# On (T,L)- Identification Function

**Aqeel Ketab Al-Khafaji**
*Department of Mathematics, Collage of education For Pure Sciences*
*University of Babylon*
aqeelketab@gmail.com

## Abstract:

In this paper, we study and introduce the notion of $(T,L)$-***identification function*** which is a function $f$ from a topological space $(X,\tau)$ to another topological space $(Y,\sigma)$ and $T,L$ be two operators associative with $\tau,\sigma$ respectively, then $f$ is called $(T,L)$-identification function if and only if $f$ is onto and $V$ is $L$-open set in $Y$ if and only if $f^{-1}(V)$ is $T$-open set in $X$.

**Keywords:** Identification function, $(T,L)$-identification function, Operator, $T$-open set,(T,L)-continuous function, $(T,L)$-contra continuous function.

**الخلاصة:**

في هذا البحث قدمنا مفهوم دالة الهوية من النوع $(T,L)$ والتي هي الدالة $f$ من الفضاء التبولوجي $(X,\tau)$الى فضاء تبولوجي اخر $(Y,\sigma)$ وكل من $T,L$ مؤثر على $\tau,\sigma$ على التوالي فنقول ان الدالة $f$ هي دالة هوية من النوع $(T,L)$ اذا وفقط اذا كانت شاملة والصورة العكسية للمجوعة $V$ تكون $T$−open في $X$ اذا وفقط اذا كانت المجموعة $V$ هي $L$−open في $Y$.

**الكلمات المفتاحية:** دالة الهوية من النوع $(T,L)$، المجموعة المفتوحة من النوع $T$، الدالة المستمرة من النوع $(T,L)$، الدالــة ضد المستمرة من النوع $(T,L)$.

## 1. Introduction

In 1966 HS [ **Hu,1966]** introduced the concept of an identification function (Definition 3.1) and in 1997 Al-K utaibi [**Alkutabibi,1997**] introduced the notion of semi-identification and some other types of identification. In this paper we introduce the notion of $(T,L)$-identification function using the concepts of an operator associated to a topology (Definition 2.1) and $T$-open set (Definition 2.2) was introduced by Kasahara **[ Kasahara ,1979]** and Ogata [ **Ogata, 1991**] respectively .

## 2. Fundamental Concepts

In this section we recall the basic definitions needed in this work.

**<u>2.1. Definition</u>** [ **Kasahara, 1979]**

Let $(X,\tau)$ be a topological space, let $P(X)$ be the power set of $X$. Let $T:\tau \longrightarrow P(X)$, we say that $T$ is an operator associated with the topology $\tau$ on $X$ if $U \subseteq T(U)$ for every open set $U$ in $X$ .

We denote by $(X,\tau,T)$ as a topological space with an operator $T$ associated with the topology $\tau$ and we will call it operator topological space.

**<u>2.2. Example</u>** [ **Mustafa, 2014**]

Let $(X,\tau)$ be a topological space, Let $T:\tau \longrightarrow P(X)$ be defined as follows, Let $T(A) = Int\ cl(A), A \subseteq X$, where Let $Int(A) = interior\ of\ A, cl(A) = closure\ of\ A$. Notice that if Let $U$ is open in $X$, then $U \subseteq Int\ cl(U) = T(U)$. So that $T$ is an operator associated with the topology $\tau$ on $X$ and the triple $(X,\tau,T)$ is an operator topological space. If $T$ is the identity operator $(T(A) = A)$ then the triple $(X,\tau,T)$ will reduces to $(X,\tau)$ , so that the operator topological space is the ordinary topological space.

**2.3. Definition** [ Ogata,1991]

Let $(X,\tau,T)$ be an operator topological space, A subset $A$ of $X$ is said to be $T$-open set if for each $x \in A$, there exists an open set $U$ containing $x$ such that $T(U) \subset A$. A subset $B$ is said to be $T$-closed set if it its complement is $T$-open set.

It is clear that every $T$-open set is open, but the converse is not necessarily true as shown in the following example :

**2.4. Example**

Let $(X,\tau,T)$ be an operator topological space, where $X=\{a,b,c\}, \tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $T:\tau \longrightarrow P(X)$ is a closure operator. Consider $A=\{a,b,\}$, we claim that $A$ is open but not $T$-open. Now, let $a \in A$ also $a \in \{a\}$ which is open in $X$. Let $W=\{a\}$ then $T(W)=cl(W)=cl(a)=\{a,c\}$, that means $a \in W=\{a\} \subseteq T(W)=\{a,c\}$. But $T(W)=\{a,c\} \not\subset A=\{a,b\}$, which means that $A$ is not $T$-open .

