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H_∞ Robust T-S Fuzzy Design for Uncertain Nonlinear Systems with State Delays Based on Sliding Mode Control

X.Z. Zhang, Y.N. Wang, X.F. Yuan

Xizheng Zhang
Hunan University
P.R.China, 410082 Changsha, Yuelushan, and
Hunan Institute of Engineering
P.R.China, 411104 Xiangtan, 88 Fuxing East Road
E-mail: z_x_z2000@163.com

Yaonan Wang, Xiaofang Yuan
Hunan University
P.R.China, 410082 Changsha, Yuelushan
E-mail: yaonan@hnu.cn, yuanxiaof@21cn.com

Abstract: This paper presents the fuzzy design of sliding mode control (SMC) for nonlinear systems with state delay, which can be represented by a Takagi-Sugeno (T-S) model with uncertainties. There exist the parameter uncertainties in both the state and input matrices, as well as the unmatched external disturbance. The key feature of this work is the integration of SMC method with H_∞ technique such that the robust asymptotically stability with a prescribed disturbance attenuation level γ can be achieved. A sufficient condition for the existence of the desired SMC is obtained by solving a set of linear matrix inequalities (LMIs). The reachability of the specified switching surface is proven. Simulation results show the validity of the proposed method.
Keywords: sliding mode control, T-S fuzzy model, time-delayed system, H_∞ control.

1 Introduction

Recently, the dynamic T-S fuzzy model has become a popular tool and has been employed in most model-based fuzzy analysis approaches [1]. Moreover, the ordinary T-S fuzzy model has been further extended to deal with nonlinear uncertain systems with time-delays [2]. The stability analysis and stabilization controller design for fuzzy time-delayed systems have attracted much attention over the past few decades due to their extensive applications in mechanical systems, economics, and other areas. A large number of results on this topic have been reported in the literature, see, e.g. [3–5]. Note that the uncertainties may exist in the real systems, or come from the fuzzy modeling procedure. Hence, the robust stabilization problems have recently been investigated in [6] for nonlinear uncertain fuzzy systems.

In practice, the inevitable uncertainties may enter a nonlinear system in a much more complex way. The uncertainty may include modeling error, parameter perturbations, fuzzy approximation errors, and external disturbances. In such circumstances, especially in the existence of external disturbances, the above established methods to control fuzzy time-delay systems could not work well any more. However, it is well known that the sliding mode control (SMC) is a reasonable approach to take effect if the lumped uncertainties are known to be bounded by smooth functions. In a more detail, the SMC system could drive the trajectories onto the so-called switching surface in a finite time and maintain on it thereafter, and on the switching surface the system is insensitive to internal parameter perturbations and external disturbances [7]. SMC approach has been successfully adopted in the control of time-delay systems these years. Quite recently, SMC approach has been also applied to solve the stabilization and tracking problems for fuzzy systems with matched uncertainties [8]. However, the sliding motion cannot be detached from the effect of unmatched parameter uncertainties, especially, unmatched external disturbances [9]. This means that the unmatched external disturbances make the design of SMC complex and challenging.

On the other hand, the H_∞ control, in the past decades, has been widely employed to deal with the uncertain systems with external disturbance [10, 11]. The goal of this problem is to design a controller to stabilize a given system while satisfying a prescribed level of disturbance attenuation. The H_∞ control for uncertain time-delayed systems has been considered by some researchers [12, 13]. In [14], Peng and Yue investigated the H_∞ controller design for uncertain T-S fuzzy systems with time-varying interval delay by using a new Lyapunov-Krasovskii functionals and an innovative integral inequality. The output feedback controller in [15] was designed for uncertain fuzzy systems such that the closed-loop systems are robustly asymptotically stable and satisfy a prescribed H_∞ performance. Lin et al. [16] further presented the mixed H_2/H_∞filter design for nonlinear discrete-time systems with state-dependent noise.

Motivated by the above discussion, it is certain that the integration of the SMC method with H_∞ technique would have a great potential in extending the SMC to the systems with unmatched uncertainties and obtain a better dynamic performance. Therefore, in this paper, by utilizing the H_∞ technique to attenuate the effect of unmatched external disturbance, we proposes a novel SMC controller that can ensure the robust stability with a prescribed disturbance attenuation level γ for the fuzzy time-delayed system, irrespective of parameter uncertainties and unmatched external disturbance. The controller design method is presented in terms of LMIs.

The notations used in this paper are quite standard: $\Re^n$ denotes the n-dimensional real Euclidean space; $\boldsymbol{I}$ is the identity matrix with appropriate dimensions; $\boldsymbol{W} < 0$ ($\boldsymbol{W} > 0$) means that $\boldsymbol{W}$ is symmetric and negative (positive) definite; $L_2[0,\infty]$denotes the space of square-integrable vector functions over $[0,\infty]$; The superscript "T" represents the transpose of a matrix, and the notation "$*$" is used as an ellipsis for terms that are induced by symmetry; $\|\cdot\|$ denotes the spectral norm; Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

2 Problem Formulation

As stated in Introduction, T-S fuzzy models can provide an effective representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning applied to a set of linear input-output submodels. Hence, in this work, a class of nonlinear time-delay systems is represented by a T-S model. As in [2], the T-S fuzzy time-delay system with uncertainties is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. The i-th rule of the fuzzy model is formulated in the following equation:

