INTERNATIONAL JOURNAL OF COMPUTERS COMMUNICATIONS & CONTROL ISSN 1841-9836, 12(6), 748-789, December 2017.

# Fuzzy Logic Is Not Fuzzy: World-renowned Computer Scientist Lotfi A. Zadeh

I. Dzitac, F.G. Filip, M.J. Manolescu



Lotfi A. Zadeh (1921-2017)

Ioan Dzitac<sup>1,2</sup>, Florin Gheorghe Filip<sup>3</sup>, Misu-Jan Manolescu<sup>2,\*</sup>
1. Aurel Vlaicu University of Arad
310330 Arad, Elena Dragoi, 2, Romania, ioan.dzitac@uav.ro
2. Agora University of Oradea
410526 Oradea, P-ta Tineretului 8, Romania
\*Corresponding author: mmj@univagora.ro
3. Romanian Academy
Calea Victoriei 125, Sector 1,
010071 Bucharest, Romania, ffilip@acad.ro

Abstract: In 1965 Lotfi A. Zadeh published "Fuzzy Sets", his pioneering and controversial paper, that now reaches almost 100,000 citations. All Zadeh's papers were cited over 185,000 times. Starting from the ideas presented in that paper, Zadeh founded later the Fuzzy Logic theory, that proved to have useful applications, from consumer to industrial intelligent products. We are presenting general aspects of Zadeh's contributions to the development of Soft Computing(SC) and Artificial Intelligence(AI), and also his important and early influence in the world and in Romania. Several early contributions in fuzzy sets theory were published by Romanian scientists, such as: Grigore C. Moisil (1968), Constantin V. Negoita & Dan A. Ralescu (1974), Dan Butnariu (1978). In this review we refer the papers published in "From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence" (2008, Eds.: L.A. Zadeh, D. Tufis, F.G. Filip, I. Dzitac), and also from the two special issues (SI) of the International Journal of Computers Communications & Control (IJCCC, founded in 2006 by I. Dzitac, F.G. Filip & M.J. Manolescu; L.A. Zadeh joined in 2008 to editorial board). In these two SI, dedicated to the 90th birthday of Lotfi A. Zadeh (2011), and to the 50th anniversary of "Fuzzy Sets" (2015), were published some papers authored by scientists from Algeria, Belgium, Canada, Chile, China, Hungary, Greece, Germany, Japan, Lithuania, Mexico, Pakistan, Romania, Saudi Arabia, Serbia, Spain, Taiwan, UK and USA.

**Keywords:** fuzzy sets, fuzzy languages, fuzzy logic, Romanian early contributions, SC, AI, IJCCC.

# 1 Introduction

"Fuzzy logic is not fuzzy. Basically, fuzzy logic is a precise logic of imprecision and approximate reasoning." (L.A. Zadeh [139])

In 2008, Professor Lotfi A. Zadeh was a keynote speaker at the International Conference on Computers Communications and Control (ICCCC), organized by Agora University of Oradea, Romania (ICCCC2008).

Under the umbrella of the ICCCC2008, on the occasion of an exploratory workshop we coedited the volume "From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence" (Editors: Lotfi A. Zadeh, Dan Tufis, Florin Gheorghe Filip, Ioan Dzitac) [145]<sup>1</sup>



I. Dzitac, M.J. Manolescu, L.A. Zadeh and F.G. Filip (ICCCC2008)

The members of the Agora University and the Editorial Team of the  $IJCCC^2$ , were very honoured that a world-famous computer scientist, Lotfi A. Zadeh, was their collaborator and they are dedicating to his memory this work.

We have made a literature review in the fields of fuzzy sets, fuzzy logic and their applications, consisting in main papers of Lotfi A. Zadeh, and also of the several important international followers and Romanian contributors [1-151].

<sup>&</sup>lt;sup>1</sup>"Dear Professor Dzitac: I received copies of the Proceedings and the Journal. I was highly impressed in all respects. You and your colleagues have done an outstanding job. I was very pleased with the inclusion of my powerpoint presentation in the Appendix of the Proceedings. Please accept my compliments and congratulations. I should like to take this opportunity to thank you again for the very warm welcome which you extended to me. Participation in your Conference was a very stimulating as well as pleasant experience. With my warm regards. Sincerely, Lotfi Zadeh." (From: Lotfi A. Zadeh [zadeh@eecs.berkeley.edu]; Sent: Thursday, May 07, 2009 9:51 AM, To: Ioan Dzitac; Subject: Congratulations/Dzitac).

<sup>&</sup>lt;sup>2</sup>http://univagora.ro/jour/index.php/ijccc/

# 2 Lotfi A. Zadeh (1921 - 2017): A brief biographical note

"The question really isn't whether I'm American, Russian, Iranian, Azerbaijani, or anything else. I've been shaped by all these people and cultures and I feel quite comfortable among all of them." (Lotfi A. Zadeh [15])

Lotfi A. Zadeh, inventor of fuzzy logic, multidisciplinary and interdisciplinary scientist (engineer, mathematician, computer scientist, logician, scientist in artificial intelligence), was born on February 4, 1921, in Baku, Azerbaijan, as Lotfi Aliaskerzadeh<sup>3</sup>. Lotfi enrolled the elementary school no.16 in Baku.

In 1905, Lotfi's mother, Fanya Korenman, with her family moved from Russia to Baku and settled there. She graduated from Tbilisi (Georgia) Women Gymnasium in 1918, and after she graduated from Azerbaijan State University, Faculty of Medicine, Pediatric Department. Lotfi's father, Rahim Aleskerzade, was from Ardabil (South Azerbaijan, Iranian Azerbaijani ethnicity) graduated from Azerbaijan State University, faculty of Journalism. They met each other during university time and married in 1920 in Baku.

When Lotfi turned 10, in 1931, Zadeh's family moved from Azerbaijan to Tehran, Iran. Here Zadeh was enrolled in Alborz College, which was a Presbyterian missionary school, where he was educated for the next eight years, and where he met his future wife, Fay.

In 1942, Zadeh graduated from the University of Tehran with a degree in Electrical Engineering.

In 1943 he emigrated to the US and obtained his Master in the famous Massachusetts Institute of Technology (MIT) and his Ph.D. at Columbia University in 1949, where he lectured for a year, then becoming a professor at the University of California, Berkeley.

Zadeh taught for ten years at Columbia University, was promoted to Full Professor in 1957, and has taught at the University of California, Berkeley since 1959. He was also director of the Berkeley Initiative in Soft Computing (BISC) from 1991 to his death. His research was supported by Omron Grant, Tekes Grant, The Ministry of Communications and Information Technologies of Azerbaijan Republic, Grant from Azerbaijan University and BISC grant.

He was lucid and active almost until the last minute. Lotfi A. Zadeh, father of Fuzzy Sets, Fuzzy Logic and Fuzzy Mathematics, dies at 96, on September 6, 2017, in Berkeley, California, USA. On September 29, the Azerbaijan National Academy of Sciences hosted a worldwide famous Azerbaijani scientist, Lotfi Zadeh's funeral. Lotfi A. Zadeh was laid to rest in the 1st Alley of Honor in Baku, Azerbaijan. President of Azerbaijan Republic Ilham Aliyev participated at this ceremony.

Professor Shahnaz Shahbazova, a very good friend of the Zadeh's family, was legal granted by Norman Zadeh, Lotfi Zadeh's son, for transfer from USA the Lotfi's body to be buried in Baku. Both daughters of Shahnaz Shahbazova (Chimnaz and Sabina) worked with Professor Zadeh since 2012 at Berkeley. There it is some memories about Zadeh's family: "I have visited Lotfi and Fay every week and was with them until their last days. I also assisted him with his research and with every need they had. I loved visiting them and spending time with both Lotfi and Fay, listening to their life stories and experience." (Chimnaz); "He always pleased to receive emails from colleagues and friends and tried to answer to them at the same day. He also appreciated to friends who contacted him via phone as well."(Sabina).

<sup>&</sup>lt;sup>3</sup>While in the United States, he changed his name to Lotfi Asker Zadeh.



Lotfi Zadeh's funeral:

Prof. Shahnaz Shahbazova<sup>4</sup>, Prof. Vadim L. Stefanuk<sup>5</sup>, and Prof. Valentina E. Balas<sup>6</sup> at the 1st Alley of Honor in Baku.

<sup>4</sup>Dr. Shahnaz Shahbazova is a Professor at Azerbaijan Technical University. She is an academician of the International Academy of Sciences named after Lotfi A. Zadeh, 2002 and vice president of the same academy, 2014. Her research interests include Artificial Intelligence, Soft Computing, Intelligent System, Machine Learning Techniques to Decision Making and Fuzzy Neural Network. She is a member of Berkeley Initiative in Soft Computing group (BISC), New York Academy of Sciences, member of "Defined Candidate Dissertation" Society and member of the Board of Directors of North American Fuzzy Information Processing Society (NAFIPS), International Women Club, Azerbaijan [1]. In 2007, Professor Shahbazova won Fulbright Visiting Scholar Grant and had conducted her research at the University of California, Berkeley, in Computer Science Department under the supervision of Prof. Lotfi Zadeh. They worked on several projects together which were great success. Since then, she became very close to Zadeh and his family. Being treated as daughter, she visited them often and was with Fey Zadeh and Prof. Lotfi A.Zadeh until his last days. Per Shahnaz's initiative, the World Conference on Soft Computing has launched, and was held every year worldwide. Her daughters Chimnaz and Sabina, who are both in the US since 2012, were very close to Lotfi Zadeh, and were with him until his last day.

<sup>5</sup>Dr. Vadim Stefanuk is a Full Professor of Russian University for People's Friendship and a Full member of Russian Academy for Natural Sciences. He authored over 250 papers. He is interested in scientific fields, such as: Man-Machine Systems, Artificial Intelligence, Fuzzy Sets, Collective Behavior of Learning Automata, Information Theory, Information Safety. He is well known in literature for "Markov - Stefanuk chain". Vadim L. Stefanuk translated about 25 books into Russian language, including very second paper on Fuzzy Sets by L.A. Zadeh.

<sup>6</sup>Dr. Valentina E. Balas is currently Professor at the University Aurel Vlaicu of Arad, Romania. She is author of more than 250 research papers in refereed journals and International Conferences. Her research interests are in Intelligent Systems, Fuzzy Control, Soft Computing, Smart Sensors, Information Fusion, Modeling and Simulation. She is Editor-in Chief to International Journal of Advanced Intelligence Paradigms (IJAIP) and International Journal of Computational Systems Engineering (IJCSysE), member in Editorial Boards for national and international journals, serves as reviewer for many International Journals. She is General Co-Chair to seven editions of International Workshop Soft Computing Applications (SOFA) starting from 2005. As Honorary Chair of SOFA Conferences, Prof. Lotfi A. Zadeh participated to the 2005 and 2007 editions in Arad and Oradea. Dr. Balas was editor for more than 20 books in Springer and Elsevier. She is series editor for the work entitled Elsevier Biomedical Engineering from October 2017. She was Vice-president (Awards) of IFSA International Fuzzy Systems Association Council (2013-2015), responsible with recruiting to European Society for Fuzzy Logic and Technology - EUSFLAT (2011-2013), Senior member IEEE, etc. Dr. Balas invited Prof. Lotfi A. Zadeh to Arad in July 2003 when he was awarded the title of Doctor Honoris Causa at Aurel Vlaicu University of Arad, Romania. She was very good friend of Lotfi A. Zadeh and Fay Zadeh. Zadeh's daughter, Stella  $(1947-2006)^7$ , and his wife, Fay<sup>8</sup> (1920-2017), died before him. Lotfi A. Zadeh is survived by his son, Norman Zadeh.<sup>9</sup>



Lotfi Zadeh's funeral at the Azerbaijan National Academy of Sciences

#### 2.1 Awards

"The "Grigore Moisil" Prize in Fuzzy Systems comes to give a new encouragement to fuzzy system and artificial intelligence scientific community in the name of one of the greatest mathematicians and logicians of this century. It also provides once again a recognition of fuzzy logic, and of Zadeh's work, by the followers of Grigore Moisil."<sup>10</sup>

Lotfi A. Zadeh received many awards and medals, such as: the IEEE Education Medal, the IEEE Richard W. Hamming Medal, the IEEE Medal of Honor, the ASME Rufus Oldenburger Medal, the B. Bolzano Medal of the Czech Academy of Sciences, the Kampe de Feriet Medal, the AACC Richard E. Bellman Control Heritage Award, the Honda Prize, the Okawa Prize, the AIM Information Science Award, the IEEE-SMC J.P. Wohl Career Achievement Award, the SOFT Scientific Contribution Memorial Award of the Japan Society for Fuzzy Theory, the IEEE Millennium Medal, the ACM 2001 Allen Newell Award, the Norbert Wiener Award of the IEEE Systems, Man and Cybernetics Society, the Civitate Honoris Causa by Budapest Tech Polytechnical Institution, the V. Kaufmann Prize (International Association for Fuzzy-Set Management and Economy), the Nicolaus Copernicus Medal of the Polish Academy of Sciences, the J. Keith

<sup>10</sup>Lotfi Zadeh was a recipient of a prize awarded by the Romanian Academy: Historical note by I. Bogdan, Al.P. Tacu, H.-N. Teodorescu, E. Sofron, presented at Kyushu Institute of Technology Iizuka, Fukuoka, Japan, 3 AUG 1994 and published in "Fuzzy Sets and Systems", 8, p.367, 1994).

<sup>&</sup>lt;sup>7</sup>Stella Zadeh graduated in 1969 from Radcliffe College and earned a master's degree in journalism in 1971 from the University of California, Los Angeles.

<sup>&</sup>lt;sup>8</sup>Fay Zadeh authored the book: "My Life and Travels with the Father of Fuzzy Logic", Hardcover, 1998. [128] <sup>9</sup>Today he is know as Norman Zada and is founder of Perfect 10. "Prior to starting Perfect 10, Zada obtained a doctorate in operations research at the University of California, Berkeley and worked at IBM and was an adjunct mathematics professor at Stanford University, Columbia University, UCLA and University of California, Irvine, writing articles on applied mathematics as well as the 1974 book Winning Poker Systems. After teaching, he won both backgammon and sports handicapping championships. He later became a money manager, and in the 1980s ran a number of financial competitions, including the U.S. Investing Championship. Zada made headlines in 1996 when he offered 400,000 US dollars for anyone successfully refuting his claim that balancing the United States federal budget over a multi-year period without an accompanying substantial trade surplus would be effectively mathematically impossible." Source: https://en.wikipedia.org/wiki/Norman\_Zada

Brimacombe IPMM Award, the Silicon Valley Engineering Hall of Fame, the Heinz Nixdorf Museums Forum Wall of Fame, the Egleston Medal, the Franklin Institute Medal, the Medal of the Foundation by the Trust of the Foundation for the Advancement of Soft Computing, the High State Award "Friendship Order" (from the President of the Republic of Azerbaijan), the Transdisciplinary Award and Medal of the Society for Design and Process Sciences, the BBVA Foundation Frontiers of Knowledge Award in the Information and Communication Technologies<sup>11</sup>, the Nizami Gancavi Gold Medal (Azerbaijan, 2016), Golden Goose Award (USA, 2017), and other awards and twenty-four honorary doctorates.<sup>12</sup>

In 2003 Lotfi A. Zadeh became a Doctor Honoris Causa of the Aurel Vlaicu University in Arad, Romania, at the suggestion of prof. V.E. Balas. In 2007 Lotfi Zadeh was elected as a member of the Academy of Technical Sciences of Romania (in the same day as one of co-authors of this paper - F.G. Filip), at the proposal of prof. M. Petrescu.

#### 2.2 Lotfi A. Zadeh: The inventor of fuzzy sets and fuzzy logic

"I believe that my paper would have been rejected if I were not on the Editorial Board. In large measure, comments of my paper were skeptical or hostile. An exception was Japan." (L.A. Zadeh [143])

Lotfi A. Zadeh published his seminal work on fuzzy sets in 1965, in which he detailed the mathematics of fuzzy set theory. In 1973 he proposed his theory of fuzzy logic.

In [143] Lotfi A. Zadeh said about the birth of fuzzy sets: "In July of 1964, I was attending a conference in New York and was staying at the home of my parents. They were away. I had a dinner engagement but it had to be canceled. I was alone in the apartment. My thoughts turned to the unsharpness of class boundaries. It was at that point that the simple concept of a fuzzy set occurred to me. It did not take me long to put my thoughts together and write a paper on the subject. This was the genesis of fuzzy set theory.

