On vague Order and Decomposition Mapping by α-Open Sets Using Nano

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Abstract

In this paper the concept of nano topology is studied in a different way . The notation of nano α-open is established. Also the concept α-covering dimension is defined. The aim of this paper is to introduce and define a new type of covering dimension by using nano α-open sets namely, a nano α-covering dimension in a nano topological space and find some relations to other concepts. Some properties and characterization of this covering dimension are obtained.

Keywords: Nano α-open , Nano α-closure , Nano α-interior , Nano α-normal , Nano α-continuous $Ndim_{\alpha}U$.

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الخلاصة

بواسطة مفهوم المجموعات المفتوحة من النمط في الفضاءات التبولوجية من (nano topological spaces) النمط (nano α -open sets) في هذا العمل قدمنا موضوع جديد في نظرية البعد (حسب علمنا) هو($Ndim_{\alpha}U$) و بالإضافة لذلك سوف سندرس سلوك هذه الفضاءات تحت تأثير نوع معين من الدوال المستمرة و التي تدعى - (nano α continuous mapping) ومن خلال بحثنا في الخواص المقدمة في البحث رأينا ان هذه الخواص موازية الى بعض خواص نظرية البعد في التبولوجيا العامة.

الكلمات المفتاحية :- نانو , α -open، و نانو α -closure، و نانو α -interior، و نانو α -normal، و نانو- α continuous. $Ndim_{\alpha}U$

Introduction

The[Lellis and Carmel, 2013] introduced of nano topological space in relation to a subset X of a universal U around a lower and upper approximations of X . The boundary region of X with respect to an equivalence relation R is the set of all objects[Alias and HajiM, 2011].

The elements of a nano topological space are known nano open sets , and also close sets, nano interior and nano closure of a set[Cadas *et al*.,2003]. was also introduced by the same other the weak forms of nano open set namely, α-open set. Throughout this paper U and U' are non-empty sets, a finite universes $(X \subseteq U)$ and($Y \subseteq U'$;U/R)and$\left(\frac{U'}{R'}\right)$; denote the families of equivalence classes by equivalence relations (R) and (R') respectively on (U) and (V. $(U, \tau_R(X))$ and $(U', \tau_R{}'(Y))$ are the nano topological spaces on U and U' with respect to X and Y resp. [Lellis and Carmel,2013]. It is clear that each open set is α-open but the contrary is not true. [Najasted,2013] has shown that the family of all α-open sets is a topology on U. The dimension function was investigated by Pears A. [Pears, 1975]. The aim of this paper is a new notation for nano α-open sets are discussed.

2. Basic definitions and notations:

We introduce some elementary concept which we need in our work.

2.1. Definition: [Lellis and Carmel,2013]

Let (U) be a non-empty finite set of objects called the universe and R be an equivalence relation on (U) named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another , i.e. (U/R) denotes the family of equivalence classes of $(U$ by R).

The pair (U, R) is said to be the approximation space. Let ($X \subseteq U$).

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as (X) with respect to R and it is denoted by[$L_R(X)$].

[$L_R(X)] = [\bigcup_{x\in U}\{R(x)][:R(x) \subseteq X] = \{ x \in U: [x] \subseteq X \}$,where $R(x)$ denotes the equivalence class determined by x .

(ii) The upper approximation of X with respect to R is the set of all objects which can be for possibly classified as X with respect to R and it is denoted by $U_R(X)$.

,$[U_R(X)] = (\bigcup_{x\in U}) \{ R(x) : R(x) \cap X \neq \emptyset \} = \{ x \in U: [x] \cap X \neq \emptyset \}$ That is

(iii) The boundary region of X with respect to R is the set of all objects , which can be classified neither as X nor as not -X with respect to R and denoted by $[B_R(X) , B_R(X)] = [U_R(X) - L_R(X).$]

2.2. Theorem :- [Lellis and Carmel, 2013]

If (U, R) is an approximation space and($X, Y \subseteq U$) , then:

(i) $[L_R(X) \subseteq X \subseteq U_R(X)]$;

(ii) $[L_R(\emptyset) = U_R(\emptyset) = \emptyset$; $L_R(U) = U_R(U) = U]$;

(iii) $[U_R(X \cup Y) = U_R(X) \cup U_R(X)]$;

(iv) $[U_R(X \cap Y) \subseteq U_R(X) \cap U_R(X)]$;

