Genetic Algorithm with Modified Crossover for Grillage Optimization

M. Ramanauskas, D. Sesok, R. Belevicius, E. Kurilovas, S. Valentinavicius

Mikalojus Ramanauskas, Dmitrij Sesok
Rimantas Belevicius, Saulius Valentinavicius
Vilnius Gediminas Technical University,
Dept. of Information Technology, Vilnius, Lithuania
Vilnius, LT-10223 Sauletekio al. 11, Lithuania
mikalojus.ramanauskas@vgtu.lt

Eugenijus Kurilovas
1. Vilnius Gediminas Technical University,
Dept. of Information Technology, Vilnius, Lithuania
Vilnius, LT-10223 Sauletekio al. 11, Lithuania
jevgenij.kurilov@vgtu.lt
2. Vilnius University Institute of Mathematics and Informatics
Vilnius, LT-08663 Akademijos str. 4, Lithuania
*Corresponding author: jevgenij.kurilov@vgtu.lt

Abstract: Modified genetic algorithm with special phenotypes' selection and crossover operators with default specified rules is proposed in this paper thus refusing the random crossover. The suggested crossover operator enables wide distribution of genes of the best phenotypes over the whole population. During selection and crossover, the best phenotypes of the newest population and additionally the genes of the best individuals of two previous populations are involved. The effectiveness of the modified algorithm is shown numerically on the real-life global optimization problem from civil engineering - the optimal pile placement problem under grillage-type foundations. This problem is a fair indicator for global optimization algorithms since the ideal solutions are known in advance but with unknown magnitudes of design parameters. Comparison of the proposed algorithm with 6 other stochastic optimization algorithms clearly reveals its advantages: at similar accuracy level the algorithm requires less time for tuning of genetic parameters and provides narrower confidence intervals on the results than other algorithms.

Keywords: genetic algorithm; crossover operator; grillage optimization.

1 Introduction

In this paper, we propose a new genetic algorithm with modified crossover operator and compare it with other well-known stochastic optimization algorithms. As a benchmark the problem of pile placement optimization under grillage-type foundations is chosen. The grillage foundations consist of piles driven to the ground, and connected on the top with girders. The in-plane configuration of girders may be complex, and the number of piles may reach several tens (few examples of grillages are provided in the Appendix). Here we consider that the optimal grillage is the grillage with minimal number of piles of given stiffness characteristics, and the reactive forces in piles are equal. This can be achieved changing the positions of piles under girders. In ideal case the reactive forces in all piles are distributed evenly and are equal to the bearing capacity of pile. Practically this is hardly possible, since the theoretical number of piles which is obtained dividing the total loading on the foundation (i.e., all active forces plus the dead weight of the erection) by bearing capacity of pile, is common case not the integer number. Also, some technological requirements can hinder achieving the ideal scheme, e.g., the given minimal

allowable distance between adjacent piles due to pile-driver characteristic, or the presence of the given immovable piles that are introduced into placement scheme by a designer (usually at the corners of girders) and do not change their position in the optimization process.

The pile placement problem is ideal for comparison of global optimization algorithms. Firstly, the global solution - the reactive force that should be evened out in all piles is known in advance. Secondly, the practical solution of the problem shows that the landscape of objective function is complex and has many local extremes. Usually, the objective function is very sensitive to the pile positions: even small changes position of one pile sometimes leads to a large alteration of objective function. All this makes the problem a complex global optimization problem.

The mathematical models of optimization of grillage-type foundations were formulated and the solution algorithms were suggested in [4], [3]. Three problems were solved: pile placement seeking for even distribution of reactive forces in piles, pile placement seeking for least bending moments in the connecting girders, and integrated problem for minimization of reactive forces and bending moments. In case of two last problems the global solution cannot be obtained in advance, therefore it is not a right choice for comparison of optimization algorithms. The first problem was solved in [10] employing all popular at that time stochastic optimization algorithms. In all these algorithms a phenotype, or an individual, is the approximate mathematical model of the whole grillage. The fitness of a phenotype is measured by a maximum reactive force magnitude among all piles, i.e., the fittest individual has the least reactive force. In [15], combination of the sizing and topology optimization is observed, however the piles are aggregated to special groups of pile. Exhaustive technical details on the design of grillages can be found, e.g. in [13]. Also, this problem was solved by the new several dimension optimization method BAcoor [8], [9]. The last method outperformed all other algorithms for the grillages where the piles have to be placed at very uneven distances. At more even distribution of piles the classical stochastic algorithms provided better results.

