 control) and Theorem2, the dimensions of the LMIs are given in Table 2. It is seen from Table 2 that the dimension of the LMIs is simplified in the proposed method of this paper, which adds the existence of feedback gains and lowers the implementation time.

Table 2: The comparison for the dimensions of LMIs (Theorem 2)

Method	[3]	Theorem 2
Dimension	20	16

By using various methods of [3] and Theorem 2, the maximum allowable upper bounds of sampling interval are given in Table 3, which show that Theorem 2 of this paper can get a larger sampling interval. This implies that the proposed method achieves a better performance.

Table 3: The maximum allowable upper bounds of sampling interval

Method	[3]	Theorem 2
$h_{\max}(t_d = 0.5)$	0.374	0.562
$h_{\max}(t_d = 1)$	0.315	0.471
$h_{\max}(t_d = 2)$	0.251	0.283

Finally, we consider the control design for time-varying delay $t_d = 1 + \sin t$. The maximum allowable upper bound of sampling interval that is obtained by Theorem 1 is 0.295.When the design parameters are given by $\mu = 1$, $d_M = 2$, $d_D = 1$ with the sampling interval $h =$ 0.295,Theorem 1 gives the fuzzy state feedback control gains

$$K_1 = [1.0319 \text{ -0.1019 } 0.0009], \quad K_2 = [1.0319 \text{ -0.1019 } 0.0009].$$

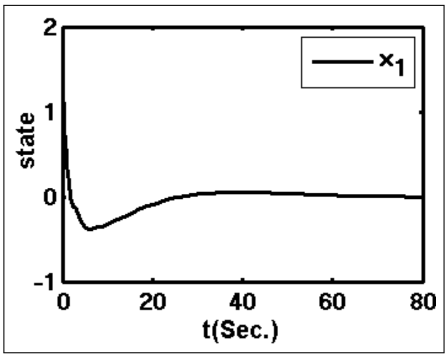

Figure 1: State response x_1

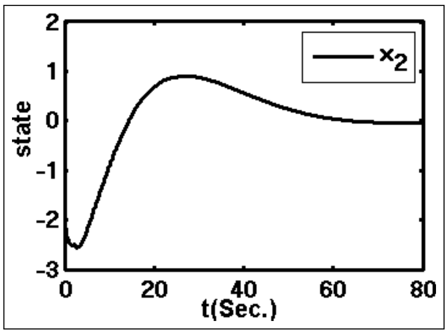

Figure 2: State response x_2

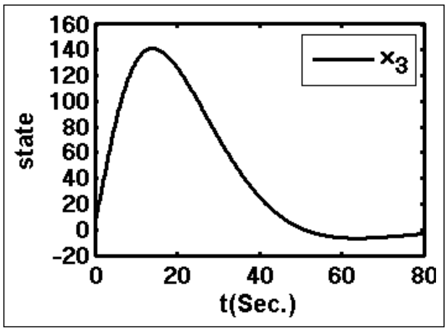

Figure 3: State response x_3

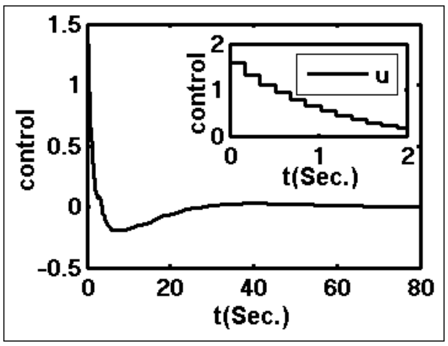

Figure 4: State response x_4

When time-varying delay t_d is $1.5 + 1.5 \sin t$, Theorem 1 gives the maximum allowable upper bound of sampling interval 0.172.With the design parameters $h = 0.172$, $\mu = 1.5$, $d_M = 3$, $d_D = 1.5$, $Q = diag\{\ 1 \quad 10 \quad 0.1\ \} \times 10^{-6}$,$R = 10^{-5}$, Theorem 1 gives the fuzzy state feedback control gains

$$K_1 = [0.9656\ \text{-}0.0657\ 0.0006]\,, \quad K_2 = [0.9656\ \text{-}0.0657\ 0.0006]\,.$$

The sampled-data fuzzy controller with the above control gains is applied to the truck trailer system, the results on the state responsesx_1, x_2, x_3 and control law$u$are shown in Figures 1- 4.

Simulation results illustrate the fuzzy H_2 guaranteed cost sampled-data control design is effective and feasible. Figs. 1-3 show the system stability, and Fig.4 shows the sampled-data control signal for the system (28).

5 Conclusion

This work considers the fuzzyH_2 GC sampled-data control problem for nonlinear systems with time-varying delay. It should be pointed that this problem is more complicated and harder to deal with due to the coexistence of feedback delay and sampled-data control. A new sufficient condition for the existence of fuzzy sampled-data controller is given in terms of LMIs.

To better demonstrate our results, a truck-trailer system with sampled-data control is given. Simulation results show the effectiveness and feasibility of sampled-data control design. Furthermore, this method could be extended to H_∞ control.

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