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A Multiple Attribute Group Decision Making Method Based on 2-D Uncertain Linguistic Weighted Heronian Mean Aggregation Operator

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Abstract: 2-Dimension uncertain linguistic variables can describe both subjective evaluation result of attributes and reliability of the evaluation results in multiple attribute decision making problems. However, it is difficult to aggregate these evaluation information and give comprehensive results. Heronian mean (HM) has the characteristic of capturing the correlations between aggregated arguments and is extended to solve this problem. The 2-dimension uncertain linguistic weighted HM aggregation(2DULWHMA) operator is employed in this paper. Firstly, the definition, properties, expectations and the operational laws the 2-dimension uncertain linguistic variables are investigated. Furthermore, the properties of the 2DULWHMA operators, such as commutativity, idempotency and monotonicity, etc. are studied. Some special cases of the generalized parameters in these operators are analyzed. Finally, an example is given to demonstrate the effectiveness and feasibility of the proposed method.
Keywords: 2-Dimension (2-D) uncertain linguistic variables; Heronian mean; aggregation operator.

1 Introduction

Multiple attribute decision making (MADM) refers to ranking and selecting the best alternatives by utilizing the known information. It has been widely employed in economic, science and technology etc. Since Churchman et al. [1] introduced the multi-attribute decision making and employed it in enterprise investment decisions, the classical multi-attribute decision making has attracted more and more research attention. However, owing to the complexity and uncertainty of objective things and the fuzziness of human thought, a large number of MADM problems are uncertain, which are called uncertain multiple attribute decision making problems. Those uncertain multiple attribute decision making problems are difficult to evaluate alternatives for decision makers using real numbers in many cases [25].It is more reasonable and natural expressed by combining linguistic information (good, fair, poor) with fuzzy term (slightly, very, mightily, extremely, obviously) . The researches on the MADM based on linguistic variables have made the fruitful achievements[614].

However, there exist a kind of the linguistic fuzzy MADM problems in practical decision-making, such as review of the science technology project, blind evaluate of economic industry system, etc. In these decision-making problems, decision makers not only assess all the indicators of evaluation objects, but also estimate the familiarity with the given results. Therefore, Zhu et al. [15] presented the concept of 2-dimension linguistic assessment information to solve this kind of decision making problems. Liu et al. [16]extended 2-dimension linguistic information to deal with the multiple attribute group decision making problems with unknown weight. 2-dimension linguistic employs two class linguistic information to describe the judgment on the object representing evaluation result and reliability of evaluation respectively. This can easily distinguish indetermination between decision making problems and subjective understanding, which is helpful to express opinions more accurately for decision makers. When 2-dimension linguistic assessment information is described by uncertain linguistic variables, it is called as 2-dimension uncertain linguistic variables.

At present, research based on 2-dimension linguistic information mainly focus on the following. Aggregation operators is a hot topic. 2-dimension uncertain linguistic power generalized weighted aggregation operator is proposed and some properties are discussed[15]. 2-dimension uncertain linguistic generalized hybrid weighted average operator is proposed and employed in technological innovation ability evaluation[17]. Liu and Qi[18] proposed some generalized dependent aggregation operators for 2-dimension linguistic information and developed a group decision making method based on these operators. 2-dimension uncertain linguistic density geometric aggregation operators and 2-dimension uncertain linguistic density generalized aggregation operators is proposed and used in an example[19]. Another topic is operational rules, the operational rules of 2-dimension uncertain linguistic variables are improved by transferring it into intuitionistic fuzzy numbers to make the operations more accurate[20]. Yu et al. [21] transformed 2-dimension linguistic information into generalized triangular fuzzy number, and proposed 2-dimension linguistic weighted average (2DLWA) operator and 2-dimension linguistic ordered weighted average (2DLOWA) operator.

Although the 2-dimension linguistic variables can reflect the evaluation of decision makers on objects, some information to be aggregated can still be omitted easily, which makes the integrated result cannot be better to express alternatives synthetically. However Heronian mean operator can capture the correlations of the aggregated arguments, so we combine 2-dimension linguistic variables with Heronian mean and propose some 2-dimension uncertain linguistic Heronian mean aggregation operators and weighted Heronian mean aggregation operators, further apply them to solve the MADM problems.

