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Interval Certitude Rule Base Inference Method using the Evidential Reasoning

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Abstract: Development of rule-based systems is an important research area for artificial intelligence and decision making, as rule base is one of the most general purpose forms for expressing human knowledge. In this paper, a new rule-based representation and its inference method based on evidential reasoning are presented based on operational research and fuzzy set theory. In this rule base, the uncertainties of human knowledge and human judgment are designed with interval certitude degrees which are embedded in the antecedent terms and consequent terms. The knowledge representation and inference framework offer an improvement of the recently developed rule base inference method, and the evidential reasoning approach is still applied to the rule fusion. It is noteworthy that the uncertainties will be defined and modeled using interval certitude degrees. In the end, an illustrative example is provided to illustrate the proposed knowledge representation and inference method as well as demonstrate its effectiveness by comparing with some existing approaches.
Keywords: interval certitude rule, knowledge representation, uncertainty inference, evidential reasoning.

1 Introduction

There are several types of uncertainties of knowledge such as incompleteness, randomness, fuzziness, inexactness and ignorance. The inference method which is based on uncertain knowledge is the uncertainty inference method. Because of the uncertainty is ubiquitous, the uncertainty knowledge representation and uncertainty inference method have become the most visible and fastest growing branch of artificial intelligence [38].

In the existed methods of knowledge representation, rule base seems to be the most common form for expressing various types of knowledge [27]. The rule bases can be used to support decision making with both quantitative and qualitative knowledge under various types of uncertainties. Specifically, different types of data such as exact number, fuzzy number, interval value, even subjective judgment can be represented by rule.

In the last four decades, the development of uncertainty methods have received extensive attention. Many methods have been proposed for showing uncertain knowledge and information. Such as the certainty factor based inference method [36], inference method using the Dempster-Shafer theory [12], *Belief Rule Base Inference Methodology using the Evidential Reasoning* (RIMER) [31, 38–40], subjective Bays method [36], fuzzy inference method [29], et al.

In these methods, inference method using the Dempster-Shafer theory has a shortcoming in the aspect of dealing with conflict evidence, the subjective Bayes method is mainly used in the randomness but does not apply to other uncertainties, fuzzy inference method is used in the vagueness.

With the Dempster-Shafer theory, fuzzy set theory and If-then rule, Yang et al [38] have researched and given RIMER. The uncertain knowledge is described by *Belief Rule Base* (BRB), and it is able to handle many types of information. *Evidential Reasoning* (ER) approach is an uncertainty inference method which make up for the deficiencies of the inference method using the Dempster-Shafer theory. However, there are still some defects [17].

Liu et al. [5, 21] proposed the *Extended Belief Rule Base* (EBRB) based on BRB and the *Extended Belief Rule Base Inference Methodology using the Evidential Reasoning* (RIMER+). And the uncertainty of antecedent is described with belief structure and different evaluation grade set, but there is only one consequent attribute and knowledge should be converted to belief structure.

The fuzzy set has been widely researched and applied in various fields [20, 30, 34]. Interval number is an important fuzzy number, and it has been widely used in a lot of fields such as decision making science [2,32,37,41], operational research [6,9,19], [8,33]. The main reasons are as follows:

- In the theoretical research, interval numbers are closely linked with other forms of uncertainty.
- In the practical application, in order to evaluate the things, people are mainly inclined to give the upper and the lower bounds of information, and the interval number's expression is nearly accord with this uncertainty characteristics of human thought.

According to the above analysis, the certitude rule base is not accord with actual situation. Because of the limitations of human cognition and the complexity of the objective things, decision maker can not always be confident enough to provide subjective judgment with exact certitude degrees. But at times, a range of the certitude degree can be assessed, such range is referred to as interval certitude degree. In order to apply the interval uncertainty, the knowledge representation which is based on the interval certitude rule and its inference method should be proposed. [16]

This paper is organized as follows. In Section 2, the *Interval Certitude Rule Base Inference Method using the Evidential Reasoning* (ICRIMER) is given. First, the structure and representation of *Interval Certitude Rule Base* (ICRB) is presented. Then a new generic knowledge base inference method ICRIMER is proposed. In Section 3, the performances of ICRIMER is demonstrated by comparing with some existing approaches using a case study of classifications with eight data sets of UCI Machine Learning Repository. The paper is concluded in Section 4.

2 Interval certitude rule base inference method

In this section, the basic knowledge of interval value fuzzy number is introduced, include concept, ranking method and similarity measure. Then the interval certitude rule is proposed as knowledge representation method. In the end of this section, the interval certitude rule base inference method using the evidential reasoning with nonlinear programming is presented.

2.1 Interval value fuzzy number

Note $a = \left[a^L, a^U\right]$ be a bounded closed interval, and satisfies $a^L \le a^U$, a^L is the lower bound of a, a^U is the upper bound of a. If $a^L, a^U \in R$, then $a = \left[a^L, a^U\right]$ is an interval number; if

$a^L, a^U \in [0,1]$, then $a = \left[a^L, a^U\right]$ is an interval value fuzzy number (for short, interval number). Specially, if $a^L = a^U$, then $a = \left[a^L, a^U\right]$ is an exact number, note $a = a^L$.

A ranking method based on the technique for order preference by similarity to ideal solution (TOPSIS) and distance is given in order to rank the interval number.