In all cases genetic algorithm (GA) with carefully tuned genetic parameters provided best results or results close to the best solutions. Among these parameters, the crossover operator plays significant role [4], [7] since it combines the information contained in the previous individuals in order to obtain fitter phenotypes. Therefore, in this paper, the main attention is given to the crossover operator. The crossover was investigated, e.g., in [5] where a gender was assigned to each individual and the crossover was performed only between individuals of opposite genders. In many cases the crossover operator is dedicated for a particular type of problems, e.g., operator EAX for traveling salesman problems [15], [11].

There are five chapters and appendices in this paper. In the second chapter, the ideas of new crossover operator and its implementation are described. The third chapter depicts the problem under consideration - the analysis of grillage-type foundations via finite element method. In the fourth chapter, the optimization problem along with all requirements for fair comparison of optimization algorithms are given, and the obtained results are discussed. In the final section, some general conclusions are drawn.

2   Modified crossover operator

In the classical genetic algorithm, the initial population of a given length N is created. Then, using selection and crossover operators the new individual generations are generated. During the selection, the fitter individuals have better chances to be included into the next generation. The probability of an individual to be selected for a next generation is usually directly proportional to the ratio of its objective function value with the best function value in the present population. Instead of individuals that do not enter into the new generation, the new random individuals are created. After the selection, the random individuals interchange genes between them during

the crossover operation. This does not guarantee that after the selection all best genes will survive and will get into the new generation. If the non-elitist strategy is employed, always a certain probability exists that even the individual representing the global solution will not pass the selection. It also should be noted, that in classical algorithm the crossover operator is applied only to the individuals of newest generation. Thus, the individuals of previous generations are completely lost. We suggest the following modified selection and crossover operations:

- The new population is created not only of crossbred individuals, but also of the best individuals of two previous generations. Thus in the current population coexist parents and grandparents.

- One-third of the best individuals participate in the crossover, and each individual is inter-bred twice with in advance definite individuals.

- Thus, less random genetic parameters must be chosen for the algorithm; only the breeding point and mutation probability have to be selected.

The crossover rules are summarized as follows:

1. Initial population of N individuals $A_0 = \{a_0, a_1, ..., a_{N-1}\}$.

2. The individual a_i' of a succeeding population A_j $(j \neq 0)$ is obtained breeding N best individuals of the generation A_{j-1} according to the equations:

- if $i < \frac{N}{2}, a_i' = a_i \times a_{i+\frac{N}{2}}$

- if $i \geqslant \frac{N}{2}, a_i' = a_i \times a_{N-i-1}$

where the $\times$ denotes the crossover operations.

3. Population A_j $(j > 1)$ consists of $3 \cdot N$ individuals: N crossbred individuals of generation A_{j-1} and two groups of size N of the best individuals from generations A_{j-1} and A_{j-2}.

Thus, the recursive function for creation of populations is: $A_0 = \{a_0, a_1, a_2, ..., a_{N-1}\}$,

$A_1 = A_0 \bigcup A_0^\times$,

$A_2 = A_0 \bigcup TOP_N (A_1) \bigcup TOP_N \left(A_1^\times\right)$,

$A_3 = TOP_N (A_1) \bigcup TOP_N (A_2) \bigcup TOP_N \left(A_2^\times\right)$,

$\ldots$

$A_i = TOP_N (A_{i-2}) \bigcup TOP_N (A_{i-1}) \bigcup TOP_N \left(A_{i-1}^\times\right)$,

where $TOP_N (A_i)$ is the set of N individuals of the ith generation with the best objective function values $A_i^\times$, is the crossover operator breeding the individuals of the ith generation according to the rules shown. Thus, the proposed rules do not require tuning of selection parameters. The third rule also guaranties a special elitist strategy, i.e., the individual with the best objective function value is retained from generation to generation. On the other hand, passing into the next generations only the fittest individuals of preceding populations may narrow the search space. Sufficient diversity of a population therefore is achieved by a classical mutation with a rather high probability that should be tuned for the problem under consideration.

3   Grillage optimization model

One individual of a routine population is the approximate mathematical model of the grillage-type foundation. Grillage is discretized by the finite element method into 2D beam element mesh with out-of-plane boundary conditions instead of piles. Out-of-plane active forces consist of the self-weight of the erection, plus all active forces according to Eurocodes. The girders of

a grillage are approximated as two-node beam elements with fixed cross-section and material characteristics. The piles are represented as the supports with specified displacements (zero displacements are the most common case). Alternatively, piles are regarded as supports with specified stiffness characteristics. Generally, the maximum reactive force among all piles is treated as the objective function.