In order to do so, the remainder of this paper is shown as follows. In Sect. 2, we briefly introduce the operational rules of 2-dimension linguistic variables and the Heronian mean. In Sect. 3, some Heronian mean aggregation operators based on 2-dimension linguistic variables are proposed and commutativity, idempotency and monotonicity are studied.In Sect.4 detailed calculating steps are given to solve the group decision making problem with 2-dimension uncertain linguistic information. In Sect. 5, we use an illustrate example to verify the efficiency of the proposed method, and some special cases are also discussed. In the last section, the conclusions are given.

2 2-Dimension uncertain linguistic variable and Heronian mean

2.1 Uncertain linguistic variable

When decision makers need to express the qualitative information, generally it is necessary to set an appropriate linguistic assessment set in advance. Let the linguistic assessment set be $S = \{s_l \,|l = 1, 2, \cdots, L-1\}$, where s_l represents a linguistic variable, L is odd number. when $L = 7$, it is represented as = (extremely poor, very poor, poor, medium, good, very good, extremely good).

2.2 2-Dimension uncertain linguistic variable

In many cases, decision makers should give both evaluation conclusion of evaluation objects and other similar the reliability of their evaluation. In order to reliably describe decision makers judgment to the evaluated object, 2-dimension uncertain linguistic variable $\tilde{s} = (s_F, s_T)$ has been proposed[22]. s_F= $\{\dot{s}_i \,|i = 1, 2, \cdots, n-1\}$represents decision makers judgment to an evaluated object and s_T= $\{\ddot{s}_i \,|i = 1, 2, \cdots, m-1\}$ represents the subjective evaluation on the reliability of their given results.

Definition 1. Let $\tilde{s} = ([\dot{s}_a, \dot{s}_b], [\ddot{s}_c, \ddot{s}_d])$ where $\dot{s}_a, \dot{s}_b \in s_F$ is I class uncertain linguistic information, $\ddot{s}_c, \ddot{s}_d \in s_T$ is II class uncertain linguistic information [21]. In order to minimize the loss of linguistic information, the discrete linguistic assessment sets of 2-dimension uncertain linguistic information are extended to continuous linguistic assessment sets, such that $\dot{s}_a, \dot{s}_b \in s_F = \{s_i \,|i \in [0, t]\}$ and Let $\ddot{s}_c, \ddot{s}_d \in s_T = \{s_j \,|j \in [0, t]\}$ be the set of all 2-dimension uncertain linguistic variables.

Consider any three uncertain linguistic variables $\tilde{s} = ([\dot{s}_a, \dot{s}_b][\ddot{s}_c, \ddot{s}_d])$,$\tilde{s}_i = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}])$ and $\tilde{s}_j = ([\dot{s}_{a_j}, \dot{s}_{b_j}][\ddot{s}_{c_j}, \ddot{s}_{d_j}])$.let $\lambda \geq 0$,then the operational rules are defined as follows[15][23˜24]:

$$\tilde{s}_i \oplus \tilde{s}_j = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}]) \oplus ([\dot{s}_{a_j}, \dot{s}_{b_j}][\ddot{s}_{c_j}, \ddot{s}_{d_j}]) = ([\dot{s}_{a_i+a_j}, \dot{s}_{b_i+b_j}][\ddot{s}_{\min(c_i,c_j)}, \ddot{s}_{\min(d_i,d_j)}]) \tag{1}$$

$$\tilde{s}_i \otimes \tilde{s}_j = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}]) \otimes ([\dot{s}_{a_j}, \dot{s}_{b_j}][\ddot{s}_{c_j}, \ddot{s}_{d_j}]) = ([\dot{s}_{a_i\times a_j}, \dot{s}_{b_i\times b_j}][\ddot{s}_{\min(c_i,c_j)}, \ddot{s}_{\min(d_i,d_j)}]) \tag{2}$$

$$\lambda\tilde{s} = \lambda([\dot{s}_a, \dot{s}_b][\ddot{s}_c, \ddot{s}_d]) = ([\dot{s}_{\lambda a}, \dot{s}_{\lambda b}][\ddot{s}_c, \ddot{s}_d]) \tag{3}$$

$$\tilde{s}^{\lambda} = ([\dot{s}_a, \dot{s}_b][\ddot{s}_c, \ddot{s}_d])^{\lambda} = ([\dot{s}_{a^{\lambda}}, \dot{s}_{b^{\lambda}}][\ddot{s}_c, \ddot{s}_d]) \tag{4}$$

