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# Design of Congestion Control Scheme for Uncertain Discrete Network Systems

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**Abstract:** For a class of uncertain discrete network systems, a sliding mode control algorithm is presented for active queue management (AQM) in order to solve the problem of congestion control in transmission control protocol (TCP) communication. First, the sliding surface is designed based on linear matrix inequality (LMI) technique. Then, we analyze the mechanism of chattering for the discrete-time exponential approximation law, a modified one is presented and applied to the network systems. Simulation results demonstrate that the proposed controller has good stability and robustness with respect to the uncertainties of the number of active TCP sessions, link capacity and the round-trip time.
**Keywords:** sliding mode control, network systems, linear matrix inequality(LMI)

## 1 Introduction

As the rapid expansion of network scale, congestion control has become an important issue. AQM is a router-based control mechanism, which can implement the end system to demand quality of service. So the combination of TCP and AQM is the main ways to solve the problems of current network congestion control.

Random early detection (RED) as the earliest well-known AQM algorithm is sensitive to parameter variations [2]. So some improved RED methods are presented [3-5]. However, these algorithms can not guarantee high network utilization and low packet loss[6-7]. Recently, some AQM algorithms have been proposed based on mathematical models, which give the basis for control theory research. In [6], a fluid-flow model for TCP/AQM networks has been introduced. The proportional-integral (PI) controller is designed in [8], also some robust control schemes [9-12], such as intelligent PID, variable structure sliding mode controller, H-infinity controller, and so on. These methods can obtain well performance for practical network systems.

With the rapid development of computer technology and digital signal processing chips, the study of discrete-time control theory is rather important. Sliding mode control (SMC) is a robust technique for its unique ability to withstand external disturbance, it has achieved fruitful for continuous system. However, sliding mode control schemes are relatively small for discrete-time system [13-15], a few discrete algorithms are applied to the network control. In this paper, a robust discrete-time sliding mode controller is designed for TCP network model with uncertain disturbance. The aim is to avoid network congestion.

## 2 Problem Statement and Preliminaries

In [6], a model of TCP connection through a congested AQM router is developed.

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R(t)} - \dfrac{W(t)W(t-R(t))}{2R(t)}p(t-R(t)) \\ \dot{q}(t) = \dfrac{N(t)}{R(t)}W(t) - C(t) \end{cases} \tag{1}$$

where $C(t)$ is the capacity of link, $W(t)$ is the size of TCP congestion window, $q(t)$is the length of queue in buffer, $p(t)$ is packet-dropping probability function $(0 \leq p(t) \leq 1)$, $N(t)$ is the number of active TCP link, $R(t)$ is the transfer delay and $R(t) = T_p + q(t)/C(t)$.

To linearize(1), we first assume$R(t) = R_0$, $N(t) = N$and $C(t) = C$is normal value of $R(t)$, $N(t)$ and $C(t)$, the equilibrium point $(W_0, q_d, p_0)$ is defined by $\dot{W} = 0$ and $\dot{q} = 0$. Let $\delta W(t) = W(t) - W_0$, $\delta q(t) = q(t) - q_d$, $\delta p(t) = p(t) - p_0$. A linearized model is given.

$$\begin{aligned} \delta\dot{W}(t) &= -\frac{2N}{R_0^2 C}\delta W(t) - \frac{R_0 C^2}{2N^2}\delta p(t - R_0) \\ \delta\dot{q}(t) &= \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t) \end{aligned} \tag{2}$$

Let $x(t) = \begin{pmatrix} \delta q(t) & \delta \dot{q}(t) \end{pmatrix}^T = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T, u(t) = \delta q(t), (-p_0 \leq u(t) \leq 1 - p_0)$. We have

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) \tag{3}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \bar{A} = \begin{bmatrix} 0 & 1 \\ -\dfrac{2N}{R_0^3 C} & -\left(\dfrac{1}{R_0} + \dfrac{2N}{R_0^2 C}\right) \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ -\dfrac{C^2}{2N} \end{bmatrix}.$$