I knew that the word "fuzzy" would make the theory controversial. Knowing how the real world functions, I submitted my paper to Information and Control because I was a member of the Editorial Board. There was just one review-which was very lukewarm. I believe that my paper would have been rejected if I were not on the Editorial Board. [...] My paper was a turning point in my research. Since 1965, almost all of my papers relate to fuzzy set theory and fuzzy logic. As I expected, my 1965 paper drew a mixed reaction, partly because the word "fuzzy" is generally used in a pejorative sense, but, more substantively, because unsharpness of class boundaries was not considered in science and engineering. In large measure, comments of my paper were skeptical or hostile. An exception was Japan. In 1968, I began to receive letters from Japan expressing interest in application of fuzzy set theory to pattern recognition. In the years which followed, in Japan fuzzy set theory and fuzzy logic became objects of extensive research and wide-ranging application, especially in the realm of consumer products. A very visible application was the subway system in the city of Sendai - a fuzzy logic based system designed by Hitachi and Kawasaki Heavy Industry. The system began to operate in 1987 and is considered to be a great success."

The departure of fuzzy logic from the classical Aristotelian logic, with a tradition of over two millennia, several mathematicians and scientists has made fuzzy logic an object of controversy, skepticism and hostility. In [116] E. Trillas said: "... important aspect of Zadeh's life is his way

<sup>&</sup>lt;sup>11</sup>This category has been granted in this fifth edition to the electrical engineer Lotfi A. Zadeh, "for the invention and development of fuzzy logic." This "revolutionary" breakthrough, affirms the jury in its citation, has enabled machines to work with imprecise concepts, in the same way humans do, and thus secure more efficient results more aligned with reality. In the last fifty years, this methodology has generated over 50,000 patents in Japan and the U.S. alone (2013).

<sup>&</sup>lt;sup>12</sup>https://www2.eecs.berkeley.edu/Faculty/Homepages/zadeh.html

of confronting criticism. Not only did he never avoid criticism of his work, but always encouraged people to criticize his idea; and he did it in a very polite and gentle form. It should be recalled that since Zadeh's first ideas on fuzzy logic fell down the wall of crisp bivalent logic and the mystics of precision, by introducing contextual and purpose driven representations of imprecise concepts, he received strong criticism coming from prestigious researchers. Nevertheless, forty years later, it can be said that Zadeh's ideas not only resisted criticism but imposed on them."

In [139] Lotfi A. Zadeh said about mathematical rigour of fuzzy logic theory: "Fuzzy logic is not fuzzy. Basically, fuzzy logic is a precise logic of imprecision and approximate reasoning. More specifically, fuzzy logic may be viewed as an attempt at formalization/mechanization of two remarkable human capabilities. First, the capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, conflicting information, partiality of truth and partiality of possibility - in short, in an environment of imperfect information. And second, the capability to perform a wide variety of physical and mental tasks without any measurements and any computations [137, 138].

In fact, one of the principal contributions of fuzzy logic - a contribution which is widely unrecognized - is its high power of precisiation. Fuzzy logic is much more than a logical system. It has many facets. The principal facets are: logical, fuzzy-set-theoretic, epistemic and relational. Most of the practical applications of fuzzy logic are associated with its relational facet".

Since then many authors have developed the theory of fuzzy set and its applications. Especially, many mathematicians tried to extend in fuzzy context classical mathematics results.

The success of the research undertaken has been demonstrated in a variety of areas such as: artificial intelligence, computer science, quantum particle physics, control engineering, robotics and many more. Perhaps the main reason for this rapid development is that the world that surrounds us is full of uncertainty, the data we collect from the environment are, in general, vague and incorrect. So the notion of fuzzy set allows us to study the degree of uncertainty mentioned above in a purely mathematical way [36].

Nowadays, in Quantum computing is used Quantum Logic. Similarities and differences between Fuzzy and Quantum Logic are presented in [105]: "Quantum logic was developed in the context of quantum mechanics. In contrast to fuzzy logic, the logic is not based on membership values but on vector subspaces identified by projectors. The lattice of all projectors provides us with a lattice operations interpreted as conjunction and disjunction. Interestingly, there are relations between both theories. The interaction of a projector with a normalized vector produces a value which can be interpreted directly as fuzzy membership value. This paper shows, that under some circumstances the conjunction of projectors directly corresponds to the t-norm algebraic product in fuzzy logic. However, in contrast to fuzzy logic which is defined on fuzzy sets, quantum logic takes the producing projectors into consideration. As result, we are able to overcome the problem of idempotence for the algebraic product. Furthermore, if we restrict projectors to be mutually commuting we obtain a logic obeying the rules of the Boolean algebra. Thus, quantum logic gives us more insights into the semantics behind the fuzzy norms algebraic product and algebraic sum."

# 3 Zadeh's visits in Romania

"Zadeh has been a friend of Romanian science, which he prized publicly several times. Zadeh dedicated a single paper to the memory of a scientist -namely, to Grigore C. Moisil, whom he named his friend and supporter. The topic he chose for the paper, approximate reasoning, was cherished by both of them." (H.-N. Teodorescu [113]). Professor Ioana Moisil<sup>13</sup> let us know that she met Zadeh in 1967 on a visit he had made in Bucharest to her uncle Grigore C. Moisil. This visit was amazing and productive. Zadeh was in Romania at the invitation of Prof. Mircea Stelian Petrescu.<sup>14</sup>

Grigore C. Moisil  $(1906-1973)^{15}$  himself, on 1968 in the paper "Lukasiewiczian Algebras" [68] writes the following:

Acknowledgment: "During professor L. Zadeh visit to Bucharest, in the autumn 1967, I became acquainted with his work about "fuzzy sets" as a set theory in a logic with totaly ordered set of logical values. The present work exposes the logic of propositions with a totaly ordered set of logical values. The models of this logic of propositions uses an algebraic technic very closed to that given by us in the study of models for propositional logic with a finite number of logic values". (This preprint of the Gr. C. Moisil was reprinted in his book [69], 311-324. His paper [67] was also reprinted in [69], 195-232).

Several Romanian scientists started to study fuzzy theory and their applications, obtaining valuable results, the best performers in the field were Constantin Virgil Negoita<sup>16</sup> and Dan Ralescu<sup>17</sup>, who published in collaboration two books, in 1974 [76] (in Romanian), and in 1975 [77](in English). "The first published book ever, specifically devoted to fuzzy sets is the one (in French) by Kaufmann (1973), translated into English in 1975 ([55]), closely followed by a mathematical treatise by Negoita and Ralescu (1975, [77]), based on a 1974 monograph in Romanian ([76])."(Dubois and Prade [34]).

Since 1978, Dan Butnariu<sup>18</sup> published some early papers in fuzzy mathematics field: [19–22]. Anca L. Ralescu<sup>19</sup> and Dan A.Ralescu have many papers published in the field, more quoted being [95](1984), and [96] (1997).

<sup>14</sup>Dr. Mircea S. Petrescu (born 1933) is an Emeritus Professor of the University Politehnica of Bucharest, Romania. He is a honor member of the Romanian Academy. He was a visiting professor at the University of California, Berkeley (1969-1990). He was a very good friend of Lotfi A. Zadeh.

<sup>15</sup>In 1996, Grigore C. Moisil received posthumously the IEEE Computer Pioneer Award for the development of polyvalent logic switching circuits, the Romanian School of Computing, and support of the first Romanian computer. Moisil, one of the greatest Romanian intellectuals, was first of all a gifted mathematician. [70]

<sup>16</sup>Dr. Constantin Virgil Negoita (born 1936 in Bucharest, Romania) is a Romanian computer scientist and writer. He published the first in the world monograph inspired from Zadeh's pioneering work: "Fuzzy Sets and Their Applications" (in Romanian, with Dan Ralescu, Bucharest, 1974) [76]. Since 1982 he served as a professor in Mexico and USA. Since 2009 he is a Professor Emeritus at Department of Computer Science, Hunter College, City University of New York. Other referential books authored by C.V. Negoita are [77–80].

<sup>17</sup>Dr. Dan Ralescu (graduated from University of Bucharest, Romania, 1972) is a Professor at the Department of Mathematical Sciences, University of Cincinnati, Ohio, USA. He is the coauthor of the first comprehensive monograph on fuzzy sets and systems, published in the early 1970s. He has authored and coauthored more than 60 papers in scientific Journals. In the late 1970s he has initiated the theory of fuzzy random variables and mixed models of uncertainty. His recent interests are in statistical decision making under various kinds of uncertainty. He was awarded the IFSA Fuzzy Pioneer in 2003.

<sup>18</sup>Dr. Dan Butnariu (1951-2008) was born in Romania and in 1984 he emigrated to Israel. He received his PhD from A.I. Cuza University of Iasi, Romania and was a post-doctoral fellow at the Weizmann Institute of Science. He was a professor at the University of Haifa and visiting professor in Linz, Texas, Rio, and CUNY. He published over 70 papers in approximation theory, fuzzy game theory, fuzzy topology, etc.

<sup>19</sup>Dr. Anca Ralescu (graduated from University of Bucharest, Romania, 1972) is a Professor at the Department of Mathematical Sciences, University of Cincinnati, Ohio, USA. During 1990-1995 she worked in Japan in the area of Fuzzy Engineering, in connection with the Laboratory for International Fuzzy Engineering (LIFE), including holding the LIFE Endowed Chair of Fuzzy Theory in the Systems Science Department at the Tokyo Institute of Technology.

<sup>&</sup>lt;sup>13</sup>Dr. Ioana I. Moisil is a Professor Honoris Causa of Agora University. She obtained the M.Sc. in mathematicsmechanics at the University of Bucharest, and the Ph.D. in mathematics-statistics and probabilities at the Romanian Academy. She has, as a second specialisation, informatics, and she obtained the scientific degree from the School of Public Health-Universite Libre de Bruxelles, Belgium. She started her career at the Institute for Research in Informatics - ICI Bucharest, then moved on to the Carol Davila University of Medicine, the Ministry of Health and since 1999 at the Lucian Blaga University of Sibiu. She was a full professor at the Lucian Blaga University of Sibiu.

After 2000 has been published lots of papers and books in fuzzy theory and applications fields, (co)authored by several Romanian scientists. We can enumerate some contributions quoted in this work: Alina Alb-Lupas [2], Razvan Andonie [3], Marius Balas [6–8], Valentina E. Balas [6–8, 58], Adrian I. Ban [9], Boldur E. Barbat [38], Barnabas Bede [11,12], Lucian Coroianu [9], Otilia Dragomir [30], Ioan Dzitac [22,36–41,73,88,107,118,119,145], Simona Dzitac [22,39–41,118,119], Ioan Felea [39–41,118], Sorin G. Gal [12], George Georgescu [44], Vasile Lupulescu [59], Dorel Mihet [65], Sorin Nadaban [22,71–74], Georgia I. Oros [86,87], Gheorghe Oros [2,86,87], Bogdana Pop-Stanojevic [88,106,107], Emil M. Petriu [90], Radu-Emil Precup [89–91], Stefan Preitl [89], Ioan M. Stancu-Minasian [106], Sergiu Rudeanu [100], Horia-Nicolai L. Tedorescu [111–113], Marius L. Tomescu [91], Tiberiu Vesselenyi [39–41,118,119] etc.

In 2003, Lotfi A. Zadeh became Doctor Honoris Causa of Aurel Vlaicu University in Arad at the suggestion of professor Valentina E. Balas, who met him at the IMPU 2002 conference in Annecy (France), was due to come to Arad in 2005 (SOFA2005) and to Oradea (SOFA2007), where Ioan Dzitac was very fortunate to meet him through prof. Balas. Lotfi A. Zadeh accepted to be a guest of Agora University in Oradea at ICCCC2008.

### 3.1 Remembering the beginnings

"The admiration of his followers is well known, but the picture which the critics drew of him was not always a pleasing one." (C.V. Negoita [81])

The following appreciations, authored by Constantin V. Negoita, are reprinted from [81], partially.

"[...] We know Zadeh as we know no other eminent man who have made the last half of the last century memorable in science history. A mere glance at the materials to which we have access still suffice to show that our information regarding him is of such a kind as to leave scarcely anything to be desired. In the first place we have his papers. They are written with talent. He has not only left a minute record of his research during a space of nearly five decades but he found real pleasure in communicating on paper.

As a scientist, he was consistent. His memoir of himself remains unfortunately a fragment, but enough was completed to illustrate that portion of his career during which fuzzy sets were promoted. But if we owe much to the communicativeness of Zadeh himself, we owe much also to the communicativeness of his peers. The admiration of his followers is well known, but the picture which the critics drew of him was not always a pleasing one. They saw him not as he presented himself to the fascinated eye of friendship.

It should be remembered that they knew him only from a vivid immediacy that derives from the speech of witnesses, full of slang of the moment yet extremely serious because the fuzzy set seemed of the utmost cultural significance. What they painted was what they understood, and what they understood was very little.

It is common for logicians to give truth conditions for predicates in terms of set theory. "John is tall" is defined to be true just in case the individual John is in the set of tall men. Zadeh used the same path and said that "tall men" is a fuzzy set when the membership is a matter of degree.

In presenting the theory of fuzzy sets, we hoped to break through the bars of the prison of set theory. To understand a fuzzy set, imagine a two-dimensional world called Flatland. Each Flatlander is incarcerated in a flat set. We can peel him off and place him back somewhere else. If we fling a Flatlander into our three-dimensional world, he can see only two-dimensional cross sections of our world, a family of crisp sets. Simply put, by adding another dimension, we can capture more features. This is what a fuzzy set does. It adds a new dimension: our evaluation of the membership. Using classical flat mathematics, a fuzzy set can be represented by a family of crisp sets, projected on the Flatland.

The classical rules of logic are represented by operations on the set with only two elements: true and false (0 and 1). In the universe of all sets - call it the category of sets - this important set is called classifier. When we tried to investigate the category of fuzzy sets, it was impossible to find a similar classifier. In fact in fuzzy set theory there is no fuzzy set of fuzzy subsets of a fuzzy set. The point is that there are two predicates in set theory: membership and equality. In the category of generalized sets, both can be fuzzy, but in fuzzy set theory only membership is allowed to be. This fact puzzled a lot of people.

Some critics bordered on the vituperative, and the tenets of fuzzy logic were dismissed as comical. Its arguments were declared frivolous and idle exercises in irrelevance and blasphemy. In 1977 Arbib wrote bad reviews for the periodicals, and in 1984 Zeleny published a paper on the (ir)relevancy of fuzzy set theory. That, no doubt, explains why Herbert Toth, in 1987, in his PhD Thesis at the University of Vienna suspected that probably something has gone wrong in the development and interpretation of the theory.

Toth didn't deny Zadeh's original definition to be natural, immediate and elegant. This assertion was sufficiently justified by the vast amount of literature in an epidemically growing number of papers and books.

Perhaps to deter us fuzzy people from further abuse, or perhaps only to improve our connection, Jim Bezdeck established our annual meetings. These are the origins of the international meetings that have today become pilgrimages, where all controversies were clarified when fuzzy sets were represented as families of crisp sets, and the Japanese started to implement fuzzy comptrollers, and, later, fuzzy systems were implemented as neural networks. But, still then, some mathematicians never understood fuzziness, because, for them, the precise specification of a set could be given only by binary logic.

The anatomy of a boom is simple. Over time, most ideas will rise in value. As this happens, people are attracted to them and this causes the ideas to rise more. This further gain attracts more people and gradually, perhaps over a period of a century, the number of people looking for this increase in value comes to determine what ideas are worth. The knowledgeable man, as he unwisely considers himself, is now concerned with the way an idea is attracting interest. That, rightly for the moment, determines its value.

The binary logic, beautiful, useful, and promising, determined the modern era, obsessed with mathematical models. Scientific truths became the pillars of progress. Zadeh, speaking of degrees of truth, shocked the foundation of modernity, and became the postmodern of information sciences. More than that, he defined postmodernity as a return to premodernity.

Why do postmodern scientists, in their advancing years, when love for precision survives, but only barely, cleave to the fluid vagueness as though it were cable for rappelling and not a tightrope any longer?

To embrace the whole from one point of view. The remarks made are less detailed, but more sure. You perceive each object less distinctly, but you describe the facts with more certainty. The details of the immense picture are lost in the shade, but you conceive a clear idea of the entire object.