According to above equation from (1) to (4),following relationship can be easily proved:

$$\tilde{s}_i \oplus \tilde{s}_j = \tilde{s}_j \oplus \tilde{s}_i \tag{5}$$

$$\tilde{s}_i \otimes \tilde{s}_j = \tilde{s}_j \otimes \tilde{s}_i \tag{6}$$

$$\lambda(\tilde{s}_i \oplus \tilde{s}_j) = \lambda\tilde{s}_i \oplus \lambda\tilde{s}_j \tag{7}$$

$$\lambda_1\tilde{s} \oplus \lambda_2\tilde{s} = (\lambda_1 + \lambda_2)\tilde{s} \tag{8}$$

$$\tilde{s}^{\lambda_1} \oplus \tilde{s}^{\lambda_2} = \tilde{s}^{(\lambda_1+\lambda_2)} \tag{9}$$

Definition 2. Let $\tilde{s} = ([\dot{s}_a, \dot{s}_b][\ddot{s}_c, \ddot{s}_d])$ be a 2-dimension uncertain linguistic variable, m is the length of s_F and n is the length of s_T ,then the expectation $E(\tilde{s})$ of $\tilde{s}$ is defined as

$$E(\tilde{s}) = \frac{a+b}{2(m-1)} \times \frac{c+d}{2(n-1)} \tag{10}$$

For any 2-dimension uncertain linguistic variable $\tilde{s}_i$ and $\tilde{s}_j$, if $E(\tilde{s}_i) \geq E(\tilde{s}_j)$ then $\tilde{s}_i \geq \tilde{s}_j$,or vice versa.

2.3 Heronian mean

Heronian Mean is one of aggregation methods, which can reflect the interrelationship of the input arguments[25][26].

Definition 3. Let $s_i (i = 1, 2, \cdots, n)$ be a collection of nonnegative numbers, then HM is defined as

$$HM(s_1, s_2, \cdots, s_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \sqrt{a_i a_j} \tag{11}$$

Definition 4. Let $p, q \geq 0$ and p , q do not take the value 0 simultaneously, then generalized Heronian mean(GHM) is defined

$$GHM(s_1, s_2, \cdots, s_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} a_i^p a_j^q \right)^{\frac{1}{p+q}} \tag{12}$$

3 The 2-dimension uncertain linguistic weighted Heronian mean aggregation operators

Definition 5. Let $p, q \geq 0$ and p, q do not take the value 0 simultaneously. $\tilde{s}_i = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}])$ be a collection of 2-dimension uncertain linguistic variables. 2-dimension uncertain linguistic generalized Heronian mean aggregation (2DULGHMA) operators $:\Omega^n \to \Omega$ is defined as

$$2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i^p \tilde{s}_j^q \right)^{\frac{1}{p+q}} \tag{13}$$

According to the operational rules of 2-dimension uncertain linguistic variables, formula(13) can be transformed into the following formŁş

$$\begin{aligned} &2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = \left(\tfrac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i^p \tilde{s}_j^q \right)^{\frac{1}{p+q}} \\ &= \left(\left[\dot{s}_{\left(\frac{2}{n(n+1)} \sum\limits_{i=1,j=i}^{n} a_i^p a_j^q\right)^{\frac{1}{p+q}}}, \dot{s}_{\left(\frac{2}{n(n+1)} \sum\limits_{i=1,j=i}^{n} b_i^p b_j^q\right)^{\frac{1}{p+q}}} \right], \left[\min_i \ddot{s}_{c_i}, \min_i \ddot{s}_{d_i} \right] \right) \end{aligned} \tag{14}$$

Theorem 6. *(Commutativity). Let $(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n)$ is any permutation of $(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)$ then $2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = 2DULGHMA(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n)$*

Proof: Since $(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n)$ is any permutation of $(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)$,then

$$\begin{aligned} &2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = \left(\tfrac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i^p \tilde{s}_j^q \right)^{\frac{1}{p+q}} \\ &= \left(\tfrac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i{'}^p \tilde{s}_j{'}^q \right)^{\frac{1}{p+q}} = 2DULGHMA(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n) \end{aligned} \tag{15}$$

□

Theorem 7. *(Idempotency). Let* $\tilde{s}_i = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}])(i = 1, 2, \cdots, n)$ *be a collection of uncertainty linguistic variables. If all* $\tilde{s}_i$ *are equal, for all* i *,*$\tilde{s}_i = \tilde{s} = ([\dot{s}_a, \dot{s}_b][\ddot{s}_c, \ddot{s}_d])$ *then* $2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = \tilde{s}$