(v) $[L_R(X \cup Y) \supseteq L_R(X) \cup L_R(X)]$;

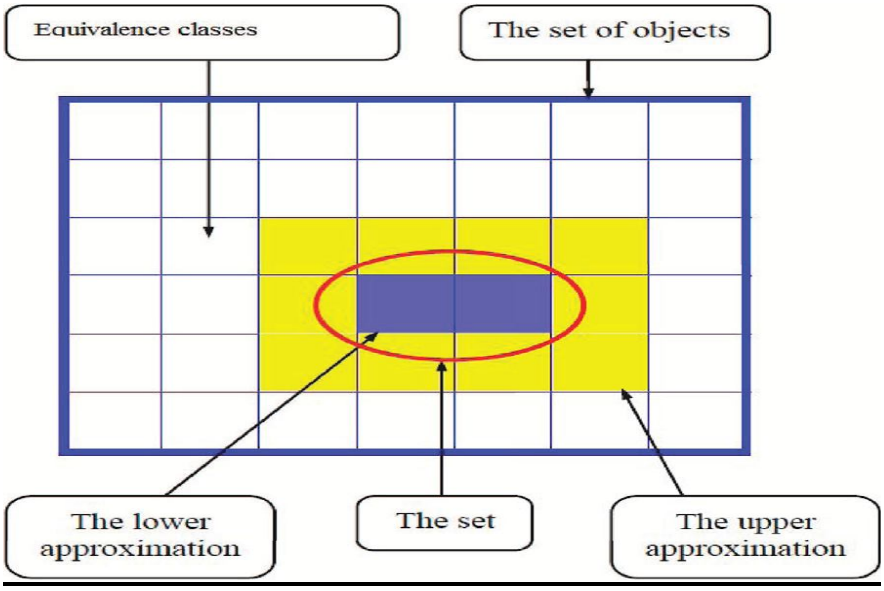
(vi)[$L_R(X \cap Y) \subseteq L_R(X) \cap L_R(X)]$;

(vii) If [$X \subseteq Y$, then $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)]$.

(viii)[$U_R(X^c)] = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.

(ix)[$[U_R U_R(X)] = [L_R U_R(X) = U_R(X)]$;[$L_R L_R(X) = U_R L_R(X) = L_R(X)]$;

i.e. The following diagram shows that the relations among the difference types of sets above .



2.3. Theorem :- [Lellis and Carmel,2013]

Let(U) be the universe, R be an equivalence relation on U and $X \subseteq U$, $\tau_R(X) = \{\emptyset, U, L_R(X), U_R(X), B_R(X)\}$. Then satisfies the following axioms :

;$\emptyset$, $U \in \tau_R(X)$}**(i)** }

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus[$\tau_R(X)$] is a topology called nano topology on U with respect to X . We call $(U, \tau_R(X))$ is a nano topological space . The elements of[$\tau_R(X)$] are called nano open , $[\tau_R(X)]^c$ is called the dual nano topology of[$\tau_R(X)$].

2.4. Example:

Let $(U) = \{a, b, c, d\}$ and $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ where $(X) = \{a, b\}$ with the nano topology[$\tau_R(X)] = \{\emptyset, U, \{a\}, \{a, b, d\}, \{b, d\}\}$,then the nano closed sets are($\emptyset$, U0), $\{b, c, d\}$, $\{c\}$ and $\{a, c\}$.

2.5. Definition:- [Lellis M.T.and Carmel R. , 2013]

Let $(U, \tau_R(X))$ be a nano topological space with respect to(X) and if ($A \subseteq U$), then the nano

interior of(A) is defined as the union of all nano open subsets of A and it is denoted by $Nint(A)$.

Also, the nano closure of A is defined as the intersection of all nano closed sets containing (A) and it is denoted by $Ncl(A)$. Now, we say that A is nano α-open set , if ($A \subseteq$)$Nint(Ncl(Nint(A)))$. The collection of all nano α-open subsets of U will be denoted by $N_\alpha O(U, X)$ and its need not to be the

forms a nano α-topology on U. The complement of the nano α-open sets is nano α-closed subset

i.e.(A) is nano α-closed set , if{ $Ncl(Nint(NclA)) \subseteq A$} .

2.6. Remark:

The intersection of two a nano α-open subsets need not to be a nano α-open, as the following example shows:

Let $(U) = \{a, b, c\}$, with$(\frac{U}{R}) = \{\{a\}, \{b, c\}\}$ and $(X) = \{a, c\}$.