According to TOPSIS [22], for interval number, the positive ideal solution is $[1,1]$, the negative ideal solution is $[0,0]$. The relative similarity degree η of interval number $a = \left[a^L, a^U\right]$ is as follows:

$$\eta(a) = \frac{d^-(a)}{d^+(a) + d^-(a)}$$

where $d^-(a) = \left|\frac{a^L+a^U}{2} - 0\right| = \frac{a^L+a^U}{2}$ is the distance [23] between $a = \left[a^L, a^U\right]$ and negative ideal solution $[0,0]$, $d^+(a) = \left|\frac{a^L+a^U}{2} - 1\right| = 1 - \frac{a^L+a^U}{2}$ is the distance [23] between $a = \left[a^L, a^U\right]$ and positive ideal solution $[1,1]$.

The relative similarity degree η satisfies that the larger the relative similarity degree is the larger the interval number will be. So the ranking of interval number can be given based on the relative similarity degree:

- If $\eta(a) > \eta(b)$ then $a \succ b$, '$\succ$' denotes the fuzzified version of '$>$' and has the linguistic interpretation 'greater than'.
- If $\eta(a) = \eta(b)$ then $a = b$, means that the ranking orders of $a = \left[a^L, a^U\right]$ and $b = \left[b^L, b^U\right]$ are identical, $a = \left[a^L, a^U\right]$ is equal to $b = \left[b^L, b^U\right]$, but $a^L = b^L$ and $a^U = b^U$ may be not always realized.

The similarity measure of interval numbers is also an important content of interval type data processing. According to the Lukasiewicz implication algebra on $[0,1]$, suppose $a = \left[a^L, a^U\right]$ and $b = \left[b^L, b^U\right]$ are interval numbers, the similarity measure $S_{[]}$ is as follows:

$$S_{[]}(a,b) = \left[S_{[]}^L(a,b), S_{[]}^U(a,b)\right]$$

$$S_{[]}{}^L(a,b) = \min\left\{\min\left\{1 - a^L + b^L, 1 - b^L + a^L\right\}, \min\left\{1 - a^U + b^U, 1 - b^U + a^U\right\}\right\}$$

$$S_{[]}^U(a,b) = \max\left\{\min\left\{1 - a^L + b^L, 1 - b^L + a^L\right\}, \min\left\{1 - a^U + b^U, 1 - b^U + a^U\right\}\right\}$$

2.2 Interval certitude rule

An interval certitude rule base (ICRB) with K rules can be represented as follows:

$$R = \left\langle (X,A), (Y,C), ICD_{[]}, \Theta, W, F \right\rangle$$

where $X = \{X_i \,|i = 1, 2, \cdots, I\}$ is the set of antecedent attributes, the relationship among the antecedent attributes is taken as '$\wedge$', '$\wedge$' is the logical connective; $A_i = \left\{A_{i,I_i} \,\middle|\, I_i = 1, 2, \cdots, L_i^A\right\}$ is the set of attribute values for antecedent attribute X_i $(i = 1, 2, \cdots, I)$, note $A = \{A_i \,|i = 1, 2, \cdots, I\}$ be the set of antecedent attribute value sets; $Y = \{Y_j \,|j = 1, 2, \cdots, J\}$ is the set of consequent attributes, the relationship among the consequent attributes is taken as '$\wedge$'; $C_j = \left\{C_{j,J_j} \,\middle|\, J_j = 1, 2, \cdots, L_j^C\right\}$ is the set of attribute values for consequent attribute Y_j $(j = 1, 2, \cdots, J)$, note $C = \{C_j \,|j = 1, 2, \cdots, J\}$ be the set of consequent attribute value sets; $\Theta = \left\{\theta^k \,|0 \leq \theta^k \leq 1, k = 1, 2, \cdots, K\right\}$ is the set of the importance degree of each rule, θ^k is the rule weight of the kth rule; $W = \left\{ w_i \middle| 0 \le w_i \le 1, i = 1, \cdots, I, and \sum_{i=1}^{I} w_i = 1 \right\}$ is the set of the antecedent attribute weights, w_i is the weight of X_i ; $ICD_{[]} = \{Icd(\Delta) \,|Icd(\Delta) \subseteq [0,1]\}$ is interval certitude degree set, $Icd(\Delta)$ is the interval certitude degree of event Δ, $Icd(\Delta)$ is called interval certitude degree. $Icd(\Delta)$ satisfies the more higher the degree of certainty is, the more large $Icd(\Delta)$ will be. $Icd(\Delta) = [0,0]$ means completely uncertainty, $Icd(\Delta) = [1,1]$ means completely certainty, $Icd(\Delta) = [0,1]$ means that we know nothing about the uncertainty of event Δ. F is a logical function.

More specifically, the kth rule in the ICRB R can be written as R^k $(k = 1, 2, \cdots, K)$:

If $\left(X_1 = A_1^k, Icd^k\left(X_1 = A_1^k\right)\right) \wedge \cdots \wedge \left(X_I = A_I^k, Icd^k\left(X_I = A_I^k\right)\right)$
then $\left(Y_1 = C_1^k, Icd^k\left(Y_1 = C_1^k\right)\right) \wedge \cdots \wedge \left(Y_J = C_J^k, Icd^k\left(Y_J = C_J^k\right)\right)$
with rule interval certitude degree $Icd^k\left(R^k\right)$, rule weight θ^k and antecedent attribute weights $(w_1, w_2, \cdots, w_I)$

where '=' means 'is'; θ^k is the weight of the kth rule R^k; $(w_1, w_2, \cdots, w_I)$ are the weights of the antecedent attributes.

For $i = 1, 2, \cdots, I$, A_i^k is the referential value of the ith antecedent attribute X_i that is used in the kth rule R^k , $A_i^k \in A_i$ or $A_i^k = \phi$, (ϕ is default, means that the ith attribute A_i of the kth rule. R^k has no effect on the consequent); $Icd^k\left(X_i = A_i^k\right) \subseteq [0,1]$ is the interval certitude degree of that A_i^k is the referential value of X_i in R^k , means the degree of A_i^k influence on the consequent; if $A_i^k = \phi$ then $Icd^k\left(X_i = A_i^k\right) = 0$ and this antecedent attribute can be left out.