Proof:

$$\begin{aligned} & 2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = 2DULGHMA(\tilde{s}, \tilde{s}, \cdots, \tilde{s}) \\ & = \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}^p \tilde{s}^q\right)^{\frac{1}{p+q}} = \left(\frac{2}{n(n+1)}(n + (n-1) + \cdots 1)\tilde{s}^{p+q}\right)^{\frac{1}{p+q}} \\ & = \tilde{s} \end{aligned} \tag{16}$$

□

Theorem 8. *(Monotonicity). Let* $\tilde{s}_i = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}])$ *and* $\tilde{s}'_i = ([\dot{s}'_{a_i}, \dot{s}'_{b_i}][\ddot{s}'_{c_i}, \ddot{s}'_{d_i}])(i = 1, 2, \cdots, n)$ *be a collection of uncertainty linguistic variables. If all* $\tilde{s}_i$ *are 2-dimension uncertain linguistic variables. For all* i*,if* $\tilde{s}_i \leq \tilde{s}_i{}'$ *,then* $2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) \leq 2DULGHMA(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n)$

Proof:

$$\begin{aligned} & 2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) \\ & = \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i^p \tilde{s}_j^q\right)^{\frac{1}{p+q}} \leq \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i{}'^p \tilde{s}_i{}'^q\right)^{\frac{1}{p+q}} \\ & = 2DULGHMA(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n) \end{aligned} \tag{17}$$

□

Theorem 9. *(Boundedness).Let 2DULGHMA operator* $\tilde{s}_i = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}])(i = 1, 2, \cdots, n)$ *,then* $\min_i(\tilde{s}_i) \leq 2DULGHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) \leq \max_i(\tilde{s}_i)$

Proof: Let $\tilde{s}_{\min} = \min(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)$ $\tilde{s}_{\max} = \max(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)$,since $\tilde{s}_{\min} \leq \tilde{s}_i \leq \tilde{s}_{\max}$,then

$$\left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_{\min}^p \tilde{s}_{\min}^q\right)^{\frac{1}{p+q}} \leq \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_i^p \tilde{s}_j^q\right)^{\frac{1}{p+q}} \leq \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \tilde{s}_{\max}^p \tilde{s}_{\max}^q\right)^{\frac{1}{p+q}} \tag{18}$$

□

Eq.(14) assumes that all of arguments being aggregated are of equal importance. However, in many real cases, the importance degrees are not equal. Thus, we need to assign different weights for different arguments, and further define a new aggregation operator to process this case.

Because unfair evaluation information may be provided by some decision makers, the weight of evaluation information are larger when they are consistent with other evaluation information given by other decision makers. Otherwise, smaller weight will be set. As result, the influence of outlier values can be reduced, and the decision results based on these operators will be more reasonable and reliable.

Definition 10. Let $p, q \geq 0$ and p, q do not take the value 0 simultaneously. $\tilde{s}_i = ([\dot{s}_{a_i}, \dot{s}_{b_i}][\ddot{s}_{c_i}, \ddot{s}_{d_i}])$ be a collection of 2-dimension uncertain linguistic variables whose weight vector is $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_j = 1$, then 2DULWHMA is called the 2-dimension uncertain linguistic weighted HM aggregation (2DULWHMA) operator: $\Omega^n \to \Omega$.

If

$$\begin{aligned}
&2DULWHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)\\
&= \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} (\omega_i \tilde{s}_i)^p (\omega_j \tilde{s}_j)^q\right)^{\frac{1}{p+q}}\\
&= \left(\left[\dot{s}_{\left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} (\omega_i a_i)^p (\omega_j a_j)^q\right)^{\frac{1}{p+q}}}, \dot{s}_{\left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} (\omega_i b_i)^p (\omega_j b_j)^q\right)^{\frac{1}{p+q}}}\right], \left[\min_i \ddot{s}_{c_i}, \min_i \ddot{s}_{d_i}\right]\right)
\end{aligned} \tag{19}$$

Similar to Theorems 69,we can prove that the 2DULWHMA operator with the following properties:

Theorem 11. *(Commutativity) Let* $(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_2)$ *is any permutation of* $(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)$ *then* $2DULWHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = 2DULWHMA(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_2)$

Theorem 12. *(Idempotency) Let* $\tilde{s}_i = \tilde{s}(i = 1, 2, \cdots, n)$ *then* $2DULWHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = \tilde{s}$