Then nano topology on(U) is $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c\}\}$, then the sets $\{a, c\}$ and $\{b, c\}$ are nano(α-)
open sets in(U), but their intersection is not nano α-open .

Also, the intersection of two a nano α-closed sets need not to be a nano α-closed as the following example shows:

Let($U) = \{a, b, c, d, e\}$, with $U/R = \{\{a, b\}, \{c, e\}, \{d\}\}$ and $(X) = \{a, d\}$.

$\tau_R(X) = \{\emptyset, U, \{d\}, \{a, b, d\}, \{a, b\}\}$, the sets $\{c, e\}$ and $\{c, d, e\}$ are nano α-closed sets but their intersection is not nano α-closed.

i.e. [$N_\alpha O(U, X)$] is supra nano topology on(U).

2.7. Definition:

A nano topological space $(U, \tau_R(X))$ is called nano α-multiplicative , if arbitrary intersection of nano α-open (nano α-closed) sets of U is also nano α-open (nano α-closed) set and it is denoted by (IN_α-space).

If $(U, \tau_R(X))$ be an IN_α- space, then the[$N_\alpha O(U, X)$] becomes nano topology on(U) and called nano α-topology denoted by $\tau_R^\alpha(X)$.

2.8. Definition :- [Revathy and Ilango, 2015]

Let $(U, \tau_R(X))$ be a nano topological space with respect to X and if $A \subseteq U$, then the nano α -interior of A is defined as the union of all nano α-open subsets of A and it is denoted by $Nint_\alpha(A)$. Also , the nano α-closure of A is defined as the intersection of all nano α-closed sets containing A and it is denoted by $Ncl_\alpha(A)$.

Also , the nano α-closure of A is defined as the intersection of all nano α-closed sets containing A and it is denoted by $Ncl_\alpha(A)$.

For any subset A of a nano topological space $(U, \tau_R(X))$, the following are satisfied:

(i)[$Ncl_\alpha(\emptyset) = \emptyset$] .

(ii)[$Ncl_\alpha(A)$] need not to be a nano α-closed for all($A \subseteq U$) .

(iii){ $Ncl_\alpha\left(Ncl_\alpha(A)\right)$} = [$Ncl_\alpha(A)$] for all($A \subseteq U$) .

(iv)[$Ncl(A) \subseteq Ncl_\alpha(A)$] for all($A \subseteq U$) .

(v) If($A \subseteq B$), then[$Ncl_\alpha(A) \subseteq Ncl_\alpha(B)$] for all $(A, B \subseteq U)$.

(vi) $Nint_\alpha(A)$ is a nano α-open for all $(A \subseteq U)$.

2.9. Remark :- [Lellis and Carmel, 2013]

(i) Every nano open subset of U is nano α-open.

(ii) Every nano closed subset of U is nano α-closed.

(iii) If[$U_R(X) = U$], then $\tau_R(X) = \tau_R^\alpha(X)$.

The converse (i , ii) of the remark above is not true as the following example shows:

2.10. Example:

Let($U) = \{a, b, c, d, e\}$, with$(\frac{U}{R}) = \{\{a,b\},\{c,e\},\{d\}\}$ and $X = \{a, d\}$.
$\tau_R(X) = \{\emptyset, U, \{d\}, \{a,b,d\}, \{a,b\}\}$, the nano closed sets are $\emptyset$, U, $\{a, b, c\}$, $\{c\}$ and $\{c, d\}$.
Clearly, $\tau_R^{\alpha}(X) = \tau_R(X) \cup \{\{a,b,c,d\}, \{a,b,d,e\}\}$.

2.11. Theorem:

If A and B be a nano α-open sets in a space $(U, \tau_R(X))$, then $A \cup B$ it is clear nano α-open in U by **Proof:** Since A and B are α-open sets,($A \subseteq Nint(Ncl(Nint(A)))$ and $B \subseteq Nint(Ncl(Nint(B)))$.
Then:
$(A \cup B \subseteq)$ $Nint(Ncl(Nint(A))) \cup Nint(Ncl(Nint(B))) \subseteq$
$Nint(Ncl(Nint(A)) \cup Ncl(Nint(B))) = Nint(Ncl(Nint(A \cup B)))$
, we have $(A \cup B)$ is nano α-open in(U).