In the philosophy of science this fact has been known for a long time. In the sixth century, Leontius from Bizantium observed that our impression of the world is vague, not revealing the details. If we attempt to particularize by division into species and individuals, the general view is lost: we are heading not towards truth but towards an infinite regress. In 1906, in France, Pierre Duhem, in a book about physics, its object and structure, distinguished between practical facts expressed in vague, ordinary language and theoretical facts expressed in precise, quantitative language. He argued that confidence in the truth of a vague assertion may be justified just because of its vagueness, which makes the assertion compatible with a whole range of observed facts. There is a balance between precision and certainty: one is increased only to the detriment of the other. [...]

Berkeley was the place where Zadeh was preaching his gospel, but his onslaughts were mercy compared with those terrible philippics in which, at Vanderbilt University in Nashville, Georgescu Roegen gave vent to his rage against arithmomorphism, the worship of numbers. In 1971, in an extraordinary book, The Entropy Law and the Economic Process, among the thesis that he defends is the claim that concepts are not arithmomorphic. They do not overlap. Concepts like "good" or "tall" have no boundaries. Instead they are surrounded by a penumbra within which they overlap with their opposites. At a particular historical moment, he notes, a nation may be both a democracy and a dictatorship just as there is an age when a man is both young and old. To the category of concepts we cannot apply the fundamental law of the binary logic, the principle of excluded middle (X cannot be both A and non-A). [...]

Basically, fuzzy logic is based on the same feeling, and its applications allow engineers to create machines that approach human responses to stimuli, working with incomplete and unclear data to generate positive actions. Using fuzzy logic, Japanese washing machines are able to decide how dirty clothes are, how much water and soap should be used to wash them, and how long it should take to get them clean, all things an experienced launderer would know how to do instinctively. To me, the most impressive accomplishment was a fuzzy system built by Michio Sugeno, in 1985, at the Tokyo Institute of Technology, when he stabilized a helicopter that lost a rotor blade. No human pilot can manage that, and no mathematical model either.

Fuzzy systems, linguistically inspired, are a direct consequence of the seminal papers of Zadeh, published in the 1970s.

In 1975 classical fuzzy set theory had reached its apogee, since solutions of its basic problems were now at hand. Classical fuzzy set theory then changed from an heroic phase, in which we addressed ourselves to hitherto unfathomable questions, to an academic phase, in which a wealth of detail, albeit most important detail, was worked on by an army of competent scholars and technologists following well-established lines. The period of trail blazing was over, though most of the practical benefits were yet to be reaped."

## 3.2 On the meaning of approximate reasoning: An unassuming subsidiary to Lotfi Zadeh's paper dedicated to the memory of Grigore Moisil

"As Grigore Moisil has also did, Zadeh emphasized in all his work how much language is fundamentally creative." (H.-N. Teodorescu [111])

The following text, authored by H.-N. Teodorescu<sup>20</sup>, is reprinted from [111], partially.

"The concept of "approximate reasoning" is central to Zadeh's contributions in logic. Standard fuzzy logic as we use today is only one potential interpretation of Zadeh's concept. I discuss various meanings for the syntagme "approximate reasoning" as intuitively presented in the paper Zadeh dedicated to the memory of Grigore C. Moisil in 1975 [134].

I perceive three central ideas in Zadeh's wide conceptual construction in his work until now. The first one is that words and propositions in natural languages, consequently human thoughts are not representable by standard sets and standard binary predicates. The second central idea in many of Zadeh's papers is that humans perform computations in an approximate manner that

<sup>&</sup>lt;sup>20</sup>Dr. Horia-Nicolai L. Teodorescu is a Romanian computer scientist, full member of the Romanian Academy. He is a member in editorial board of IJCCC. He have several important contributions in fuzzy/neuro-fuzzy systems, and also he publish many applications of fuzzy methods in engineering, medicine, social sciences and artificial life. He has authored or co-authored over 300 journal and conference papers, holds 24 national and international patents and has received numerous national and international awards and prizes.

real numbers can not represent well. The third idea, which Zadeh presented in his more recent works, is that of granularity of human mental representations and reasoning. These three strong points were represented by Zadeh in various forms and synthesized in the title of his paper on "computing with words" [136].

Most frequently - and Zadeh himself is doing no exception - authors cite as the initial paper clearly stating the approximate reasoning model the one published in the journal Information Sciences, July 1975 [133]. However, another paper published in Synthese [134], during the same year, deals with the main ideas in approximate reasoning, not to mention a conference paper published in 1974.

The paper published in Synthese is important in several ways. First, it offers clear explanations of the meanings associated by Zadeh to the syntagme "approximate reasoning". Second, the paper was published in a well-established journal that "spans the topics of Epistemology, Methodology and Philosophy of Science", thus boldly reaching to a large audience in a set of fundamental disciplines and daringly affirming the importance of the new concepts. Third, the paper is important for the Romanian scientists because it re-enforces the understanding of Grigore Moisil's contributions and the deep connections between Zadeh and Moisil. [...]

Zadeh starts the paper [134] from the common-sense remark that "It is a truism that much of human reasoning is approximate rather than precise in nature." From that remark, he builds a broad program for research.

The program that Zadeh establishes remains, in my opinion, unfinished. In the abstract of the paper discussed here, Zadeh states "Since T [the truth-value set] is not closed under the operations of negation, conjunction, disjunction and implication, the results of an operation on truth-values in T requires, in general, a linguistic approximation by a truth value in T." With this clarification, classic fuzzy logic as we know today is not a proper representation of Zadeh's original ideas.

There are two ways to interpret the above quotation. The first interpretation is in the frame of the classic thinking and runs as follows. Because language is a set of propositions (we restrict the discussion to truth-valuable propositions only), under whatever logic, all simple and composed propositions have a truth value. Denoting the set of truth values by T, the above remark by Zadeh has no effect. Moreover, when defining the truth-valuation function we already need to know the set of truth values, which again makes ineffective Zadeh's remark. Standard fuzzy logic pursue this direction of thinking, as it starts with. Under this approach, as T is given, what is needed is to define the logic operators for the respective logic.

The second interpretation is constructive. It may start with the assumption that language is not predetermined and must be constructed as a recursive, dynamic process, as poets, writers, other language professionals, and laymen do every day. Whenever a new proposition is invented, it is assigned a meaning. The meaning includes what we conveniently name "truth", a coverage degree of the reality that we need for making inferences. The *truth* may or may not be numerically representable. Moreover, the truth of a composed proposition may not be representable by the truth of each of the initial propositions. Therefore, the "set of truth values" evolves continuously. That, in turn, creates a stumbling block. Because we are supposed to know the truth of the original sentences, but not of the result, how are we supposed to infer? The answer proposed by Zadeh is that we still have an approximate truth in the initial set of truth values, approximating the truth of the new proposition. He hypothesizes that there is always an inverse application, which I will name *projection*, from the new truth set to the original one, indicating a truth degree in the original set that approximates the truth degree of the composed proposition. That makes our reasoning possible, if approximate.

This way of thinking, more or less directly suggested by Zadeh, opens the door to several formal descriptions. I sketch below a loose formalism for the recursive approach.

Consider a language  $L = (\{p\}, c_1, c_2, \cdots, c_r)$ , where p denotes extant propositions and  $c_k$  logic operators (either unary or binary). Notice that L is an initial language, meaning that the initial set  $\{p\}$  is evolvable, that is, it is recursively increased by adding propositions correctly formed from the initial ones through logic operations. Assume that any proposition has a characteristic named truth value. Thus, there is some set T of truth values (I use for this set the same notation as Zadeh's), as well as an application such that for any proposition p there is a truth value in T. Also assume that any propositions can be concatenated to produce a new or extant proposition. Whenever such proposition is new in the language, it has a characteristic truth value which is not necessarily in T. We can regard the creation of new propositions (including those used for reasoning) as producing an application  $T \to TI$ , as for the negation operator, or  $T \times T \times \cdots \to TI$ , as for the connectives. The new T-set which includes the original one. Moreover, for satisfying Zadeh's hypothesis, there is a projection operation  $\vartheta$ . Notice that whenever a new proposition is produced, L is modified.[...]

The main requirements for the  $T^*$ -set are: i) The truth-attribute of a proposition is represented by an application  $\theta: L \to T$ , where T is measurable. ii) For any two extant propositions  $p_1$  and  $p_2$  and for any connective c, the new proposition  $p_1cp_2$  has a truth degree such that  $T \cup \{\theta(p_1cp_2)\}$  is measurable and includes the measurable space T. iii) For any valid linguistic construction that uses logic operators, there is an application  $\vartheta: T' \to T'$ , named back-projection. iv) For any connective c, there is a formula  $f_c$  such that  $\vartheta(\theta(p_1cp_2))$  and  $f_c(\theta(p_1), \theta(p_2))$  are two points close enough in T.

Standard fuzzy logic obviously satisfies the above conditions, with T = T'. Replacing the measurable space with a metric one, we obtain another construction, which may be closer to Zadeh's intuition in [134]. Using distances  $d: T \to T$ , the conditions to satisfy are:

$$\begin{aligned} d(\vartheta(\neg p), 1 - \theta(p)) < \varepsilon \\ d(\vartheta(p \bigwedge q), \min(\theta(p), \theta(q)) < \varepsilon \\ d(\vartheta(p \bigvee q), \max(\theta(p), \theta(q)) < \varepsilon, \end{aligned}$$

where  $\varepsilon$  is the allowed approximation error.

As Grigore Moisil has also did, Zadeh emphasized in all his work how much language is fundamentally creative. By stating that " $T \cdots$  is not closed under the logic operations", he highlights that there is always generation of new meaning, or at least an evolution of the meaning, whenever a sentence is uttered. I believe that this is one of the fundamental contributions made by Zadeh's work until now.

Understanding the creative process in languages and in reasoning was significantly elucidated by Zadeh's work, yet much remains to be done for deriving conclusions and theoretical developments in the directions pointed to by his works. The developments informally suggested above may indicate several such directions, yet others may be put forward in a more formal manner during the years to come."

## 4 Visit of Lotfi A. Zadeh at the Agora University

In 2008 L.A. Zadeh was a keynote speaker at the International Conference on Computers Communications and Control (ICCCC2008)<sup>21</sup>, organized by the Agora University of Oradea, Romania.



D. Tufis, I. Dzitac, L.A. Zadeh, M.J. Manolescu and F.G. Filip (ICCCC2008, Agora University)

After ICCCC2008 we published in vol. 3 of IJCCC/2008, in a supplementary issue, several papers on topics related to fuzzy theory and its applications, authored by: I. Dzitac [37]; L.A. Zadeh [140]; M.M. Balas, and V.E. Balas [6]; D.G. Radojevic [92]; A. Bazoula , M.S Djouadi., and H. Maaref [10]; G.C.Crisan, and E. Nechita [25]; O. Dragomir, R. Gouriveau, and N. Zerhouni [30]; S. Dzitac , I. Felea, I. Dzitac, and T. Vesselenyi [40]; N. Nikolova, S. Ahmed, K. Tenekedjiev [82].

Under the umbrella of the ICCCC2008, on the occasion of an exploratory workshop, we edited the volume "From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence", co-edited by Lotfi A. Zadeh, Dan Tufis, Florin Gheorghe Filip, Ioan Dzitac [145]. In this volume there were published 14 papers and one appendix, by authors from Bulgaria, Chile, Hungary, Romania, Serbia and Spain: Marius M. Balas, and Valentina E. Balas [7,8]; Ioan Buciu, and Ioan Nafornita [18]; Camelia Chira, Camelia M. Pintea, and D. Dumitrescu [27]; Felisa M. Cordova, and Luis E. Quezada [28]; Simona Dzitac, Ioan Felea, Ioan Dzitac and Tiberiu Vesselenyi [41]; Gaston Lefranc [57]; Natalia Nikolova, Daniela Toneva, Sevda Ahmed, and Kiril

<sup>&</sup>lt;sup>21</sup>"Dear Professor Dzitac, Please accept my apology for not writing to you earlier. I was away on travel much of the time during the past three weeks. With regard to sending you a paper for publication in your book, I do not have a written version of my presentation. I have other papers, which are attached, which are scheduled for publication in journals. Currently, I am working on a paper but it is on a subject different from the one that I presented in your Conference. I appreciate your including my name as a co-editor of your book "From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence." However, be sure to list my name after the names of Professor Tufis, Filip and yourself. I should like to take this opportunity again to extend to you, Professor Tufis, Professor Filip and your associates my great appreciation for the very warm welcome which was extended to me in Oradea. With my warmest regards, Sincerely, Lotfi Zadeh." (From: Lotfi A. Zadeh, Sent: Wednesday, July 23, 2008 2:21 AM; To: Ioan Dzitac).



Volume co-edited by L.A. Zadeh, D. Tufis, F.G. Filip and I. Dzitac (2008, Romanian Academy) [145]



First slide of the plenary lecture of Lotfi A. Zadeh at ICCCC2008, Agora University of Oradea, Romania

Tenekedjiev [83]; Dragan G. Radojevic [93]; Emilio del Rosal, Rafael Nunez, Carlos Castaneda, and Alfonso Ortega, [94]; Imre J. Rudas, and Janos Fodor [97]; Gheorghe Stefanescu, and Camelia Chira, [108]; Dan Tufis [115] and Lotfi A. Zadeh [141], [142].

Also since 2008, he accepted to join the editorial board of our IJCCC.

IJCCC has published in Zadeh's honour two special issues:

1. Special Issue on Fuzzy Sets and Systems Dedicated to the 90th Birthday of Prof. Lotfi A. Zadeh, Guest Editors: Valentina Emilia Balas, Ioan Dzitac and Horia-Nicolai Teodorescu (IJCCC, Year: 2011, Volume: 6, Number: 3).

In the first special issue papers have been published by the following authors: L.A. Zadeh [144]; H. Benitez-Perez, F. Cardenas-Flores, F. Garcia-Nocetti [14]; I. Dzitac, T. Vesselenyi, R.C. Tarca [39]; T.C. Lin, C.H. Kuo, V.E. Balas [58]; A. Meyer, H.-J. Zimmermann [63]; G.I. Molnarka, L.T. Koczy [66]; C.V. Negoita [81]; C.N. Nyirenda, F. Dong, K. Hirota [84]; M.Z. Reformat, R.R. Yager, Z. Li, N. Alajlan [98]; N. Sahab, H. Hagras [101]; R. Seising [104]; D.E. Tamir, A. Kandel [109]; H.-N.L. Teodorescu [111].

2. Special Issue on Fuzzy Sets and Applications Dedicated to the 50th Anniversary of "Fuzzy Sets" Published by Lotfi A. Zadeh in 1965 (Year: 2015, Volume: 10, Issue: 6).

In this second special issue the following authors have published articles: R.R. Yager [127]; I. Dzitac [36]; S. Ashraf, A. Rehman, E.E. Kerre [4]; O. Bologa, R.E. Breaz, S.G. Racz [16]; Z.-B. Du, T.-C. Lin, T.-B. Zhao [31]; V. Kreinovich, C. Stylios [56]; S. Nadaban [71]); R.-E. Precup, M.L. Tomescu, E.M. Petriu [91]; X. Tang, L. Shu [110]; H.-N.L. Teodorescu [112]; Z. Turskis, E.K. Zavadskas, J. Antucheviciene, N. Kosareva [117]; H.-D. Wang, S.-C. Guo, S.M. Hosseini Bamakan, Y. Shi [120]; T. Wang, G. Zhang, M.J. Perez-Jimenez [121]; H. Xu, R. Vilanova [126].

IJCCC also published several papers on applications of fuzzy methods, such as [118], and [119].

### 4.1 Lotfi A. Zadeh: Foreword (IJCCC, Issue 3, 2011)

"In the early nineties, a thought that began to crystallize in my mind was that in most of the applications of fuzzy logic linguistic concepts play an important, if not very visible role." (L.A. Zadeh [144])

The following text, authored by Lotfi A. Zadeh, is reprinted integral from [144].

"I feel honored by the dedication of the Special Issue of IJCCC to me. I should like to express my deep appreciation to the distinguished Co-Editors and my good friends, Professors Balas, Dzitac and Teodorescu, and to distinguished contributors, for honoring me. The subjects which are addressed in the Special Issue are on the frontiers of fuzzy logic.