For $j = 1, 2, \cdots, J$, C_j^k is the referential value of the jth consequent attribute Y_j that is used in R^k , $C_j^k \in C_j$ or $C_j^k = \phi$ (means that the kth rule R^k and its antecedent have no effect on the jth consequent C_j), $Icd^k\left(Y_j = C_j^k\right) \subseteq [0,1]$ is the interval certitude degree of that C_j^k is the referential value of Y_j in R^k, means the degree of that Y_j is C_j^k , if $C_j^k = \phi$ then $Icd^k\left(Y_j = C_j^k\right) = 0$ and this consequent attribute can be left out; $Icd^k\left(R^k\right) \subseteq [0,1]$ is the interval certitude degree of R^k, means the degree of that R^k is true.

For shortly, the kth rule R^k can be written as:

If $\left(A_1^k, Icd^k\left(A_1^k\right)\right) \wedge \cdots \wedge \left(A_I^k, Icd^k\left(A_I^k\right)\right)$
then $\left(C_1^k, Icd^k\left(C_1^k\right)\right) \wedge \cdots \wedge \left(C_J^k, Icd^k\left(C_J^k\right)\right)$
with rule certitude degree $Icd^k\left(R^k\right)$, rule weight θ^k and antecedent attribute weights $(w_1, w_2, \cdots, w_I)$

Note:

$$\alpha_i^k = Icd^k\left(A_i^k\right) = \left[\left(\alpha_i^k\right)^L, \left(\alpha_i^k\right)^U\right], \beta_j^k = Icd^k\left(C_j^k\right) = \left[\left(\beta_j^k\right)^L, \left(\beta_j^k\right)^U\right]$$

$$\gamma^k = Icd^k\left(R^k\right) = \left[\left(\gamma^k\right)^L, \left(\gamma^k\right)^U\right]$$

where $0 \le \left(\alpha_i^k\right)^L \le \left(\alpha_i^k\right)^U \le 1, 0 \le \left(\beta_j^k\right)^L \le \left(\beta_j^k\right)^U \le 1, 0 \le \left(\gamma^k\right)^L \le \left(\gamma^k\right)^U \le 1$; and the kth rule R^k can be given as:

If $\left(A_1^k, \alpha_1^k\right) \wedge \cdots \wedge \left(A_I^k, \alpha_I^k\right)$, then $\left(C_1^k, \beta_1^k\right) \wedge \cdots \wedge \left(C_J^k, \beta_J^k\right)$.
with rule certitude degree $\left[\left(\gamma^k\right)^L, \left(\gamma^k\right)^U\right]$, rule weight θ^k and antecedent attribute weights $(w_1, w_2, \cdots, w_I)$

Note:

$$A^k = \left\{ A_i^k \left| A_i^k \in A_i or A_i^k = \phi, i = 1, 2, \cdots, I \right. \right\}$$

$$C^k = \left\{ C_j^k \left| C_j^k \in C_j or C_j^k = \phi, j = 1, 2, \cdots, J \right. \right\}$$

$$\wedge A^k = A_1^k \wedge A_2^k \wedge \cdots \wedge A_I^k$$

$$\wedge C^k = C_1^k \wedge C_2^k \wedge \cdots \wedge C_J^k$$

where A^k is the set of antecedent attribute values of the kth rule R^k; $\wedge A^k$ is the antecedent of the kth rule R^k; C^k is the set of consequent attribute values of the kth rule R^k; $\wedge C^k$ is the consequent of the kth rule R^k.

The kth rule R^k can be given as:

$$\text{If } \left(\wedge A^k, \alpha^k\right) \text{ then } \left(\wedge C^k, \beta^k\right)$$

with rule certitude degree γ^k, rule weight θ^k and antecedent attribute weights $(w_1, w_2, \cdots, w_I)$

where $\alpha^k = \left[\left(\alpha^k\right)^L, \left(\alpha^k\right)^U\right] \subseteq [0,1]$ is the interval certitude degree of $\wedge A^k$, $\beta^k = \left[\left(\beta^k\right)^L, \left(\beta^k\right)^U\right] \subseteq [0,1]$ is the interval certitude degree of $\wedge C^k$, $\gamma^k = \left[\left(\gamma^k\right)^L, \left(\gamma^k\right)^U\right] \subseteq [0,1]$ is the interval certitude degree of the kth rule R^k, θ^k is the weight of the kth rule R^k, $(w_1, w_2, \cdots, w_I)$ are the weights of the antecedent attributes.

2.3 Inference method

The interval certitude rule base inference method using the evidential reasoning is given as follows.

The input actual vector can be noted as

$$Input() = \{(a_1, \alpha_1), (a_2, \alpha_2), \cdots, (a_I, \alpha_I)\}$$

where $a_i \in A_i$ or $a_i = \phi\,(i = 1, 2, \cdots, I)$ is one-to-one correspondence with the ith antecedent attribute X_i; $\alpha_i = \left[(\alpha_i)^L, (\alpha_i)^U\right] \left(0 \le (\alpha_i)^L \le (\alpha_i)^U \le 1, i = 1, 2, \cdots, I\right)$ is the interval certitude degree of a_i. If $a_i = \phi$ then $\alpha_i = 0$.

Suppose that the input fact $Input()$ and the kth rule R^k match successfully, for all $i = 1, 2, \cdots, I$, a_i is equal to A_i^k without $A_i^k = \phi$. But the interval certitude degrees of input fact value and antecedent attribute value may be different; so according to the similarity measure, there is a similarity degree should be given as activation certitude factors.