2.12. Theorem:

If A and B be a (nano open) nano α-open sets respectively in an IN_{α}-space $(U, \tau_R(X))$ respectively, then $A \cap B$ is nano α-open set in U.

Proof: Since A be a nano open set and B be a nano α-open, we have A^c is nano closed and B^c is nano α-closed in U. By remark (2.9.ii), then A^c and B^c are nano α-closed sets in U.

Then
$[Ncl_{\alpha}Nint_{\alpha}Ncl_{\alpha}(A^c)] \cup [Ncl_{\alpha}Nint_{\alpha}Ncl_{\alpha}(B^c)] =$
$Ncl_{\alpha}[(Nint_{\alpha}Ncl_{\alpha}(A^c)) \cup (Nint_{\alpha}Ncl_{\alpha}(B^c))]$
$= Ncl_{\alpha}Nint_{\alpha}[(Ncl_{\alpha}(A^c)) \cup (Ncl_{\alpha}(B^c))] = Ncl_{\alpha}(Nint_{\alpha}(Ncl_{\alpha}(A^c \cup B^c))) \subseteq A^c \cup B^c = (A \cap B)^c$ i. e. $(A \cap B)^c$ is nano α-closed set. This implies that $A \cap B$ is nano α-open.
Similarity, if A and B be a (nano closed) nano α-closed sets respectively in an IN_{α}-space above, we have($A \cap B$) is nano α-closed set in(U).

2.13. Corollary:

If $(A \subseteq Y)$ and Y be nano open in an(IN_{α}-space $(U, \tau_R(X))$, then($A \cap Y$) is nano α-open set in Y.

Proof: Follows from theorem (2.12).

2.14. Definition:

A nano topological space $(U, \tau_R(X))$ is said to be nano α-normal, if for each pair of disjoint nano α-closed sets(A) and(B) of(U) , there are disjoint nano α-open sets (W) and(V) such that($A \subseteq W$) and($B \subseteq V$).

2.15. Theorem:

A nano topological space $(U, \tau_R(X))$ is nano α-normal if and only if for every nano α -closed set(F) in(U) and each nano α-open sets (W) in(U) such that $(F \subseteq W)$, there is a nano α-open set($V \subseteq U$)such that [$F \subseteq W \subseteq Ncl_{\alpha}(W) \subseteq V$].

Proof: Straightforward.

2.16. Definition:

A family $\{A_\lambda : \lambda \in \Lambda\}$ of subsets of a nano topological space $(U, \tau_R(X))$ is said to be a nano α-
locally finite family, if for each point $u \in U$, there is nano α-open set W of U such that the set
$\{\lambda \in \Lambda : W \cap A_\lambda \neq \emptyset\}$ is finite .

2.17. Definition:

Let $(U, \tau_R(X))$ be a nano topological space and $\{G_\lambda : \lambda \in \Lambda\}$ be a cover of U, then $\{V_\sigma : \sigma \in \Gamma\}$
is called a nano α-refinement of $\{G_\lambda : \lambda \in \Lambda\}$ if, and only if ,it's a cover of U and for each $\sigma \in \Gamma$, there is $\lambda \in \Lambda$ such that $V_\sigma \subseteq G_\lambda$.

3. Nano α-Covering Dimension:

In this section we will introduce necessary definition and every nano topological space is an IN_α- space. Let U be a set and $\mathcal{V}$ be a non-empty family of subsets of U, the order of is the largest integer n such that$(\bigcap_{i=1}^{n+1} V_i \neq \emptyset)$for some$(n+1)$ members $(V_1, V_2, V_3, \dots\dots, V_{n+1})$ of $\mathcal{V}$ [Pears R. 1975].

3.1. Definition:

The nano α-covering dimension of a nano topological space $(U, \tau_R(X))$ is the least positive integer n such that every finite nano α-open cover of U has a nano α-open refinement of order$\leq n$.

We shall denote the nano α-covering dimension of U by $Ndim_\alpha U$.

(i) $(Ndim_\alpha U = -1)$, whenever $(U = \emptyset)$,

(ii)$(Ndim_\alpha U = n)$ if and only if $(Ndim_\alpha U \leq n)$, but it is no $(Ndim_\alpha U \leq n-1)$.

(iii)$(Ndim_\alpha U = \infty)$ if and only if $(Ndim_\alpha U = n)$ for no n .