The Foreword gives me an opportunity to share with the readers of the Journal my recent thoughts regarding a subject which I have been pondering about for many years - fuzzy logic and natural languages. The first step toward linking fuzzy logic and natural languages was my 1973 paper," Outline of a New Approach to the Analysis of Complex Systems and Decision Processes." Two key concepts were introduced in that paper. First, the concept of a linguistic variable - a variable which takes words as values; and second, the concept of a fuzzy if- then rule - a rule in which the antecedent and consequent involve linguistic variables. Today, close to forty years later, these concepts are widely used in most applications of fuzzy logic.

The second step was my 1978 paper, "PRUF - a Meaning Representation Language for Natural Languages." This paper laid the foundation for a series of papers in the eighties in which a fairly complete theory of fuzzy - logic-based semantics of natural languages was developed. My theory did not attract many followers either within the fuzzy logic community or within the linguistics and philosophy of languages communities. There is a reason. The fuzzy logic community is largely a community of engineers, computer scientists and mathematicians - a community which has always shied away from semantics of natural languages. Symmetrically, the linguistics and philosophy of languages communities have shied away from fuzzy logic.

In the early nineties, a thought that began to crystallize in my mind was that in most of the applications of fuzzy logic linguistic concepts play an important, if not very visible role. It is this thought that motivated the concept of Computing with Words (CW or CWW), introduced in my 1996 paper "Fuzzy Logic = Computing with Words." In essence, Computing with Words is a system of computation in which the objects of computation are words, phrases and propositions drawn from a natural language. The same can be said about Natural Language Processing (NLP.) In fact, CW and NLP have little in common and have altogether different agendas.

In large measure, CW is concerned with solution of computational problems which are stated in a natural language. Simple example. Given: Probably John is tall. What is the probability that John is short? What is the probability that John is very short? What is the probability that John is not very tall? A less simple example. Given: Usually Robert leaves office at about 5 pm. Typically it takes Robert about an hour to get home from work. What is the probability that Robert is home at 6:15 pm.? What should be noted is that CW is the only system of computation which has the capability to deal with problems of this kind. The problem-solving capability of CW rests on two key ideas. First, employment of so-called restriction-based semantics (RS) for translation of a natural language into a mathematical language in which the concept of a restriction plays a pivotal role; and second, employment of a calculus of restrictions - a calculus which is centered on the Extension Principle of fuzzy logic.

What is thought-provoking is that neither traditional mathematics nor standard probability theory has the capability to deal with computational problems which are stated in a natural language. Not having this capability, it is traditional to dismiss such problems as ill-posed. In this perspective, perhaps the most remarkable contribution of CW is that it opens the door to empowering of mathematics with a fascinating capability - the capability to construct mathematical solutions of computational problems which are stated in a natural language. The basic importance of this capability derives from the fact that much of human knowledge, and especially world knowledge, is described in natural language.

In conclusion, only recently did I begin to realize that the formalism of CW suggests a new and challenging direction in mathematics - mathematical solution of computational problems which are stated in a natural language. For mathematics, this is an unexplored territory."

### 4.2 Ronald R. Yager: Foreword (IJCCC, Issue 6, 2015)

"Interestingly a number of researchers from Romania, the home of this journal, were among the early supporters of ideas presented by Zadeh." (R.R. Yager [127])

Following text, authored by Ronald R. Yager<sup>22</sup>, is reprinted integral from [127].

"Here we are celebrating the fiftieth anniversary of Lotfi Zadeh's pioneering paper Fuzzy Sets that appeared in Information and Control in 1965. While the paper was clear, direct and easy to understand the ideas presented were revolutionary and ground breaking. This article now has close to sixty thousand citations as noted in Google Scholar. Clarity and simplicity are the hallmark of the writing of Zadeh. This has always reminded me of the writings of Sigmund Freud. One rarely needs to draw on complex mathematics to read Zadeh's papers.

The capacity of fuzzy sets to represent and manage imprecise linguistic concepts has proven to

<sup>&</sup>lt;sup>22</sup>Ronald R. Yager is a Professor of Information Systems and Director of the Machine Intelligence Institute at Iona College. He is among the world's most highly cited researchers with over 61,000 citations to his work in Google Scholar. He has published over 500 papers and edited over 30 books in areas related to fuzzy sets, human behavioral modeling, decision-making under uncertainty and the fusion of information.

be of great use in the modern technological world where there is now a great interest in building intelligent systems that can model human reasoning but take advantage of the vast amount of information available on the Internet. If the idea of fuzzy sets was not introduced in Zadeh's ground breaking paper in the 1960's it would have naturally arisen in early 2000's as we moved into intelligent systems. However the early reception of fuzzy sets was not very promising both the Artificial Intelligence community and the probabilistic community were very dubious of the worth of this new field.

Interestingly a number of researchers from Romania, the home of this journal, were among the early supporters of ideas presented by Zadeh. Zadeh persevered in the face of adversity, describing himself as thick skinned, until mid 1980's when the Japanese engineers provided significant applications of fuzzy sets in control systems. Particularly notable among these applications was the use of fuzzy control to the Sendai subway. These applications brought a new appreciation to the possibilities of fuzzy sets and clearly changed its history.

The editors of this Special Issue of the International Journal of Computers Communications & Control dedicated to the 50th anniversary of the publication of Lotfi Zadeh's pioneering paper Fuzzy Sets have provided a collection of papers representative of the current state of the field of fuzzy sets. Included in this issue are papers investigating some current theoretical issues and applied papers in domains in which fuzzy sets has introduced some benefits.

The editorial team is to be congratulated for providing a wonderful anniversary gift to Professor Zadeh and a useful collection of articles for the community."

# 5 Several concepts of fuzzy mathematics

Following concepts and results are partial reprinted from papers [36], [73], [71], [74], and [107] (authored/co-authored by I. Dzitac, S. Dzitac, S. Nadaban and B. Stanojevic).

#### 5.1 Fuzzy sets

Definition 1. [129] A fuzzy set in X is a function  $\mu : X \to [0, 1]$ . We denote by  $\mathcal{F}(X)$  the family of all fuzzy sets in X.

Remark 2. In fact  $\mu$  is the membership function of a fuzzy set A of X and the value  $\mu(x)$  represents "the grade of membership" of x to fuzzy set A. But, in this paper, we adopt the convention to identify fuzzy sets with their membership functions. This identification was first used by J.A. Goguen [46].

Remark 3. As any subset of X can be identified with its characteristic function we remark that fuzzy sets generalize subsets.

Definition 4. [129] Let  $\mu, \nu$  be fuzzy sets in X. The union of fuzzy sets  $\mu \notin \nu$ , denoted  $\mu \vee \nu$ , the intersection of fuzzy sets  $\mu \notin \nu$ , denoted  $\mu \wedge \nu$ , the complement of fuzzy set  $\mu$ , denoted  $1 - \mu$ , are fuzzy sets in X, defined by

$$(\mu \lor \nu)(x) = \max\{\mu(x), \nu(x)\}$$

$$(1)$$

$$(\mu \wedge \nu)(x) = \min\{\mu(x), \nu(x)\}$$
(2)

$$\mathcal{C}(\mu)(x) = 1 - \mu(x) \tag{3}$$

Definition 5. The union of the fuzzy sets  $\{\mu_i\}_{i \in I}$  is defined by

$$\left(\bigvee_{i\in I}\mu_i\right)(x) = \sup\{\mu_i(x) : i\in I\}.$$

The intersection of the fuzzy sets  $\{\mu_i\}_{i\in I}$  is defined by

$$\left(\bigwedge_{i\in I}\mu_i\right)(x) = \inf\{\mu_i(x) : i\in I\}.$$

Definition 6. Let  $\alpha \in (0,1]$ , and let  $\mu$  be a fuzzy set in X. The  $\alpha$ -level set  $[\mu]_{\alpha}$  is defined by

$$[\mu]_{\alpha} := \{ x \in X : \mu(x) \ge \alpha \} .$$

The support of  $\mu$  is

supp 
$$\mu := \{x \in X : \mu(x) > 0\}$$

Definition 7. [129] Let X be a vector space over a field  $\mathbb{K}$  (where  $\mathbb{K}$  is  $\mathbb{R}$  or  $\mathbb{C}$ ). A fuzzy set  $\mu$  is called convex if

$$\mu(\lambda x_1 + (1 - \lambda)x_2) \ge \min\{\mu(x_1), \mu(x_2)\}, \ (\forall)x_1, x_2 \in X, (\forall)\lambda \in [0, 1].$$

The extension principle is undoubtedly one of the most important of Zadeh's contribution in fuzzy set theory, allowing to extend in a fuzzy context almost any mathematical concept. The extension principle was introduced by Zadeh [129] in 1965, and then it suffered many changes: Zadeh [133]; Jain [52]; Dubois & Prade [32]. [...]

Let  $X = X_1 \times X_2 \times \cdots \times X_r$  and  $\mu_1, \mu_2, \cdots, \mu_r$  be fuzzy sets in  $X_1, X_2, \cdots, X_r$ , respectively. Let  $f: X \to Y$ . The extension principle allows us to define a fuzzy set in Y by

$$\mu(y) = \begin{cases} \sup_{(x_1, \dots, x_r) \in f^{-1}(y)} \min\{\mu_1(x_1), \dots, \mu_r(x_r)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

#### 5.2 Fuzzy relations

It is well known that the fuzzy relations play an important role in fuzzy modeling and fuzzy control and they also have important applications in relational databases, approximate reasoning, preference modeling, medical diagnosis.

The concept of fuzzy relation was introduced by L.A. Zadeh:

A fuzzy relation T between two nonempty sets X and Y is a fuzzy set in  $X \times Y$ , i.e. it is a mapping  $T: X \times Y \to [0,1]$ . We denote by FR(X,Y) the family of all fuzzy relations between X and Y.

For  $x \in X$  we denote by  $T_x$  the fuzzy set in Y defined by  $T_x(y) = T(x, y)$ . Thus, a fuzzy relation can be seen as a mapping  $X \ni x \mapsto T_x \in \mathcal{F}(Y)$ , where  $\mathcal{F}(Y)$  represents the family of all fuzzy sets in Y. [36]

#### 5.3 Fuzzy real numbers and fuzzy interval

Definition 8. A fuzzy set in  $\mathbb{R}$ , namely a mapping  $x : \mathbb{R} \to [0,1]$ , with the following properties:

- 1. x is convex, i.e.  $x(t) \ge \min\{x(s), x(r)\}$ , for  $s \le t \le r$ ;
- 2. x is normal, i.e.  $(\exists)t_0 \in \mathbb{R}$  :  $x(t_0) = 1$ ;
- 3. x is upper semicontinuous, i.e.

 $(\forall)t \in \mathbb{R}, (\forall)\alpha \in (0,1] : x(t) < \alpha,$ 

$$(\exists)\delta > 0$$
 such that  $|s - t| < \delta \Rightarrow x(s) < \alpha$ 

is called a fuzzy real number. We will denote by  $\mathbb{R}(I)$  the set of all fuzzy real numbers.

Remark 9. Let  $x \in \mathbb{R}(I)$ . For all  $\alpha \in (0, 1]$ , the  $\alpha$ -level sets  $[x]_{\alpha} = \{t : x(t) \geq \alpha\}$  are closed intervals  $[a_{\alpha}, b_{\alpha}]$ , where the values  $a_{\alpha} = -\infty$  and  $b_{\alpha} = \infty$  are admissible. When  $a_{\alpha} = -\infty$ , the interval  $[a_{\alpha}, b_{\alpha}]$  will be denoted by  $(-\infty, b_{\alpha}]$ .

Definition 10. A fuzzy real number x is called non-negative if  $x(t) = 0, (\forall)t < 0$ . The set of all non-negative real numbers will be denoted by  $\mathbb{R}^*(I)$ .

*Remark* 11. For each  $r \in \mathbb{R}$  we can consider the fuzzy real number  $\overline{r}$  defined by

$$\overline{r}(t) = \begin{cases} 1 & \text{if } t = r \\ 0 & \text{if } t \neq r \end{cases}$$

These fuzzy numbers are called crisp. Thus  $\mathbb{R}$  can be embedded in  $\mathbb{R}(I)$ .

Definition 12. [64] The arithmetic operations  $+, -, \cdot, /$  on  $\mathbb{R}(I)$ , are defined by:

$$(x+y)(t) = \bigvee_{s \in \mathbb{R}} \min\{x(s), y(t-s)\}, \ (\forall)t \in \mathbb{R}$$

$$(4)$$

$$(x-y)(t) = \bigvee_{s \in \mathbb{R}} \min\{x(s), y(s-t)\}, \ (\forall)t \in \mathbb{R}$$
(5)

$$(xy)(t) = \bigvee_{s \in \mathbb{R}^*} \min\{x(s), y(t/s)\}, \ (\forall)t \in \mathbb{R}$$
(6)

$$(x/y)(t) = \bigvee_{s \in \mathbb{R}} \min\{x(ts), y(s)\}, \ (\forall)t \in \mathbb{R}$$
(7)

Remark 13. Previous definitions are special cases of Zadeh's extension principle.

*Remark* 14. The additive and multiplicative operations are associative and commutative with the identities  $\overline{0}$  and  $\overline{1}$ , where

$$\overline{0}(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}, \ \overline{1}(t) = \begin{cases} 1 & \text{if } t = 1 \\ 0 & \text{if } t \neq 1 \end{cases}$$

Remark 15. It is obvious that

1.  $-x = \overline{0} - x;$ 2. (-x)(t) = x(-t);3. x - y = x + (-y);4. -(x + y) = (-x) + (-y).

Definition 16. The absolute value |x| of  $x \in \mathbb{R}(I)$  is defined by

$$|x|(t) = \begin{cases} \max\{x(t), x(-t)\} & \text{if } t \ge 0\\ 0 & \text{if } t < 0 \end{cases}$$

Proposition 17. [54] The equations  $a + x = \overline{0}$  and  $ax = \overline{1}$  have unique solutions if and only if a is crisp.

Definition 18. [42] A partial ordering on  $\mathbb{R}(I)$  is defined by

$$x \leq y$$
 if  $a_{\alpha}^1 \leq a_{\alpha}^2$  and  $b_{\alpha}^1 \leq b_{\alpha}^2$ ,  $(\forall) \alpha \in (0, 1]$ ,

where  $[x]_{\alpha} = [a^1_{\alpha}, b^1_{\alpha}]$  and  $[y]_{\alpha} = [a^2_{\alpha}, b^2_{\alpha}]$ .

Proposition 19. [54] If  $[a_{\alpha}, b_{\alpha}], 0 < \alpha \leq 1$ , are the  $\alpha$ -level sets of a fuzzy real number x, then:

- 1.  $[a_{\alpha_1}, b_{\alpha_1}] \supseteq [a_{\alpha_2}, b_{\alpha_2}], \ (\forall) 0 < \alpha_1 \leq \alpha_2;$
- 2.  $[\lim_{\substack{k\to\infty\\\alpha,}} a_{\alpha_k}, \lim_{k\to\infty} b_{\alpha_k}] = [a_{\alpha}, b_{\alpha}],$  where  $\{\alpha_k\}$  is an increasing sequence in (0, 1] converging to

Conversely, if  $[a_{\alpha}, b_{\alpha}]$ ,  $0 < \alpha \leq 1$ , is a family of non-empty intervals which satisfy the conditions (1) and (2), then the family  $[a_{\alpha}, b_{\alpha}]$  represents the  $\alpha$ -level sets of a fuzzy real number.

Remark 20. As  $\alpha$ -level sets of a fuzzy real number is an interval, there is a debate in the nomenclature of fuzzy real numbers. D. Dubois and H. Prade [32] suggested to call this fuzzy interval.

# 6 Fuzzy logic and fuzzy languages

The following subsections, authored by Rudolf  $\text{Seising}^{23}$  in 2011, are reprinted from [104], partially.

### 6.1 From logic of inexact concepts to fuzzy logic

"It should be noted that a membership function may be regarded as a predicate in a multivalued logic in which the truth values range over [0, 1]." (L.A. Zadeh)

To understand what happened from coming from Fuzzy Sets and Systems to the idea of the computational theory of perceptions (CTP) we have go once again back to the roots.

In the 1960s Zadeh looked for applying fuzzy sets in linguistics. This idea led to interdisciplinary scientific exchange on the campus of the University of California at Berkeley between him and the mathematician Joseph Goguen (1941-2006) - who was a Ph. D. student of his, his Berkeley-colleague Hans-Joachim Bremermann (1926-1996), who was then in the mathematics department on the one hand and between the psychologist Eleanor Rosch (Heider) (born 1938) and the Berkeley-linguist George Lakoff (born 1941) on the other.