Supports of the first type are rather non-realistic representations and sometimes yield misleading analysis results. For example, when multiple supports are needed to carry large concentrated load, this kind of supports will lead to a logjam. If odd number of supports is placed under load, the central support will be located just beneath the load and will take all the force. In case of even number of supports the "saw-teeth" like distribution of reactions is observed, and the more supports will be installed, the larger in absolute value reactions will arise.

The optimization problem is defined as in

$$\min_{x \in D} f(x).$$ (1)

Here $f(x)$ is the objective function, D is the feasible shape of structure, which is defined by the type of certain supports, the given number and layout of different crosssections as well as different materials in the structure.

$f(x)$ is defined by the maximum difference between vertical reactive force at a support and allowable reaction for this support, thus allowing us to achieve different reaction at supports on different beams, or even at particular supports on the same beam:

$$f(x) = \max_{x \in D} \max_{1 \leqslant i \leqslant N_i} |R_i - c_i R_{allow}|.$$ (2)

Here N_s denote the number of supports, R_{allow} is allowable reaction, c_i are factor to this reaction and R_i are reactive forces in each support.

Finite element matrices and sensitivity analysis

The problem has to be solved in statics and in linear stage

$$[K]\{u\} = \{F\}.$$ (3)

Here $[K]$ is the stiffness matrix of grillage, $\{u\}$ are the displacement of grillage nodes, and $\{F\}$ - the loadings. The reactive forces at a rigid supports are obtained using equation

$$R_i = \sum_j K_{ij} u_j, i = 1, 2, ..., N_S,$$ (4)

where a part of nodal displacements (displacements of free nodes) are already obtained via, and the displacements of nodes representing the rigid supports are specified (usually - zero). If the supports have finite stiffness k_i,

$$R_i \approx k_i u_i, i = 1, 2, ..., N_S.$$ (5)

If the local search around the certain solution obtained by stochastic optimization algorithm is implemented, the sensitivity information is the must. The sensitivity analysis is performed using the pseudo-load approach; thus, the numerical calculation of derivatives can be avoided. Denoting the support positions by $x_i, i = 1, 2, ..., N_S$

$$R_{i,x_i} = [K]_{,x_i} \{u\} + [K] \{u\}_{,x_i}.$$ (6)

Here the derivative of stiffness matrix is obtained analytically, while the derivative of displacements supposes solution of the general sensitivity equation:

$$[K]\{u\}_{,x_i} = \{F\}_{,x_i} - [K]_{,x_i}\{u\}.$$

(7)

The derivatives of load vector are obtained also in a closed form, analytically. A simple two-node beam element with 6 d.o.f's at a node (three displacements and three rotations about local element axes) is employed in the analysis. Details on the stiffness matrices of an element may be found in many textbooks, e.g., [16].

Program. Original Fortran program is used for obtaining the objective function value. First of all, the finite element mesh along with all needed data is prepared by a special pre-processor. Some initial data for the pre-processor are constant and do change in the optimization process - the configuration of the grillage, self-weight of the structure and active loadings, material characteristics. The locations of piles are obtained from the guess of optimization algorithm. Since supports have to be placed under the girders of grillage, first of all the grillage is "unfolded" to one-dimensional construct, and locations of all supports are freely chosen along the whole length of this construct. After that, the initial configuration is restored. From all this input, the pre-processor automatically prepares the finite element mesh introducing nodes at support points, discontinuities of material and cross-sections properties, etc. The third independent program analyses the finite element results and provides the objective function value. Model transformation patterns are obtained by using the Formal Concept Analysis [10], where relations and element meta-classes of target and source models are linked together based on model classification group links that have similarities between them.

4   Numerical results and discussion

10 different grillages possessing from 17 to 55 piles, i.e., the optimization variables were optimized. Data for these problems (see Appendix 1) are obtained from several Dutch design bureaus which use the professional software package MatrixFrame (http://www.matrix-software.com/Uk/structuralengineering/matrixframe/index.html) for structural engineering. Seven different optimization algorithms are compared. 28 independent numerical experiments were performed with each algorithm. In order to have fair comparison, the objective function was evaluated 5000 times in each experiment. The following algorithms were employed [10]: modified random search (MRS), simulated annealing (SA), simplex (SM), the variable metric method NEWUOA [10], [12], [14], BAcoor, and the proposed genetic algorithm with modified crossover (MCGA).

