INTERNATIONAL JOURNAL OF COMPUTERS COMMUNICATIONS & CONTROL
ISSN 1841-9836, 14(1), 56-62, February 2019.

IoT Devices Signals Processing based on Multi-dimensional Shepard Local Approximation Operators in Riesz MV-algebras

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Abstract: In this article we continue the study started in [8] to use Riesz MV-algebras for IoT devices signals processing. The Shepard local approximation operators introduced in [8] were defines such that to approximate single variable functions. In real industrial usage, the signals coming from IoT devices may be influenced by mode than a parameter, and thus we introduce generalized Shepard local approximation operators that can approximate multi-dimensional functions and some numerical experiments are considered.
Keywords: IoT devices; signal processing; Shepard local approximation operators; local approximation operators; approximation algorithms; Riesz MV-algebras, vectorial MV-algebras.

1 Introduction

With the aim to provide new mathematical tools that are useful to develop algorithms that are suitable for IoT devices signals processing, in [1] were introduced Shepard local approximation operators that can approximate one-dimension functions. Since in real industrial applications, the IoT devices signals are not depending on one single parameter, there is a real need to introduce generalized Shepard local approximation operators that can approximate multi-dimensional functions. This new approximation operators can be later used to develop software algorithms that act as input validators for industrial automated control systems [10, 11] based on the Riesz MV-algebra structure of the IoT devices signals [8].

In 1958, multivalued algebras, shortly named MV-algebras, were introduced by Chang [2, 3] as the algebraic structures corresponding to the ∞ - valued Lukasiewicz logic.

Definition 1. An MV-algebra is a structure $\mathcal{A} = (A, \oplus, \neg, 0_A)$ if and only if the following axioms are fulfilled:

$$(A, \oplus, \neg, 0_A) \text{ is an abelian monoid,}$$

$$\neg\neg x = x,$$

$$x \oplus \neg 0_A = \neg 0_A,$$

$$\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x.$$

In an MV-algebra $\mathcal{A}$, the constant 1_A and the binary operations $\odot$ and $\ominus$ can be defined as follows:

$$1_A = \neg 0A,$$

$$x \odot y = \neg(\neg x \oplus \neg y),$$

$$x \ominus y = x \odot \neg y.$$

Also we can define a distance function $d : A \times A \to A$ as follows:

$$d(x, y) = (x \ominus y) \oplus (y \ominus x).$$

This distance as it is defined is a metric and plays a very important role in image and signal processing.

By introducing an additional external operation, in 2003, was defined the concept of Vectorial MV-algebras [6], shortly named VMV-algebras. Let consider an MV-algebra $\mathcal{A}$ and an external operation defined as follows:

$$\bullet : \mathbb{R}_+ \times A \to A.$$

Definition 2. The MV algebra $\mathcal{A}$ is an VMV-algebra if and only if the following axioms are fulfilled:

$$1 \bullet x = x, \forall x \in A,$$

$$(a + b) \bullet x = a \bullet x \oplus b \bullet x, \forall x \in A \text{ and } \forall a, b \in \mathbb{R}_+,$$

$$a \bullet (b \bullet x) \leq (a \cdot b) \bullet x, \forall x \in A \text{ and } \forall a, b \in \mathbb{R}_+,$$

$$d(a \bullet x, a \bullet y) \leq a \bullet d(x, y), \forall x, y \in A \text{ and } \forall a \in \mathbb{R}_+.$$

VMV-algebras inspired new algebraic structures, MV-modules [4] and Riesz MV-algebras [1].

Definition 3. An MV-algebra $\mathcal{A}$ is a truncated MV-module over the unital latticeal ring (R, v) if there is defined an external operation $\bullet : \mathbb{R}_+ \times A \to A$, such that the following properties are fulfilled for $\forall \alpha, \beta \in \mathbb{R}_+$ and $\forall x, y \in A$.

$$(\alpha + \beta) \bullet \ x = \alpha \bullet \ x \oplus \beta \bullet \ x,$$

$$\alpha \bullet (x \oplus y) = \alpha \bullet x \oplus \alpha \bullet y, \text{ if } x \leq \neg\, y,$$

$$\alpha \bullet (\beta \bullet x) = (\alpha \cdot \beta) \bullet \ x, \text{ if } \alpha, \beta \in [0, v].$$

If property

$$v \bullet x = x,$$

is also fulfilled, then $\mathcal{A}$ is an unital MV-module over unital ring $(\mathbb{R}, v)$.

Definition 4. If an MV-algebra is a truncated unital module over $(\mathbb{R}, 1)$, it is a Riesz MV-algebra.

Theorem 5. *In any Riesz MV-algebra, the following properties are fulfilled [4]:*

$$\alpha \bullet (\beta \bullet x) \leq (\alpha \cdot \beta) \bullet x,$$

$$d(\alpha \bullet x, \alpha \bullet y) \leq \alpha \bullet d(x, y),$$

$$0 \bullet x = 0,$$

$$\alpha \bullet 0 = 0,$$

$$x \leq y \Rightarrow \alpha \bullet x \leq \alpha \bullet y,$$

$$\alpha \leq \beta \Rightarrow \alpha \bullet x \leq \beta \bullet x,$$

$$\alpha \bullet (x \oplus y) \leq \alpha \bullet x \oplus \alpha \bullet y,$$

for any $x, y \in A$ and $\alpha, \beta \in \mathbb{R}_+$.