Then the discrete-time uncertain system can be expressed as the sampling period $T$.

$$x(k+1) = (\tilde{A} + \Delta\tilde{A})x(k) + (\tilde{B} + \Delta\tilde{B})u(k) \tag{4}$$

where $\tilde{A} = e^{\bar{A}T}$, $\tilde{B} = (\int_0^T e^{AT})\bar{B}, \Delta\tilde{A}$ and $\Delta\tilde{B}$ are depending on network parameters.

In the process of designing controller, the following assumptions are taken.

**$A_0$.** The pair $(\tilde{A}, \tilde{B})$ is controllable, and $\tilde{B} = \begin{pmatrix} \tilde{B}_1 & \tilde{B}_2 \end{pmatrix}$, $det(\tilde{B}_2) \neq 0$.

**$A_1$.** The matrix $\Delta\tilde{A}(k)$ satisfies mismatch condition and $\Delta\tilde{B}$ satisfies $\Delta\tilde{B} = \tilde{B} \times \Delta\hat{B}$.

We do a linear transformation as follows:

$$z = Tx = \begin{bmatrix} I_{n-m} & -\tilde{B}_1\tilde{B}_2^{-1} \\ 0 & \tilde{B}_2^{-1} \end{bmatrix} x \tag{5}$$

The system (4) is written as follows.

$$z(k+1) = (A + \Delta A)z(k) + B(I + \Delta\hat{B})u(k) \tag{6}$$

where $z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}$, $A = T\hat{A}T^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $\Delta A = T\Delta\tilde{A}T^{-1} = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix}$,
$B = T\tilde{B} = \begin{bmatrix} 0 \\ I_m \end{bmatrix}$, $z_1(k) \in R^{n-m}$, $z_2(k) \in R^m$.

We have

$$z_1(k+1) = A_{11}z_1(k) + A_{12}z_2(k) + \Delta A_{11}z_1(k) + \Delta A_{12}z_2(k) \tag{7}$$

$$z_2(k+1) = A_{21}z_1(k) + A_{22}z_2(k) + \Delta A_{21}z_1(k) + \Delta A_{22}z_2(k) + I_m\left(I + \Delta\hat{B}\right)u(k) \tag{8}$$

# 3 Design of Controller for Discrete-time Network Systems

## 3.1 Designing Sliding Mode Surface

Without loss of generality, we suppose that the sliding surface is

$$s(k) = \bar{M}z(k) = \begin{bmatrix} -M & I_m \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} = 0 \tag{9}$$

where $\bar{M} \in R^{m\times n}$, $M \in R^{m\times(n-m)}$. Substituting (9) into (7) gives the sliding motion

$$z_1(k+1) = (A_{11} + \Delta A_{11} + A_{12}M + \Delta A_{12}M)\, z_1(k) \tag{10}$$

**A$_2$.** The $\Delta A_{11}(k)$ and $\Delta A_{12}(k)$ satisfy $\Delta A_{11}(k) = DF(k)E_1$, $\Delta A_{12}(k) = DF(k)E_2$, $D$ and $E_i(i{=}1, 2)$ are constant matrices of appropriate dimensions, $F(k)$ satisfies $F^T(k)F(k) \le I$.

**Lemma 1**[16]**.** *Given constant matrices$D, E$and symmetric matrix$Y$ of appropriate dimensions, the following inequality holds$Y + DFE + E^TF^TD^T < 0$. where $F$satisfy $F(k)^TF(k) \le I$, if and only if for some constant$\varepsilon > 0$, we have$Y + \varepsilon DD^T + \varepsilon^{-1}E^TE < 0$.*

**Theorem 1.** *If there exists a symmetric and positive definite matrix$P$, some matrix $W$and some scalar $\varepsilon$ such that the following LMI(11) is satisfied, then the reduced-order discrete-time system(6) is asymptotically stable by the sliding mode surface(9).*

$$\begin{bmatrix} -X & * & * \\ A_{11}X + A_{12}W & -X + \varepsilon DD^T & * \\ E_1X + E_2W & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{11}$$

where$X = P^{-1}$,$W = MP^{-1}$ and * denotes the transposed elements in the symmetric positions.