Plant rule i: IF θ_1 is η_1^i and θ_2 is η_2^i $\cdots$ and θ_p is η_1^p, THEN

$$\begin{cases} \dot{\boldsymbol{x}}(t) = [(\boldsymbol{A}_i + \Delta\boldsymbol{A}_i(t))\boldsymbol{x} + (\boldsymbol{A}_{di} + \Delta\boldsymbol{A}_{di}(t))\boldsymbol{x}(t-\tau)] + (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t) + \boldsymbol{B}_{wi}\boldsymbol{w}(t) \\ \boldsymbol{z}(t) = \boldsymbol{C}_i\boldsymbol{x}(t) \\ \boldsymbol{x}(t) = \varphi(t), t \in [-\tau(t), 0], i = 1, 2, ., r \end{cases} \tag{1}$$

where η_j^i is the fuzzy set, $\boldsymbol{\theta} = [\theta_1(t), \theta_2(t), \cdots, \theta_p(t)]^T$ is the premise variable vector, r is the number of rules of this T-S fuzzy. $\boldsymbol{x}(t) \in \Re^n$ is the state vector, $\boldsymbol{u}(t) \in \Re^m$ is the control input vector, $\boldsymbol{z}(t) \in \Re^l$ is the controlled output, $\boldsymbol{w}_i(t) \in \Re^p$ denotes the unknown external disturbances or modeling error. $\boldsymbol{A}_i$, $\boldsymbol{A}_{di}$, $\boldsymbol{B}_i$, $\boldsymbol{B}_{wi}$ and $\boldsymbol{C}_i$ are known real constant matrices with appropriate dimensions. $\Delta\boldsymbol{A}_i(t)$, $\Delta\boldsymbol{A}_{di}(t)$, $\Delta\boldsymbol{B}_i$ are unknown time-varying matrices representing parameter uncertainties. τ is the time-varying delay for the state vector satisfying $0 < \tau(t) < d < \infty$, $\dot{\tau}(t) < h < 1$, where d and h are known real constant scalars. $\varphi(t)$ is a continuous vector-valued initial function.

The overall fuzzy model achieved by fuzzy blending of each plant rule is represented as follows:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\theta})[(\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) + (\boldsymbol{A}_{di} + \Delta\boldsymbol{A}_{di})\boldsymbol{x}(t-\tau) + (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t) + \boldsymbol{B}_{wi}\boldsymbol{w}(t)] \tag{2}$$

where $h_i(\boldsymbol{\theta}) = \frac{\alpha_i(\boldsymbol{\theta})}{\sum_{j=1}^{r}\alpha_i(\boldsymbol{\theta})}$,$\alpha_i(\boldsymbol{\theta}) = \prod_{j=1}^{p}\eta_j^i(\boldsymbol{\theta})$, in which $\eta_j^i(\boldsymbol{\theta})$ is the membership grade of θ_j in η_j^i. According to the theory of fuzzy sets, we have $\boldsymbol{\theta} \geq 0$ and $\sum_{i=1}^{r}\boldsymbol{\theta} \geq 0$. Therefore, it implies that $h_i(\boldsymbol{\theta}) \geq 0$ and $\sum_{i=1}^{r} h_i(\boldsymbol{\theta}) = 1$. In this work, the following assumptions are introduced.

(Assumption.1) The time-varying uncertainties $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{A}_{di}$ are assumed to be norm-bounded, that is,

$$[\Delta\boldsymbol{A}_i, \Delta\boldsymbol{A}_{di}] = \boldsymbol{H}_i\boldsymbol{F}_i(t)[\boldsymbol{E}_{1i}, \boldsymbol{E}_{2i}] \tag{3}$$

where $\boldsymbol{H}_i$,$\boldsymbol{E}_{1i}$, $\boldsymbol{E}_{2i}$ are known constant matrices, and $\boldsymbol{F}_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $\boldsymbol{F}_i^T(t)\boldsymbol{F}_i(t) \leq \boldsymbol{I}$,$\forall t$.

(Assumption.2) It is assumed that the matrices $\boldsymbol{B}_i$ satisfy $\boldsymbol{B}_1 = \boldsymbol{B}_1 = \cdots = \boldsymbol{B}_r = \boldsymbol{B}$. Moreover, the pair $(\boldsymbol{A}_i, \boldsymbol{B})$ is controllable and the input matrix $\boldsymbol{B}$ has full-column rank m and $m < n$.

(Assumption.3) The uncertainty matrix $\Delta\boldsymbol{B}_i$ is assumed to be matched, i.e., there exists a matrix $\boldsymbol{\delta}_i(t) \in \Re^{m\times m}$ such that $\Delta\boldsymbol{B}_i = \boldsymbol{B}_i\boldsymbol{\delta}_i(t)$ with $\|\boldsymbol{\delta}_i(t)\| \leq \rho_B < 1$, where ρ_B is a positive constant.

(Assumption.4) The upper bound for $\boldsymbol{w}_i(t)$ is known.

It is noted that there exists parameter uncertainties in both the state and control input matrices and unmatched external disturbance $\boldsymbol{w}_i(t)$ in the systems under consideration.

Remark 1: Assumptions 1 4 are standard assumptions in the study of variable structure control.

Before proceeding, some standard concepts and lemma are given as follows, which are useful for the development of our result.

Definition 1. The uncertain fuzzy time-delayed systems in (2) is said to be robustly asymptotically stable if the system with $\boldsymbol{u}(t) = 0$ and $\boldsymbol{w}_i(t) = 0$ is asymptotically stable for all admissible parameter uncertainties.

Definition 2. Given a scalar $\gamma > 0$, the unforced fuzzy system in (2) with $\boldsymbol{u}(t) = 0$ is said to be robustly stable with disturbance attenuation γ if it is robustly stable and and under zero initial condition, $\|\boldsymbol{z}(t)\|_{E2} \leq \gamma\|\boldsymbol{w}(t)\|_2$ for all non-zero and all admissible uncertainties, where

$$\|\boldsymbol{z}(t)\|_{E2} = \sqrt{\int_0^t |\boldsymbol{z}(t)|^2 dt} \tag{4}$$

(Lemma.1 Choi [10]): Let $\boldsymbol{E}$, $\boldsymbol{H}$, and $\boldsymbol{F(t)}$ be real matrices of appropriate dimensions with $\boldsymbol{F(t)}$ satisfying $\boldsymbol{F}_i^T(t)\boldsymbol{F}_i(t) \leq \boldsymbol{I}$. Then, we have

(i) For any scalar $\varepsilon \leq 0$, $\boldsymbol{EF}(t)\boldsymbol{H} + \boldsymbol{H}^T\boldsymbol{F}^T(t)\boldsymbol{E}^T \leq \varepsilon^{-1}\boldsymbol{EE}^T + \varepsilon\boldsymbol{H}^T\boldsymbol{H}$

(ii) For any matrix $P > 0$, $-2\boldsymbol{E}^T\boldsymbol{H} \leq \boldsymbol{E}^T\boldsymbol{PE} + \boldsymbol{H}^T P^{-1}\boldsymbol{H}$.