Zadeh had served as first reviewer for Goguens's Ph.D. thesis "Categories of Fuzzy Sets" [45] and Bremermann served as the second.

In this work, Goguen generalized the fuzzy sets to so-called "L-sets" [47]. An L-set is a function that maps the fuzzy set carrier X into a partially ordered set  $L: A: X \to L$ .

The partially ordered set L Goguen called the "truth set" of A. The elements of L can thus be interpreted as "truth values"; in this respect, Goguen then also referred to a *Logic of Inexact Concepts* [48].

Since Zadeh's earlier definition had established this truth set as the unit interval, Fuzzy Set Theory was very soon associated with multi-valued logics, and also Lotfi Zadeh mentioned this in later papers, e.g.: "It should be noted that a membership function may be regarded as a predicate in a multivalued logic in which the truth values range over [0, 1]."

Goguen's generalization of the set of values to a set L for which the only condition was to be partially ordered cleared up these misunderstandings. Goguen's work was laid out in terms of

<sup>&</sup>lt;sup>23</sup>Rudolf Seising received his MS degree in mathematics from the Ruhr University of Bochum in 1986, and PhD from the Ludwig Maximilians University (LMU) of Munich in 1995. In 2008 and from 2014 to 2016 he was acting as a professor for the history of science at the Friedrich-Schiller-University Jena. In 2010 he was acting as a professor for science history at the Ludwig-Maximilians-University Munich. He has been Visiting Scholar at the University of California at Berkeley in 2000, 2001 and 2002 and at the University of Turku, Finland in 2008. From 2009 to 2014 he was Adjoint Researcher at the European Centre for Soft Computing in Mieres, Spain.

logical algebra and category theory, and his proof of a representation theorem for L-sets within category theory justified Fuzzy Set Theory as an expansion of set theory.

### 6.2 Fuzziness for biology and computer science. Fuzzy algorithms

Also in 1969 Zadeh gave a talk on "Biological Applications of the Theory of Fuzzy Sets and Systems" [130] where he proposed his new mathematical theory to the life scientists, when he wrote:

"The great complexity of biological systems may well prove to be an insuperable block to the achievement of a significant measure of success in the application of conventional mathematical techniques to the analysis of systems [130]".

"By 'conventional mathematical techniques' in this statement, we mean mathematical approaches for which we expect that precise answers to well-chosen precise questions concerning a biological system should have a high degree of relevance to its observed behavior. Indeed, the complexity of biological systems may force us to alter in radical ways our traditional approaches to the analysis of such systems. Thus, we may have to accept as unavoidable a substantial degree of fuzziness in the description of the behavior of biological systems as well as in their characterization." [130]

In the same year he wrote more generally:

"What we still lack, and lack rather acutely, are methods for dealing with systems which are too complex or too ill-defined to admit of precise analysis. Such systems pervade life sciences, social sciences, philosophy, economics, psychology and many other 'soft' fields."

Since that time, Zadeh is inspired by the "remarkable human capability to perform a wide variety of physical and mental tasks without any measurements and any computations", e. g. parking a car, playing golf, deciphering sloppy handwriting, and summarizing a story.

However, in 1970, 20 years after later then his first paper on "Thinking machines", Zadeh was aware of Turing's philosophical article when he presented his paper "Fuzzy Languages and their Relations to Human and Machine Intelligence" at the conference "Man and Computer" in Bordeaux, France: "The question of whether or not machines can think has been the subject of many discussions and debates during the past two decades." He continued:

"As computers become more powerful and thus more influential in human affairs, the philosophical aspects of this question become increasingly overshadowed by the practical need to develop an operational understanding of the limitations of the machine judgment and decision making ability. Can computers be relied upon to match people, decide on promotions and dismissals, make medical diagnoses, prescribe treatments, act as teachers, formulate business, political and military strategies, and, more generally, perform intellectual tasks of high complexity which in the past required expert human judgment? Clearly, this is already a pressing issue which is certain to grow in importance in the years ahead."

He called it a paradox that the human brain is always solving problems by manipulating "fuzzy concepts" and "multidimensional fuzzy sensory inputs" whereas "the computing power of the most powerful, the most sophisticated digital computer in existence is not able to do this". Therefore, he stated that "in many instances, the solution to a problem need not be exact, so that a considerable measure of fuzziness in its formulation and results may be tolerable. The human brain is designed to take advantage of this tolerance for imprecision whereas a digital computer, with its need for precise data and instructions, is not."

One year later these arguments should culminate in Zadeh's *Principle of Incompatibility* whereas here he intended to push his theory of fuzzy sets to model the imprecise concepts and directives:

"Indeed, it may be argued that much, perhaps most, of human thinking and interaction with the outside world involves classes without sharp boundaries in which the transition from membership to non-membership is gradual rather than abrupt."

He stated:

"Although present-day computers are not designed to accept fuzzy data or execute fuzzy instructions, they can be programmed to do so indirectly by treating a fuzzy set as a data-type which can be encoded as an array [...]."

Granted that this is not a fully satisfactory approach to the endowment of a computer with an ability to manipulate fuzzy concepts, it is at least a step in the direction of enhancing the ability of machines to emulate human thought processes. It is quite possible, however, that truly significant advances in artificial intelligence will have to await the development of machines that can reason in fuzzy and non-quantitative terms in much the same manner as a human being."

In August 1967, the Filipino electrical engineer William Go Wee (born 1937) at Purdue University in Indiana had submitted his dissertation "On Generalizations of Adaptive Algorithms and Application of the Fuzzy Sets Concept to Pattern Classification" that he had written under King Sun Fu, one of the pioneers in the field of pattern recognition.

Wee had applied the fuzzy sets to iterative learning procedures for pattern classification and had defined a finite automaton based on Zadeh's concept of the fuzzy relation as a model for nonsupervised learning systems:

"The decision maker operates deterministically. The learning section is a fuzzy automaton. The performance evaluator serves as an unreliable "teacher" who tries to teach the "student" to make right decisions."

The fuzzy automaton representing the learning section implemented a "nonsupervised" learning fuzzy algorithm and converged monotonously. Wee showed that this fuzzy algorithm could not only be used in the area of pattern classification but could also be translated to control and regulation problems. He also demonstrated that the fuzzy automaton he had defined contained the concepts of deterministic and non-deterministic automata as special cases: "Based on the concept of fuzzy relation defined by Zadeh, a class of fuzzy automata is formulated similar to that of Mealy's definition. A fuzzy automaton behaves in a deterministic fashion. However, it has many properties similar to that of a probabilistic automaton."

Working with his doctoral advisor, Wee presented his findings in the article "A Formulation of Fuzzy Automata and its Applications as a Model of Learning Systems".

At the end of the same year the Chinese student Chin-Liang Chang completed his dissertation "Fuzzy Sets and Pattern Recognition" that was an advancement of Zadeh's thoughts on the separation problem in pattern recognition. This was the first Ph D dissertation on Fuzzy Sets that was supervised by Lotfi Zadeh. Chang had also had contact with Professor King Sun Fu to whom he expresses gratitude for the conversations they shared.

Two years later, in "Towards a Theory of Fuzzy Systems" that was first printed as a report in 1969, Zadeh's goal was a theory for all systems - including those that were too complex or poorly defined to be accessible to a precise analysis. Alongside the systems of the "soft" fields, the "non-soft" fields were replete with systems that were only "unsharply" defined, namely "when the complexity of a system rules out the possibility of analyzing it by conventional mathematical means, whether with or without the computers".

As he would also do in the same year in Bordeaux, Zadeh was already pointing out here the usefulness of fuzzy sets in computer science: In describing their fields of application, he enumerated the problems that would be solved by future computers. Alongside pattern recognition, these included traffic control systems, machine translation, information processing, neuronal networks and games like chess and checkers. We had lost sight of the fact that the class of non-trivial problems for which one could find a precise solution algorithm was very limited, he wrote. Most

real problems were much too complex and thus either completely unsolvable algorithmically or – if they could be solved in principle – not arithmetically feasible. In chess, for instance, there was in principle an optimal playing strategy for each stage of the game; in reality, however, no computer was capable of sifting through the entire tree of decisions for all of the possible moves with forward and backward repetitions in order to then decide what move would be the best in each phase of the game. The set of good strategies for playing chess had fuzzy limits similar to the set of tall men - these were fuzzy sets. By far the most systems that remained to be solved were fuzzy systems, and in a footnote Zadeh remarks that the automata proposed by Wee and his supervisor Eugene Santos were also considered examples of fuzzy systems.

To make fuzziness a part of system theory, Zadeh presented in 1968 "fuzzy algorithms". With that, he had fuzzified the central concept of computer sciences. "The concept in question will be called a fuzzy algorithm because it may be viewed as a generalization, through the process of fuzzification, of the conventional (nonfuzzy) conception of an algorithm."

Algorithms depend upon precision. An algorithm must be completely unambiguous and errorfree in order to result in a solution. The path to a solution amounts to a series of commands which must be executed in succession. Algorithms formulated mathematically or in a programming language are based on set theory. Each constant and variable is precisely defined, every function and procedure has a definition set and a value set. Each command builds upon them. Successfully running a series of commands requires that each result (output) of the execution of a command lies in the definition range of the following command, that it is, in other words, an element of the input set for the series. Not even the smallest inaccuracies may occur when defining these coordinated definition and value ranges.

After 1965, when Zadeh had fuzzified input, output and state in system theory and had thus founded a theory of fuzzy systems, it was obvious to him how to go about fuzzifying algorithms. The commands needed to be fuzzified and so, of course, did their relations!

"I began to see that in real life situations people think certain things. They thought like algorithms but not precisely defined algorithms."

Inspired by this idea, he wrote the article "Fuzzy Algorithms" for *Information and Control* in 1968 which uncharacteristically contained neither theorems nor proofs. Many years later he said that "it is not really a mathematical paper. And the reason why it appeared there is because, again, I was on the editorial board. So it could be published quickly. And I do say it's not a mathematical paper but the idea. But then other people who were mathematicians have developed that and added more mathematical and so forth. So, my function was not that of coming up with very precise. It's just an idea. That's little bit like a composer who just hums something, a sort of orchestrazing." [...]

To illustrate, fuzzy algorithms may contain fuzzy instructions such as: (a) "Set y approximately equal to 10 if x is approximately equal to 5," or (b) "If x is large, increase y by several units," or (c) "If x is large, increase y by several units; if x is small, decrease y by several units; otherwise keep y unchanged." The sources of fuzziness in these instructions are fuzzy sets which are identified by their underlined names.

All people function according to fuzzy algorithms in their daily life, Zadeh wrote – they use recipes for cooking, consult the instruction manual to fix a TV, follow prescriptions to treat illnesses or heed the appropriate guidance to park a car. Even though activities like this are not normally called algorithms: "For our point of view, however, they may be regarded as very crude forms of fuzzy algorithms".

In that time Zadeh wrote also a paper with the title "Toward Fuzziness in Computer Systems. Fuzzy Algorithms and Languages". [...]

Therefore this paper was not written before 1969. The next section in this article is titled "Fuzzy Languages". It has also just two pages and a footnote says:

"A more detailed discussion of fuzzy languages appears in a forthcoming paper by E. T. Lee and the writer." [...]

However, the association of fuzziness and computers in the title of this paper must have sounded surprisingly in the late 1960s and referring to that Zadeh set in the introduction to this paper: "At first glance, it may appear highly incongruous to mention computers and fuzziness in the same breath, since fuzziness connotes imprecision whereas precision is a major desideratum in computer design."

In the following paragraphs Zadeh justified this with arguing that the future computer systems will have to perform many more complex information processing tasks than that kind of computers that he and his contemporaries in the 1960s knew. He expected that the future computers have to process more and more imprecise information! "Fuzziness, then, is a concomitant of complexity. This implies that as the complexity of a task, or a system for performing that task, exceeds a certain threshold, the system must necessarily become fuzzy in nature. Thus, with the rapid increase in the complexity of the information processing tasks which the computers are called upon to perform, a point is likely to be reached – perhaps within the next decade - when the computers will have to be designed for processing of information in fuzzy form. In fact, it is this capability - a capability which present-day computers do not possess - that distinguishes human intelligence from machine intelligence. Without such capability we cannot build computers that can summarize written text, translate well from one natural language to another, or perform many other tasks that humans can do with ease because of their ability to manipulate fuzzy concepts."

For that purpose, Zadeh pointed out, "intriguing possibilities for computer systems" are offered by fuzzy algorithms and fuzzy languages!

#### 6.3 Fuzzy algoritm and fuzzy languages

"Roughly speaking, a fuzzy algorithm is an ordered set of instruction containing names of fuzzy sets. An example of such an instruction is "If x is large, set y equal to 2. Otherwise, set y equal to 1. All languages, whether natural of artificial, tend to evolve and rise in level through the addition of new words to their vocabulary. These new words are, in effect, names for ordered subsets of names in the vocabulary to which they are added". (L.A. Zadeh)

Real world phenomena are very complex and rich of members. To characterize or picture these phenomena in terms of our natural languages we use our vocabulary and because this set of words is restricted, Zadeh argued that this process leads to fuzziness: "Consequently, when we are presented with a class of very high cardinality, we tend to group its elements together into subclasses in such a way as to reduce the complexity of the information processing task involved. When a point is reached where the cardinality of the class of subclasses exceeds the information handling capacity of the human brain, the boundaries of the subclasses are forced to become imprecise and fuzziness becomes a manifestation of this imprecision. This is the reason why the limited vocabulary we have for the description of colors makes it necessary that the names of colors such as red, green, bleu, purple, etc. be, in effect, names of fuzzy rather than non-fuzzy sets. This is why natural languages, which are much higher in level than programming languages, are fuzzy whereas programming languages are not."

Here, Zadeh argued explicitly for programming languages that are - because of missing rigidness and preciseness and because of their fuzziness – more like natural languages. He mentioned the concept of *stochastic languages* that was published by the Finnish mathematician Paavo Turakainen in *Information and Control* in the foregoing year, being such an approximation to our human languages using randomizations in the productions, but however, he preferred fuzzy productions to achieve a formal fuzzy language. Then, he presented a short sketch of his program to extend non-fuzzy formal languages to fuzzy languages which he published in elaborated form with the co-author Edward T.-Z. Lee in "Note on Fuzzy Languages."

His definition in these early papers was given in the terminology of the American computer scientists John Edward Hopcroft and Jeffrey David Ullman that was published in the same year.

L is a fuzzy language if it is a fuzzy set in the set  $V_T^*$  ( $V_T^*$  is called the Kleene closure of  $V_T$ , named after the American mathematician Stephen Kleene (1909-1994)) of all finite strings composed of elements of the finite set of terminals  $V_T$ , e.g.  $V_T = \{a, b, c, ..., z\}$ .

The membership function  $\mu_L(x): V_T^* \to [0,1]$  associates with each finite string x, composed of elements in  $V_T$ , its grade of membership in L. Here is one of the simple examples that he gave in the early article ([131], p. 16): "Assume that  $V_T = \{0, 1\}$ , and take L to be the fuzzy set  $L = \{(0, 0.9), (1, 0.2), (00, 0.8), (01, 0.6), (10, 0.7), (11, 0.3)\}$  with the understanding that all the

other strings in  $V_T^*$  do not belong to L (i.e., have grade of membership equal to zero)."

In general the language L has high cardinality and therefore it is not usual to define it by a listing of its elements but by a finite set of rules for generating them. Thus, in analogy to the case of non-fuzzy languages Zadeh defined a *fuzzy grammar* as "a quadruple  $G = (V_N, V_T, P, S)$ , where  $V_N$  is a set of variables (non-terminals) disjoint from  $V_T$ , P is a set of [fuzzy] productions and S is an element of  $V_N$ . The elements of  $V_N$  (called *[fuzzy] syntactic categories*) and S is an abbreviation for the syntactic category "sentence". The elements of P define conditioned fuzzy sets in  $(V_T \cup V_N)^*$ ."