It was also proved that any Riesz MV-algebra is an VMV-algebra, but the reciprocal statement isn't true, but the set of values of IoT devices signals can be organized as Riesz MV-algebras [8].

In [1] was proved that Riesz MV-algebras are algebraic and topological structures for data processing, because:

Theorem 6. *Any method developed in the classical numerical analysis is applicable in Riesz MV-algebras if the Riesz MV-algebras operations are used.*

Based on this statement, in [8] were introduced Shepard local approximation operators on Riesz MV-algebras, to approximate one-dimension functions. Were considered a Riesz MV-algebra $\mathcal{A}$ and a function $f : [0, n] \to A$ and a Shepard kernel [9], which is a strictly decreasing function $K : [0, 1] \to \mathbb{R}_+$. Also, was considered the set

$$B(x, r) = \{y \in [0, n] \mid |x - y| \leqslant r\} \tag{1}$$

Definition 7. A Shepard local approximation operator, is a function $S : [0, n] \to A$ defined as follows:

$$S(f, x) = \oplus_{x_i \in B(x,r)} \frac{K\left(\frac{|x - x_i|}{r}\right)}{\sum_{x_i \in B(x,r)} K\left(\frac{|x - x_i|}{r}\right)} \bullet f(x_i)$$

where $\oplus$ and $\bullet$ are the Riesz MV-algebra operations.

2 Generalized Shepard local approximation operators for IoT devices signal processing

We us consider a Riesz MV-algbra $\mathcal{A}$ and a function $f : [0, n_1] \times [0, n_2] \times ... \times [0, n_m] \to A$. In this generalized situation, in the definition 1 of the set B the distance has to be replaced by a norm. The norms we consider in the numerical experiments are:

$$\|x\| = \sqrt{c_1^2 + c_2^2 + ... + c_m^2} \qquad \text{(euclidean norm)}$$

$$\|x\| = \max\left(|c_1|, |c_2|, ..., |c_m|\right) \qquad \text{(supremum norm)}$$

$$\|x\| = |c_1| + |c_2| + ... + |c_m| \qquad (l^1 \text{ norm})$$

where $x = (c_1, c_2, ..., c_m) \in [0, n_1] \times [0, n_2] \times ... \times [0, n_m]$.

Considering this, we can define the generalized Shepard local approximation operators as follows:

Definition 8. A generalized Shepard local approximation operator, is a function $Sg : [0, n_1] \times [0, n_2] \times ... \times [0, n_m] \to A$ defined as follows:

$$Sg\left(f, x\right) = \oplus_{x_i \in Bg(x,r)} \frac{K\left(\frac{\|x - x_i\|}{r}\right)}{\sum_{x_i \in Bg(x,r)} K\left(\frac{\|x - x_i\|}{r}\right)} \bullet f\left(x_i\right)$$

where $\oplus$ and $\bullet$ are the Riesz MV-algebra operations and

$$Bg\left(x, r\right) = \{y \in [0, n_1] \times [0, n_2] \times ... \times [0, n_m] \mid \|x - y\| \leqslant r\}. \qquad (2)$$

In [1] was considered that the set of possible values for IoT devices signals is the interval $[0, 2^t - 1]$, where t is the number of bits used to store these values. In [10] was proved that the structure $([0, 2^t - 1], \oplus, \neg, 0)$ is a MV-algebra, if the following definitions are used:

$$x \oplus y =_{def} \min(2^t - 1, x + y),$$

$$\neg\, x =_{def} 2^t - 1 - x,$$

$\forall x, y \in [0, 2^t - 1]$.

In [11] was proved that if we consider the external operation $\bullet : \mathbb{R}_+ \times [0, 2^t - 1] \to [0, 2^t - 1]$, defined as follows:

$$a \bullet x =_{def} \min(2^t - 1, a \bullet x),$$

$\forall a \in \mathbb{R}_+$ and $\forall x \in [0, 2^t - 1]$, the structure $([0, 2^t - 1], \oplus, \neg, 0, \bullet)$ is a vectorial MV-algabra and is easy to see that this is also a Riesz MV-algebra, as mentioned in [8].

If we use the above definition of $\oplus$ and $\bullet$ operations and the formula of the general Shepard local approximation operator from Definition 8, we can define an algorithm that can be used to fill in the missing data of signals received from IoT devices.