Theorem 13. *(Monotonicity) Let* $\tilde{s}_i$ *and* $\tilde{s}'_i$ $(i = 1, 2, \cdots, n)$ *are 2-dimension uncertain linguistic variables. For all* i*, if* $\tilde{s}_i \leq \tilde{s}'_i$ *,then* $2DULWHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) \leq 2DULWHMA(\tilde{s}'_1, \tilde{s}'_2, \cdots, \tilde{s}'_n)$

Theorem 14. *(Boundedness) The operator lies between the max and min operators:*
$\min(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) \leq 2DULWHMA(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) \leq \max(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n)$

4 A Method for group decision making based on Weighted HM under 2-dimension uncertain linguistic environment

In this section, we consider a group decision making problem with 2-dimension uncertain linguistic information. 2DULWHMA proposed in section 3 will be used to solve a group decision making problem.
Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives and $C = \{C_1, C_2, \cdots, C_n\}$ be the set of attributes, whose weighting vector is $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}^T$ such that $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$. Let $D = \{D_1, D, \cdots, D_t\}$ be the set of decision makers, and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_t)$ is the expert weight vector such that $\lambda_k \geq 0$ and $\sum_{k=1}^{t} \lambda_k = 1$,The decision matrix $D_k = [\widetilde{d}_{ij}^k]_{m\times n}$ is 2-dimension uncertain linguistic variable about the attribute C_j for the alternative A_i and $\widetilde{d}_{ij}^k = ([\dot{s}_{a_{ij}^k}, \dot{s}_{a_{ij}^k}][\ddot{s}_{c_{ij}^k}, \ddot{s}_{d_{ij}^k}])$ where $\dot{s}_a, \dot{s}_b \in s_F$ and $\ddot{s}_c, \ddot{s}_d \in s_T$ is uncertain linguistic information. Then the ranking of alternatives is required. The method involves the following steps:
Step 1: Utilize the 2DULWHMA to aggregate the evaluation values of each expert $(j = 1, 2, \cdots, n)$

$$\begin{aligned}\widetilde{d}_{ij} &= 2DULWHMA(\widetilde{d}^1_{ij}, \widetilde{d}^2_{ij}, \cdots, \widetilde{d}^t_{ij}) \\ &= \left(\frac{2}{t(t+1)} \sum_{k=1,l=k}^{t} (\lambda^k_{ij} \widetilde{d}^k_{ij})^p (\lambda^l_{ij} \widetilde{d}^l_{ij})^q \right)^{\frac{1}{p+q}} \\ &= \left(\left[[\dot{s}_{\left(\frac{2}{t(t+1)} \sum_{k=1,l=k}^{t} (\lambda^k_{ij} a^k_{ij})^p (\lambda^l_{ij} a^l_{ij})^q\right)^{\frac{1}{p+q}}}, \dot{s}_{\left(\frac{2}{t(t+1)} \sum_{k=1,l=k}^{t} (\lambda^k_{ij} b^k_{ij})^p (\lambda^l_{ij} b^l_{ij})^q\right)^{\frac{1}{p+q}}} \right], \left[\min_k \ddot{s}_{c_{ik}}, \min_k \ddot{s}_{d_{ik}}\right] \right)\end{aligned} \tag{20}$$

Step 2: Aggregate the evaluation information of each attribute by 2DULWHMA operator based on the following formula $(i = 1, 2, \cdots, m)$

$$\begin{aligned}\widetilde{d}_i &= 2DULWHMA(\widetilde{d}_{i1}, \widetilde{d}_{i2}, \cdots, \widetilde{d}_{im}) \\ &= \left(\frac{2}{m(m+1)} \sum_{j=1,k=j}^{m} (\omega_j \widetilde{d}_{ij})^p (\omega_k \widetilde{d}_{ik})^q \right)^{\frac{1}{p+q}} \\ &= \left(\left[\dot{s}_{\left(\frac{2}{m(m+1)} \sum_{j=1,k=j}^{m} (\omega_j a_{ij})^p (\omega_k a_{ik})^q\right)^{\frac{1}{p+q}}}, \dot{s}_{\left(\frac{2}{m(m+1)} \sum_{j=1,k=j}^{n} (\omega_j b_{ij})^p (\omega_k b_{ik})^q\right)^{\frac{1}{p+q}}} \right], \left[\min_j \ddot{s}_{c_{ij}}, \min_j \ddot{s}_{d_{ij}}\right] \right)\end{aligned} \tag{21}$$

Step 3: Calculate the expectation $E(\widetilde{d}_i)$ of 2-dimension uncertain linguistic variable according to Equation(10).