3 Controller design

The objective of this work is to design a SMC law such that the desired control performance for the resulting closed-loop system is obtained despite of parameter uncertainties and unmatched external disturbance. In this section, a SMC law is first synthesized such that the closed-loop systems are robustly asymptotically stable with disturbance attenuation γ. It is further proven that the reachability of the specified switching (sliding) surface $s(t) = 0$ can be ensured by the proposed SMC law. Thus, it is concluded that the synthesized SMC law can guarantee the state trajectories of uncertain systems (2) to be driven onto the sliding surface, and asymptotically tend to zero along the specified sliding surface.

3.1 Sliding mode controller design

Essentially, a SMC design is composed of two phases: hyperplane design and controller design. There are various methods for designing hyperplane, however in this paper the switching surface is defined as

$$s(t)=\sum_{i=1}^{r}h_i(\boldsymbol{\theta}(t))\boldsymbol{G}_i\boldsymbol{x}(t) \tag{5}$$

where $\boldsymbol{G}_i\in\Re^{m\times n}$ is designed so that $\boldsymbol{G}_i\boldsymbol{B}_i$ is not singular. Furthermore, we design the VSC control law as follows

$$\begin{cases}\boldsymbol{u}(t)=\boldsymbol{u}_s(t)+\boldsymbol{u}_r(t)\\ \boldsymbol{u}_s(t)=-\sum_{i=1}^{r}\boldsymbol{K}_i\boldsymbol{x}(t)\\ \boldsymbol{u}_r(t)=-\sum_{i=1}^{r}h_i(\boldsymbol{\theta}(t))\boldsymbol{G}_i\left[\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{A}_{di}\boldsymbol{x}(t-\tau(t))\right]-\sum_{i=1}^{r}h_i(\boldsymbol{\theta}(t))\rho_i(\boldsymbol{x},t)sgn(s(t))\end{cases} \tag{6}$$

where $\boldsymbol{K}_i\in\Re^{m\times n}$ is chosen such that $(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)$ is Hurwitz, $sgn()$is a sign function and $\rho_i(\boldsymbol{x},t)$is a positive scalar function given as

$$\begin{aligned}\rho_i(\boldsymbol{x},t) \;\geq\; & \frac{2}{1-\rho_B^2}\Big\{\left[\|\Phi(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)\|+\|\Phi\boldsymbol{H}_i\boldsymbol{E}_{1i}\|+\rho_B\|\boldsymbol{K}_i\|+(1+\rho_B)\|\boldsymbol{G}\boldsymbol{A}_i\|\right]\|\boldsymbol{x}(t)\|\\ &+\left[\|\Phi\boldsymbol{A}_{di}\|+\|\Phi\boldsymbol{H}_i\boldsymbol{E}_{2i}\|+(1+\rho_B)\|\boldsymbol{G}\boldsymbol{A}_{di}\|\right]\|\boldsymbol{x}(t-\tau)\|\\ &+\|s\|\|\Phi\boldsymbol{B}_{wi}\|\|\boldsymbol{w}\|+\beta\Big\}\end{aligned} \tag{7}$$

with $\Phi=(\boldsymbol{G}_i\boldsymbol{B}_i)^{-1}\boldsymbol{G}_i$ and $\beta>0$ is a small known scalar.

Thus, substituting (6) into (2), we obtain the closed-loop system as follows

$$\begin{aligned}\dot{\boldsymbol{x}}(t) \;=\; & \sum_{i=1}^{r}h_i(\boldsymbol{\theta})\Big\{\left[\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i+\Delta\boldsymbol{A}_i(t)-\Delta\boldsymbol{B}_i(t)\boldsymbol{K}_i\right]\boldsymbol{x}(t)+\left[\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di}(t)\right]\boldsymbol{x}(t-\tau(t))\\ &+\left[\boldsymbol{B}_i+\Delta\boldsymbol{B}_i(t)\right]\boldsymbol{u}_r(t)+\boldsymbol{B}_{wi}\boldsymbol{w}(t)\Big\}\end{aligned} \tag{8}$$

The above expression Eq.(8) is the sliding-mode dynamics of the fuzzy uncertain system (2) in the specified sliding surface $s(t)=0$.

3.2 Stability of the sliding mode motion

In this subsection, we analyze the dynamic performance of the closed-loop system described by (8), and derives some sufficient conditions for the asymptotically stability of the sliding dynamics via LMI method. The following theorem shows that system (2) in the defined switching surface is robustly stabilizable with disturbance attenuation level γ.