Comparison of all algorithms is provided in the Fig.1. Since total number of objective function evaluations is 5000, small populations of 15 individuals were created in 333 generations. The mutation probability after few numerical experiments was set to 15%. All numerical results of 28 independent experiments are rendered in Appendix 2. Thus, in three cases the proposed algorithm outperforms all other algorithms. Compared to the classical genetic algorithm, the results of MCGA is better for almost all problems considered. The ideal solution was not found for any problem, however, the differences compared to the ideal solutions do not exceed 5% for problems No. 2, 5 and 7. Results for problems No. 3, 4, 6, 7 and 9 differ from global solutions till 8%. Summarizing, the SA showed the performance for the grillage optimization problems, however, the NEWUOA, MCGA and GA (excluding the simplest structures No. 1 and 3) are not far behind. GA outperforms the MCGA only for most complex grillages No. 8 and 10 with 34 and 55 design parameters. Better results may be expected with larger populations. However, in this case we cannot fairly compare results with other algorithms. More important is, the
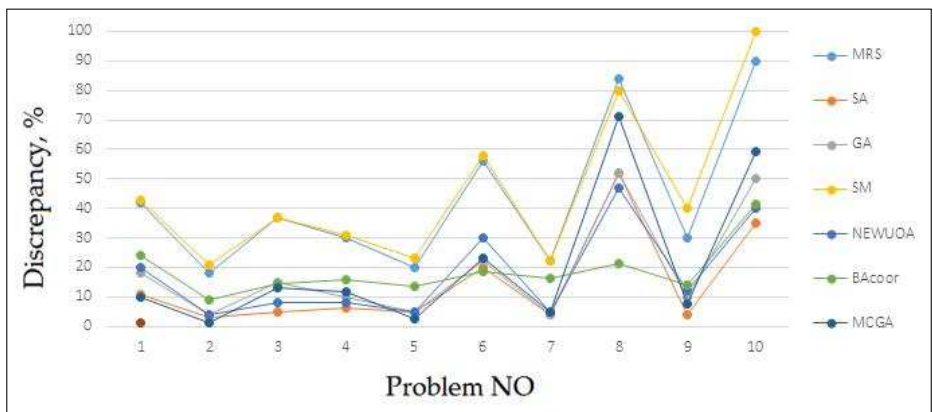
Figure 1: The best objective function values in 28 runs obtained by MRS, SA, GA, SM, NEWUOA, BAcoor and MCGA, normalized to Rideal .

confidence intervals of objective function value are much narrower for results of MCGA than for results of BAcoor (we do not have the confidence intervals for other algorithms) (see in Fig. 2). Also, for all problems the upper values of MCGA confidence intervals are better than ones of BAcoor. In some problems the upper values are even better than the lower values of confidence intervals of BAcoor. All numerical results - objective function values and confidence intervals are provided in the Tables 1 and 2.
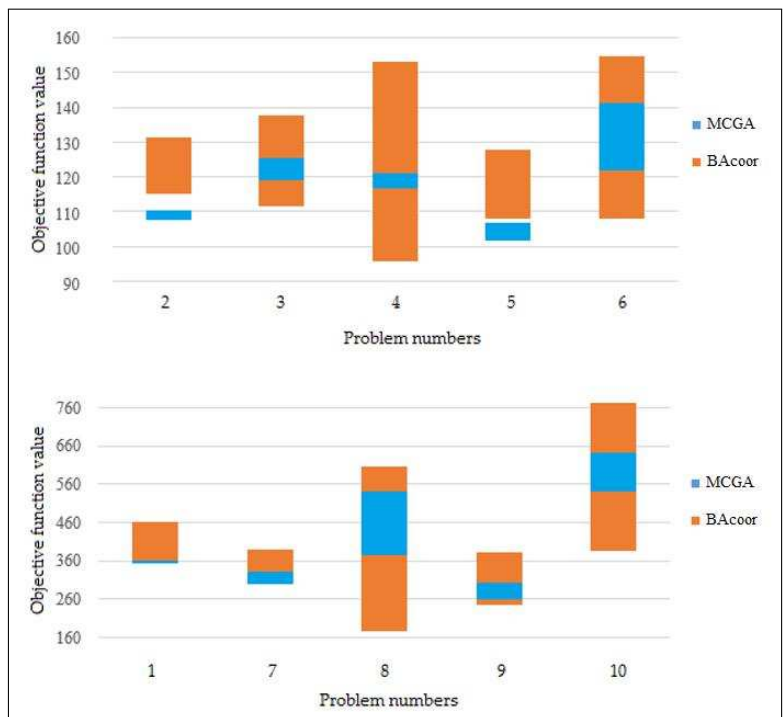

Figure 2: Confidence intervals.