## 7 Fuzzy control

"One special moment of the 7th European Congress on Intelligent Techniques & Soft Computing EU-FIT'99, Aachen, Germany, September 13-16, 1999, was a Debate on Fuzzy versus Conventional Control. This debate has been organized as one of the plenary sessions. The debate featured Prof. Lotfi A. Zadeh of the University of California at Berkeley taking the side of fuzzy control, and Prof. Michael Athans of the MIT taking the side of conventional control. Prof. Zadeh has argued the merits of fuzzy control, and his opinions have represented a serious step forward towards continuing the research in fuzzy control." (Radu-Emil Precup)<sup>24</sup>

The following text, co-authored by A.  $Meyer^{25}$  and H.-J. Zimmermann<sup>26</sup>, is reprinted from

<sup>&</sup>lt;sup>24</sup>Dr. Radu-Emil Precup is a Professor at the Politehnica University of Timisoara, Romania. He was the recipient of the "Grigore Moisil" Prize from the Romanian Academy, two times, in 2005 and 2016, for his contribution on fuzzy control and the optimization of fuzzy systems. He was listed as one of the top 10 researchers in Artificial Intelligence and Automation (according to HoT World as of July 2017). He wrote a valuable survey about fuzzy control applications in industry [90].

<sup>&</sup>lt;sup>25</sup>Andreas Meyer: After his studies of Computer Science at "Saarland University" and "Darmstadt University of Technology", Andreas Meyer started his career in 1997 as product manager in an IT Security company. Being in charge of the development of IT products to secure internet banking as Director for Research and Development since 2003, he became two years later Vice President for Sales and Marketing with world-wide responsibility. Working as an executive, he received in parallel his PhD in Public Key Cryptography at Darmstadt University of Technology. In 2008 he joined INFORM as the Director of the Risk & Fraud Division and a member of the executive board.

<sup>&</sup>lt;sup>26</sup>Hans-Jürgen Zimmermann: He studied in Frankfurt, Darmstadt, Berlin and Oxford, received his masters in engineering and his PhD in Operations Research and Management Science from the Technical University of Berlin. He received honorary doctors from universities in Belgium and Finland. He spent appr. 10 years in industry in different companies such as Siemens, IT&T Europe (SEL), Caterpillar International etc. and became Assoc. Professor of Operations Research and Industrial Administration at the University of Illinois in Urbana in 1964. 1967 he accepted the chair for Operations Research of the Aachen Institute of Technology. He was founding president of IFSA, editor of Fuzzy Sets and Systems and the European Journal for Operational Research and Assoc. Editor of 10 other international journals. He founded INFORM in 1969 and published 260 papers and 35

[63], partially.

"In his first paper on Fuzzy Sets L.A. Zadeh [129] already mentioned as one motivation of his Theory of Fuzzy Sets: "The fact remains that such imprecisely defined 'classes' play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction." This statement has become even more true in the meantime since we have moved from a time of scarce data into the period of data warehouses etc., i.e., into a world of abundance of data, in which people try hard to extract useful information from masses of data ([142]).

From the point of view of applications we still consider as some of the most important goals of fuzzy set theory to extract information from data and model it visually or otherwise in such a way, that people can understand it, communicate it and to model problems adequately. They can use it to solve their problems better than by a purely dichotomous modeling language. In doing this professionally, they can combine the high computing power of EDP with human experience and creativity.

Extremely successful fuzzy models were first used in engineering intelligence in areas such as Fuzzy Control ([60,142,147,149]). Controlling cranes, cement kilns, video cameras, washing machines, ABS, and even subway systems by fuzzy control turned out to be almost sensational. Most of these applications had one feature in common: These were manmade systems, the control of which was often nonlinear (and therefore difficult to model traditionally). However the controls could be decomposed into linear systems by modeling human experience by fuzzy technology ([148]) and one could then determine the adequate parameters, operators ([33]) and membership functions, as well as the defuzzification models, by trial and error. Practitioners loved these controllers because they used predominantly rather basic fuzzy set theory and operators as well as membership functions could be defined rather than determined on the basis of human knowledge. Many of these models have become regular teaching material in control engineering courses.

In the meantime fuzzy set theory was further developed, it became more powerful mathematically and it became more strenuous to learn and understand it. When one started to apply it to business intelligence and to human decision making another problem became visible: Many applications do not permit a trial and error calibration as in fuzzy control because the results of a fuzzy model cannot easily be compared with the results or the behavior of the real system.

Think of strategic decisions, of evaluations of long term vulnerability of companies or persons, the determination of the creditworthiness ([151]) of persons or institutions. Here the human knowledge that goes into the fuzzy model has to be modeled properly in advance. That means, that operators ([99]), membership functions, inference methods ([148]) etc. have to properly map the counterparts in the human mind, in which they are very often very context dependent. This is no longer only a mathematical problem but predominantly a problem of psycho-linguistics or similar disciplines ([146]). Unluckily this part of science is much less developed than the mathematics of fuzzy set theory. Hence, in applications one often has to rely again on assumptions rather than on scientific results when modeling operators ([114, 150]), membership functions ([29, 50]) and other parts of fuzzy models. The justification of assumptions, of course, also depends on whether one wants to build descriptive or prescriptive models."

## 8 Fuzzy logic systems

"The Fuzzy Logic Systems are general knowledge base systems with linguistic rules that can be constructed using the knowledge of experts in a given field." (D. Wu & W.W. Tan [122])

books in different areas. In 2011 he received the IEEE Fuzzy Systems Pioneer Award.

This part is co-authored by Nazanin Sahab and Hani Hagras<sup>27</sup>, and is reprinted from [101], partially.

"Professor Lotfi Zadeh introduced fuzzy sets in 1965. He has written several papers since then, but his comprehensive paper on the concept of linguistic variables became very famous as a reference on fuzzy logic theory. He introduced fuzzy logic for the application of approximate reasoning and mentioned the need of a humanistic system whose behavior is strongly influenced by human judgment, perception or emotion for the purpose of computing with words to solve the problems of human-oriented fields such as philosophy, psychology, politics, law, sociology and literature. Professor Zadeh also described the use of his fuzzy logic approach in [132] in describing the behaviors of too complex or too ill-defined systems. This has inspired control engineers from over the world to investigate the power of fuzzy logic to control applications which are difficult to mathematically model.

In 1975, the first Fuzzy Logic Controller (FLC) was developed by Mamdani and Assilian for controlling a steam engine [61]. Since then Fuzzy Logic Systems (FLSs) have been applied with great success to many real world applications. It was reported in 1995 that over the last two decades, the number of applications of FLSs have dramatically increased. Several industrial applications of fuzzy logic were reported from which we can mention cement kilns [43], steel furnaces [43], aircraft engine control, steam turbines, power supply controllers, etc [17]. FLSs have also been used domestically in elevators, washing machines [17], fridges [24], air conditioners, automobiles [43], automatic transmission, camera autofocus control, etc [17]. [...]

According to the John Lewis website about a top brand washing machine from AEG and reporting about the fuzzy logic washing machine, the website mentions "fuzzy logic circuit detects when the laundry is out of balance and rejig it accordingly, ensuring minimum wear and tear to the drum bearings. The fuzzy logic also detects half loads, if too much detergent has been added and adds extra rinses if required" [123]. Hence, for the past thirty years fuzzy logic and its applications became embedded in our everyday environments.

Fuzzy logic laid the basis for a successful method to model uncertainty, vagueness, and imprecision [53]. The FLSs are general knowledge base systems [122] with linguistic rules that can be constructed using the knowledge of experts in a given field. During more than four decades from the invention of fuzzy logic, great progress in using FLS has been achieved. While traditionally type-1 FLSs have been widely employed in real world applications, recent years have shown a rapidly growing interest in the research and application of type-2 FLSs. This is because, it has become apparent that type-1 FLSs cannot fully handle the large amounts of uncertainties encountered in real world environments and applications. Interval type-2 FLSs have shown to provide better performance when compared to type-1 FLSs in applications with high levels of uncertainty. The difference in performance has been attributed to the nature of interval type-2 fuzzy sets which can better account for the uncertainties as they incorporate a Footprint of Uncertainty (FOU) and they can be assumed to embed a large number of type-1 fuzzy sets. However, the majority of the type-2 FLSs [102] handle the linguistic and input numerical uncertainties using singleton interval type-2 FLSs that mix the numerical and linguistic uncertainties to be handled only by the linguistic labels type-2 fuzzy sets. Input numerical uncertainties are associated with the noise, imprecision and uncertainty associated with sensors and input devices. However, the linguistic uncertainties are associated with human words and perceptions. Hence, it seems paradoxical to use singleton type-2 FLSs to handle the input numerical uncertainties, as this ignores the fact that if input numerical uncertainties were present,

<sup>&</sup>lt;sup>27</sup>Professor Hani Hagras is a Director of the Computational Intelligence Centre, Head of the Intelligent Environments Research Group, Head of the Fuzzy Systems Research GroupSchool of Computer Science and Electronic Engineering, University of Essex, UK.

they should affect the incoming inputs to the FLS. Thus we cannot treat the incoming FLS inputs as perfect signals as in the case of singleton FLSs. Hence, there is a need to employ non-singleton FLSs to handle the input numerical uncertainties by modeling the inputs as fuzzy inputs rather than crisp values.

[...] The work done so far in (type-1 and type-2) non-singleton FLSs use predefined shapes for the uncertainty distribution affecting the FLSs inputs (mainly Gaussian and triangular) which might not reflect the real uncertainty distribution associated with the given sensor. For example in the case of a sonar sensor, the numerical input uncertainties depend on many factors such as the shape and type of objects reflecting the sonar signal as well as wind speed, humidity and temperature. Moreover, the uncertainty distribution also varies with the measured values where for the case of a sonar sensor, the amount of uncertainty affecting measurement readings at 20cm distance could be much less than the uncertainties affecting the measurement at large distances such as 3m. Hence, it is not fair to consider that there is specific shape for the uncertainty distribution (triangular, Gaussian, etc.) with fixed parameters (average, standard deviation, etc.) for all the measured values. [102]"

### 8.1 Fuzzy logic based applications in communication networks

"The application of fuzzy technology is widely known as a technological revolution." (K. Hirota [51])

Following text is reprinted from [84] (co-authored by C.N. Nyirenda, F. Dong, and K. Hirota), partialy.

"The number of fuzzy logic based applications in communication networks is increasing rapidly. This development is motivated by the difficulties experienced when modeling communication networks by using conventional analytical methods. Some of the fuzzy applications include power control [23] in cellular systems; congestion control in IP networks [26], [85]; routing [5] and data fusion [62] in wireless sensor networks; and Quality of Service management in wireless sensor and actuator networks [125]. Input parameters are, generally, sampled at a fixed rate and the fuzzy system is triggered accordingly. In some cases, an external signal is used in order to trigger the systems. The fuzzy computations are invoked even when there are no significant differences between the subsequent input parameters, at the expense of precious CPU and memory resources. Furthermore, for systems that employ a sampling rate, the rate is chosen by trial and error such that it is difficult to tell if it is optimal."

Kaoru Hirota<sup>28</sup> has coauthored many papers in this field within scientists from Canada, China, Japan and Romania.

<sup>&</sup>lt;sup>28</sup>Prof. Kaoru Hirota, a member of the editorial board of IJCCC, was born in Japan (1950). He received the B.E., M.E., and Dr.Eng. degrees in electronics from Tokyo Institute of Technology, Tokyo, Japan, in 1974, 1976, and 1979, respectively. From 1979 to 1982, he was with Sagami Institute of Technology, Fujisawa, Japan. From 1982 to 1995, he was with the College of Engineering, Hosei University, Tokyo. Since 1995, he has been with the Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology. Dr. Hirota is a member of the International Fuzzy Systems Association and the Japan Society for Fuzzy Theory and Systems. In present he is a head of the Beijing Representative Office at Japan Society for the Promotion of Science (Beijing Institute of Technology; JSPS Beijing Office), and an editor in chief of Int. J. of Advanced Computational Intelligence and Intelligent Informatics. His research interests include fuzzy systems, intelligent robot, image understanding, expert systems, hardware implementation and multimedia intelligent communication. Kaoru Hirota was a good fiend of Lotfi Zadeh.

# 9 Artificial intelligence, soft computing, and decision making

### 9.1 Artificial intelligence

In the near future Artificial Intelligence (AI) will surpass human intelligence in more and more domains. Indeed, based on soft computing, fuzzy control, bio-inspired computing, computational theory of perceptions and computation in natural language, Artificial Intelligent computers can write their own programs as they encounter situations and try different ways to achieve a goal. In the near future Artificial Intelligent machines (net-centric automobiles, intelligent aircraft, intelligent home utilities, intelligent learn labs, entertainment devices, military defense arms, health applications), will be a commonplace. Humans have a remarkable capability to reason and make decisions in an environment of imprecision, uncertainty and partiality of knowledge, truth and class membership. It is this capability that is needed to achieve human-level machine intelligence. Achievement of human-level machine intelligence is beyond the reach of existing AI techniques and more of these are based on fuzzy sets theory and fuzzy logic.

AI has made slow but steady advances in all its subfields, overcoming unexpected obstacles and taking advantage of a long period of exponentially increasing computer power. AI problems that had begun to seem impossible in 1970 have been solved and the solutions are now used in successful commercial usually products. Experts are divided on whether it is possible even in theory to build an AI system with a human-level of intelligence.

We can see today many examples already increasing to strong AI, such as: robot Sophia (a humanoid robot developed by Hong Kong-based company Hanson Robotics); self-driving car (The Google project Waymo); AlphaGo (a computer program that plays the board game Go, developed by Alphabet Inc.'s Google DeepMind in London), etc.

### 9.2 Soft computing

In accordance with Zadeh's definition, *Soft Computing* (SC) consist of computational techniques in computer science, machine learning and some engineering disciplines, which study, model, and analyze very complex reality: those for which more traditional methods have unusable or inefficiently.

SC uses soft techniques, contrasting it with classical artificial intelligence, *Hard Computing* (HC) techniques), and includes: Fuzzy Logic, Neural Computing, Evolutionary Computation, Machine Learning, and Probabilistic Reasoning.

HC is bound by a Computer Science (CS) concept called *NP-Complete*, which means that there is a direct connection between the size of a problem and the amount of resources needed to solve that called "grand challenge problem". SC aids to surmount NP-complete problems by using inexact methods to give useful but inexact answers to intractable problems.

SC became a formal CS area of study in the early 1990's. Earlier computational approaches could model and precisely analyze only relatively simple systems. More complex systems arising in biology, medicine, the humanities, management sciences, and similar fields often remained intractable to HC. It should be pointed out that simplicity and complexity of systems are relative, and many conventional mathematical models have been both challenging and very productive. SC techniques resemble biological processes more closely than traditional techniques, which are largely based on formal logical systems, such as Boolean logic, or rely heavily on computer-aided numerical analysis (as in finite element analysis).

SC techniques are intended to complement HC techniques. Unlike HC schemes, which strive for exactness and full truth, soft computing techniques exploit the given tolerance of imprecision, partial truth, and uncertainty for a particular problem. The inductive reasoning plays a larger role in SC than in HC. SC and HC can be used together in certain fusion techniques.

Hard Computing	Soft Computing		
Deterministic environment	Nondeterministic environment		
(closed, static, known)	(open, dynamic, uncertain)		
Well-defined problem	Fuzzy-defined Situation		
(quantity, precision, certainty)	(quality, imprecision, uncertainty)		
Solving accurately problems	Managing "Just In Time" situations		
(imperative, firm, reliable)	(descriptive, flexible, robust)		
Optimal, lasting, solution	Suboptimal, temporary, answer		
(algorithmic, apodictic, general)	(non-algorithmic, revisable, local)		
Technocentric design	Anthropocentric design		
Software entity: <i>PROGRAM</i>	Software entity: $AGENT$		
(object devised as tool)	(process devised as interactant)		
Client-Server paradigm	"Computing as Interaction" paradigm		
(object-oriented, sequential)	(agent-oriented, parallel)		

Table 1:	Hard	Computing vs.	Soft	Computing
----------	------	---------------	------	-----------

Soft Computing can deal with ambiguous or noisy data and is tolerant of imprecision, uncertainty, partial truth, and approximation. In effect, the role model for SC is the human mind. Artificial Intelligence and Computational Intelligence based on SC provide the background for the development of smart management systems and decisions in case of ill-posed problems.

In Table 1 we present the summarized conclusions of the Hard Computing paradigm versus Soft Computing paradigm, adapted from [38].