The activation certitude degree of $A_i^k\,(i = 1, 2, \cdots, I)$ under the input fact is given with similarity measure $S_{[]}$ as follows:

$$\tilde{\alpha}_i^k = \begin{cases} S\left(A_i^k, a_i\right) & A_i^k \in A_i \\ 1 & A_i^k = \phi \end{cases}$$

where $S\left(A_i^k, a_i\right) = S_{[]}\left(\alpha_i^k, \alpha_i\right)$, α_i^k is the interval certitude degree of A_i^k, α_i is the interval certitude degree of a_i, $S\left(A_i^k, a_i\right)$ is the similarity degree of A_i^k and a_i. $A_i^k{=}\phi$ is a specific case, if $A_i^k{=}\phi$ then the value of will not influence the matched conclusions, whether a_i is default or some other values, the ith antecedent attribute in the kth rule always can be satisfied.

Note

$$\tilde{\mathrm{A}}^k = \left\{ \tilde{\alpha}_i^k \left| i = 1, 2, \cdots, I \right. \right\}$$

$\tilde{\mathrm{A}}^k$ is the collection of activation certitude degree.

The activation weight set of the kth rule R^k is given as follows:

$$W^k = \left\{ w_i^k \left| i = 1, 2, \cdots, I \right. \right\}$$

where

$$w_i^k = \begin{cases} w_i & A_i^k \in A_i \\ 0 & A_i^k = \phi \end{cases}$$

after normalization, the activation weight set is given as

$$\tilde{W}^k = \left\{ \tilde{w}_i^k \left| i = 1, 2, \cdots, I \right. \right\}$$

where

$$\tilde{w}_i^k = \frac{w_i^k}{\sum\limits_{t=1}^{I} w_t^k}.$$

Because that the '$\wedge$' connective is used in rules, so the aggregation function of antecedent is referred to as the T-norm operator. The interval certitude degree of antecedent $\wedge A^k$ can be obtained as follows:

$$\tilde{\alpha}^k = \left[\left(\tilde{\alpha}^k\right)^L, \left(\tilde{\alpha}^k\right)^U \right] = \left[\prod_{i=1}^{I} \sqrt[I]{\left[\left(\tilde{\alpha}_i^k\right)^L \right]^{\bar{w}_i^k}}, \prod_{i=1}^{I} \sqrt[I]{\left[\left(\tilde{\alpha}_i^k\right)^U \right]^{\bar{w}_i^k}} \right]$$

where

$$\bar{w}_i^k = \frac{\tilde{w}_i^k}{\max\limits_{l=1,\cdots,I} \left\{ \tilde{w}_l^k \right\}}$$

$\bar{w}_i^k$ means that the smaller the weight is, the less the impact on certitude degree $\left(\left(\tilde{\alpha}_i^k\right)^{\bar{w}_i^k}\right.$ is approaches the limit of 1), if $\tilde{w}_i^k = 0$ then $\sqrt[I]{\left[\left(\tilde{\alpha}_i^k\right)^L \right]^{\bar{w}_i^k}} = \sqrt[I]{\left[\left(\tilde{\alpha}_i^k\right)^U \right]^{\bar{w}_i^k}} = 1$ [23].

According to ER approach, $\left\{ \wedge C^k \right\}$ is the set of the consequent of the kth rule R^k as the frame of discernment, let $\Omega^k = \left\{ \emptyset, \left\{ \wedge C^k \right\} \right\}$, R^k and $\wedge A^k$ are evidences, the weights of them should be confirmed, the new weight is called credibility degree. By the definition of ICRB, $\gamma^k = \left[\left(\gamma^k\right)^L, \left(\gamma^k\right)^U \right]$ is the degree of that the rule R^k is true, it can be seem as the degree of R^k influence on $\wedge C^k$ when R^k is true; $\tilde{\alpha}^k = \left[\left(\tilde{\alpha}^k\right)^L, \left(\tilde{\alpha}^k\right)^U \right]$ is the degree of $\wedge A^k$ influence on $\wedge C^k$ when $\wedge A^k$ is true. Based on the above analysis, the credibility degree w_R^k of R^k and the credibility degree w_A^k of $\wedge A^k$ can be obtained as follows:

$$w_R^k = \left[\left(w_R^k\right)^L, \left(w_R^k\right)^U \right] = \left[\frac{\left(\gamma^k\right)^L}{\left(\tilde{\alpha}^k\right)^U + \left(\gamma^k\right)^L}, \frac{\left(\gamma^k\right)^U}{\left(\tilde{\alpha}^k\right)^L + \left(\gamma^k\right)^U} \right]$$

$$w_A^k = \left[\left(w_A^k\right)^L, \left(w_A^k\right)^U \right] = \left[\frac{\left(\tilde{\alpha}^k\right)^L}{\left(\tilde{\alpha}^k\right)^L + \left(\gamma^k\right)^U}, \frac{\left(\tilde{\alpha}^k\right)^U}{\left(\tilde{\alpha}^k\right)^U + \left(\gamma^k\right)^L} \right]$$