**Proof:** For system (6), we choose the following Lyapunov- Krasovskii function

$$v(k) = z_1^T(k)\, P z_1(k) \tag{12}$$

Differential equation along the trajectory of the system in (10) is given by

$$\begin{aligned} \Delta v(k) &= v(k+1) - v(k) \\ &= z_1^T(k)\left(Q^TPQ - P\right) z_1(k) \end{aligned}$$

where $Q = A_{11} + A_{12}M + DFE_1 + DFE_2M$.If the $Q^TPQ - P < 0$, we can the Theorem 1.

## 3.2 Design of discrete-time sliding mode controller

Discrete-time approximate law is given in [17].

$$s(k+1) = (1 - \eta T)\, s(k) - \lambda T sgns(k) \tag{13}$$

where $\lambda > 0$, $\eta > 0$, $0 < \eta T < 1$, $T$ is sampling period.

The (13) can not guarantee the system reaches to point. The modified reaching law is given.

$$s(k+1) = (1 - \eta T)\, s(k) - \left(1 - e^{-|s(k)|} - \eta\right) |s(k)|\, T sgn\,(s(k)) \tag{14}$$

According to the equations (6), (9) and (14), we get the control law as follows.

$$u(k) = (\bar{M}B)^{-1}[\bar{M}Ax(k) + (1-\eta T)s(k) - (1 - e^{-|s(k)|} - \eta)|s(k)|Tsgn(s(k)) - \bar{M}f(k)] \tag{15}$$

where$f\left(k\right)=\Delta Az\left(k\right)+\Delta Bu\left(k\right)$. it is an unknown number, the controller can not be achieved.

The system existing unknown disturbances of the dynamics is much slower compared with the sampling frequency, so we have

$$f(k-1)=z(k)-Az(k-1)-Bu(k-1) \tag{16}$$

Considering the practical network system, we can obtain the following controller.

$$\begin{array}{l} u\left(k\right)=-p_0\ ,\ u\left(k\right)<-p_0 \\ u\left(k\right)=\ \left(\bar{M}B\right)^{-1}\left[-\bar{M}Az\left(k\right)+\left(1-\eta T\right)s\left(k\right)-\left(1-e^{-\left|s\left(k\right)\right|}-\eta\right)\left|s\left(k\right)\right|Tsgn\left(\mathbf{s}\left(k\right)\right)\right. \\ \left.-\ \bar{M}\left(z\left(k\right)-Az\left(k-1\right)-Bu\left(k-1\right)\right)\right]\ ,\ -\ p_0\leq u\left(k\right)\leq 1-p_0 \\ u\left(k\right)=1-\ p_0\ ,\ u\left(k\right)>1-\ p_0 \end{array} \tag{17}$$

**Remark:** 1) when $s(k)\geq 0$, the modified reaching law (14) is rewritten as follows.

$$s(k+1)-s(k)=-(1-e^{-|s(k)|})s(k)T\leq 0$$

2) when$s\left(k\right)\leq 0$, the modified reaching law (14) is rewritten as follows.

$$s(k+1)-s(k)=(1-e^{-|s(k)|})s(k)T\leq 0$$

We can see that the design control law satisfying the sliding mode reaching condition.