3.2. Example:

Let$(U) = \{a, b, c, d\}$ and $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ where $X = \{a, b\}$ with the nano topology $\tau_R(X) = \{\emptyset, U, \{a\}, \{a, b, d\}, \{b, d\}\}$, clearly that $\tau_R^\alpha(X) = \tau_R(X)$ and$(Ndim_\alpha U = 0)$

3.3. Theorem:

Let $(U, \tau_R(X))$ be a nano topological space , if $dim_\alpha U = 0$, then it is nano α-normal.

Proof: Let(F_1, F_2) be two nano α-closed sets in U, with$(F_1 \cap F_2 = \emptyset)$. Then $\{F_1^c, F_2^c\}$ is a nano (α-open cover) of(U) , since$(Ndim_\alpha U = 0)$,then it is has nano α-open refinement $\{H_1, H_2\}$ of order 0 .

Hence there are nano α-open sets H_1 and H_2 such that$(H_1 \cap H_2 = \emptyset)$ with $(H_1 \cup H_2 = U)$, so that $(H_1 \subseteq F_1^c$, $H_2 \subseteq F_2^c$).Thus $(F_1 \subseteq H_1^c = H_2)$ and $(F_2 \subseteq H_2^c = H_1)$, this is complete prove.

3.4. Theorem:

Let $(U, \tau_R(X))$ be a nano topological space , A be a nano open set of U, then $dim_\alpha A \leq dim_\alpha U$.

Proof: Suppose $(Ndim_\alpha U \leq n)$. Let $(\mathcal{H} = \{H_1, H_2, \dots, H_k\})$ be a nano α-open cover of A . Now
for each $(i = 1, 2, \dots., k)$, $(H_i = V_i \cap A)$, where (V_i) is nano (α-open) set in) U). Since $dim_\alpha U \leq n)$,
then a nano α-open cover of U has a nano α-open refinement $\mathcal{F}$ in U of order $\leq n$.

Let $\mathcal{L}) = \{ F_i \cap A : F_i \in \mathcal{F} \}$ be a nano α-open refinement of $\mathcal{H}$ of order$\leq n$ So $Ndim_\alpha A \leq n$.

Recall that a family $(\mathcal{A} = \{A_\lambda : \lambda \in \Lambda\})$ is a reduce of a family$(\mathfrak{C} = \{D_\lambda : \lambda \in \Lambda\})$, if($D_\lambda \subseteq A_\lambda$) for each($\lambda \in \Lambda$) [Dugundji , 1966].

3.5. Theorem:

Let $(U, \tau_R(X))$ be a nano topological space , the following statements are equivalent:

(i)($Ndim_\alpha U \leq n$).

(ii) Every finite nano α-open cover of U can be reduced to a nano α-open cover of U of order $\leq n$.

Proof: $(i) \Rightarrow (ii)$ Suppose that($Ndim_\alpha U \leq n$) . Let($\wp = \{G_1, G_2, \dots, G_k\}$) be a nano α-open cover of(U) . Then($\wp$) has a nano α-open refinement $(\mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \})$ which it is of (order $\leq n$. $\forall\ V_\lambda \in \mathcal{V}$, $\exists\ G_i \in \wp$) such that $(V_\lambda \subseteq G_i$, $i = 1, 2, \dots, k$. Let $W_i = \bigcup_{\lambda \in \Lambda} V_\lambda \subseteq G_i)$ thus $(\mathcal{W} = \{W_i : i = 1, 2, \dots, k\})$ is a nano α-open reduction of $\wp$ which it is nano α-open cover of(U of order $\leq n$) .

$(ii) \Rightarrow (i)$, suppose that the condition is holds and $\mathcal{M} = \{M_1, M_2, \dots, M_k\}$ be a finite nano α-open covering of U . Since every nano α-open reduction of $\mathcal{M}$ which it is nano α-open covering of U is nano α-open refinement. Thus $\mathcal{M}$ has a nano α-open refinement which it is of order $\leq n$. The other result holds immediately.

3.6. Corollary:

Let $(U, \tau_R(X))$ be a nano topological space, the following statements are equivalent:

(i) Every finite nano α-open cover of U has a nano α-open refinement of (U of order $\leq n$) .

(ii) Every finite nano α-open cover of U can be reduced to a nano α-open cover of U order $\leq n$.

Proof: It is easy to show that $(Ndim_\alpha U \leq n)$. Now apply theorem (3.5).