Table 1: Summary of numerical results

Problem No	Best value found in MCGA	Best value found in [10]	Exact solution
1	337.15	339.30	307.47
2	105.34	106.36	104.12
3	115.02	107.25	101.85
4	113.29	106.80	101.24
5	100.00	101.05	97.51
6	120.28	115.45	97.53
7	301.48	298.11	287.35
8	404.29	286.22	236.28
9	263.12	253.00	244.71
10	556.11	463.34	349.05

Table 2: Summary of confidence intervals

Problem No	MCGA lower	MCGA upper	BAcoor lower	BAcoor upper
1	351.98	361.85	360.29	461.26
2	107.46	110.48	115.29	131.41
3	119.01	125.32	111.67	137.62
4	116.65	121.18	95.61	153.22
5	101.72	106.96	107.92	127.86
6	121.87	141.36	108.08	154.73
7	300.02	332.12	319.70	390.43
8	375.90	541.41	174.99	606.98
9	260.63	302.23	246.69	384.22
10	539.95	642.83	387.85	772.54

5   Conclusions

In rather simple configurations of grillages where the optimal distribution of piles due to the geometry and given loadings is more or less even, the proposed method MCGA outperforms other popular stochastic optimization algorithms. Additionally, this algorithm requires less effort in tuning of algorithm parameters. The crossover and selection parameters are forecasted in advance by recursive crossover rules. Also the confidence intervals for results of MCGA are always narrower than those of BAcoor. Only for grillage configurations requiring larger number of supports and their uneven distributions, it loses to NEWUOA and BAcoor. Thus, the proposed method can be treated as a new global optimization algorithm that is simpler to use as the classical genetic algorithm but providing better or similar level of accuracy.

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Appendices

Table 3: Appendix 1: Characteristics of problems

Problem No	Number of supports	Foundation length	R_{allw}	R_{ideal}
1	25	172,90	325	307,47
2	18	52,90	110	104,12
3	31	84,10	105	101,85
4	31	84,90	105	101,24
5	30	63,90	100	97,51
6	37	80,10	100	97,53
7	23	129,10	300	287,35
8	34	137,90	250	236,28
9	17	97,60	250	244,71
10	55	315,61	350	349,05

Table 4: Appendix 2: Optimization results for all 10 problems in 28 independent numerical experiments

Probl/Exper	1	2	3	4	5	6	7	8	9	10
1	375,87	108,33	122,88	119,18	105,21	136,06	308,04	540,96	283,54	614,04
2	359,09	111,01	124,15	118,72	107,04	129,00	312,08	520,77	277,54	591,94
3	391,24	107,81	127,80	121,81	110,12	129,23	313,24	434,80	275,06	580,37
4	356,93	111,85	117,99	119,09	102,96	122,32	321,58	414,26	282,97	606,22
5	341,47	107,69	123,78	118,68	105,85	126,39	313,00	404,29	287,37	579,35
6	368,95	106,81	120,46	113,29	105,79	120,28	319,00	425,89	265,36	563,57
7	352,35	107,28	124,94	119,06	101,28	133,34	315,42	419,62	309,76	658,62
8	358,81	105,86	121,88	123,05	105,82	127,83	310,84	538,27	268,19	593,13
9	340,49	108,43	124,31	119,79	107,36	122,32	305,63	407,56	290,50	576,15
10	348,00	112,49	119,78	117,50	103,19	138,99	319,01	523,01	299,97	556,11
11	352,52	107,82	123,30	120,11	105,54	141,05	327,97	532,61	273,09	590,31
12	349,46	107,75	118,99	119,69	101,86	125,69	341,05	432,63	284,85	567,98
13	376,92	110,01	126,53	118,31	105,97	126,22	301,49	525,76	279,00	607,32
14	351,34	105,45	118,58	121,63	102,85	136,55	315,65	420,27	286,73	563,72
15	341,32	106,93	115,70	118,35	103,23	131,61	312,04	429,79	276,59	606,96
16	352,12	117,91	123,82	119,26	106,82	143,18	318,38	425,28	270,80	602,00
17	337,15	108,61	123,21	121,01	107,00	127,17	305,69	442,52	263,12	638,64
18	345,64	107,62	118,43	114,78	105,90	138,09	311,00	439,41	276,29	613,01
19	364,22	109,22	117,90	121,67	101,88	143,99	313,38	408,72	298,48	578,94
20	344,76	111,06	128,80	115,69	103,36	134,57	354,81	445,75	268,88	612,41
21	369,08	110,98	115,02	113,43	100,00	124,37	316,54	457,68	290,61	566,76
22	370,84	106,44	117,24	119,15	103,18	147,41	319,63	449,23	266,89	593,30
23	354,59	105,34	131,69	117,51	105,93	132,50	307,67	537,75	279,97	557,41
24	361,88	110,66	118,33	120,48	103,45	136,64	311,88	446,24	278,44	590,55
25	355,11	108,59	127,67	118,72	103,24	122,41	315,93	441,27	290,06	563,00
26	376,52	106,98	120,09	122,02	102,57	126,20	309,44	534,65	280,92	578,71
27	341,46	112,65	120,74	120,05	103,12	136,93	315,33	423,97	290,76	621,52
28	355,41	109,58	126,61	117,59	101,11	124,86	314,31	419,46	284,29	586,92