## 4 Simulation Results

Let we choose parameters. $N=50$, $C=300$ packets/s, $R_0=0.5$s, $W_0=3$, $p_0=0.22$. $\tilde{A}=\begin{bmatrix}0.9999 & 0.0098\\ -0.0263 & 0.9671\end{bmatrix}$, $\tilde{B}=\begin{bmatrix}-0.0445\\ -8.8514\end{bmatrix}$,$\Delta\tilde{A}=\begin{bmatrix}0.0001sin(0.1\pi k) & 0\\ 0 & 0.0005sin(0.1\pi k)\end{bmatrix}$,$\Delta\tilde{B}=\begin{bmatrix}-0.0013 & -0.2655\end{bmatrix}$, $\varepsilon=0.01$. Using LMI toolbox in the matlab, we can get $M=-8.3$. Then choosing $\lambda=0.1$,$T=0.01$s, $\eta=5$, $x\left(0\right)=\left[\begin{array}{cc}30 & 2\end{array}\right]^T$. Fig.1 is the control law response curve based on the proposed control law, which can have much lower packet-dropping probability, satisfying the demand of system response.

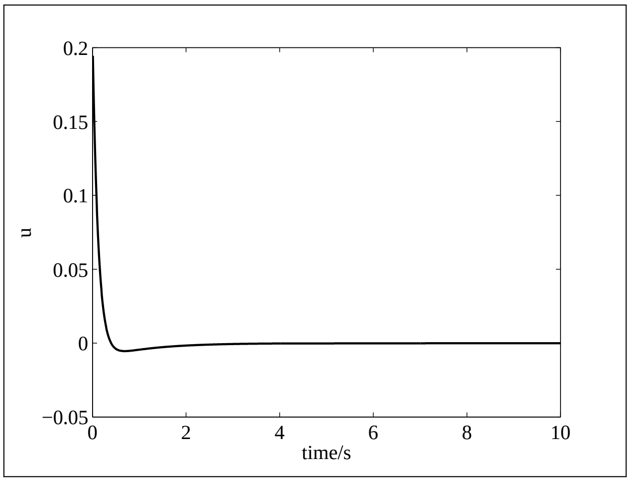


Figure 1: The motion curve with designed control law u(k) in this paper

We give the system state responses curve based on reaching law (13)and (14). The proposed control scheme can obtain much better performance both response time and chattering from Fig.2 and Fig.3.

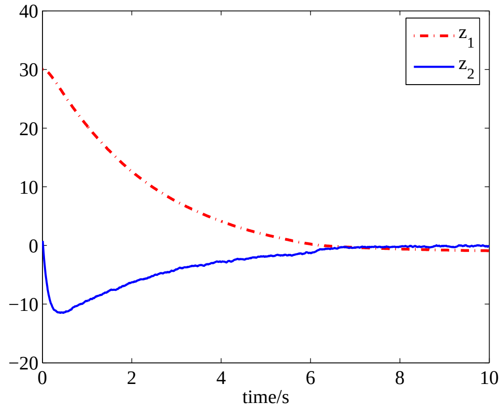


Figure 2: State responses with control law (13)

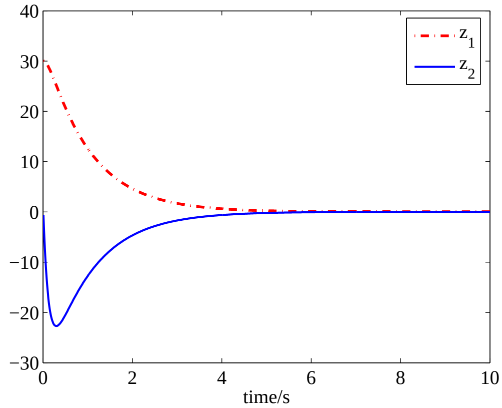


Figure 3: State responses with control law (14)

# 5 Conclusion

This paper gives a discrete sliding mode control algorithm for network systems. A modified reaching law is presented and applied to the system. Simulation results show that the controller has better stability and robustness, which can get a faster transient response and smaller steady state error. The scheme can effectively avoid network congestion.

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