3.7. Theorem:

Let $(U, \tau_R(X))$ It is known I guess be a nano topological space , the following statements are equivalent:

(i)($Ndim_\alpha U \leq n$).

(ii) Every nano α-open cover of(U with $n + 2$) sets has a reduction which a nano α-open cover of(U with $n + 2$) sets and empty intersection .

Proof: $(i) \Rightarrow (ii)$ Suppose that($Ndim_\alpha U \leq n$) , then every finite nano α-open cover of U has a (nano α-open) reduction which cover of(order $\leq n$) by theorem (3.5) . The number of the elements of this cover is ($n + 2$) , then the number its reduction is also($n + 2$) with(order $\leq n$) . The intersection is not empty for at most ($n + 1$) elements , i.e. the intersection of all($n + 2$)with (order $\leq n$) is empty .

$(ii) \Rightarrow (i)$ let $\mathfrak{R} = \{R_1, R_2, \dots, R_k\}$ be a finite nano α-open cover of(U). We shall prove that this cover has a reduction of order $\leq n$. Suppose that order $\mathfrak{R} > n$, i.e.($\exists\ R_1, R_2, \dots, R_{n+1}, R_{n+2}$) such that ($\bigcap_{i=1}^{n+2} R_i \neq \emptyset$). Put ($R^* = R_{n+2} \cup R_{n+3} \cup \dots \cup R_k$) . Then $\{ R_1, R_2, \dots, R_{n+1}, R^* \}$ be a finite nano α-open cover of U with $n + 2$ sets has a reduction $\{P_1, P_2, \dots, P_{n+1}, P^*\}$ which it is a nano α-open cover of U and empty intersection .

$\{ P_1, P_2, \dots, P_{n+1}, P^*, , P^* \cap R_{n+2}, \dots, P^* \cap R_k\}$ is a nano α-open reduction of $\mathfrak{R}$ of order $\leq n$.

The intersection of the first $n + 1$ sets of this cover is not empty intersection . But the intersection of all the sets of this cover has empty intersection . By continuous this operation finite numbers at time , we will get the reduction with $n+2$ sets and empty intersection . This completes the proof .

3.8. Definition:

Let $(U, \tau_R(X))$ and $(U', \tau_{R'}(Y))$ be a nano topological spaces . Then a mapping($f: U \longrightarrow U'$) is said to be a nano α-continuous on(U) , if the inverse image of every nano α-open set in U' is nano(α-open in U).

3.9. Definition:

A nano(α-continuous mapping($f: U \longrightarrow U'$) is said to have a vague order at most n , if for each finite nano (α-open)cover$\{M_1, M_2, \dots, M_k\}$ of(U'),there is a finite (nano α-open) cover$\{N_1, N_2, \dots, N_k\}$
of U such that :

(i) [$f(N_i) \subseteq M]_i$ for all($i = 1, 2, \dots, k$).

(ii)($f^{-1}(q) \cap N_i \neq \emptyset$) holds for at most($n+1$) values of i and for every ($q \in U'$).

3.10. Theorem:

Let($f: U \longrightarrow U'$)and ($g: U' \longrightarrow U''$) be two a(nano α-continuous), onto mappings, if the nano α-vague order of g is at most n , then the nano α- vague order of gof is at most n.

Proof: Suppose the condition is holds and let [$\mathcal{W} = \{W_1, W_2, \dots, W_k\}$] be a finite nano α-open cover of $(U')'$. Then there is $\{\mathcal{V} = \{V_1, V_2, \dots, V_k\}$ be a finite nano α-open cover of (U) such that($g(V_i) \subseteq W_i \quad \forall i = 1, 2, \dots, k$) and ($g^{-1}(q) \cap V_i \neq \emptyset$) for at most($n+1$) values of($i \ \forall q \in U''$). Then($f^{-1}(\mathcal{V})$ is a finite nano α-open cover of(U) , where $gof(f^{-1}(V_i)) = g(V_i) \subseteq W_i \ \forall i = 1, 2, \dots, k$).Suppose that

$(gof)^{-1}(q) \cap f(V_i) \neq \emptyset \ \forall i = 1, 2, \dots, n+2 \Rightarrow \exists x_i \in V_i \ni f^{-1}(x_i) \in (gof)^{-1}(q) \Rightarrow$

$f(f^{-1}(x_i)) \in f(f^{-1}(g^{-1}(q))) \subseteq g^{-1}(q) \ \forall i = 1, 2, \dots, n+2 \Rightarrow g^{-1}(q) \cap V_i \neq \emptyset \ \forall i = 1, 2, \dots, n+2$,

which is a contradiction.