**2.5. Definition** [ Ogata. 1991]

Let $(X,\tau,T)$ and $(Y,\sigma,L)$ be two operator topological spaces. We say that a function $f:(X,\tau,T) \longrightarrow (Y,\sigma,L)$ is (T,L)- continuous function if for each point $x \in X$ and every open set $V$ *in* $Y$ containing $f(x)$ ther exists an open set $U$ *in* $X$ containing $x$ such that $f(T(U)) \subseteq L(V)$ .

**2.6. Theorem** [ Mustafa, 2014]

Let $f:(X,\tau,T) \longrightarrow (Y,\sigma,L)$ is $(T,L)$- continuous function, then the inverse image of each $L$-open set in $Y$ is $T$-open set in $X$.

**Proof:** Let $W$ be $L$-open set in $Y$, to prove that, $f^{-1}(W)$ is $T$-open set in $X$.Let $x \in f^{-1}(W)$ then, $f(x) \in W$ but $W$ is $L$-open set then there exists an open set $V$ in $Y$ such that $f(x) \in V \subseteq L(V) \subseteq W$. Now $f$ is $(T,L)$-continuous at $x$ then there exists an open set $U$ in $X$ containing $x$ such that, $f(T(U)) \subseteq L(V)$ so, $T(U) \subseteq f^{-1}(L(V))$.

Now,$x \in U \subseteq T(U) \subseteq f^{-1}(L(V)) \subseteq f^{-1}(W)$that is,$x \in U \subseteq T(U) \subseteq f^{-1}(W)$ which means that $f^{-1}(W)$ is $T$-open.

**2.7. Definition** [ Ogata, 1991]

Let $(X,\tau,T)$ and $(Y,\sigma,L)$ be two operator topological spaces. We say that a function $f:(X,\tau,T) \longrightarrow (Y,\sigma,L)$ is $(T,L)$-contra continuous function if $f^{-1}(A)$ is $T$-closed set in $X$ for all $L$-open set $A$ in $Y$.

## 3.The Main Results

**3.1. Definition** [ Hu, 1966]

Let $(X,\tau),(Y,\sigma)$ be two topological spaces. A function $f:(X,\tau) \longrightarrow (Y,\sigma)$ is called ***identification function*** if and only if (i) $f$ is onto and (ii) $V$ is open set in $Y$ if and only if $f^{-1}(V)$ is an open set in $X$ .

**3.2. Example**

Let $(X,\tau)$and $(Y,\sigma)$be tow topological spaces where $X=\{1,2,3\}, \tau=\{\phi,X,\{1\},\{2\},\{1,2\}\}$ and $Y=\{a,b,c\}$, $\sigma=\{\phi,y,\{b\},\{c\},\{b,c\}\}$ , let $f:(X,\tau) \longrightarrow (Y,\sigma)$ be define by $f(1)=b, f(2)=c, f(3)=a$ , then $f$ is identification function .

**3.3. Definition**

A function $f$ from the operator topological space $(X,\tau,T)$ to another topological space $(Y,\sigma,L)$ is called $(\boldsymbol{T},\boldsymbol{L})$***-identification function*** if and only if $f$ is onto and $V$ is $L$-open set in $Y$ if and only if $f^{-1}(V)$ is $T$-open set in $X$.

**3.4. Example**

Let $(X,\tau,T)$ and $(Y,\sigma,L)$ be two operator topological spaces, where $X=\{1,2,3\}, \tau=\{\phi.X,\{1\}.\{2\},\{1,2\}\}, T:\tau\longrightarrow P(X)$ such that

$$T(A)=\begin{cases}A & if\ A\neq\{2\}\\ \{2,3\} & if\ A=\{2\}\end{cases}$$ and

$Y=\{a,b,c\}$, $\sigma=\{\phi.Y,\{a,b\}\}, L:\sigma\longrightarrow P(X)$ $L(A)=A$. Now the set of all $T$-open set $=\{\phi.X,\{1\},\{1,2\}\}$ and the set of all $L$-open set $=\{\phi.Y,\{a,b\}\}$ , let $f:(X,\tau,T)\longrightarrow(Y,\sigma,L)$ be define by $f(1)=a, f(2)=b, f(3)=c$, then $f$ is $(T,L)$-identification function.

**3.5. Remark**

(i) Observe that if $T$ and $L$ are the identity operators on $X$ and $Y$ respectively, then the Definition 2.3 is reduced to the definition *identification function* (Definition 2.1).

(ii) Identification function and $(T,L)$-identification function are independent. And the following two examples will show that.