The basic probability masses are:

$$m_R\left(\wedge C^k\right) = w_R^k \gamma^k = \left[m_R^L\left(\wedge C^k\right), m_R^U\left(\wedge C^k\right) \right] = \left[\left(w_R^k\right)^L \left(\gamma^k\right)^L, \left(w_R^k\right)^U \left(\gamma^k\right)^U \right]$$

$$m_R\left(\Omega^k\right)=1-m_R\left(\wedge C^k\right)=\left[m_R^L\left(\Omega^k\right),m_R^U\left(\Omega^k\right)\right]=\left[1-\left(w_R^k\right)^U\left(\gamma^k\right)^U,1-\left(w_R^k\right)^L\left(\gamma^k\right)^L\right]$$

$$\bar{m}_R\left(\Omega^k\right)=1-w_R^k=\left[\bar{m}_R^L\left(\Omega^k\right),\bar{m}_R^U\left(\Omega^k\right)\right]=\left[1-\left(w_R^k\right)^U,1-\left(w_R^k\right)^L\right]$$

where $m_R\left(\wedge C^k\right)$ is the basic probability mass of R^k; $m_R\left(\Omega^k\right)$ is the remaining probability mass that is unassigned to C^k which is caused by R^k; $\bar{m}_R\left(\Omega^k\right)$ is amount of remaining support left uncommitted by the weight of R^k.

$$m_A\left(\wedge C^k\right)=w_A^k\tilde{\alpha}^k=\left[m_A^L\left(\wedge C^k\right),m_A^U\left(\wedge C^k\right)\right]=\left[\left(w_A^k\right)^L\left(\tilde{\alpha}^k\right)^L,\left(w_A^k\right)^U\left(\tilde{\alpha}^k\right)^U\right]$$

$$m_A\left(\Omega^k\right)=1-m_A\left(\wedge C^k\right)=\left[m_A^L\left(\Omega^k\right),m_A^U\left(\Omega^k\right)\right]=\left[1-\left(w_A^k\right)^U\left(\tilde{\alpha}^k\right)^U,1-\left(w_A^k\right)^L\left(\tilde{\alpha}^k\right)^L\right]$$

$$\bar{m}_A\left(\Omega^k\right)=1-w_A^k=\left[\bar{m}_A^L\left(\Omega^k\right),\bar{m}_A^U\left(\Omega^k\right)\right]=\left[1-\left(w_A^k\right)^U,1-\left(w_A^k\right)^L\right]$$

where $m_A\left(\wedge C^k\right)$ is the basic probability mass of $\wedge A^k$, $m_A\left(\Omega^k\right)$ is the remaining probability mass that is unassigned to $\wedge C^k$ which is caused by $\wedge A^k$; $\bar{m}_A\left(\Omega^k\right)$ is amount of remaining support left uncommitted by the weight of $\wedge A^k$.

With the Dempster combination method [12], the upper bound $\left(\tilde{\beta}^k\right)^U$ and the lower bound $\left(\tilde{\beta}^k\right)^L$ of the activation degree $\tilde{\beta}^k$ of consequent $\wedge C^k$ are given as follows:

$$Max/Min\tilde{\beta}^k$$

$$S.t.\tilde{\beta}^k=\frac{m_R^*\left(\wedge C^k\right)m_A^*\left(\wedge C^k\right)+m_R^*\left(\wedge C^k\right)m_A^*\left(\Omega^k\right)+m_R^*\left(\Omega^k\right)m_A^*\left(\wedge C^k\right)+m_R^*\left(\Omega^k\right)m_A^*\left(\wedge C^k\right)}{1-\bar{m}_R^*\left(\Omega^k\right)\bar{m}_A^*\left(\Omega^k\right)}$$

$$m_R^*\left(\wedge C^k\right)+m_R^*\left(\Omega^k\right)=1$$

$$m_A^*\left(\wedge C^k\right)+m_A^*\left(\Omega^k\right)=1$$

$$\bar{m}_R^*\left(\Omega^k\right)+\bar{m}_A^*\left(\Omega^k\right)=1$$

the solutions of the optimization problem are the upper bound $\left(\tilde{\beta}^k\right)^U$ and lower bound $\left(\tilde{\beta}^k\right)^L$ of the activation degree $\tilde{\beta}^k$.

$$\tilde{\beta}^k=\left[\left(\tilde{\beta}^k\right)^L,\left(\tilde{\beta}^k\right)^U\right]$$

$$\left(\tilde{\beta}^k\right)^L=Min\quad\tilde{\beta}^k$$

$$\left(\tilde{\beta}^k\right)^U=Max\quad\tilde{\beta}^k$$

Then the interval certitude degree of consequent attribute value $C_j^k\ (j=1,2,\cdots,J)$ is obtained based on the similarity measure $S_{[]}$:

$$\tilde{\beta}_j^k=\left[\left(\tilde{\beta}_j^k\right)^L,\left(\tilde{\beta}_j^k\right)^U\right]$$

$$\left(\tilde{\beta}_j^k\right)^L = \begin{cases} \left[\min\left\{1-\left(\beta_j^k\right)^L+\left(\tilde{\beta}^k\right)^L, 1-\left(\beta_j^k\right)^U+\left(\tilde{\beta}^k\right)^U\right\}\right]\left(\beta_j^k\right)^L \tilde{\alpha}^k=1 \\ \min\left\{\left(\tilde{\beta}^k\right)^L\left(\beta_j^k\right)^L, 1\right\}\tilde{\alpha}^k<1 \end{cases}$$

$$\left(\tilde{\beta}_j^k\right)^U = \begin{cases} \left[\max\left\{1-\left(\beta_j^k\right)^L+\left(\tilde{\beta}^k\right)^L, 1-\left(\beta_j^k\right)^U+\left(\tilde{\beta}^k\right)^U\right\}\right]\left(\beta_j^k\right)^U \tilde{\alpha}^k=1 \\ \min\left\{\left(\tilde{\beta}^k\right)^U\left(\beta_j^k\right)^U, 1\right\}\tilde{\alpha}^k<1 \end{cases}$$