#### 9.3 Decision making in fuzzy environments

In many real-world situations, the problems of decision making are subjected to some constraints, objectives and consequences that are not accurately known. After Bellman and Zadeh [13] introduced for the first time (1970) fuzzy sets within multiple-criteria decision-making (MCDM), many researchers have been preoccupied by decision making in fuzzy environments. The fusion between MCDM and fuzzy set theory has led to a new decision theory, known today as fuzzy multi-criteria decision making (FMCDM), where we have decision-maker models that can deal with incomplete and uncertain knowledge and information [22, 35].

The most important thing is that, when we want to assess, judge or decide we usually use a natural language in which the words do not have a clear, definite meaning. As a result, we need fuzzy numbers to express linguistic variables, to describe the subjective judgement of a decision maker in a quantitative manner. Fuzzy numbers (FN) most often used are triangular FN, trapezoidal FN and Gaussian FN [36], [107].

We highlight that the concept of linguistic variable introduced by Lotfi A. Zadeh in 1975 allows computation with words instead of numbers and thus linguistic terms defined by fuzzy sets are intensely used in problems of decision theory for modelling uncertain information.

## 10 Conclusion

Several key notions in Zadeh's thinking that are worth mentioning, namely optimality, uncertainty, reasoning, and meaning; also, the concept of discrete variables and its tension with the concept of continuum plays a central part in his work. Lotfi A. Zadeh has been a system theorist, a computer scientist, a physicist, and an engineer. He published the vast majority of his papers as single author, but he has also worked together with John R. Ragazzini (1912-1988), Richard E. Bellman (1920-1984), Charles A. Desoer (1926-2010), and a few other scientists [113].

A number of researchers from Romania were among the early supporters and followers of ideas presented by Lotfi A. Zadeh, such as: Grigore C. Moisil (1906-1973), Constantin V. Negoita (born 1936), Dan A. Ralescu (born 1949), Dan Butnariu (1951-2008), and many others.

His seminal paper "Fuzzy Sets" has already about 100,000 citations in Google Scholar and all Zadeh's papers have over 185,000 citations (Dec. 1, 2017).

Fuzzy Logic have nowadays many useful applications, from consumer smart products to industrial intelligent products, such as: washing machines, air conditioners, cameras, camcorders, fuzzy/neuro-fuzzy expert systems, fuzzy data/information mining, fuzzy search engines, microcontrollers, microprocessors, signal processing, fuzzy/neuro-fuzzy controllers, edge detectors, speech recognition, fuzzy decision making in economy/medecine, knowledge management, fuzzy thinking, etc.

Lotfi A. Zadeh, the inventor of Fuzzy Logic, dies at 96, but Fuzzy Logic is alive and is in progress through his followers.

#### Acknowledgements

Many thanks to all contributors for development of fuzzy theory and their applications. In this work we had use papers published in IJCCC (2008, 2011, 2015), and also in [145], by several authors from Algeria (A. Bazoula, M.S. Djouadi, H. Maaref); Bulgaria (Natalia Nikolova, Sevda Ahmed, Kiril Tenekedjiev); Chile (Felisa Cordova, Gaston Lefranc); Japan (Fangyan Dong, Kaoru Hirota, Clement N. Nyirenda), Germany (Andreas Meyer, Hans-Jürgen Zimmermann), Hungary (Imre Rudas, Janos Fodor); Romania (Marius M. Balas, Valentina E. Balas, Ioan Buciu, Gloria Cerasela Crisan, Ioan Dzitac, Simona Dzitac, Ioan Felea, Ioana Moisil, Ioan Nafornita, Elena Nechita, Horia Nicolai Teodorescu, Tiberiu Vesselenyi), Serbia (Dragan Radojevic); Spain (Carlos Castaneda, Rafael Nunez, Alfonso Ortega, Emilio del Rosal, Rudolf Seising), UK (Hani Hagras, Nazanin Sahab), and USA (Abraham Kandel, Constantin V. Negoita, Dan E. Tamir, Ronald R. Yager, Lotfi A. Zadeh). We address our special thanks to Prof. Shahnaz Shahbazova for her accurate information about Zadeh's biography.

# Bibliography

- Abbasov A.M., Shahbazova S.N. (2015); Functional Solution of the Knowledge Level Control Problem: The Principles of Fuzzy Logic Rules and Linguistic Variables, in D.E. Tamir et al. (eds.), Fifty Years of Fuzzy Logic and its Applications, Studies in Fuzziness and Soft Computing, Springer International Publishing, Switzerland, 25-38, 2015.
- [2] Alb-Lupas A., Oros G. (2015); On special fuzzy differential subordinations using Salagean and Ruscheweyh operators, *Applied Mathematics and Computation*, 261, 119-127, 2015.
- [3] Andonie R., Sasu L.M., Cataron A. (2009); A Novel Fuzzy ARTMAP Architecture with Adaptive Feature Weights based on Onicescu's Informational Energy, *International Journal* of Computers Communications & Control, 4(2), 104-117, 2009.
- [4] Ashraf S., Rehman A., Kerre E.E. (2015); Group Decision Making with Incomplete Intervalvalued Fuzzy Preference Relations Based on the Minimum Operator, International Journal of Computers Communications & Control, 10(6), 789-802, 2015.
- [5] Balakrishnan M., Johnson E.E. (2006); Fuzzy diffusion analysis: Decision significance and applicable scenarios, Proc. of IEEE Military Communications Conference, 1, 2175-2181, 2006.

- [6] Balas M.M., Balas V.E. (2008); World Knowledge for Control Applications by Fuzzy-Interpolative Systems International Journal of Computers Communications & Control, 3(Supplement: S), 28-32, 2008.
- [7] Balas M.M., Balas V.E. (2008), World Knowledge for Controllers by Fuzzy-Interpolative Systems, 11-21, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 2008.
- [8] Balas V.E., Balas M.M. (2008), A Survey on the Constant Time to Collision Techniques, 22-31, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 2008.
- [9] Ban A.I., Coroianu L. (2012); Nearest interval, triangular and trapezoidal approximation of a fuzzy number preserving ambiguity, *International Journal of Approximate Reasoning*, 53(5), 805-836, 2012.
- [10] Bazoula A., Djouadi M.S., Maaref H. (2008); Formation Control of Multi-Robots via Fuzzy Logic Technique, *International Journal of Computers Communications & Control*, 3(Supplement: S), 179 -184, 2008.
- [11] Bede B. (2013); Mathematics of Fuzzy Sets and Fuzzy Logic, Book Series: Studies in Fuzziness and Soft Computing, 295, 1-276, 2013.
- [12] Bede B., Gal S.G. (2005); Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations, *Fuzzy Setts and Systems*, 151(3), 581-599, 2005.
- [13] Bellman R.E., Zadeh L.A. (1970); Decision making in a fuzzy environment, Management Science, 17, 141-164, 1970.
- [14] Benitez-Perez H., Cardenas-Flores F., Garcia-Nocetti F. (2011); Reconfigurable Takagi-Sugeno Fuzzy Logic Control for a Class of Nonlinear System considering Communication Time Delays on Peripheral Elements, *International Journal of Computers Communications* & Control, 6(3), 387-402, 2011.
- [15] Blair B. (1994); Interview with Lotfi Zadeh, Creator of Fuzzy Logic, Azerbaijada International, http://www.azer.com/aiweb/categories/magazine/24\\_folder/24\\_articles/ 24\\_fuzzylogic.html
- [16] Bologa O., Breaz R.E., Racz S.G. (2015); A Fuzzy-based Decision Support Tool for Engineering Curriculum Design, International Journal of Computers Communications & Control, 10(6), 803-811, 2015.
- [17] Bonissone P.P., Badami V., Chiang K.H., Khedkar P.S., Marcelle K.W., Schutten M.J. (1995), Industrial Applications of Fuzzy Logic at General Electric, *Proceedings of IEEE*, 83(3), 450-465, 1995.
- [18] Buciu I., Nafornita I. (2008), Non-Negative Matrix Factorization Methods and their Applications, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 32-50, 2008.

- [19] Butnariu D. (1978); Fuzzy games. A description of the concept, Fuzzy Sets and Systems, 1, 181-192, 1978.
- [20] Butnariu D. (1979); Solution concepts for n-person fuzzy games, in M.M. Gupta et al. (eds), Advances in Fuzzy Sets. Theory and Applications North-Holland, Amsterdam, 339-359, 1979.
- Butnariu D. (1982); Fixed-points for fuzzy mappings, Fuzzy Sets and Systems, 7(2), 191-207, 1982.
- [22] Butnariu D. (1983); Additive Fuzzy Measures and Integrals I, Journal of Mathematical Analysis and Applications, 93, 436-452, 1983.
- [23] Chen Y.-L., Wang J.-W., Lin Y.-S., Wen J.-H. (2011); Combined Fuzzy-Based Power Control with Window-Based Transmission Rate Management in Multimedia CDMA Cellular Systems, *International Journal of Electronics and Communications*, 65(4), 377-383, 2011.
- [24] Choi B., Han S., Hong S. (1998), Refrigerator Temperature Control Using Fuzzy Logic and Neural Network, *IEEE International Symposium on Industrial Electronics*, 1, 186-191, 1998.
- [25] Crisan G.C., Nechita E. (2008); Solving Fuzzy TSP with Ant Algorithms, International Journal of Computers Communications & Control, 3(Supplement: S), 228-231, 2008.
- [26] Chrysostomou C., Pitsillides A., Sekercioglu A. (2009); Fuzzy Explicit Marking: A Unified Congestion Controller for Best-effort and Diff-serv Networks, *Computer Networks Journal* (COMNET), 53(5), 650-667, 2009.
- [27] Chira C., Pintea C.M., Dumitrescu D. (2008), A Multi-Agent Stigmergic Model for Complex Optimization Problems, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 51-62, 2008.
- [28] Cordova F.M., Quezada L.E. (2008), Conceptual Model of an Intelligent Supervising System for Evaluating Policies in the Operation of an Underground Mine, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 63-83, 2008.
- [29] Dishkant H. (1981), About membership function estimation, Fuzzy Sets and Systems, 5, 141-147, 1981.
- [30] Dragomir O., Gouriveau R., Zerhouni N. (2008); Adaptive Neuro-Fuzzy Inference System for Mid Term Prognostic Error Stabilization, *International Journal of Computers Communications & Control*, 3(Supplement: S), 271-276, 2008.
- [31] Du Z.-B., Lin T.-C., Zhao T.-B. (2015); Fuzzy Robust Tracking Control for Uncertain Nonlinear Time-Delay System, International Journal of Computers Communications & Control, 10(6), 812-824, 2015.
- [32] Dubois D., Prade H. (1980); Fuzzy Sets and Systems: Theory and Applications, Academic Press, 1980.
- [33] Dubois D., Prade H. (1985), A review of fuzzy set aggregation connectives, Information Science, 36, 85-121, 1985.

- [34] Dubois D., Prade H. (eds.) (2000); Fundamentals of Fuzzy Sets, Kluwer Academic Publishers, 2000.
- [35] Duta L., Zamfirescu C.B., Filip, F.G. (2014); Mathematical Decision Model for Reverse Supply Chains Inventory, International Journal of Computers Communications & Control, 9(6), 686-7693, 2014.
- [36] Dzitac I., The Fuzzification of Classical Structures: A General View, International Journal of Computers Communications & Control, 10(6), 772-788, 2015.
- [37] Dzitac I. (2008); ICCCC 2008 & EWNLC 2008 Celebrates Bardeen's Centenary and Welcomes Professor Zadeh, International Journal of Computers Communications & Control, 3(Supplement: S), 16-25, 2008.
- [38] Dzitac I., Barbat B.E.(2008); Artificial Intelligence + Distributed Systems = Agents, International Journal of Computers Communications & Control, 4(1), 17-26, 2009.
- [39] Dzitac I., Vesselenyi T., Tarca R.C. (2011), Identification of ERD using Fuzzy Inference Systems for Brain-Computer Interface, *International Journal of Computers Communications* & Control, 6(3), 403-417, 2011.
- [40] Dzitac S., Felea I., Dzitac I., Vesselenyi T. (2008); An Application of Neuro-Fuzzy Modelling to Prediction of Some Incidence in an Electrical Energy Distribution Center, *International Journal of Computers Communications & Control*, 3(Supplement: S), 287-292, 2008.
- [41] Dzitac S., Felea I., Dzitac I., Vesselenyi T. (2008), Neuro-Fuzzy Modeling of Event Prediction using Time Intervals as Independent Variables, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 84-99, 2008.
- [42] Eklund P., Gahler W. (1988); Basic notions for fuzzy topology, Fuzzy Sets and Systems, 26: 333–356, 1988.
- [43] Feng G. (2006), A Survey on Analysis and Design of Model-Based Fuzzy Control Systems, IEEE Transactions on Fuzzy Systems, 14(5), 676-697, 2006.
- [44] Georgescu G., Popescu A. (2004); Non-dual fuzzy connections, Archive for Mathematical Logic, 43(8), 1009-1039, 2004.
- [45] Goguen J. (1968), Categories of Fuzzy Sets: Applications of a Non-Cantorian Set Theory, Ph.D. Thesis. University of California at Berkeley, June 1968.
- [46] Goguen J. (1973); The fuzzy Tychonoff theorem, J. of Math. Analysis and Applications, 43, 734–742, 1973.
- [47] Goguen J. (1967), L-Fuzzy Sets, Journal of Mathematical Analysis and Applications, 18, 145-174, 1967.
- [48] Goguen J. (1969), The Logic of Inexact Concepts, Synthese, 19, 325-373, 1969.
- [49] Hammerbacher J.M., Yager R.R. (1981), The personalization of security: An application of fuzzy set theory, *Fuzzy Sets and Systems*, 5, 1-9, 1981.
- [50] Hersh H.M., Caramazza A., Brownell H.H. (1979), Effects of context on fuzzy membership functions, in: M.M. Gupta et. al. (eds) Advances in Fuzzy Set Theory and Applications, Amsterdam, New York, Oxford, 1979.

- [51] Hirota K. (1993); Industrial Applications of Fuzzy Technology, Springer, 1993.
- [52] Jain R. (1976); Tolerance analysis using fuzzy sets, Int. J. Syst, 7(12), 1393–1401.
- [53] John R., S. Coupland S. (2007); Type-2 Fuzzy Logic: A Historical View, IEEE Computational Intelligence Magazine, 2(1), 57-62, 2007.
- [54] Kaleva O., Seikkala S. (1984); On fuzzy metric spaces, Fuzzy Sets and Systems, 12, 215–229, 1984.
- [55] Kaufmann A. (1975); Introduction to the Theory of Fuzzy Subsets: Fundamental theoretical elements, Academic Press, 1975.
- [56] Kreinovich V., Stylios C. (2015); Why Fuzzy Cognitive Maps Are Efficient, International Journal of Computers Communications & Control, 10(6), 825-833, 2015.
- [57] Lefranc G. (2008), Towards to Colony of Robots, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 100-128, 2008.
- [58] Lin T.C., Kuo C.H., Balas V.E. (2011); Uncertain Fractional Order Chaotic Systems Tracking Design via Adaptive Hybrid Fuzzy Sliding Mode Control, International Journal of Computers Communications & Control, 6(3), 418-428, 2011.
- [59] Lupulescu V. (2009); On a class of fuzzy functional differential equations, Fuzzy Sets and Systems, 160(11), 1547-1562, 2009.
- [60] Mamdani E.H. (1977), Application of fuzzy logic to approximate reasoning, *IEEE Trans. Comput.*, 26, 1182-1191, 1977.
- [61] Mamdani E.H., Assilian S. (1975), An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller, International Journal of Man-Machine Studies, 7(1), 1-13, 1975.
- [62] Marin-Perianu M., Havinga P.J.M. (2007); D-FLER: A distributed fuzzy logic enginefor rulebased wireless sensor networks, Proc. of International Symposium on Ubiquitous Computing Systems (UCS), 86-101, 2007.
- [63] Meyer A., Zimmermann H.-J. (2011), Applications of Fuzzy Technology in Business Intelligence, International Journal of Computers Communications & Control, 6(3), 428-441, 2011.
- [64] Mizumoto M., Tanaka J. (1979); Some properties of fuzzy numbers, in M.M. Gupta et al. (eds.), Advances in Fuzzy Set Theory and Applications, 153–164, 1979.
- [65] Mihet D. (2004); A Banach contraction theorem in fuzzy metric spaces, Fuzzy Sets and Systems, 144(3), 431-439, 2004.
- [66] Molnarka G.I., Koczy L.T. (2011), Decision Support System for Evaluating Existing Apartment Buildings Based on Fuzzy Signatures, International Journal of Computers Communications & Control, 6(3), 442-457, 2011.
- [67] Moisil G.C. (1940), Recherches sur les logiques non-chrysippiennes (in French), Ann. Sci. Univ. Jassy, 26, 413-466, 1940.
- [68] Moisil G.C. (1968), Lukasiewiczian algebras, Preprint, Computing Center, Univ. of Bucharest, 1-25, 1968.