3.11. Theorem:

Let ($f: U \longrightarrow U'$) and ($g: U' \longrightarrow U''$) be two a (nano α-continuous), onto mappings and(f) is a(nano α-open) , if the(nano α- vague order) of gof is at most (n) , then the nano α- vague order of(g) is at most(n) .

Proof: suppose that the(nano α- vague) order of gof is at most n. We shall prove that the nano α- vague order of g is at most n .

Let ($\mathcal{W} = \{W_1, W_2, \dots, W_k\}$) be a finite nano (α- open covering) of (U''). Then there is ($\mathcal{V} = \{V_1, V_2, \dots, V_k\}$) be a finite nano (α- open covering)of(U) such that $(gof)(V_i) \subset W_i \ \forall i = 1, 2, \dots, k$, and $(gof)^{-1}(q) \cap V_i \neq \emptyset$ hold at most n+1 values of I and for every $q \in U''$. Now since f is nano α-open ,then the collection $\{f(V_i): i = 1, 2, \dots, k\}$ is a finite nano α-open covering of U' satisfying: $g(f(V_i)) = (gof)(V_i) \subset W_i$,$i = 1, 2, \dots, k$. To prove $g^{-1}(q) \cap f(V_i) \neq \emptyset$ for at most n+1 vales of i and for every $q \in U''$. Suppose for some $q \in U''$ there is $x_i \in V_i$ such that $f(x_i) \in g^{-1}(q)$ for n+2 values of I that is $(f^{-1}(f(x_i)) \in f^{-1}(g^{-1}(q))$ then ($x_i \in (gof)^{-1}(q)$ for n+2 values of i .This means that $(gof)^{-1}(q) \cap V_i \neq \emptyset$) hold for at most n+2 values of i which is contradiction with theorem (3.10). Thus the nano α-vague order of g is at most n .

3.12. Theorem:

Let $(U, \tau_R(X))$ be a nano topological space, the following statements are equivalent:

(i)($Ndim_\alpha U \leq n$) .

(ii) The(nano α- vague) order of the identity mapping(I_U of U) is at most n .

(iii) The (nano α- vague) order of every(nano α-continuous) onto mapping with range(U) is at most(n) .

Proof: (i ⇒ ii) the following result follows immediately from definition (3.9) and we obtain (ii ⇒ iii) by theorem (3.10) . Also , (iii ⇒ ii) by theorem (3.11) .

Now, we prove that (ii ⇒ i) . Let($\mathcal{H} = \{ H_1 , H_2, \dots , H_k \}$be a nano α-open cover of U. Since I_U is nano α- vague order at most n . Then($\xi = \{E_1 , E_2 , \dots , E_k\}$ be a nano α-open cover of U such that :

•($I_U (E_i) = E_i \subseteq H_i \ \forall i = 1 , 2 , \dots , n + 1)$.

•($I_U^{-1}(q) \cap E_i \neq \emptyset \ \forall i = 1 , 2 , \dots , n + 1)$. i.e. order($\xi \leq n$) , we get the desired result .

3.13. Theorem:

Let($Ndim_\alpha U = 0$) and ($f: U \longrightarrow U'$) be a nano α-continuous , onto mapping with($f^{-1}(q)$ contains
at most($n + 1$) points of(U) for all ($q \in U'$), then the(nano α- vague) order of(f) is at most n .

Proof: Let($\mathfrak{R} = \{ R_1, R_2 , \dots , R_k\}$) be a finite(nano α-open cover) of(U'). Then ($f^{-1}(\mathfrak{R})$ is a finite(nano α-open) cover of(U) . Since ($Ndim_\alpha U = 0$) , then there exist ($\mathcal{M} = \{M_1, M_2, \dots , M_k\}$)be a disjoint nano α-open refinement which cover of(U) with $M_i \subseteq f^{-1}(R_i) \Rightarrow f(M_i) \subseteq R_i \ \forall i = 1, 2, \dots , k)$.
Now since($f^{-1}(q)$ contains at most($n + 1$) points of(U), then($f^{-1}(q) \cap M_i \neq \emptyset$) for at most $n + 1$) values of i for all($q \in U'$).

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