In [1] were considered the following types of kernels:

$$K(u) = \frac{1}{u^{2\lambda}}, \qquad \text{(Shepard kernel)}$$

$$K(u) = e^{-\lambda u^2}, \qquad \text{(Exponential kernel)}$$

$$K(u) = \left(\frac{\sin\left(q\pi u\right)}{\sin\left(\pi u\right)}\right)^{2\lambda}, \qquad \text{(Shepard-Jackson kernel)}$$

where λ is a parameter that can influence the performance of obtained results, and q is the degree of the Shepard-Jackson kernel. Also in [1] was determined by numerical experiments that the best results are given by Shepard and exponential kernels, thus only these will be considered in the following numerical experiments.

Let now consider an industrial rectangle shape grid of IoT sensors that collect temperature of an environment. The temperatures collected at a certain moment can be represented by a two-dimension function

$$f : [0, n_1] \times [0, n_2] \to [0, 2^t - 1].$$

In the formula of Definition 8, $x_i = (c_{i_r}, c_{i_c})$ are the sensors located on row c_{i_r} and column c_{i_c} have transmitted data and $x = (c_r, c_c)$ is the sensor located in row c_r and column c_c that was not transmitted data.

The fill in algorithm has the following steps:

1. A kernel has to be selected;
2. The parameter λ is set;
3. The radius r, that influence how many received values are considered in the approximation of missing values, is set;
4. A grid traversing method is selected and each missing value is approximated.

3 Numerical results

In the approximation process, there is also the possibility to use the previously new approximated values or to ignore them. Also to reduce the computational complexity, we replace the ball defined in 2 with a square having the side length $2r + 1$ and we used the supremum norm.

A grid of 31×31 sensors was considered, and we assumed that the collected values should be $f(x) = \frac{(c_r+c_c)}{2} \cdot \sin(\frac{\pi(c_r+c_c)}{40})$. It was considered a continuous function, because one of the purposes of this algorithm is to be used to approximate missing values collected by the new fiber optic Bragg grating sensors system designed to monitor the ethanol fermentation during the bioethanol and wine production. This new fiber optic Bragg grating sensor system was developed using financing through grant PN-III-P2-2.1-PED-2016-1955.

The approximation error was determined using the formula

$$P = \sum_{x \in [0,n_1] \times [0,n_2]} |f(x) - Sg(f, x)|,$$

because we are interested in the overall error cumulated error.

3.1 Experiment 1

In this experiment we assume that we received only data transmitted by sensors that are located on rows and columns that are both even numbers. Several parameterizations are considered. After running the tests, we get the approximation errors listed as follows:

Parametrization	*Shepard kernel*	*Exponential kernel*
$r = 2, \lambda = 2$	48.1403	74.1296
$r = 2, \lambda = 10$	36.3588	36.3465
$r = 2, \lambda = 20$	36.3587	36.3587
$r = 3, \lambda = 2$	58.0658	183.302
$r = 3, \lambda = 10$	36.3588	42.9482
$r = 3, \lambda = 20$	36.3587	36.5376
$r = 5, \lambda = 2$	68.9431	476.092
$r = 5, \lambda = 10$	36.3588	110.7
$r = 5, \lambda = 20$	36.3587	54.642
$r = 10, \lambda = 2$	80.0381	1431.53
$r = 10, \lambda = 10$	36.3588	463.169
$r = 10, \lambda = 20$	36.3587	237.138

3.2 Experiment 2

In this experiment we assume that we couldn't receive data transmitted by sensors that are located on rows and columns that are both odd numbers. Same parameterizations like in the previous experiment are considered. After running the tests, we get the approximation errors listed as follows:

Parametrization	*Shepard kernel*	*Exponential kernel*
$r = 2, \lambda = 2$	15.1187	20.5379
$r = 2, \lambda = 10$	13.1902	13.2046
$r = 2, \lambda = 20$	13.1902	13.1902
$r = 3, \lambda = 2$	18.6535	58.1864
$r = 3, \lambda = 10$	13.1902	14.2934
$r = 3, \lambda = 20$	13.1902	13.2233
$r = 5, \lambda = 2$	21.7479	152.12
$r = 5, \lambda = 10$	13.1902	33.5822
$r = 5, \lambda = 20$	13.1902	16.5552
$r = 10, \lambda = 2$	24.8594	455.862
$r = 10, \lambda = 10$	13.1902	146.969
$r = 10, \lambda = 20$	13.1902	74.6371

4 Conclusion

Nowadays, industrial information systems are depending on signals received form IoT devices. There can be several problems in acquiring data from these IoT devices, problems that can led to missing values. Without a complete set of data, the automation of processes isn't possible or is not satisfying enough. The algorithm proposed in this paper has the role to fill in the missing values of signals sent by IoT devices. As mentioned in [8], for industrial usage of the algorithm, this methods should be further developed to determine the proper set of parameters for each of the kernels, based on the particularities of the industrial processes handled and on the amount of missing values. Depending on the constrains of the real processes that have to be modeled, other error measures can be considered as well.

Funding

This research was funded by Unitatea Executiva pentru Finantarea Invatamantului Superior, a Cercetarii, Dezvoltarii si Inovarii (UEFISCDI) Romania, grant number 198PED/2017, grant code: PN-III-P2-2.1-PED-2016-1955.

Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

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