- [69] Moisil G.C. (1972), *Essais sur les logiques non chrysippiennes* (in French), Editions de l'Academie, Bucharest, 1972.
- [70] Moisil I. (2016), The wonderful adventures of the mathematician in logic-land: From Lukasiewicz-Moisil logic to computers, Computers Communications and Control (ICCCC), 2016 6th International Conference on, 1-9, DOI: 10.1109/ICCCC.2016.7496730, 2016
- [71] Nadaban S. (2015); Fuzzy Continuous Mappings in Fuzzy Normed Linear Spaces, International Journal of Computers Communications & Control, 10(6), 834-842, 2015.
- [72] Nadaban S. (2014); Fuzzy pseudo-norms and fuzzy F-spaces, Fuzzy Sets and Systems, 282, 99-114, 2016.
- [73] Nadaban S., Dzitac I. (2014), Atomic Decompositions of Fuzzy Normed Linear Spaces for Wavelet Applications, *Informatica*, 25(4), 643-662, 2014.
- [74] Nadaban S., Dzitac I. (2016), Some Properties and Applications of FuzzyQuasi-Pseudo-Metric Spaces, *Informatica*, 27(1), 141-150, 2016.
- [75] Nadaban S., Dzitac S., Dzitac I. (2016); Fuzzy TOPSIS: A General View, Procedia Computer Science, 91, 823-831, 2016.
- [76] Negoita C.V., Ralescu D.A. (1974), Fuzzy Sets and Their Applications (in Romanian), Ed. Tehnica, Bucharest, 1974.
- [77] Negoita C.V., Ralescu D.A. (1975), Applications of Fuzzy Sets to Systems Analysis, Springer Basel, 1975.
- [78] Negoita C.V. (1981), Fuzzy Systems, Abacus Press, Tunbridge Wells, UK, 1981.
- [79] Negoita C.V.(1985), Expert Systems and Fuzzy Systems, Benjamin Cummings, Menlopark, CA, 1985.
- [80] Negoita C.V. (2000), Fuzzy Sets, Newfalcon Publications, Tempe, AZ, 2000.
- [81] Negoita C.V. (2011); Remembering the Beginnings, International Journal of Computers Communications & Control, 6(3), 458-461, 2011.
- [82] Nikolova N., Ahmed S., Tenekedjiev K. (2008); Generic Procedure for Construction of a Multi-Dimensional Utility Function under Fuzzy Rationality, *International Journal of Computers Communications & Control*, 3(Supplement: S), 422-426, 2008.
- [83] Nikolova N., Toneva D., Ahmed S., Tenekedjiev K. (2008), Constructing Multi-dimensional Utility Function under Different Types of Preferences over the Fundamental Vector Attributes, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 129-149, 2008.
- [84] Nyirenda C.N., Dong F., Hirota K. (2011); Distance Based Triggering and Dynamic Sampling Rate Estimation for Fuzzy Systems in Communication Networks, *International Journal* of Computers Communications & Control, 6(3), 462-472, 2011.
- [85] Nyirenda C.N., Dawoud D.S. (2006); Multi-objective Particle Swarm Optimization for Fuzzy Logic Based Active Queue Management, Proc. of the IEEE International Conference in Fuzzy Systems, 2231-2238, 2006.

- [86] Oros G.I., Oros G. (2011); The notion of subordination in fuzzy sets theory, General Mathematics, 19(4), 97-103, 2011.
- [87] Oros G.I., Oros G. (2012); Dominants and best dominants in fuzzy differential subordinations, Stud. Univ. Babes-Bolyai Math., 57(2), 239-248, 2012.
- [88] Pop B., Dzitac I. (2006); On a fuzzy linguistic approach to solving multiple criteria fractional programming problem, *International Journal of Computers Communications & Control*, 1(Supplement: S), 381-385, 2006.
- [89] Precup R.-E., Preitl S. (2006); Stability and Sensitivity Analysis of Fuzzy Control Systems. Mechatronics Applications, *CActa Polytechnica Hungarica*, 3(1), 61-76, 2006.
- [90] Precup R.-E., Hellendoorn H. (2011); A survey on industrial applications of fuzzy control, Computers in Industry, 62(3), 213-226, 2011.
- [91] Precup R.-E., Tomescu M.L., Petriu E.M. (2015); A Unified Anti-Windup Technique for Fuzzy and Sliding Mode Controllers, *International Journal of Computers Communications* & Control, 10(6), 843-854, 2015.
- [92] Radojevic D.G. (2008); Fuzzy Set Theory in Boolean Frame, International Journal of Computers Communications & Control, 3(Supplement: S), 121-131, 2008.
- [93] Radojevic D.G. (2008), Real Sets as Consistent Boolean Generalization of Classical Sets, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 150-171, 2008.
- [94] del Rosal E., Nunez R., Castaneda C., Ortega A. (2008), Simulating NEPs in a Cluster with jNEP, pp. 172-184, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 150-171, 2008.
- [95] Ralescu A.L., Ralescu D.A. (1984); Probability and fuzziness, Information Sciences, 34(2), 85-92, 1984.
- [96] Ralescu A.L., Ralescu D.A. (1997); Extensions of fuzzy aggregation, Fuzzy Sets and Systems, 86(3), 321-330, 1997.
- [97] Rudas I.J., Fodor J. (2008), Soft Computing for Intelligent Systems, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 185-195, 2008.
- [98] Reformat M.Z., Yager R.R., Li Z., Alajlan N. (2011); Human-inspired Identification of Highlevel Concepts using OWA and Linguistic Quantifiers, *International Journal of Computers Communications & Control*, 6(3), 473-502, 2011.
- [99] Ruan D. (1990); A critical study of widely used fuzzy implication operators and the inference rules in fuzzy expert systems, Ph.D. Thesis, Gent 1990.
- [100] Rudeanu S. (1993); On Lukasiewicz-Moisil algebras of fuzzy sets, Studia Logica, 52(1), 95-111, 1993.

- [101] Sahab N., Hagras H. (2011); Adaptive Non-singleton Type-2 Fuzzy Logic Systems: A Way Forward for Handling Numerical Uncertainties in Real World Applications, International Journal of Computers Communications & Control, 6(3), 503-529, 2011.
- [102] Sahab N., Hagras H. (2011), An Adaptive Type-2 Input Based Non-Singleton Type-2 Fuzzy Logic System for Real World Applications, *IEEE International conference on Fuzzy Systems*, 509-516, 2011.
- [103] Salski A. (1999), Ecological Modeling and Data Analysis, in: H.-J. Zimmermann (ed.): Practical Applications of Fuzzy Technologies, Kluwer Academic Publ., Boston, 247-266, 1999.
- [104] Seising R. (2011), From Electrical Engineering and Computer Science to Fuzzy Languages and the Linguistic Approach of Meaning: The non-technical Episode: 1950-1975, International Journal of Computers Communications & Control, 6(3), 530-561, 2011.
- [105] Schmitt I., Nurnberger A., Lehrack S. (2009); On the Relation between Fuzzy and Quantum Logic, In: Seising R. (eds) Views on Fuzzy Sets and Systems from Different Perspectives. Studies in Fuzziness and Soft Computing, vol 243, Chapter 20, Springer, Berlin, Heidelberg, 2009.
- [106] Stancu-Minasian I.M., Pop B. (2003); On a fuzzy set approach to solving multiple objective linear fractional programming problem, *Fuzzy Sets and Systems*, 134(3), 397-405, 2003.
- [107] Stanojevic B., Dzitac I., Dzitac S. (2015), On the ratio of fuzzy numbers exact membership function computation and applications to decision making, *Technological and Economic Development of Economy*, 21(5), 815-832, 2015.
- [108] Stefanescu G., Chira C. (2008), New Parallel Programming Language Design: A Bridge between Brain Models and Multi-Core/Many-Core Computers?, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 196-210, 2008.
- [109] Tamir D.E., Kandel A. (2011); Axiomatic Theory of Complex Fuzzy Logic and Complex Fuzzy Classes, International Journal of Computers Communications & Control, 6(3), 562-576, 2011.
- [110] Tang X., Shu L. (2015); An Improved Attribute Reduction Algorithm based on Granular Computing, International Journal of Computers Communications & Control, 10(6), 856-864, 2015.
- [111] Teodorescu H.-N.L. (2011); On the meaning of approximate reasoning: An unassuming subsidiary to Lotfi Zadeh's paper dedicated to the memory of Grigore Moisil, International Journal of Computers Communications & Control, 6(3), 577-580, 2011.
- [112] Teodorescu H.-N.L. (2015); A Retrospective Assessment of Fuzzy Logic Applications in Voice Communications and Speech Analytics, International Journal of Computers Communications & Control, 10(6), 865-872, 2015.
- [113] Teodorescu H.-N.L. (2015); In Memoriam Lotfi A. Zadeh, Memoirs of the Scientific Sections of the Romanian Academy, Tome XL, 2017.
- [114] Thole U., Zimmermann H.-J., Zysno P. (1979), On the suitability of minimum and product operators for the intersection of fuzzy sets, *Fuzzy Sets and Systems*, 2, 167-180, 1979.

- [115] Tufis D. (2008), Playing with Word Meanings, in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 211-223, 2008.
- [116] Trillas E. (2011); Lotfi A. Zadeh: On the man and his work, *Scientia Iranica*, 18(3), 574-579, 2011.
- [117] Turskis Z., Zavadskas E.K., Antucheviciene J., Kosareva N. (2015); A Hybrid Model Based on Fuzzy AHP and Fuzzy WASPAS for Construction Site Selection, *International Journal* of Computers Communications & Control, 10(6), 873-888, 2015.
- [118] Vesselenyi T., Dzitac S., Dzitac I., Manolescu M.J. (2007), Fuzzy and neural controllers for a pneumatic actuator, *International Journal of Computers Communications & Control*, 2(4), 375-387, 2007.
- [119] Vesselenyi T., Dzitac I., Dzitac S., Vaida V. (2008), Surface Roughness Image Analysis using Quasi-Fractal Characteristics and Fuzzy Clustering Methods, *International Journal* of Computers Communications & Control, 3(3), 304-316, 2008.
- [120] Wang H.-D., Guo S.-C., Hosseini Bamakan S.M., Shi Y. (2015); Homeomorphism Problems of Fuzzy Real Number Space and The Space of Bounded Functions with Same Monotonicity on [-1,1], International Journal of Computers Communications & Control, 10(6), 889-903, 2015.
- [121] Wang T., G. Zhang G., Perez-Jimenez M.J. (2015); Fuzzy Membrane Computing: Theory and Applications, International Journal of Computers Communications & Control, 10(6), 904-935, 2015.
- [122] Wu D., Tan W.W. (2006), Genetic Learning and Performance Evaluation of Interval Type-2 Fuzzy Logic Controllers, Engineering Applications of Artificial Intelligence, 19(8), 829-841, 2006.
- [123] http://www.johnlewis.com/
- [124] http://www.gandhiappliances.com/index.php?main\\_page=product\\_info\&cPath= 10\\_224\&products\\_id=5847
- [125] Xia F., Zhao W.H., Sun Y.X., Tian Y.C. (2007); Fuzzy Logic Control Based QoS Management in Wireless Sensor/Actuator Networks, Sensors, 7(12), 3179-3191, 2007.
- [126] Xu H., Vilanova R. (2015); PI and Fuzzy Control for P-removal in Wastewater Treatment Plant, International Journal of Computers Communications & Control, 10(6), 936-951, 2015.
- [127] Yager R.R. (2015); Foreword, International Journal of Computers Communications & Control, 10(6), 771-771, 2015.
- [128] Zadeh F. (1998); My Life and Travels with the Father of Fuzzy Logic, Hardcover, 1998
- [129] Zadeh L.A. (1965), Fuzzy Sets, Information and Control, 8, 338-353, 1965.
- [130] Zadeh L.A. (1969), Biological Application of the Theory of Fuzzy Sets and Systems, in: Proctor, LD. (ed.): The Proceedings of an International Symposium on Biocybernetics of the Central Nervous System, London: Little, Brown and Comp., 199-206, 1969.

- [131] Zadeh L.A. (1971), Toward Fuzziness in Computer Systems: Fuzzy Algorithms and Languages, in: Architecture and Design of Digital Computers, G. Boulaye (ed.), Paris: Dunod, 9-18, 1971.
- [132] Zadeh L.A. (1973), Outline of a New Approach to the Analysis of Complex Systems and Decision Processes, *IEEE Transactions on Systems*, Man, and Cybernetics, 28-44, 1973.
- [133] Zadeh, L.A. (1975); The concept of a linguistic variable and its application to approximate reasoning, parts 1,2 and 3, Inf. Sciences, 8: 199–249, 8: 301–357, 9: 43–80, 1975.
- [134] Zadeh L.A. (1975), Fuzzy Logic and Approximate Reasoning (In Memory of Grigore Moisil), Synthese, 30, 407-428, 1975.
- [135] Zadeh L.A. (1988); Fuzzy Logic, *IEEE Computer*, 21(4), 83-93, 1988.
- [136] Zadeh L.A. (1996), Fuzzy logic = computing with words, Fuzzy Systems, IEEE Transactions on, 4(2), 103–111, 1996.
- [137] Zadeh L.A. (1999), From computing with numbers to computing with words from manipulation of measurements to manipulation of perceptions, *IEEE Transactions on Circuits* and Systems, 45, 105-119, 1999.
- [138] Zadeh L.A. (2001), A new direction in AI toward a computational theory of perceptions, AI Magazine, 22(1), 73-84, 2001.
- [139] Zadeh L.A., Is there a need for fuzzy logic?, Information Sciences, 178, 2751-2779, 2008.
- [140] Zadeh L.A. (2008); A New Frontier in Computation Computation with Information Described in Natural Language, International Journal of Computers Communications & Control, 3(Supplement: S), 26-27, 2008.
- [141] Zadeh L.A. (2008), An Extended Abstract of Workshop's Keynote Lecture: "A New Frontier in Computation - Computation with Information Described in Natural Language", in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 224-225, 2008.
- [142] Zadeh L.A. (2008), APPENDIX 1: Slides of PowerPoint Presentation of Workshop's Keynote Lecture: "A New Frontier in Computation - Computation with Information Described in Natural Language", in: L.A.Zadeh, Dan Tufis, F.G.Filip, I.Dzitac (Eds); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Ed. House of Romanian Academy, 227-267, 2008.
- [143] Zadeh L.A. (2011); My Life and Work A Retrospective View, Aplied and Computational Mathematics, Special Issue on Fuzzy Set Theory and Applications, Dedicated to the 90th Birthday of prof. Lotfi A. Zadeh, 10(1), 4-9, 2011.
- [144] Zadeh L.A. (2011); Foreword, International Journal of Computers Communications & Control, 6(3), 385-386, 2011.
- [145] Zadeh, L.A., Tufis, D., Filip, F.G., Dzitac I. (2008); From Natural Language to Soft Computing: New Paradigms in Artificial Intelligence, Editing House of the Romanian Academy, 2008.

- [146] Zimmermann H.-J. (1980), Testability and meaning of mathematical models in social sciences, *Mathematical Modelling*, 1, 123-139, 1980.
- [147] Zimmermann H.-J. (1987); Fuzzy Sets, Decision Making, and Expert Systems, Kluwer, Boston, Dodrecht, Lancaster, 1987.
- [148] Zimmermann H.-J. (1988), Fuzzy set theory and inference mechanism, in: G.Mitra (ed.) Mathematical models for decision support, Berlin, Heidelberg, 1988.
- [149] Zimmermann H.-J. (2001), Fuzzy Set Theory and its Applications, fourth edition, Boston, Dodrecht, London, 2001.
- [150] Zimmermann H.-J., Zysno P. (1980), Latent connectives in human decision making, Fuzzy Sets and Systems, 4, 37-51, 1980.
- [151] Zimmermann H.-J., Zysno P. (1983), Decisions and evaluations by hierarchical aggregation of information, *Fuzzy Sets and Systems*, 10, 243-266, 1983.