Theorem 3. *Consider the fuzzy uncertain systems (2) with Assumptions* 1 4*, with the prescribed switching function, if there exist matrices* $\boldsymbol{P}>0$, $\boldsymbol{Q}>0$, *and positive scalars* ε_1, ε_2 *and* ε_3 *such that the LMI shown in (11) holds, with*

$$\Theta_1=\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^T\boldsymbol{P}+\boldsymbol{Q}+\varepsilon_1\boldsymbol{E}_{1i}^T\boldsymbol{E}_{1i}+\varepsilon_3\rho_B^2\boldsymbol{K}_i^T\boldsymbol{K}_i+\boldsymbol{C}_i^T\boldsymbol{C}_i \tag{9}$$

$$\Theta_2=-(1-h)\boldsymbol{Q}+\varepsilon_2\boldsymbol{E}_{2i}^T\boldsymbol{E}_{2i} \tag{10}$$

for $i=1,2,\cdots,r$*, then, by choosing* $\boldsymbol{G}_i=\boldsymbol{B}_i^T\boldsymbol{P}$*, the sliding-mode dynamics (8) is robust asymptotically stable with disturbance attenuation* γ.

$$\begin{pmatrix} \Theta_1 & * & * & * & * & * \\ \boldsymbol{A}_{di}^T\boldsymbol{P} & \Theta_2 & * & * & * & * \\ \boldsymbol{B}_{wi}^T\boldsymbol{P} & 0 & -\gamma^2\boldsymbol{I} & * & * & * \\ \boldsymbol{H}_i^T\boldsymbol{P} & 0 & 0 & \varepsilon_1\boldsymbol{I} & * & * \\ \boldsymbol{H}_i^T\boldsymbol{P} & 0 & 0 & 0 & \varepsilon_2\boldsymbol{I} & * \\ \Theta_1 & 0 & 0 & 0 & 0 & \varepsilon_3\boldsymbol{I} \end{pmatrix} < 0 \tag{11}$$

Proof: To analyze the stability of the sliding-mode dynamics (8), we consider the fuzzy uncertain system (2) with $\boldsymbol{w}(t) = 0$ and choose the following Lyapunov functional candidate

$$V(\boldsymbol{x},t) = \boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{x}(t) + \int_{t-\tau}^{t} \boldsymbol{x}(m)^T\boldsymbol{P}\boldsymbol{x}(m)dm \tag{12}$$

By differentiating the given Lyapunov function, we obtain the differential along the trajectories as

$$\begin{aligned} \dot{V} \;=\; & \sum_{i=1}^{r} h_i(\boldsymbol{\theta})\Big\{\boldsymbol{x}^T(t)\big[\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^T\boldsymbol{P}+\boldsymbol{Q}+2\boldsymbol{P}(\Delta\boldsymbol{A}_i+\Delta\boldsymbol{B}_i\boldsymbol{K}_i)\big]\boldsymbol{x}(t) \\ & +2\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di})\boldsymbol{P}\boldsymbol{x}(t-\tau)-2s^T[\boldsymbol{I}+\delta(t)]\{\rho_i sgn(s))+\boldsymbol{B}_i^T\boldsymbol{P}[\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{A}_{di}\boldsymbol{x}(t-\tau)]\}\Big\} \\ & -(1-\dot{\tau})\boldsymbol{x}^T(t-\tau)\boldsymbol{Q}\boldsymbol{x}(t-\tau) \end{aligned} \tag{13}$$

.

Noting the definition of switching function $s(t)$ and the control law (6), we have

$$\begin{aligned} \dot{V} \;=\; & \sum_{i=1}^{r} h_i(\boldsymbol{\theta})\Big\{\boldsymbol{x}^T(t)\big[\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^T\boldsymbol{P}+\boldsymbol{Q}\big]\boldsymbol{x}(t) \\ & +2\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di})\boldsymbol{P}\boldsymbol{x}(t-\tau)+2\boldsymbol{x}^T(t)\boldsymbol{P}(\Delta\boldsymbol{A}_i-\Delta\boldsymbol{B}_i\boldsymbol{K}_i)\boldsymbol{P}\boldsymbol{x}(t) \\ & -2s^T(t)[\boldsymbol{I}+\delta(t)]\boldsymbol{B}_i\boldsymbol{P}\big[\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{A}_{di}\boldsymbol{x}(t-\tau)\big] \\ & -2s^T(t)[\boldsymbol{I}+\delta(t)]\rho_i sgn(s)\Big\}-(1-\dot{\tau})\boldsymbol{x}^T(t-\tau)\boldsymbol{Q}\boldsymbol{x}(t-\tau) \end{aligned} \tag{14}$$

.

By Lemma 1, we obtain that for $\varepsilon_i > 0$, the following inequalities hold.

$$2\boldsymbol{x}^T\boldsymbol{P}\Delta\boldsymbol{A}_i\boldsymbol{x}(t) \le \varepsilon_1^{-1}\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{H}_i\boldsymbol{H}_i^T\boldsymbol{P}\boldsymbol{x}(t)+\varepsilon_1\boldsymbol{x}^T(t)\boldsymbol{E}_{1i}^T\boldsymbol{E}_{1i}\boldsymbol{x}(t) \tag{15}$$

$$2\boldsymbol{x}^T(t)\boldsymbol{P}\Delta\boldsymbol{A}_{di}\boldsymbol{x}(t-\tau) \le \varepsilon_2^{-1}\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{H}_i\boldsymbol{H}_i^T\boldsymbol{P}\boldsymbol{x}(t)+\varepsilon_2\boldsymbol{x}^T(t-\tau)\boldsymbol{E}_{2i}^T\boldsymbol{E}_{2i}\boldsymbol{x}(t-\tau) \tag{16}$$

$$2\boldsymbol{x}^T(t)\boldsymbol{P}\Delta\boldsymbol{B}_i\boldsymbol{K}_i\boldsymbol{x}(t) \le \varepsilon_3^{-1}\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{B}_i\boldsymbol{B}_i^T\boldsymbol{P}\boldsymbol{x}(t)+\varepsilon_3\rho_B^2\boldsymbol{x}^T(t)\boldsymbol{K}_i^T\boldsymbol{K}_i\boldsymbol{x}(t) \tag{17}$$

$$-2s^T[\boldsymbol{I}+\delta(t)]\rho_i sgn(s) \le -2\rho_i\|s\|+\rho_i[s^T\delta\delta^Ts+s^Ts]\|s\|^{-1} \le \rho_i(\rho_B^2-1)\|s\| \tag{18}$$