Grillage No 1

Grillage No 2

Grillage No 3

Grillage No 4

Grillage No 5

Grillage No 6

Grillage No 7
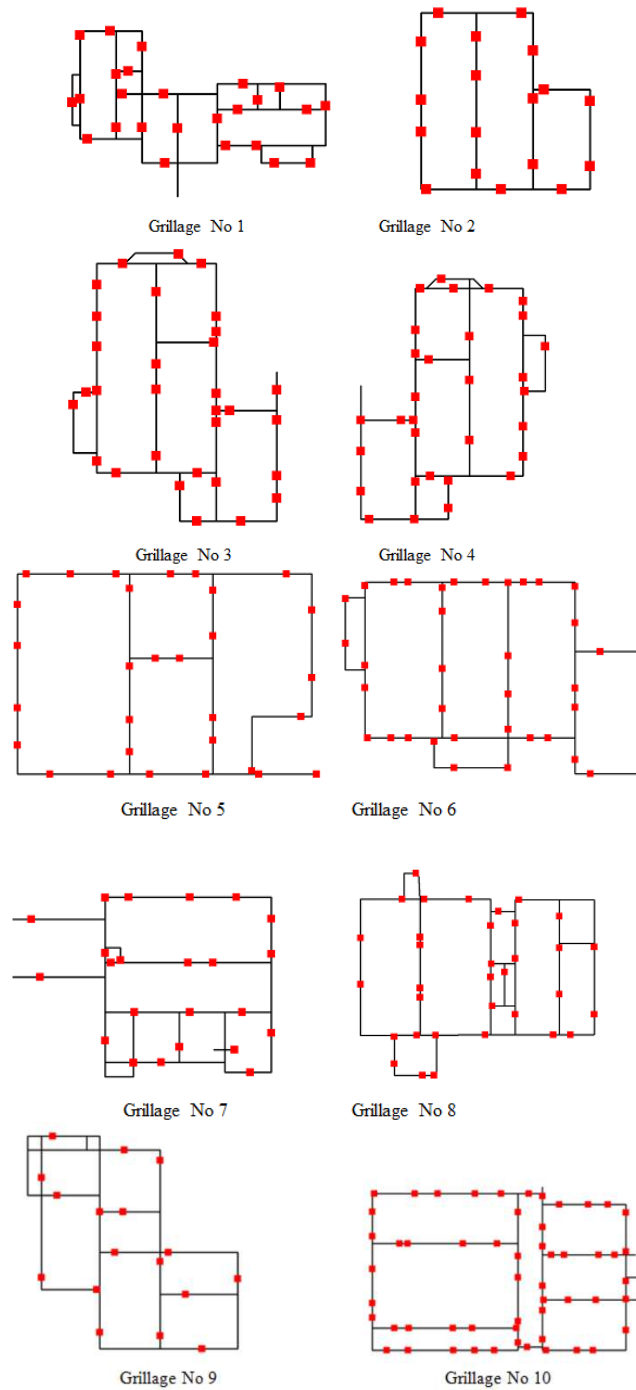
Grillage No 8

Grillage No 9

Grillage No 10

Figure 3: Appendix 3: Best pile placement schemes found with MCGA