Step 4: Rank all the alternatives and select the best ones in accordance with the ranking of $E(\widetilde{s})$.

5 Examples illustration and discussion

In this section, the method are illustrated through an application case of evaluation in extra-efficient economic industry system of Shandong province. From this example, we explains the actual application and the effectiveness of the proposed method. In order to evaluate industry ecological level, four typical industries are selected and expressed by $\{a_1, a_2, a_3, a_4\}$. a_1 is for ecological agriculture,a_2 is for environmentally friendly industry, a_3 is for energy saving industry, and a_4 is for circular economy. Three experts $\{e_1, e_2, e_3\}$ were invited to evaluate these projects by following indexes expressed by $\{ c_1, c_2, c_3, c_4\}$. c_1 expressed *Industrial efficiency* reflectting the quantity and quality of industry economic growth, and it could be evaluated by the energy consumption, material consumption and water consumption, and etc. c_2 expressed *industrial structure coordination* reflecting industry coordination among the economic benefits, environmental benefits and social benefits, and it could be evaluated by industrial spatial distribution harmony, industrial structure harmony and R&D to investment industrial gross output etc. c_3 expressed *environment benefits* reflecting impacts on the industry environmental over entire product life cycle ,it could be evaluated by cleaner production, pollution control, reclamation of wastes and condition of work safety etc. c_4 expressed *performance indicator* measuring the industry development contribution for social progress ,it could be evaluate by total tax and profit payment, total assets contribution and green GDP per person etc.

Supposed that $\lambda = (0.243, 0.514, 0.243)$ is the weight vector of the five experts, and $\omega = (0.25, 0.27, 0.25, 0.23)$ is the index weight. The index values given by the experts take the form of 2- dimension uncertain linguistic variables, and they are shown in Tables 1-3.

The experts adopted I class linguistic set and the II class linguistic set.

According to step 1- step 4, we got the following result as table 4.

The 2DULWHMA were utilized to aggregate the arguments, some different overall attribute value $\widetilde{d}_i$ of the alternatives $a_i\,(i = 1, 2, 3, 4)$ were list in Table 4. From this table, we could find that the overall attribute values of each alternative depended on the choice of the parameters p and q,but the ranking was kept unchanged. According to table 4,a1 has higher ecological level.

Conclusions and future works

The multiple attribute decision making problems based on 2-dimension uncertain linguistic variables have been applied to a wide range of areas. Compared with the traditional uncertain linguistic variables, 2-dimension uncertain linguistic variables add a subjective evaluation on the reliability of the evaluation results given by decision makers, so they can better express fuzzy information.

In addition, the HM can take all the decision arguments and their relationships into account. Based on HM and 2-dimension uncertain linguistic variables, 2DULWHMA operators are developed in this paper. Some desirable properties, such as commutativity, idempotency, monotonicity and boundedness are discussed. Moreover, different p, q cannot affect ranking of alternatives , which proved that it is a flexible multiple attribute decision making method in that the decision makers, and they can choose different values of the parameters p and q according to their actual needs. Finally, to demonstrate the effectiveness and feasibility of the developed method, an example about industry ecological level is given. In further research, the proposed operators and methods can be extended to other fuzzy information.

Table 1: the index values of industry given by expert e1

industry	attribute(c1)	attribute (c2)	attribute (c3)	attribute (c4)
a1	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_2, \dot{s}_3], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_5], [\ddot{s}_4, \ddot{s}_4])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_1, \ddot{s}_2])$
a2	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_5], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_4, \ddot{s}_4])$	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_1, \ddot{s}_2])$
a3	$([\dot{s}_2, \dot{s}_3], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_4, \ddot{s}_4])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_1, \ddot{s}_2])$
a4	$([\dot{s}_5, \dot{s}_6], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_1, \dot{s}_2], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_4, \ddot{s}_4])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_1, \ddot{s}_2])$