**3.6. Example**

Let $X=Y=\{a,b,c\}, \tau=\{\phi.X,\{a\}.\{b\},\{a,b\}\}$, and $T:\tau\longrightarrow P(X)$ defined as $T(A)=\begin{cases}A & if\ A\neq\{b\}\\ \{b,c\} & if\ A=\{b\}\end{cases}$, and $L:\tau\longrightarrow P(X)$ defined as $L(A)=A$, then the set of all $T$-open set $=\{\phi.X,\{a\},\{a,b\}\}$ and the set of all $L$-open set $=\{\phi.Y,\{a\}.\{b\},\{a,b\}\}$. Let $f:(X,\tau,T)\longrightarrow(Y,\tau,L)$ is identity function, then $f$ is identification function but not $(T,L)$-identification function [take $W=\{b\}$ is $L$-open set in $Y$, then $f^{-1}(W)=W=\{b\}$ which is not $T$-open set in $X$] .

**3.7. Example**

Let $(X,\tau,T)$ and $(Y,\sigma,L)$ be two operator topological spaces, where $X=\{a,b,c\}, \tau=\{\phi.X,\{a\}.\{b\},\{a,b\}\}, T:\tau\longrightarrow P(X)$ such that

$$T(A)=\begin{cases}A & if\ A\neq\{b\}\\ \{b,c\} & if\ A=\{b\}\end{cases}$$ and

$Y=\{a,b,c\}$, $\sigma=\{\phi.Y,\{c\}\}, L:\sigma\longrightarrow P(X)$ such that $L(A)=\begin{cases}A & if\ A\neq\{c\}\\ \{a,c\} & if\ A=\{c\}\end{cases}$.

Now the set of all $T$-open set $=\{\phi.X,\{a\},\{a,b\}\}$ and the set of all $L$-open set $=\{\phi.Y,\}$, let $f:(X,\tau,T)\longrightarrow(Y,\sigma,L)$ is identity function, then $f$ is$(T,L)$-identification function but not identification function[take $V=\{c\}$ *is open set in* $Y$, *then* $f^{-1}(V)=V=\{c\}$ which is not open set in $X$ ] .

**3.8. Theorem**

An onto function $f:(X,\tau,T)\longrightarrow(Y,\sigma,L)$ is $(T,L)$-identification function if and only if $W$is $L$-closed set in $Y$ if and only if $f^{-1}(W)$ is $T$-closed set in $X$.

**Proof:** $(\Longrightarrow)$ Let $W$ be $L$-closed set in $Y$, then $W^C$ is $L$-open set in $Y$, but $f$ is $(T,L)$-identification function, then $f$ is onto and $f^{-1}(W^C)=(f^{-1}(W))^C$ is $T$-open set in $X$, thus $f^{-1}(W)$ is $T$-closed in $X$. Similarly, if $f^{-1}(W)$ is $T$-closed set in $X$, then$(f^{-1}(W))^C=f^{-1}(W^C)$ is $T$-open set in $X$, and since $f$ is $(T,L)$-identification function , then $W^C$ is $L$-open set in $Y$ and hence $W$ is $L$-closed set in $Y$.

$(\Longleftarrow)$ Let $W$ be $L$-open set in $Y$, then $W^C$ is $L$-closed set in $Y$, then $f^{-1}(W^C)=(f^{-1}(W))^C$ is $T$-closed set in $X$. That is $f^{-1}(W)$ is $T$-open set in $X$.

Similarly, if $f^{-1}(W)$ is $T$-open set in $X$, then $(f^{-1}(W))^C = f^{-1}(W^C)$ is $T$-closed set in $X$, and then $W^C$ is $L$-closed set in $Y$ and hence $W$ is $L$-open set in $Y$. Since $f$ is onto then $f$ is $(T,L)$-identification function.

**3.9. Definition**

A function $f$ from the operator topological space $(X,\tau,T)$ to another topological space $(Y,\sigma,L)$ is called $(T,L)$- ***open (closed)*** if and only if the image of every $T$-open ($T$-closed) set in $X$ is $L$-open ($L$- closed) set in $Y$.

**Example 3.10**

Let $(X,\tau,T)$ and $(Y,\sigma,L)$ be two operator topological spaces, where $X = \{1,2,3\}, \tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}\}$ and $Y = \{a,b,c\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ , where $T$ and $L$ are defined as follows: $T(U) = L(U) = U$ , then the set of all $T$-open set $= \{\phi, X, \{1\}, \{2\}, \{1,2\}\}$ and the set of all $L$-open set $= \{\phi, Y, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ let $f: (X,\tau,T) \longrightarrow (Y,\sigma,L)$ be define by $f(1) = a, f(2) = b, f(3) = c$, then $f$ is $(T,L)$-open function.