Noting that (3) and $\sum_{i=1}^{r} h_i(\boldsymbol{\theta}(\boldsymbol{t})) = 1$, and substituting the above inequalities into (14) results in

$$\dot{V} \le \sum_{i=1}^{r} h_i(\boldsymbol{\theta})\begin{bmatrix} \boldsymbol{x}^T(t) & \boldsymbol{x}^T(t-\tau) \end{bmatrix} \times \Pi \times \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-\tau) \end{bmatrix} \tag{19}$$

where $\Pi = \begin{pmatrix} \Xi_1 & \boldsymbol{P}\boldsymbol{A}_{di} \\ \boldsymbol{A}_{di}^T\boldsymbol{P} & \Xi_2 \end{pmatrix}$, with

$$\begin{aligned}\Xi_1 &= \boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^T\boldsymbol{P}+\boldsymbol{Q}+\varepsilon_1^{-1}\boldsymbol{P}\boldsymbol{H}_i\boldsymbol{H}_i^T\boldsymbol{P}+\varepsilon_1\boldsymbol{E}_{1i}^T\boldsymbol{E}_{1i}\\ &\quad+\varepsilon_2^{-1}\boldsymbol{P}\boldsymbol{H}_i\boldsymbol{H}_i^T\boldsymbol{P}+\varepsilon_3^{-1}\boldsymbol{P}\boldsymbol{B}_i\boldsymbol{B}_i^T\boldsymbol{P}+\varepsilon_3\rho_B^2\boldsymbol{K}_i^T\boldsymbol{K}_i \end{aligned} \tag{20}$$

$$\Xi_2=-(1-h)\boldsymbol{Q}+\varepsilon_2\boldsymbol{E}_{2i}^T\boldsymbol{E}_{2i} \tag{21}$$

In the following, it will be shown that the LMI (11) implies $\Pi<0$. By Schur's complement, $\Pi<0$ is equivalent to the LMI shown in (24), with

$$\Xi_3=\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^T\boldsymbol{P}+\boldsymbol{Q}+\varepsilon_1^{-1}\boldsymbol{P}\boldsymbol{H}_i\boldsymbol{H}_i^T\boldsymbol{P}+\varepsilon_1\boldsymbol{E}_{1i}^T\boldsymbol{E}_{1i}+\varepsilon_3\rho_B^2\boldsymbol{K}_i^T\boldsymbol{K}_i \tag{22}$$

$$\Xi_4=\Xi_2 \tag{23}$$

$$\begin{pmatrix}\Theta_3 & * & * & * & *\\ \boldsymbol{A}_{di}^T\boldsymbol{P} & \Theta_4 & * & * & *\\ \boldsymbol{H}_i^T\boldsymbol{P} & 0 & -\varepsilon_1\boldsymbol{I} & * & *\\ \boldsymbol{H}_i^T\boldsymbol{P} & 0 & 0 & -\varepsilon_2\boldsymbol{I} & *\\ \boldsymbol{B}_i^T\boldsymbol{P} & 0 & 0 & 0 & -\varepsilon_3\boldsymbol{I}\end{pmatrix}<0 \tag{24}$$

It is shown that the LMI (11) implies the above matrix inequality (24). Together with (19) implies that for all $\begin{bmatrix}\boldsymbol{x}^T(t) & \boldsymbol{x}^T(t-\tau)\end{bmatrix}\neq 0$, we have

$$\dot V(\boldsymbol{x}(t),t)\le 0 \tag{25}$$

This means that the closed-loop fuzzy system (8) with $\boldsymbol{w}(t)=0$ is robustly asymptotically stable. Next, we shall show that the fuzzy uncertain system (2) satisfies

$$\|\boldsymbol{z}(t)\|_{E2}\le\gamma\|\boldsymbol{w}(t)\|_2 \tag{26}$$

for all non-zero $\boldsymbol{w}(t)\in L_2[0,\infty]$. To this end, we assume zero initial condition, that is, with $\boldsymbol{x}(t)=0$ for all $t\in[-d,0]$. Then, we can rewritten the Lyapunov function candidate as follows:

$$\begin{aligned}\dot V &= \sum_{i=1}^{r}h_i(\boldsymbol{\theta})\Big\{\boldsymbol{x}^T(t)\big[\boldsymbol{P}(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)+(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)^T\boldsymbol{P}+\boldsymbol{Q}\big]\boldsymbol{x}(t)\\ &\quad+2\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di})\boldsymbol{P}\boldsymbol{x}(t-\tau)+2\boldsymbol{x}^T(t)\boldsymbol{P}(\Delta\boldsymbol{A}_i-\Delta\boldsymbol{B}_i\boldsymbol{K}_i)\boldsymbol{P}\boldsymbol{x}(t)\\ &\quad+2\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{B}_{wi}\boldsymbol{w}(t)-2s^T(t)[\boldsymbol{I}+\delta(t)]\boldsymbol{B}_i^T\boldsymbol{P}\big[\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{A}_{di}\boldsymbol{x}(t-\tau)\big]\\ &\quad-2s^T(t)[\boldsymbol{I}+\delta(t)]\rho_i sgn(s)\Big\}-(1-h)\boldsymbol{x}^T(t-\tau)\boldsymbol{Q}\boldsymbol{x}(t-\tau)\end{aligned} \tag{27}$$

.