Table 2: the index values of industry given by expert e2

industry	attribute(c1)	attribute (c2)	attribute (c3)	attribute (c4)
a1	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_5, \dot{s}_6], [\ddot{s}_3, \ddot{s}_4])$
a2	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_3], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_3, \ddot{s}_3])$
a3	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_2, \dot{s}_3], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_3, \ddot{s}_4])$
a4	$([\dot{s}_5, \dot{s}_5], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_2, \dot{s}_2], [\ddot{s}_3, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_3, \ddot{s}_4])$

Table 3: the index values of industry given by expert e3

industry	attribute(c1)	attribute (c2)	attribute (c3)	attribute (c4)
a1	$([\dot{s}_5, \dot{s}_5], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_3, \dot{s}_3], [\ddot{s}_2, \ddot{s}_2])$	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_1, \ddot{s}_1])$
a2	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_2, \ddot{s}_2])$	$([\dot{s}_1, \dot{s}_2], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_3, \dot{s}_3], [\ddot{s}_1, \ddot{s}_1])$
a3	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_5, \dot{s}_5], [\ddot{s}_2, \ddot{s}_2])$	$([\dot{s}_2, \dot{s}_3], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_4, \dot{s}_4], [\ddot{s}_1, \ddot{s}_1])$
a4	$([\dot{s}_3, \dot{s}_4], [\ddot{s}_2, \ddot{s}_3])$	$([\dot{s}_2, \dot{s}_3], [\ddot{s}_2, \ddot{s}_2])$	$([\dot{s}_4, \dot{s}_5], [\ddot{s}_3, \ddot{s}_4])$	$([\dot{s}_3, \dot{s}_5], [\ddot{s}_1, \ddot{s}_1])$

Table 4: aggregate attribute values by the 2DULWHMA and the rankings of the industry