**3.11. Theorem**

If $f: (X,\tau,T) \longrightarrow (Y,\sigma,L)$ is, $(T,L)$- open, onto and $(T,L)$- continuous function, then $f$ is $(T,L)$-identification function.

**Proof:** Let $W$ be a subset of $Y$, such that $f^{-1}(W)$ is $T$-open set in $X$. Since $f$ is onto, we have $f\left(f^{-1}(W)\right) = W$ , since $f^{-1}(W)$ is $T$-open set in $X$ and $f$ is $(T,L)$- open, then $W$ is $L$-open set in $Y$. Now $W$ is $L$-open set in $Y$, since $f$ is $(T,L)$- continuous function, then $f^{-1}(W)$ is $T$-open set in $X$. That is $f$ is $(T,L)$-identification function.

**3.12. Corollary**

A function $f: (X,\tau,T) \longrightarrow (Y,\sigma,L)$ is $(T,L)$-identification function, if it is, $(T,L)$- closed, onto and $(T,L)$- continuous function.

**Proof:** Clear ■

**3.13. Theorem**

Let $(X,\tau,T)$, $(Y,\sigma,L)$ and $(Z,\rho,K)$ are operator topological spaces, then if: $f: (X,\tau,T) \longrightarrow (Y,\sigma,L)$ is $(T,L)$-identification function, and $g: (Y,\sigma,L) \longrightarrow (Z,\rho,K)$ is $(L,K)$-identification function,then $gof: (X,\tau,T) \longrightarrow (Z,\rho,K)$ is $(T,K)$-identification function .

**Proof:** Clear that, the composition of two onto function is onto. Now , let $W$ be any $K$-open set in $Z$, since $g$ is $(L,K)$-identification function, then $g^{-1}(W)$ is $L$-open set in $Y$, and $f$ is $(T,L)$- identification function, we have $f^{-1}\left(g^{-1}(W)\right) = (gof)^{-1}(W)$ is $T$-open set in $X$ ■

Similarly, if $(gof)^{-1}(W) = f^{-1}\left(g^{-1}(W)\right)$ is $T$-open set in $X$, since $f$ is $(T,L)$-identification function, then $g^{-1}(W)$ is $L$-open set in $Y$, and since $g$ is $(L,K)$-identification function, then $W$ be any $K$-open set in $Z$. Thus $gof$ is $(T,K)$-identification function ■

**3.14. Theorem**

Let $f: (X,\tau,T) \longrightarrow (Y,\sigma,L)$ is $(T,L)$-identification function, and $g: (Y,\sigma,L) \longrightarrow (Z,\rho,K)$ is a function, then the following statements are valid:

(i) If $gof$ is $(T,K)$- continuous function, then $g$ is $(L,k)$- continuous function.

(ii) If $gof$ is $(T,k)$-contra continuous function, then $g$ is $(L,k)$-contra continuous function.

**Proof**: Assume that $h = gof$

(i) let $W$ be $K$-open set in $Z$, put $V = g^{-1}(W)$ and $U = f^{-1}(V)$, since $h = gof$, we have $h^{-1}(W) = f^{-1}[g^{-1}(W)] = U$. Since $W$ is $K$-open set in $Z$ and $h$ is $(T,K)$-continuous function, then $h^{-1}(W)$ is $T$-open set in $X$, this means that $f^{-1}(V)$ is $T$-open set in $X$. But $f$ is $(T,L)$-identification function, then $V$ is $L$-open set in $Y$, that is $g^{-1}(W)$ is $L$-open set in $Y$. Thus $g$ $is(L,k)$- continuous function ■

(ii) let $W$ be $K$-open set in $Z$, put $V = g^{-1}(W)$ and $U = f^{-1}(V)$, since $h = gof$, we have $h^{-1}(W) = f^{-1}[g^{-1}(W)] = U$. Since $h$ is $(T,k)$-contra continuous function, then $h^{-1}(W)$ is $T$-closed set in $X$, that is $U$ is $T$-closed set in $X$. Since $U = f^{-1}(V)$, then $f^{-1}(V)$ is $T$-closed set in $X$. But $f$ is $(T,L)$-identification function, then $V$ is $L$-closed set in $Y$ (Theorem 3.8), that is $g^{-1}(W)$ is $L$-closed set in $Y$. Thus $g$ $is(L,k)$- contra continuous function ■

## 4.Future works:

We will discuss the following concepts :

1- $(T,L)$ –semi-identification function. (Using the concepts operator $T$ and semi-open sets)

2- $(T,L)$- pre-identification function. (Using the concepts operator $T$ and preopen sets)

3- We can use the concept of ***b-open*** set and the concept of **operator $T$** to define $(T,L)$-$b$-identification function.

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