Now, set

$$J(t)=\int_0^t[\boldsymbol{z}^T(m)\boldsymbol{z}(m)-\gamma^2\boldsymbol{w}^T(m)\boldsymbol{w}(m)]dm \tag{28}$$

with $t>0$. It is easy to show that

$$\begin{aligned}J(t) &= \int_0^t[\boldsymbol{z}^T(m)\boldsymbol{z}(m)-\gamma^2\boldsymbol{w}^T(m)\boldsymbol{w}(m)+\dot V(\boldsymbol{x},t)]dm-V(\boldsymbol{x},t)\\ &\le\int_0^t[\boldsymbol{z}^T(m)\boldsymbol{z}(m)-\gamma^2\boldsymbol{w}^T(m)\boldsymbol{w}(m)+\dot V(\boldsymbol{x},t)]dm\end{aligned} \tag{29}$$

Hence, noting (15)-(19), it follows from (29) that

$$J(t) \leq \int_0^t \begin{bmatrix} \boldsymbol{x}^T(m) & \boldsymbol{x}^T(m-\tau) & \boldsymbol{w}^T(m) \end{bmatrix} \times \Omega \times \begin{bmatrix} \boldsymbol{x}(m) & \boldsymbol{x}(m-\tau) & \boldsymbol{w}(m) \end{bmatrix} dm \tag{30}$$

with $\Omega = \begin{pmatrix} \Theta_1 & \boldsymbol{PA}_{di} & 0 \\ * & \Theta_2 & 0 \\ * & * & -\gamma^2 \boldsymbol{I} \end{pmatrix}$, where Θ_1 and Θ_2 are given as in (9) and (10). By Schur's complement, it can be shown that $\Omega < 0$ is ensured by LMI (11). This together with (30) implies that $J(t) < 0$ for all $t > 0$. Hence, we obtain (26) from (30). □

Remark 2: It is noted that the condition in Theorem 1 is delay independent, which might be conservative when the time delay is known and small. Hence, it would be appropriate to extend the current study to delay-dependent issues in future research.

3.3 Reachability of the sliding-mode

As the last step of design procedure, we will further prove that the VSC controller in (6) ensures the reachability of the specified switching surface. It is known from [17] that the solution of the system (2) is given by

$$\begin{aligned} J(t) &= \boldsymbol{x}(t) = \varphi(0) + \int_0^t \sum_{i=1}^r h_i(\boldsymbol{\theta}) \big[(\boldsymbol{A}_i + \Delta \boldsymbol{A}_i)\boldsymbol{x} + (\boldsymbol{A}_{di} + \Delta \boldsymbol{A}_{di})\boldsymbol{x}(m-\tau) \\ &\quad + (\boldsymbol{B}_i + \Delta \boldsymbol{B}_i)\boldsymbol{u}(m) + \boldsymbol{B}_{wi}\boldsymbol{w} \big] dm \end{aligned} \tag{31}$$

Hence, the switching function $s(t)$ can be expressed as

$$\begin{aligned} s(t) &= \sum_{i=1}^r h_i(\boldsymbol{\theta}) \boldsymbol{B}_i^T \boldsymbol{P} \varphi(0) + \int_0^t \sum_{i=1}^r h_i(\boldsymbol{\theta}) \boldsymbol{B}_i^T \boldsymbol{P} \big[(\boldsymbol{A}_i + \Delta \boldsymbol{A}_i)\boldsymbol{x}(m) + (\boldsymbol{A}_{di} + \Delta \boldsymbol{A}_{di})\boldsymbol{x}(m-\tau) \\ &\quad + (\boldsymbol{B}_i + \Delta \boldsymbol{B}_i)\boldsymbol{u}(m) + \boldsymbol{B}_{wi}\boldsymbol{w} \big] dm. \end{aligned} \tag{32}$$

This means that $s(t)$ varies finitely. That is, it is rational to take the time derivation of $s(t)$. Hence, we have

$$\dot{s}(t) = \sum_{i=1}^r h_i(\boldsymbol{\theta}) \boldsymbol{B}_i^T \boldsymbol{P} \big[(\boldsymbol{A}_i + \Delta \boldsymbol{A}_i)\boldsymbol{x}(t) + (\boldsymbol{A}_{di} + \Delta \boldsymbol{A}_{di})\boldsymbol{x}(t-\tau) + (\boldsymbol{B}_i + \Delta \boldsymbol{B}_i)\boldsymbol{u}(t) + \boldsymbol{B}_{wi}\boldsymbol{w} \big] \tag{33}$$

and then, the reachability of the specified sliding surface $s(t) = 0$ can be obtained in the following theorem.

Theorem 4. *For the uncertain fuzzy time-delay systems (2) with the given switching function (5) where* $\boldsymbol{G}_i = \boldsymbol{B}_i^T \boldsymbol{P}$ *and* $\boldsymbol{P}, \boldsymbol{Q}$, $\varepsilon_i (i = 1, 2, 3)$ *is the solution of LMIs (11). Then, it can be shown that the state trajectories of the system (2) will be driven onto the switching surface* $s(t) = 0$ *for all* $\boldsymbol{w}(t) \in L_2[0, \infty]$ *by the above VSC law (6).*

Proof: For purpose of design integrity, a simple stability analysis based on Lyapunov direct method is carried out. Define the Lyapunov function