p,q	industry	$\widetilde{d}_i(i=1,2,3,4)$	$E(\widetilde{d})$	Ranking
p=0.5,q=0.5	a1	$([\dot{s}_{0.188},\dot{s}_{0.336}],[\ddot{s}_1,\ddot{s}_1])$	0.01091	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.131},\dot{s}_{0.240}],[\ddot{s}_1,\ddot{s}_1])$	0.007745	
	a3	$([\dot{s}_{0.091},\dot{s}_{0.229}],[\ddot{s}_1,\ddot{s}_1])$	0.006672	
	a4	$([\dot{s}_{0.115},\dot{s}_{0.248}],[\ddot{s}_1,\ddot{s}_1])$	0.007554	
p=1,q=1	a1	$([\dot{s}_{0.324},\dot{s}_{0.375}],[\ddot{s}_1,\ddot{s}_1])$	0.014547	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.300},\dot{s}_{0.349}],[\ddot{s}_1,\ddot{s}_1])$	0.013526	
	a3	$([\dot{s}_{0.267},\dot{s}_{0.342}],[\ddot{s}_1,\ddot{s}_1])$	0.012687	
	a4	$([\dot{s}_{0.282},\dot{s}_{0.347}],[\ddot{s}_1,\ddot{s}_1])$	0.013122	
p=5,q=5	a1	$([\dot{s}_{0.765},\dot{s}_{0.770}],[\ddot{s}_1,\ddot{s}_1])$	0.0319827	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.763},\dot{s}_{0.768}],[\ddot{s}_1,\ddot{s}_1])$	0.031904	
	a3	$([\dot{s}_{0.759},\dot{s}_{0.767}],[\ddot{s}_1,\ddot{s}_1])$	0.031798	
	a4	$([\dot{s}_{0.761},\dot{s}_{0.768}],[\ddot{s}_1,\ddot{s}_1])$	0.031843	
p=10,q=10	a1	$([\dot{s}_{0.873},\dot{s}_{0.874}],[\ddot{s}_1,\ddot{s}_1])$	0.036389	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.872},\dot{s}_{0.873}],[\ddot{s}_1,\ddot{s}_1])$	0.036367	
	a3	$([\dot{s}_{0.871},\dot{s}_{0.873}],[\ddot{s}_1,\ddot{s}_1])$	0.036336	
	a4	$([\dot{s}_{0.872},\dot{s}_{0.873}],[\ddot{s}_1,\ddot{s}_1])$	0.036349	
p=1,q=0.5	a1	$([\dot{s}_{0.252},\dot{s}_{0.326}],[\ddot{s}_1,\ddot{s}_1])$	0.012046	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.218},\dot{s}_{0.286}],[\ddot{s}_1,\ddot{s}_1])$	0.010498	
	a3	$([\dot{s}_{0.180},\dot{s}_{0.277}],[\ddot{s}_1,\ddot{s}_1])$	0.009517	
	a4	$([\dot{s}_{0.199},\dot{s}_{0.285}],[\ddot{s}_1,\ddot{s}_1])$	0.010091	
p=1,q=2	a1	$([\dot{s}_{0.444},\dot{s}_{0.474}],[\ddot{s}_1,\ddot{s}_1])$	0.019121	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.430},\dot{s}_{0.460}],[\ddot{s}_1,\ddot{s}_1])$	0.018551	
	a3	$([\dot{s}_{0.407},\dot{s}_{0.455}],[\ddot{s}_1,\ddot{s}_1])$	0.017963	
	a4	$([\dot{s}_{0.417},\dot{s}_{0.458}],[\ddot{s}_1,\ddot{s}_1])$	0.01824	
p=1,q=5	a1	$([\dot{s}_{0.647},\dot{s}_{0.658}],[\ddot{s}_1,\ddot{s}_1])$	0.0272	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.643},\dot{s}_{0.654}],[\ddot{s}_1,\ddot{s}_1])$	0.02701	
	a3	$([\dot{s}_{0.633},\dot{s}_{0.652}],[\ddot{s}_1,\ddot{s}_1])$	0.02677	
	a4	$([\dot{s}_{0.637},\dot{s}_{0.653}],[\ddot{s}_1,\ddot{s}_1])$	0.026875	
p=1,q=10	a1	$([\dot{s}_{0.783},\dot{s}_{0.787}],[\ddot{s}_1,\ddot{s}_1])$	0.032726	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.782},\dot{s}_{0.786}],[\ddot{s}_1,\ddot{s}_1])$	0.03266	
	a3	$([\dot{s}_{0.778},\dot{s}_{0.785}],[\ddot{s}_1,\ddot{s}_1])$	0.03257	
	a4	$([\dot{s}_{0.780},\dot{s}_{0.785}],[\ddot{s}_1,\ddot{s}_1])$	0.032608	
p=0.5,q=1	a1	$([\dot{s}_{0.252},\dot{s}_{0.786}],[\ddot{s}_1,\ddot{s}_1])$	0.012046	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{0.218},\dot{s}_{0.286}],[\ddot{s}_1,\ddot{s}_1])$	0.010498	
	a3	$([\dot{s}_{0.180},\dot{s}_{0.277}],[\ddot{s}_1,\ddot{s}_1])$	0.009517	
	a4	$([\dot{s}_{0.199},\dot{s}_{0.285}],[\ddot{s}_1,\ddot{s}_1])$	0.010091	
p=2,q=1	a1	$([\dot{s}_{1.233},\dot{s}_{1.234}],[\ddot{s}_1,\ddot{s}_1])$	0.051394	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{1.106},\dot{s}_{1.160}],[\ddot{s}_1,\ddot{s}_1])$	0.047219	
	a3	$([\dot{s}_{0.917},\dot{s}_{1.106}],[\ddot{s}_1,\ddot{s}_1])$	0.042142	
	a4	$([\dot{s}_{1.039},\dot{s}_{1.164}],[\ddot{s}_1,\ddot{s}_1])$	0.045891	
p=5,q=1	a1	$([\dot{s}_{1.108},\dot{s}_{1.108}],[\ddot{s}_1,\ddot{s}_1])$	0.046167	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{1.050},\dot{s}_{1.074}],[\ddot{s}_1,\ddot{s}_1])$	0.044247	
	a3	$([\dot{s}_{0.958},\dot{s}_{1.050}],[\ddot{s}_1,\ddot{s}_1])$	0.041829	
	a4	$([\dot{s}_{1.016},\dot{s}_{1.075}],[\ddot{s}_1,\ddot{s}_1])$	0.043543	
p=10,q=1	a1	$([\dot{s}_{1.057},\dot{s}_{1.057}],[\ddot{s}_1,\ddot{s}_1])$	0.044041	$a_1>a_2>a_4>a_3$
	a2	$([\dot{s}_{1.027},\dot{s}_{1.039}],[\ddot{s}_1,\ddot{s}_1])$	0.043031	
	a3	$([\dot{s}_{0.977},\dot{s}_{1.027}],[\ddot{s}_1,\ddot{s}_1])$	0.041738	
	a4	$([\dot{s}_{1.008},\dot{s}_{1.039}],[\ddot{s}_1,\ddot{s}_1])$	0.04264	

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