If there are T rules which are matching successfully with the input fact and having the same consequent attribute value $C\prime$, then the combination method should be given. First of all, the tth rules can be given as R_t:

$$\text{If } \left(\wedge A^t, \tilde{\alpha}^t\right) \text{ then } \left(C\prime, \tilde{\beta}_n^t\right) \wedge \cdots \wedge \left(C_{n-1}^t, \tilde{\beta}_{n-1}^t\right) \wedge \left(C_{n+1}^t, \tilde{\beta}_{n+1}^t\right) \wedge \cdots \wedge \left(C_J^t, \tilde{\beta}_J^t\right)$$
$$\text{with rule certitude degree } \gamma^t\text{, rule weight } \theta^t \text{ and antecedent attribute weights } w_1, w_2, \cdots, w_I$$

where $= 1, 2, \cdots, T$, $n \in \{1, 2, \cdots, J\}$ and n is determinate; $\wedge A^t$ is the antecedent of rule R_t, $\tilde{\alpha}^t = \left[\left(\tilde{\alpha}^t\right)^L, \left(\tilde{\alpha}^t\right)^U\right] \subseteq [0,1]$ is the interval certitude degree of $\wedge A^t$; $C\prime$ and C_j^t $(j = 1, \cdots, n-1, n+1, \cdots, J)$ are the consequent attribute values of rule R_t, $\tilde{\beta}_n^t = \left[\left(\tilde{\beta}_n^t\right)^L, \left(\tilde{\beta}_n^t\right)^U\right] \subseteq [0,1]$ is the interval certitude degree of $C\prime$ under the input fact, $\tilde{\beta}_j^t = \left[\left(\tilde{\beta}_j^t\right)^L, \left(\tilde{\beta}_j^t\right)^U\right] \subseteq [0,1]$ $(j = 1, \cdots, n-1, n+1, \cdots, J)$ is the interval certitude degree of C_j^t under the input fact; $\gamma^t = \left[\left(\gamma^t\right)^L, \left(\gamma^t\right)^U\right] \subseteq [0,1]$ is the rule certitude degree of rule R_t, $\theta^t \in [0,1]$ is the rule weight of rule R_t, the activation weights of rules are under the influence of $\Theta = \{\theta_t \,|t = 1, 2, \cdots, T\}$, $w_1, w_2, \cdots, w_I$ are antecedent attribute weights.

The rule weights are the relative weights (preference weights), according to the preference weight method [25], the credibility degrees of rules are given as $\varpi = \{\varpi_t \,|t = 1, 2, \cdots, T\}$, and ϖ_t is the credibility degree of the tth rule R_t.

According to the evidential reasoning approach, $\{C\prime\}$ is frame of discernment, let $\Omega\prime = \{\ \emptyset,\ \ \{C\prime\}\ \}$, the basic probability masses are given as follows:

$$m_t\left(C\prime\right) = \varpi_t \tilde{\beta}_n^t = \left[m_t^L\left(C\prime\right), m_t^U\left(C\prime\right)\right] = \left[\varpi_t\left(\tilde{\beta}_n^t\right)^L \varpi_t\left(\tilde{\beta}_n^t\right)^U\right]$$

$$m_t\left(\Omega\prime\right) = 1 - m_t\left(C\prime\right) = 1 - \varpi_t \tilde{\beta}_n^t = \left[m_t^L\left(\Omega\prime\right), m_t^U\left(\Omega\prime\right)\right] = \left[1 - \varpi_t\left(\tilde{\beta}_n^t\right)^U, 1 - \varpi_t\left(\tilde{\beta}_n^t\right)^L\right]$$

$$\bar{m}_t\left(\Omega\prime\right) = 1 - \varpi_t$$

where $m_t\left(C\prime\right)$ is the basic probability mass of R_t, $m_t\left(\Omega\prime\right)$ is the remaining probability mass that is unassigned to $C\prime$ which is caused by the incompleteness of rule R_t; $\bar{m}_t\left(\Omega\prime\right)$ is amount of remaining support left uncommitted by the weight of R_t.

Suppose $m_{\Lambda(t)}\left(C\prime\right)$ is the combined basic probability mass of $C\prime$ by aggregating the first $t\,(t = 1, 2, \cdots, T)$ rules $(R_1, \cdots, R_t)$, and $m_{\Lambda(t)}\left(\Omega\prime\right)$ is unassigned to $C\prime$ which is caused by the incomplete-ness of the first t rules $(R_1, \cdots, R_t)$; $\bar{m}_{\Lambda(t)}\left(\Omega\prime\right)$ is amount of remaining support left uncommitted by the weight of the first t rules $(R_1, \cdots, R_t)$.

Obviously, for the first $t = 1$,

$$m_{\Lambda(1)}\left(C\prime\right) = m_1\left(C\prime\right) = \varpi_1 \tilde{\beta}_j^1 = \left[m_{\Lambda(1)}^L\left(C\prime\right), m_{\Lambda(1)}^U\left(C\prime\right)\right] = \left[\varpi_1\left(\tilde{\beta}_n^1\right)^L, \varpi_1\left(\tilde{\beta}_n^1\right)^U\right]$$

$$m_{\Lambda(1)}(\Omega\prime) = m_1(\Omega\prime) = \left[m^L_{\Lambda(1)}(\Omega\prime), m^U_{\Lambda(1)}(\Omega\prime)\right] = \left[1 - \varpi_1\left(\tilde{\beta}^1_n\right)^U, 1 - \varpi_1\left(\tilde{\beta}^1_n\right)^L\right]$$

$$\bar{m}_{\Lambda(1)}(\Omega\prime) = 1 - \varpi_1$$

The combined masses of the first t rules are given as follows:

- $m_{\Lambda(t+1)}(C\prime)$

$$Max/Min \quad m_{\Lambda(t+1)}(C\prime) = m^*_{\Lambda(t)}(C\prime)\, m^*_{t+1}(C\prime) + m^*_{\Lambda(t)}(C\prime)\, m^*_{t+1}(\Omega\prime) + m^*_{\Lambda(t)}(\Omega\prime)\, m^*_{t+1}(C\prime)$$

$$S.t. \quad m^*_{\Lambda(t)}(C\prime)\, m^*_{t+1}(C\prime) + m^*_{\Lambda(t)}(\Omega\prime)\, m^*_{t+1}(\Omega\prime) + m^*_{\Lambda(t)}(C\prime)\, m^*_{t+1}(\Omega\prime) + m^*_{\Lambda(t)}(\Omega\prime)\, m^*_{t+1}(C\prime) = 1$$

$$m^*_{t+1}(C\prime) + m^*_{t+1}(\Omega\prime) = 1$$

The result of solving the optimization model is

$$m_{\Lambda(t+1)}(C\prime) = \left[m^L_{\Lambda(t+1)}(C\prime), m^U_{\Lambda(t+1)}(C\prime)\right]$$

$$m^L_{\Lambda(t+1)}(C\prime) = Min \quad m_{\Lambda(t+1)}(C\prime)$$

$$m^U_{\Lambda(t+1)}(C\prime) = Max \quad m_{\Lambda(t+1)}(C\prime)$$

- $m_{\Lambda(t+1)}(\Omega\prime)$

$$Max/Min \quad m_{\Lambda(t+1)}(\Omega\prime) = m^*_{\Lambda(t)}(\Omega\prime)\, m^*_{t+1}(\Omega\prime)$$

$$S.t. \quad m^*_{\Lambda(t)}(C\prime)\, m^*_{t+1}(C\prime) + m^*_{\Lambda(t)}(\Omega\prime)\, m^*_{t+1}(\Omega\prime) + m^*_{\Lambda(t)}(C\prime)\, m^*_{t+1}(\Omega\prime) + m^*_{\Lambda(t)}(\Omega\prime)\, m^*_{t+1}(C\prime) = 1$$

$$m^*_{t+1}(C\prime) + m^*_{t+1}(\Omega\prime) = 1$$

The result of solving the optimization model is

$$m_{I(t+1)}(\Omega\prime) = \left[m^L_{I(t+1)}(\Omega\prime), m^U_{I(t+1)}(\Omega\prime)\right]$$

$$m^L_{I(t+1)}(\Omega\prime) = Min \quad m_{I(t+1)}(\Omega\prime)$$

$$m^U_{I(t+1)}(\Omega\prime) = Max \quad m_{I(t+1)}(\Omega\prime)$$

- $\bar{m}_{\Lambda(t+1)}(\Omega\prime)$

$$\bar{m}_{\Lambda(t+1)}(\Omega\prime) = \bar{m}_{\Lambda(t)}(\Omega\prime)\, \bar{m}_{t+1}(\Omega\prime)$$

The composition certitude degree of the consequent attribute value $C\prime$ is given as follows:

$$\beta\prime = \left[(\beta\prime)^L, (\beta\prime)^U\right], (\beta\prime)^L = \frac{m^L_{\Lambda(T)}(C\prime)}{1 - \bar{m}_{\Lambda(T)}(\Omega\prime)}, (\beta\prime)^U = \frac{m^U_{\Lambda(T)}(C\prime)}{1 - \bar{m}_{\Lambda(T)}(\Omega\prime)}.$$

Obviously, interval certitude degree satisfies that the interval certitude degree is more greater the consequent attribute value is more certain. If there are more than one matches and values of each consequent attribute, then the consequent attribute value with the greater interval certitude degree is the final consequent attribute value under the actual input vector.

3 Illustrative examples

In this section, a case study of classification with UCI data sets is provided. This example is used to illustrate the effectiveness of ICRIMER by comparing with some existing approaches.

In order to compare ICRIMER inference method with other methods which are based on different data types as well as missing values, eight data sets from the UCI Machine Learning Repository are used. **Table 1** shows for each data set its name, the numbers of instances, attributes, linear attributes, nominal attributes, linguistic attributes, classes, and the percentage of the missing values.

Table 1: Properties of eight data sets from UCI

Dataset	Instances	Attributes	Linear Attributes	Nominal Attributes	Linguistic Attributes	Classes	Missing Values (%)
cancer	699	10	0	0	10	2	0.25
glass	214	9	9	0	0	6	0
horse	368	27	7	20	0	2	24
ionosphere	351	34	34	0	0	2	0
iris	150	4	4	0	0	3	0
liver	345	6	6	0	0	2	0
pima	768	8	8	0	0	2	0
wine	178	13	13	0	0	2	0

These data sets are used to compare the performances of Interval Certitude Rule Base Inference Method using Evidential Reasoning approach (ICRIMER), Rule Based Inference Method using Dempster-Shafer theory (RIMDS) [15], Feature Interval Learning algorithm (FIL) [11], k-Nearest Neighbor algorithm (k-NN) [4] Logistic Regression (LoR) [3,26], Naive Bayesian classifier (NB) [7], Pruning Decision Tree (PDT) [13].

ICRIMER and RIMDS are interval feature projection based logical inference algorithms. FIL is an interval feature projection based algorithm. LoR and NB are probability based algorithm. PDT is a structure based algorithm and decision tree can be used to describe If-then rules. k-NN is a famous machine learning method and give excellent results on many real-world examples [24].