$$V(t) = \frac{1}{2} s^T (\boldsymbol{G}_i \boldsymbol{B}_i)^{-1} s \tag{34}$$

Noting that (6), the expressions of ρ and $\dot{s}$, thus, we have

$$\begin{aligned}\dot{\boldsymbol{V}}(t) \leq\ & s^T(t)\sum_{i=1}^{r} h_i(\boldsymbol{\theta})(\boldsymbol{G}_i\boldsymbol{B}_i)^{-1}\boldsymbol{G}_i\Big\{(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i)\boldsymbol{x}(t)+(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di})\boldsymbol{x}(t-d)\\ & +s^T(t)[\boldsymbol{I}+\delta(t)]\boldsymbol{u}(t)+s^T(t)(\boldsymbol{G}_i\boldsymbol{B}_i)^{-1}\boldsymbol{G}_i\boldsymbol{B}_{wi}\boldsymbol{w}(t)\}\\ \leq\ & s^T(t)\sum_{i=1}^{r} h_i(\boldsymbol{\theta})(\boldsymbol{G}_i\boldsymbol{B}_i)^{-1}\boldsymbol{G}_i\Big\{(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i+\Delta\boldsymbol{A}_i)\boldsymbol{x}(t)-s^T(t)\delta(t)\boldsymbol{K}_i\boldsymbol{x}(t)\\ & +s^T(\boldsymbol{G}_i\boldsymbol{B}_i)^{-1}\boldsymbol{G}_i(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di})\boldsymbol{x}(t-d)-s^T[\boldsymbol{I}+\delta(t)]\boldsymbol{G}_i[\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{A}_{di}\boldsymbol{x}(t-d)]\\ & -s^T(t)[\boldsymbol{I}+\delta(t)]\rho_i(\boldsymbol{x},t)sgn(s(t))+s^T(t)(\boldsymbol{G}_i\boldsymbol{B}_i)^{-1}\boldsymbol{G}_i\boldsymbol{B}_{wi}\boldsymbol{w}(t)\Big\}\end{aligned} \tag{35}$$

By (18), we have

$$\begin{aligned}\dot{\boldsymbol{V}}(t) \leq\ & \|s(t)\|\sum_{i=1}^{r} h_i(\boldsymbol{\theta})\Big\{\big[\|\Phi(\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_i)\|+\|\Phi\boldsymbol{H}_i\|\|\boldsymbol{E}_{1i}\|+\rho_B\|\boldsymbol{K}_i\|\big]\|\boldsymbol{x}(t)\|\\ & +\big[\|\Phi\boldsymbol{A}_{di}\|+\|\Phi\boldsymbol{H}_i\|\|\boldsymbol{E}_{2i}\|\big]\|\boldsymbol{x}(t-d)\|\\ & +(1+\rho_B)\big[\|\boldsymbol{B}_i^T\boldsymbol{P}\boldsymbol{A}_i)\|\|\boldsymbol{x}(t)\|+\|\boldsymbol{B}_i^T\boldsymbol{P}\boldsymbol{A}_{di}\|\|\boldsymbol{x}(t-d)\|\big]\\ & +\|\Phi\boldsymbol{B}_{wi}\|\|\boldsymbol{w}(t)\|-0.5\rho(\boldsymbol{x},t)(1-\rho_B^2)\Big\}\end{aligned} \tag{36}$$

Then, it follows from (36) that for $s(t)\neq 0$

$$\dot{V}(t)\leq -\beta\|s(t)\|<0 \tag{37}$$

which implies that the reachability of the specified switching surface is guaranteed, and the trajectories of the fuzzy uncertain system (2) are globally driven onto the specified switching surface $s(t)=0$ for all $\boldsymbol{w}(t)\in L_2[0,\infty]$. Moreover, it is seen that the existence domain of the sliding mode is the whole switching surface. □

Remark 3: In fact, the design strategy of the sliding-mode controller (6) accords with the so-called parallel distributed compensation (PDC) scheme [3, 5, 6, 10, 12]. This idea is that the overall controller is a fuzzy blending of each individual controller for each local linear model. The PDC method has been widely utilized in fuzzy control, and is proven to be a very appealing approach.

4 Simulation Studies

In this section, a simple design example is used to illustrate the approach proposed in this paper. Consider a T-S fuzzy uncertain stated-delay system with the following model

Plant rule i: IF $x_2(t)$ is η_{ii}, THEN

$$\begin{cases}\dot{\boldsymbol{x}}(t)=[(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i)\boldsymbol{x}+(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di})\boldsymbol{x}(t-\tau)]+(\boldsymbol{B}_i+\Delta\boldsymbol{B}_i)\boldsymbol{u}(t)+\boldsymbol{B}_{wi}\boldsymbol{w}(t)\\ \boldsymbol{z}(t)=\boldsymbol{C}_i\boldsymbol{x}(t)\end{cases}$$

where $i=1,2$. The model parameters are given as $\boldsymbol{A}_1=\begin{bmatrix}0.1 & 0\\ 0 & -2\end{bmatrix}$,$\boldsymbol{A}_2=\begin{bmatrix}-0.3 & 0\\ 1 & -3\end{bmatrix}$,$\boldsymbol{A}_{d1}=\begin{bmatrix}0.1 & 0.1\\ 0 & 0.1\end{bmatrix}$,$\boldsymbol{A}_{d2}=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}$,$\boldsymbol{B}_1=\boldsymbol{B}_2=\begin{bmatrix}1\\ 1\end{bmatrix}$,$\boldsymbol{B}_{w1}=\begin{bmatrix}2\\ 0\end{bmatrix}$,,$\boldsymbol{B}_{w2}=\begin{bmatrix}1\\ 0\end{bmatrix}$,$\boldsymbol{C}_1=\boldsymbol{C}_2=\begin{bmatrix}2 & 0\\ 0 & 1.5\end{bmatrix}$.

The uncertainties are set to be $\Delta\boldsymbol{A}_1=\begin{bmatrix}0 & 0.08\sin t\\ 0 & 0.06\sin t\end{bmatrix}$, $\Delta\boldsymbol{A}_2=\begin{bmatrix}0.06\sin t & 0\\ 0.02\sin t & 0.01\sin t\end{bmatrix}$, $\Delta\boldsymbol{A}_{d1}=\begin{bmatrix}0 & 0.06\sin t\\ 0 & 0.06\sin t\end{bmatrix}$,

$\Delta A_{d2} = \begin{bmatrix} 0.01\cos t & 0 \\ 0 & 0.06\sin t \end{bmatrix}$, $\Delta B_1 = \begin{bmatrix} 0.1\cos t \\ 0.1\cos t \end{bmatrix}$, $\Delta B_2 = \begin{bmatrix} 0.1\sin t \\ 0.1\sin t \end{bmatrix}$, and the time-varying delay $\tau(t) = 0.5 + 0.5\sin t$ with $d = 1$ and $h = 0.5$. When choosing the matrix function as $F_i(t) = 0.02\sin t$, one can easily obtain the real constant matrices H_i, E_{1i} and E_{2i} from Assumption 1. It is also obviously that $\rho_B = 0.1$ with $\delta(t) \leq \rho_B$. The membership functions are selected as $\eta_{11} = \sin^2(x_2)$ and $\eta_{22} = \cos^2(x_2)$. Let the initial state $x = [0.9, 0.9]^T, t \in [-1, 0]$.

The problem at hand is to design a sliding mode controller such that the sliding motion in the specified switching surface is robustly stable, and the state trajectories can be driven onto the switching surface. To this end, we select the attenuation level $\gamma = 0.5$ and the matrices as follows: $K_1 = [1.5, 2.3], K_2 = [0.2, 3.4]$.

By solving LMIs (11), we obtain: $P = \begin{bmatrix} 1.5757 & -0.0144 \\ -0.0144 & 1.6211 \end{bmatrix}$, $Q = \begin{bmatrix} 0.0729 & 0.127 \\ 0.127 & 0.5216 \end{bmatrix}$.

Hence, the switching surface can be obtained as $s = [0.6405, 0.6223]x(t)$. It following from Theorem 2 that the desired VSC law can be obtained. The simulation results are given in Figures 1-3. Since it is well known that the chattering phenomenon is undesirable as it may incite high-frequency un-modeled dynamics and even leads to the instability of controlled system, we replace $sgn(\cdot)$ by $s/(\|s\| + \varepsilon)$(ε is the thickness of boundary layer) in the previous VSC law so as to prevent the control signals from chattering. However, it should also be pointed out that such an approach may lead to delay or make the controller less robust. Recently, to avoid chattering the use of high order and adaptive sliding mode is receiving more attentions; see, e.g., [10] for more details. It is seen that the reachability of the sliding motion can be guaranteed. Furthermore, the simulation results also show that our present design effectively attenuates the effect of both parameter uncertainties and external disturbances.

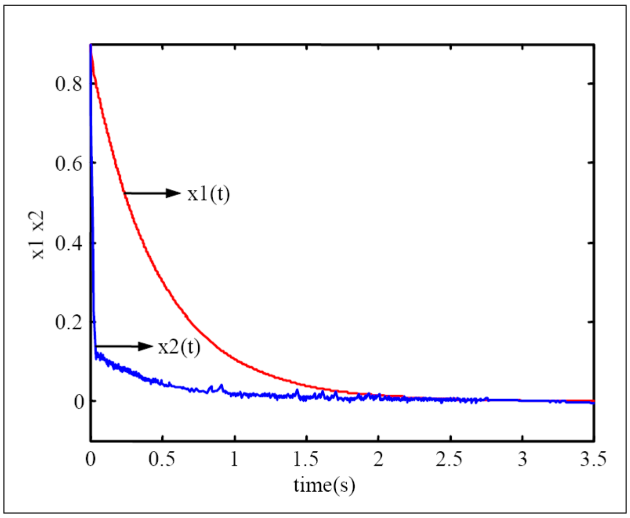

Figure 1: Trajectories of state x_1, x_2

5 Conclusions

This paper has firstly generalized the T-S model to represent a class of nonlinear uncertain systems. Then, a novel robust VSC method integrated with H_∞ technique, has been proposed for the fuzzy time-delayed system with parameters uncertainties and unmatched external disturbances. Moreover, by means of LMIs, a sufficient condition for the robustly stability of sliding motion with H_∞ disturbance attenuation level γ has been derived. It has been shown that both the switching surface and the VSC controller have been obtained by means of the feasibility of LMIs.

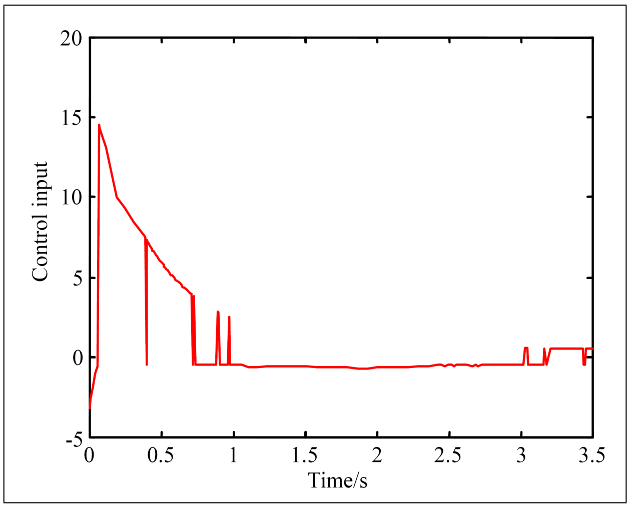

Figure 2: Switching surface $s(t)$

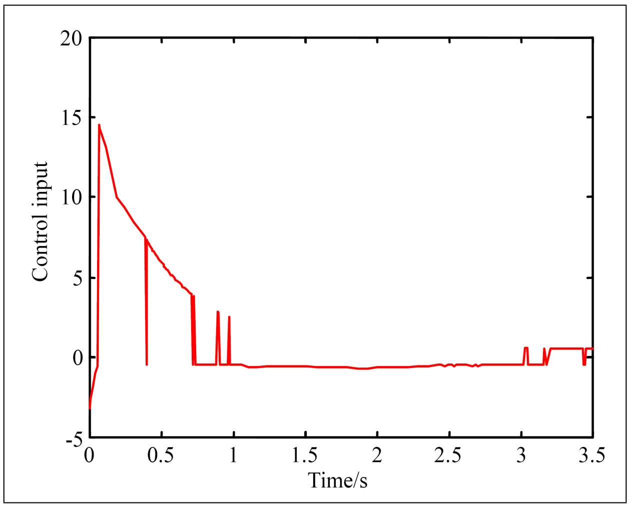

Figure 3: Control effort $\boldsymbol{u}(t)$

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