To illustrate the significance of the accuracy of the rule base, a comparison rule base is given, and the interval certitude degrees and weights of rules are given based on some artificial restrictions. For example, if two rules which have the same antecedent and the different consequent, then these two rules will be given the smaller interval certitude degrees and weights; the weights of attribute are given using the feature selection method based on Relief [18]. The learning method based on ICRIMER and the comparison rule base is recorded as ICRIMER-c.

In the eight data sets, there are different data types: linear values, nominal values, linguistic values (evaluation grade) and missing values. So the instances should be transformed into interval certitude rules firstly.

Table 2 and Figure 1 report the correctly classified accuracy rate of the FIL, k-NN, LoR, NB, PDT, RIMDS and ICRIMER which are obtained by averaging the correctly classified accuracy rates over five repetitions of 5-folder cross validations. In this paper, k=5 for k-NN which exactly consistent with the case study in Ref. 29 as 5-NN gives the best correctly classified accuracy rate.

From Table 2 and Figure 1, we can find some phenomena as follows.

(i) ICRIMER may not always be the best classification method except data sets cancer and iris, but ICRIMER outperforms the interval classification methods RIMDS and FIL on five data sets, cancer, glass, horse, iris and liver. It means that ICRIMER has a higher classification capability on interval data sets.

Table 2: Correctly classified accuracy rates of the FIL, k-NN, LoR, NB, PDT, RIMDS, ICRIMER and ICRIMER-c

Learning Method	cancer	glass	horse	ionosphere	iris	liver	pima	wine
FIL	0.9688	0.5502	0.7718	0.9253	0.9426	0.6185	0.7054	0.9663
k-NN	0.9691	0.6455	0.8157	0.8501	0.9640	0.6110	0.7353	0.9540
LoR	0.9342	0.6355	0.8300	0.8775	0.8533	0.6870	0.6355	0.9607
NB	0.9740	0.5392	0.8027	0.8842	0.9293	0.6151	0.7262	0.9348
PDT	0.9299	0.6822	0.8033	0.8974	0.9600	0.6870	0.6822	0.9157
RIMDS	0.9629	0.5844	0.7700	0.8643	0.9739	0.6232	0.5195	0.9444
ICRIMER	0.9759	0.6047	0.7733	0.8942	0.9800	0.6318	0.6987	0.9055
ICRIMER-c	0.9900	0.6512	0.8132	0.8971	0.9899	0.6957	0.7532	0.9722

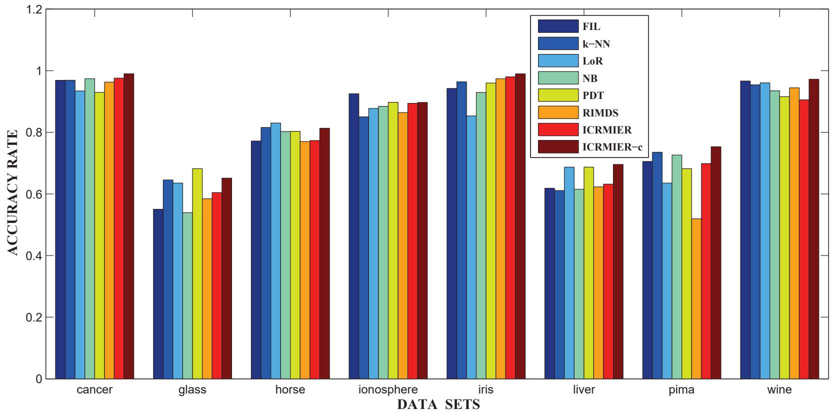

Figure 1: Correctly classified accuracy rates of the FIL, k-NN, LoR, NB, RIMDS, ICRIMER and ICRIMER-c

(ii) With regard to If-then rules, ICRIMER outperforms or a bit underperforms PDT on five data sets, cancer, ionosphere, iris, pima and wine. It means that ICRIMER has advantages in this respect.

(iii) ICRIMER-c is the best classification method on five data sets cancer, iris, liver, pima and wine, on the other three data sets the accuracy rate of ICRIMER-c is higher than the accuracy rate of ICRIMER and ICRIMER-c is the second best method in the eight methods. It means that, with a relatively accurately rule base, ICRIMER has general applicability and high classification capability.

(iv) Compare the classified accuracy rates of ICRIMER and ICRIMER-c, the classified accuracy rate of ICRIMER-c is higher than the classified accuracy rates of ICRIMER. It means that a relatively accurately rule base is an important influencing factors of ICRIMER and the rule base can be more accurately by adjusting the interval certitude degrees and weights of rules, the knowledge representation method ICRB is effective.

In general, ICRIMER is the better algorithm on this eight UCI data sets, with the relatively accurate rule bases, the results of classification would be better. And the interval certitude structure can be used to describe the uncertainties of knowledge; the ICRB can better reflect the uncertainty, correctness and importance of instance or knowledge. Moreover, if the weights and interval certitude degrees are given by domain expert or gained through training, then the results will be more rational and better.

4 Conclusions

In this paper, a new knowledge representation, interval certitude rule base (ICRB), was proposed to capture uncertainty and nonlinear causal relationships based on the If-then rule and fuzzy set theory. The inference process of interval certitude rule base was characterized by all interval certitude rules expression and was reasoning using the evidential reasoning. And an illustrative example with UCI data sets was used to illustrate the application of the proposed method.

There are several features of the interval certitude rule base inference method (ICRIMER). First, due to the use of interval certitude rule, the ICRIMER method gives a description framework of relationships between inputs and outputs which may be complete or incomplete, linear or nonlinear, discrete or continuous, or their mixture, which are based on both numerical data and human judgments. Second, the uncertainty of human judgment is characterized with interval certitude structure, and ICRIMER is a multi-output uncertainty inference method with one or more than one consequent attributes. These features are more close to the experience and perception of human.

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